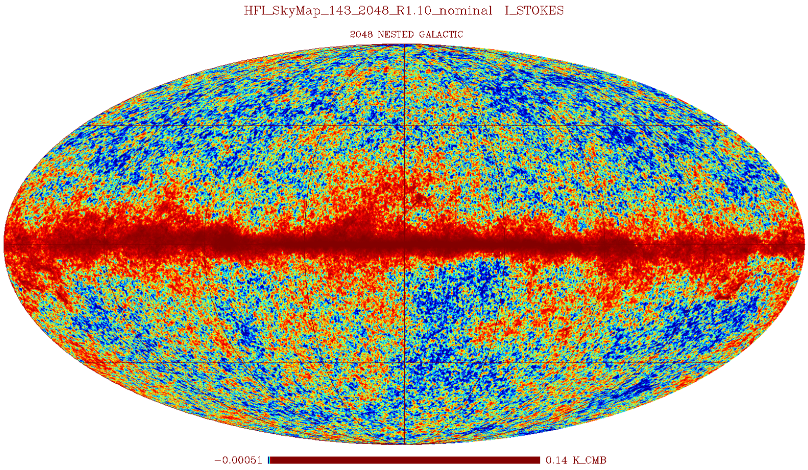
Stochastic gravitational-wave backgrounds with LISA and beyond: Challenges and Opportunities

Carlo Contaldi

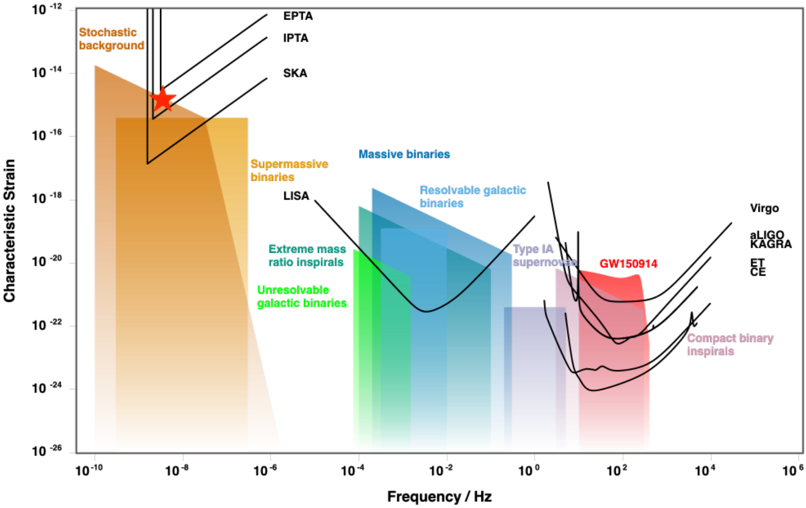
Theory Group, Imperial College London

CERN, 20th July, 2023

Background


HFI_SkyMap_143_2048_R1.10_nominal I_STOKES
2048 NESTED GALACTIC
−0.00051
0.14 K_CMB

Observational landscape



Stochastic Backgrounds

- Unresolved sources. Incoherent superposition of unresolved compact sources.

Stochastic Backgrounds

- Unresolved sources. Incoherent superposition of unresolved compact sources.
- Diffuse sources. Spatially correlated at generation due to phase transitions, topological defects, primordial perturbations, etc.

Stochastic Backgrounds

- Unresolved sources. Incoherent superposition of unresolved compact sources.
- Diffuse sources. Spatially correlated at generation due to phase transitions, topological defects, primordial perturbations, etc.
- LIGO: actively searching for stochastic component (but very specific).

Stochastic Backgrounds

- Unresolved sources. Incoherent superposition of unresolved compact sources.
- Diffuse sources. Spatially correlated at generation due to phase transitions, topological defects, primordial perturbations, etc.
- LIGO: actively searching for stochastic component (but very specific).
    - Upper limits $\Omega_{\mathrm{GW}} \sim 10^{-9}$ at $f = 25$ Hz [LVK circa 2022].

Stochastic Backgrounds

- Unresolved sources. Incoherent superposition of unresolved compact sources.
- Diffuse sources. Spatially correlated at generation due to phase transitions, topological defects, primordial perturbations, etc.
- LIGO: actively searching for stochastic component (but very specific).
  - Upper limits $\Omega_{\rm GW} \sim 10^{-9}$ at $f = 25$ Hz [LVK circa 2022].
- PTAs evidence for $A \sim 10^{-15}$ at yr^{-1} *and and* Hellings Down [June 2023]

Stochastic Backgrounds

- Unresolved sources. Incoherent superposition of unresolved compact sources.
- Diffuse sources. Spatially correlated at generation due to phase transitions, topological defects, primordial perturbations, etc.
- LIGO: actively searching for stochastic component (but very specific).
    - Upper limits $\Omega_{\rm GW} \sim 10^{-9}$ at $f = 25$ Hz [LVK circa 2022].
- PTAs evidence for $A \sim 10^{-15}$ at yr^{-1} *and and* Hellings Down [June 2023]
- LISA: guaranteed to see significant stochastic *components*.

What is a Stochastic signal?

$$h_{ab}(\vec{x},\hat{k},t) = h^{+}(\vec{x},t,\hat{k})e^{+}_{ab}(\hat{k}) + h^{\times}(\vec{x},t,\hat{k})e^{\times}_{ab}(\hat{k})\,.$$

- **Resolved**: The signal is correlated either *temporally* or *spatially* (frequency and/or direction).
- The signal is coherent and can be distinguished from random noise by "averaging" data (linear in strain $\tilde{h}$).

$$\langle h\rangle_T \neq 0, \quad \langle n\rangle_T = 0\,.$$

What is a Stochastic signal?

$$h(\vec{x}, t) = \sum_A \int_{-\infty}^{+\infty} df \int_\Omega d\Omega_{\hat{k}} \tilde{h}_A(f, \hat{k})\, e^A_{ab}(\hat{k}) e^{-i2\pi\left[f(t - \hat{k}\cdot\vec{x})\right]} .$$

- **Stochastic**: Limit where phase is uncorrelated between frequencies and/or directions e.g. due to incoherent superposition of sources or generation by random field.
- The signal is incoherent and cannot be distinguished from noise at linear level.

$$\langle \tilde{h} \rangle_T = 0, \quad \langle \tilde{n} \rangle_T = 0 .$$

$$\langle \tilde{h}\tilde{h}^\star \rangle_T \sim P_h, \quad \langle \tilde{n}\tilde{n}^\star \rangle_T \sim S_n .$$

Statistical properties

- Incoherent signal: fully stochastic backgrounds hold no phase information in strain h.
- Usually assumed to be stationary, and statistically isotropic;

$$\langle h(t,\hat{k})h^{\star}(t+\Delta t,\hat{k}')\rangle \sim \delta^{(3)}(\hat{k}-\hat{k}')\,H(\Delta t)\,,$$
$$\Updownarrow$$
$$\langle h(f,\hat{k})h^{\star}(f',\hat{k}')\rangle \sim \delta(f-f')\,\delta^{(3)}(\hat{k}-\hat{k}')\,P_h(f)\,.$$

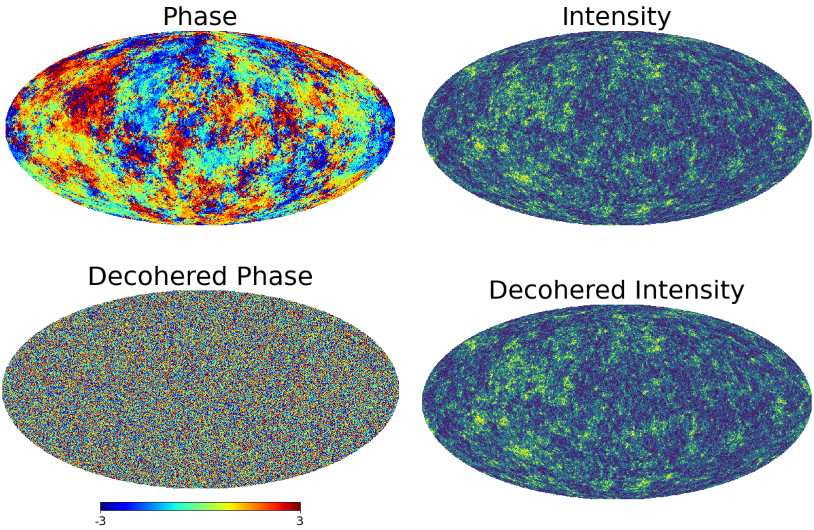
- These assumptions are very important ones for methods aimed at characterising and separating SGWBs.
- Note that statistical isotropy does not imply lack of angular correlations. The strain intensity (power) can be anisotropic and have non-trivial angular correlations

$$\langle h(f,\hat{k})h^{\star}(f',\hat{k}')\rangle \sim \delta(f-f')\,\delta^{(3)}(\hat{k}-\hat{k}')\,P_h(f,\hat{k})\,,$$
$$\langle P_h(f,\hat{k})P_h(f,\hat{k}')\rangle = \frac{1}{4\pi}\sum_{\ell}(2\ell+1)C_{\ell}(f)\mathcal{P}_{\ell}(\hat{k}\cdot\hat{k}')\,,$$

One more possibility...

- Only *one way* to generate a *diffuse* background with (temporal and/or angular) coherency i.e. $\langle\phi(f,\hat{k})\phi(f',\hat{k}')\rangle \not\propto \delta(f-f')\,\delta^{(3)}(\hat{k}-\hat{k}')$.
- GWs that have spent time outside the horizon. These will be squeezed (zero-momentum) and then start oscillating (and travelling) coherently in all directions as they re-enter the horizon.
- Unique signature of inflationary background which would lead to standing waves [Grishchuk & Sazhin 1975].
- Interferometers can distinguish between standing and travelling waves [CC & Magueijo 2018].
- Density perturbations destroy all coherence [Bartolo et al 2019, Margalit, CC, & Pieroni 2020] $\rightarrow$ no unique signature due to coherent $\vec{k}$ and $-\vec{k}$ modes.

Scalar modes are annoying foregrounds...



Non-Gaussianity

- Decoherence, or randomisation of phase correlations, affects what kind of non-Gaussianity can be observed using GWs.
- Any non-Gaussian correlations in the strain field is wiped out by the propagation through a perturbed universe eg.

$$\langle h(\vec{k}_1)h(\vec{k}_2)h(\vec{k}_3)\rangle \to 0\,.$$

- Only three-point correlations of the GW *intensity* will carry information (angular correlations) [Bartolo et al. 2019, 2020].

$$\langle P_h(\vec{k}_1)P_h(\vec{k}_2)P_h(\vec{k}_3)\rangle$$

- Mining non-Gaussianity will require spectral *and* angular resolution.
- Valuable to constrain *all* generation scenarios including astrophysical sources, cosmological phase transitions, topological defects, etc.
- ...but scalar perturbations are a foreground → tensor non-Gaussianity "polluted" by scalar non-Gaussianity. Use GWs to constrain $f_{\rm NL}$?!

Characterising backgrounds

- LISA will almost certainly observe a superposition of different SGWBs.

Characterising backgrounds

- LISA will almost certainly observe a superposition of different SGWBs.
- These should be separated using *both* spectral and angular information (see, e.g. "cyclostationary", Neil, Quentin, et al.)

Characterising backgrounds

- LISA will almost certainly observe a superposition of different SGWBs.
- These should be separated using *both* spectral and angular information (see, e.g. "cyclostationary", Neil, Quentin, et al.)
- Coherent detectors are typically good spectrometers but bad imagers.

Characterising backgrounds

- LISA will almost certainly observe a superposition of different SGWBs.
- These should be separated using *both* spectral and angular information (see, e.g. "cyclostationary", Neil, Quentin, et al.)
- Coherent detectors are typically good spectrometers but bad imagers.
- e.g. CMB radio interferometry; very successful spectral rejection of compact radio source signals but spatial (angular) rejection impossible because of sparse Fourier (uv) coverage.

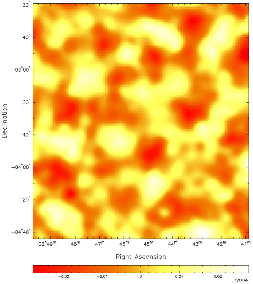
Characterising backgrounds

- LISA will almost certainly observe a superposition of different SGWBs.
- These should be separated using *both* spectral and angular information (see, e.g. "cyclostationary", Neil, Quentin, et al.)
- Coherent detectors are typically good spectrometers but bad imagers.
- e.g. CMB radio interferometry; very successful spectral rejection of compact radio source signals but spatial (angular) rejection impossible because of sparse Fourier (uv) coverage.
- GW interferometry; excellent spectral resolution and baseline but low angular resolution (in "intensity"-mode).

Characterising backgrounds

- LISA will almost certainly observe a superposition of different SGWBs.
- These should be separated using *both* spectral and angular information (see, e.g. "cyclostationary", Neil, Quentin, et al.)
- Coherent detectors are typically good spectrometers but bad imagers.
- e.g. CMB radio interferometry; very successful spectral rejection of compact radio source signals but spatial (angular) rejection impossible because of sparse Fourier (uv) coverage.
- GW interferometry; excellent spectral resolution and baseline but low angular resolution (in "intensity"-mode).
- *Not to be confused with* localisation resolution which uses time phase information to reconstruct angular position of coherent compact sources.

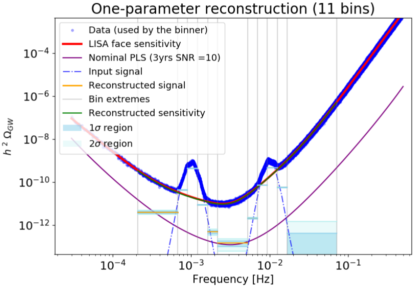
Photon Interferometry *vs* GW Interferemetry

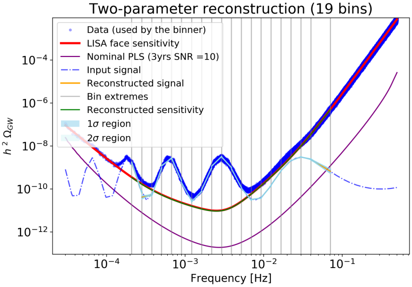


Cosmic Background Imager, Caltech, NSF

Spectral characterisation

- Coherent detectors make very good spectrometers.





Caprini at al. 2019

- ...as long as several real-world effects are taken care of...

Spectral characterisation - Challenges

- Non-stationarity in signal and noise. When does $\langle X(f)X^{\star}(f')\rangle \rightarrow \delta(f-f')$?
  - Noise: well-understood problem.
  - What is the effective high-pass frequency for LISA (10^{-4} Hz?).
  - Signal: less well-understood problem.
  - When does a superposition of signals become sufficiently "stochastic"?
  - Complicates directional searches and 'global fits'.
- Resolved source (time and angular) removal: Great feature of GW signal but will leave non-trivial residuals in the time-domain. Stochastic timestream will contain residuals *plus* significant non-stochastic contribution from SNR~ 1 signal.
  - This will complicate the spectral analysis by degrading spectral resolution making signal and noise estimation harder.
- Time-domain gaps? cosmic ray hits, time continuity, glitches, etc. - degrade spectral resolution.

Noise estimation

- LISA will have a high signal-to-noise timestream - even after resolved source removal.

Noise estimation

- LISA will have a high signal-to-noise timestream - even after resolved source removal.
- Noise power should be estimated iteratively by subtracting estimate of lowest SNR component (stochastic).

Noise estimation

- LISA will have a high signal-to-noise timestream - even after resolved source removal.
- Noise power should be estimated iteratively by subtracting estimate of lowest SNR component (stochastic).
  1. Integrate timestream into frequency *or/and* angular domain. $P_h(f, \vec{k})$. cf. "map-making" χ^2 step in CMB analysis.

Noise estimation

- LISA will have a high signal-to-noise timestream - even after resolved source removal.
- Noise power should be estimated iteratively by subtracting estimate of lowest SNR component (stochastic).
  1. Integrate timestream into frequency *or/and* angular domain. $P_h(f, \vec{k})$. cf. "map-making" χ^2 step in CMB analysis.
  2. "Re-scan" to timestream and subtract from data.

Noise estimation

- LISA will have a high signal-to-noise timestream - even after resolved source removal.
- Noise power should be estimated iteratively by subtracting estimate of lowest SNR component (stochastic).
  1. Integrate timestream into frequency *or/and* angular domain. $P_h(f, \vec{k})$. cf. "map-making" χ^2 step in CMB analysis.
  2. "Re-scan" to timestream and subtract from data.
  3. Evaluate new PSD of timestream.

Noise estimation

- LISA will have a high signal-to-noise timestream - even after resolved source removal.
- Noise power should be estimated iteratively by subtracting estimate of lowest SNR component (stochastic).
  1. Integrate timestream into frequency *or/and* angular domain. $P_h(f, \vec{k})$. cf. "map-making" χ^2 step in CMB analysis.
  2. "Re-scan" to timestream and subtract from data.
  3. Evaluate new PSD of timestream.
  4. Iterate 1-3.

Noise estimation

- LISA will have a high signal-to-noise timestream - even after resolved source removal.
- Noise power should be estimated iteratively by subtracting estimate of lowest SNR component (stochastic).
  1. Integrate timestream into frequency *or/and* angular domain. $P_h(f, \vec{k})$. cf. "map-making" χ^2 step in CMB analysis.
  2. "Re-scan" to timestream and subtract from data.
  3. Evaluate new PSD of timestream.
  4. Iterate 1-3.
  5. Fix PSD and evaluate a final "map" $(f,\ \hat{k},\ \ell m)$.

Noise estimation

- LISA will have a high signal-to-noise timestream - even after resolved source removal.
- Noise power should be estimated iteratively by subtracting estimate of lowest SNR component (stochastic).
  1. Integrate timestream into frequency *or/and* angular domain. $P_h(f, \vec{k})$. cf. "map-making" χ^2 step in CMB analysis.
  2. "Re-scan" to timestream and subtract from data.
  3. Evaluate new PSD of timestream.
  4. Iterate 1-3.
  5. Fix PSD and evaluate a final "map" (f, $\hat{k}$, ℓm).
  6. Iterative Maximum Likeliood estimate of intensity power spectrum e.g. $C_\ell(f)$.

Noise estimation

- LISA will have a high signal-to-noise timestream - even after resolved source removal.
- Noise power should be estimated iteratively by subtracting estimate of lowest SNR component (stochastic).
    1. Integrate timestream into frequency *or/and* angular domain. $P_h(f, \vec{k})$. cf. "map-making" χ^2 step in CMB analysis.
    2. "Re-scan" to timestream and subtract from data.
    3. Evaluate new PSD of timestream.
    4. Iterate 1-3.
    5. Fix PSD and evaluate a final "map" $(f,\ \hat{k},\ \ell m)$.
    6. Iterative Maximum Likeliood estimate of intensity power spectrum e.g. $C_\ell(f)$.
- Ill-conditioning problem: pre-compress harmonic space and direct to C_ℓ estimation (cf. CMB interferomery "gridding" methods)?

Noise estimation

- LISA will have a high signal-to-noise timestream - even after resolved source removal.
- Noise power should be estimated iteratively by subtracting estimate of lowest SNR component (stochastic).
  1. Integrate timestream into frequency *or/and* angular domain. $P_h(f, \vec{k})$. cf. "map-making" χ^2 step in CMB analysis.
  2. "Re-scan" to timestream and subtract from data.
  3. Evaluate new PSD of timestream.
  4. Iterate 1-3.
  5. Fix PSD and evaluate a final "map" $(f, \hat{k}, \ell m)$.
  6. Iterative Maximum Likeliood estimate of intensity power spectrum e.g. $C_\ell(f)$.
- Ill-conditioning problem: pre-compress harmonic space and direct to C_ℓ estimation (cf. CMB interferomery "gridding" methods)?
- Null or Sagnac channels are 'null-tests' not noise estimation methods. Are there other null tests for LISA (stochastic)?

Angular characterisation

- Coherent detectors make bad stochastic imagers

Angular characterisation

- Coherent detectors make bad stochastic imagers
  - Interferometer baselines observe sparse tracks in the (spherical) 'u, v' phase-space - ill-conditioned reconstruction of coordinate space.

Angular characterisation

- Coherent detectors make bad stochastic imagers
  - Interferometer baselines observe sparse tracks in the (spherical) ‘u, v’ phase-space - ill-conditioned reconstruction of coordinate space.
- Coherent detectors without ability to focus make even worse imagers.

Angular characterisation

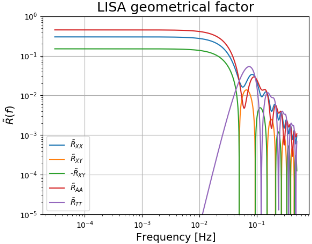
- Coherent detectors make bad stochastic imagers
  - Interferometer baselines observe sparse tracks in the (spherical) 'u, v' phase-space - ill-conditioned reconstruction of coordinate space.
- Coherent detectors without ability to focus make even worse imagers.
- LISA is "stuck" with non-compact geometric response with limited phase coverage.

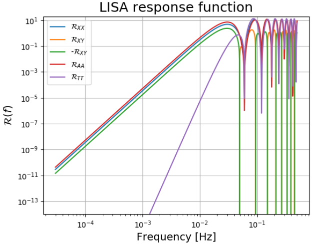
Angular characterisation

- Coherent detectors make bad stochastic imagers
  - Interferometer baselines observe sparse tracks in the (spherical) 'u, v' phase-space - ill-conditioned reconstruction of coordinate space.
- Coherent detectors without ability to focus make even worse imagers.
- LISA is "stuck" with non-compact geometric response with limited phase coverage.
- Combination of response and noise power determines spectral sensitivity at each frequency.

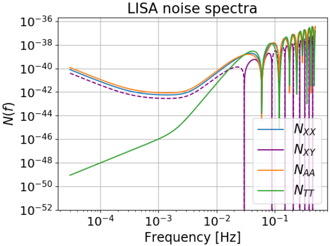
LISA Reponse and Noise

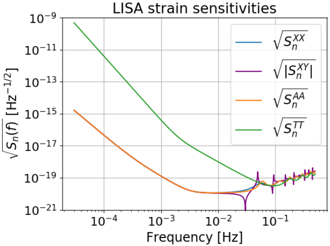
After angular integration we get:





by combining noise and response we get the the strain (bottom right):





LISA sky response

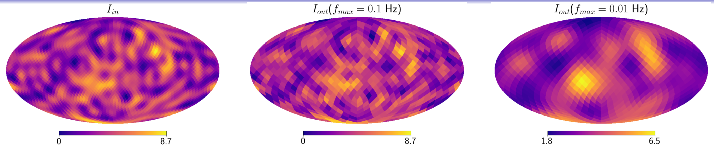


FIG. 5. An example input map from the simulations (left panel) to be compared to the final output maps obtained integrating with different frequency cutoffs, $f_{\max} = 0.1$ Hz (central panel) and $f_{\max} = 0.01$ Hz (right panel). These highlight the different resolutions the LISA channels have in different ranges of frequency.

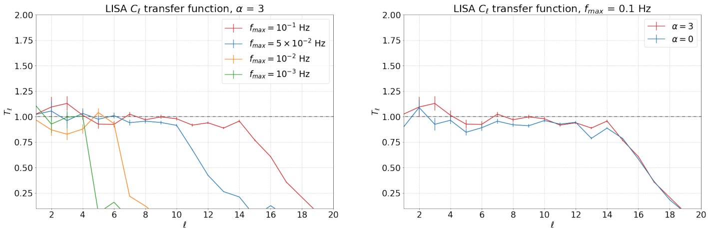
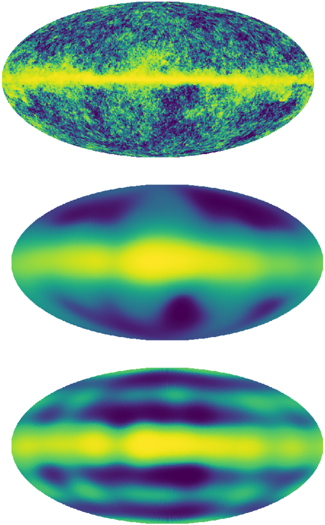


FIG. 6. Transfer functions T_ℓ for the average reconstructed C_ℓs obtained with different frequency cutoffs (left panel) and different spectral shapes, $\alpha = 3$ and $\alpha = 0$, both in the high frequency case $f_{\max} = 0.1$ Hz (right panel). Each simulation set consists of 50 maps, each a different realisation of the same C_ℓ input. There appears to be a clear one-to-one relation between the resolution $\ell_{\max}$ of the instrument and the frequency cutoff. Conversely, there is an average difference of 5% between the transfer functions obtained with different spectral weightings, however this does not affect the resolution cutoff.

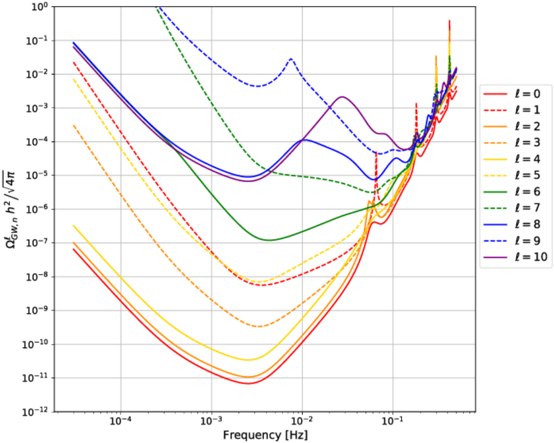
Contaldi et al. 2022

LISA "map"?



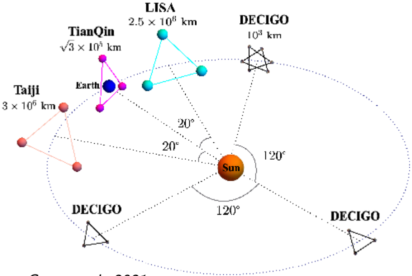
LISA Anisotropies

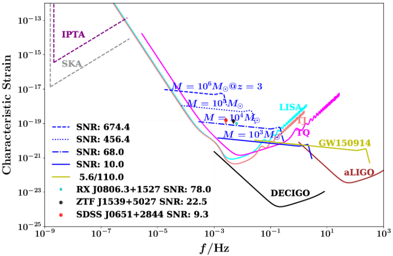
- LISA: Expected sensitivity to anisotropy multipoles in intensity.



Bartolo et al. 2022

High-ℓ SGWB from space?

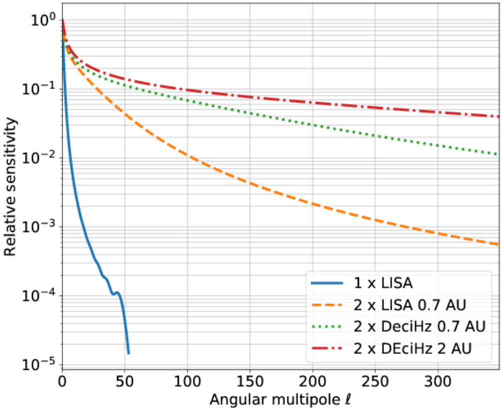




Gong et al. 2021

- Beat the fL/c factor by introducing long-baseline interferometry in space.
- Concurrent missions: LISA, TianQin, Taiji?

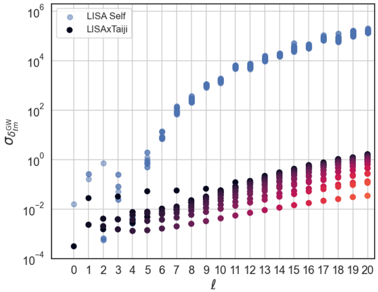
High-ℓ SGWB from space?

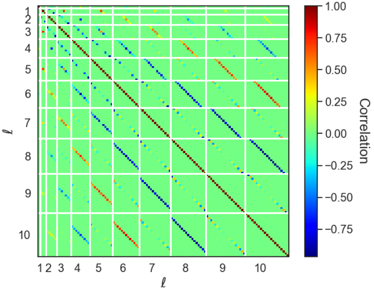


Baker et al. 2021

- Resolution dramatically improves with long baseline in space.
- Reminder: this is intensity (angular) resolution.

High-ℓ SGWB from space?

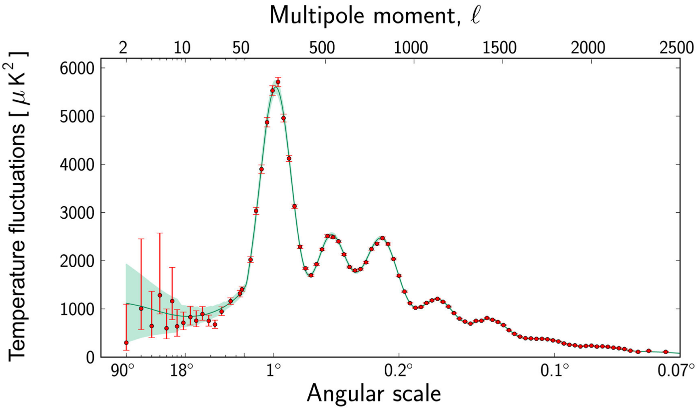




Mentasti, CC, & Peloso. Preliminary

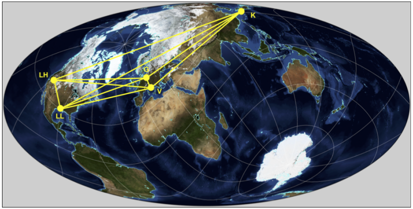
- LISA-Taiji confidence in reconstruction of individual $\delta_{\ell m}$ relative to monopole.

Cosmic Variance - a new problem?



Planck, ESA

Cosmic Variance - GW interferometry

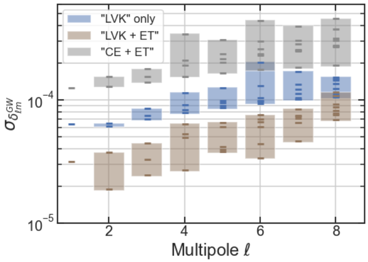


- Growing number of baselines over the next decade.
- Iterative improvement in sensitivity.
- Einstein Telescope (mid 30s?)

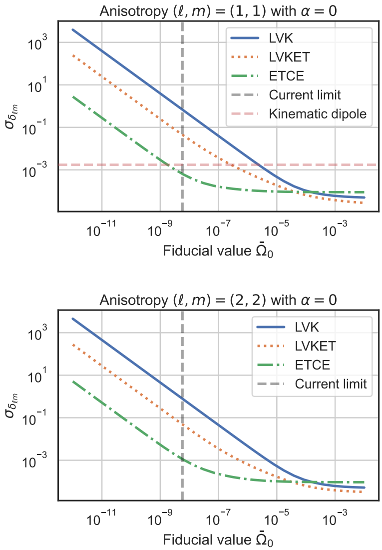
Mentasti, CC, & Peloso [2301.08074, 2304.06640]

- Consider zero-noise limit.
- Interferometers - covariance of multipoles is not diagonal, despite full-sky coverage.
  - Overlapping and finite frequency coverage.
  - Non-compact beam.
- Calculate "SNR" of anisotropies when variance is dominated by monopole.
- "How well can we measure $a_{\ell m}$'s in signal dominated limit?

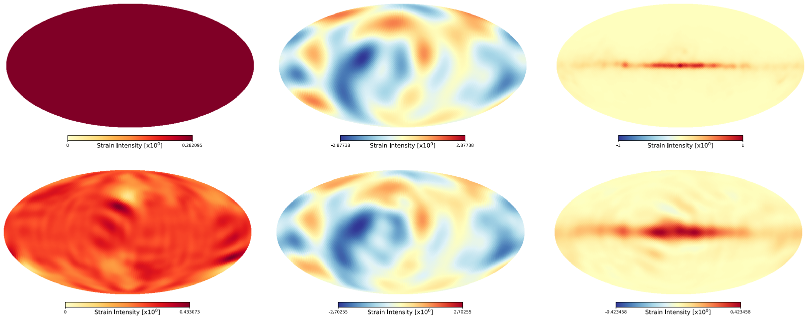
Cosmic Variance - GW interferometry



[Mentasti, CC, & Peloso, 2301.08074, 2304.06640]



Map-making?



Renzini & CC 2019

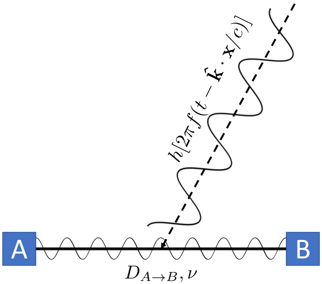
- Yes, but only if $\delta_{\ell m}^{\mathrm{GW}} > 10^{-2}$.
- ...and assuming stationarity! (see e.g. Capurri et al. 2103.12037).

Summary

- LISA: significant real-world challenges - separating stochastic components/residuals will be difficult. Exploit both frequency and angular structure.
- LISA: there will be some angular information to exploit.
- LISA-Taiji: Long baseline will make better maps ($\sim$ 1 degree?) but still be limited by sensitivity.
- Sample variance: Sets a limit on how well relative anisotropy can be mapped.
- Ground-based: can add baselines and exploit future sensitivity + time integration.

Phase vs Frequency Measurements

LISA is a phasemeter. Measures the perturbation to the *distance* between two stations (TDI - Time-Delay Interferometry).



Phase change:

$$\delta D_{A\to B} \sim c \int_{t_A}^{t_B} h\, dt \sim c\, h/f$$

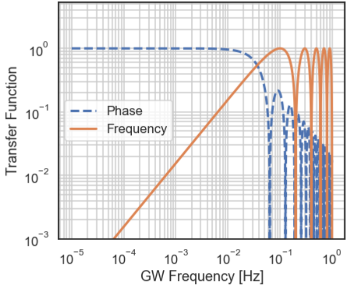
Frequency change (Doppler):

$$\frac{dD_{A\to B}}{dt} \sim \Delta\nu/\nu \sim h$$

LISA uses TDI because it cannot compare frequencies between stations - local oscillator ("clock") is not stable enough leading to overwhelming laser frequency noise.

GW Observation with space clocks?

- Lab-based optical lattice atomic clocks routinely reach 10^{-19} relative frequency stability.
- This raw sensitivity is sufficient to measure astrophysical GWs if it can be integrated on to the required frequencies.
- Measuring the Doppler shift directly may have significant advantages for the same technology and scale of e.g. LISA.



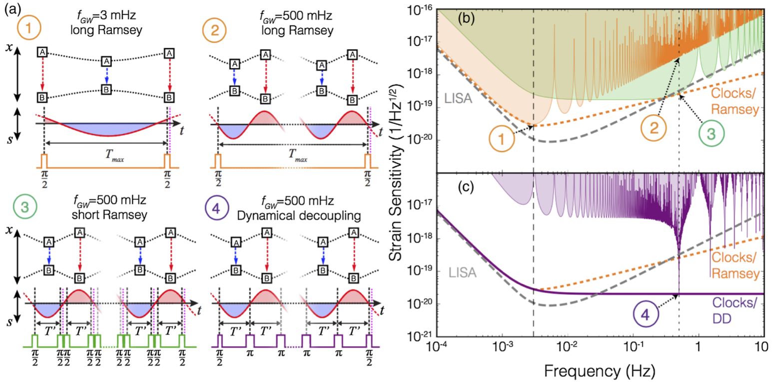
Phase:
$$d_p \sim s_p + n_p$$

Frequency:
$$d_\nu \sim s_\nu + n_\nu$$

Strain:
$$h \sim d_\nu - n_\nu \sim (d_p - n_p) f$$

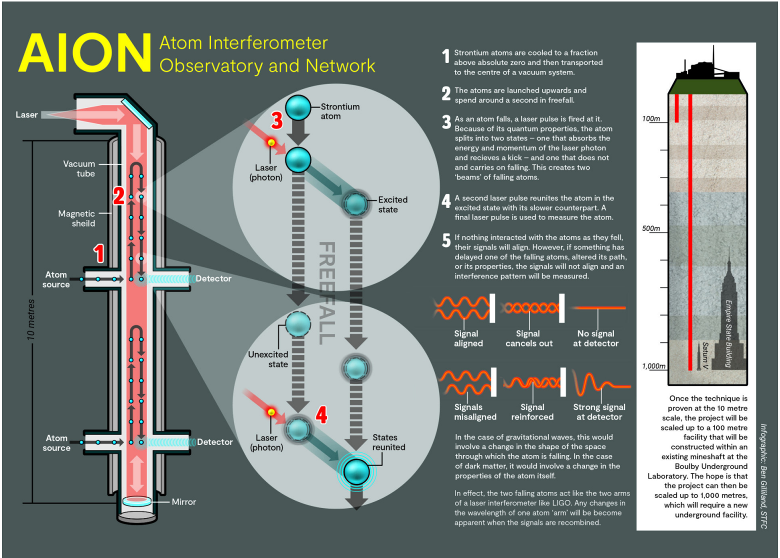
GW Observations with space clocks

Kolkowitz et al., PRD 94, 124043 (2016)



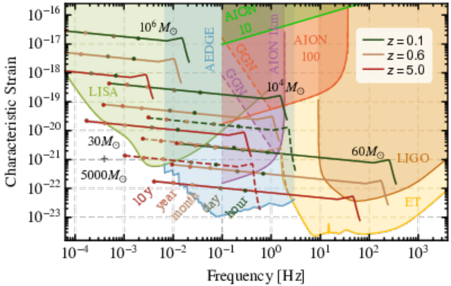
GW Observations with cold atoms

O. Buchmueller, Cold atom group, Imperial College



Fundamental physics with cold atoms in space?

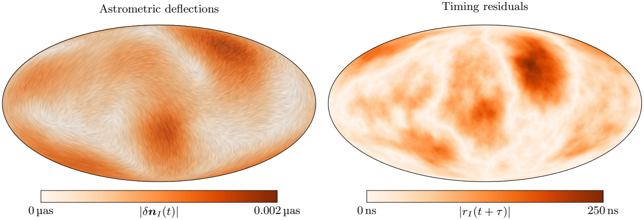
Badurina et al. 2020

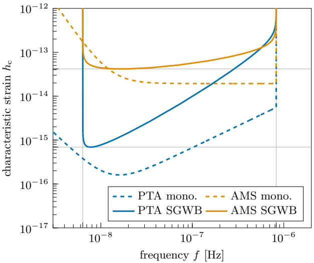


- Atom interferometry: *MAGIS* (US), *AION* (UK/EU?), AEDGE (SPACE?).
- Phase or frequency measurements?
- "Tunable" target frequency range.
- Anisotropies: higher angular resolution *cf.* LISA.
- Other tests of GR (scalar and vector modes of time dependent metric perturbations).

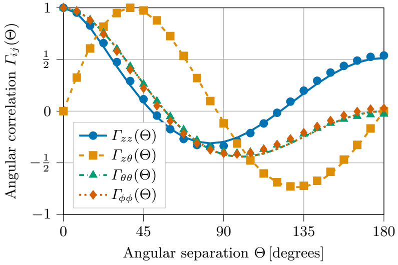
Astrometry x Timing residuals

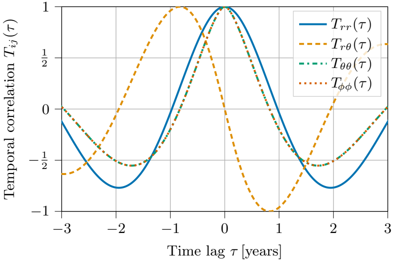
Golat & CC 2022





Astrometry x Timing residuals





Golat & CC 2022

- Generalised Hellings-Down angular correlations.
- ...and time-lagged correlations.

Summary

- Great prospects for characterisation of SWGBs over big range in frequency.
- LISA: significant real-world challenges - separation of stochastic components/residuals will be difficult. Exploit both frequency and angular structure.
- Angular resolution will improve with addition of baselines to ground-based network (but still ~ 10 degrees at current frequencies).
- Long-baseline in space (~ 1 degree?)
- Cold atoms?