Upcoming Challenges in Searching for the Stochastic Gravitational Wave Background with Terrestrial Detectors

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Outline

- SGWB searches have four dimensions to explore:
  - » Frequency, time, sky-direction, polarization
  - » Need them all for estimating the noise and for model selection!
- Terrestrial detectors:
  - » Talks by J. Romano and J. Suresh for the most recent LVK results.
  - » We don't yet fully explore the four dimensions!
- What's missing?
  - » Leverage SGWB anisotropy (including GW-EM correlations) to disentangle models/noise.
  - » Relax the assumption of a stationary background.
  - » Account for and remove the compact binary foreground.
  - » Global statistical framework?

Network of Terrestrial Detectors

Isotropic SGWB Search Strategy (1)

- Plane-wave expansion:

$$h_{ab}(t,\vec{x}) = \sum_A \int_{-\infty}^{\infty} df \int_{S^2} d\hat{\Omega}\, h_A(f,\hat{\Omega})\, e^{i2\pi f(t-\hat{\Omega}\cdot\vec{x}/c)} e^A_{ab}(\hat{\Omega})$$

- Isotropic and unpolarized background:

$$\langle h_A^*(f,\hat{\Omega}) h_{A'}(f',\hat{\Omega}')\rangle = \delta^2(\hat{\Omega},\hat{\Omega}')\,\delta_{AA'}\,\delta(f-f')H(f)$$

- Energy density:

$$\rho_{GW} = \frac{c^2}{32\pi G} < \dot{h}_{ab}\, \dot{h}^{ab} >$$

$$\Omega_{GW}(f) = \frac{1}{\rho_c} \frac{d\rho_{GW}(f)}{d\ln f}$$

(Only frequency-dependent)

$$H(f) = \frac{3H_0^2}{32\pi^3} |f|^{-3}\Omega_{\rm gw}(|f|)$$

Allen & Romano, PRD **59** 102001 (1999)

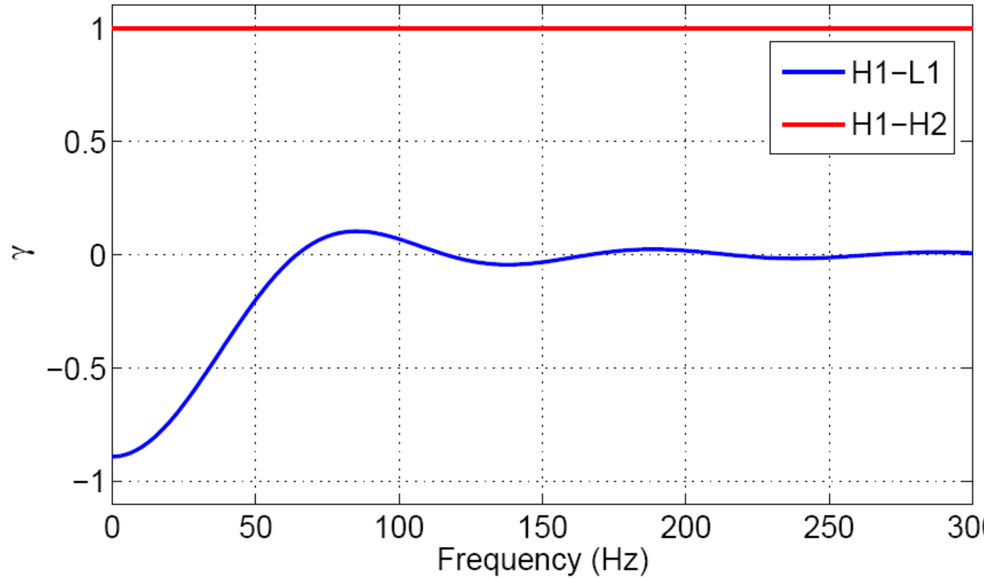
Isotropic SGWB Search Strategy (2)

- Cross-correlation estimator

$$Y = \int_{-T/2}^{+T/2} dt_1 \int_{-T/2}^{+T/2} dt_2 \, s_1(t_1) \, s_2(t_2) \, Q(t_2 - t_1)$$

$$Y = \int_{-\infty}^{+\infty} df \; \tilde{s}_1^*(f) \; \tilde{s}_2(f) \; \tilde{Q}(f)$$

Overlap Reduction Function



- Theoretical variance

$$\sigma_Y^2 \approx \frac{T}{2} \int_0^{+\infty} df \; P_1(f) \; P_2(f) \; | \, \tilde{Q}(f) \, |^2$$

- Optimal Filter

$$\tilde{Q}(f) = \frac{1}{N} \, \frac{\gamma(f) \; \Omega_t(f)}{f^3 \; P_1(f) \; P_2(f)}$$

For template: $\Omega_t(f) = \Omega_\alpha (f/100 \text{ Hz})^\alpha$

Choose *N* such that: $\langle Y \rangle = \Omega_\alpha T$

Anisotropic SGWB Search (1)

- Add directional dependence of SGWB:

$$\left\langle h_A^*(f,\widehat{\Omega})h_{A'}^*\left(f',\widehat{\Omega}'\right)\right\rangle = \frac{1}{4}\mathcal{P}(f,\widehat{\Omega})\delta_{AA'}\delta(f-f')\delta(\widehat{\Omega},\widehat{\Omega}')$$

- Separate frequency and direction dependencies:

$$\mathcal{P}(f,\widehat{\Omega}) = H(f)\mathcal{P}(\widehat{\Omega})$$

- Point source (radiometer) search: $\mathcal{P}(\hat{\Omega}) \equiv \eta(\hat{\Omega}_0)\,\delta^2(\hat{\Omega},\hat{\Omega}_0)$

- Spherical harmonic decomposition (similar to CMB analyses):

$$\mathcal{P}(\hat{\Omega}) \equiv \sum_{lm} \mathcal{P}_{lm} Y_{lm}(\hat{\Omega})$$

Anisotropic SGWB Search (2)

Cross-correlation

$$C_{IJ}(t;f) = \frac{2}{\tau}\tilde{s}_I^*(t;f)\,\tilde{s}_J^*(t;f)$$

Dirty map

$$X_\nu^{IJ} = \sum_t \sum_f (\gamma_{IJ})_\nu^*(t;f)\frac{H(f)}{P_I(t;f)P_J(t;f)}C_{IJ}(t;f)$$

Fisher matrix (dirty covariance)

$$\Gamma_{\mu\nu}^{IJ} = \sum_t \sum_f (\gamma_{IJ})_\mu^*(t;f)\frac{H^2(f)}{P_I(t;f)P_J(t;f)}(\gamma_{IJ})_\nu(t;f)$$

Clean map
(covariance = inverse Fisher)

$$\hat{\mathcal{P}}_\mu = \sum_\nu \left(\Gamma_{\mathrm{R}}^{-1}\right)_{\mu\nu} X_\nu$$

Angular (Auto-)Power Spectrum

$$\hat{C}_\ell = \left(\frac{2\pi^2 f_{\mathrm{ref}}^3}{3H_0^2}\right)^2 \frac{1}{1+2\ell}\sum_{m=-\ell}^{\ell}\left[|\hat{\mathcal{P}}_{\ell m}|^2 - (\Gamma_{\mathrm{R}}^{-1})_{\ell m,\ell m}\right]$$

SGWB Search Strategy

- Multiple baselines: formalism extents trivially!
  - » Different overlap reductions, gain sensitivity at zero-crossing frequencies.
- Supports parameter estimation formalisms.

- The formalism does not use detectors' auto-correlations.
  - » Historically: strain noise not fully understood, and much larger than the signals.
    - – Cross-correlation is much better behaved!
  - » But, auto-correlations may provide a way to estimate PSDs.
    - – This will be increasingly important as we get out of the "weak signal regime".

SGWB Polarization Search (1)

- Assume isotropic, but potentially polarized SGWB.

$$\begin{pmatrix} \langle h_R(f,\hat{\Omega}) h_R^*(f',\hat{\Omega}') \rangle \\ \langle h_L(f,\hat{\Omega}) h_L^*(f',\hat{\Omega}') \rangle \end{pmatrix} = \frac{\delta(f-f')\delta^2(\hat{\Omega}-\hat{\Omega}')}{4\pi} \begin{pmatrix} I(f,\hat{\Omega}) + V(f,\hat{\Omega}) \\ I(f,\hat{\Omega}) - V(f,\hat{\Omega}) \end{pmatrix}$$

$$\Omega'_{\mathrm{GW}} = \Omega_{\mathrm{GW}} \left[1 + \Pi(f) \frac{\gamma_V^{d_1 d_2}(f)}{\gamma_I^{d_1 d_2}(f)} \right],$$

modified energy density spectrum

$$\Pi(f) = \frac{V(f)}{I(f)}$$

$$\gamma_I^{d_1 d_2}(f) = \frac{5}{8\pi} \int d\hat{\Omega} (F_{d_1}^{+} F_{d_2}^{+*} + F_{d_1}^{\times} F_{d_2}^{\times *}) e^{2\pi i f \hat{\Omega} \cdot \Delta \vec{x}},$$

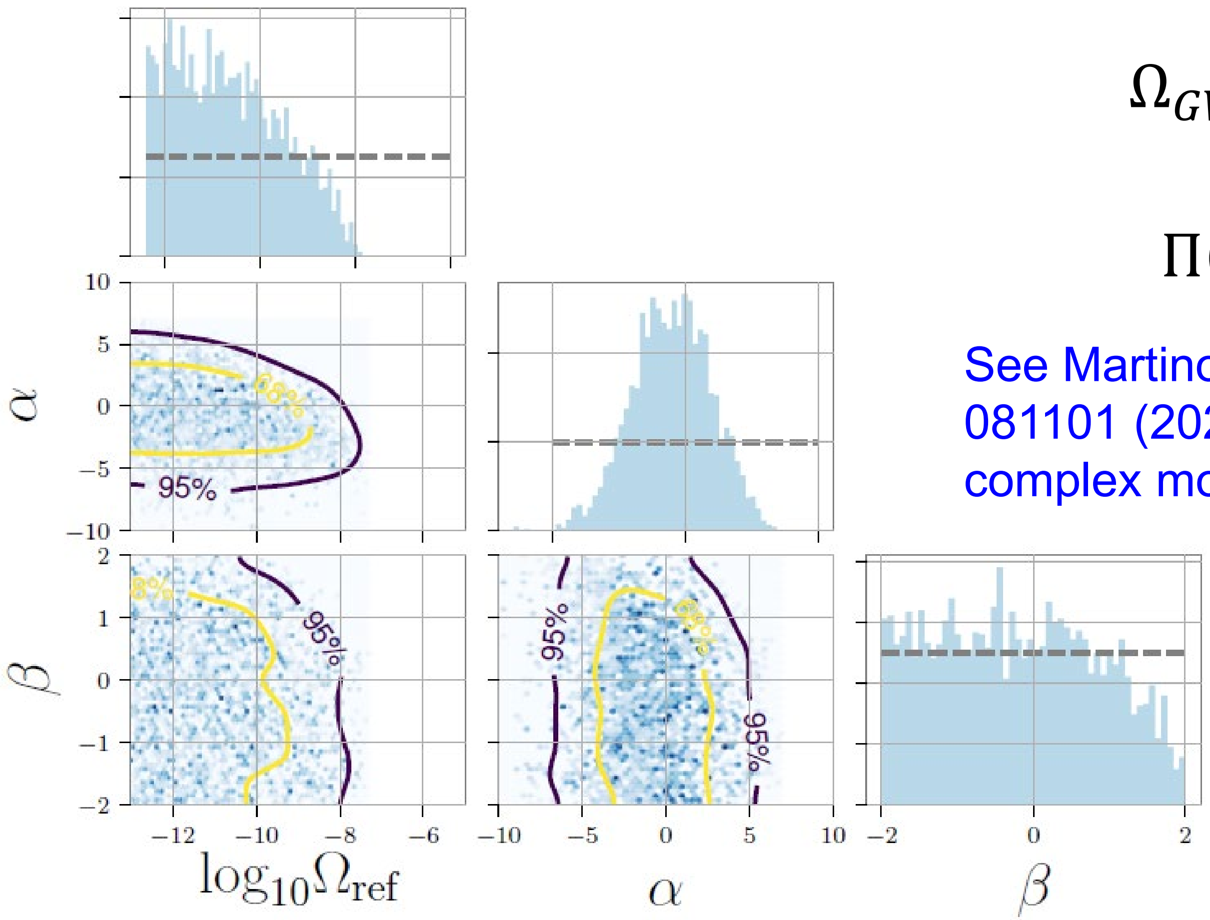
standard overlap reduction

$$\gamma_V^{d_1 d_2}(f) = -\frac{5}{8\pi} \int d\hat{\Omega} (F_{d_1}^{+} F_{d_2}^{\times *} - F_{d_1}^{\times} F_{d_2}^{+*}) e^{2\pi i f \hat{\Omega} \cdot \Delta \vec{x}}$$

“polarized” overlap reduction

- Can be done in postprocessing!
- Martinovic et al., Phys. Rev. D 104, 081101 (2021).

SGWB Polarization Search (2)



$$\Omega_{GW}(f) = \Omega_{ref}\left(\frac{f}{25\ Hz}\right)^{\alpha}$$

$$\Pi(f) = f^{\beta}$$

See Martinovic et al., Phys. Rev. D 104, 081101 (2021), for analyses of more complex models.

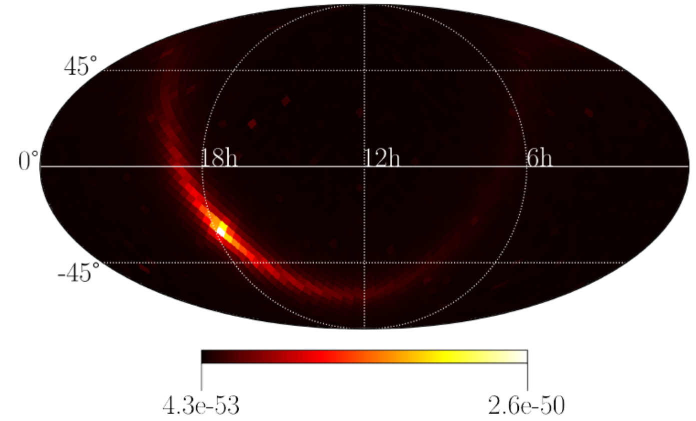
Open Problems

Inference Using Anisotropic SGWB

- Two possibilities:
  - » The anisotropy realization is modelled (e.g. Milky Way).
  - » The anisotropy realization is not modelled a priori, but its angular spectrum can be modelled.
- Case 1: anisotropy realization is modelled.
  - » Example: Anisotropy known exactly, fit only the amplitude and frequency spectrum.
    - – Agarwal et al., PRD **106**, 043019 (2022).
    - – Applied to SGWB due to galactic millisecond pulsars.

$$\mathcal{P}_\alpha(f) = A\,\bar{H}_f\,\hat{\mathcal{P}}_\alpha$$

$$\hat{A} = \frac{\mathbf{X}^\dagger \hat{\boldsymbol{\mathcal{P}}}}{\hat{\boldsymbol{\mathcal{P}}}^\dagger \boldsymbol{\Gamma} \hat{\boldsymbol{\mathcal{P}}}}$$



Case 2: Estimating Anisotropy Angular Spectrum

- Have a model for C_l's:

Multivariate Gaussian

Generalized, multivariate χ^2

$$\hat{C}_\ell = \left(\frac{2\pi^2 f_{\rm ref}^3}{3H_0^2}\right)^2 \frac{1}{1+2\ell} \sum_{m=-\ell}^{\ell} \left[|\hat{\mathcal{P}}_{\ell m}|^2 - (\Gamma_{\rm R}^{-1})_{\ell m,\ell m}\right]$$

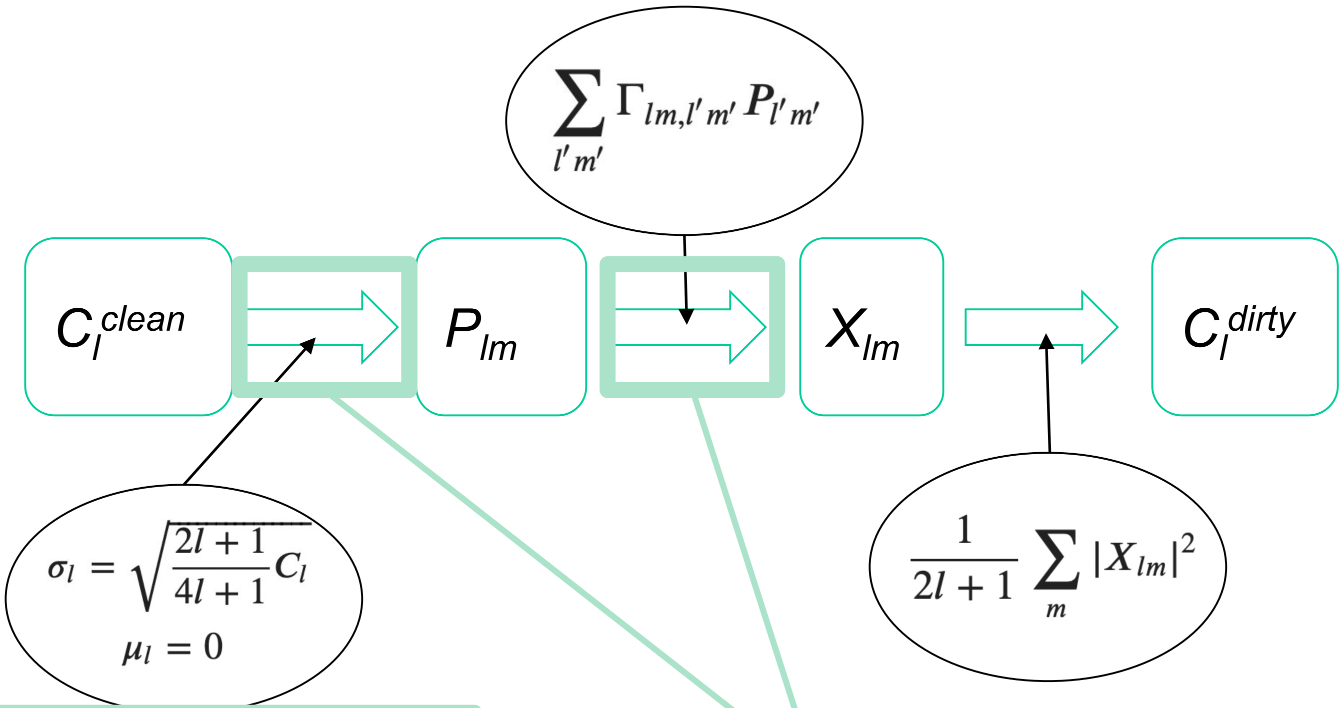
- Statistics challenge: C_l's are not Gaussian, so can't use a simple multivariate Gaussian likelihood to compare data and model.

- Data science challenge: Fisher matrix is usually not invertible, especially in narrow frequency bins.
  - » Regularization leads to bias.

- **Open problem!**

Inference in the Dirty Space?

Inference in the dirty space would avoid inverting the Fisher matrix.



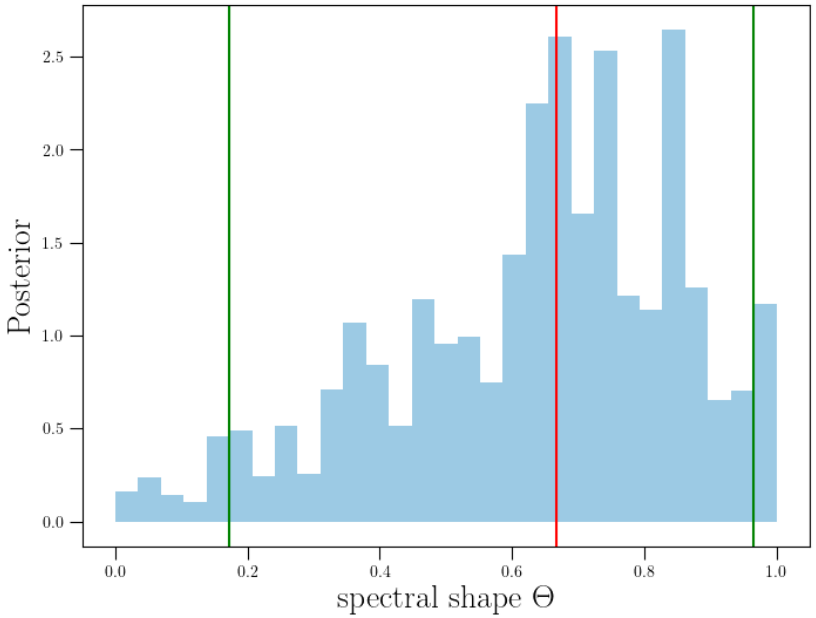
Price to pay: New contribution to the C_l covariance matrix due to draws.

$$K_{ll'} = K_{ll',draws} + K_{ll',Fisher}$$

Credit: E. Floden

Gaussian Likelihood Assumption?

- C_l's follow generalized, multivariate chi-squared distribution.
  - No explicit form, must resort to numerical approaches, non-trivial…
- Could try using Gaussian likelihood anyways.
  - How large is the bias?
- Power law model: $C_l = A\, l^{\theta}$
  - Calculate 95% confidence intervals of our posteriors, repeat the process 1000 times.
  - How often is each true parameter value within the 95% confidence interval?
    - θ: 95% of the time
    - A: 76% of the time
  - Bias: Due to dirty space or gaussian approximation?

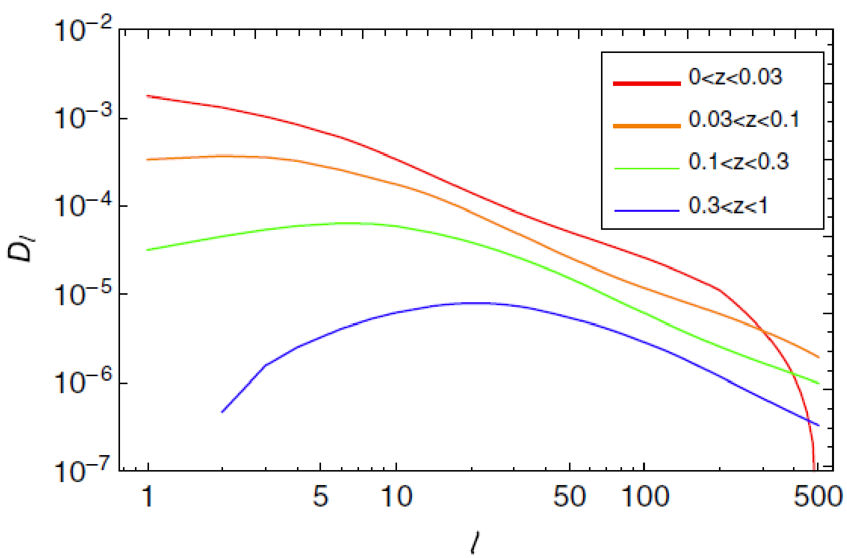


Credit: E. Floden

GW-EM Cross-power

- Correlate SGWB anisotropy with anisotropy in EM data (galaxy counts, gravitational lensing, CMB, CIB…).
  - » Possibly detect SGWB anisotropy sooner.
  - » Probe models of structure formation (eg BBH population of stellar vs primordial origin).
  - » Perhaps probe cosmology models.

- BBH population (of stellar origin) leads to a GW-GC angular power spectrum.
  - » Cusin et al., PRD 100, 063004 (2019)
- Frequency dependent spectrum!
- Shot noise: spatial and temporal realization of BBH mergers.



GW-EM Angular Cross-Power

- K. Yang et al., arXiv:2304.07621.
- Statistics is easier: $C_l \propto \sum_m \langle a_{lm}^{*GW} b_{lm}^{GC} \rangle$

$$\ln \mathcal{L}(\hat{C}_\ell | C_\ell(\theta)) = \frac{1}{2} \ln |K_C| - \frac{1}{2} (\hat{C}_\ell - C_\ell(\theta))^T K_C^{-1} (\hat{C}_\ell - C_\ell(\theta))$$

$$(K_C^{\mathrm{tot}})_{\ell\ell'} = (K_C)_{\ell\ell'} + \frac{\delta_{\ell\ell'}}{(2\ell+1)} \left[\left(C_\ell^{\mathrm{GW}}(\theta) + N_{\mathrm{shot}}^{\mathrm{GW}}(\theta) \right) \left(C_\ell^{\mathrm{GC}} + N_{\mathrm{shot}}^{\mathrm{GC}} \right) + \left(C_\ell^{\mathrm{cross}}(\theta) + N_{\mathrm{shot}}^{\mathrm{cross}}(\theta) \right)^2 \right]$$

- But inverting Fisher matrix in smaller freq. bins.
  - » Bias and problems in estimating the covariance matrix for D_l's.
  - » Limits expansion to low *l*'s.
- Empirical model of the GW-GC angular power spectrum.

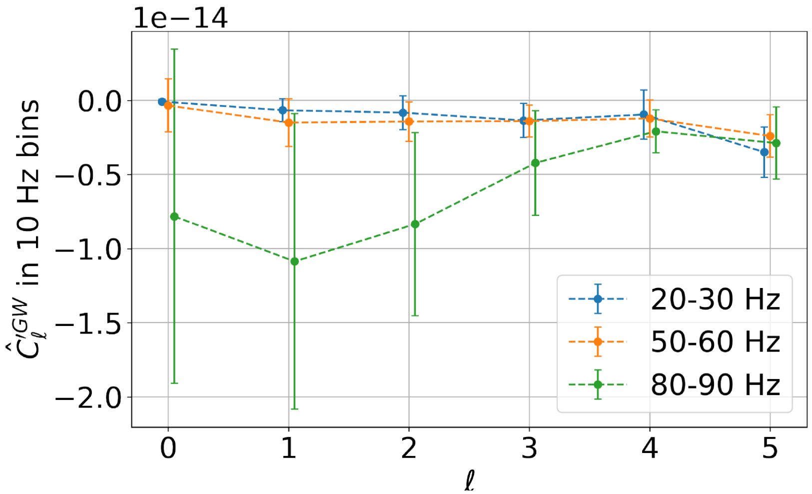
$$\partial_r \bar{\Omega}_{\mathrm{GW}}(f, r) = \frac{f}{\rho_c} \mathcal{A}(f, r)$$

$$\mathcal{A}(f, z) = \mathcal{A}(f)\, e^{-(z-z_c)^2/2\sigma_z^2} = A_{\max} f^{-1/3}\, e^{-(z-z_c)^2/2\sigma_z^2}$$

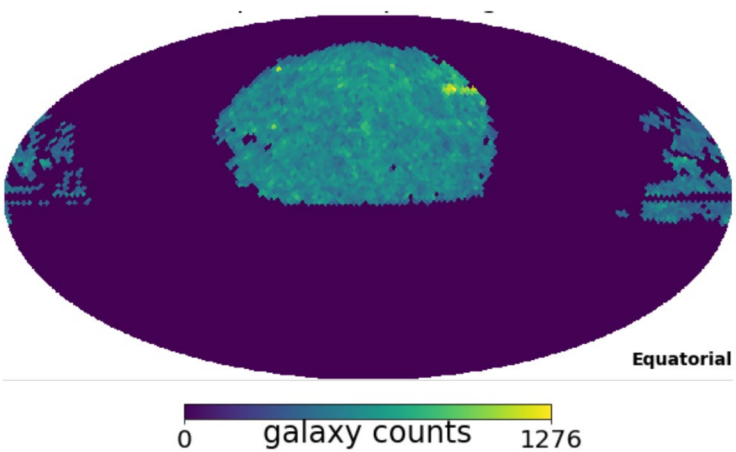
GW-EM Cross Correlation Results

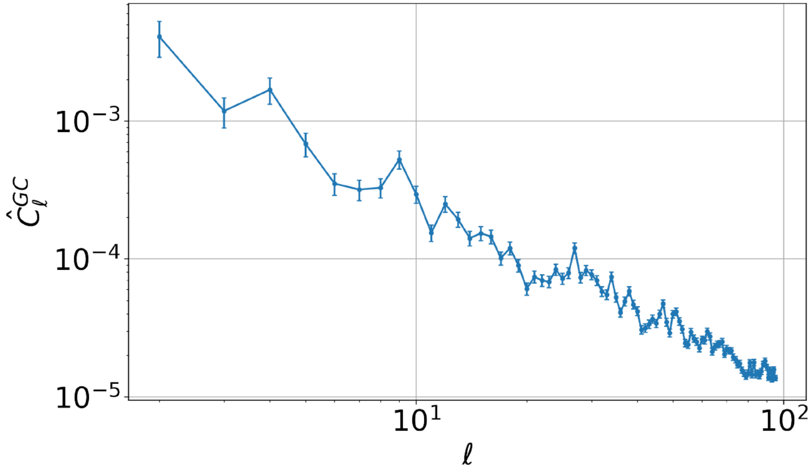
K. Yang et al., arXiv:2304.07621

O3 data used to produce GW sky-maps for 10 Hz wide frequency bins.



SDSS galaxy count sky-map and angular power spectrum





GW-EM Cross Correlation Results

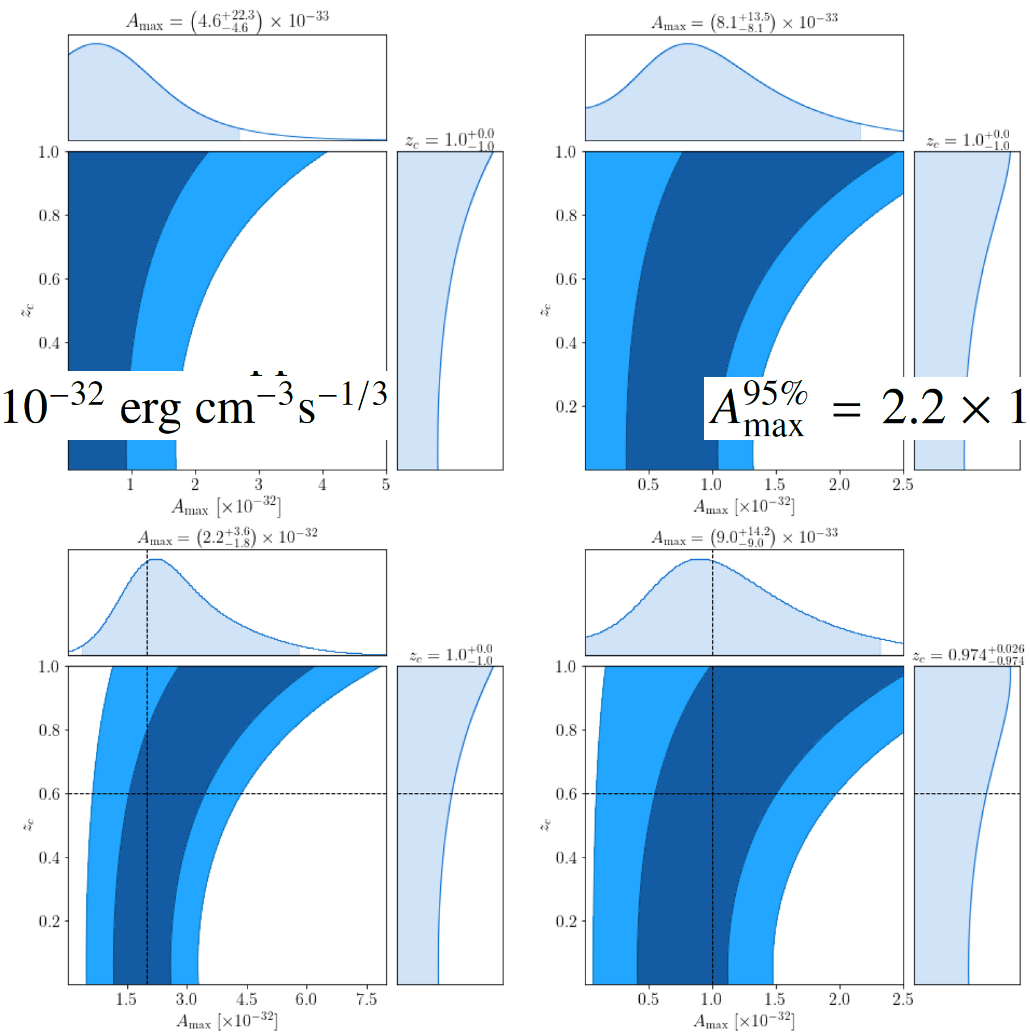
No Shot Noise | **With Shot Noise**

No Signal Added

$$A_{\rm max}^{95\%} = 2.7 \times 10^{-32}\ \mathrm{erg\ cm^{-3} s^{-1/3}}$$

$$A_{\rm max}^{95\%} = 2.2 \times 10^{-32}\ \mathrm{erg\ cm^{-3} s^{-1/3}}$$

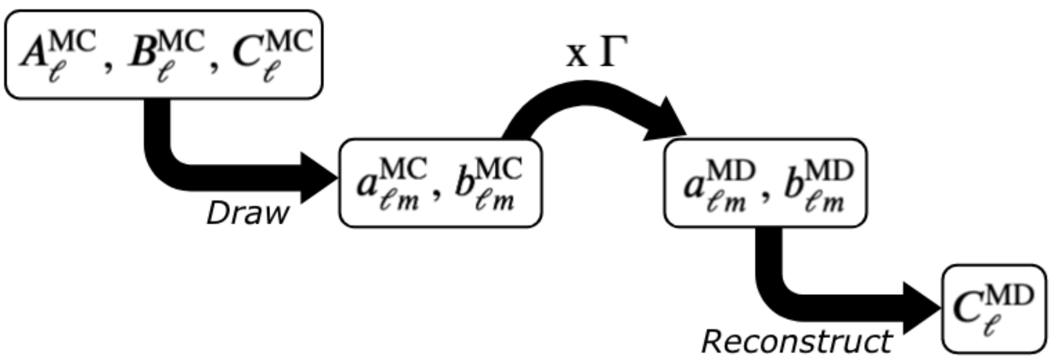
Signal Added



K. Yang et al., arXiv:2304.07621

GW-EM in the Dirty Space

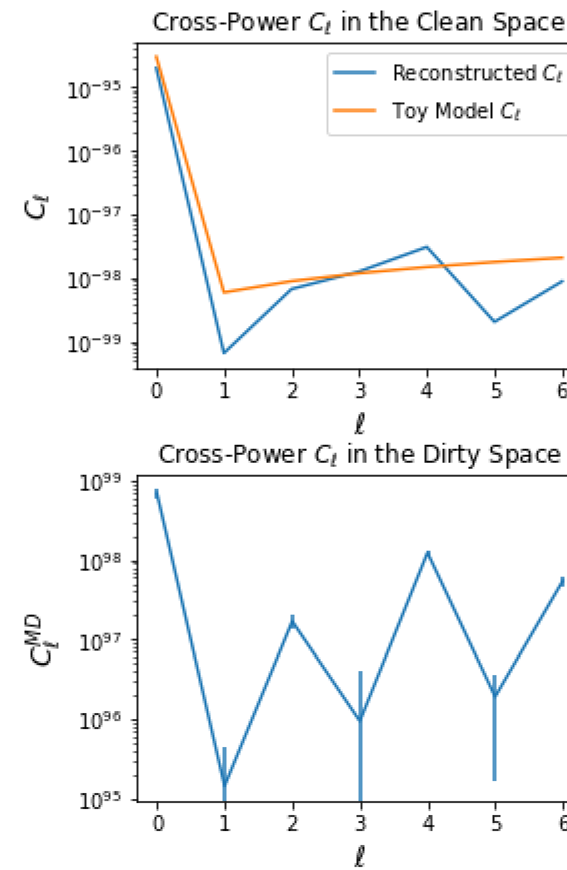
- Preliminary work by A. Granados
- Moving the model from clean to dirty space:



- Likelihood function defined in the dirty space:

$$\ln\mathcal{L}\left(\hat{C}_\ell^{DD}|C_\ell^{MD}(\theta)\right) = \frac{1}{2}\left(\hat{C}_\ell^{DD} - C_\ell^{MD}(\theta)\right)^T K^{-1}\left(\hat{C}_\ell^{DD} - C_\ell^{MD}(\theta)\right)$$

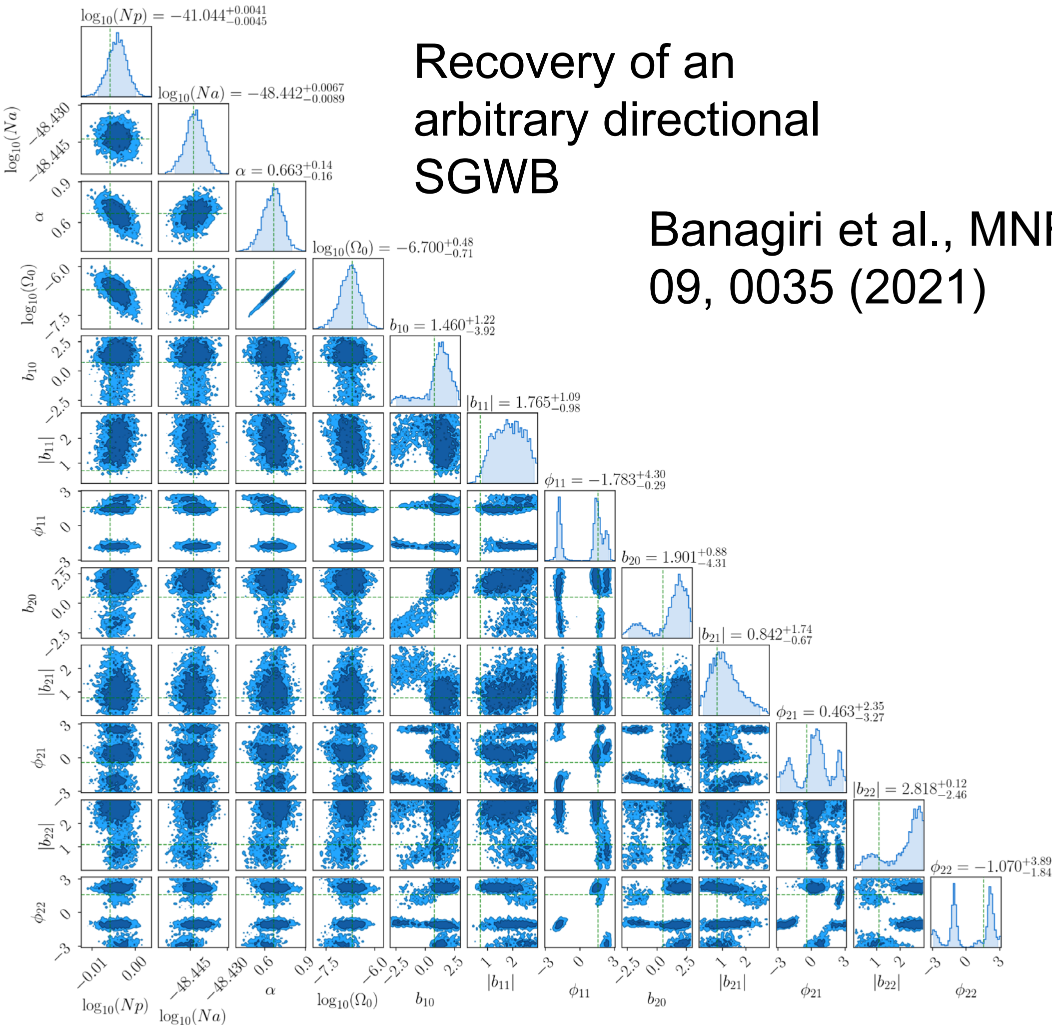
- But, covariance matrix *K* now has additional contributions due to the draws.



Anisotropy and LISA

- Need a formalism that can handle both specific anisotropy realizations and angular power spectra.
- Some components already exist:
  - » Galactic foreground, estimated based on temporal modulation or using a basis of functions on a 2-sphere.
  - » BLIP: Bayesian LISA Inference Package
    - – Python-based
    - – Modular simulation and recovery
    - – Supports multiple (an)isotropic SGWB models, noise models, multiple samplers…
    - – `https://github.com/sharanbngr/blip`
- Missing:
  - » Incorporation into the global fit.
  - » Computational limitations on increasing the angular resolution.
  - » GW-EM correlations in the LISA band.

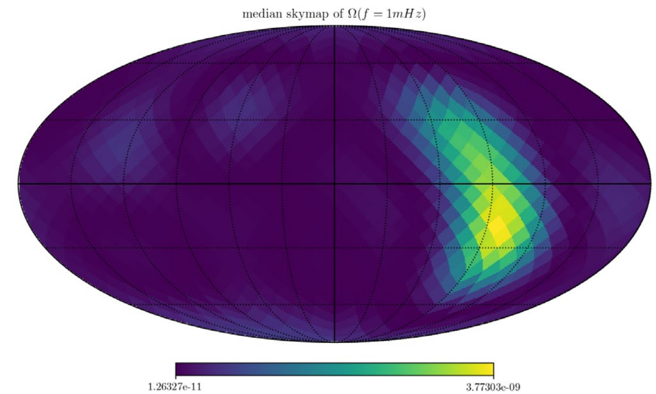
BLIP Results



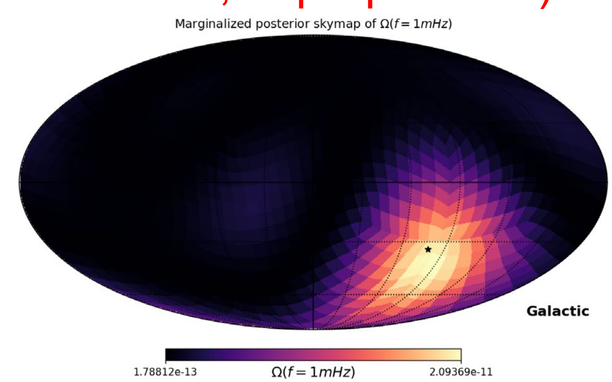
Recovery of an arbitrary directional SGWB

Banagiri et al., MNRAS 09, 0035 (2021)

Recovery of a galactic foreground

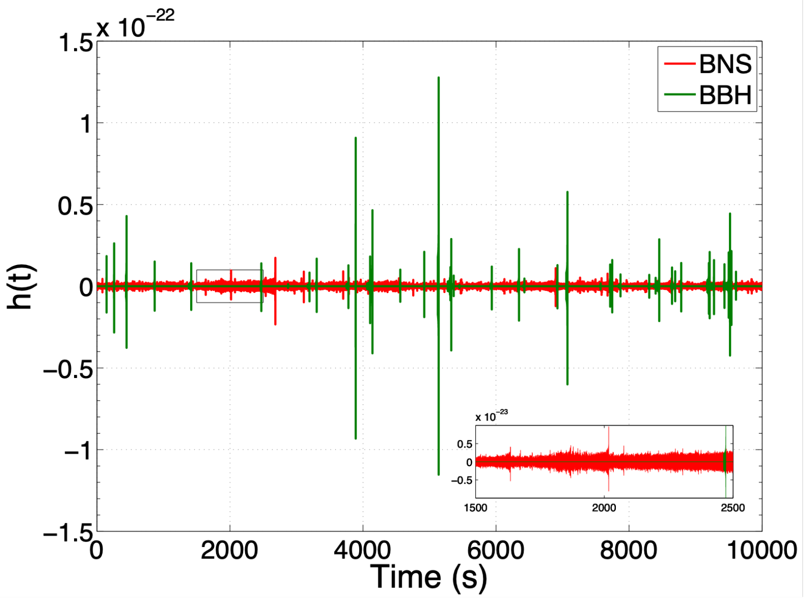


Recovery of the LMC foreground (A. Rieck et al., in preparation)



Temporal Variations

- All analyses to date assume a persistent background.
- What if this assumption is wrong?
  - » BBH Rate: 1/15 min within z<2.
- Broadband, short transients (popcorn noise):
  - » The Bayesian Search (TBS)
  - » Stochastic Search for Intermittent GWB (SSI)
- Narrowband, long transients
  - » Very Long Transient (VLT) search (time scales > 1 hour)
- Broadband, long transients
  - » No specific pipeline, but could be studied using data products of the existing pipelines



TBS & SSI

- Both searches approach the problem in a similar Bayesian framework:

Duty cycle (= prob. of a CBC in given segment)

Signal likelihood

No signal likelihood

$$\mathcal{L}_{tot} = \prod_{i}^{N_{segments}} \left[\xi \mathcal{L}_{signal,i} + (1-\xi)\mathcal{L}_{noise,i}\right]$$

- TBS case: $\mathcal{L}_{signal,i} = \mathcal{L}_i(data_i|\theta_i)$
  - » Here, θ_I denotes the binary parameters (masses, distance etc).
  - » Model-dependent approach
- SSI case: likelihood is based on comparing observed and modelled power spectra.
  - » Model-independent approach (assumes a frequency spectrum).

SSI

- Split up the data in short (4 s) segments.
- Use Gaussian mixture model likelihood with stochastic signal model

$$\mathscr{L}_{\rm tot}(d|\xi, \theta_{s,\rm pop}, \theta_n) = \prod_I \left[\xi \mathscr{L}_s(d_I|\theta_{s,\rm pop}, \theta_n) + (1-\xi)\mathscr{L}_n(d_I|\theta_n)\right]$$

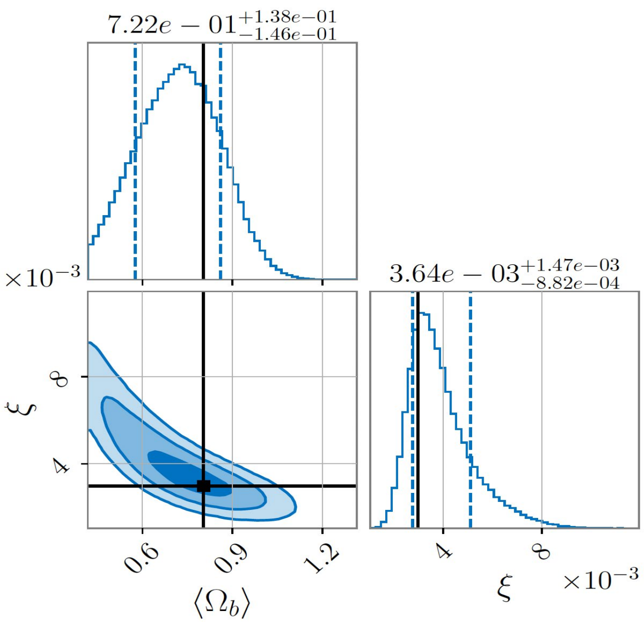
- Recovers duty cycle AND population-averaged energy density amplitude

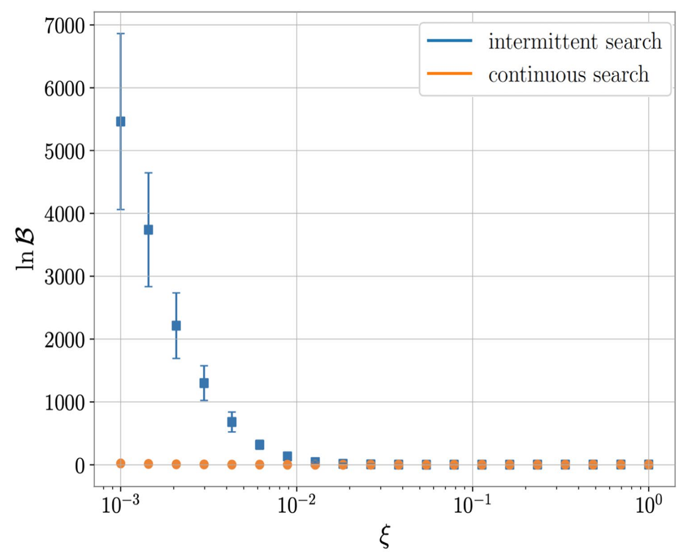
$$\underbrace{\Omega_{\rm gw}}_{\substack{\text{recovered by}\\ \text{standard search}}} = \underbrace{\xi \langle \Omega_b \rangle}_{\substack{\text{both recovered}\\ \text{by SSI}}}$$

J. Lawrence et al., Phys. Rev. D 107, 103026 (2022)
https://doi.org/10.1103/PhysRevD.107.103026

SSI

- Successful recovery of injected parameters for simplified data (i.e., simple signal distributions and white noise).
- SSI outperforms the standard (continuous) search for low duty cycles.





J. Lawrence et al., Phys. Rev. D 107, 103026 (2022)

- Ongoing developments:
  - Data processing routines (downsampling, windowing, etc.)
  - Realistic source modeling and colored noise

TBS

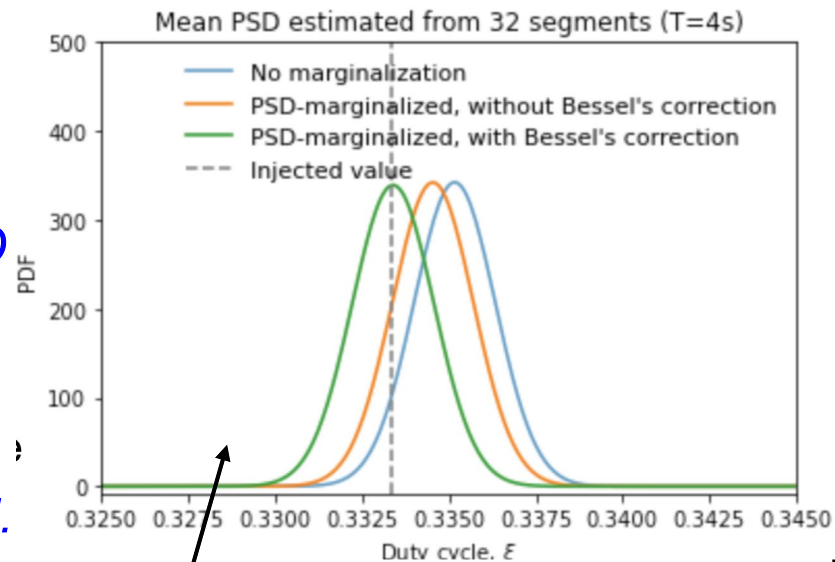
- Optimal search for compact binary mergers.
- Simple idea by Smith & Thrane, Phys. Rev. X 8, 021019 (2018):
  - » Split data into 4-sec segments.
  - » Search for CBC in each segment (posterior on 15D parameter space).
  - » Marginalize over binary parameters.
  - » Construct a likelihood for duty cycle:

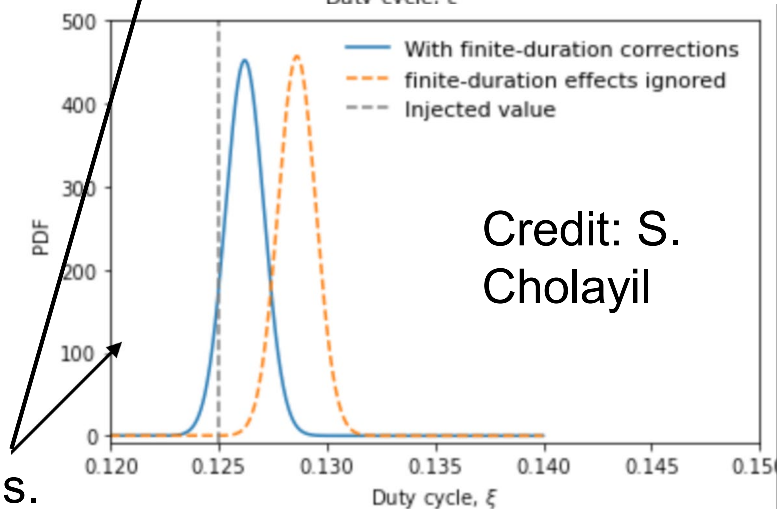
$$\mathcal{L}_{tot} = \prod_{i}^{N_{segments}} \left[\xi \mathcal{L}_{signal,i} + (1-\xi)\mathcal{L}_{noise,i}\right]$$

- Potential to improve ~1000x over the standard SGWB search.
- Could add population hyper-parameters.
  - » Extract e.g. directional distribution, mass distribution etc.

TBS Challenges

- Noise PSD not known.
  - » Assume Gaussian noise, marginalize over PSD uncertainty.
  - » Leads to Student-t distribution, *if PSD covariance matrix is diagonal*.
- PSD covariance matrix is not diagonal!
  - » Correlations between frequency bins are induced by windowing: *Talbot et al. Phys. Rev. Res. 3, 043049 (2021).*
  - » Estimate non-diagonal covariance matrix for the window function.
- Combining the two effects is a challenge.
  - » Conceptually and computationally!





Credit: S. Cholayil

Demonstrations using simulated Gaussian noise and BBH ssignals.

Empirical Approach

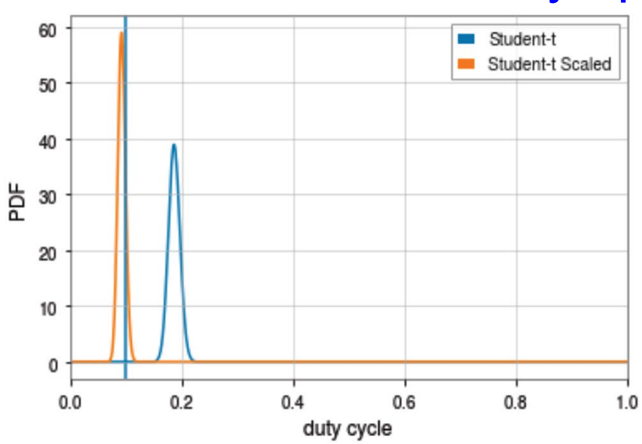
- Bypass the calculation of non-diagonal likelihood with an empirical relationship between the diagonal and non-diagonal likelihood.

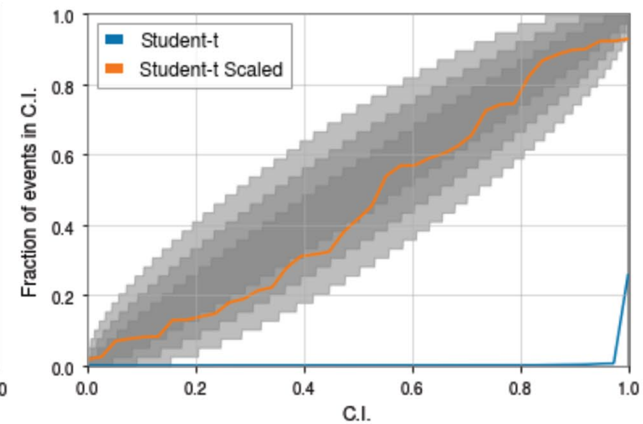
$$\mathscr{L}(\tilde{n}\,|\,\hat{C}) = \alpha \times \left(\mathscr{L}(\tilde{n}\,|\,\hat{P})\right)^{\beta}$$

Non-diagonal likelihood
marginalized over
PSD uncertainties

Diagonal likelihood
(Student=t + Bessel)
marginalized over uncertainties

- Use simulations to identify optimal α and β.





Credit: S. Cholayil

TBS Outlook

- Repeat the analysis using larger dataset with real noise.
- Other bias sources: choices of priors, segment duration etc.
- Add other (truly) stochastic background models too.
- Estimate anisotropy in the BBH SGWB, develop PE formalism, GW-EM formalism etc.

	Gaussian noise		Time-reversed Real noise
	Frequency domain	Time-domain	
Known PSD with restricted BBH prior	✅	✅	Not Applicable
Known PSD with standard BBH prior	✅	✅	Not Applicable
Unknown PSDs with restricted BBH prior	✅	✅	✅
Unknown PSDs with standard BBH prior	✅	✅	

Credit: S. Cholayil

Intermittent SGWB and LISA

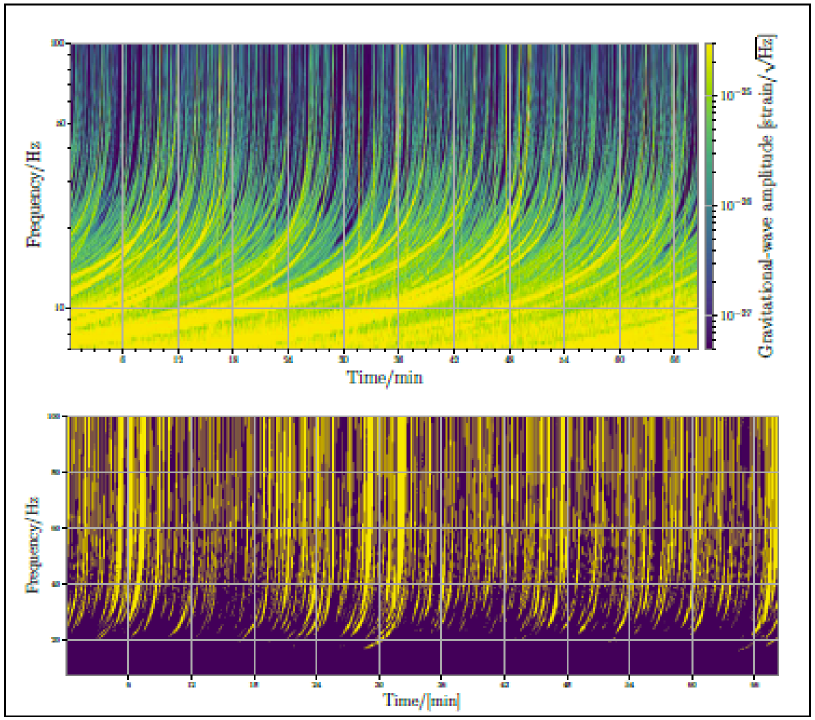
- Are there intermittent SGWB models in the LISA band?
  - » Or, perhaps, intermittent noise sources?
- If so, we may need a tool to go after them.
  - » Model-independent tool (like SSI).
  - » For specific models, could develop dedicated tools that leverage the model constraints (like TBS).
- Incorporate into the global fit…

Foreground Removal

- Currently observe a few CBC signals per week.
  - » Actual CBC rates are much higher: BBH / 15 min; BNS / 2 min, somewhere in the universe.
- Future detectors will observe many more CBCs, overlapping in time and frequency.
  - » Foreground masking cosmological signals.
- CE/ET: detect nearly all BBH and 50% of BNS mergers!

- Can we notch these signals in time-frequency maps?
  - » H. Zhong et al., Phys. Rev. D 107 064048 (2023).
  - » Simulate realistic CE strain time series, apply notches for all (or only for resolvable) CBCs.
  - » Assume CBC parameters are measured well, so the notches are ideal.

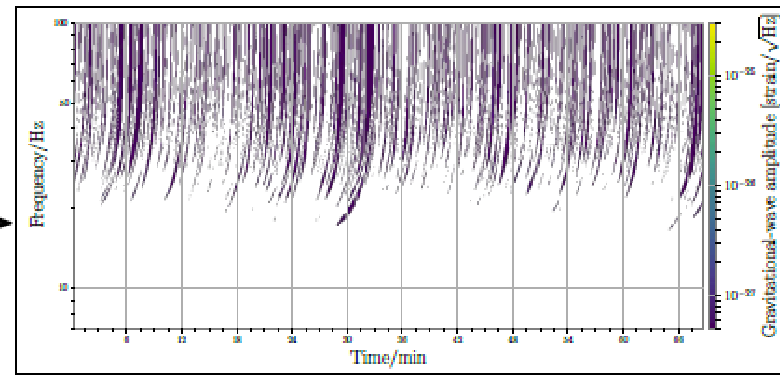
Notching Strategy



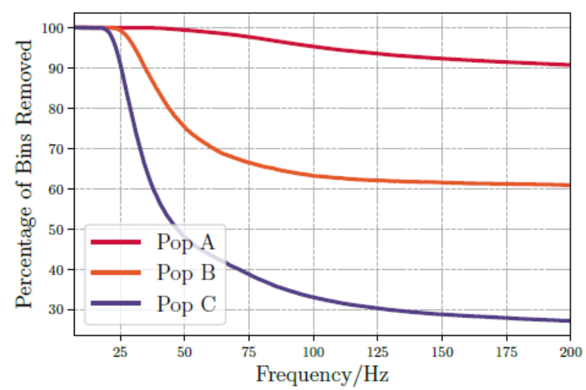
Apply

Mask

Uncertainty in the BNS rate implies different requirements on notching.



Lose nearly all bins below 25 Hz



H. Zhong et al., Phys. Rev. D 107 064048 (2023).

Notching Results

Population & Interval	Case	$\hat{C}_{IJ}$	$\hat{\sigma}_{IJ}$	SNR
	Noise+SGWB[$\alpha = 0$]	4.0×10^{-11}	9.0×10^{-14}	$\mathbf{4.5\times10^{2}}$
	Noise+SGWB[$\alpha = 4$]	4.0×10^{-11}	1.5×10^{-13}	$\mathbf{2.6\times10^{2}}$
Pop A: BNS=98.0%,BBH=2.0%; d=3s	Noise+CBC[$\alpha = 0$]	9.3×10^{-10}	9.0×10^{-14}	1.0×10^{4}
	Noise+CBC(removed)[$\alpha = 0$]	3.2×10^{-11}	5.1×10^{-11}	0.63
	Noise+CBC(removed)+SGWB[$\alpha = 0$]	7.2×10^{-11}	5.1×10^{-11}	**1.4**
	Noise+CBC(removed)+SGWB[$\alpha = 4$]	3.9×10^{-11}	1.1×10^{-12}	**35**
	Noise+CBC($\rho \leqslant 8$)[$\alpha = 0$]	$\sim 8.6\times10^{-11}$	-	-
	Noise+CBC($\rho \leqslant 8$)[$\alpha = 4$]	$\sim 5.6\times10^{-11}$	-	-
Pop B: BNS=95.9%,BBH=4.1%; d=6s	Noise+CBC[$\alpha = 0$]	6.5×10^{-10}	9.0×10^{-14}	7.2×10^{3}
	Noise+CBC(removed)[$\alpha = 0$]	1.9×10^{-12}	2.7×10^{-12}	0.68
	Noise+CBC(removed)+SGWB[$\alpha = 0$]	4.2×10^{-11}	2.7×10^{-12}	**15**
	Noise+CBC(removed)+SGWB[$\alpha = 4$]	3.9×10^{-11}	3.5×10^{-13}	$\mathbf{1.1\times10^{2}}$
	Noise+CBC($\rho \leqslant 8$)[$\alpha = 0$]	$\sim 4.2\times10^{-11}$	-	-
	Noise+CBC($\rho \leqslant 8$)[$\alpha = 4$]	$\sim 2.8\times10^{-11}$	-	-
Pop C: BNS=83.3%,BBH=16.7%; d=24s	Noise+CBC[$\alpha = 0$]	4.1×10^{-10}	9.0×10^{-14}	4.6×10^{3}
	Noise+CBC(removed)[$\alpha = 0$]	7.2×10^{-13}	1.3×10^{-12}	0.56
	Noise+CBC(removed)+SGWB[$\alpha = 0$]	4.1×10^{-11}	1.3×10^{-12}	**31**
	Noise+CBC(removed)+SGWB[$\alpha = 4$]	4.0×10^{-11}	2.4×10^{-13}	$\mathbf{1.7\times10^{2}}$
	Noise+CBC($\rho \leqslant 8$)[$\alpha = 0$]	9.1×10^{-12}	-	-
	Noise+CBC($\rho \leqslant 8$)[$\alpha = 4$]	6.0×10^{-12}	-	-

H. Zhong et al., Phys. Rev. D 107 064048 (2023).

What next?

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- *Simple notching appears sufficient to reach the SGWB floor due to unresolved BNS: $\Omega_{GW} \sim 10^{-11}$.*
  - » Repeating the analysis using better rate estimates, expected errors in CBC parameter estimation (i.e. imperfect notching) etc.
- Can we do better?
  - » Sharma & Harms, Phys. Rev. D 102, 063009 (2020).
    - – Subtraction-noise projection method, minimizes residuals in CBC subtraction. Also limited by the unresolved CBC population.
  - » Biscoveanu et al., Phys. Rev. Lett. 125, 241101 (2020).
    - – TBS-based, fit SGWB along with all CBCs.
    - – Avoids subtraction residuals, unclear what may limit the sensitivity (TBS biases, large BNS rate, noise transients…).
  - » New basis to separate CBC signals from the noise?
    - – H. Zhong: Does not seem to work, CBC and SGWB live in the same part of the parameter space…

Foreground Subtraction and LISA

- BBH/BNS/BHNS will also live in the LISA band.
  - Masking cosmological SGWB.

- Cannot individually resolve all binaries with LISA alone.
  - But perhaps we can resolve the binaries with CE/ET (or their upgrades) and then backtrack in time to remove them from LISA data.
  - How large would residuals be?

- *Global fit that includes both LISA and 3G terrestrial detectors?*

Conclusions

- CBC SGWB detection with LVK is likely in the next 3-5 years.
  - » Soon after, CBC SGWB will become a foreground.
- Need a global fit to estimate the energy budget!
  - » Leverage dependencies on frequency, direction, polarization, and duty cycle (+GW-EM correlations) to separate different contributions.
  - » Combine LISA, terrestrial, and PTA data for model inference.
- Remove astrophysical contributions to access the cosmological backgrounds.
- Share techniques/approaches between LISA and LVK.
  - » Common problems, despite differences in formalisms.
  - » Goes both ways!

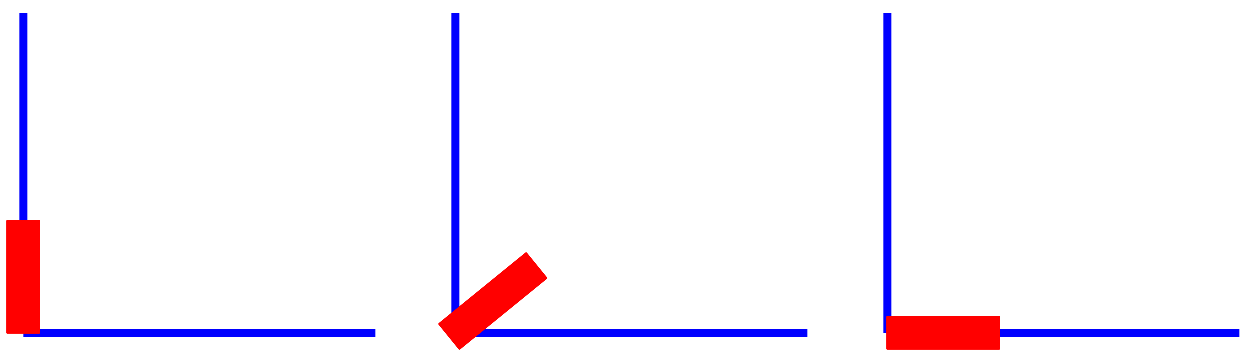
Leveraging Detector Configuration to Assess Noise

University of Minnesota

Phys. Rev. D 76, 022001, 2007

First Cross-Correlation Analysis of Interferometric and Resonant-Bar Gravitational-Wave Data for Stochastic Backgrounds

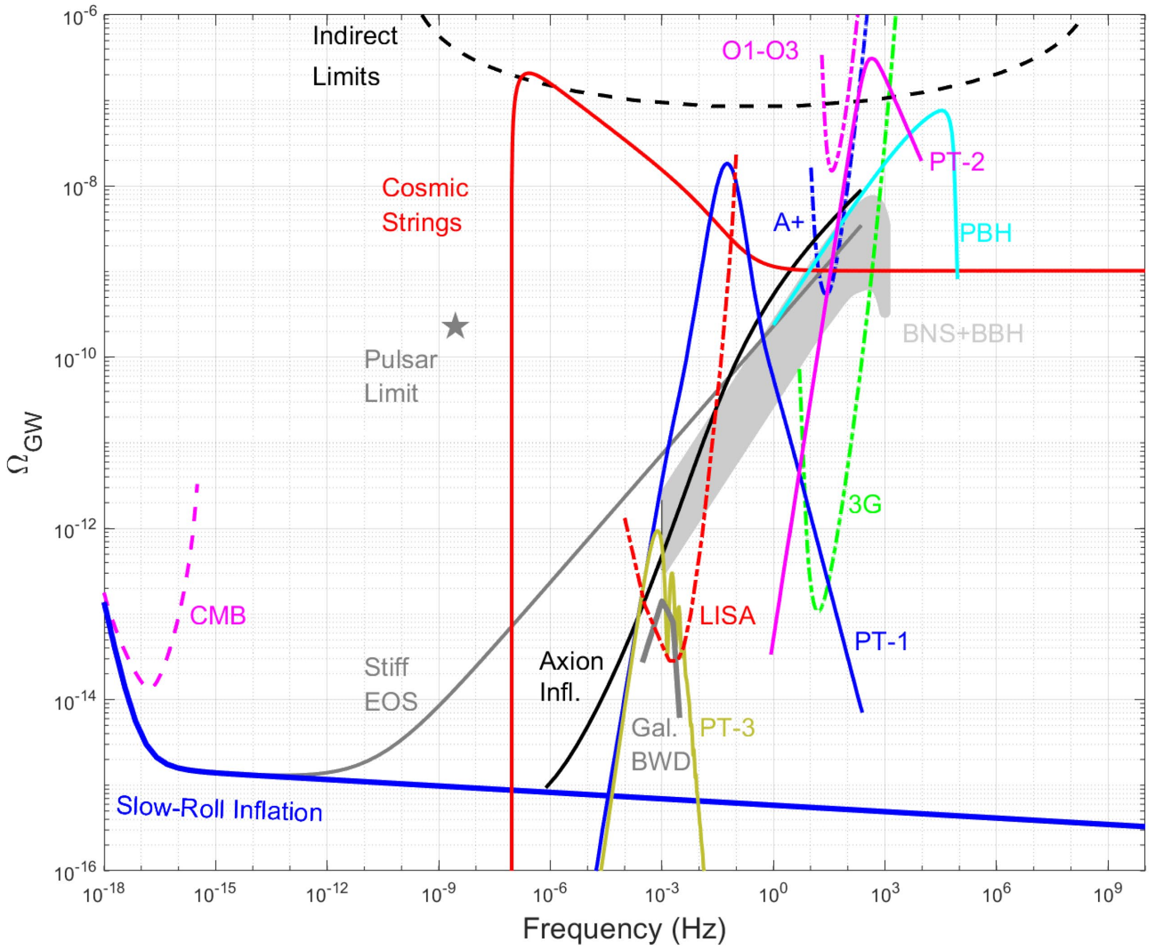
Data from the LIGO Livingston interferometer and the ALLEGRO resonant bar detector, taken during LIGO's fourth science run, were examined for cross-correlations indicative of a stochastic gravitational-wave background in the frequency range 850-950 Hz, with most of the sensitivity arising between 905 Hz and 925 Hz. ALLEGRO was operated in three different orientations during the experiment to modulate the relative sign of gravitational-wave and environmental correlations. No statistically significant correlations were seen in any of the orientations, and the results were used to set a Bayesian 90% confidence level upper limit of $\Omega_{\mathrm{gw}}(f) \leq 1.02$, which corresponds to a gravitational wave strain at 915 Hz of $1.5 \times 10^{-23}\ \mathrm{Hz}^{-1/2}$. In the traditional units of $h_{100}^2 \Omega_{\mathrm{gw}}(f)$, this is a limit of 0.53, two orders of magnitude better than the previous direct limit at these frequencies. The method was also validated with successful extraction of simulated signals injected in hardware and software.



SGWB Models

- Stochastic gravitational-wave background arises from a superposition of many uncorrelated GW sources.

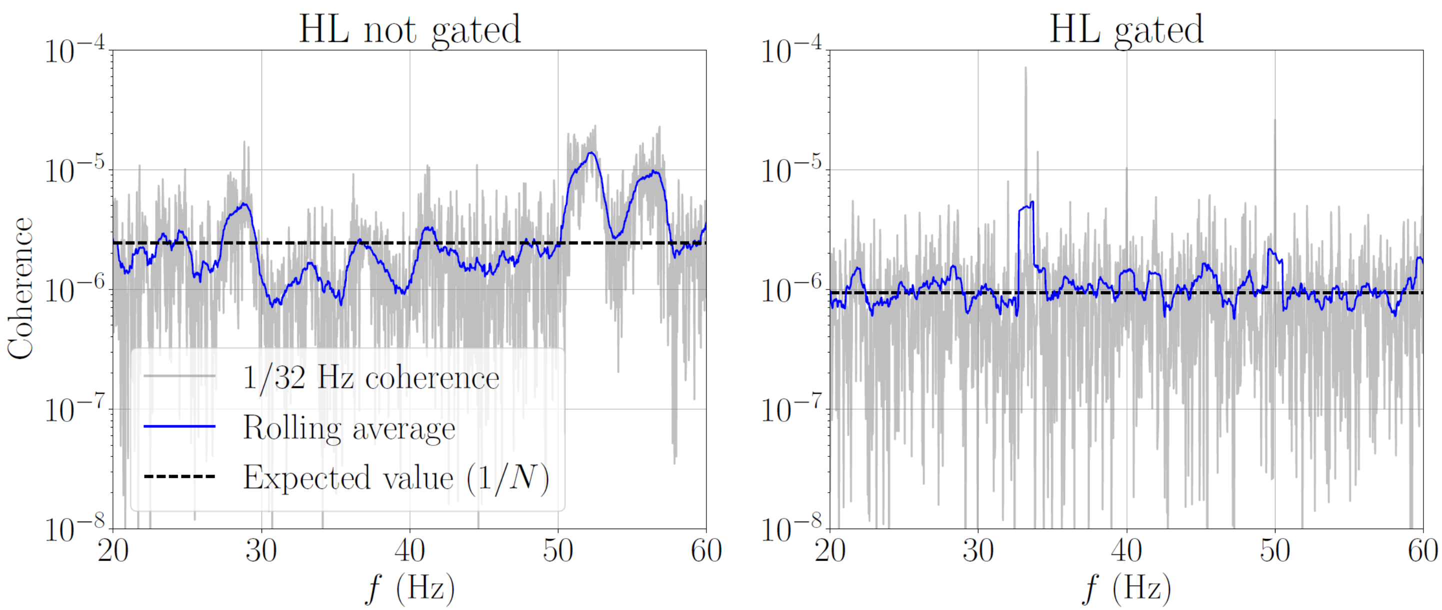
- Many models!
  - » Great physics impact!
  - » Problem: SGWB energy budget?



arXiv:2203.07972
(Snowmass white paper)

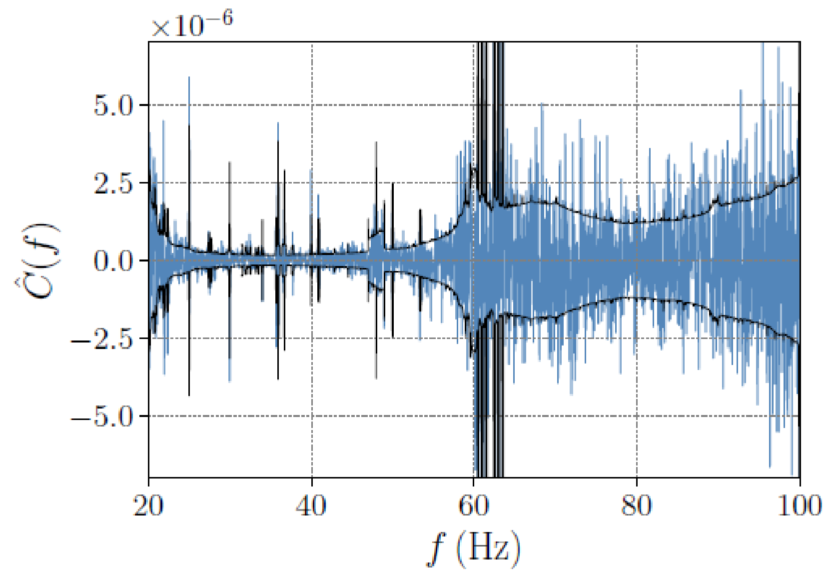
Data Quality

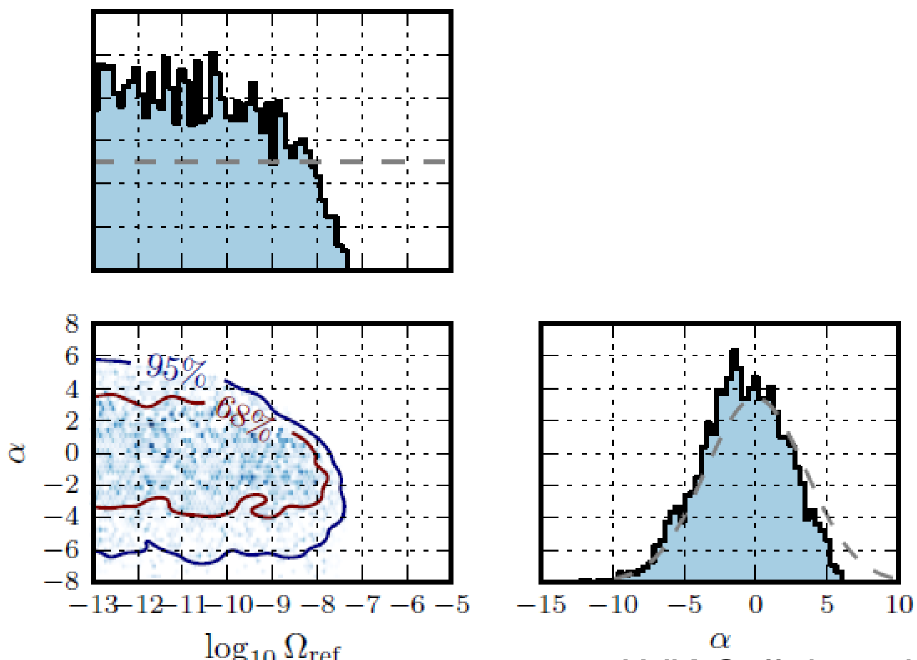
- Data quality is usually the challenge:
- Segment removal: known corrupt segments, $\Delta\sigma$ cut to remove large fluctuations.
- Line removal: Identify and notch instrumental or environmental lines.
- Gating: O3 featured many glitches, had to develop a scheme to remove them without losing much data.
- Magnetic contamination.



O1-O3 search results

- Combine all observational data to date: O1 – O3; LHO, LLO, and Virgo.
- No evidence of signal, place upper limits.





	Log-uniform prior		
α	O3	O2 [43]	Improvement
0	5.8×10^{-9}	3.5×10^{-8}	6.0
2/3	3.4×10^{-9}	3.0×10^{-8}	8.8
3	3.9×10^{-10}	5.1×10^{-9}	13.1
Marg.	6.6×10^{-9}	3.4×10^{-8}	5.1

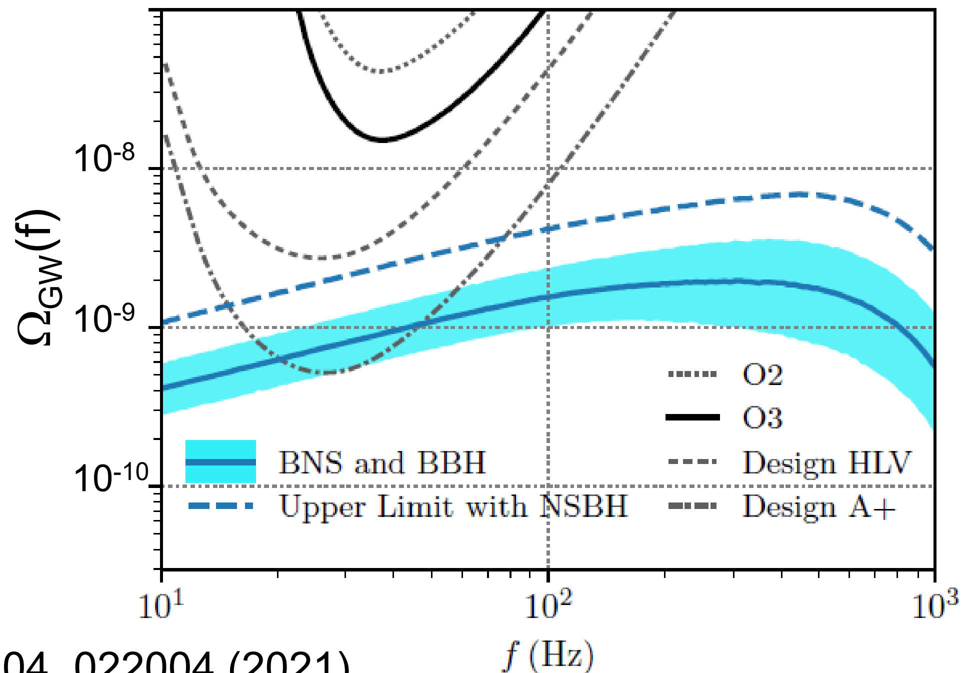
LVK Collaborations, Phys. Rev. D 104, 022004 (2021)

O1-O3 Search Results

- Can also search for non-tensor modes.
  - Amounts to using different overlap reduction functions.

Polarization	O3	O2 [43]	Improvement
Tensor	6.4×10^{-9}	3.2×10^{-8}	5.0
Vector	7.9×10^{-9}	2.9×10^{-8}	3.7
Scalar	2.1×10^{-8}	6.1×10^{-8}	2.9

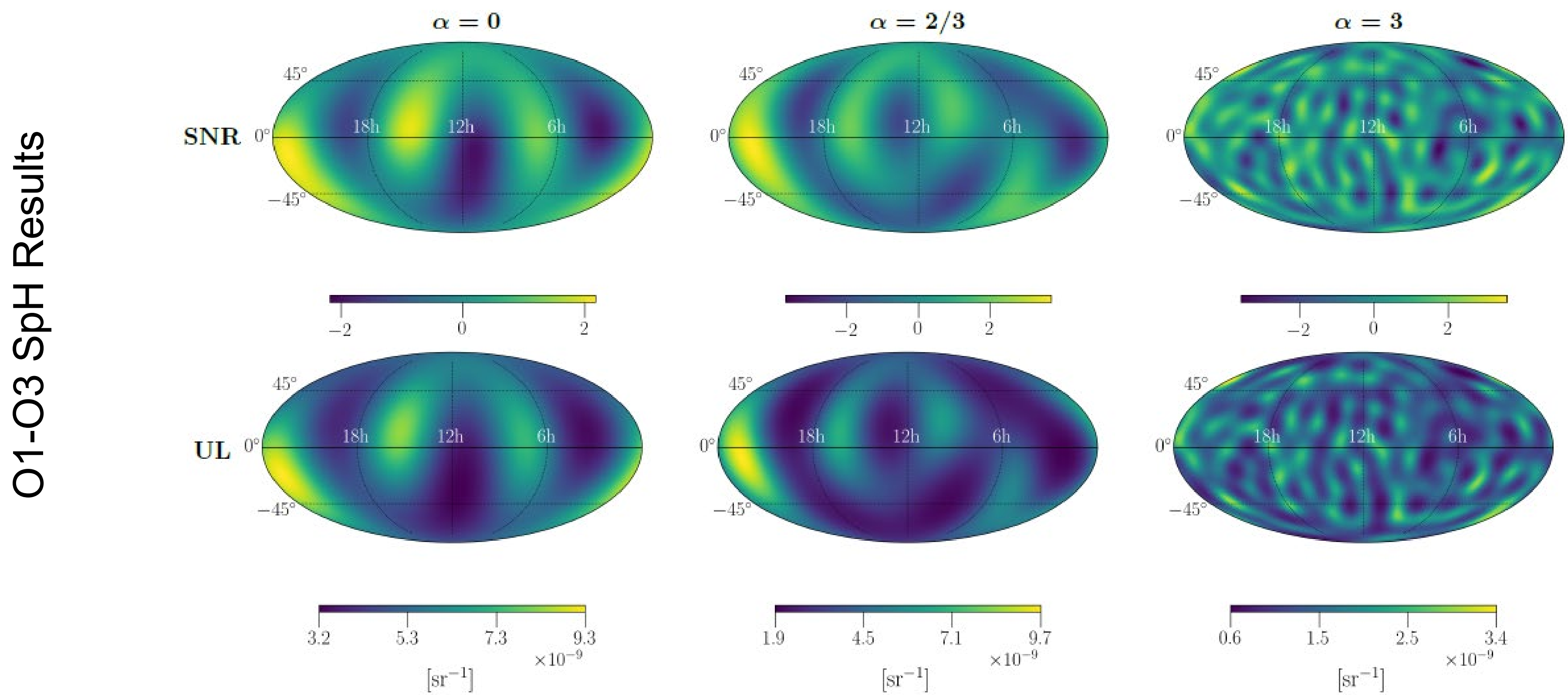
- Combine SGWB results with individual CBC rates.
  - Model dependent, similar contributions from BBH and BNS.
  - Predict likely detection of CBC SGWB in O4+O5.
  - At which point, this becomes a foreground!



LVK Collaborations, Phys. Rev. D 104, 022004 (2021)

O1-O3 Anisotropic Results (1)

- O1-O3 anisotropic analyses also yielded upper limits.
  - » Both radiometer and spherical harmonics approaches.
  - » LVK Collaborations, Phys. Rev. D 104, 022005 (2021).



O1-O3 Anisotropic Results (2)

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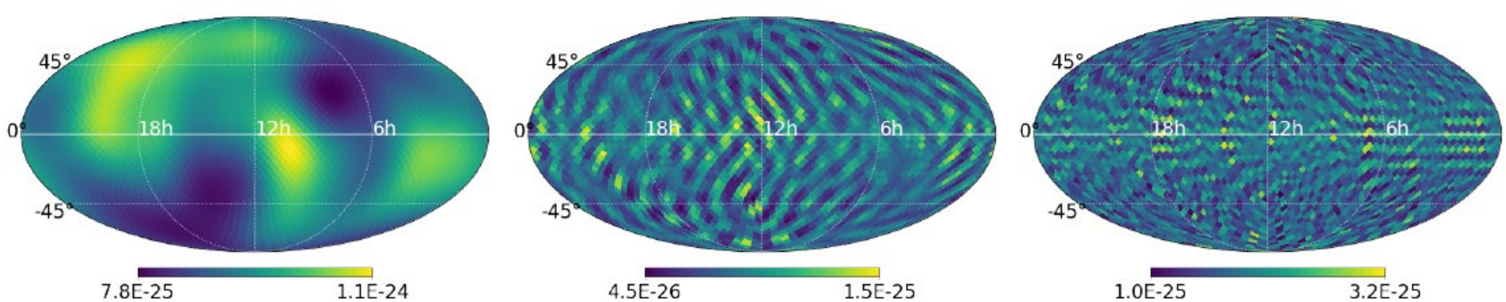
- New: All-sky-all-frequencies radiometer search
  - » Simultaneous analysis of both frequency and direction dependence.
  - » Made possible by folding cross-correlation data into one sidereal day.
  - » Dirty maps (no Fisher matrix inversion).
  - » LVK Collaborations, Phys. Rev. D 105, 122001 (2022).

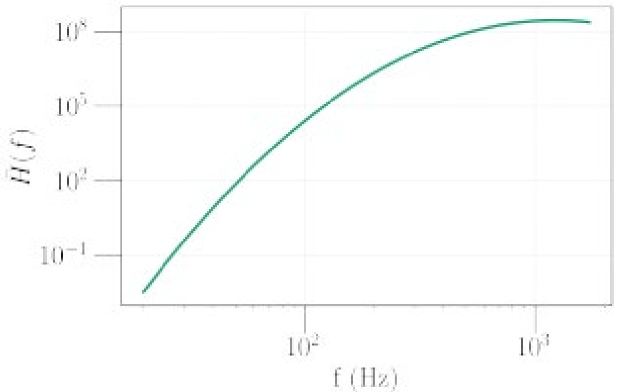
Upper Limit Maps in strain amplitude

23.0625 Hz   423.0635 Hz   1223.0625 Hz



O1-O3 Anisotropic Results (3)

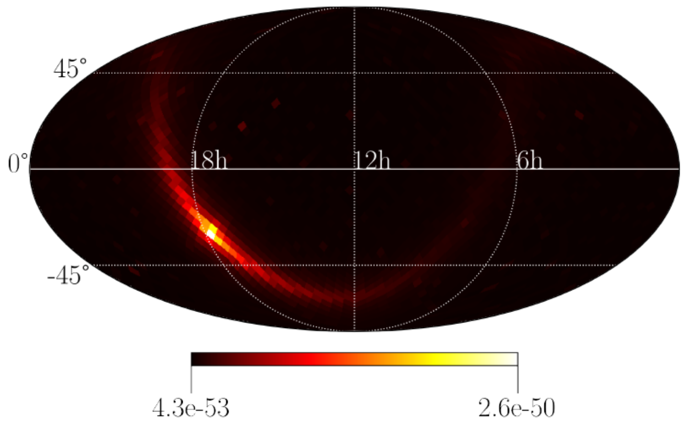
- New: Targeted anisotropic search.
  - » If the distribution in frequency and across the sky is known (e.g. galactic plane): $$\mathcal{P}_\alpha(f) = A\,\bar{H}_f\,\hat{\mathcal{P}}_\alpha$$



- Estimate only the amplitude of the model:

$$\hat{A} = \frac{\mathbf{X}^\dagger \hat{\mathcal{P}}}{\hat{\mathcal{P}}^\dagger \mathbf{\Gamma} \hat{\mathcal{P}}}$$

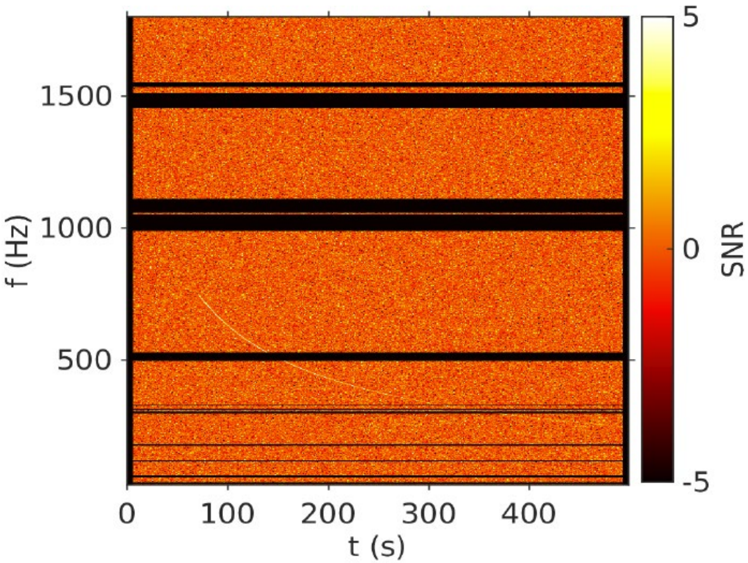
- Agarwal et al., Phys. Rev. D **106**, 043019 (2022)
  - » Applied to SGWB due to galactic millisecond pulsars.



O1+O2+O3 Results				
Baseline	Exponential radial distribution		Gaussian radial distribution	
	$(\hat{A} \pm \sigma_{\hat{A}}) \times 10^{-10}$	$\rho_{\hat{A}}$ $(p-\text{value}\%)$	$(\hat{A} \pm \sigma_{\hat{A}}) \times 10^{-10}$	$\rho_{\hat{A}}$ $(p-\text{value}\%)$
O3-HL	2.3 ± 2.5	0.92 (18)	1.4 ± 2.6	0.54 (30)
O3-HV	50 ± 22	2.3 (1.2)	39 ± 18	2.2 (1.4)
O3-LV	7 ± 1.3	0.54 (29)	24 ± 12	2.0 (2.1)
O2-HL	-6.8 ± 9.8	-0.69 (76)	-3.6 ± 1	-0.36 (64)
O1-HL	-22 ± 21	-1.1 (85)	-23 ± 22	-1.1 (86)
O1+O2+O3	**2.2 ± 2.4**	**0.92 (18)**	**2.4 ± 2.4**	**1.0 (16)**

VLT Search

- Search through the time-frequency map of cross-correlation.
  - » Target narrowband transients on different time-scales.
  - » Short transients are included in other searches.
  - » Transients on time-scales of >1 hour could bias the SGWB statistics.



- One such search performed to study the GW170817 remnant.
  - » Targeted: location on the sky and the start-time of the signal are known.
- All-sky VLT search has never been performed.
  - » Background estimation is harder than in the targeted case (cannot turn off the signal!)