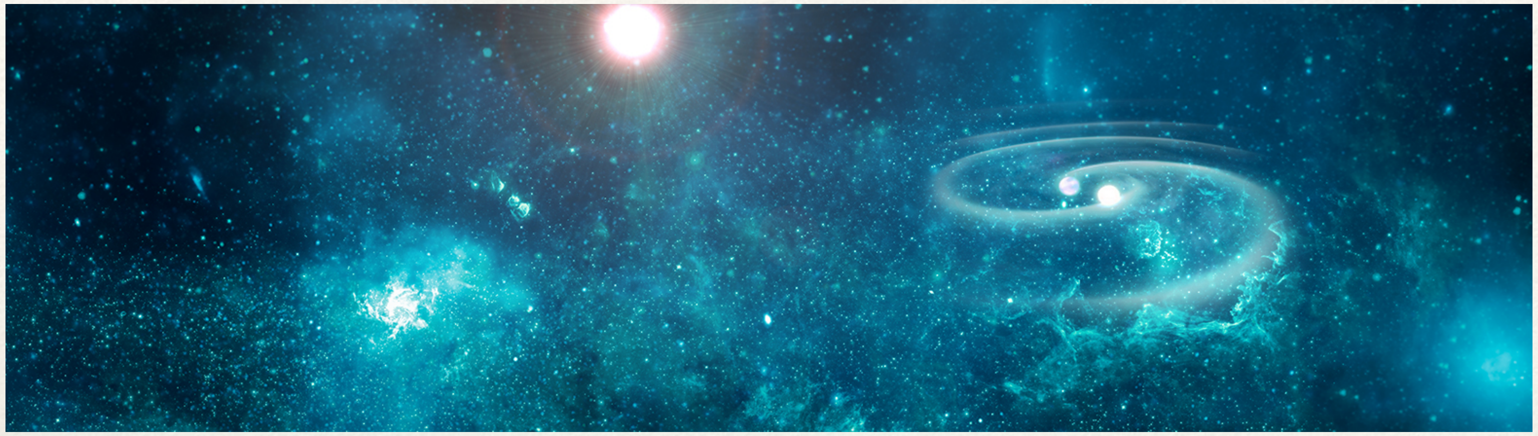
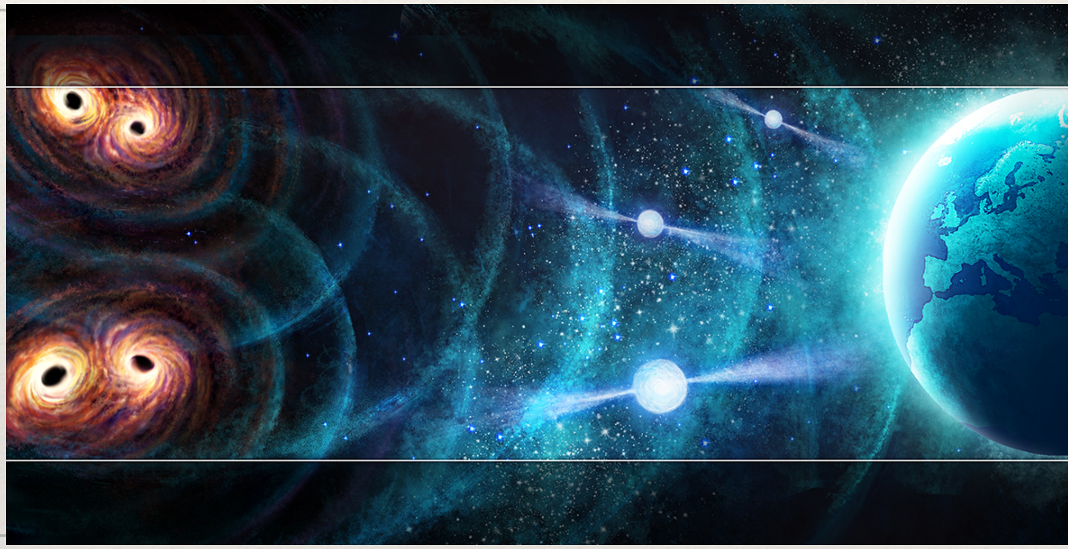
Stanislav (Stas) Babak.

AstroParticule et Cosmologie, CNRS (Paris)

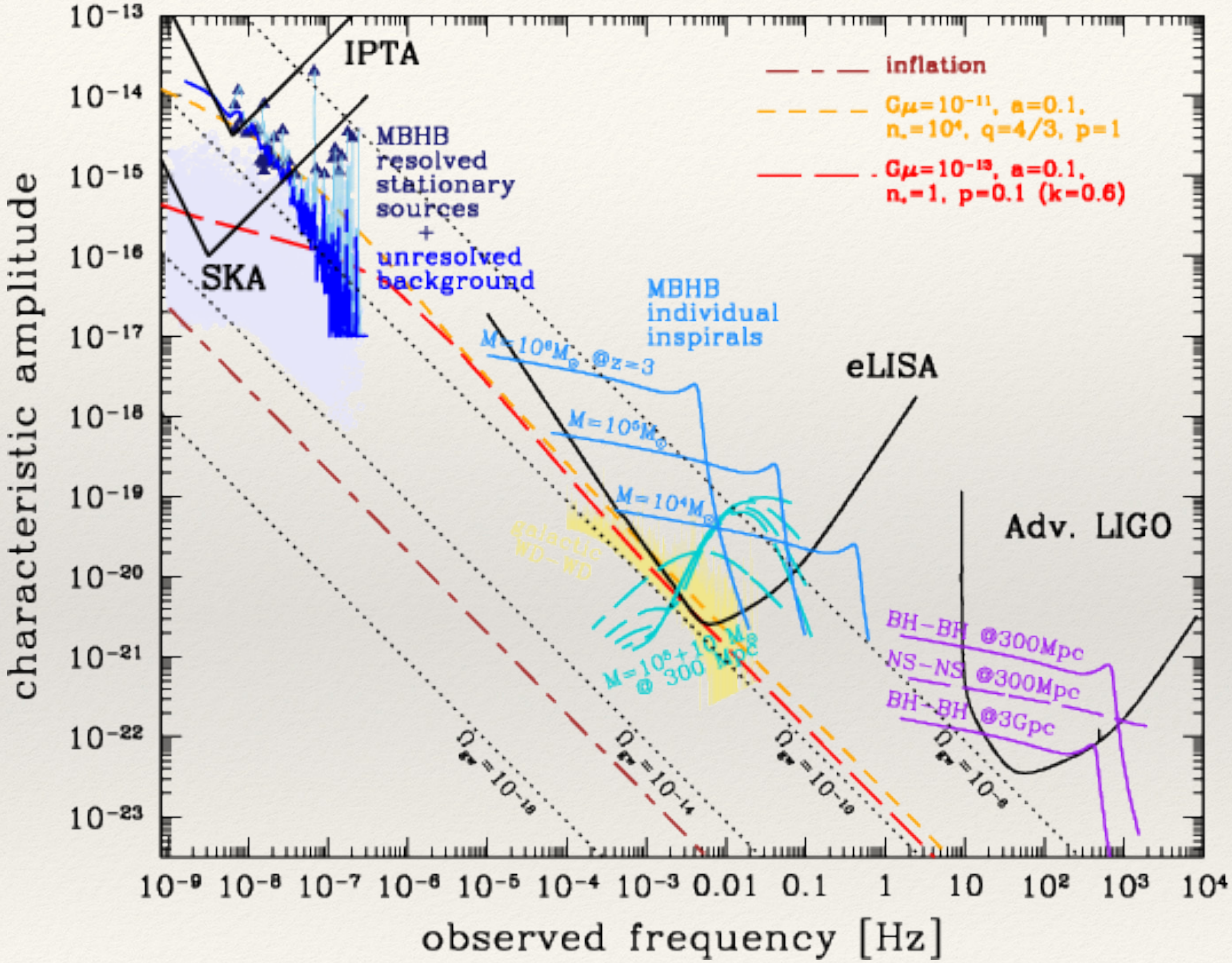


Pulsar Timing Array



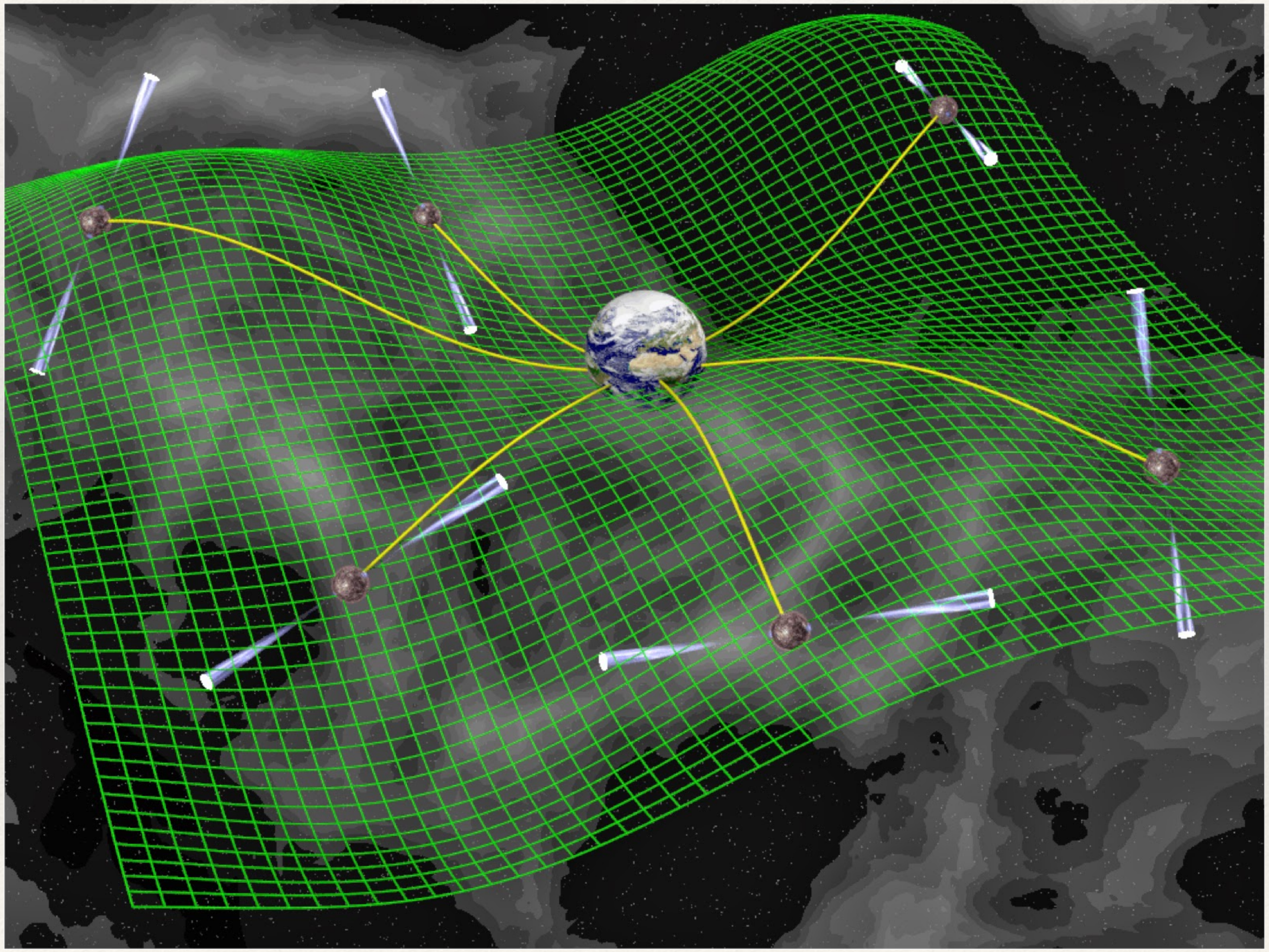
CERN 19-21 July 2023

GW landscape



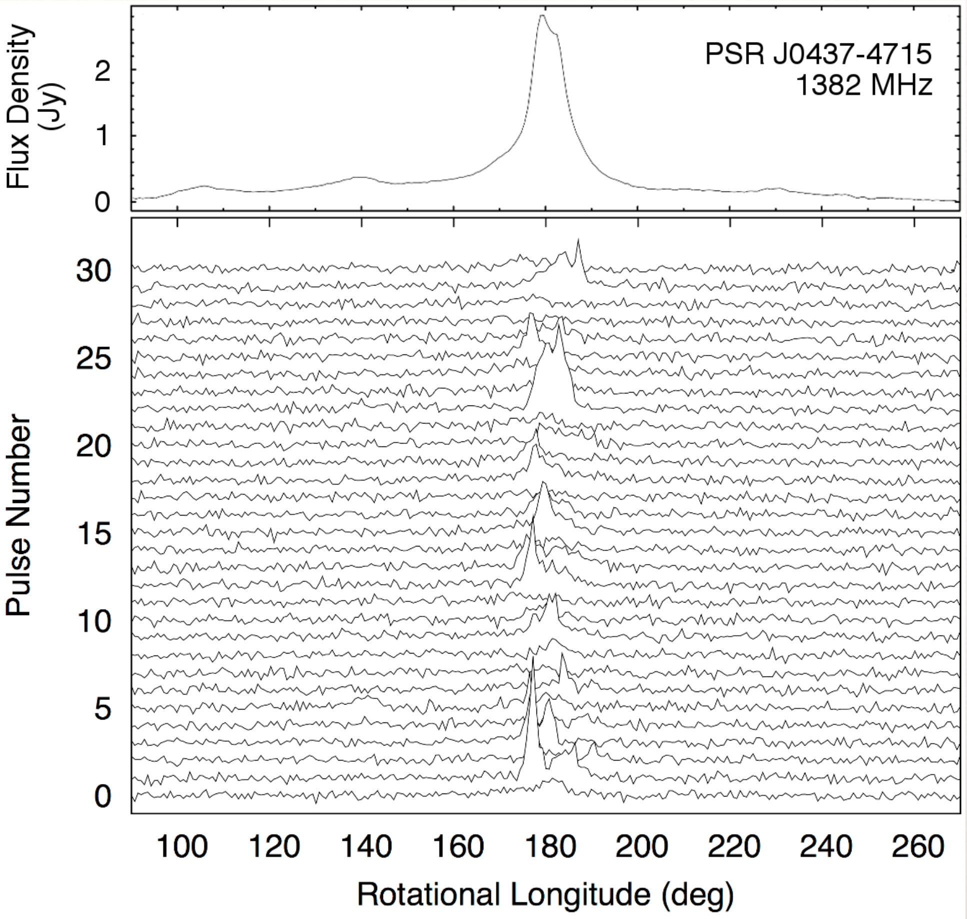
Pulsar Timing Array

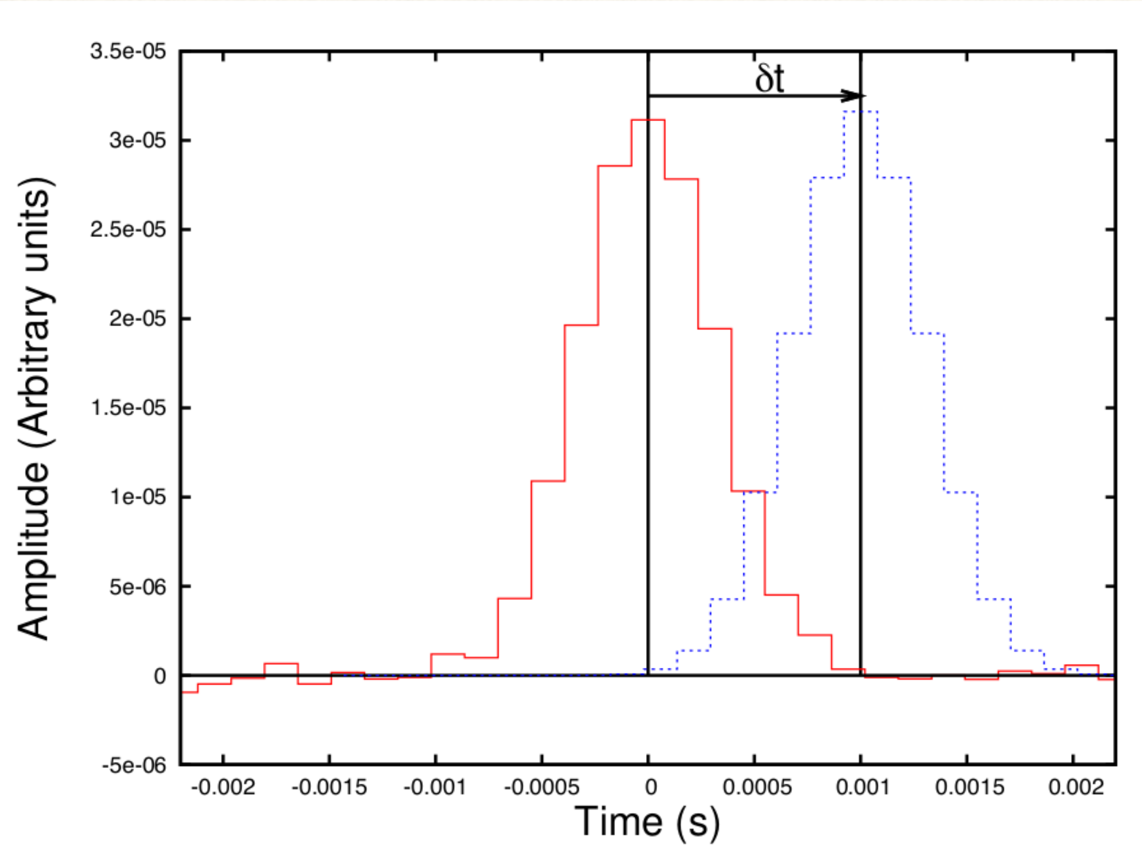
The main idea behind pulsar timing array (PTA) is to use ultra-stable millisecond pulsars as beacons (clocks sending signals) for detecting GW in the nano-Hz range (10^{-9} - 10^{-7} Hz).



[Credits: D. Champion]

Pulsar timing

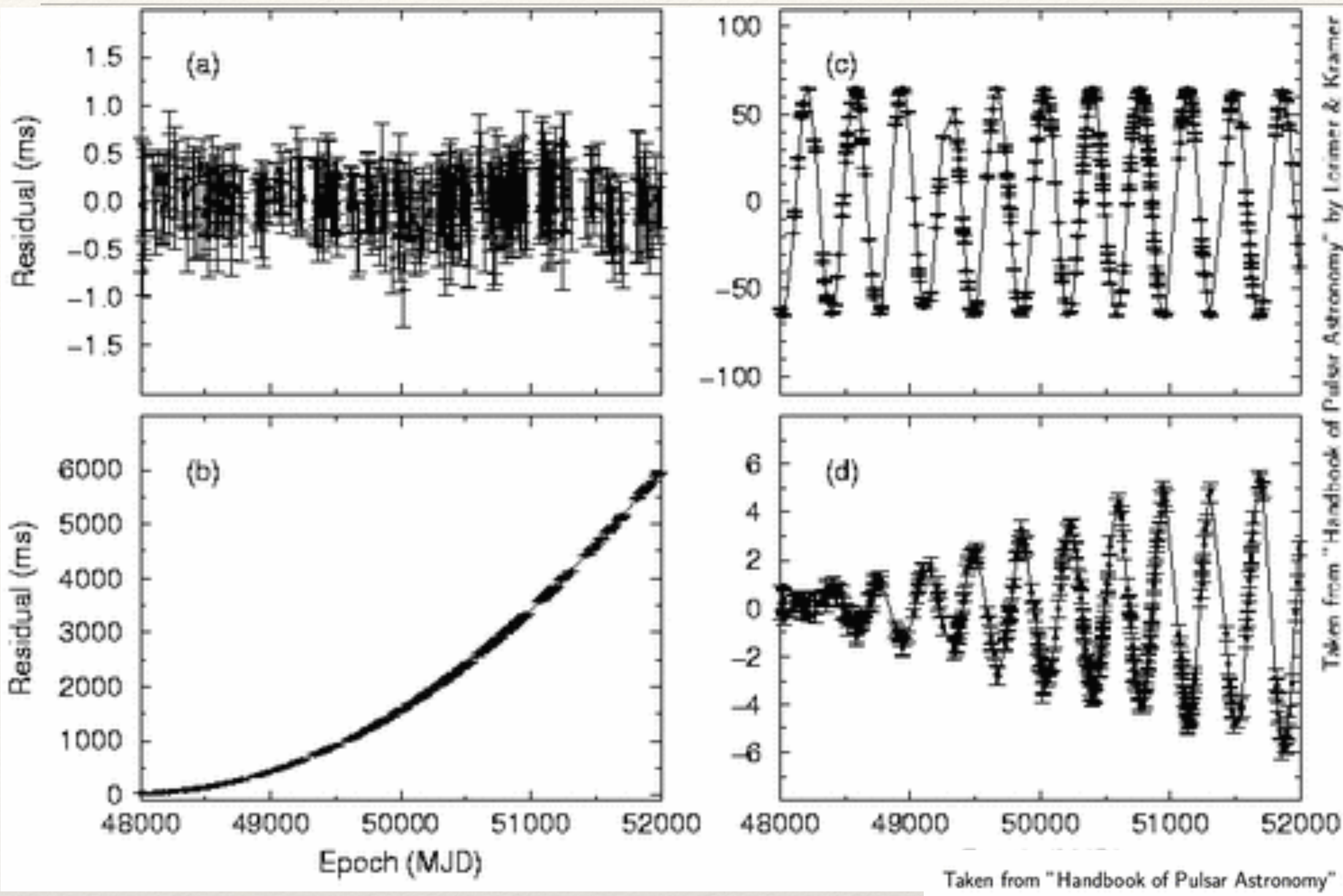




[Figs: credits
S. Burke-Spolar & L. Lentati]

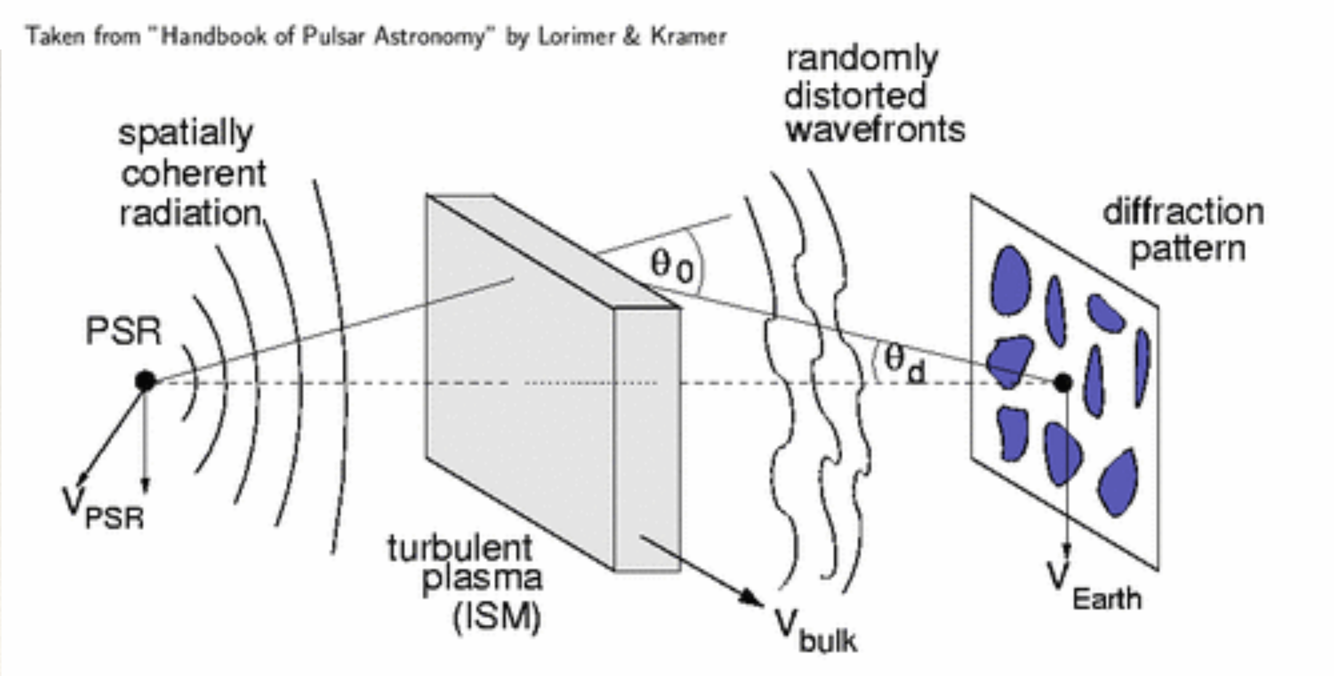
- Each observed radio pulse profile has a lot micro-structure. If we average over ~hour the (average) profile is very stable
- We can use the average pulse profile to estimate the time-of-arrival (TOA) of the pulses.
- The idea is to measure the TOA, and compare to the expected TOA. We know the spin of the pulsars, so we can predict the TOA. The difference between measure and expected TOA: ***residuals***

Timing pulsars



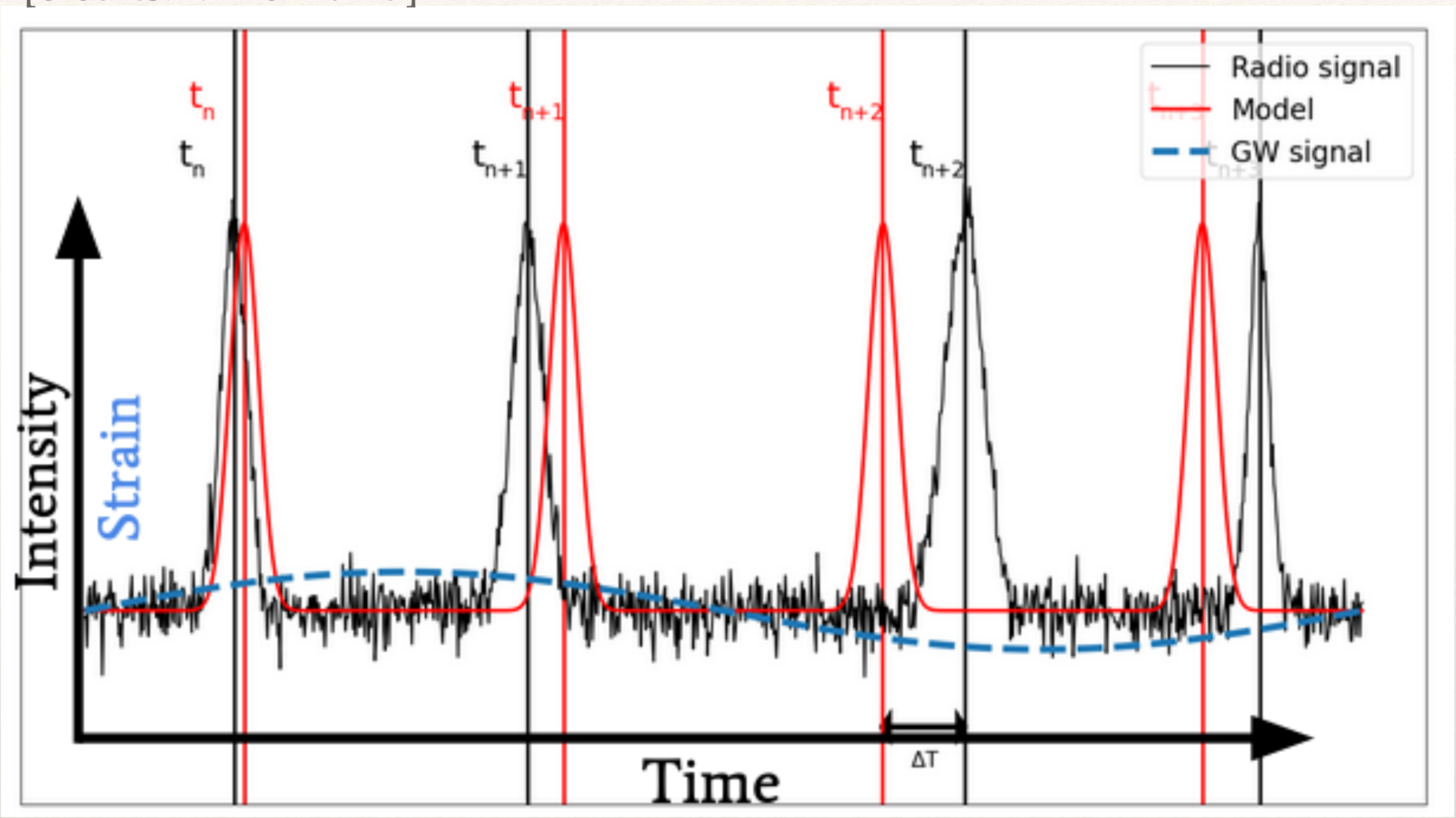
- We need to build a timing model to make accurate prediction for TOAs - take into account various physical effects
- Dispersion of e/m wave and its time dependence
- Rate of change of rotation (b)
- Sky position of the pulsar (c)
- Proper motion of a pulsar (d)

- Timing model could be quite complex if pulsar is in the binary



Timing Residuals

[credits: Mikel Falxa]



$$dt = t^{p}_{toa} - t^{o}_{toa} = dt_{errors} + \delta\tau_{GW} + noise$$

Errors in fitting the model → dt_{errors}

due to GWs → $\delta\tau_{GW}$

Detection statistic and search algorithm

- We assume that noise is Gaussian: the likelihood function (likelihood of the signal with given parameters) is

$$P(\vec{\delta t}, \vec{\theta}) = \frac{1}{\sqrt{(2\pi)^n det(C)}} \exp\left(-\frac{1}{2}(\vec{\delta t} - \vec{s})^T C^{-1} (\vec{\delta t} - \vec{s})\right),$$

- $\overrightarrow{\delta t}$ - concatenated residuals from all pulsars in the array: total size n

- $\vec{s}$ - is a model of deterministic signals (for example GW signals from individually resolvable SMBHBs)

- C is the noise variance-covariance matrix (size $n \times n$)

$$C_{\alpha i,\beta j} = C^{wn}\delta_{\alpha\beta}\delta_{ij} + C^{rn}_{ij}\delta_{\alpha\beta} + C^{dm}_{ij}\delta_{\alpha\beta} + C^{GW}_{\alpha i,\beta j} + \ldots$$

white measurement noise; red noise spin noise; dispersion variation noise; stochastic GW signal

Noise modelling in PTA

- White noise — not very interesting. Two parameters per backend per pulsar: unaccounted noise.
- Red noise: very generic noise description in freq. domain

$$S(f) = A_{rn}^2 f^{-\gamma}$$

common, uncorrelated red noise

$$S_\alpha(f) = A_{rn,\alpha}^2 f^{-\gamma_\alpha}$$

red noise in each pulsar

- DM (dispersion measurement variation) noise: depends on the radio-frequency of observation

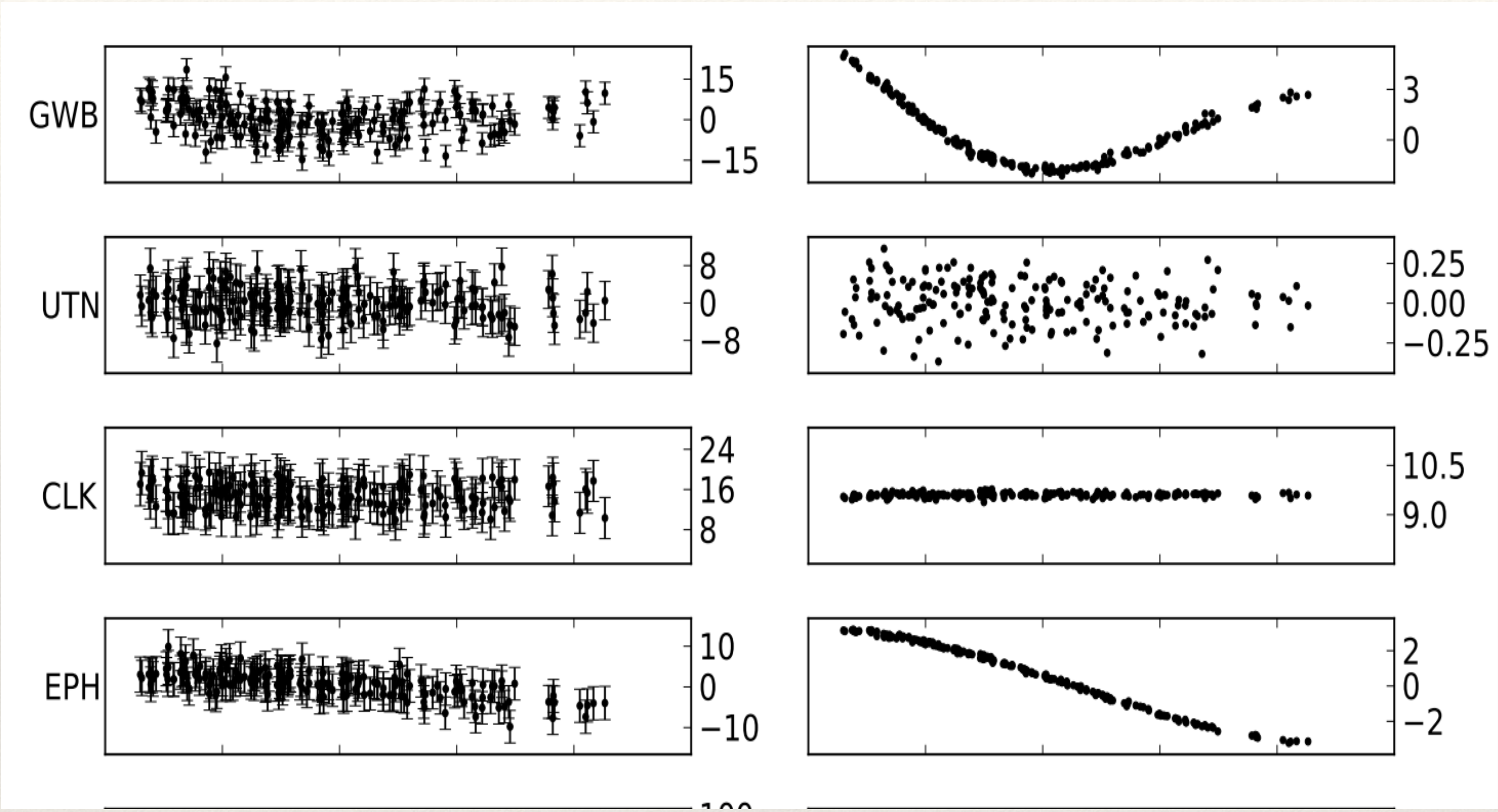
$$S_{DM}(f) \propto \frac{A_{dm}^2}{\nu^2} f^{-\gamma_{dm}}$$

- Correlated red noise processes

$$S_{\alpha\beta} = \Gamma_{\alpha\beta} A_{cor}^2 f^{-\gamma_{cor}}$$

— includes also cross spectrum between each pair of pulsars: $\Gamma_{\alpha\beta}$ - spacial correlation coefficients

Correlated noise



GWB (Quadrupole) Hellings-Downs

Uncorrelated RN

Clock error (monopole)

Ephemeris (dipole)

TOAs, time

Correlation, angular separation

stochastic GW from population of SMBHBs: $S_{\alpha\beta}^{SMBHB} = \Gamma_{\alpha\beta}^{H-D} A_{GW}^2 f^{-13/3}$

Gaussian-process approach to PTA: falling into a rabbit hole

- Applying GP to the PTA likelihood function:

Gaussian prior on weights

GP

$$p(\delta t|w_i, GP) = \frac{e^{-\frac{1}{2}\cdot\sum_{ij}(\delta t_i - \sum_a \phi_a(t_i)w_a)(C^w_{ij})^{-1}(\delta t_j - \sum_a \phi_a(t_j)w_a)}}{\sqrt{(2\pi)^n \det(C^w)}} \times \frac{e^{-\frac{1}{2}\sum_{a,b} w_a(\Sigma_{ab})^{-1}w_b}}{\sqrt{(2\pi)^m \det(\Sigma)}}$$

weight-space approach

white noise

$$p(\delta t|w_i, GP) = \frac{e^{-\frac{1}{2}\cdot\sum_{ij}\delta t_i(C^w_{ij}+C^{rn}_{ij})^{-1}\delta t_j}}{\sqrt{(2\pi)^n \det(C^w + C^{rn})}} \quad \text{with} \quad C^{rn}_{ij} = k(t_i, t_j) = \sum_{a,b}\phi_a(t_i)\Sigma_{ab}\phi_b(t_j)$$

red noise variance-covariance matrix

In time domain, uncorrelated red noise:

$$C^{rn}_{ij} = A^2(f_L/\mathrm{yr}^{-1})\left\{\Gamma(1-\gamma)\sin\left(\frac{\pi\gamma}{2}\right)(f_L\tau_{ij})^{\gamma-1} - \sum_n \frac{(-1)^n(f_L\tau_{ij})^{2n}}{(2n)!(2n+1-\gamma)}\right\}$$

where $\tau_{ij} = |t_i - t_j|$ and f_L is low freq. cut-off

Gaussian-process approach to PTA: falling into a rabbit h

- Alternatively we can use basis functions: based on the decomposition of residuals in the Fourier modes:

$$\delta t(t_i) \approx \sum_k a_k \sin 2\pi f t_i + b_k \cos 2\pi f t_i$$

weights

basis functions $\phi^F(f_a, t_i) = \phi_a^F(t_i)$

We use non-complete set of Fourier modes: covariance matrix can be approximated as

$$C_{ij}^{rn} \approx \sum_{a,b} \phi_a^F(t_i) \Sigma_{ab}^F \phi_b^F(t_j)$$ where

$$\Sigma_{ab}^F \propto \left(A_{rn}^2 f_a^{-\gamma}\right) \delta_{ab}/T$$ — red noise PSD

and for stochastic GW signal: $C_{i\alpha,j\beta}^{GW} = \sum_{i\alpha,j\beta} \Gamma_{\alpha\beta} \phi_a^F(t_{i\alpha}) \Sigma_{ab}^{F,GW} \phi_b^F(t_{j\beta})$, where

$$\Sigma_{ab}^{F,GW} = (A_{GW}^2 f_a^{-\gamma_{gw}}) \delta_{ab}/T$$

Gaussian-process approach to PTA: falling into a rabbit hole

Advantage of this description: again likelihood

$$p(\delta t|w_i, GP) = \frac{e^{-\frac{1}{2}\cdot\sum_{ij}\delta t_i (C^w_{ij}+C^{rn}_{ij})^{-1}\delta t_j}}{\sqrt{(2\pi)^n \det(C^w + C^{rn})}}$$

Data size: n - large, need to invert very large (covariance) matrices - $n \times n$
Can use Woodbury f-la

$$(C_w + C_{rn})^{-1} = (C_w + \Phi\Sigma\Phi^T)^{-1} = C_w^{-1} - C_w^{-1}\Phi\underbrace{\left(\Sigma^{-1} + \Phi^T C_w^{-1}\Phi\right)^{-1}}_{\text{inversion of } m\times m \text{ matrix}}\Phi^T C_w^{-1}$$

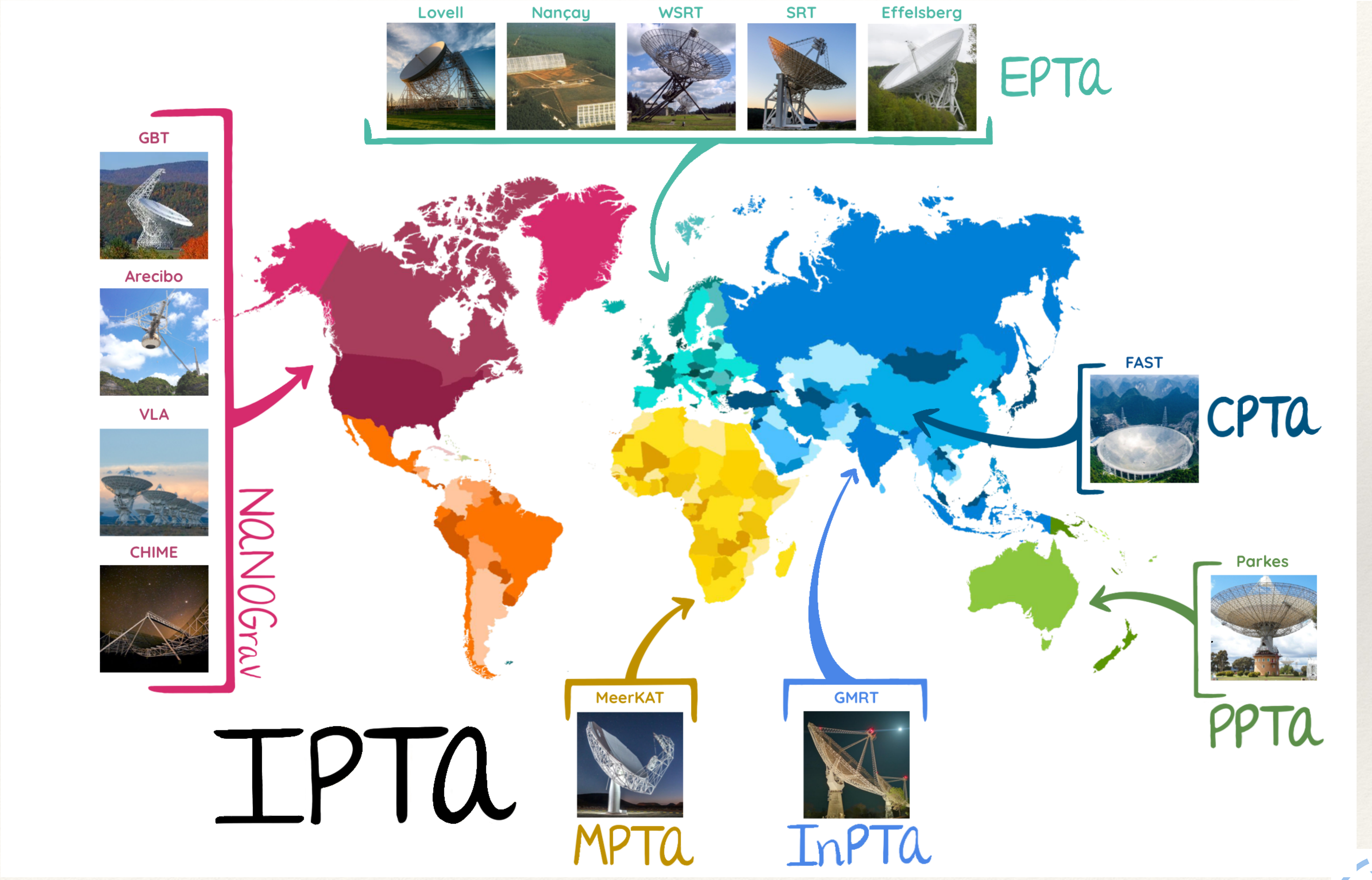
Number of modes: $m \ll n$ much faster and easier to invert, C_w is diagonal matrix

Bayesian analysis: model selection (hypothesis testing)

Odd ratio: $$O(M_1, M_2) = \frac{p(M_1|d)}{p(M_2|d)} = \frac{p(d|M_1)}{p(d|M_2)}\frac{\pi(M_1)}{\pi(M_2)}$$

Bayes factor

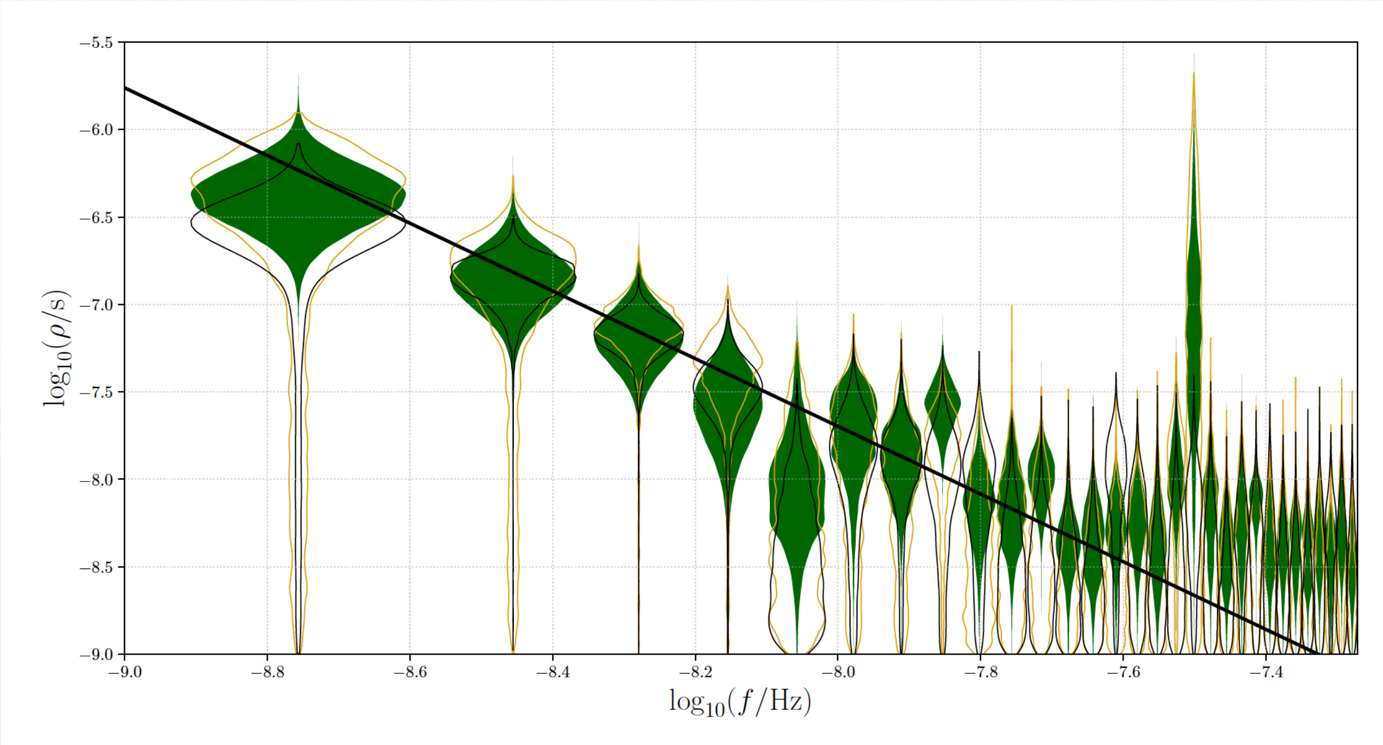
IPTA

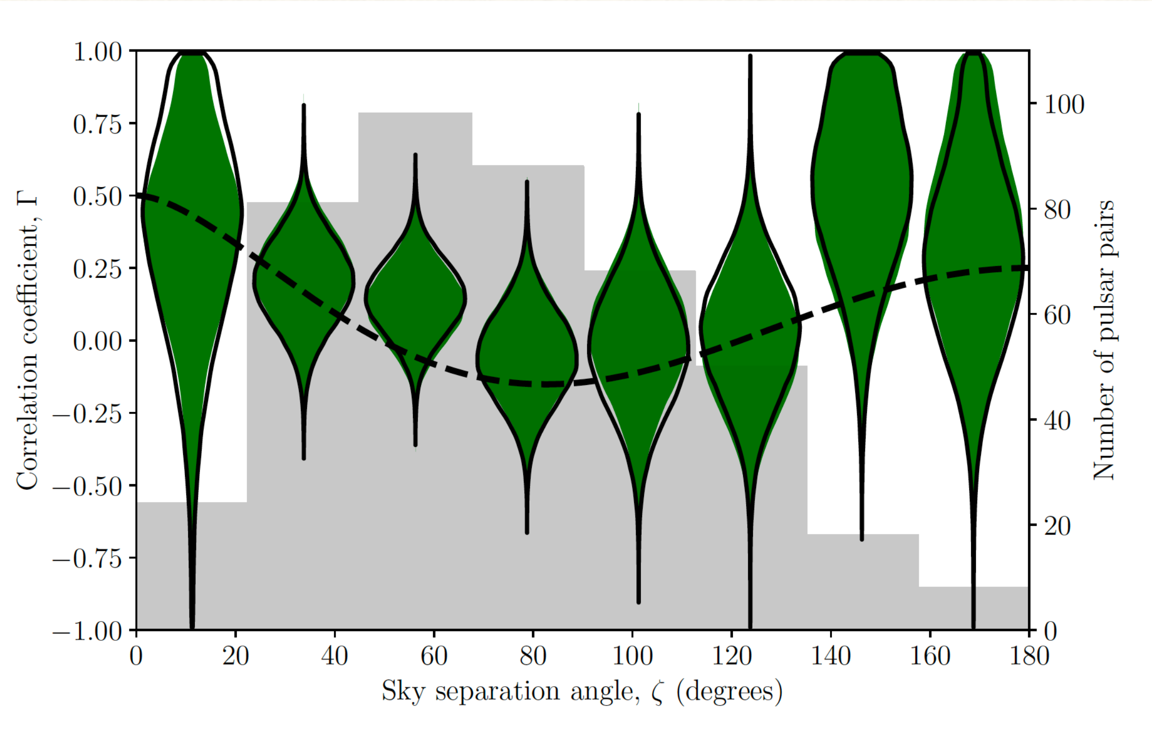

Lovell
Nançay
WSRT
SRT
Effelsberg
EPTA
GBT
Arecibo
VLA
CHIME
NANOGrav
FAST
CPTA
Parkes
PPTA
MeerKAT
MPTA
GMRT
InPTA
IPTA

PPTA results

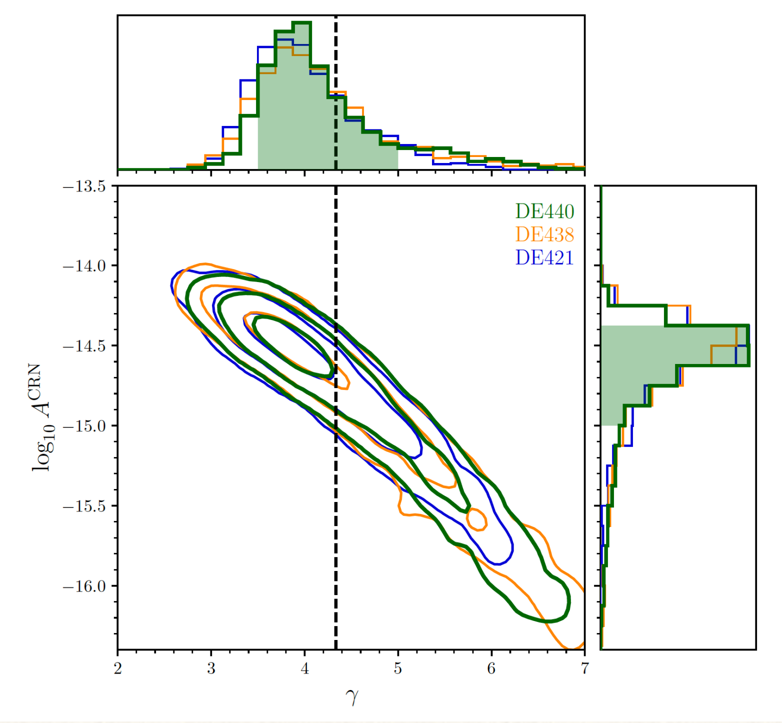
[PPTA 2306.16215]

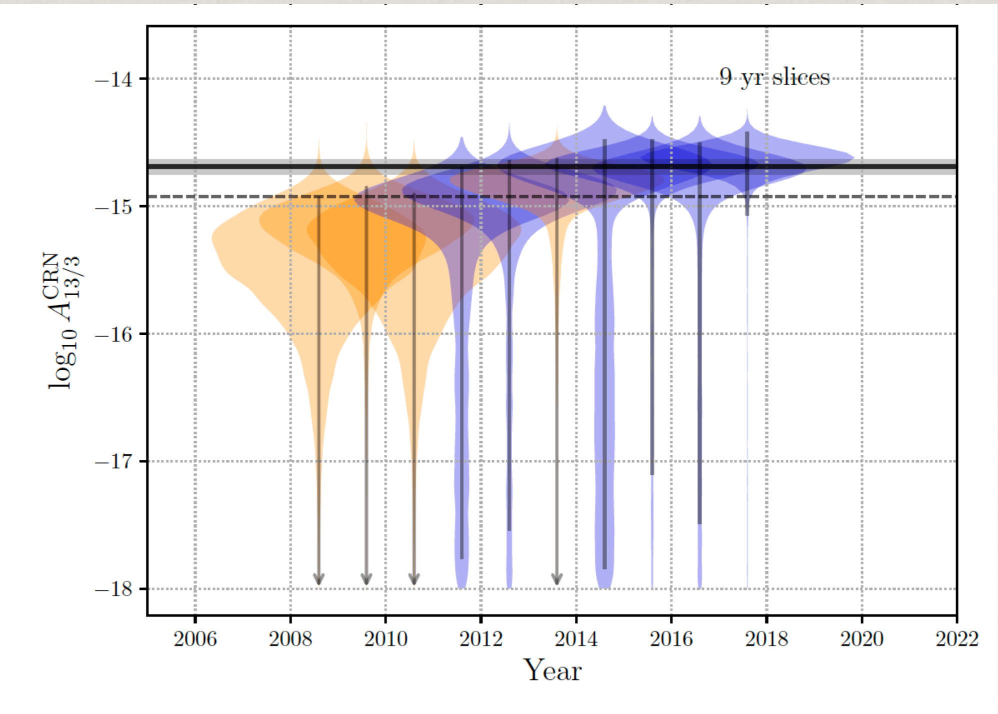
PPTA data: 18 years, 30 pulsars. 3 years of new ultra-widebandwidth radio observations





Estimating power at Fourier freq. (assuming independence).
Black: CURN, Gold: H-D



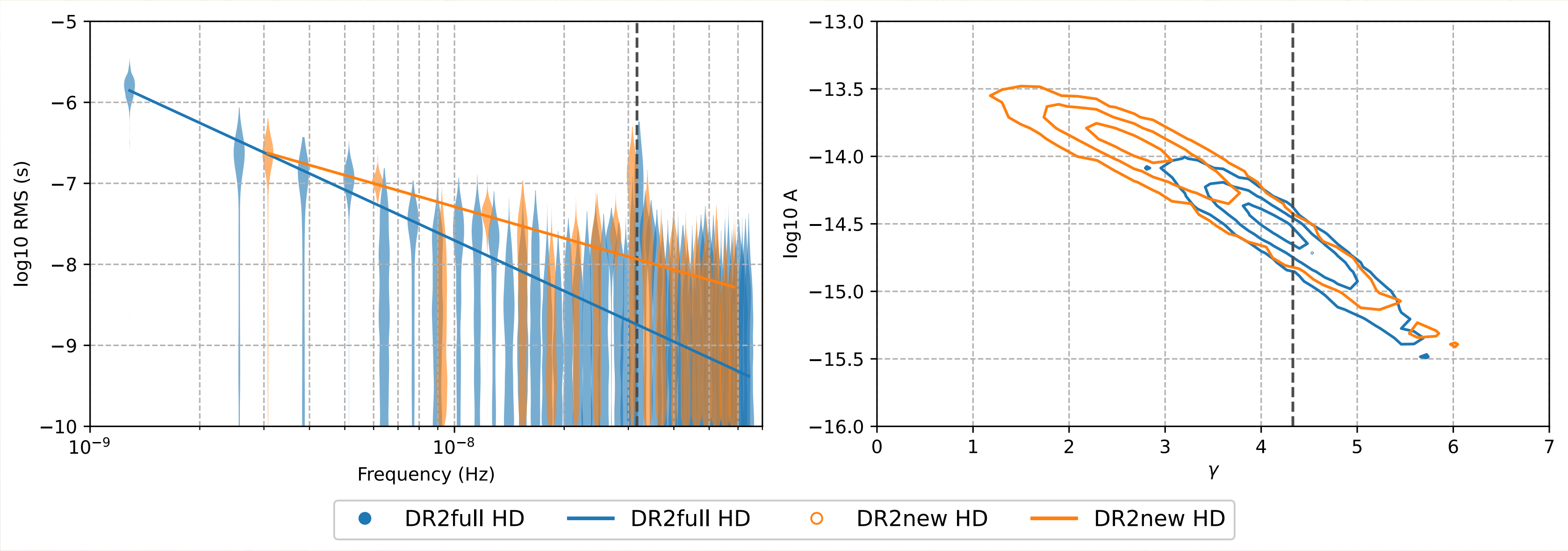


$$S^{SGWB}_{\alpha\beta} = \Gamma^{H-D}_{\alpha\beta} A^2_{GW} f^{-\gamma}$$

EPTA + InPTA

[EPTA+InPTA2306.16214]

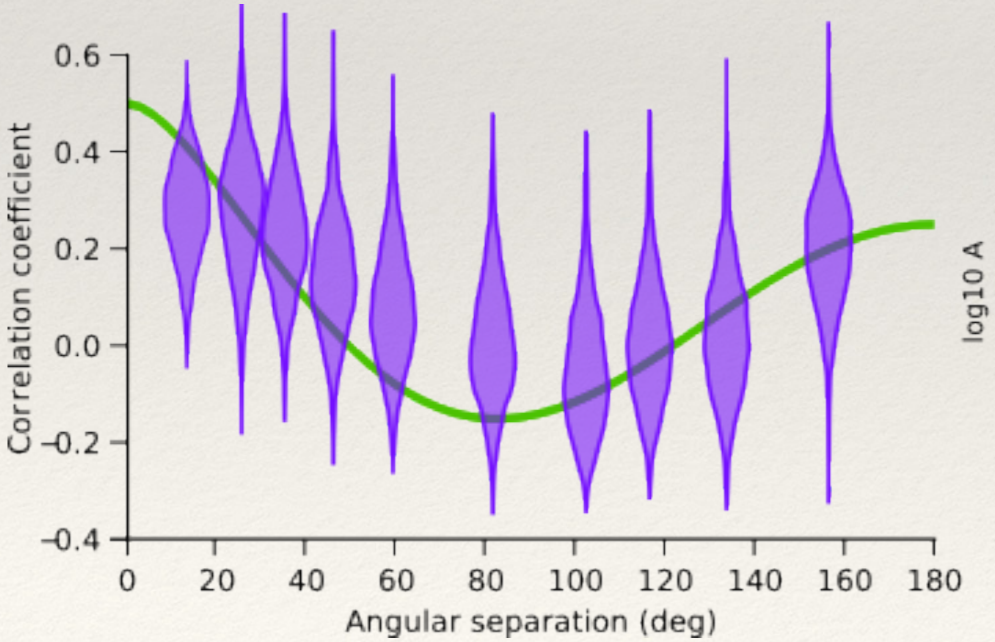
25 plsrs, DR2full: up to 25yrs, DR2new: latest 14 yrs, DR2new+ Includes InPTA data (3.5 yrs)

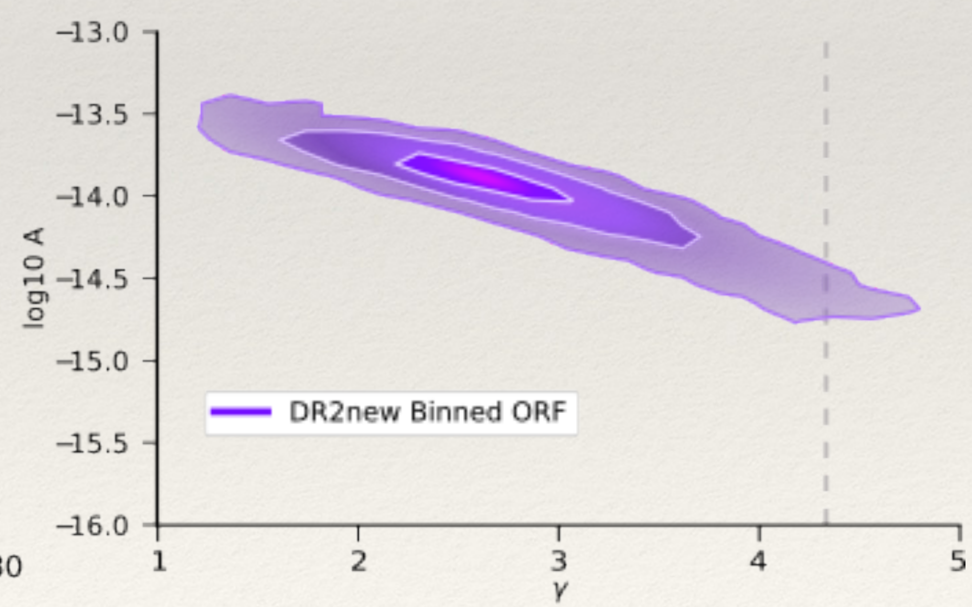


$$S_{\alpha\beta}^{SGWB} = \Gamma_{\alpha\beta}^{H-D} A_{GW}^2 f^{-\gamma}$$

Statistic	DR2full	DR2new	DRnew+
$\mathscr{B}^{HD}_{CURN}$	4	60 33(+Ephem)	65 43 (+Ephem)

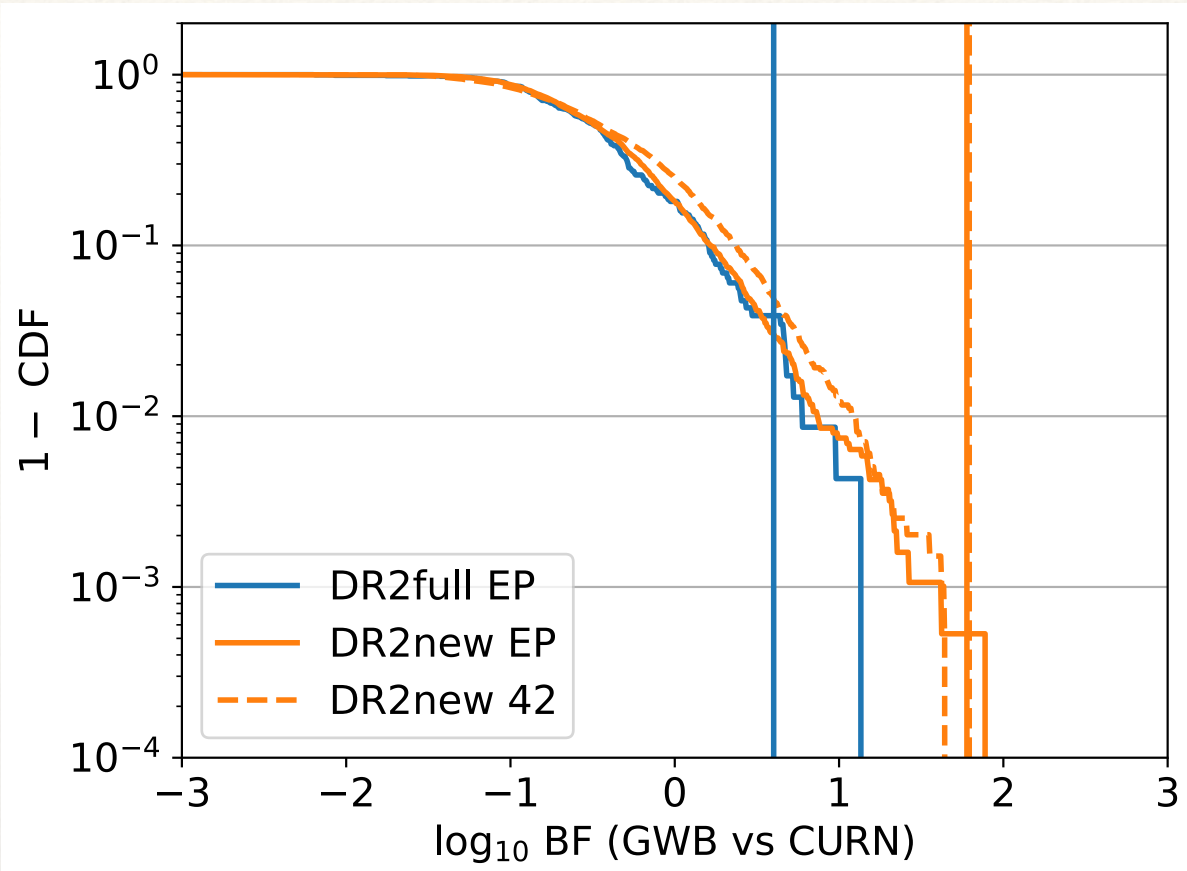
DR2new results: spatial correlations and amplitude-slope of power-law model

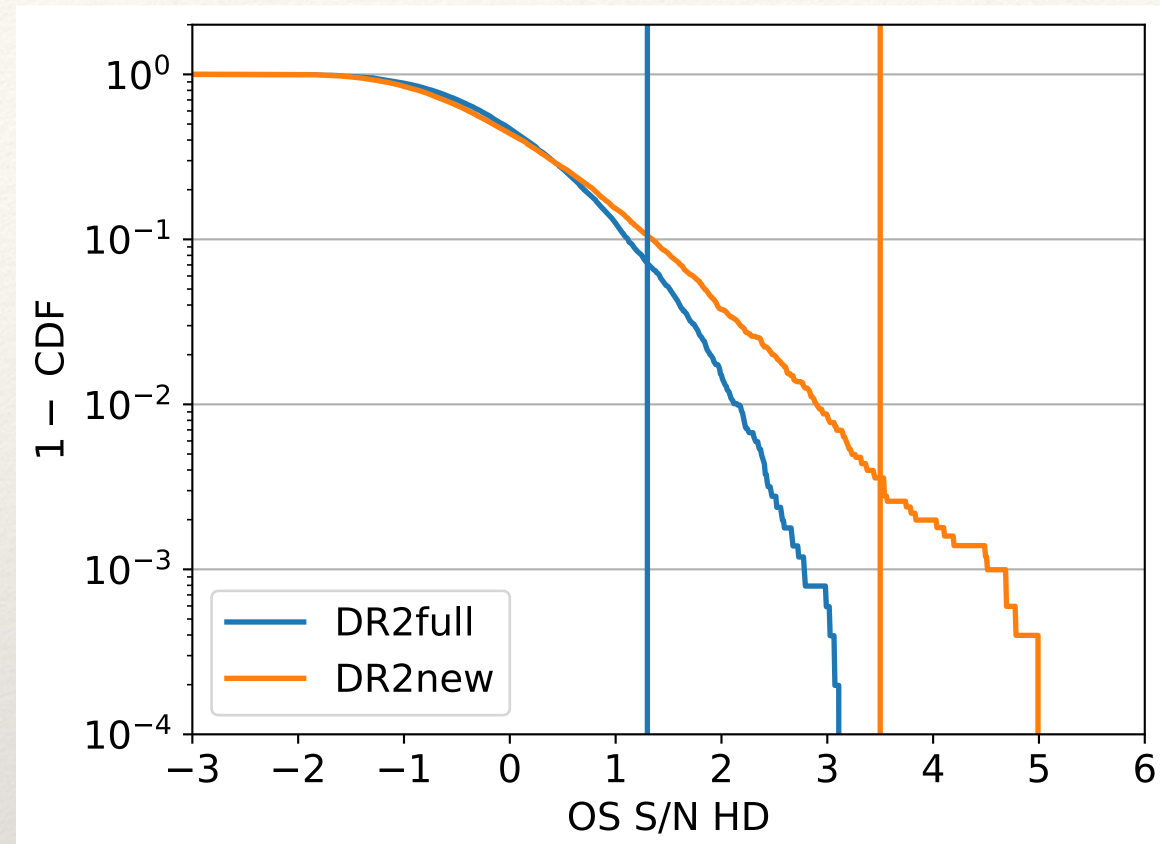




EPTA + InPTA

Significance: how likely to observe what we observe in absence (null hypothesis) of GW signal





We want [Cornish & Sampson 2015, Taylor+ 2016]

- Preserve properties of the noise (use observations)
- Data free of GW signal: not possible, instead we try to mimick measurements insensitive to GWs
  - Sky shufling: change position of pulsars: observed correlation is not consistent with GW
  - Phase shift: introduce a random shift in phase at each frequency bin: destroy correlations

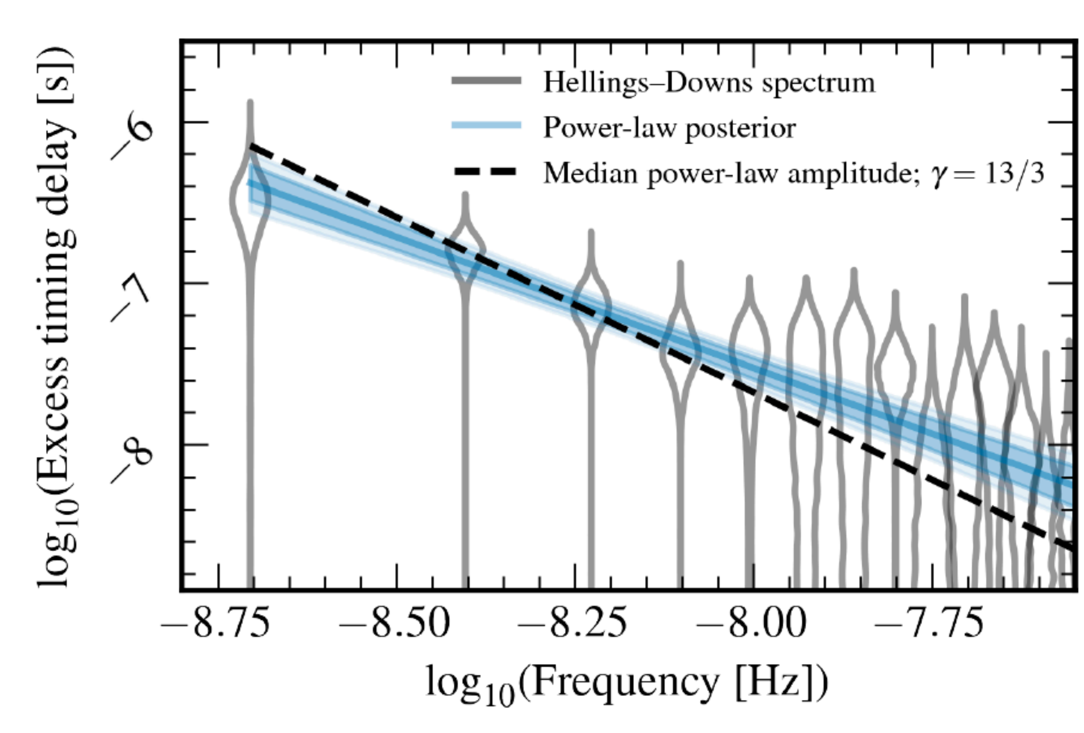
The question we are asking:

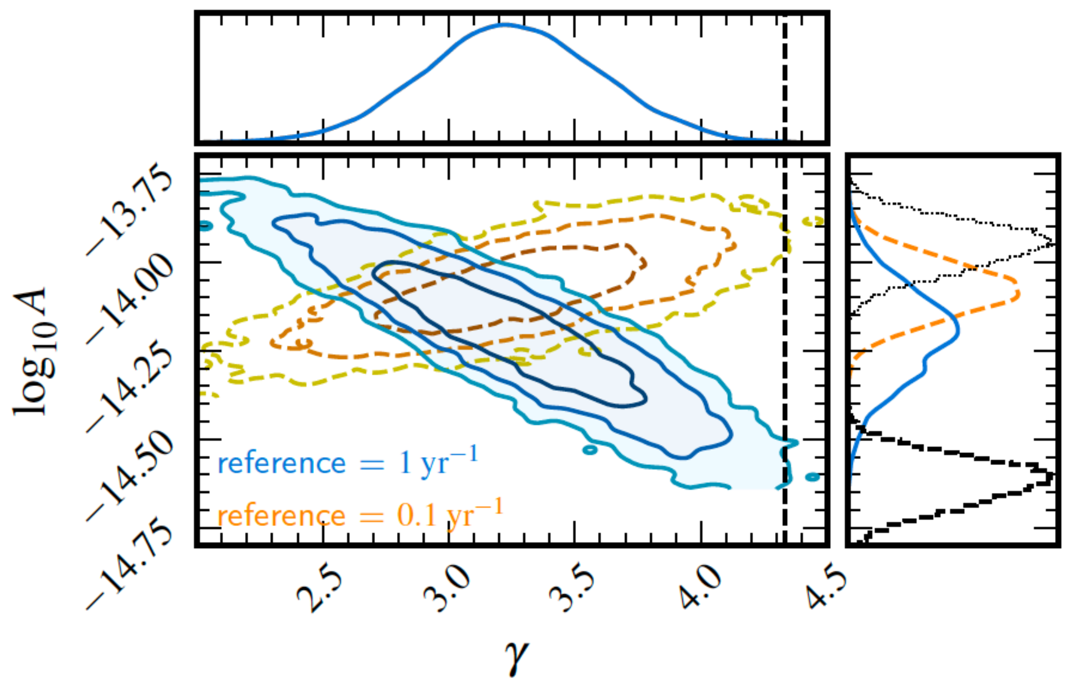
- how likely to get observed H-D pattern by randomly choosing pulsars on the sky
- How likely that the phases at low frequencies in all pulsars align to form observed H-D

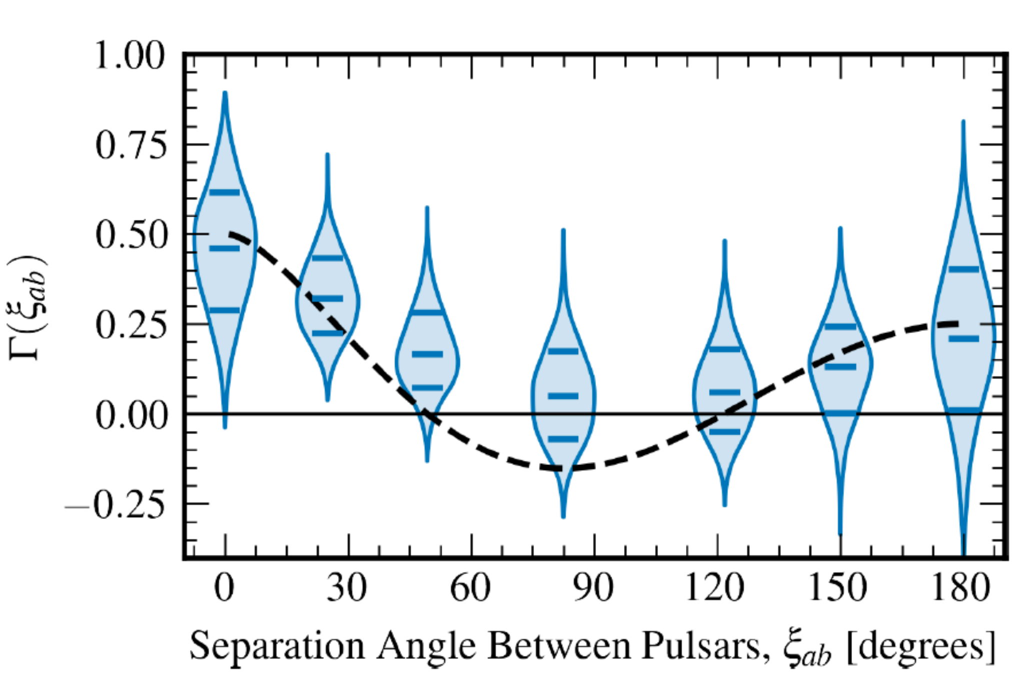
NanoGrav results

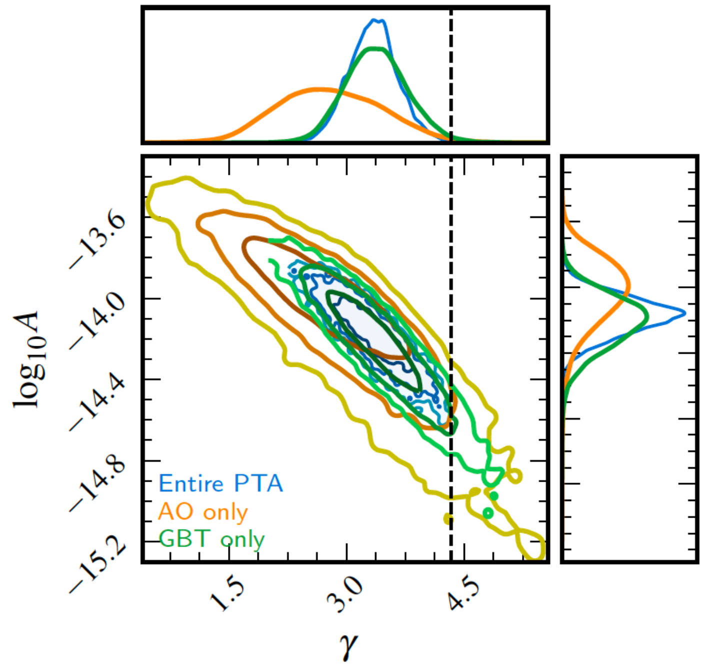
[NG 2306.16213]

NG data: 15 years of data, 67 pulsars. Arecibo + Green Bank



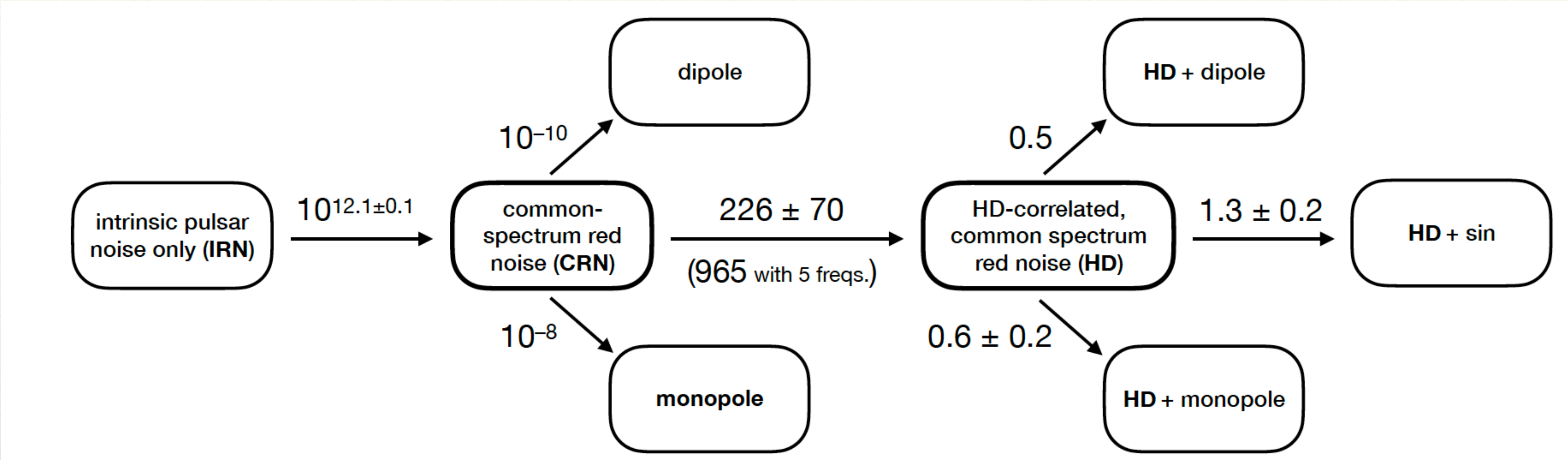




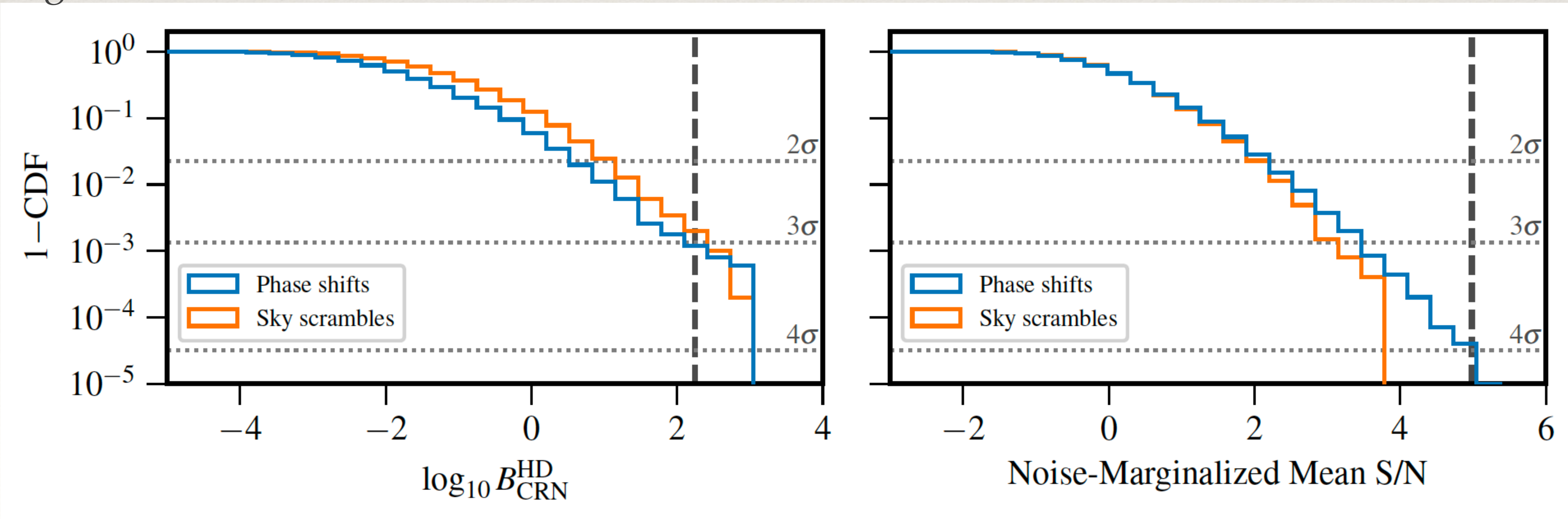


$$S_{\alpha\beta}^{SGWB} = \Gamma_{\alpha\beta}^{H-D} A_{GW}^2 f^{-\gamma}$$

NanoGrav results



Significance



Interpretation

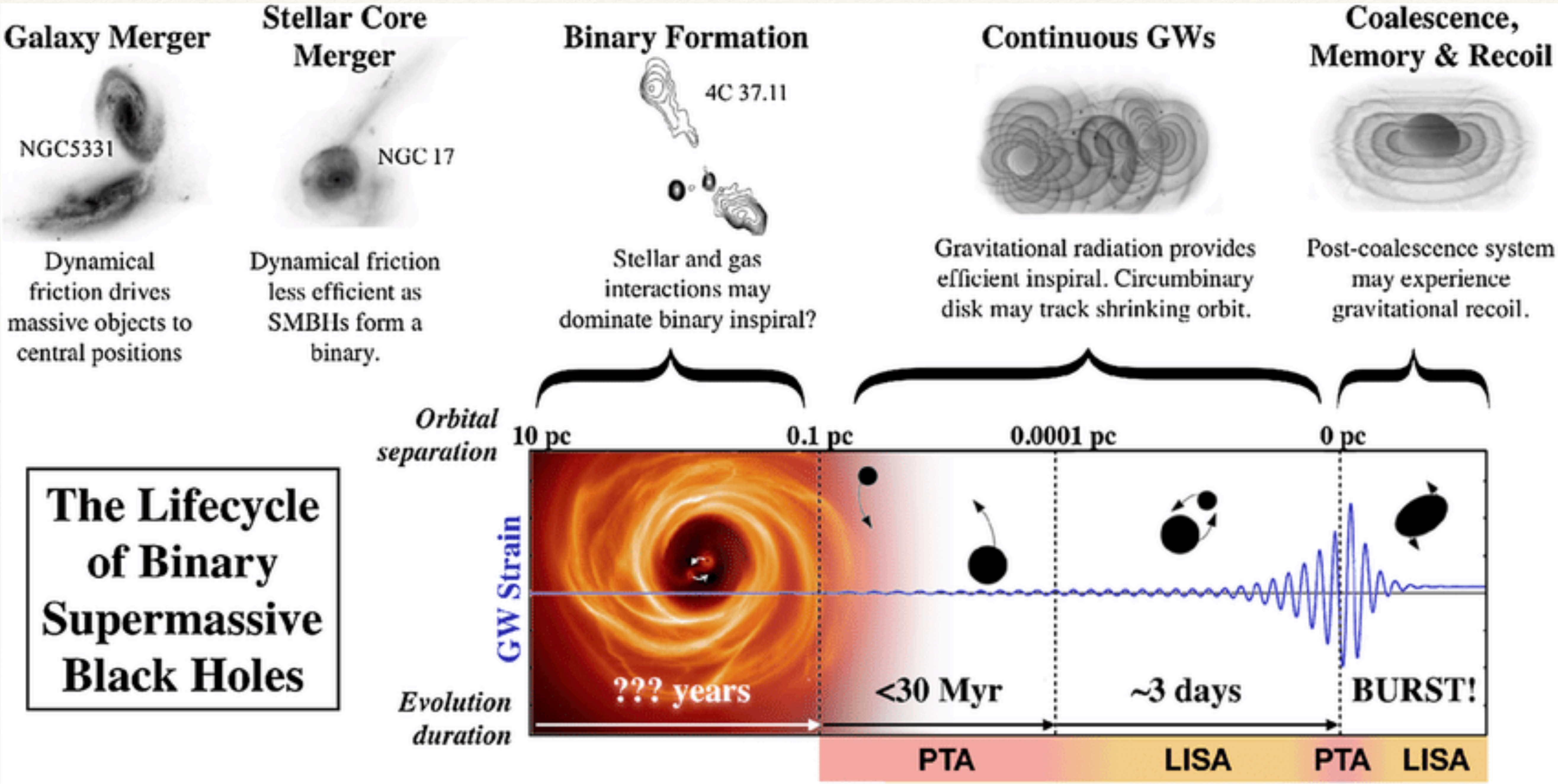
LET US ASSUME THAT WHAT WE OBSERVE IS STOCHASTIC GW BACKGROUND (SGWB)

What could produce SGWB with the power-law-like spectrum?
Apparently almost anything that falls in nHz band… and even more
I'll give only few examples

DISCLAIMER

preference in interpretation of observed signal and its siginficance: my personal view

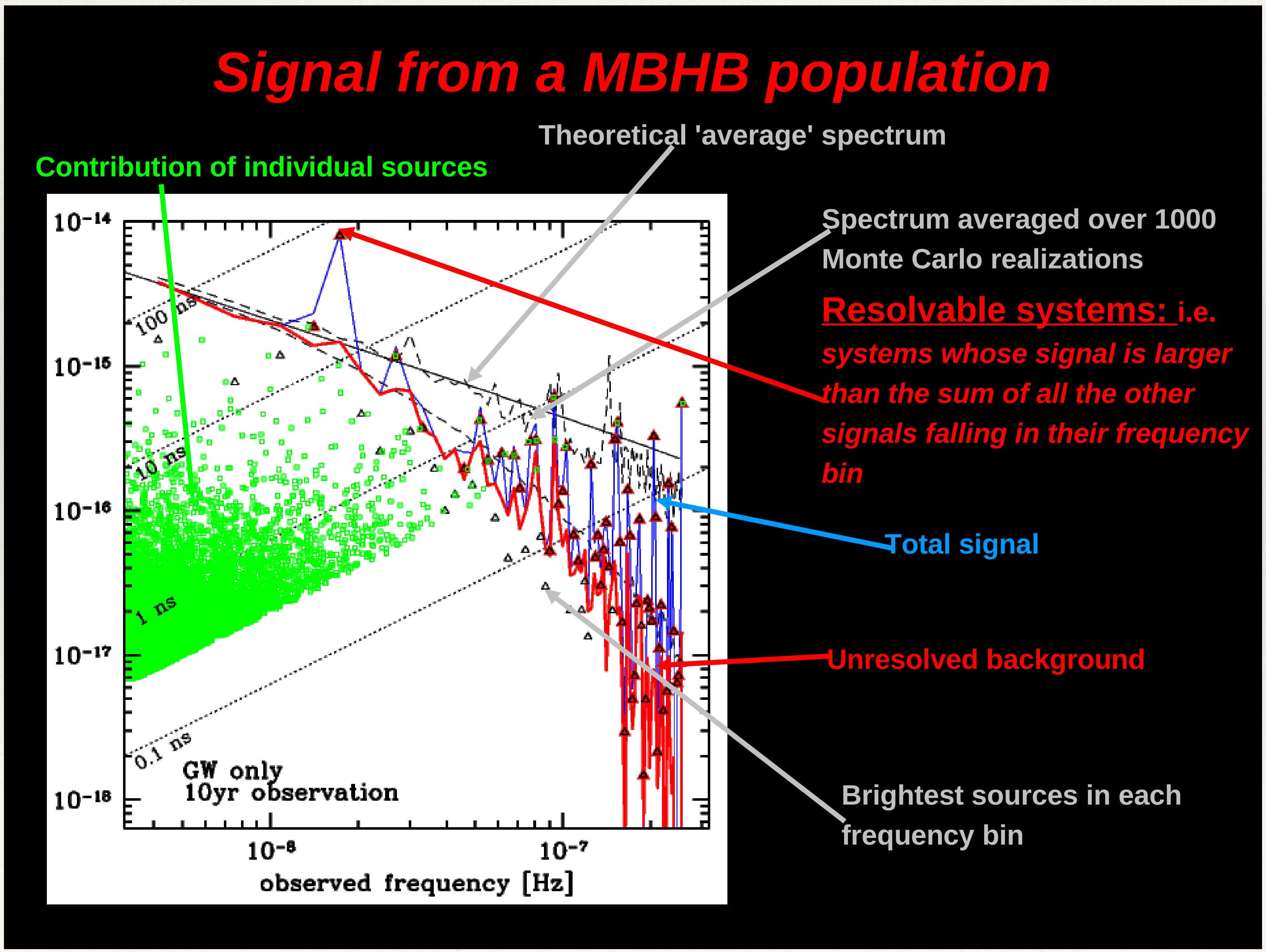
Massive black hole binaries



[S. Burke-Splolaor A&A review (2019)]

Supermassive black hole binaries

- Main sources are supermassive black hole binaries (mass 10^7 — 10^{10} solar) on very broad orbit (period ~ year(s))
- The orbital evolution due to GW emission is very slow: $\frac{dE}{dt} \propto \eta (M/r)^5$ signal is (almost) monochromatic over period of observations

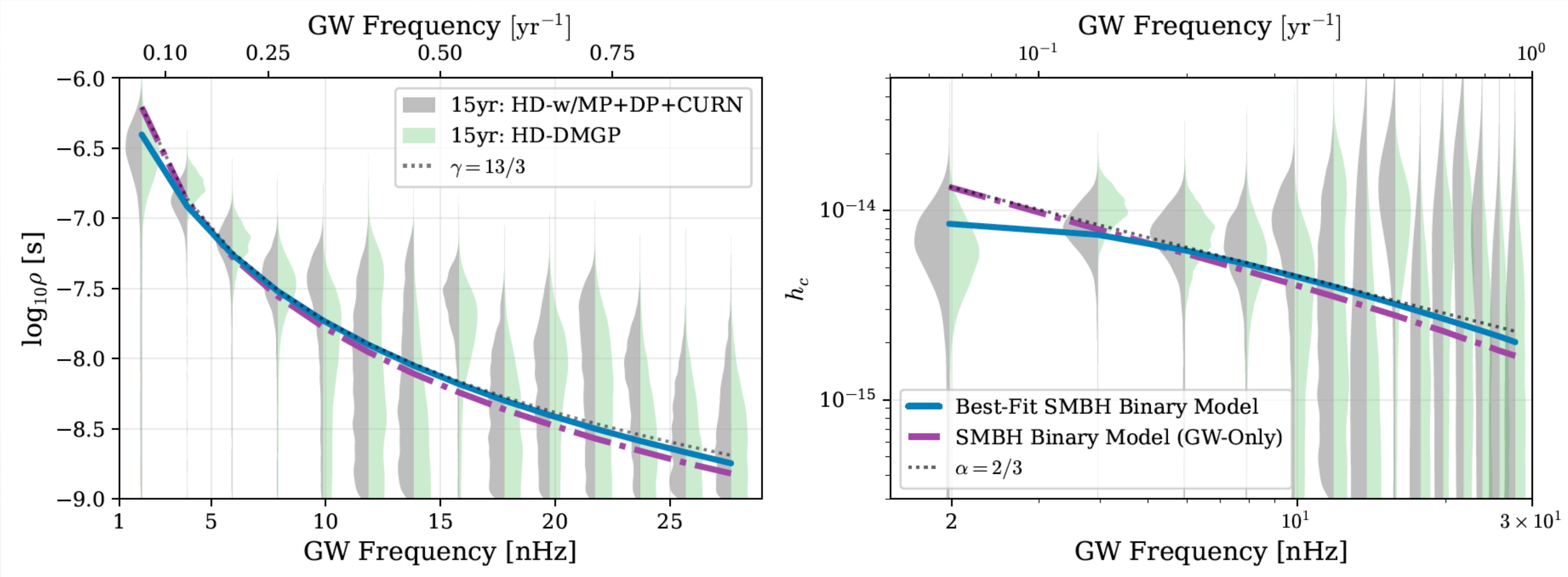


GW signal from the population of SMBH binaries: forms a stochastic signal at low freqs. (similar to Galactic binaries in LISA

[Credits: A. Sesana]

SGWB from population of SMBHBs

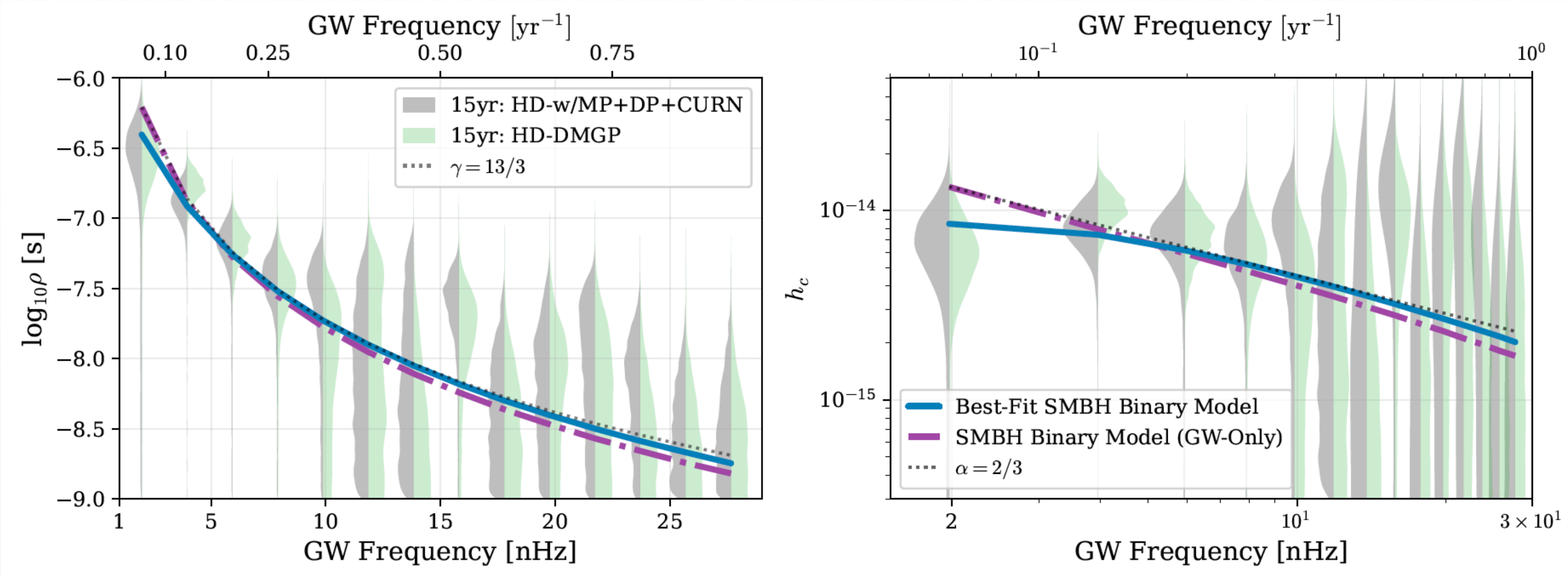
[NG: 2306.16220]



- Free spectrum (HD) : estimation of the ASD for the H-D correlated part of the noise
- SMBHB population: circular orbit + GW driven evolution: $\rho \propto f^{-13/3}$, $h_c \propto f^{-2/3}$ (black dots)
- Eccentric orbits: redistribution of GW energy towards higher modes (higher frequencies): lower amplitude, turn-over at low freq.
- Interaction with environment: dissipation of energy and angular momentum: turn-over at low freq.

SGWB from population of SMBHBs

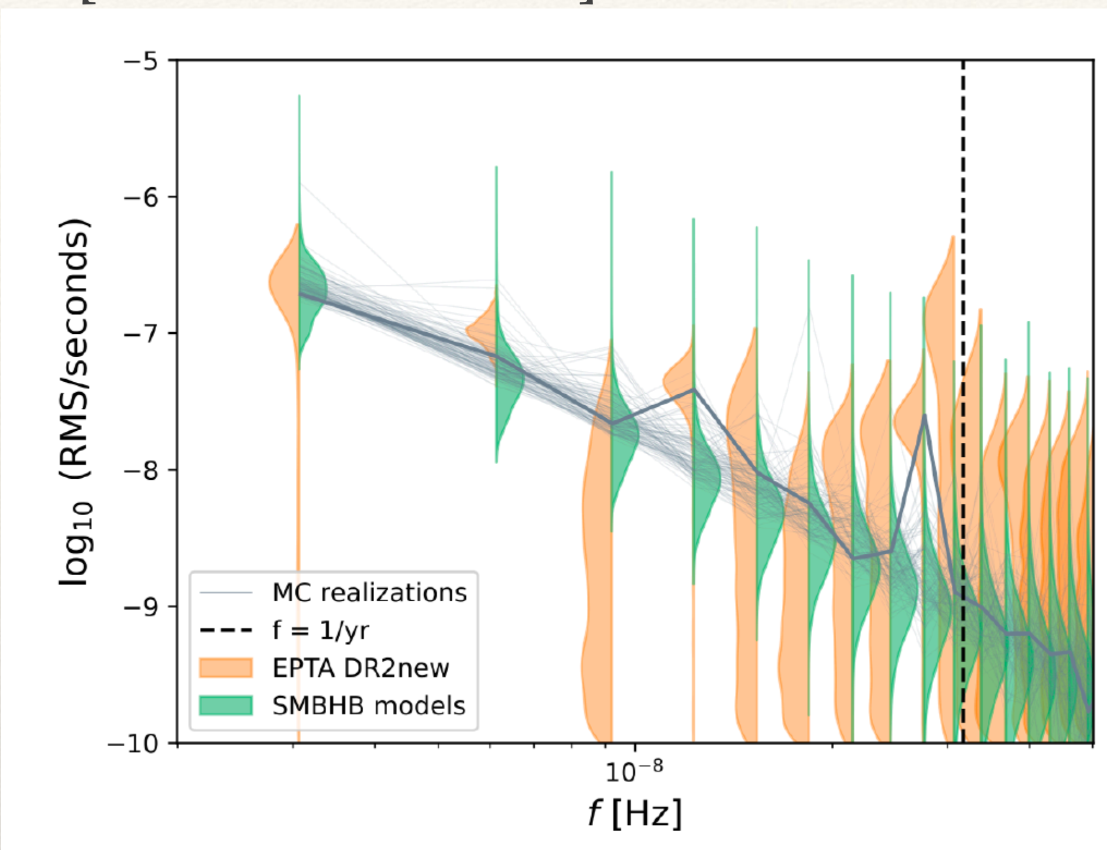
[NG: 2306.16220]

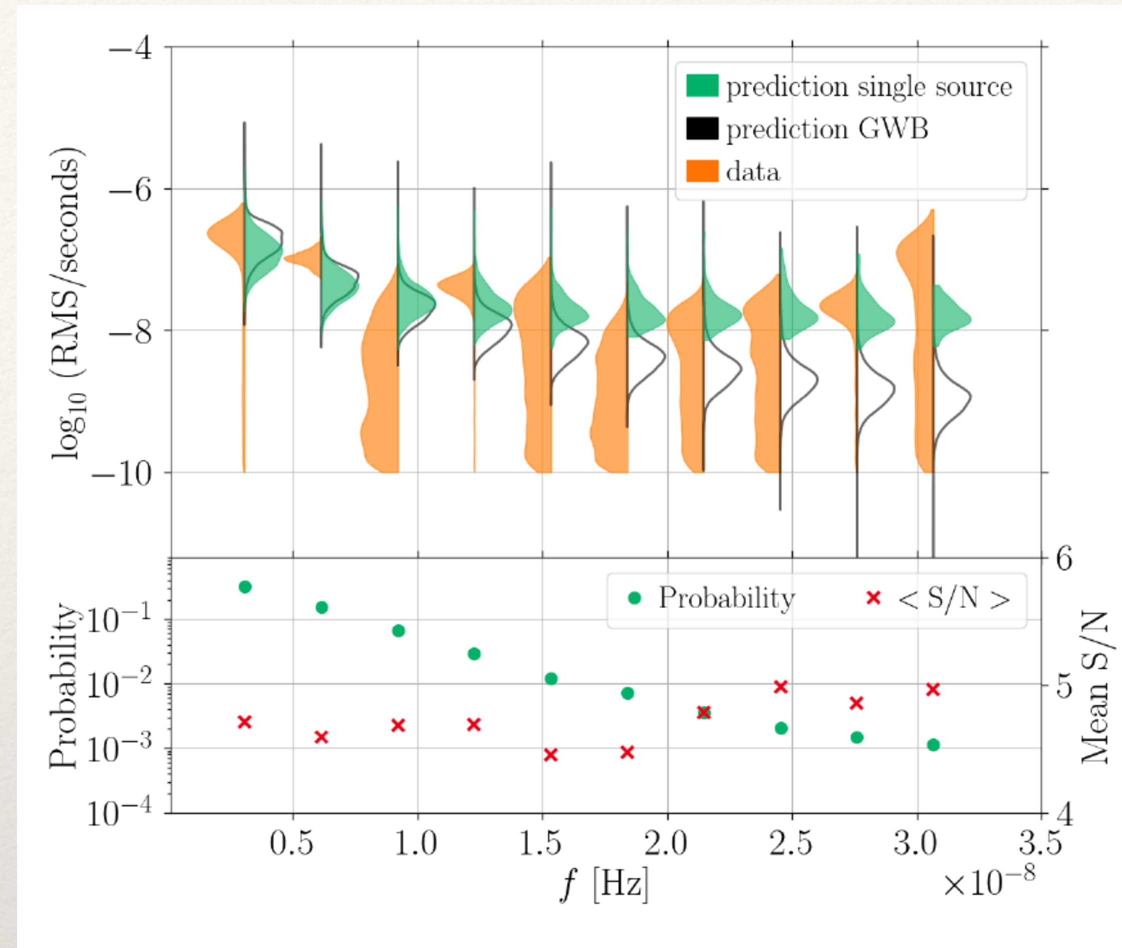


Blue: uses self-consistent SMBH binary evolution model (slightly preferred)
Purple: GW-only driven evolution model

SGWB from population of SMBHBs

[EPTA 2306.16227]





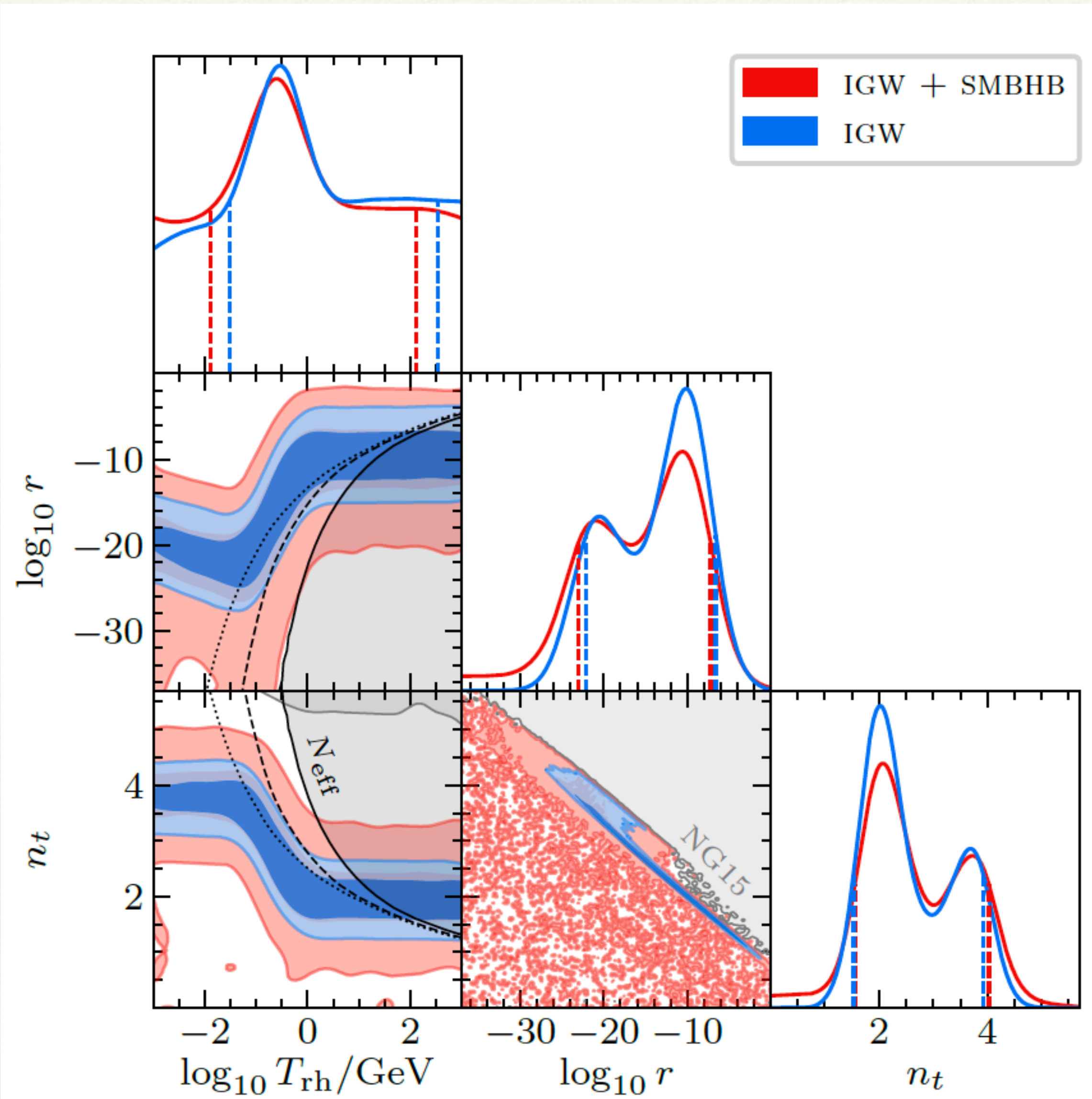
Astrophysically-informed SMBHB population model: interaction with environment, allows eccentric orbits (similar to "blue (phenom) model" of NG)

- Constrains SMBHB merger timescale
- Constrains SMBH-buldge mass relation

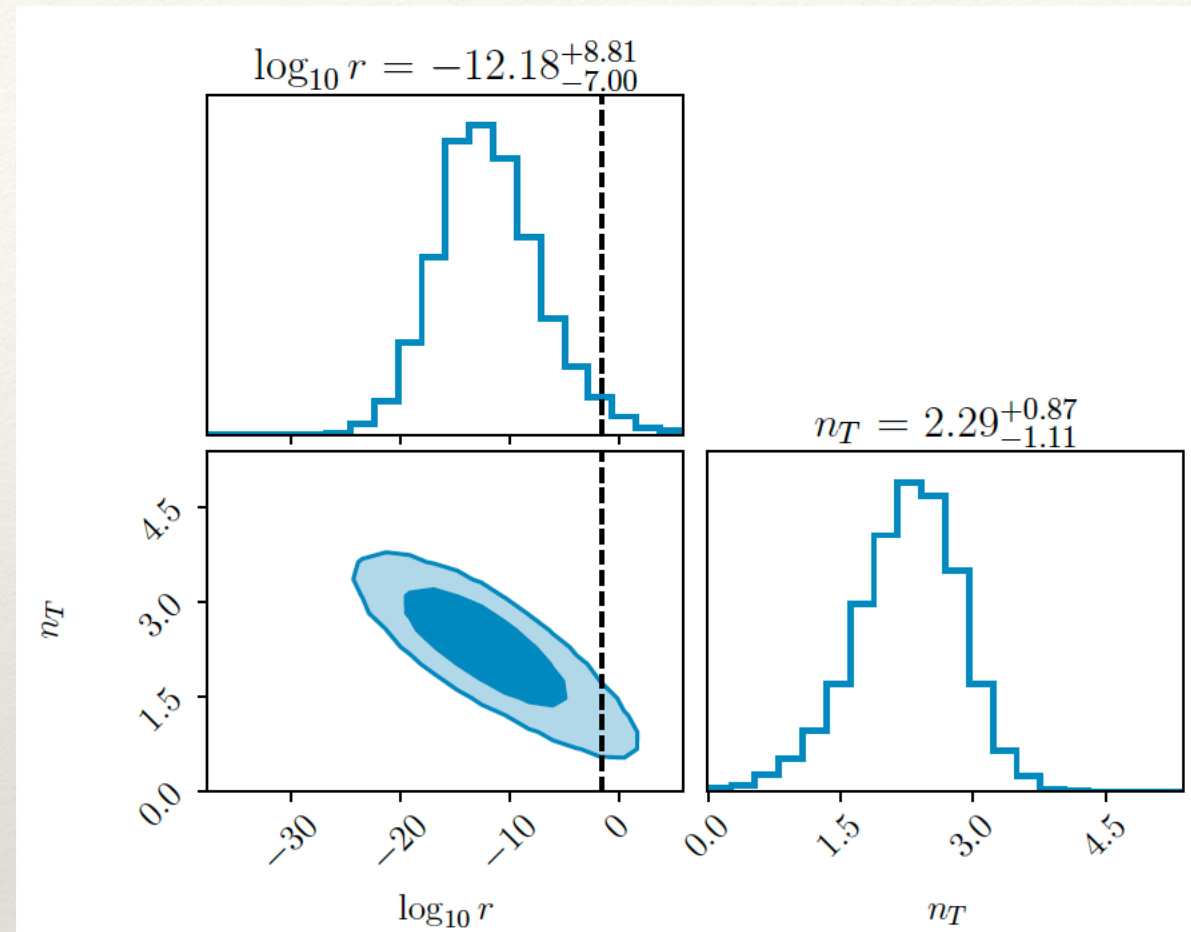
Both indicate efficient orbital decay

Relic SGWB

[NG, 2306.16219]



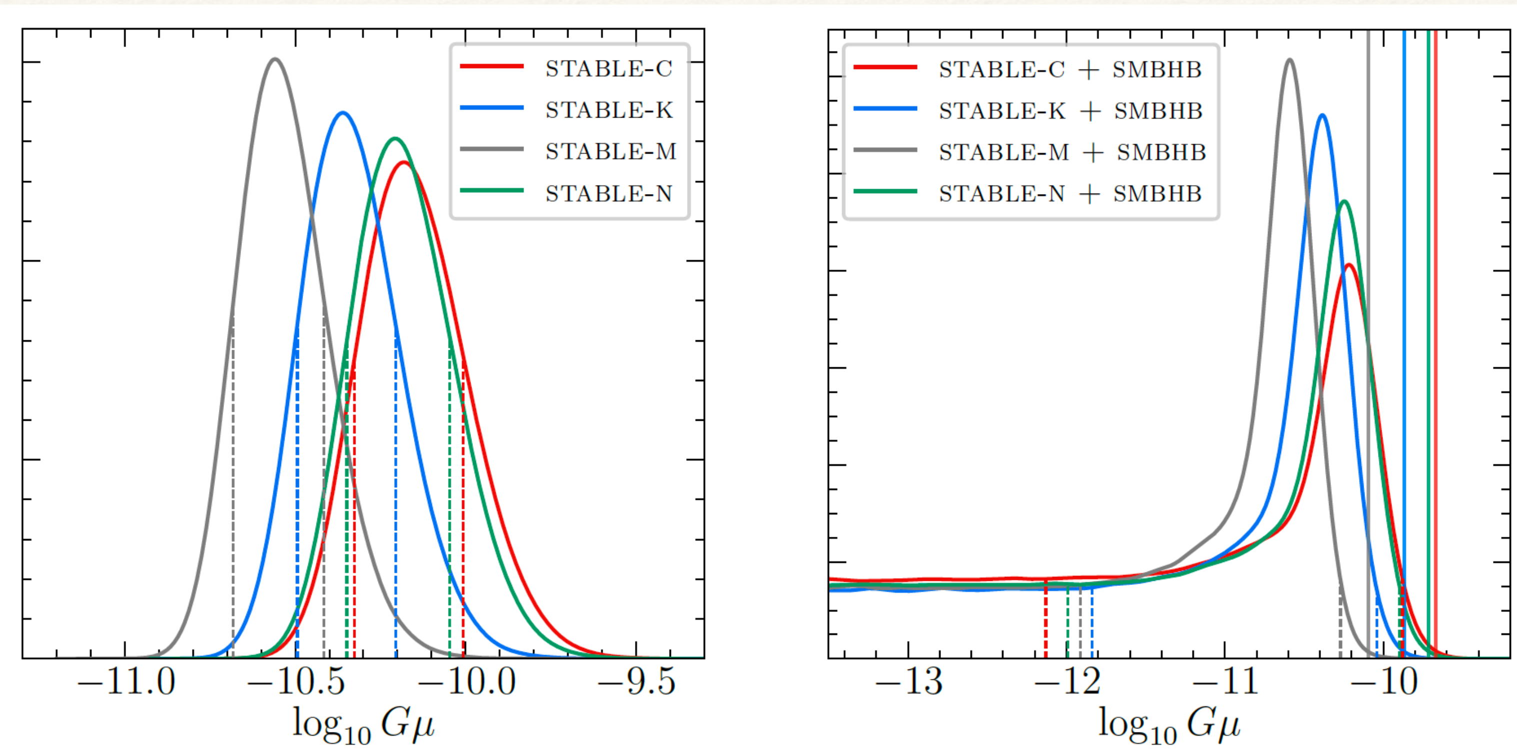
[EPTA, 2306.16227]



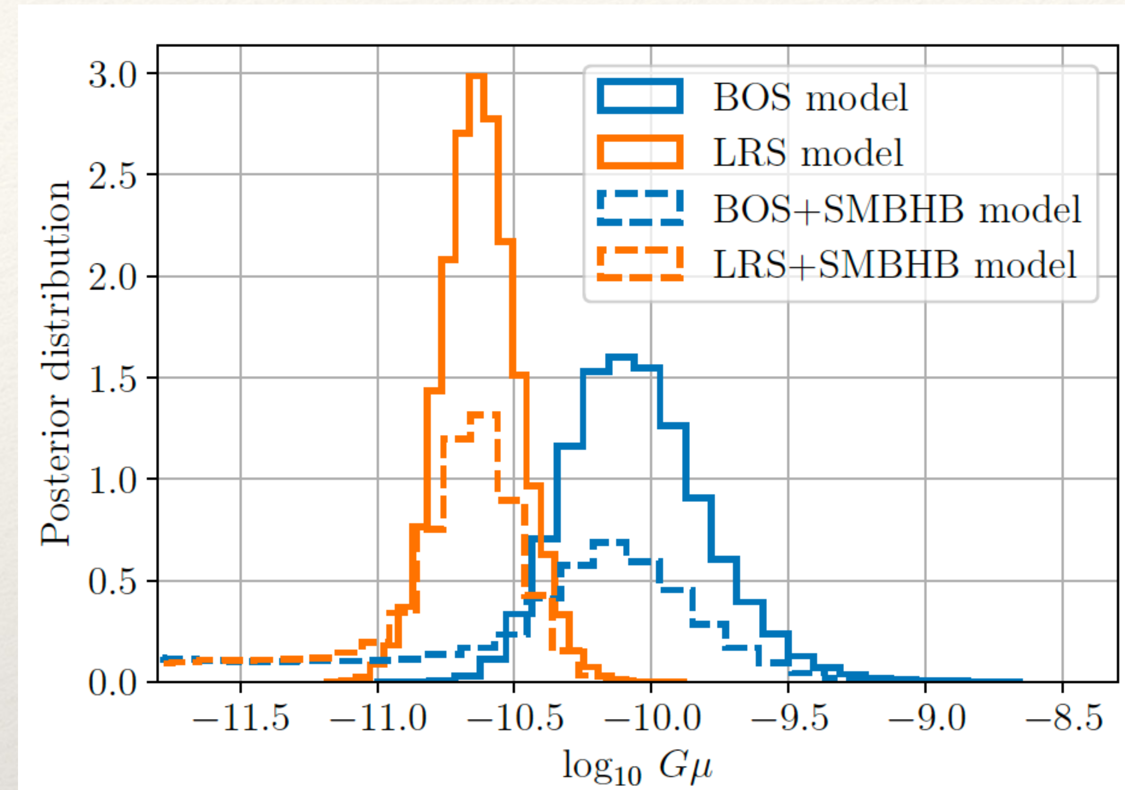
- agnostic about the microphysics of inflation and restrict ourselves to a model-independent analysis.
- the tensor-to-scalar ratio r and tensor spectral index n_t at the CMB pivot scale, reheating temperature T_{rh}

SGWB: Network of cosmic strings

[NG, 2306.16219]

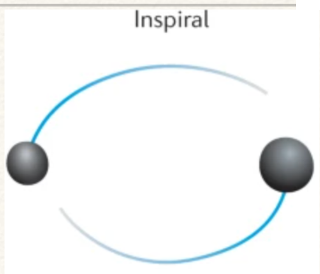


[EPTA+InPTA, 2306.16227]



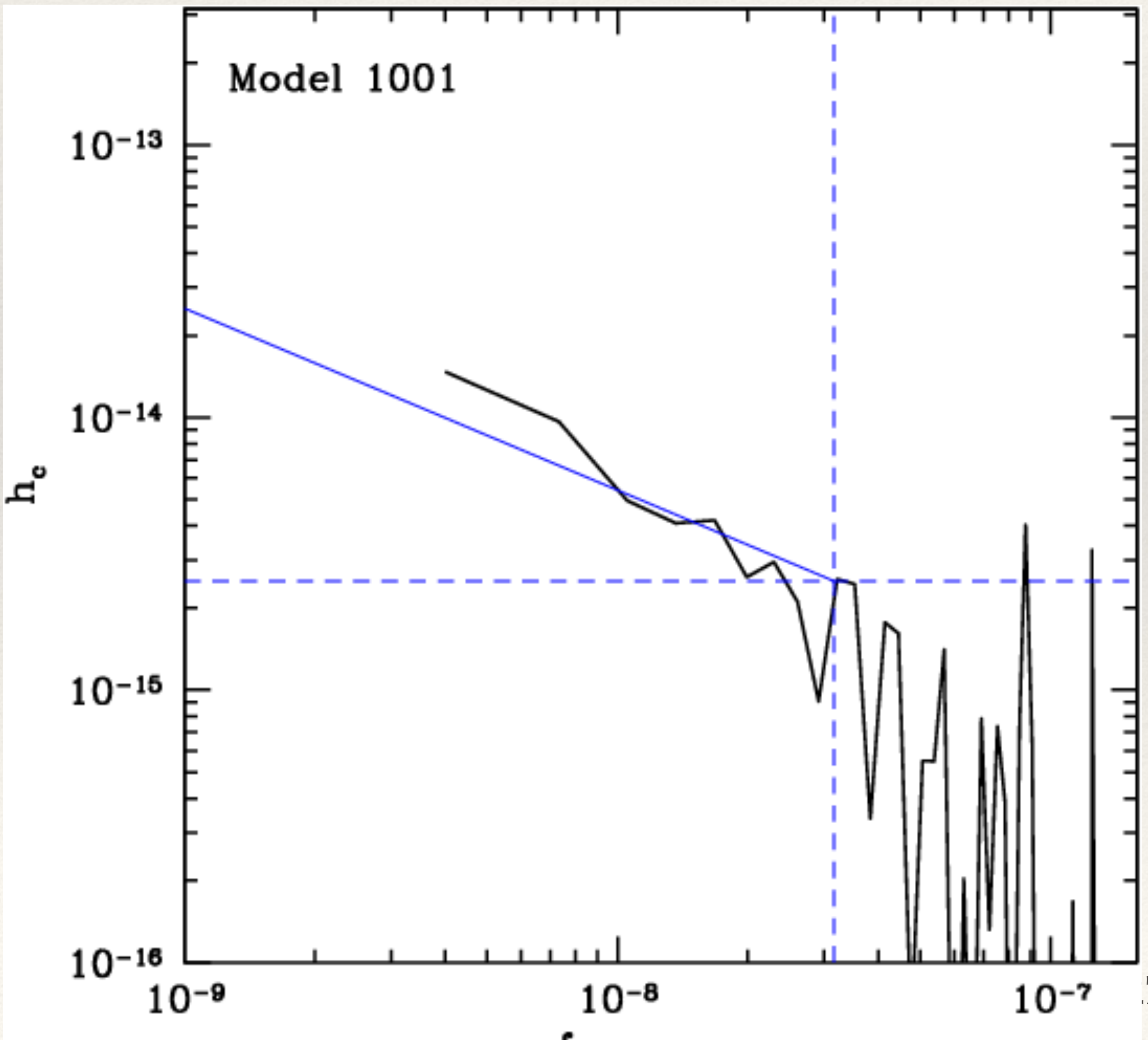
- We can constrain the tension of cosmic strings (model dependent) assuming the observed signal is entirely produced by the network of cosmic strings
- We can set un upper limit in two-component model of SGWB: CSs + SMBHBs

Search for individual MBHBs: continuous GW signal



Searching for GW signal from individual SMBHB binary:

- Assume circular orbit
- Bayesian approach
- Strategy: all-sky search with simplistic model -> follow up candidates relaxing simplified assumptions on the reduced prior range



[NG: 2306.16222]

[EPTA+InPTA: 2306.16226]

CGW signal

Consider non-spinning SMBH binary in circular orbit

- pulsar and earth terms: each is monochromatic signal
- frequency. of pulsar term might or might not coincise with the erath term: $t_p = t - L(1 + \hat{n}\,.\,\hat{k})$
- amplitude of the pulsar term is larger: $\sim \omega^{-1/3}$

$$s_\alpha = F_\alpha^+(\hat{k}, \hat{n}_\alpha)\left[\frac{h_+(t_p^\alpha, \omega_\alpha)}{2\pi f_\alpha} - \frac{h_+(t, \omega)}{2\pi f}\right] + F_\alpha^\times(\hat{k}, \hat{n}_\alpha)\left[\frac{h_\times(t_p^\alpha, \omega_\alpha)}{2\pi f_\alpha} - \frac{h_\times(t, \omega)}{2\pi f}\right]$$

α - pulsar index

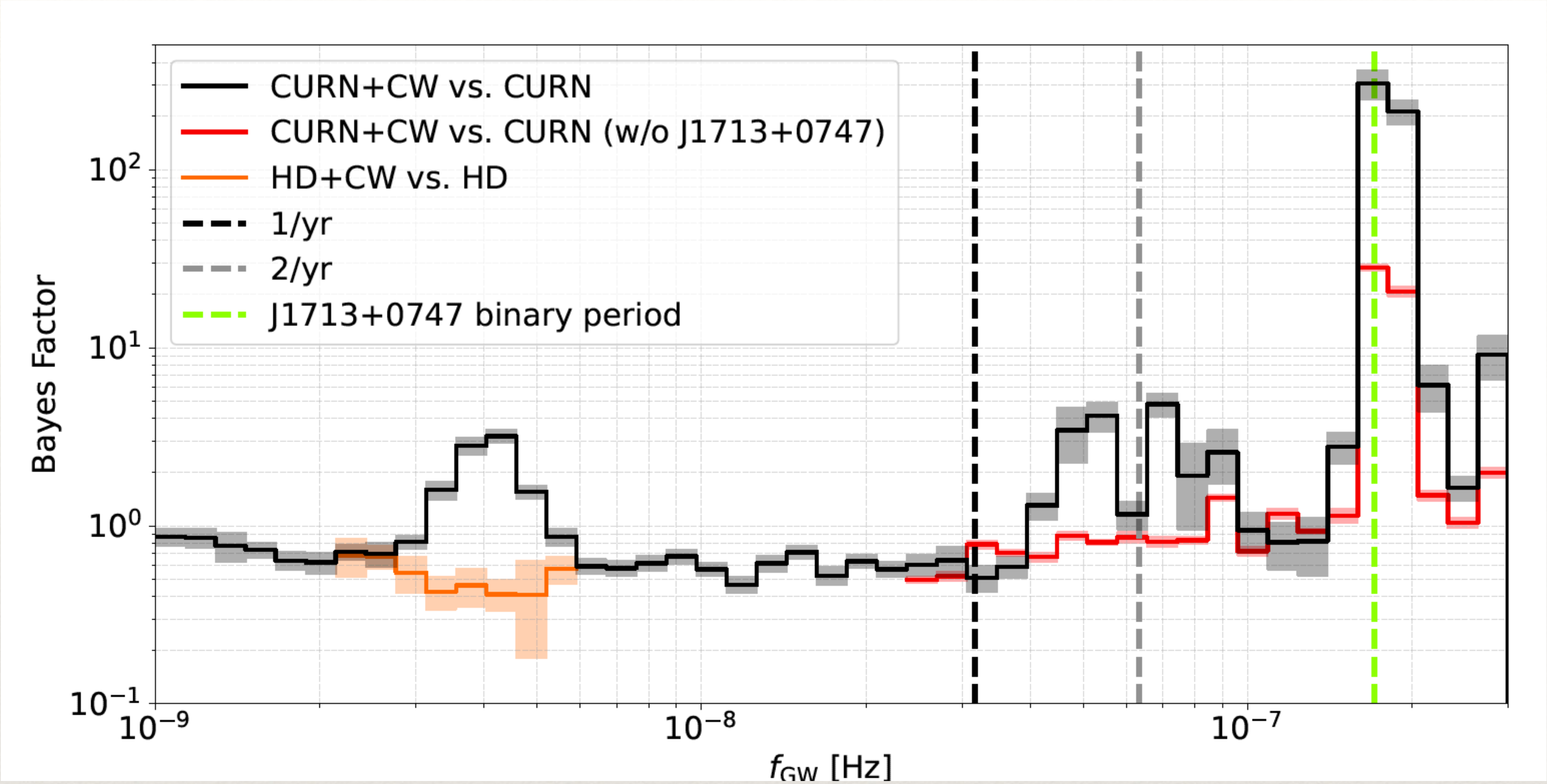
relative position
pulsar and GW source

Earth term coherent
across pulsars

Pulsar term
$\omega_\alpha = \omega(t - L_\alpha(1 + \hat{n}_\alpha\,.\,\hat{k}))$

CGW signal in NanoGrav

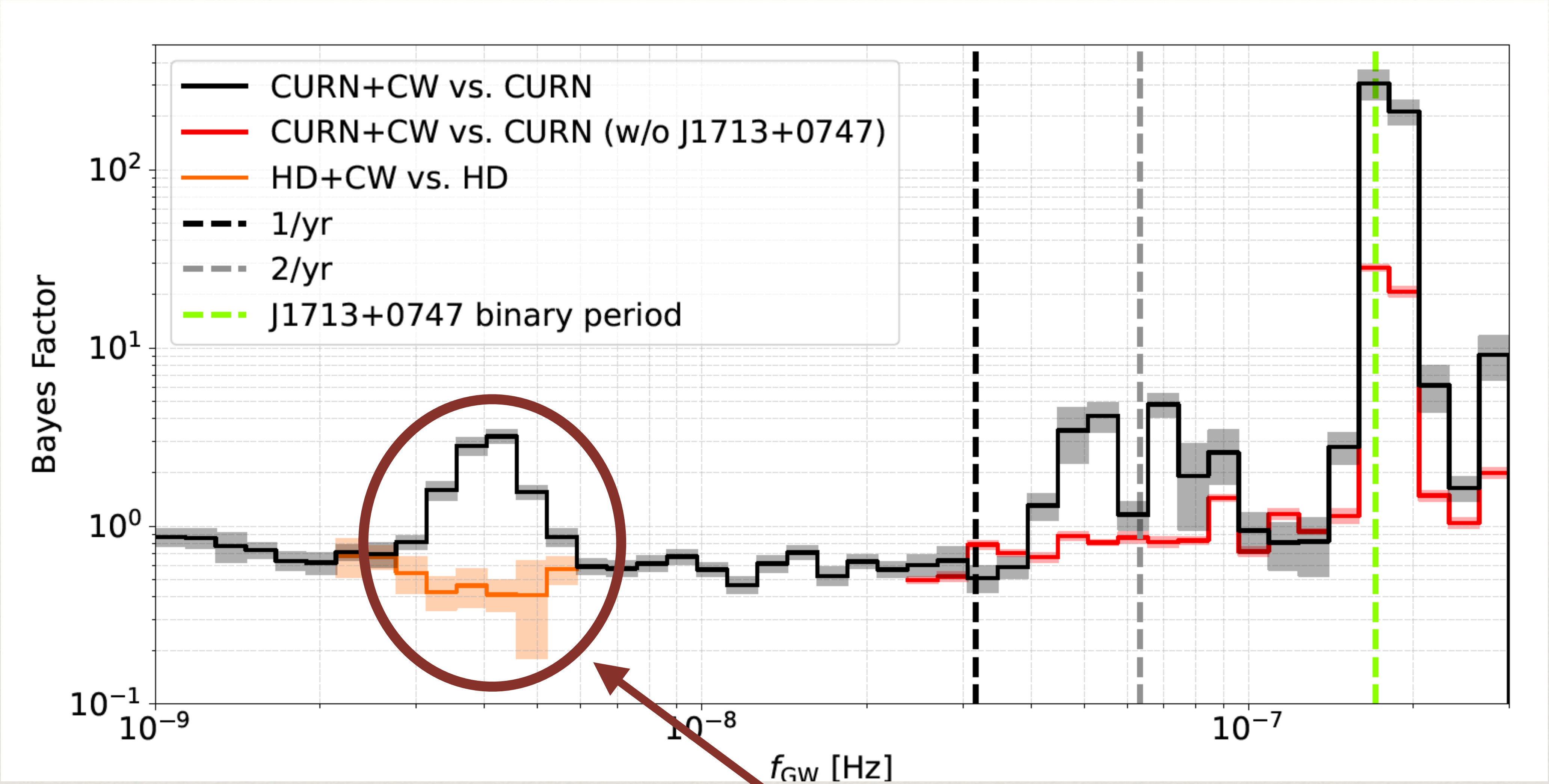
[NG: 2306.16222]



Bayesian all-sky search for a SMBHB in circular orbit: Bayes factor for presence of CGW

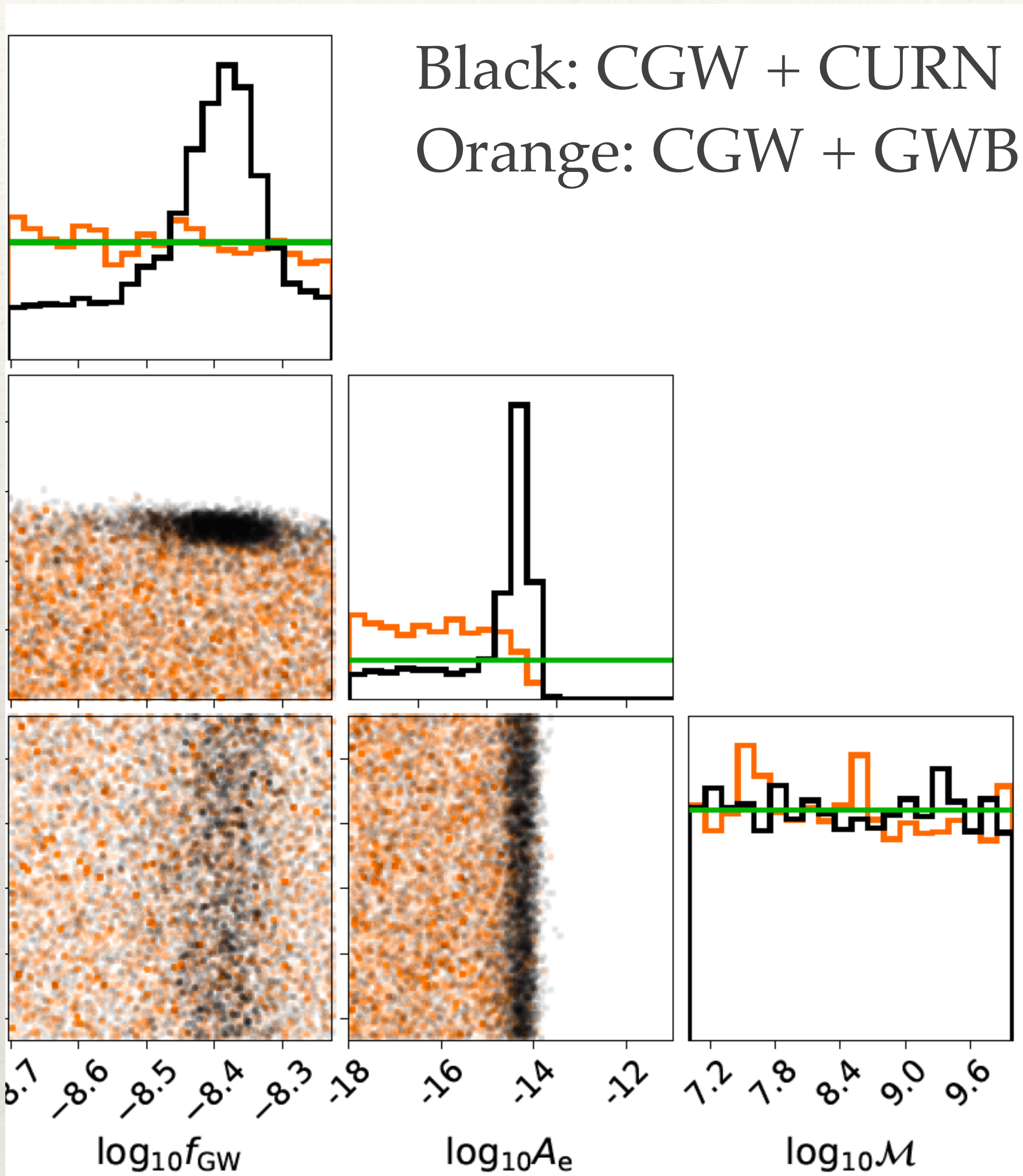
CGW signal

[NG: 2306.16222]



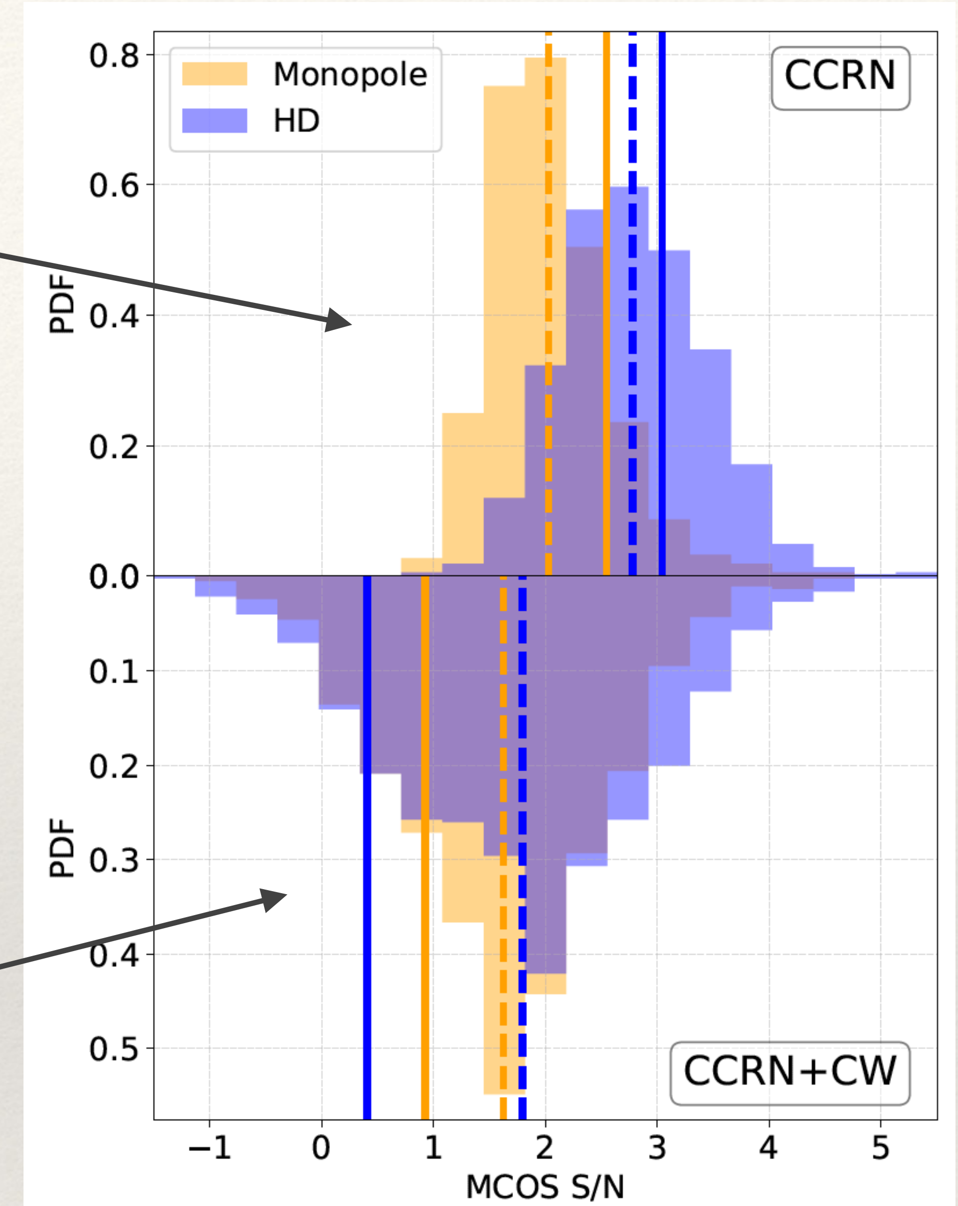
- Bayesian all-sky search for a SMBHB in circular orbit: Bayes factor for presence of CGW
- I concentrate on the low-frequency candidate

CGW in NG data

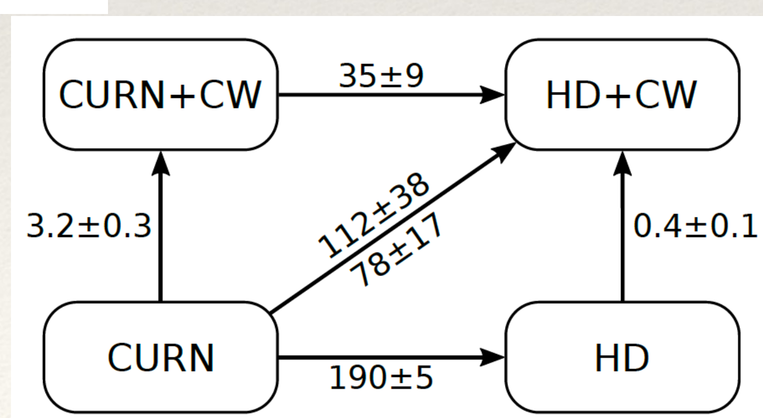


SNR of GWB and monopole in NG data

SNR of GWB and monopole in NG data **after** removing CGW



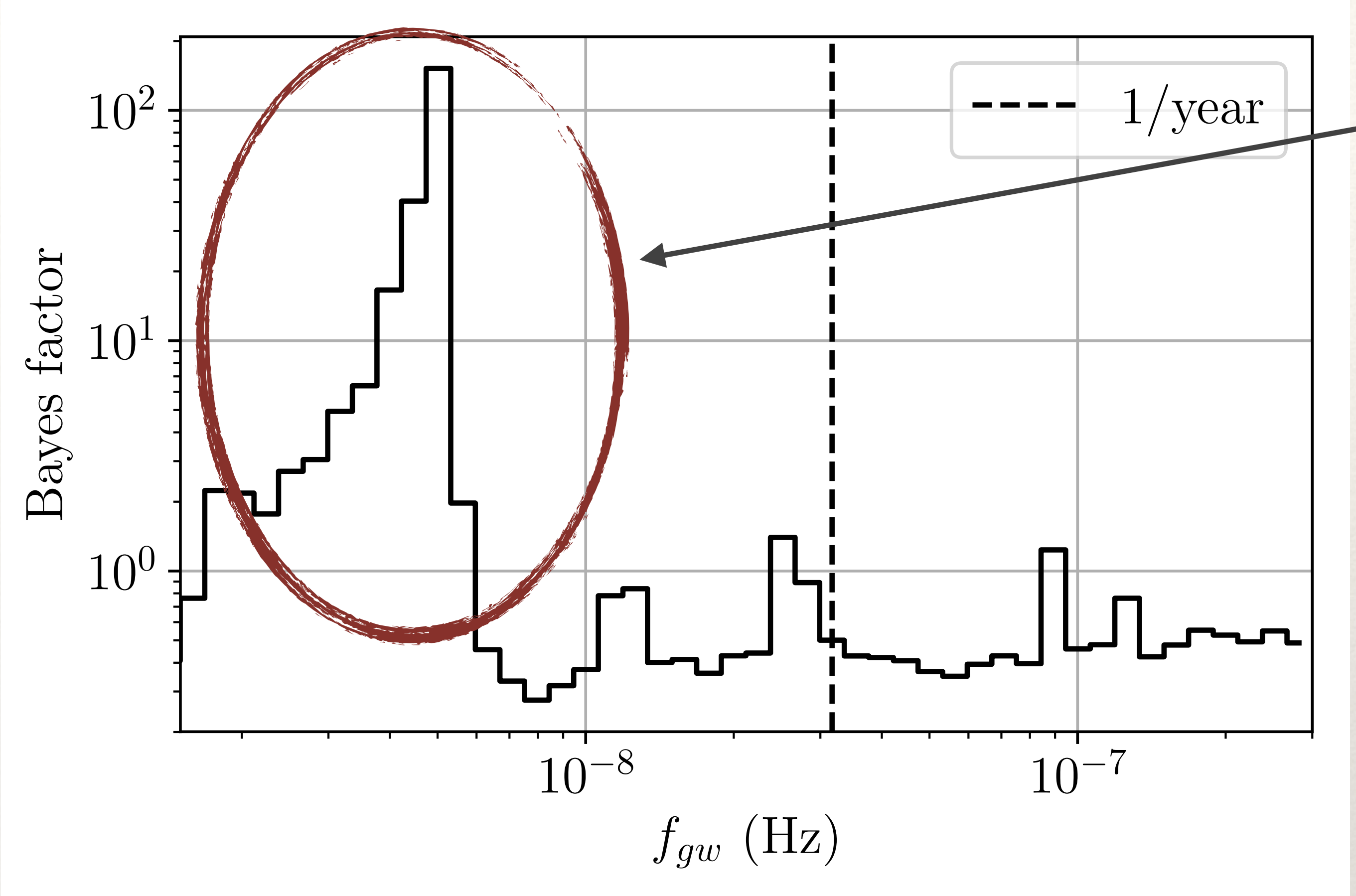
$f \in (3.2, 5.0)$ nHz



CGW signal in EPTA

BF: PSRN+CURN+CGW/PSRN+CURN

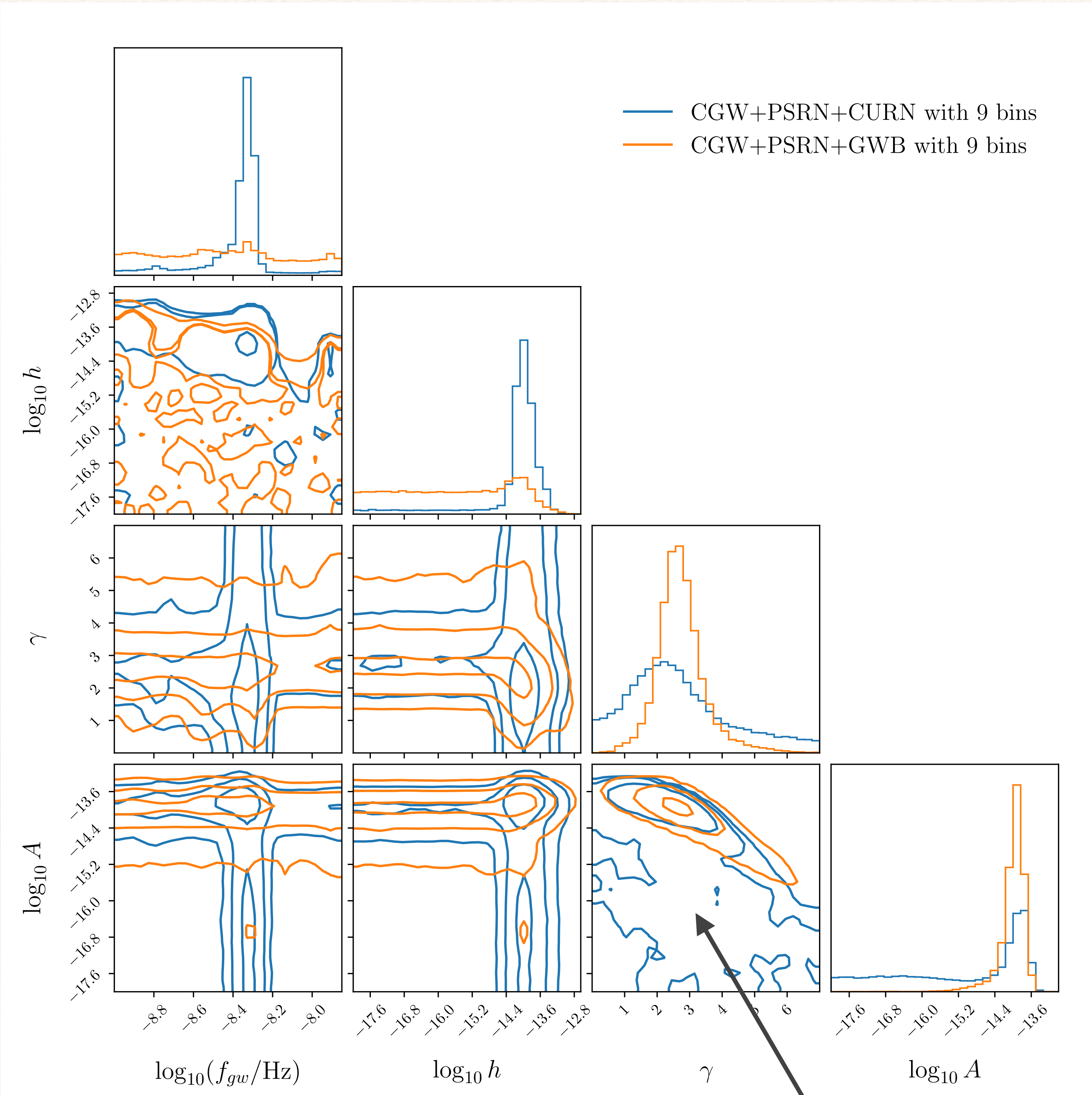
[EPTA+InPTA: 2306.16226]

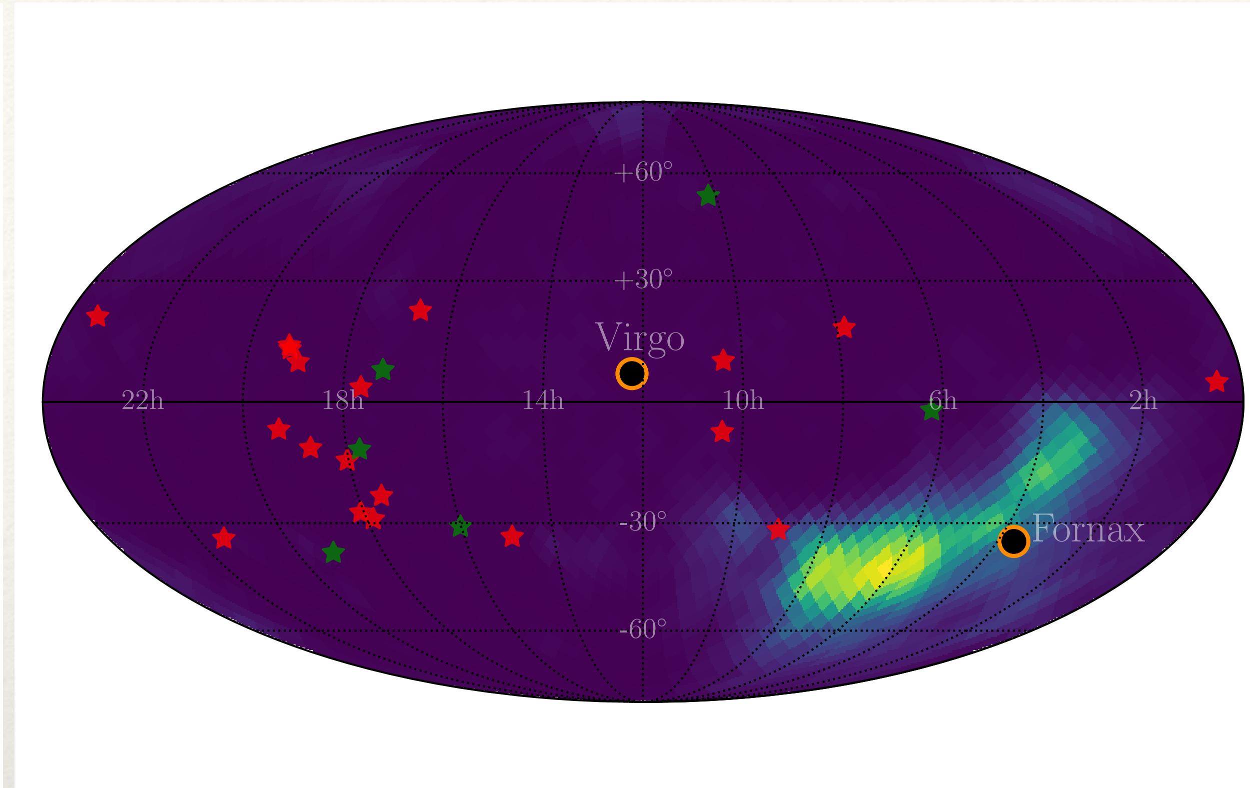


CGW candidate at low frequency

Bayesian earch for CGW using Earth term only

CGW signal in EPTA





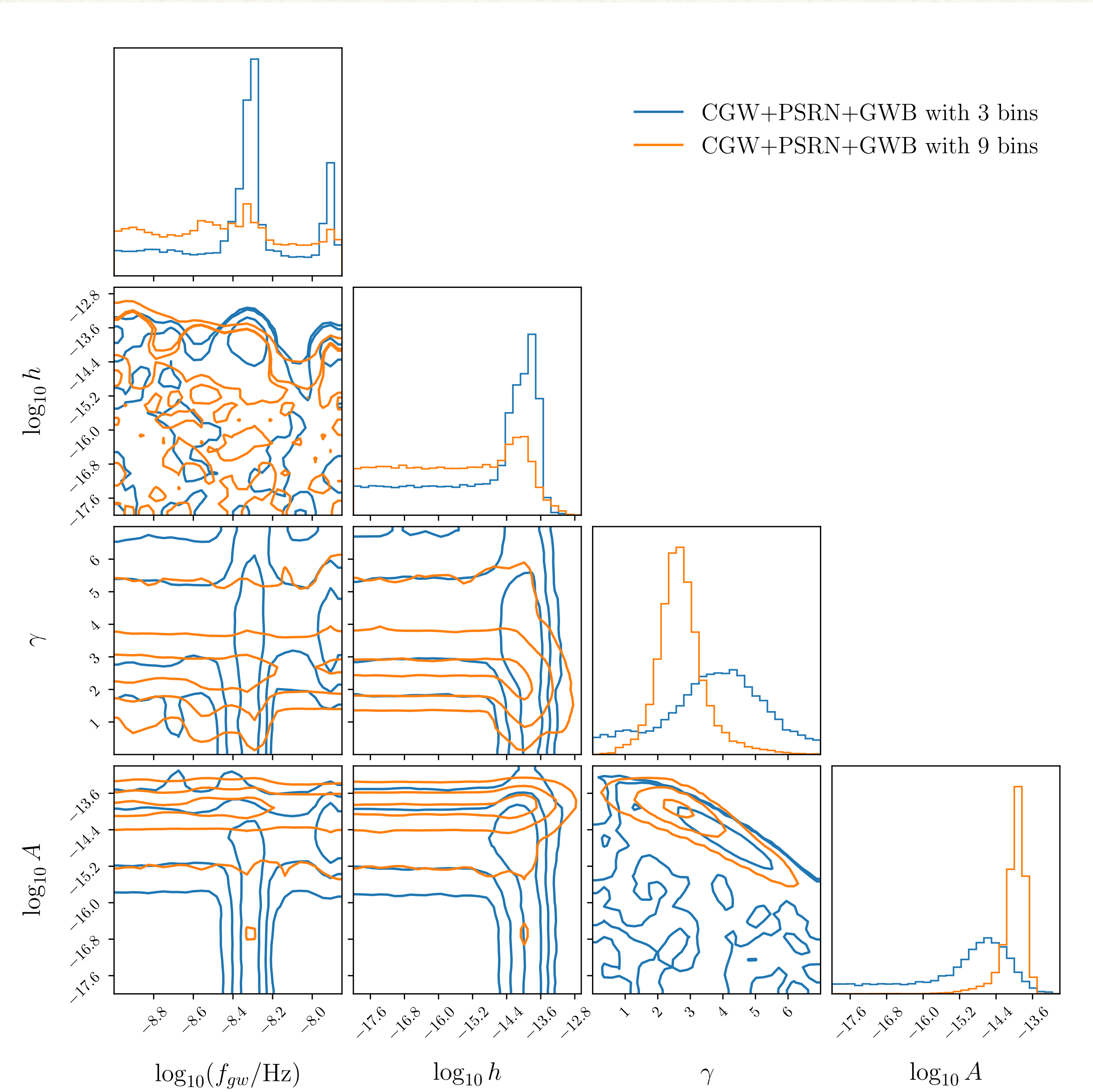
CGW: circular, Earth and Pulsar terms

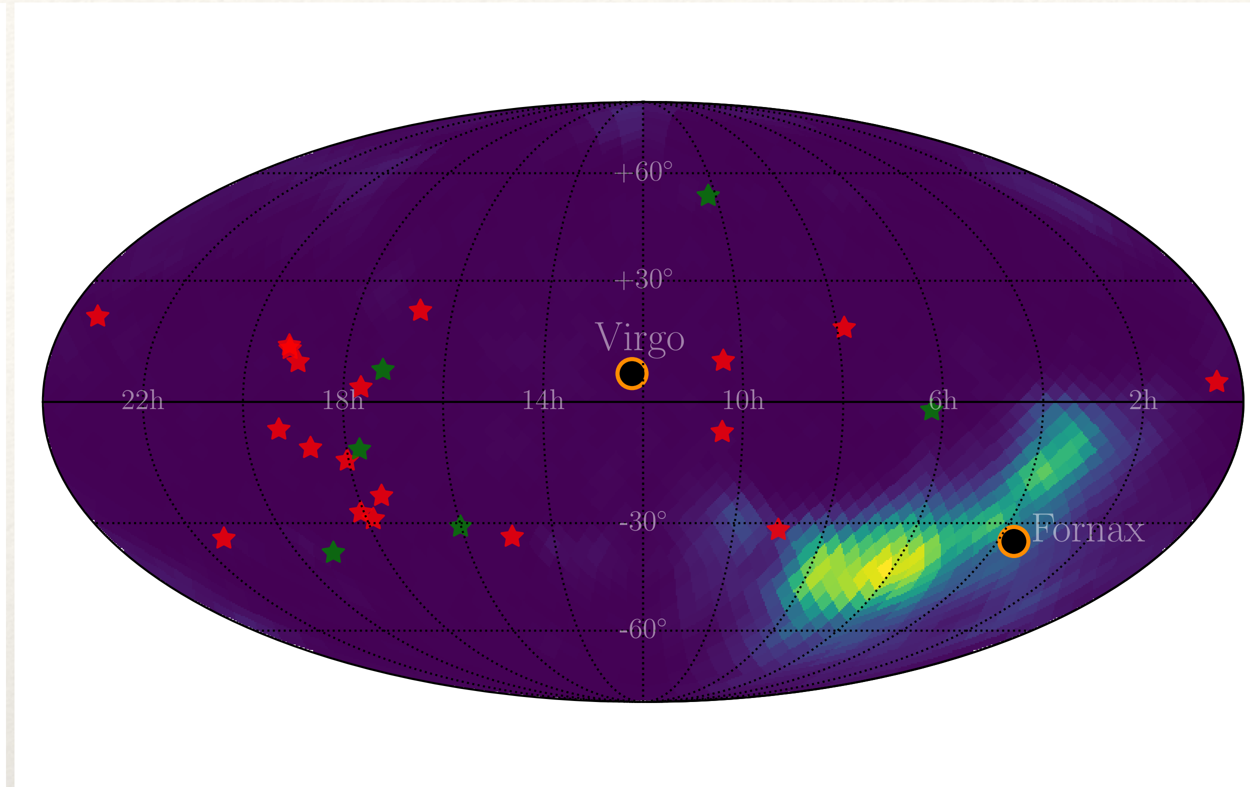
Model comparison	Bayes factor
CGW+PSRN vs PSRN	4000
CGW+PSRN+CURN vs PSRN+CURN, 3 bins	12
CGW+PSRN+CURN vs PSRN+CURN, 9 bins	4
CGW+PSRN+GWB vs PSRN+GWB, 3 bins	1
CGW+PSRN+GWB vs PSRN+GWB, 9 bins	0.7

$f \in (3.2, 6.0)$ nHz

CURN poorly constrained

CGW signal in EPTA





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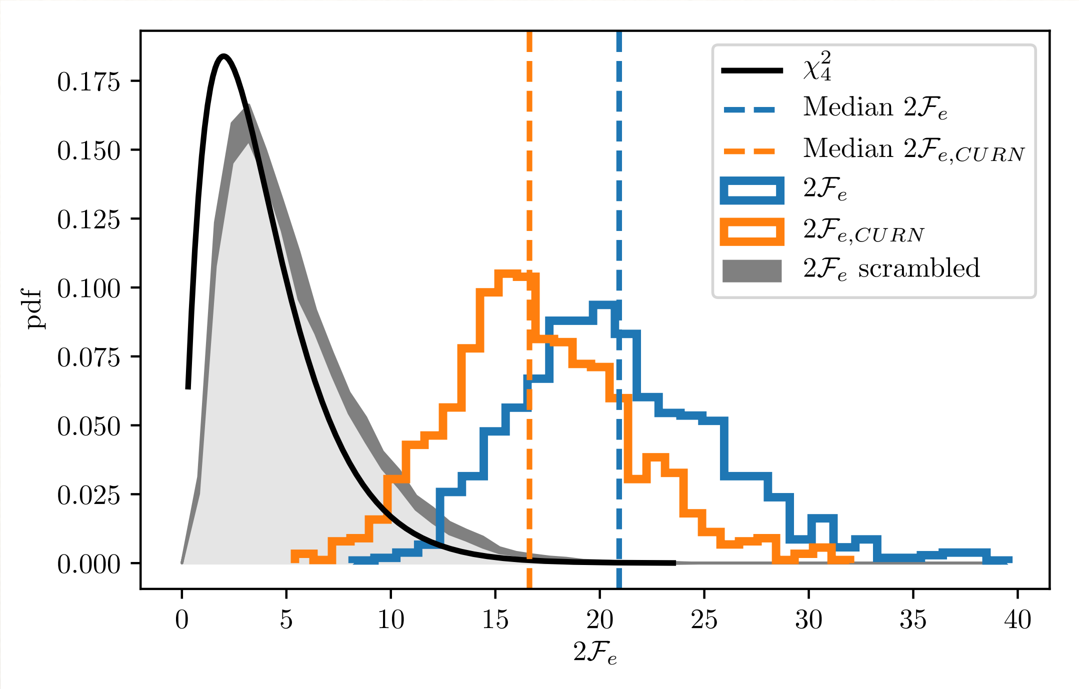
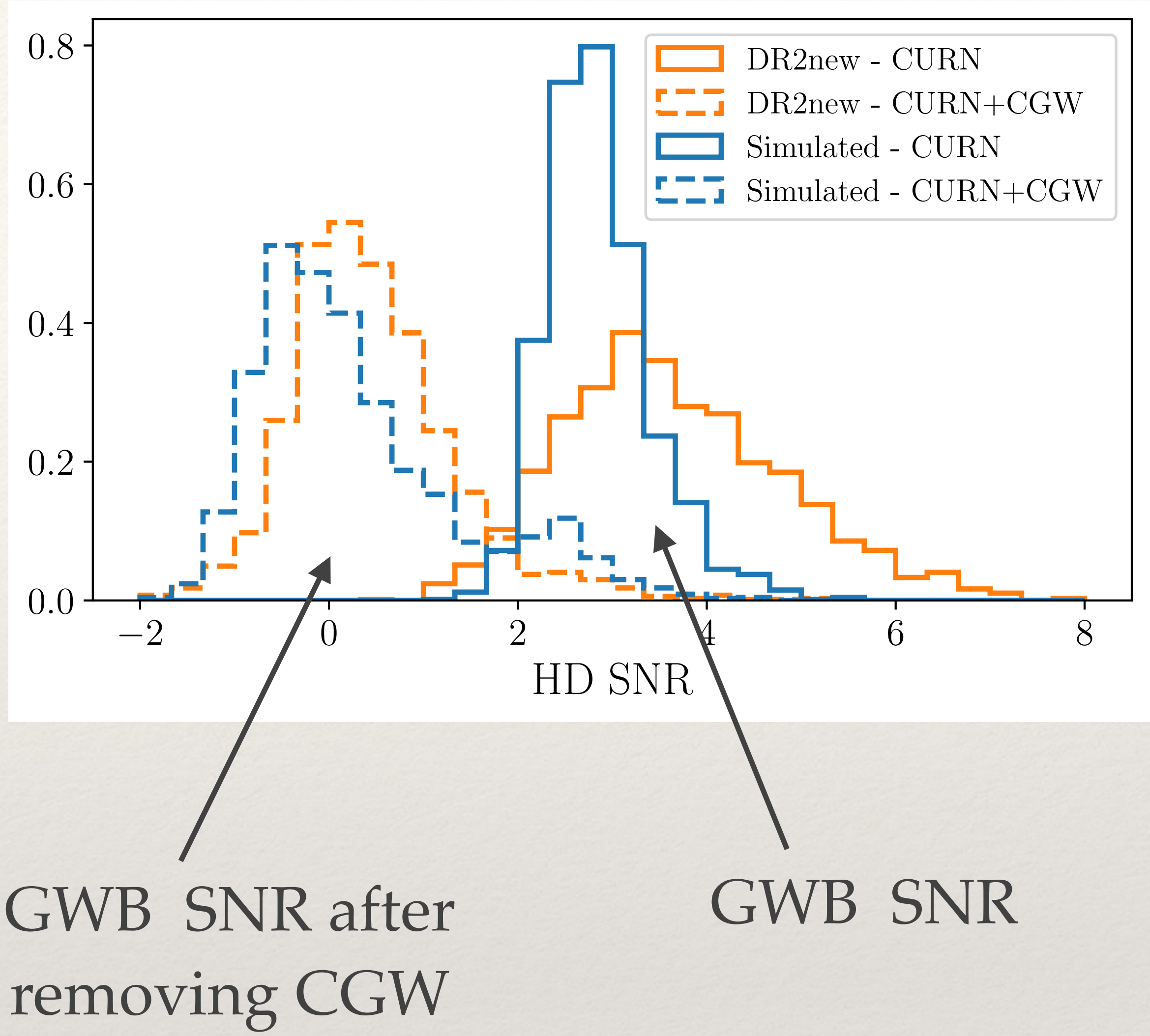
$f \in (3.2, 6.0)$ nHz

CGW signal in EPTA





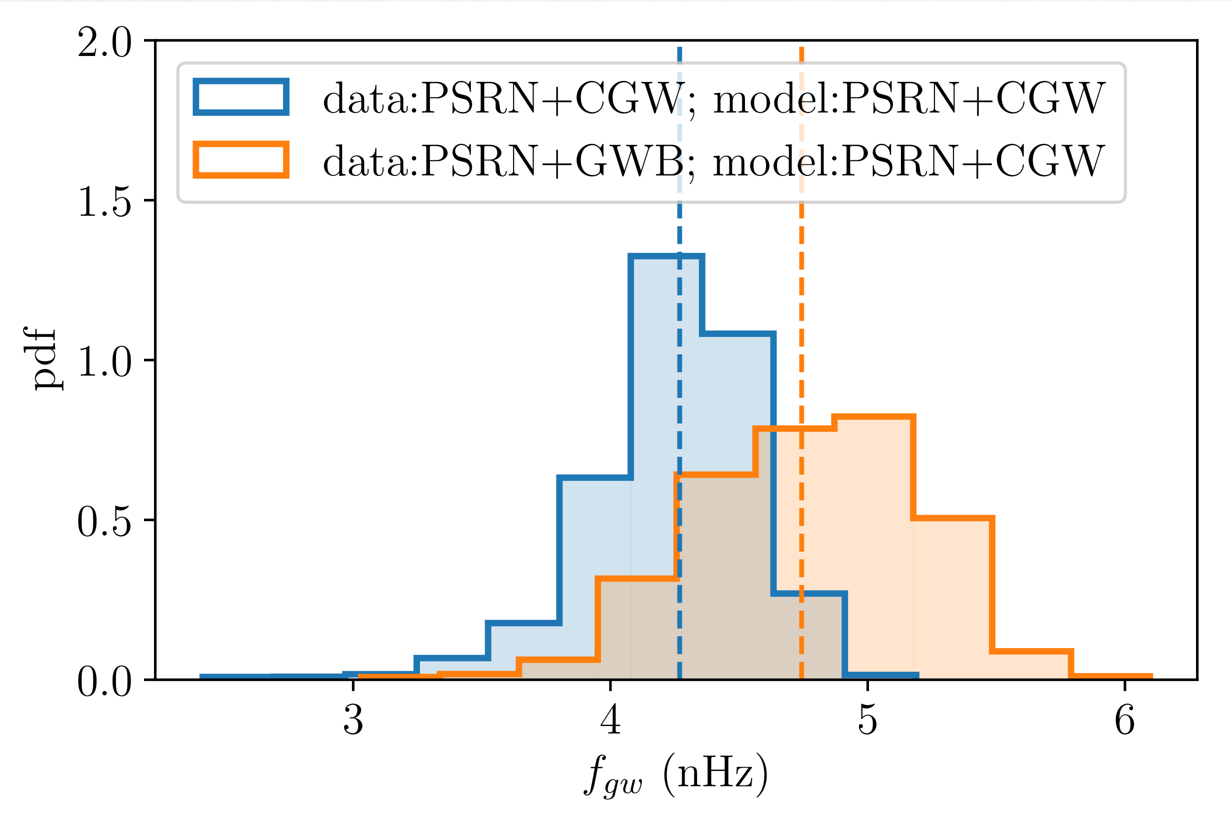
Frequentist analysis (but taking into account large uncertainties in the noise)

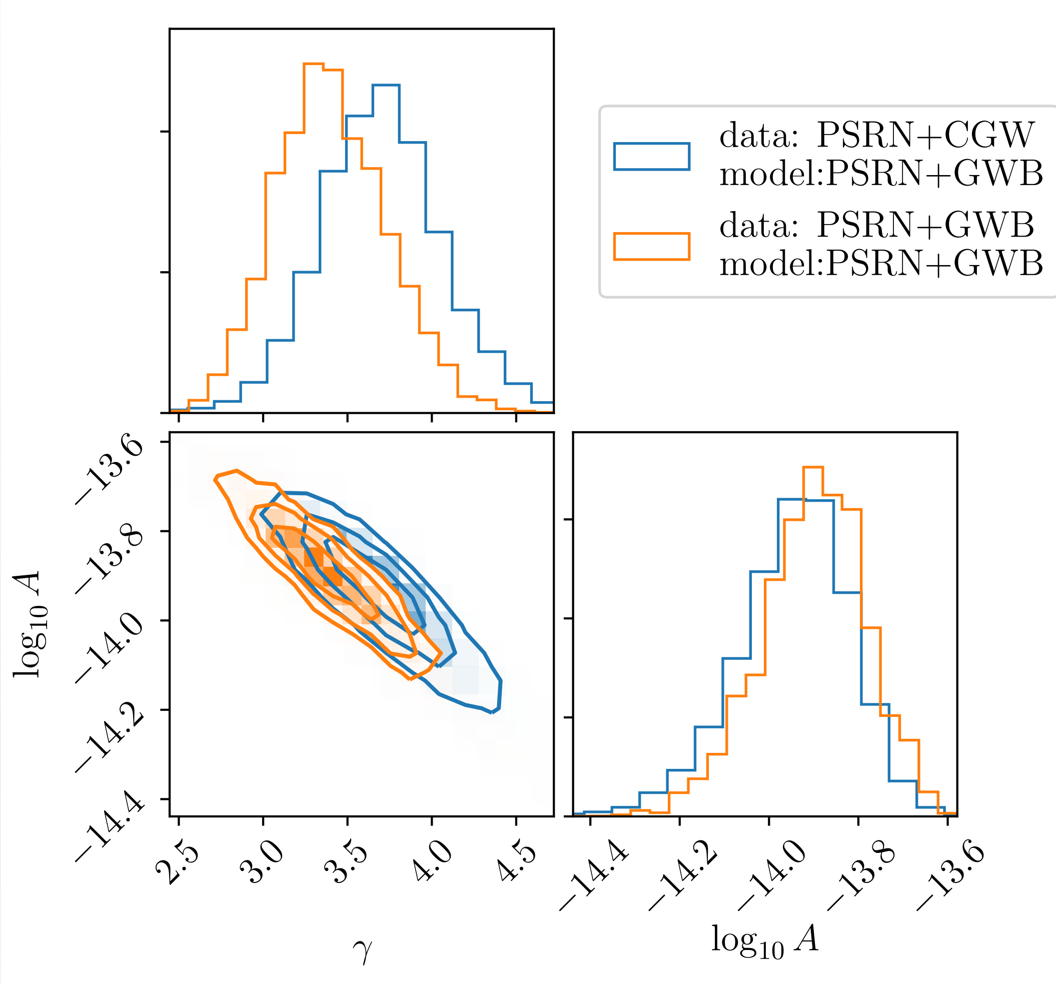
Siginificance

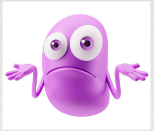
	$p(\mathcal{F}_e)$	$p(\mathcal{F}_{e,\mathrm{CURN}})$
χ^2_4	5×10^{-4}	1×10^{-3}
Sky scrambles	$(7 \pm 4) \times 10^{-4}$	$(6 \pm 1) \times 10^{-3}$

CGW signal in PTA?

Simulated data: PSRN + GWB only, Model_1: GWB, Model_2 CGW
Simulated data: PSRN + CGW only, Model_1: GWB, Model_2 CGW

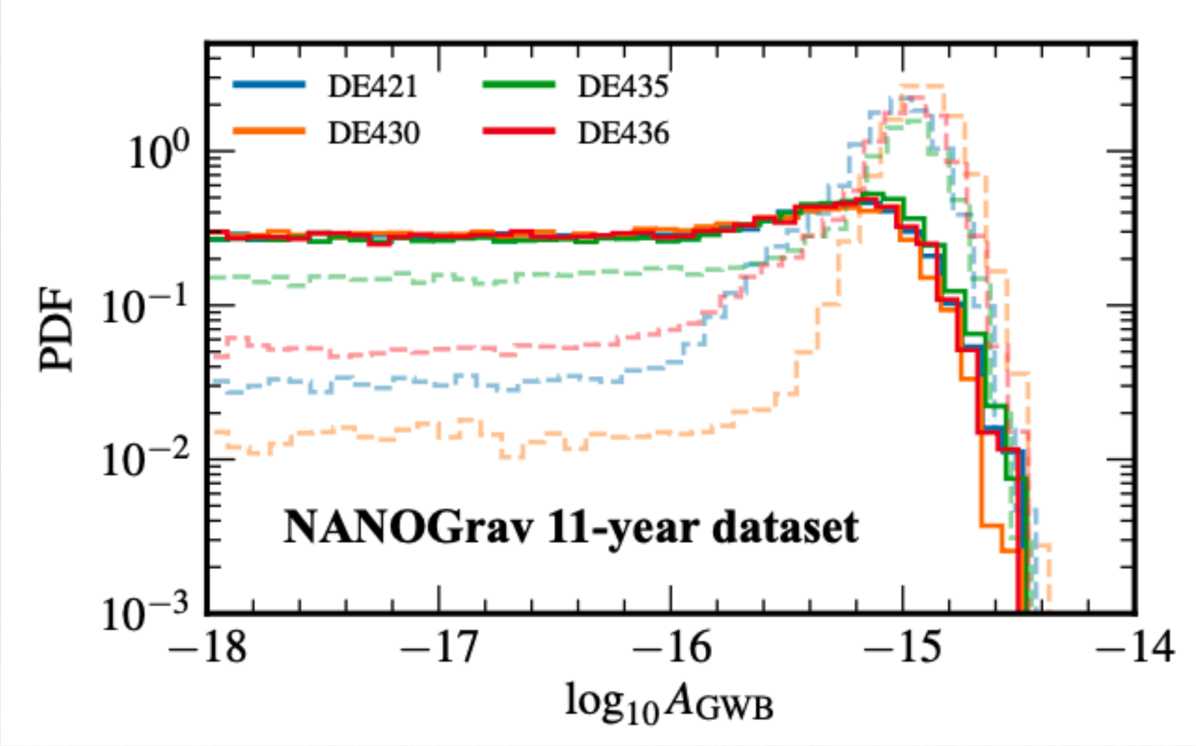




- It is easy to make mistake 
- However: GWB 2 parameters, CGW: Np+8 pars

Is it really GW signal?

- Error in ephemerids: JPL ephemerids D440, good measurment of Jupyter



[Arzoumanian+ 2018]

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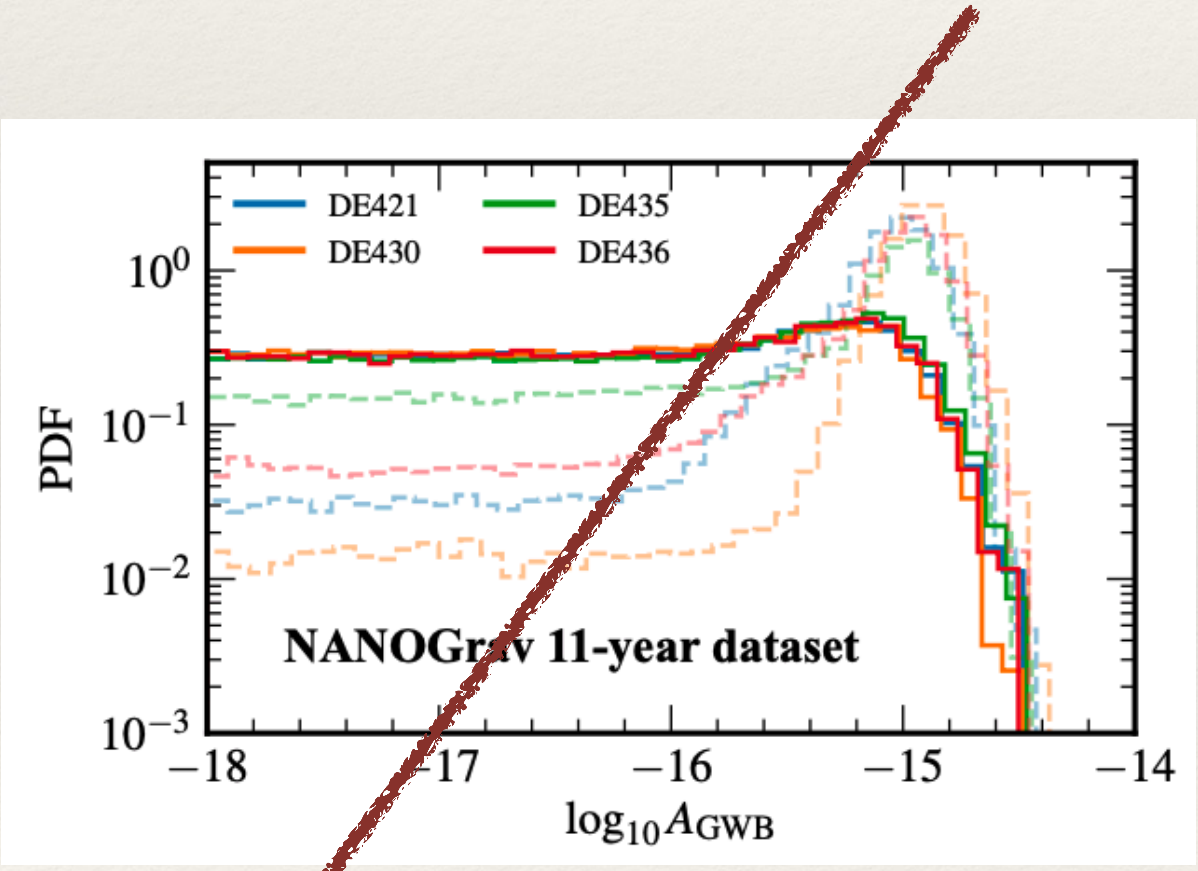
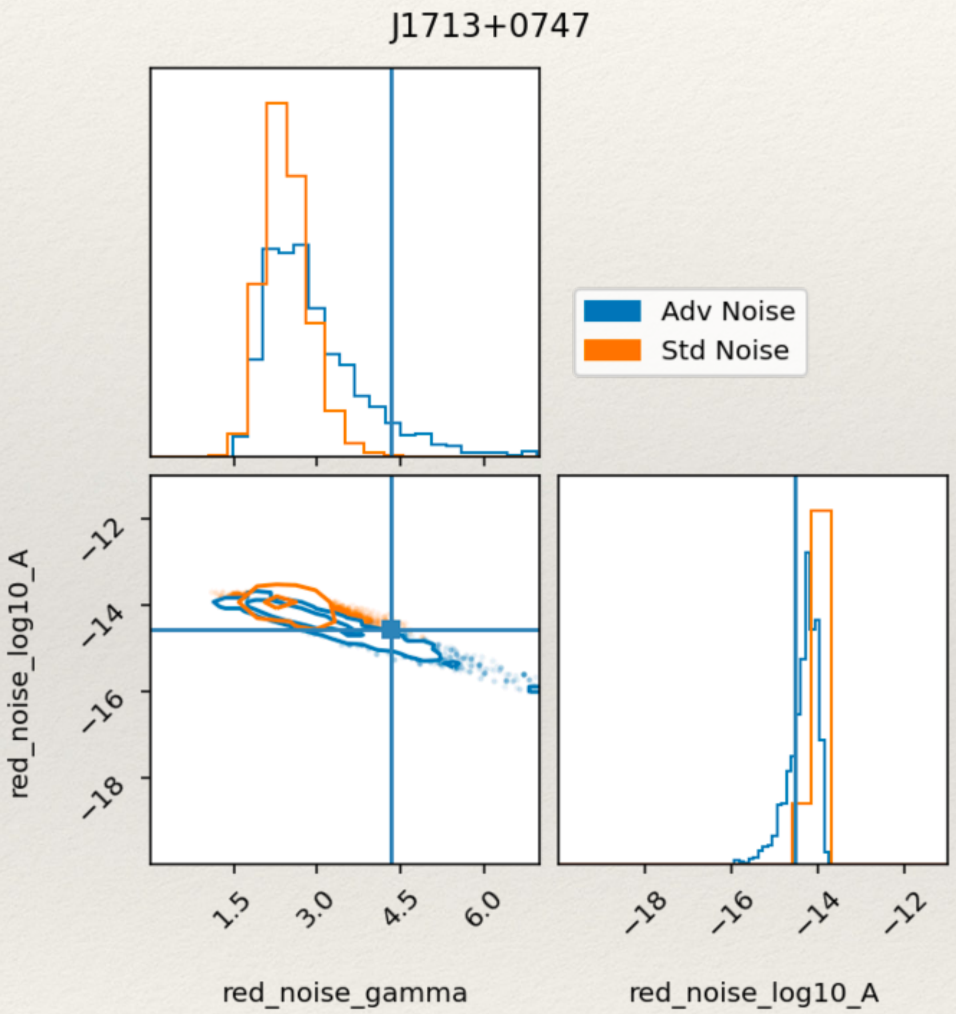
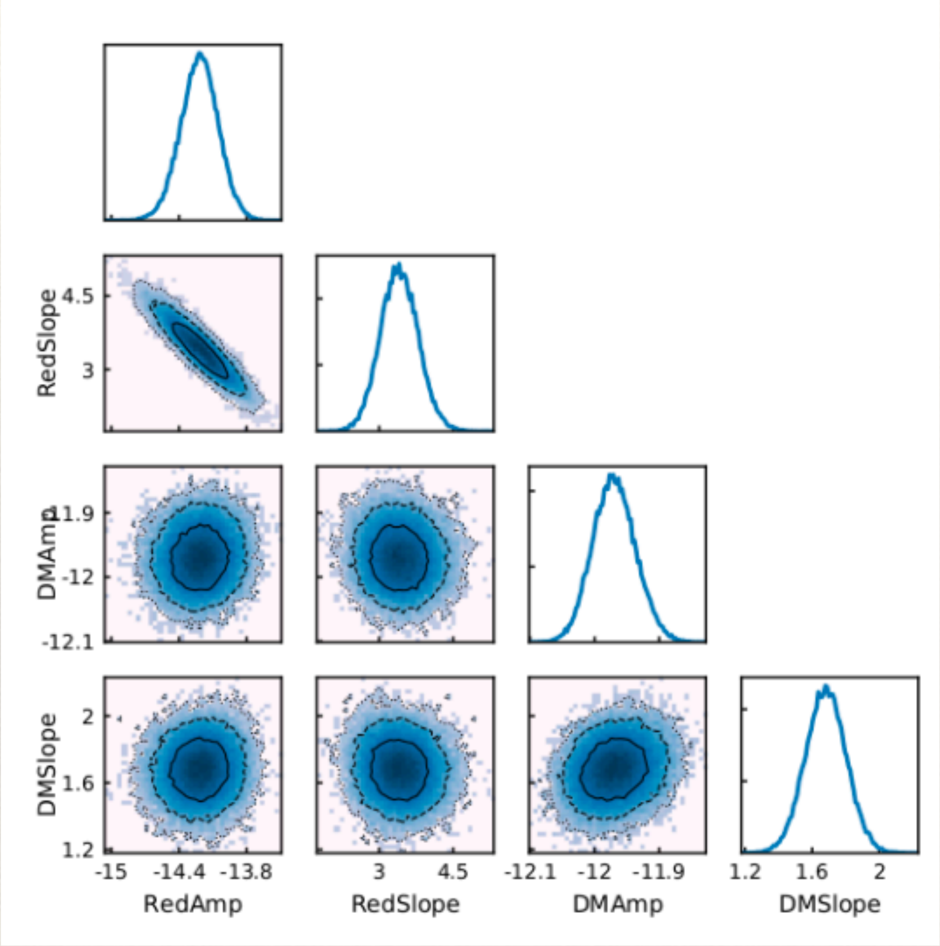


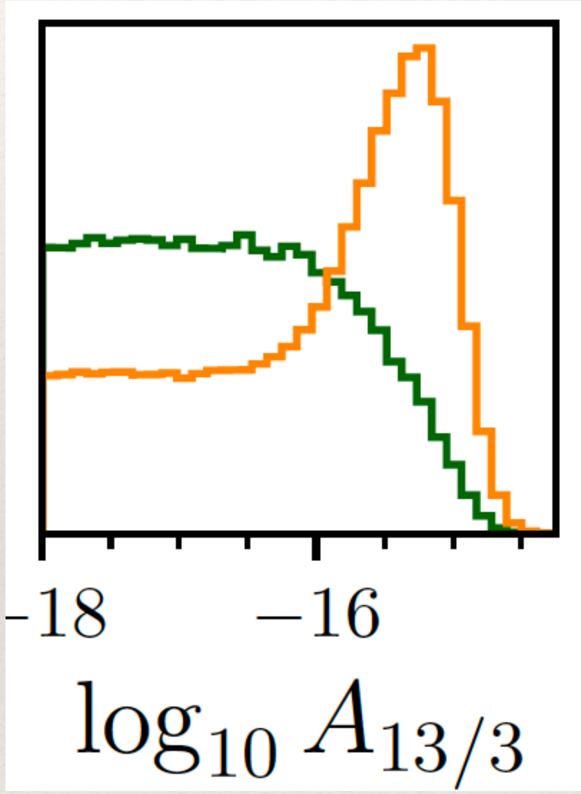
[Arzoumanian+ 2018]

Is it really GW signal?

- Error in ephemerids: JPL ephemerids D440, good measurment of Jupyter
- Modelling noise of each pulsar is very important: J1713+0747







Is it really GW signal?

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- Modelling noise of each pulsar is very important: J1713+0747

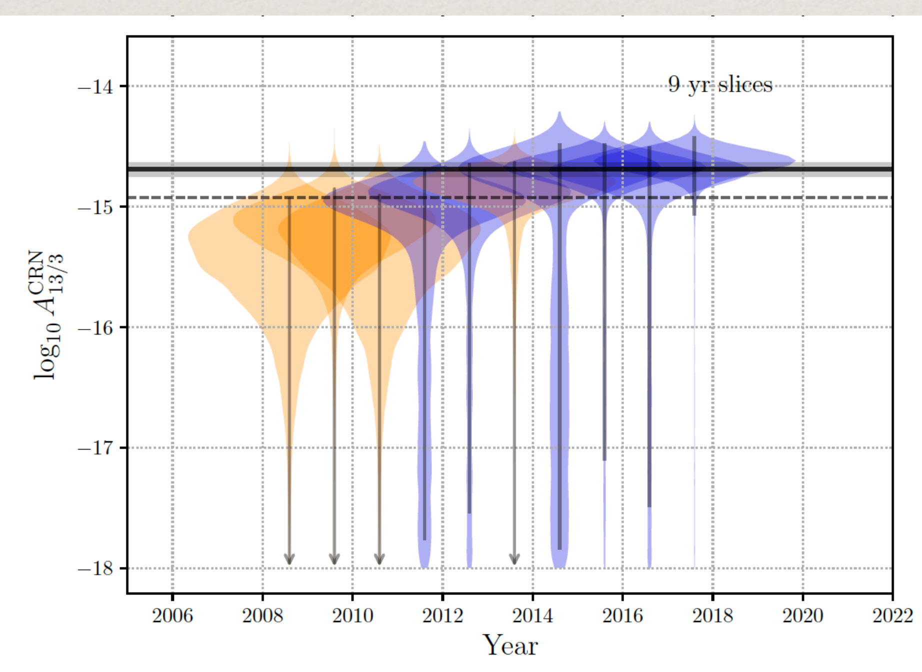
Pulsar	Sel. model
J0613-0200	*RN10 DMv30* *DMv-SN_NUP_1.4*
J1012+5307	*RN150 DMv30* *DMv-SN_NUP_1.4* *SN_NUP_2.5*
J1600-3053	*DMv30 Sv150* *SN_LEAP_1.4*
J1713+0747	*RN15 DMv150* *2 Exp. dips* *DMv-SN_NUP_1.4* *SN_JBO_1.5* *SN_LEAP_1.4* *SN_BON_2.0* *BN_Band.3*
J1744-1134	*RN10 DMv100* *DMv-SN_NUP_1.4* *BN_Band.2*
J1909-3744	*RN10 DMv100 Sv150*

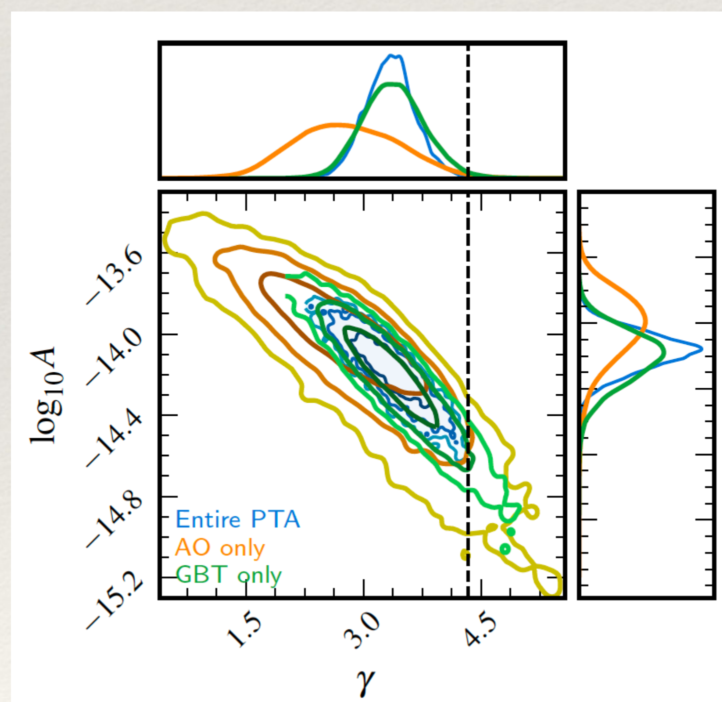
EPTA 6 best pulsars, custom noise models

[Chalumeau+ 2021]

Is it really GW signal?

- Error in ephemerids: JPL ephemerids D440, good measurment of Jupyter
- Modelling noise of each pulsar is very important: J1713+0747
- Quite different BF from each PTA: 1-2 (PPTA), 60-70 (EPTA), 230-950 (NG)
- EPTA "sees" the signal only in last 14 years, PPTA sees signs of non-stationarity
  - Is it non stationarity in the GWB?
  - or in the PSR noise model?
  - or evolution of how we deal with radioobservations?





Statistic	DR2full	DR2new	DRnew+
$\mathcal{B}^{HD}_{CURN}$	4	60 33(+Ephem)	65 43 (+Ephem)

What's next?

IPTA data combination:

- We combine the data from IPTA: EPTA, NG, InPTA, PPTA
- We use additional data (MeerKAT, Chime)
- Better coverage (dense) in time (smaller cadence)
- Better coverage in radio freq: DM and scattering variations
- Not dominated by a single radiotelescope: should see/handle systematics

Kind of summary…

- We are pretty sure that the observed signal is GW
- We are not sure about its nature
- We got so excited that made a big press release
- In relaity we need to look at IPTA data, we need longer high quality data. It is "GW detection in slow motion"