DETECTION OF ULTRA-HIGH FREQUENCY GW FROM COMPACT BINARY COALESCENSES WITH RESONANT CAVITIES

Barrau, JGB, Grenet, Martineau, arXiv: 2303.06006

UHFGW Workshop CERN

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Juan Garcia-Bellido
IFT-UAM/CSIC

Astrophysical (local) Sources

Primordial Black Holes of masses 10^{-10} to 10^{-3} $M_\odot$

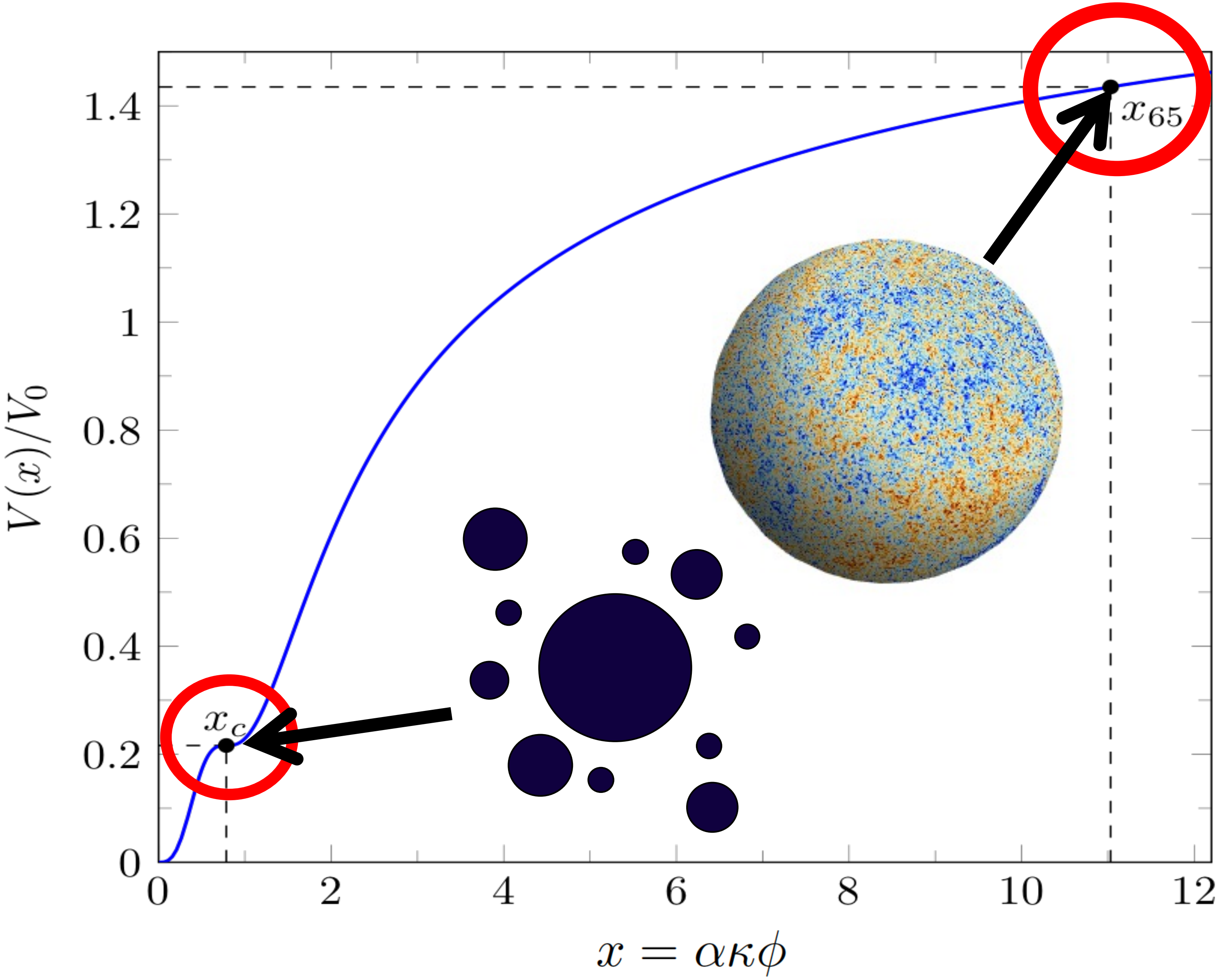
(in dense clusters in our galactic halo)

solar neighborhood - typical distance between BH

$$\rho_{\odot,\mathrm{DM}} \approx 0.008\, M_\odot/\mathrm{pc}^3 \quad \Rightarrow \quad d_{\mathrm{PBH}} \approx \frac{0.1\,\mathrm{pc}}{\delta_{\mathrm{loc}}^{1/3} f_{\mathrm{PBH}}^{1/3}} \left(\frac{M}{10^{-5}\, M_\odot}\right)^{1/3}$$

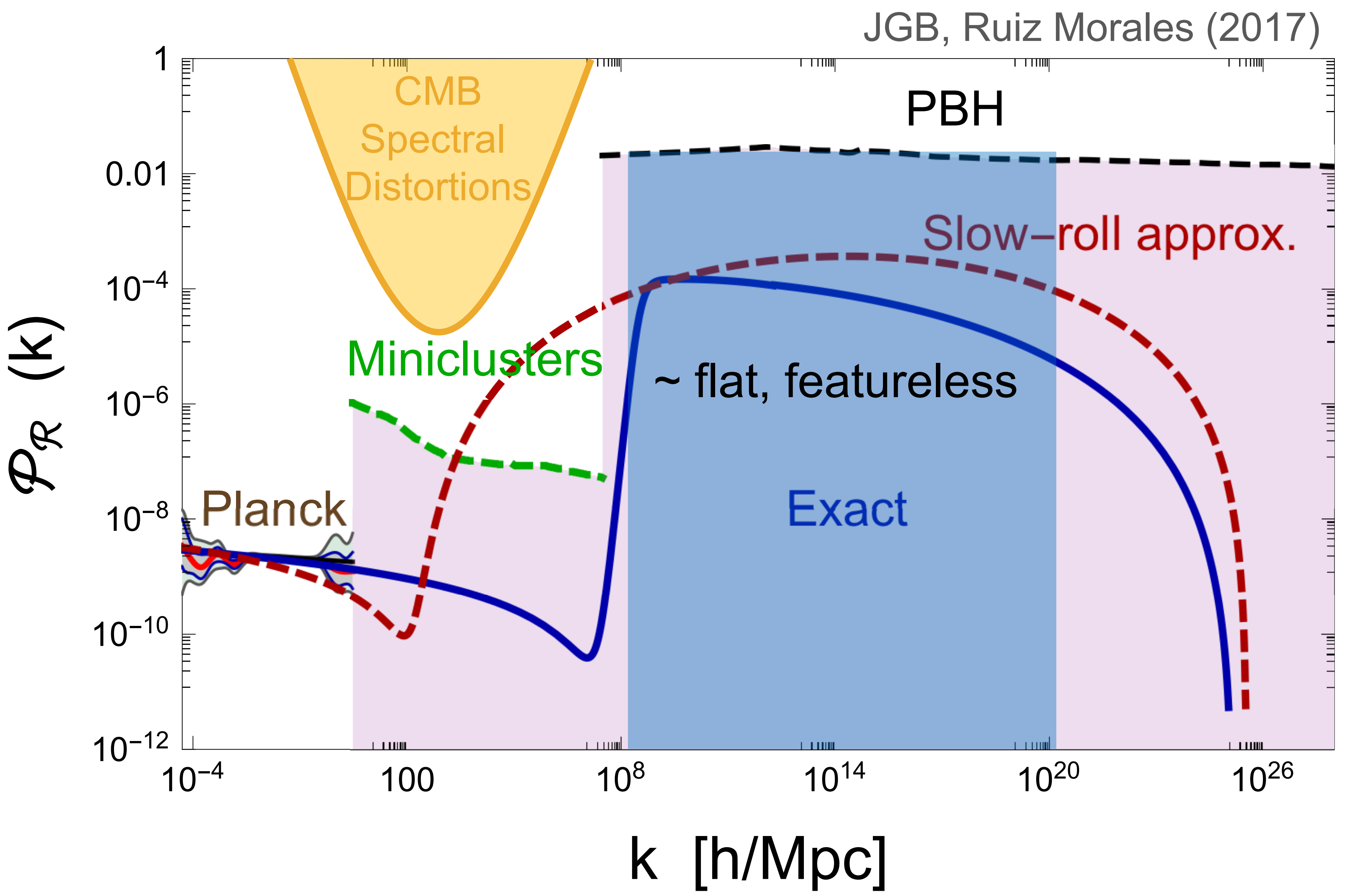
- Black Hole Binaries
- Close Hyperbolic Encounters

Primordial Black Holes



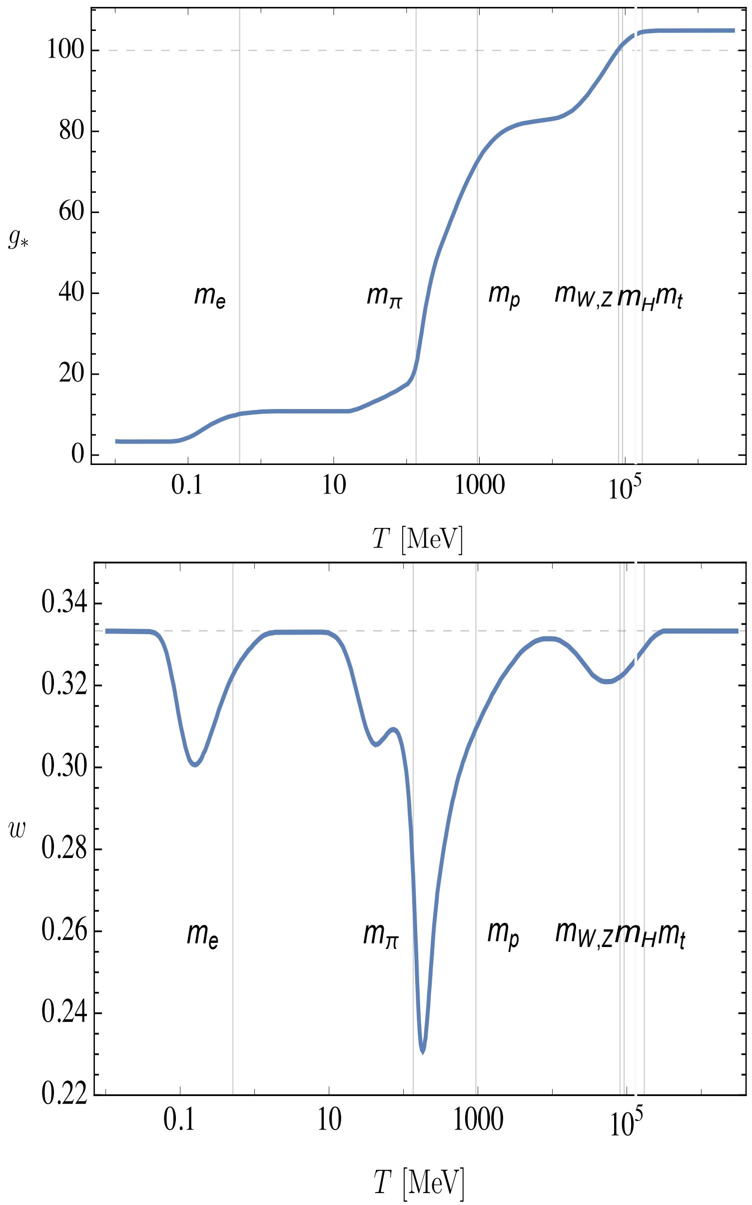
JGB & S. Clesse
1501.00460
1603.05234
1610.08479
1711.10458
JGB, B.Carr, S.Clesse
1904.11482
B.Carr, S. Clesse,
JGB & F. Kühnel
1906.08217

Primordial Power Spectrum

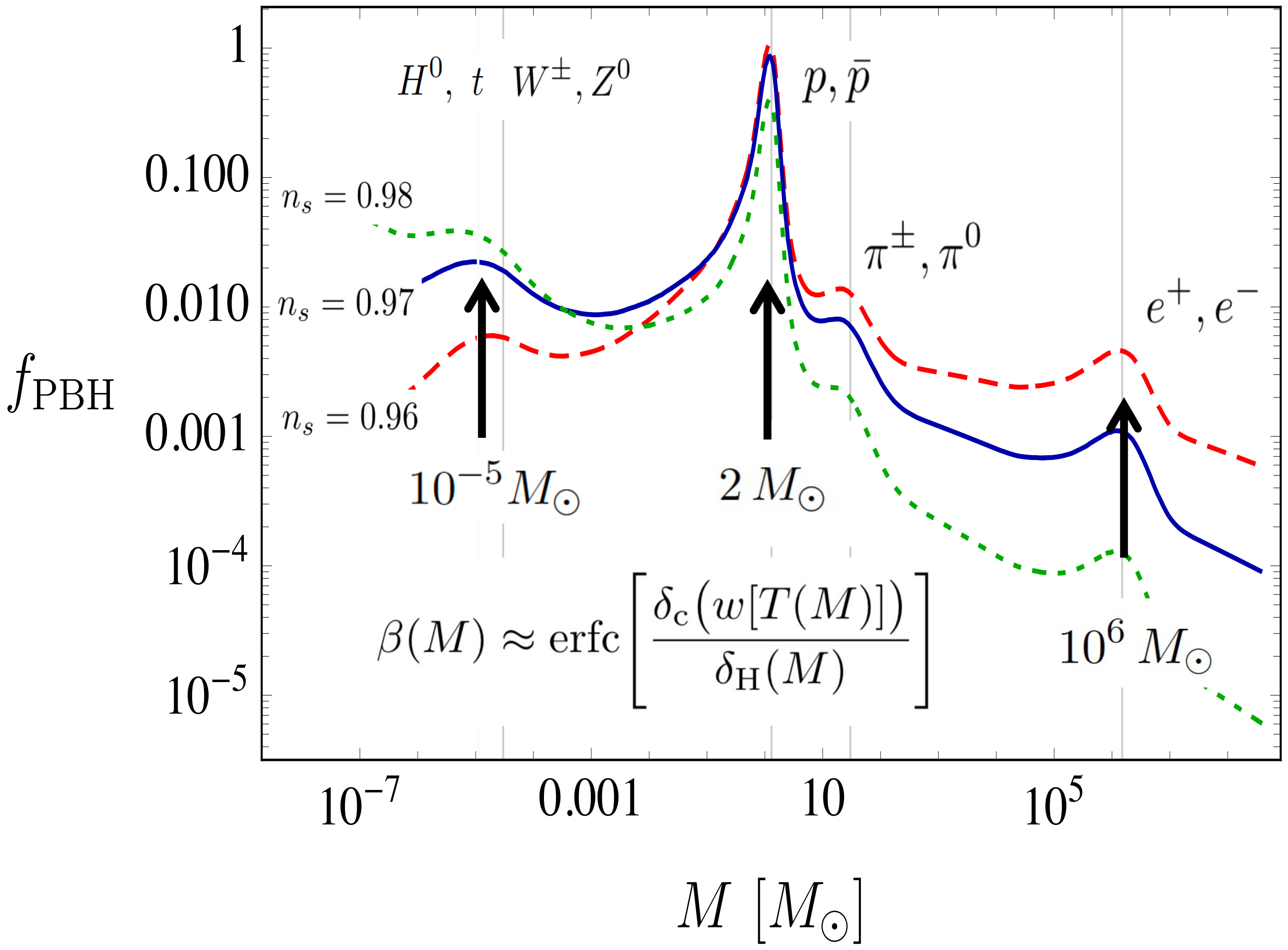


Thermal history of the universe

Carr, Clesse, JGB, Kühnel (2019)



PBH mass spectrum



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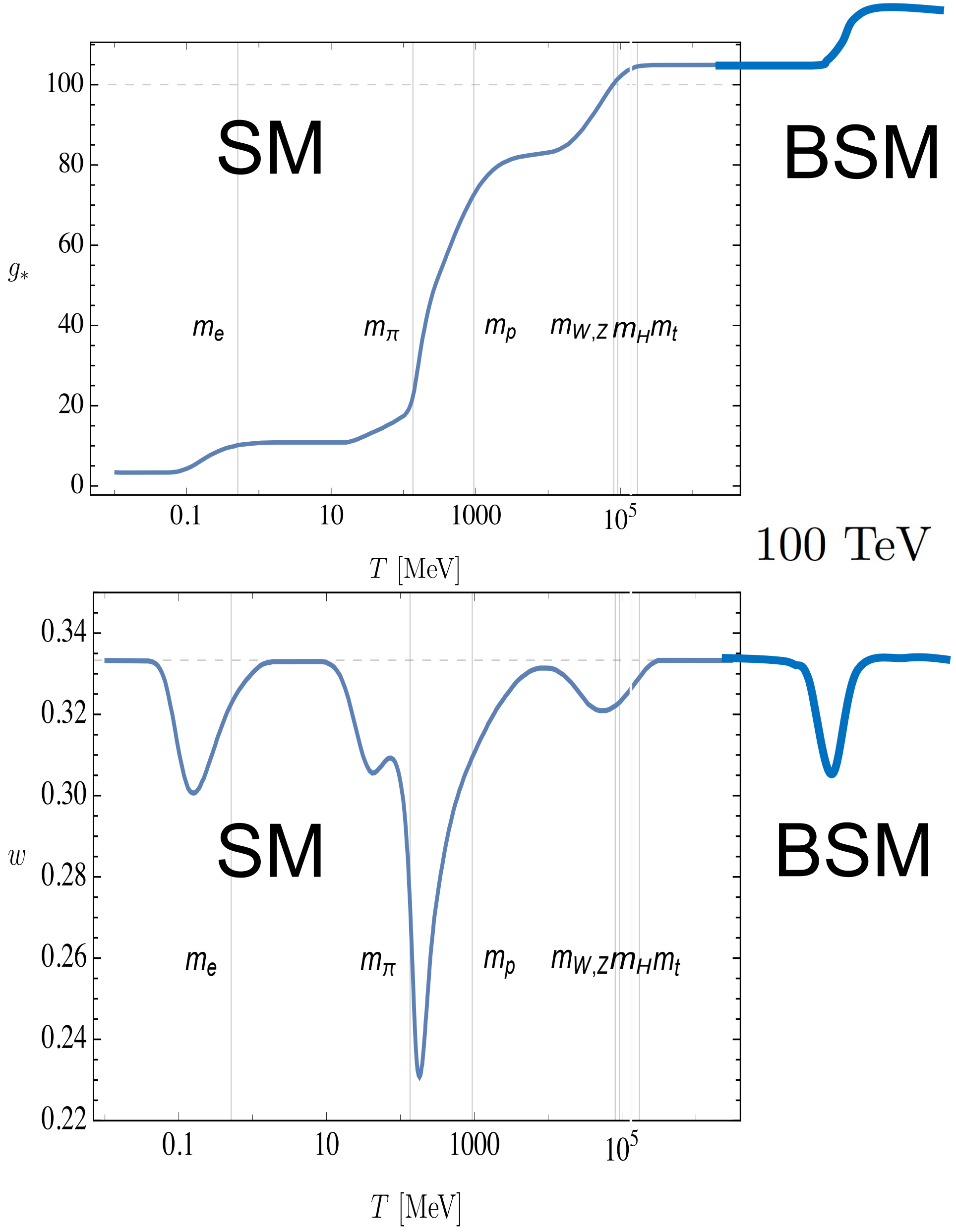
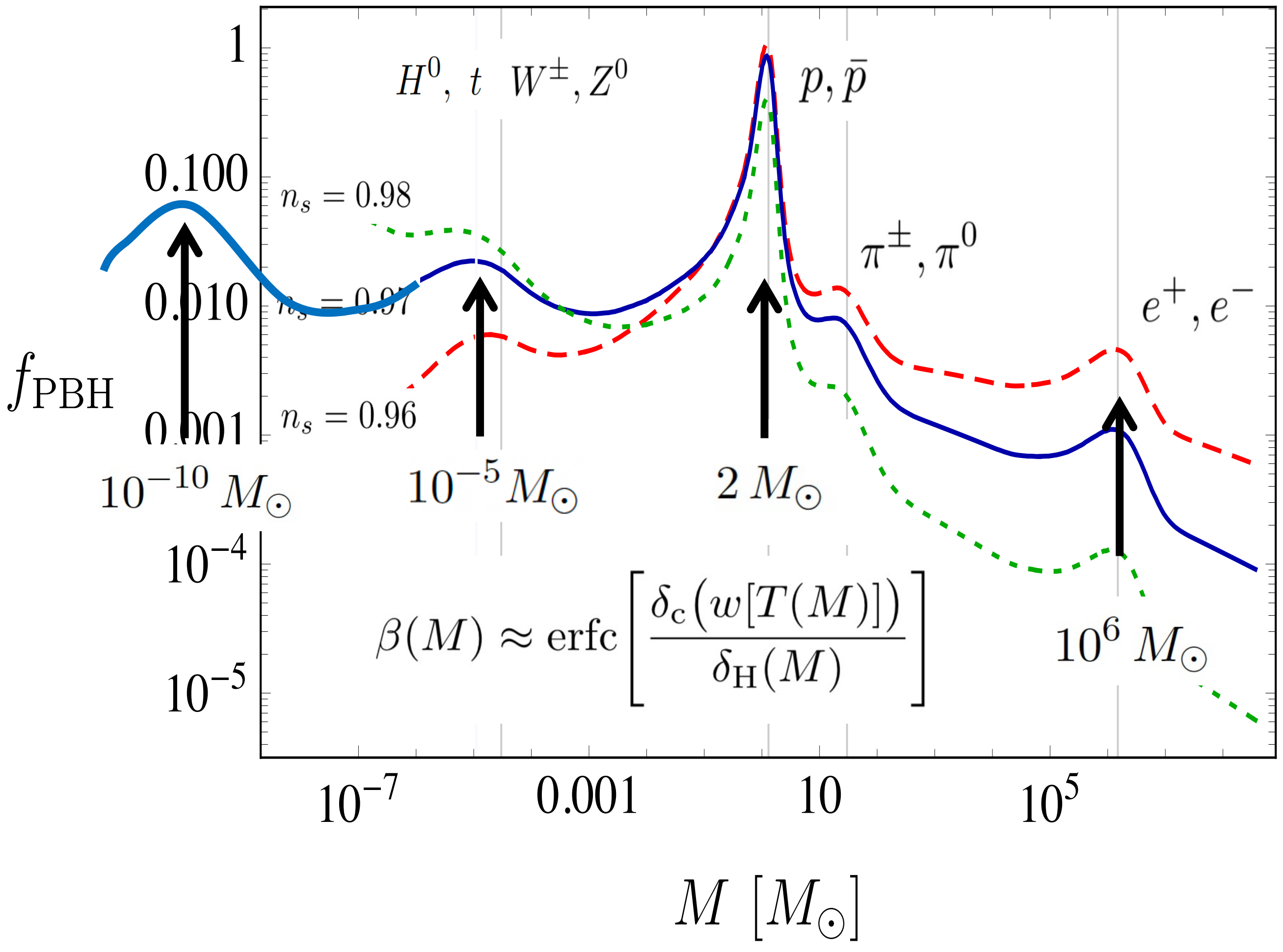


PBH mass spectrum



Black Hole Binaries

- Maximum strain at f_{ISCO} for equal mass binaries

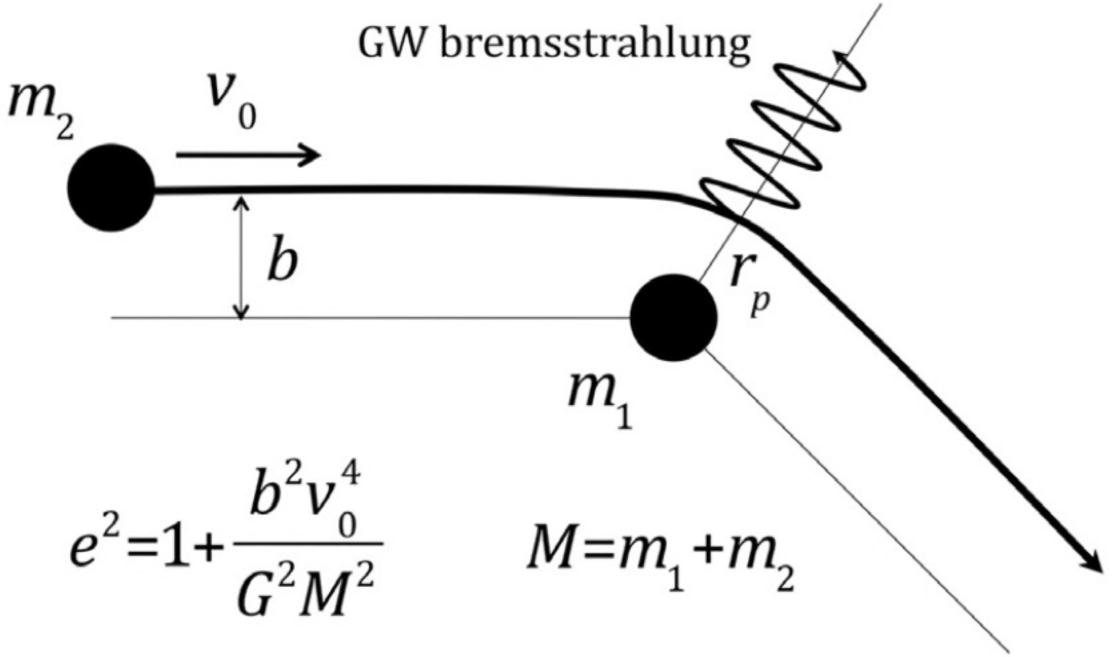
$$h_{c,\max} = 1.6 \times 10^{-23} \left(\frac{1\mathrm{Mpc}}{d_L}\right)\left(\frac{M}{10^{-3}M_\odot}\right) \quad @ \quad 2.2\,\mathrm{MHz}\left(\frac{10^{-3}M_\odot}{M}\right)$$

- Maximum GW flux received on Earth at f_{ISCO}

$$P_{\max} = \frac{32}{5}\frac{c^5}{G}\left(\frac{GM_c}{c^2}\frac{\pi f}{c}\right)^{10/3} = 2 \times 10^{48}\,W \quad \forall M$$

$$F_{\max} = \frac{P_{\max}}{4\pi d_L^2} = 1.64 \times 10^{10}\,W/\mathrm{cm}^2\left(\frac{1\mathrm{Mpc}}{d_L}\right)^2$$

Close Hyperbolic Encounters

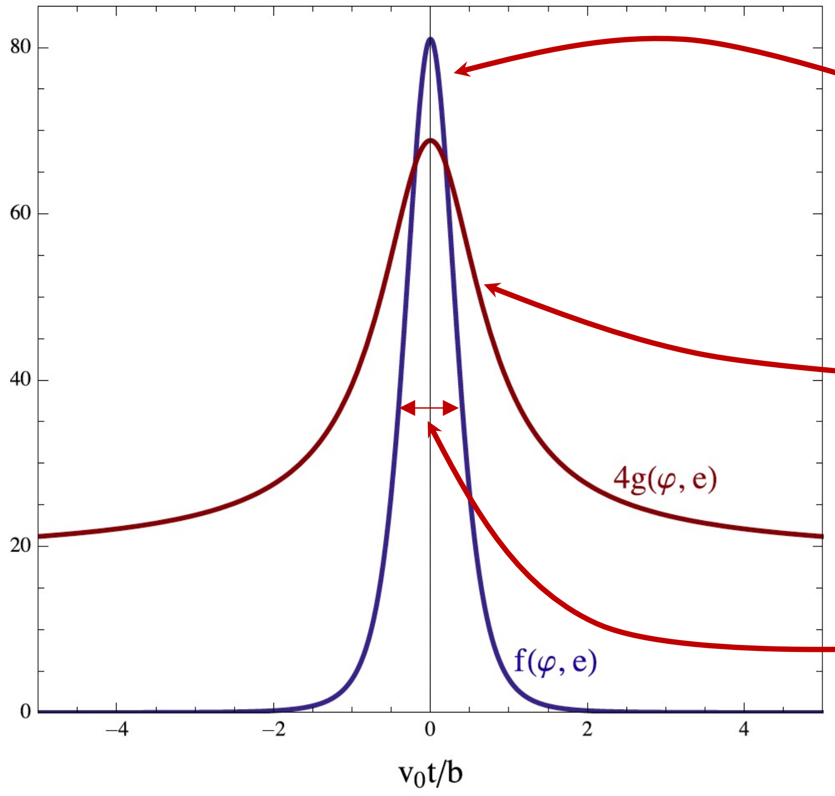


$$e^2 = 1 + \frac{b^2 v_0^4}{G^2 M^2} \qquad M = m_1 + m_2$$

$$e^2 = 1 + \left(\frac{b}{10^{-8}\mathrm{AU}}\right)^2 \left(\frac{\beta}{0.01}\right)^4 \left(\frac{10^{-6} M_\odot}{M}\right)^2$$

JGB & S. Nesseris
1706.02111
1711.09702
JGB & S. Jaraba
2106.01436
JGB, S. Jaraba &
S. Kuroyanagi
2109.11376

Close Hyperbolic Encounters



$$\frac{v_0\, t}{b} \simeq \frac{e-1}{e+1}(\varphi - \varphi_0)$$

$$P = \frac{32 G \mu^2 v_0^6}{45 c^5 b^2} f(\varphi, e)$$

Power emitted

$$h_c = \frac{2 G \mu v_0^2}{R c^4} g(\varphi, e)$$

Strain Amplitude

$$t_{1/2} \simeq 1ms \left(\frac{b}{10^{-8}\mathrm{AU}}\right)\left(\frac{0.01}{\beta}\right)(e-1)\sqrt{\frac{3\ln 2}{e+35(1+e)e}}$$

Time duration of Burst

Close Hyperbolic Encounters

- Maximum strain at maximum frequency

$$h_{c,\max} = \frac{2.43 \times 10^{-23}}{d_L(\mathrm{Mpc})} \left(\frac{M}{10^{-3} M_\odot} \right) \beta^2 \frac{2}{e-1} \sqrt{18(e+1) + 5e^2}$$

$$f_{\max} = \frac{1\mathrm{GHz}}{\pi} \left(\frac{10^{-8}\mathrm{AU}}{b} \right)^3 \left(\frac{0.01}{\beta} \right)^3 \left(\frac{M}{10^{-3} M_\odot} \right)^2 (e+1)^2$$

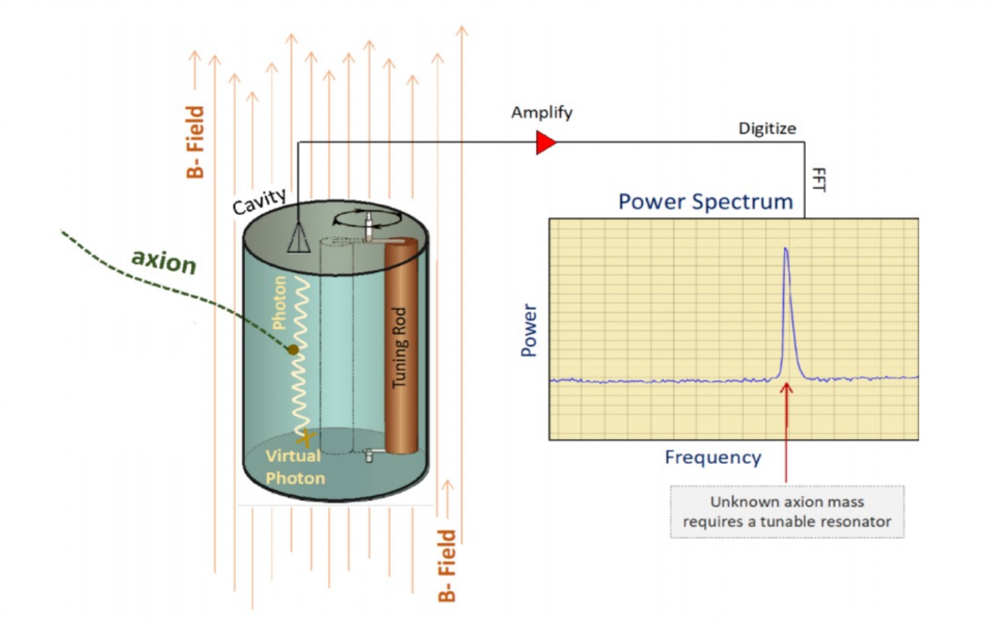
- Maximum GW flux received on Earth

$$F_{\max} = \frac{P_{\max}}{4\pi d_L^2} = 1.2 \times 10^{14}\, W/\mathrm{cm}^2 \;\beta^{10} \frac{(e+1)}{(e-1)^5} \left(\frac{1\mathrm{Mpc}}{d_L} \right)^2$$

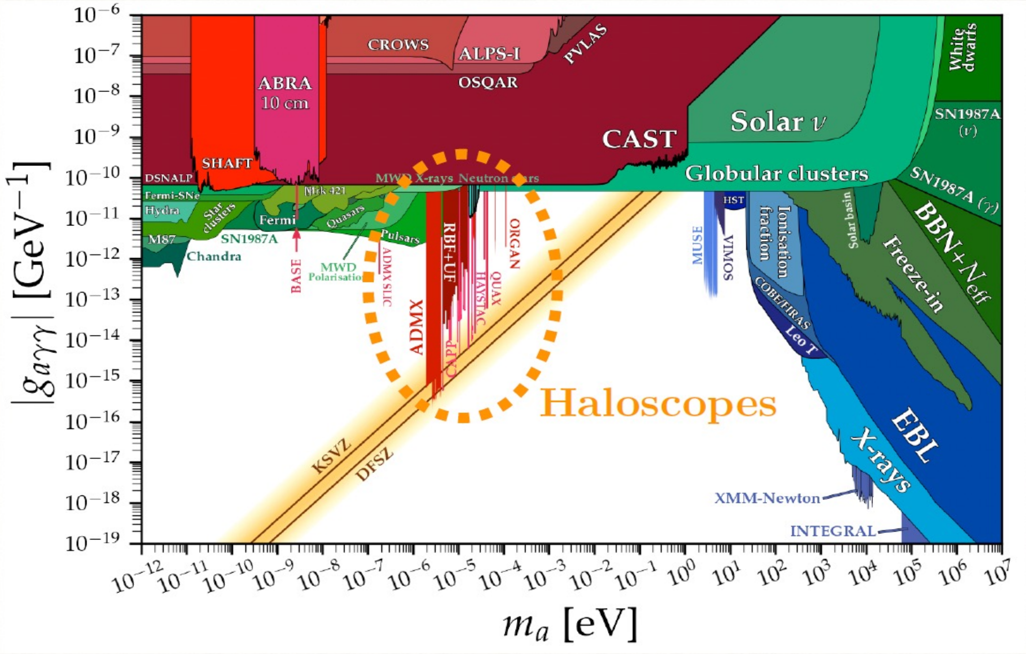
What is a haloscope?

Haloscope: experiment searching for axion dark matter in our galactic halo

Axion DM behaves like a classical oscillating field



Credits: Raphael Cervantes, University of Washington



Credits: https://cajohare.github.io/AxionLimits/

For this study: **GrAHal platform** taken as a benchmark

Grenoble Axion Haloscopes

T. Grenet et. al., arXiv:2110.14406

Different possible configurations

Field	Warm dia.	RF-cavity dia.	Frequency
43 T	34 mm	20 mm	11.5 GHz
40 T	50 mm	34 mm	6.76 GHz
27 T	170 mm	86 mm	2.67 GHz
17.5 T	375 mm	291 mm	0.79 GHz
9.5 T	800 mm	675 mm	0.34 GHz

Ideal for a haloscope

GW electrodynamics

Einstein-Maxwell action:

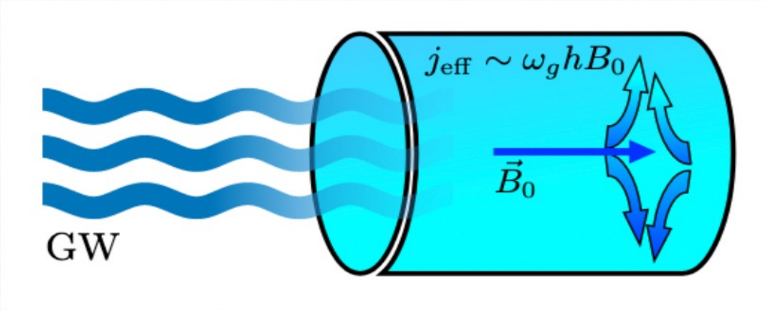
$$S_{EM} = \int d^4x \sqrt{-g} \left(-\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} \right)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \ , \ |h_{\mu\nu}| \ll 1$$

$$S_{EM} = -\frac{1}{4} \int d^4x \sqrt{-g} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \int d^4x \partial_\nu \left[\frac{h}{2} F^{\mu\nu} + h^\nu_\alpha F^{\alpha\mu} - h^\mu_\alpha F^{\alpha\nu} \right] A_\mu + \mathcal{O}\left(h^2\right)$$

Effective current: $j^\mu_{\text{eff}} = \partial_\nu \left(\frac{h}{2} F^{\mu\nu} + h^\nu_\alpha F^{\alpha\mu} - h^\mu_\alpha F^{\alpha\nu} \right)$

Result from Berlin, Blas et. al. , arXiv:2112.11465



The direction of the current depends on the GW properties $\neq$ Axionic current!

The search for hfGWs with resonant cavities

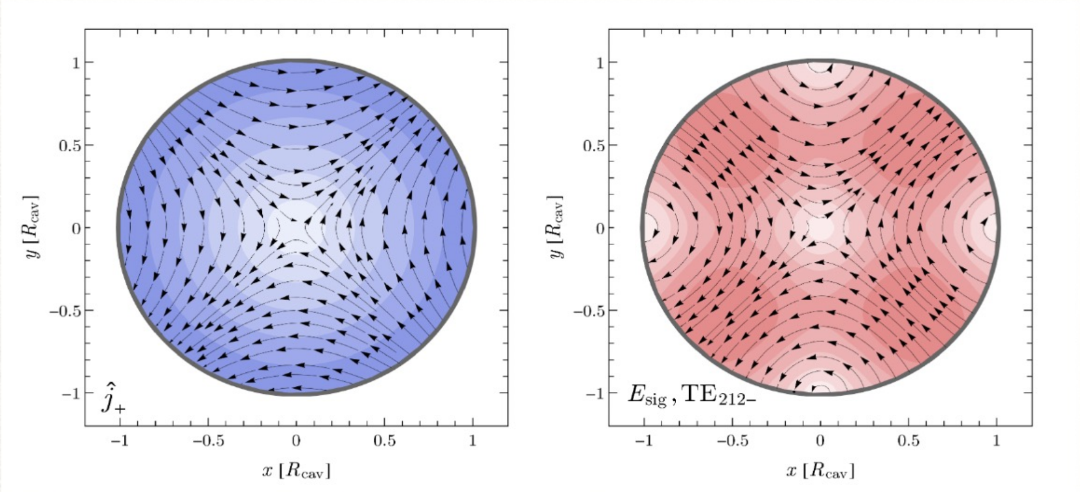
- **GW signal extracted from the cavity**

Result from Berlin et. al. , arXiv:2112.11465

$$P_{\text{sign,GW}} = \frac{1}{2}\omega_{GW}^3 Q V_{\text{cav}}^{5/3} (\eta_n h B)^2$$

Coupling coefficient between the effective current and the cavity modes

$$\eta_n \equiv \frac{|\int_{V_{\text{cav}}} d^3\vec{x}\vec{E}_n^{\star}.\hat{j}_{+,\times}|}{V_{\text{cav}}^{1/2}\left(\int_{V_{\text{cav}}} d^3\vec{x}|\vec{E}_n|^2\right)^{1/2}}$$



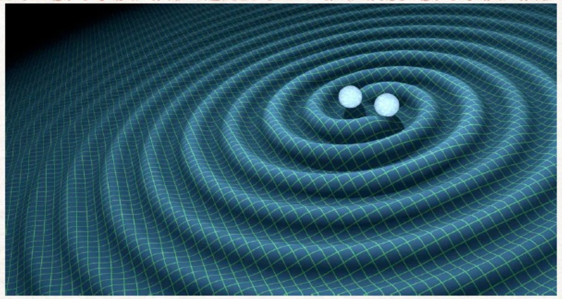
- **Signal to Noise ratio estimated by the radiometer equation:**

$$\text{SNR} \simeq \frac{P_{\text{sig}}}{k_B T_{\text{sys}}}\sqrt{\frac{t_{\text{eff}}}{\Delta\nu}}$$

The search for hfGWs with resonant cavities

- <u>Focus on binary systems of (light) black holes</u>

  A. Barrau, J.G. Bellido, T. Grenet, K. M, arXiv:2303.06006

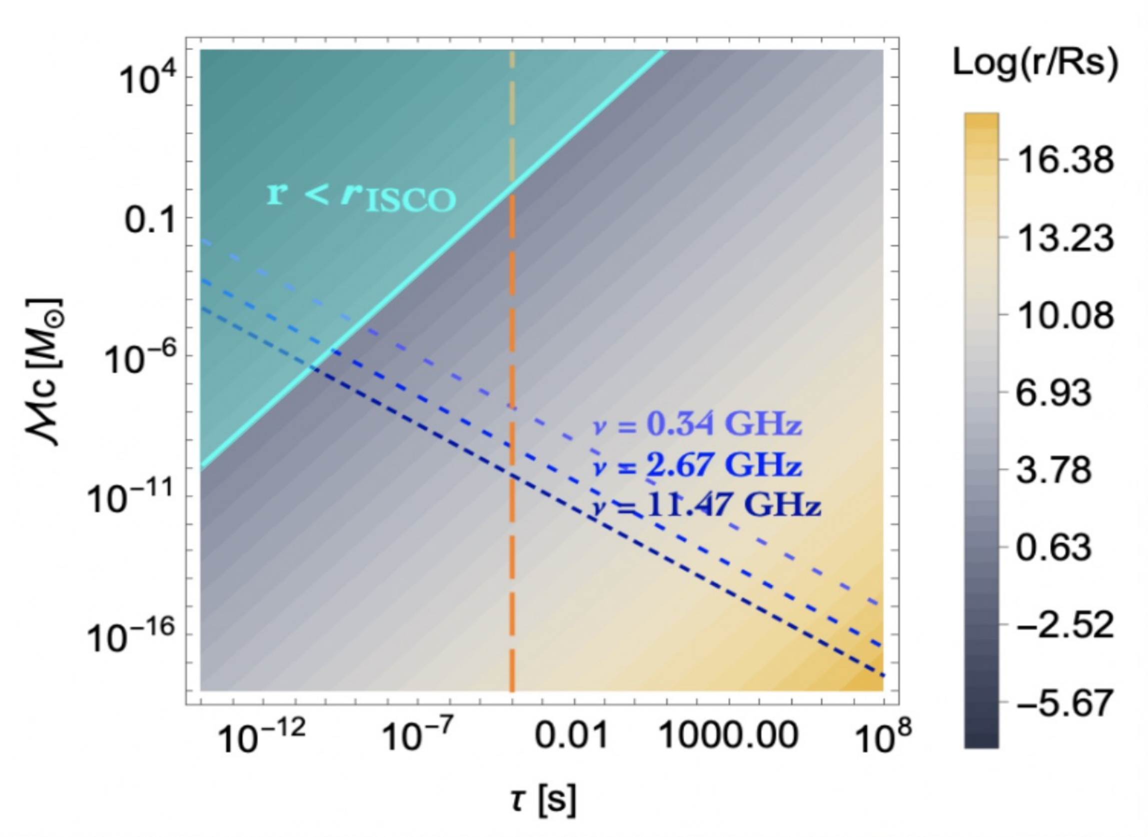


Working at fixed frequency (~ GHz) does not fix the masses!

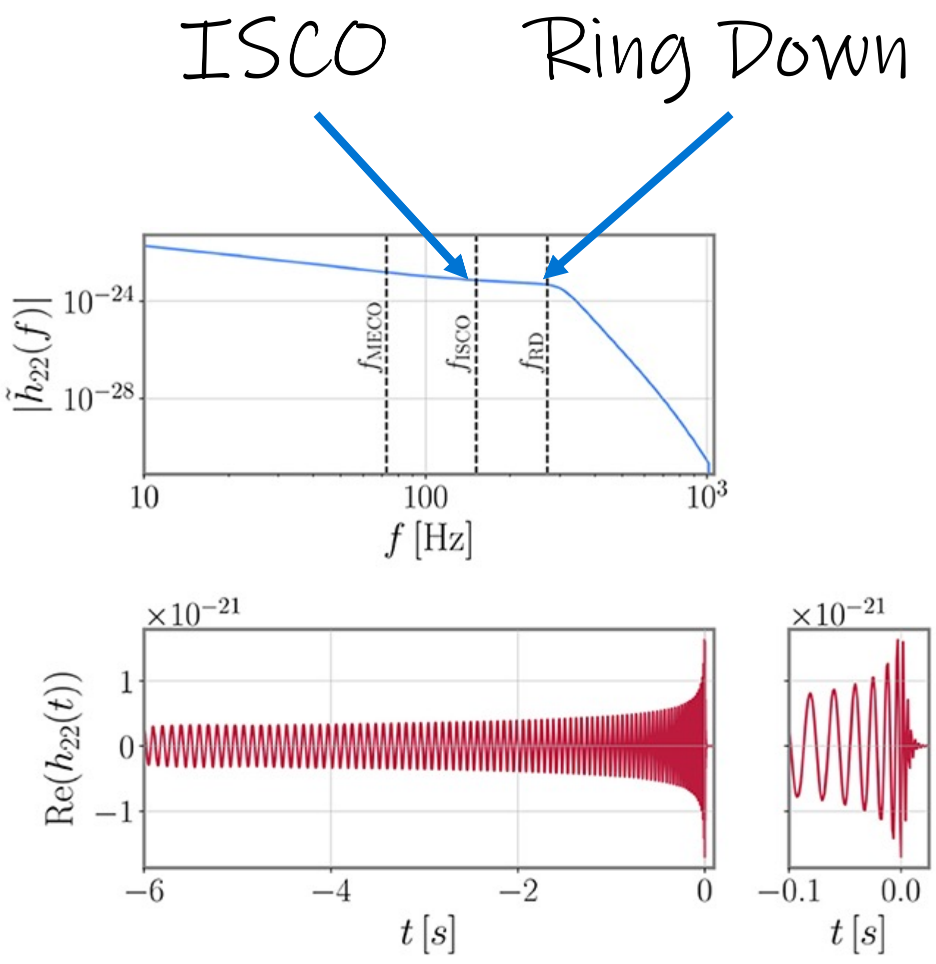
ν : resonant frequency of the detector

τ : time to merger

$$\nu = \frac{1}{\pi}\left(\frac{5}{256}\frac{1}{\tau}\right)^{\frac{3}{8}}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{-\frac{5}{8}}$$



Frequency vs time (for chirps)



$$\dot{f} = \frac{96}{5}\pi^{8/3}\left(\frac{GM_c}{c^3}\right)^{5/3} f^{11/3}$$

$$\tau = t_{\rm coal} - t = 1\,s\left(\frac{f}{134\,\mathrm{Hz}}\right)^{-8/3}\left(\frac{M}{1.4\,M_\odot}\right)^{-5/3}$$

$$= 8.24\,ns\left(\frac{f}{1\,\mathrm{GHz}}\right)^{-8/3}\left(\frac{M}{10^{-6}\,M_\odot}\right)^{-5/3}$$

$$\tau\left(f_{\rm ISCO} = \frac{4.4\,\mathrm{kHz}}{M/M_\odot}\right) = 1\,ms\,\frac{M}{M_\odot} = 1\,ns\,\frac{M}{10^{-6}M_\odot}$$

The search for hfGWs with resonant cavities

$$\mathrm{SNR} \simeq \frac{P_{\mathrm{sig}}}{k_B T_{\mathrm{sys}}}\sqrt{\frac{t_{\mathrm{eff}}}{\Delta\nu}} \quad + \quad P_{\mathrm{sign,GW}} = \frac{1}{2}\omega_{GW}^3 Q V_{\mathrm{cav}}^{5/3} \left(\eta_n h B\right)^2$$

GrAHal
Grenoble Axion Haloscopes

$\mathrm{SNR} > 1 \Rightarrow$ **Sensitivity estimates:**

$$h > 4.7\times10^{-22} \times \left(\frac{0.34\ \mathrm{GHz}}{\nu}\right)^{\frac{5}{4}}\left(\frac{0.1}{\eta}\right)\left(\frac{9\ \mathrm{T}}{B_0}\right)\left(\frac{5.01\times10^{-1}\ \mathrm{m}^3}{V_{\mathrm{cav}}}\right)^{\frac{5}{6}}\left(\frac{10^5}{Q}\right)^{\frac{3}{4}}\left(\frac{T_{\mathrm{sys}}}{0.3\ \mathrm{K}}\right)^{\frac{1}{2}}\left(\frac{1\ \mathrm{s}}{t_{\mathrm{eff}}}\right)^{\frac{1}{4}}$$

$$\Leftrightarrow h > 1.5\times10^{-21} \times \left(\frac{2.67\ \mathrm{GHz}}{\nu}\right)^{\frac{5}{4}}\left(\frac{0.1}{\eta}\right)\left(\frac{27\ \mathrm{T}}{B_0}\right)\left(\frac{1.83\times10^{-3}\ \mathrm{m}^3}{V_{\mathrm{cav}}}\right)^{\frac{5}{6}}\left(\frac{10^5}{Q}\right)^{\frac{3}{4}}\left(\frac{T_{\mathrm{sys}}}{0.4\ \mathrm{K}}\right)^{\frac{1}{2}}\left(\frac{1\ \mathrm{s}}{t_{\mathrm{eff}}}\right)^{\frac{1}{4}}$$

$$\Leftrightarrow h > 4.8\times10^{-21} \times \left(\frac{11.47\ \mathrm{GHz}}{\nu}\right)^{\frac{5}{4}}\left(\frac{0.1}{\eta}\right)\left(\frac{43\ \mathrm{T}}{B_0}\right)\left(\frac{4.93\times10^{-5}\ \mathrm{m}^3}{V_{\mathrm{cav}}}\right)^{\frac{5}{6}}\left(\frac{10^5}{Q}\right)^{\frac{3}{4}}\left(\frac{T_{\mathrm{sys}}}{1.0\ \mathrm{K}}\right)^{\frac{1}{2}}\left(\frac{1\ \mathrm{s}}{t_{\mathrm{eff}}}\right)^{\frac{1}{4}}$$

Extremely encouraging

But ...

What about this value?

Hypothesis made:

The signal must remain coherent and located in the experimental frequency bandwidth during at least 1s

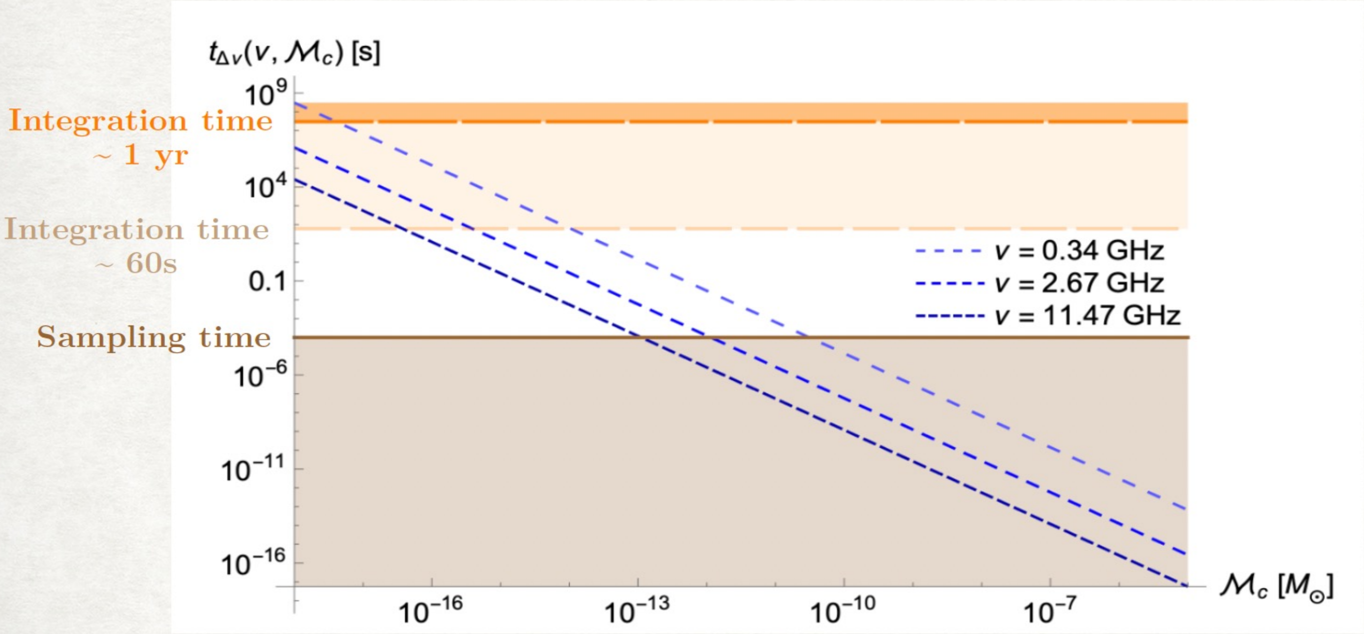
Is it really possible?

The search for hfGWs with resonant cavities

- The frequency of GWs coming from binary systems drifts with time $\dot{f}(\nu) = \frac{96}{5}\pi^{\frac{8}{3}}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{\frac{5}{3}}\nu^{\frac{11}{3}}$

- Time during which the signal drifts in the frequency sensitivity bandwidth: $t_{\Delta\nu} \sim \frac{\Delta\nu}{\dot{f}(\nu)} = \frac{\nu}{Q\dot{f}(\nu)}$

$$t_{\Delta\nu} \sim \frac{5}{96}\pi^{-\frac{8}{3}}\nu^{-\frac{8}{3}}Q^{-1}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{-\frac{5}{3}}$$



Fast decrease of the signal duration with the mass

The heavier the BHs, the closer they are to their merging

The search for hfGWs with resonant cavities

$$\mathrm{SNR} \simeq \frac{P_{\rm sig}}{k_B T_{\rm sys}}\sqrt{\frac{t_{\rm eff}}{\Delta\nu}} > 1$$

3 different regimes:

1) Effective time given by the signal frequency drift through the frequency bandwidth of the cavity

$$t_{\rm eff} = t_{\Delta\nu}$$

$$h > 2.0\times 10^{-21} \times \left(\frac{2.67\ \mathrm{GHz}}{\nu}\right)^{\frac{7}{12}}\left(\frac{0.1}{\eta}\right)\left(\frac{27\ \mathrm{T}}{B_0}\right)\left(\frac{1.83\times 10^{-3}\mathrm{m}^3}{V_{\rm cav}}\right)^{\frac{5}{6}}\left(\frac{10^5}{Q}\right)^{\frac{1}{2}}\left(\frac{T_{\rm sys}}{0.4\ \mathrm{K}}\right)^{\frac{1}{2}}\left(\frac{\mathcal{M}c}{10^{-14}M_\odot}\right)^{\frac{5}{12}}$$

2) Effective time limited by the duration of the experiment *Very small chirp masses*

The signal would spend "more time than available" within the cavity bandwidth

$$t_{\Delta\nu} > t_{\rm max} \Rightarrow t_{\rm eff} = t_{\rm max}$$

$$h > 5.3\times 10^{-22} \times \left(\frac{2.67\ \mathrm{GHz}}{\nu}\right)^{\frac{5}{4}}\left(\frac{0.1}{\eta}\right)\left(\frac{27\ \mathrm{T}}{B_0}\right)\left(\frac{1.83\times 10^{-3}\ \mathrm{m}^3}{V_{\rm cav}}\right)^{\frac{5}{6}}\left(\frac{10^5}{Q}\right)^{\frac{3}{4}}\left(\frac{T_{\rm sys}}{0.4\ \mathrm{K}}\right)^{\frac{1}{2}}\left(\frac{60\ \mathrm{s}}{t_{\rm max}}\right)^{\frac{1}{4}}$$

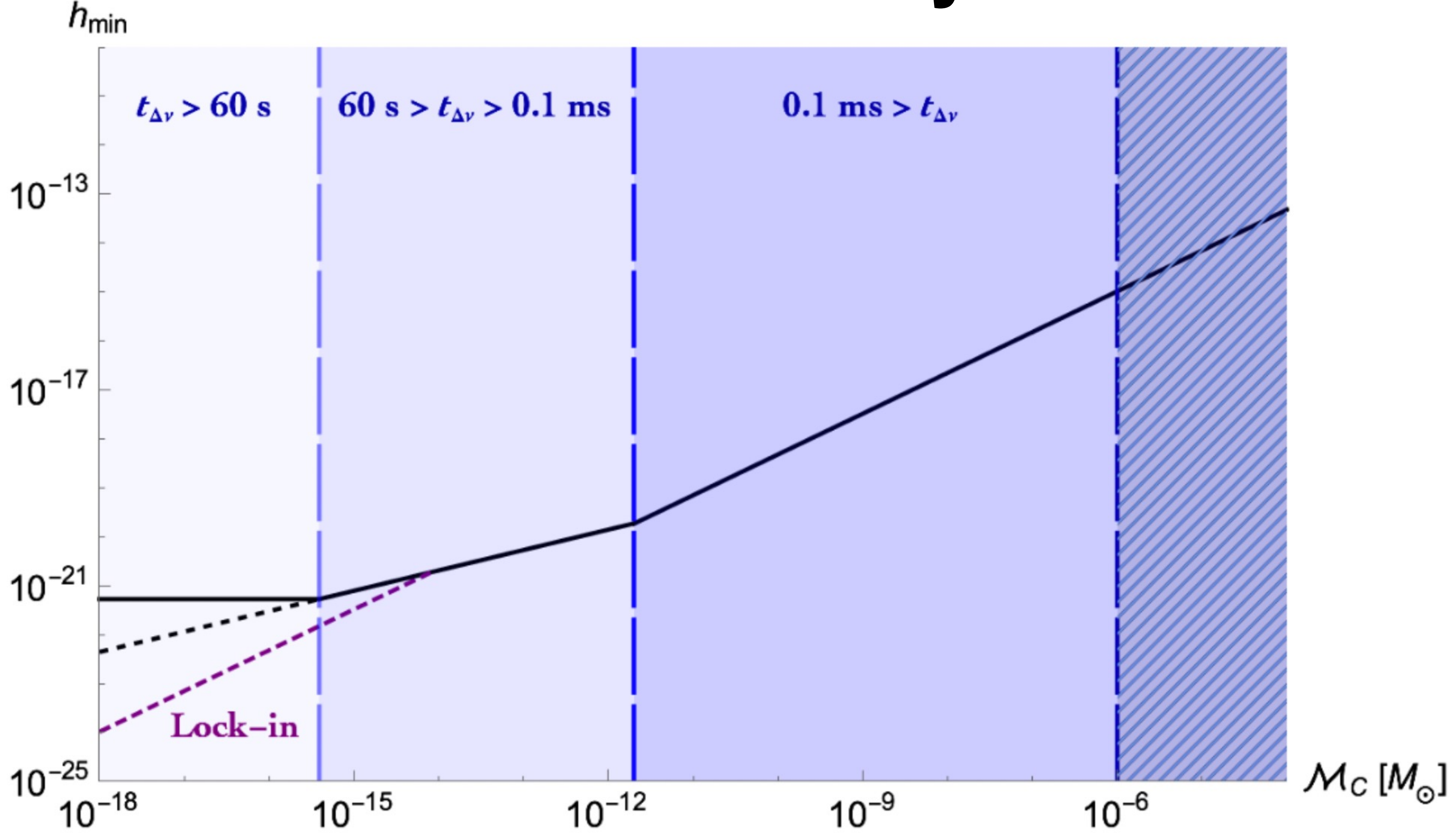
3) Effective time limited by the sampling rate *Highest chirp masses accessible*

The time spent by the signal within the experimental bandwidth is smaller than the inverse sampling frequency

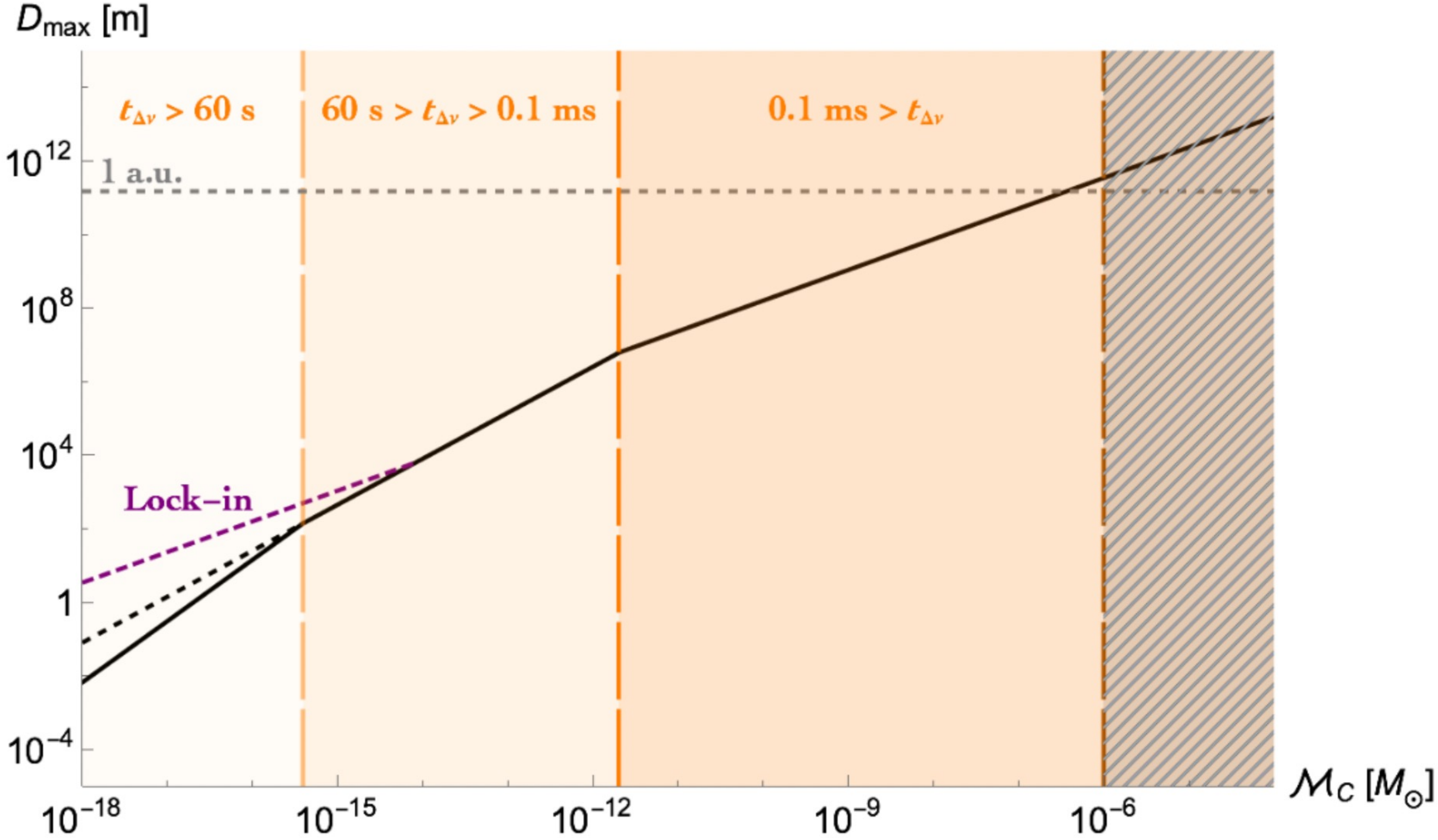
$$t_{\rm eff} = t_{\Delta\nu}^2/t_{\rm min}$$

$$h > 3.3\times 10^{-18} \times \left(\frac{2.67\ \mathrm{GHz}}{\nu}\right)^{\frac{1}{6}}\left(\frac{0.1}{\eta}\right)\left(\frac{27\ \mathrm{T}}{B_0}\right)\left(\frac{1.83\times 10^{-3}\ \mathrm{m}^3}{V_{\rm cav}}\right)^{\frac{5}{6}}\left(\frac{T_{\rm sys}}{0.4\ \mathrm{K}}\right)^{\frac{1}{2}}\left(\frac{\mathcal{M}c}{10^{-9}M_\odot}\right)^{\frac{5}{6}}$$

Strain sensitivity



Accesible distance



Summary

- PBH sources of GW in MHz-GHz range
- Typically weak and short bursts of GW
- Resonant EM cavities (e.g. GrAHal) may work
- Caveat: sampling, drift & integration time make detection of PBH mergers unlikely
    - Ways out? i) heterodyne modulation: linear in h
      ii) broad band detector