Resonant Cavities for Gravitational Waves

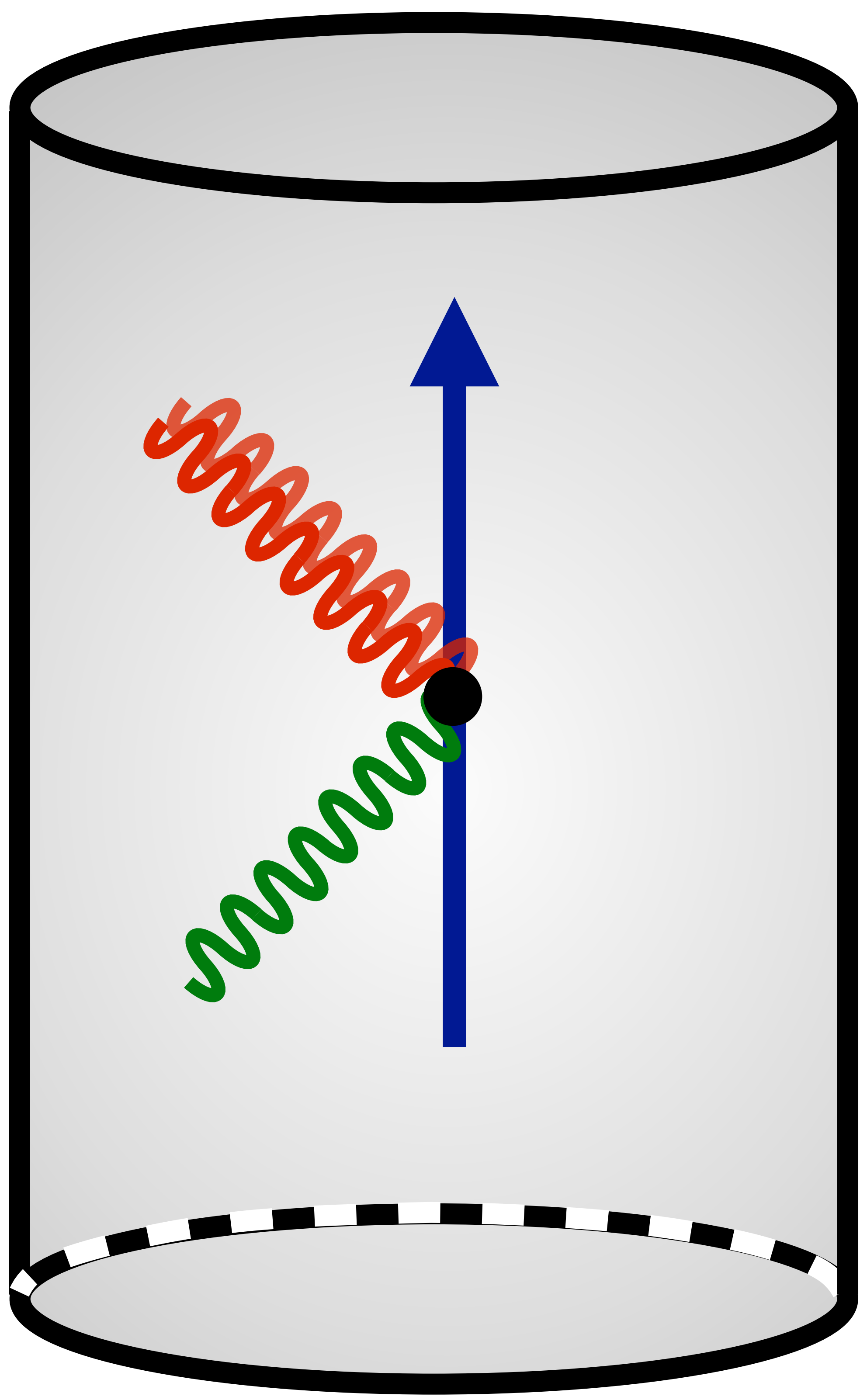
Sebastian A. R. Ellis

University of Geneva

w/ A. Berlin, D. Blas, R. T. D'Agnolo, R. Harnik, Y. Kahn, J. Schütte-Engel *& M. Wentzel*

arXiv:2112.11465 *arXiv:2303.01518*

Resonant Cavities for Gravitational Waves



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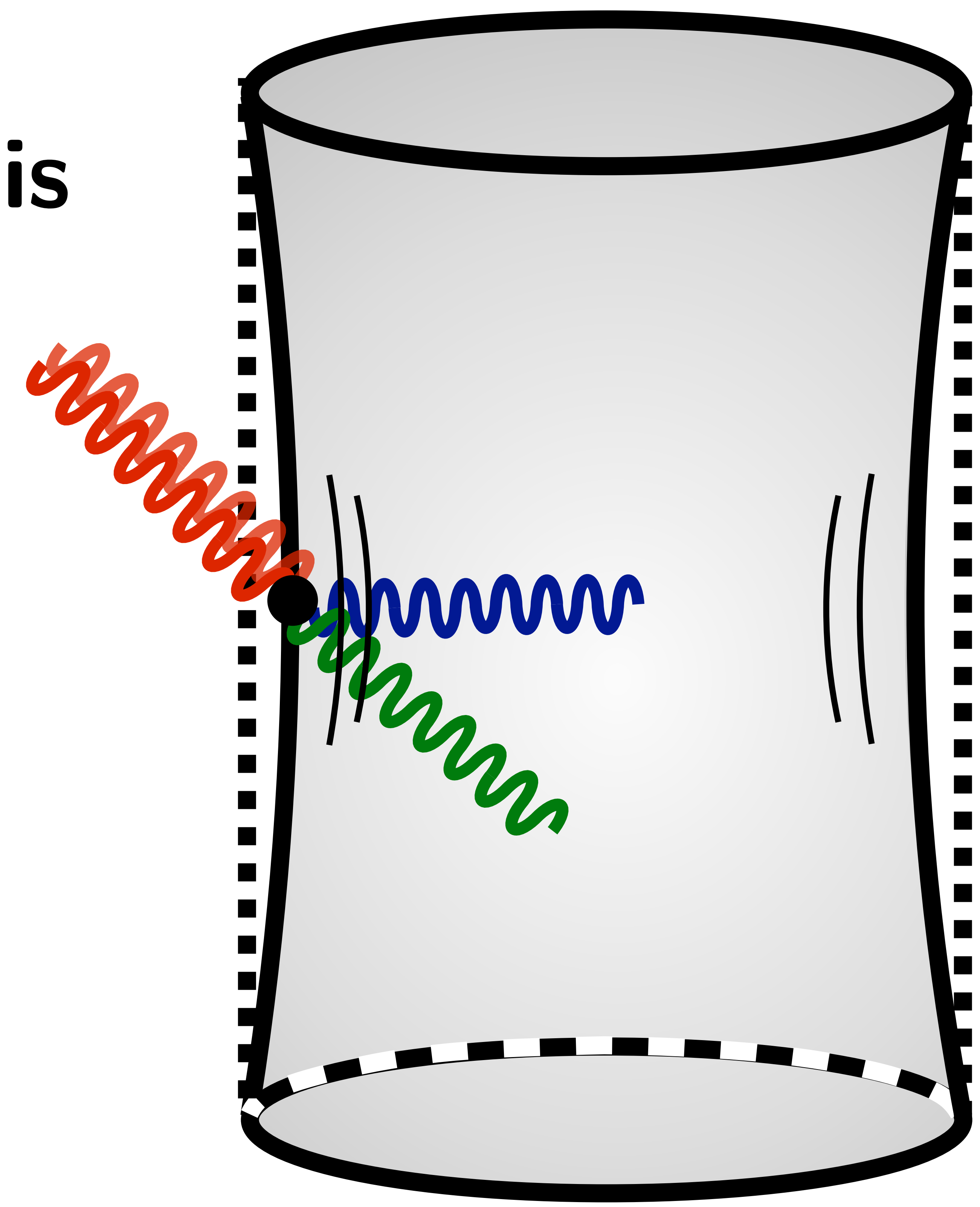

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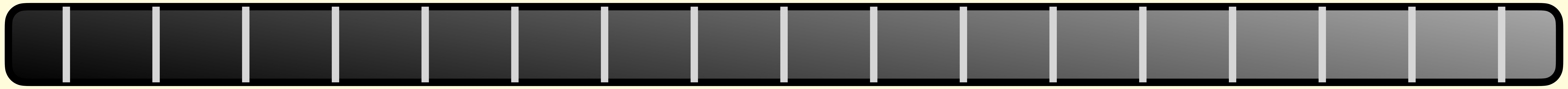
WARMUP



When is the ruler rigid?

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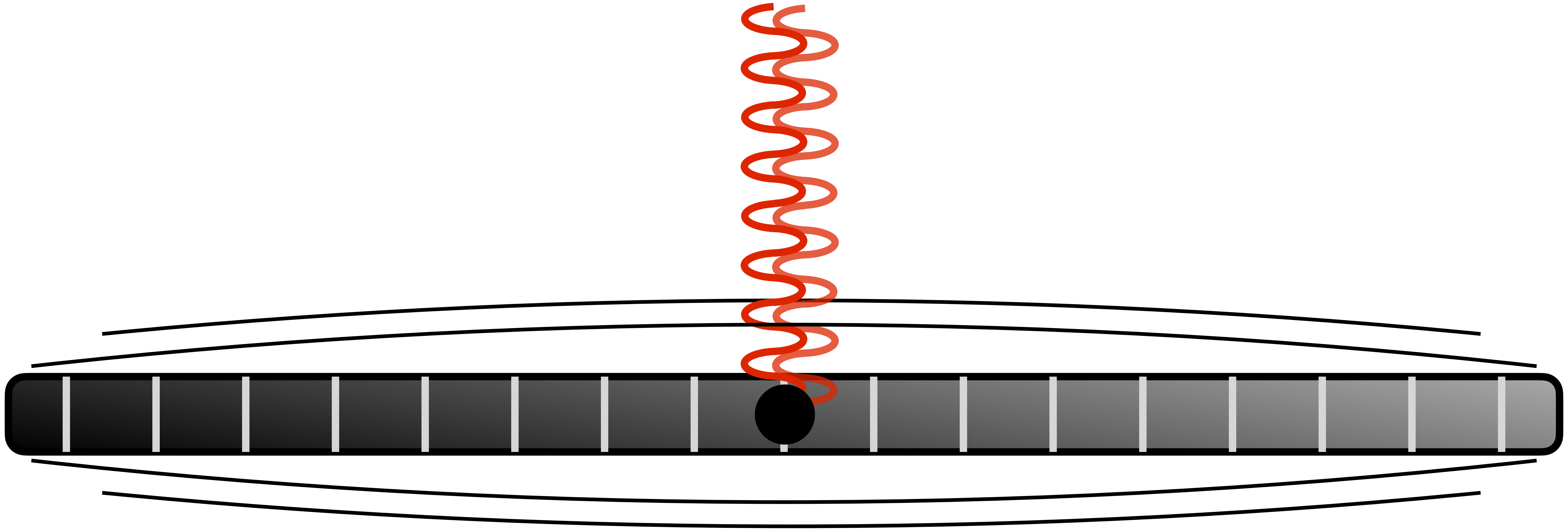
Incoming GW: long wavelength



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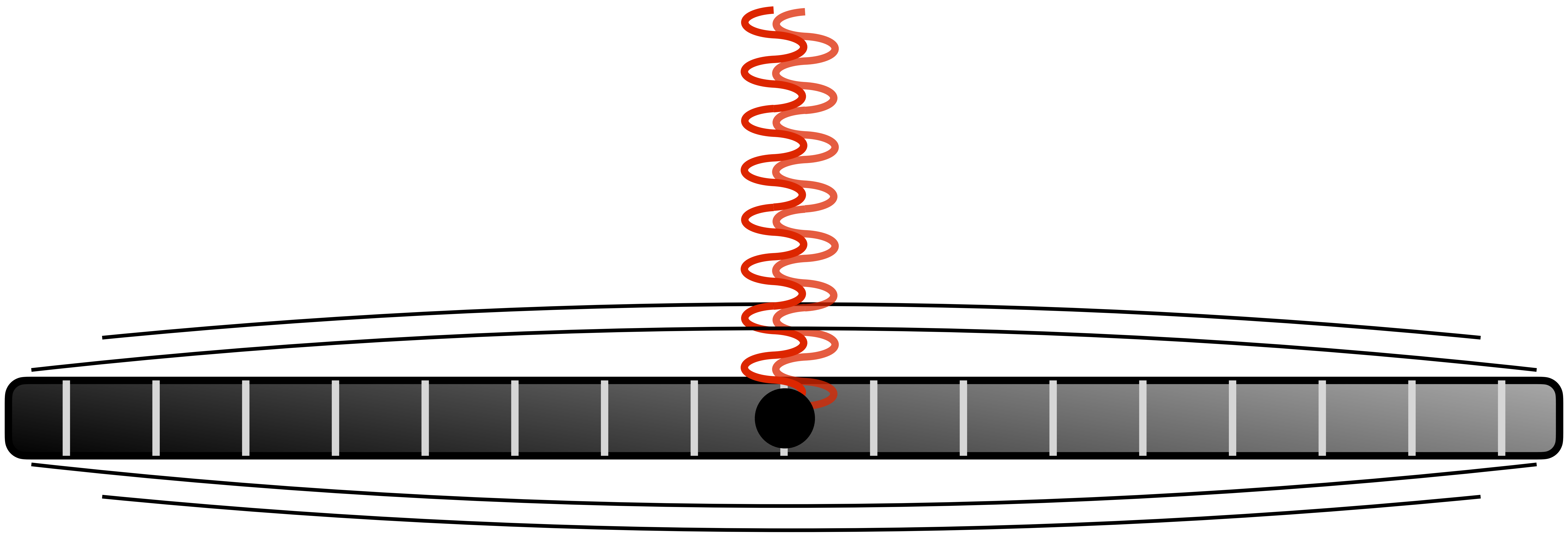

The whole object moves back and forth

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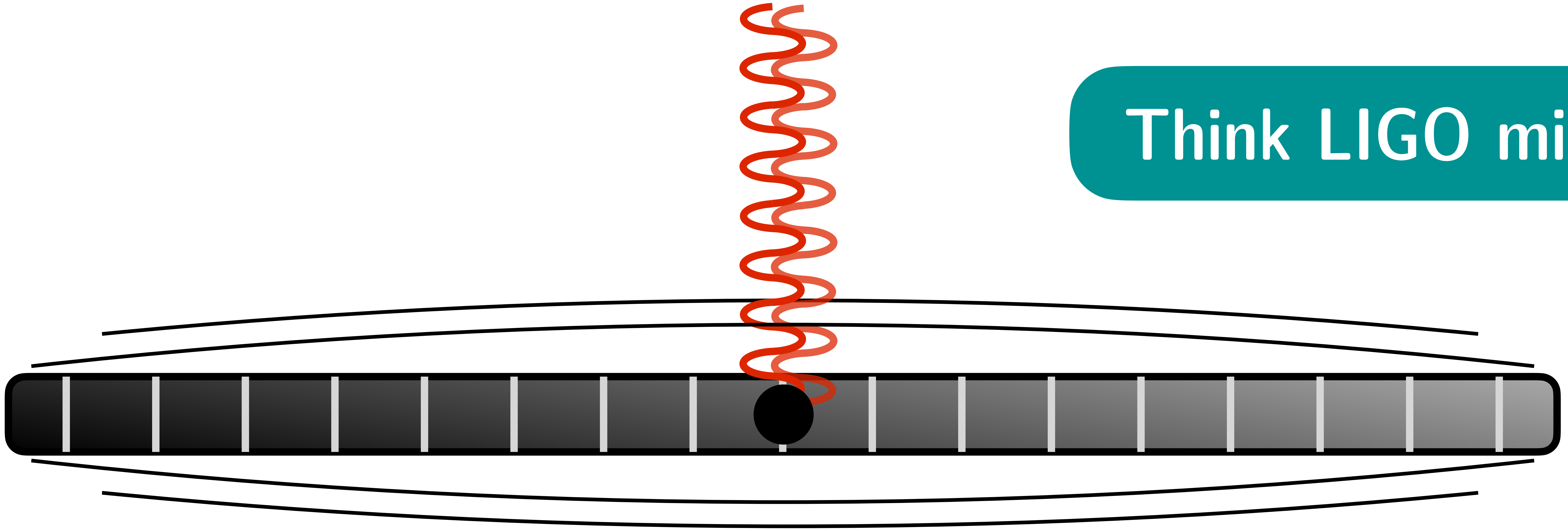
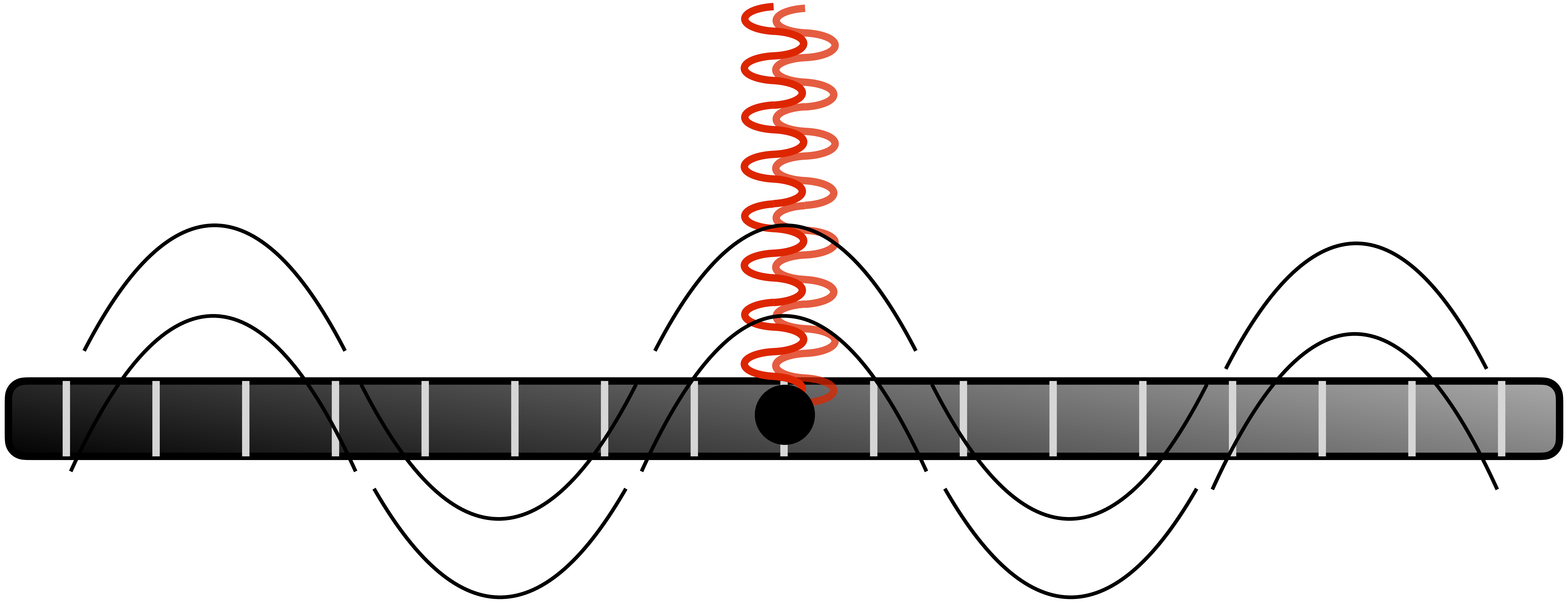


Think LIGO mirror

The whole object moves back and forth

When is the ruler rigid?

Incoming GW: matched wavelength



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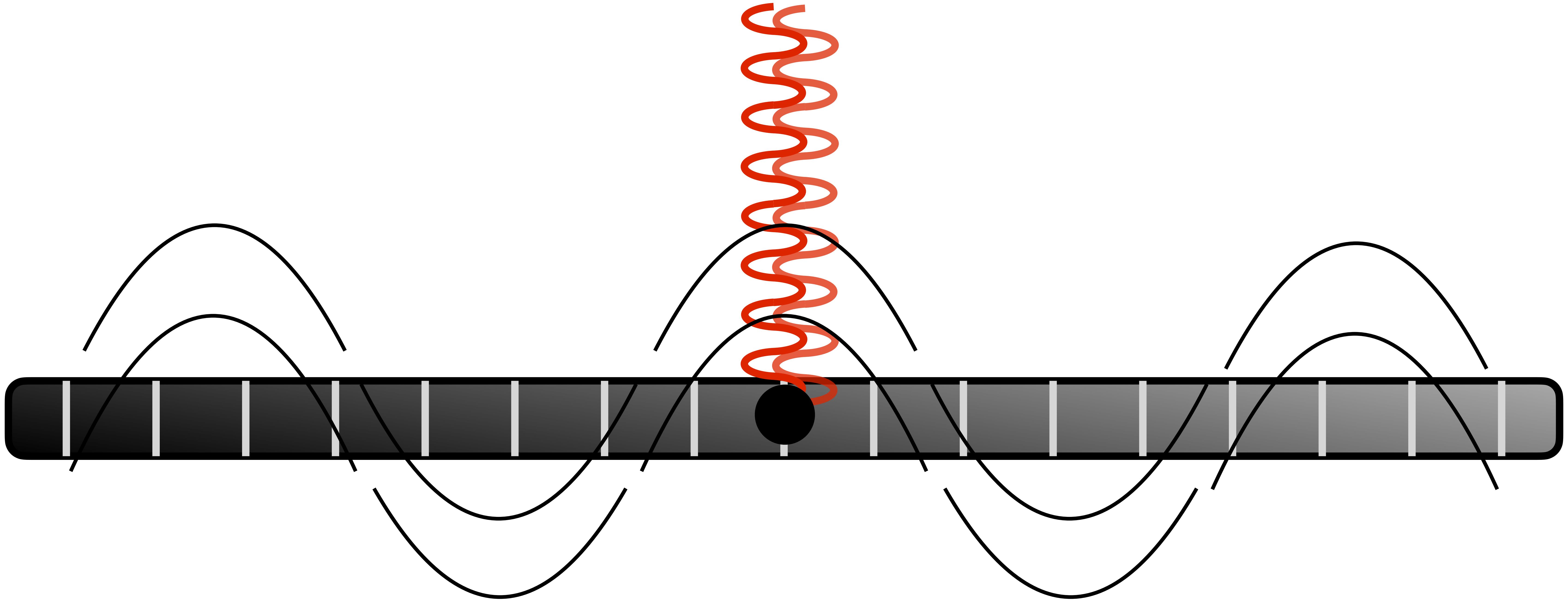

Resolve structure of the ruler, e.g. resonances

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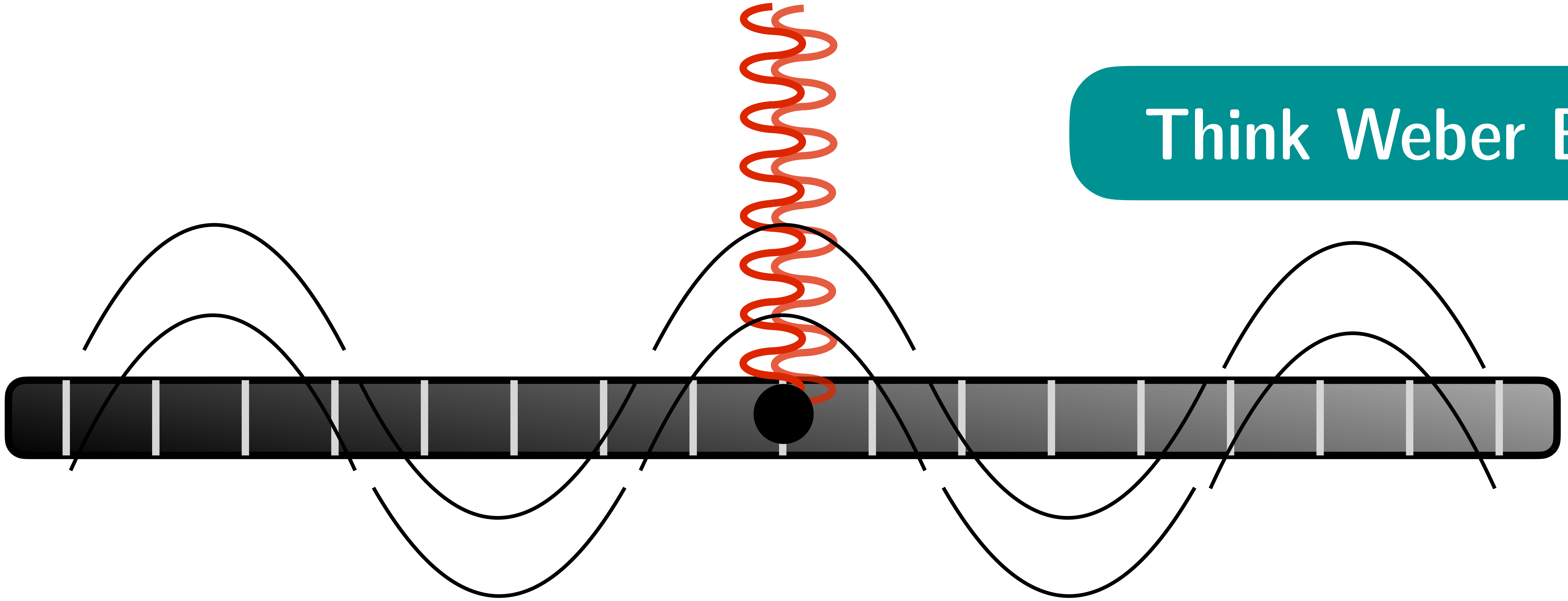
Incoming GW: matched wavelength



Think Weber Bar

Resolve structure of the ruler, e.g. resonances

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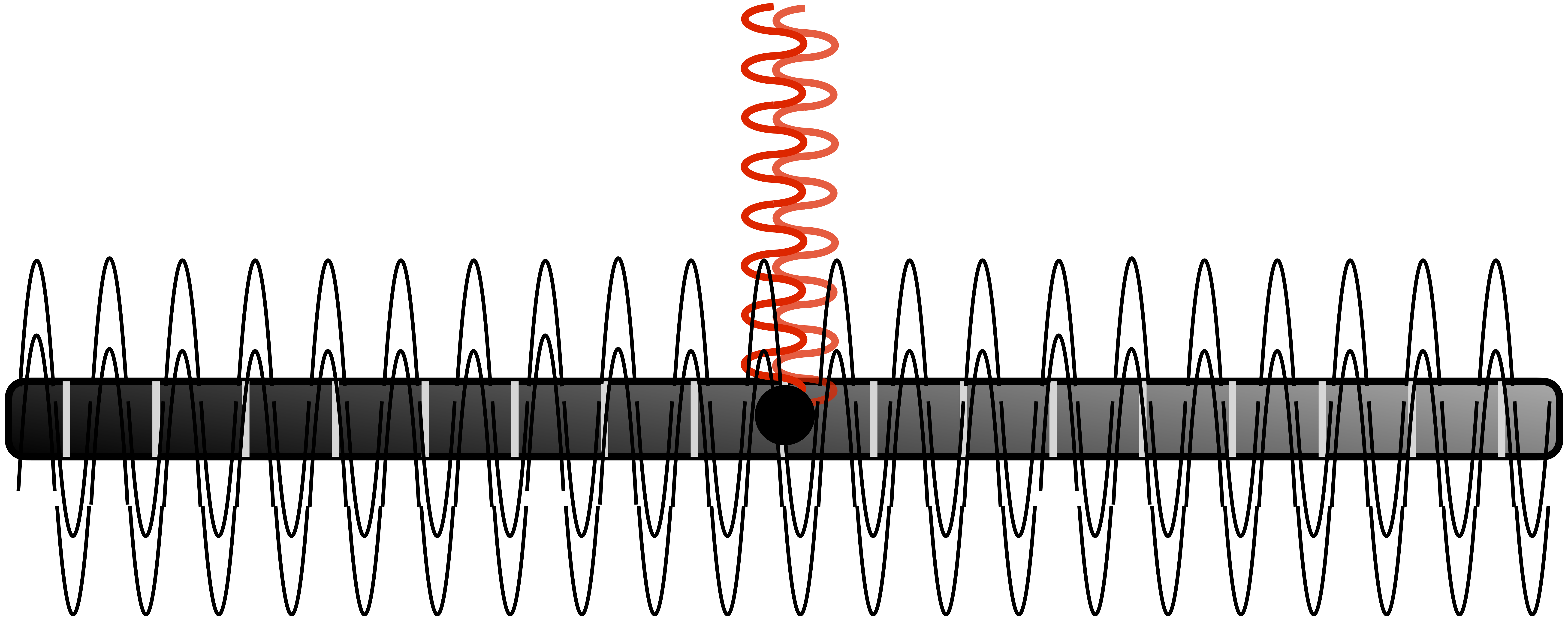

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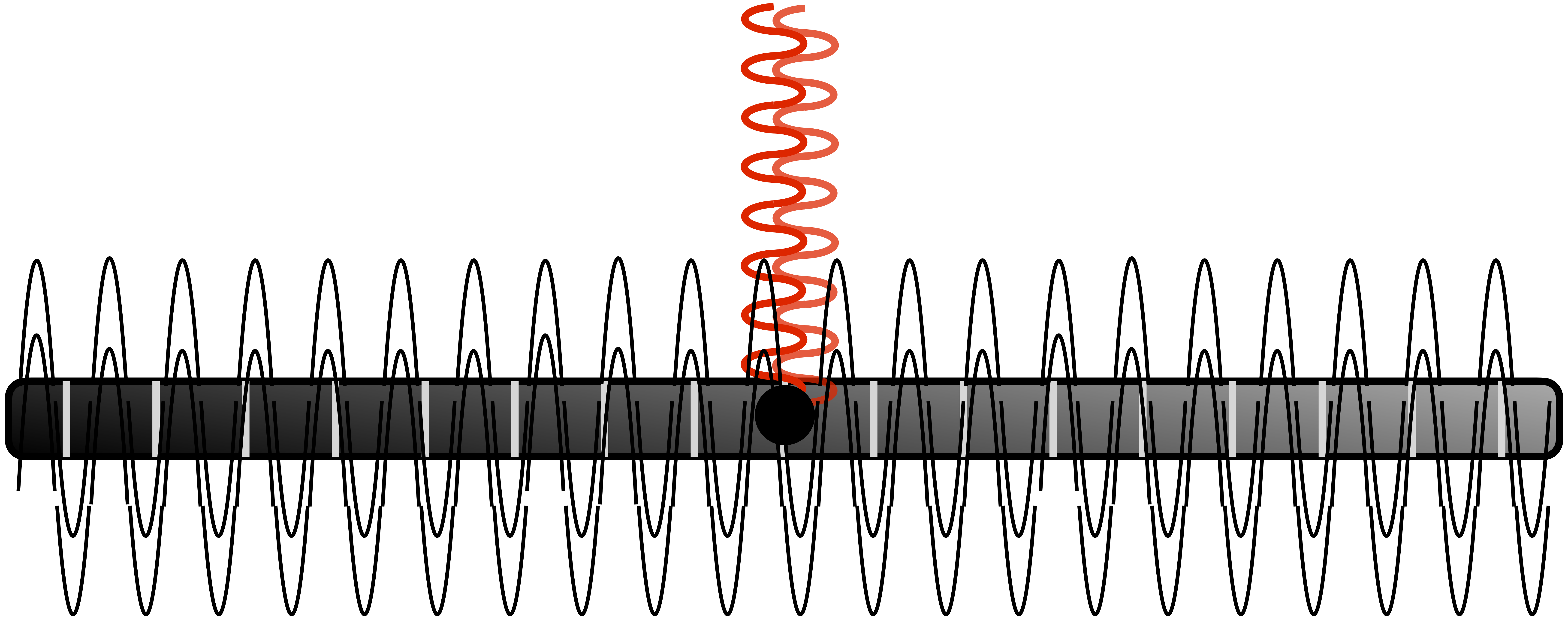
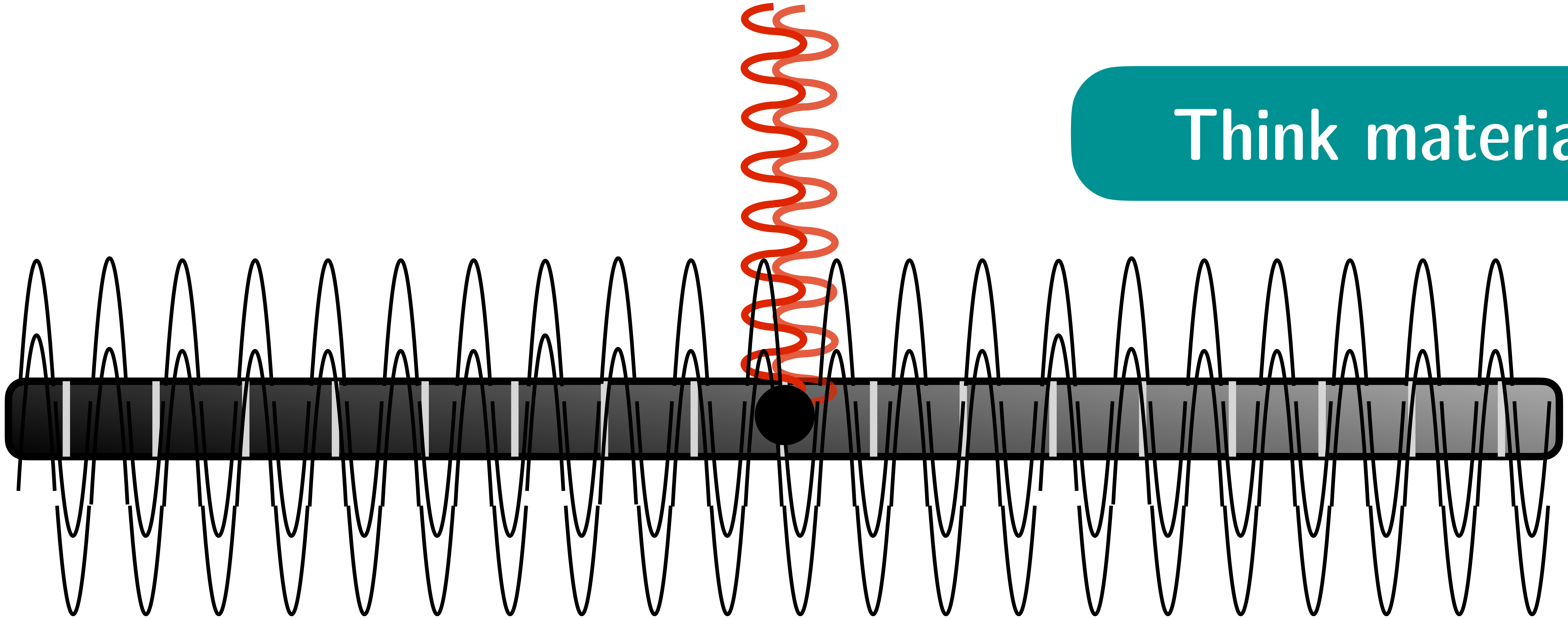


Ruler fully flexible

When is the ruler rigid?

Incoming GW: short wavelength

Think materials



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At the level of equations:

$$\partial_t^2 s_i + k^2 s_i \sim \partial_t^2 h_{ij} s_j$$

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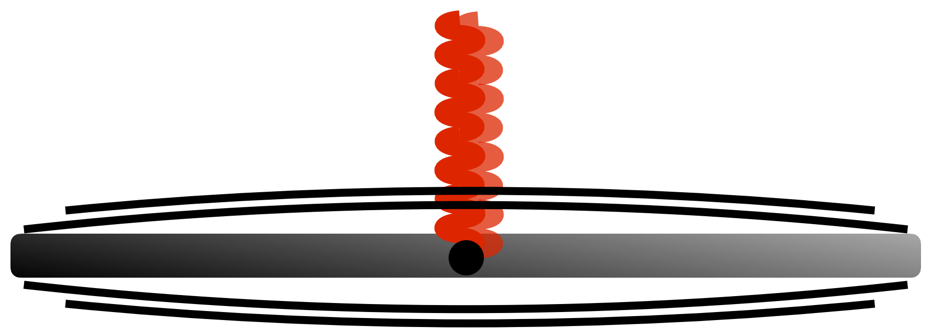

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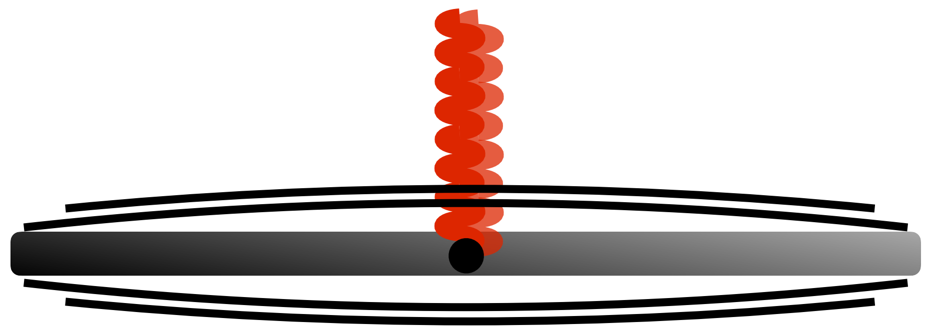

Looks like proper detector frame k~1/L

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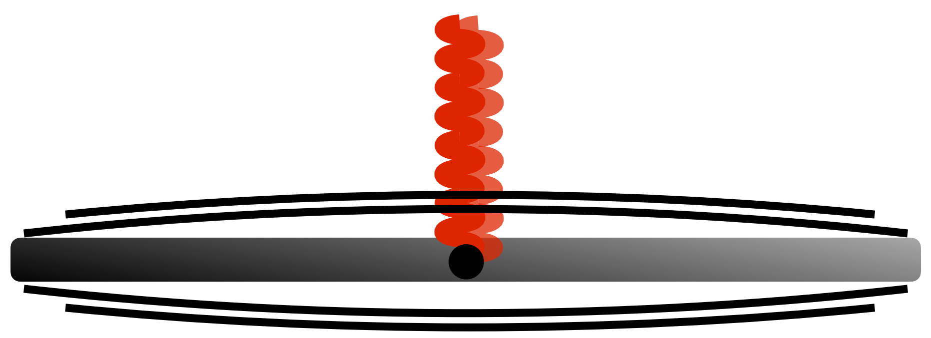

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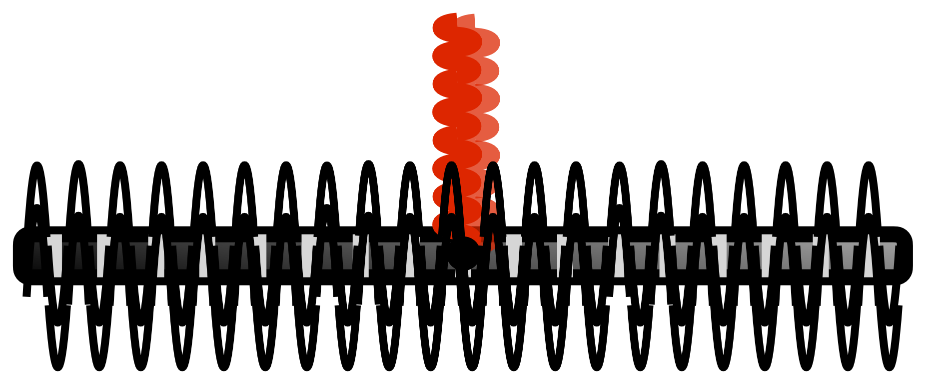

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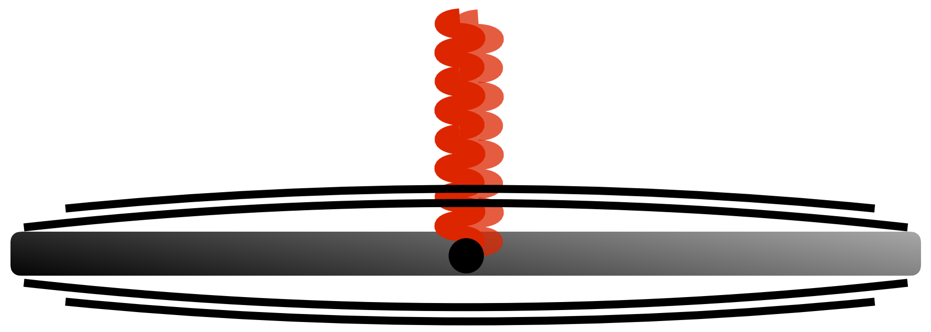

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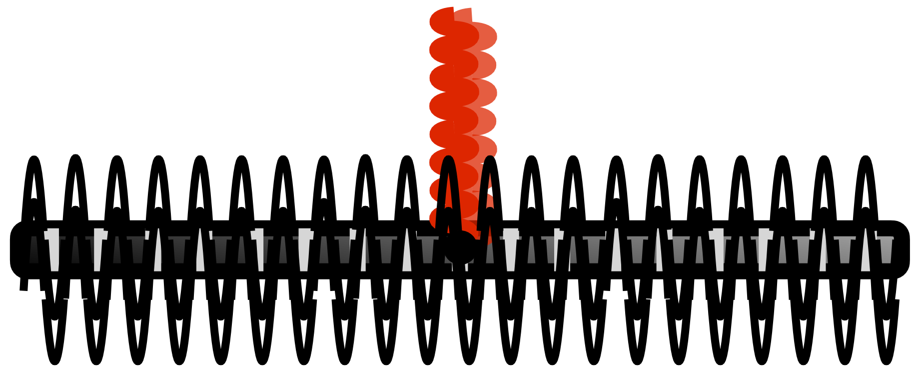

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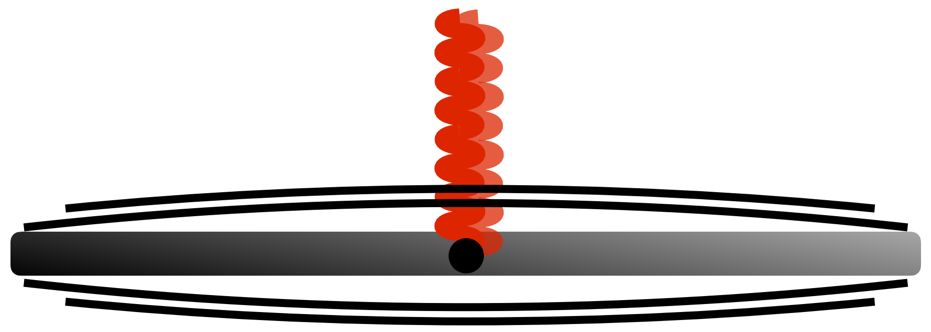

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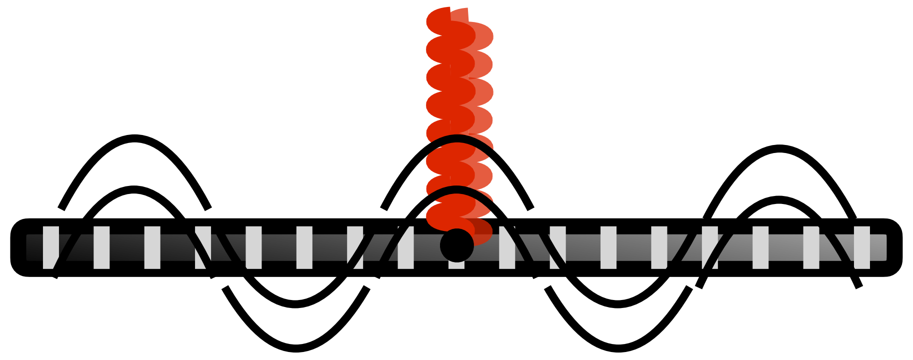
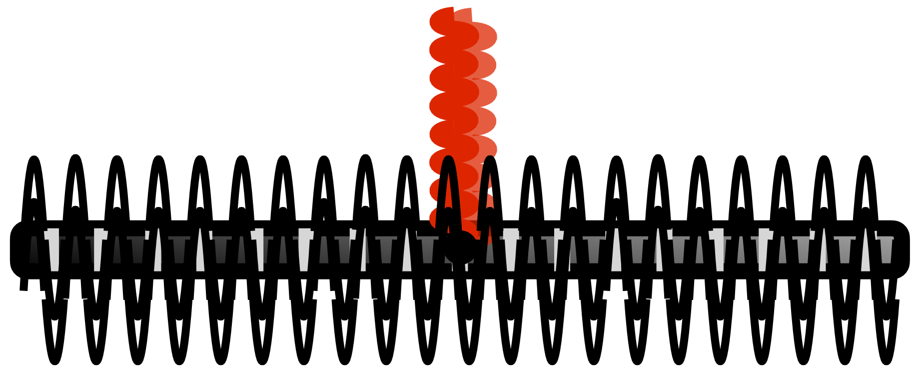


Case requires knowledge of detector

$$s_i \sim h_{ij} s_j$$



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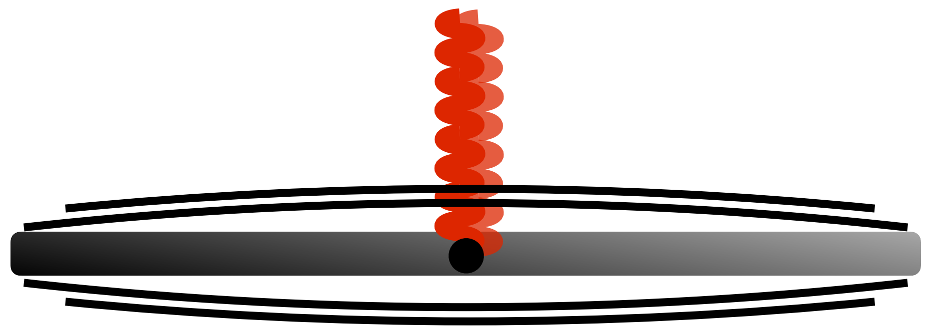

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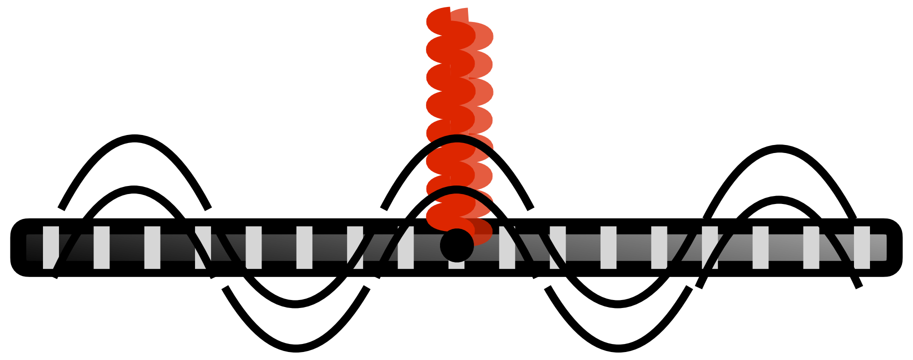

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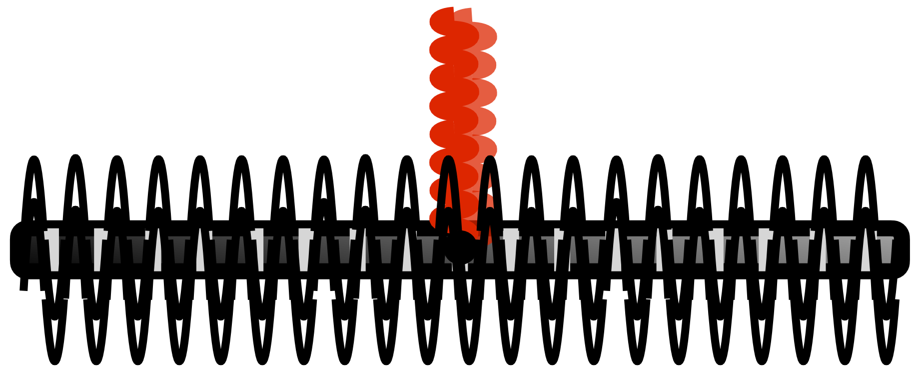

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Frame subtleties crucial!

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NB: don't include for LIGO mirror

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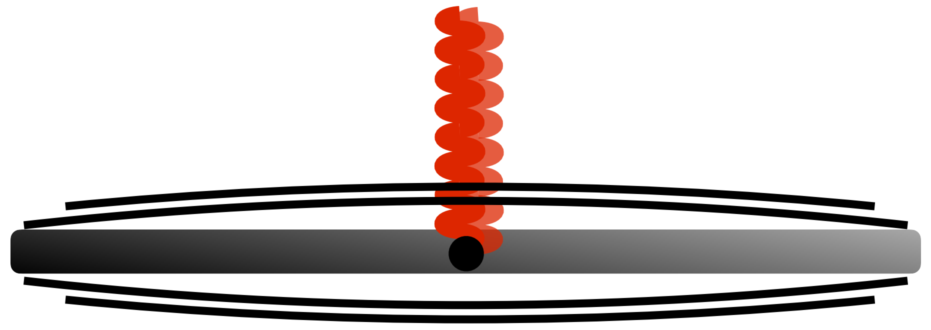

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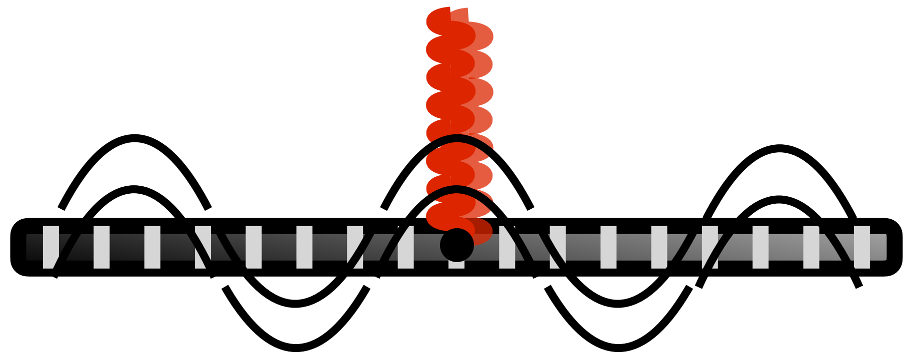

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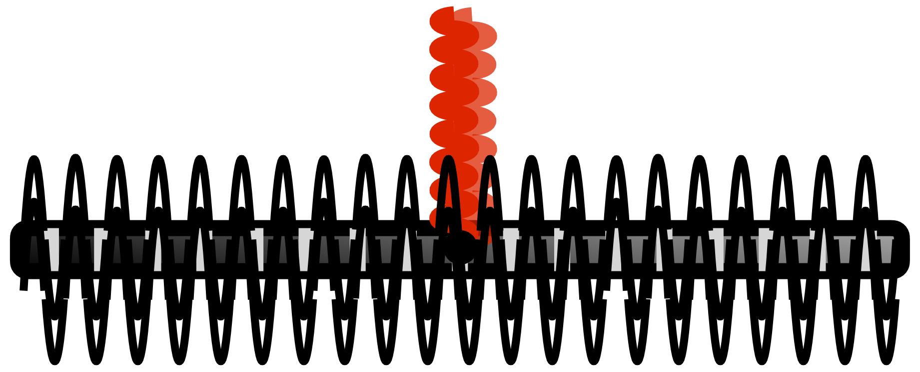

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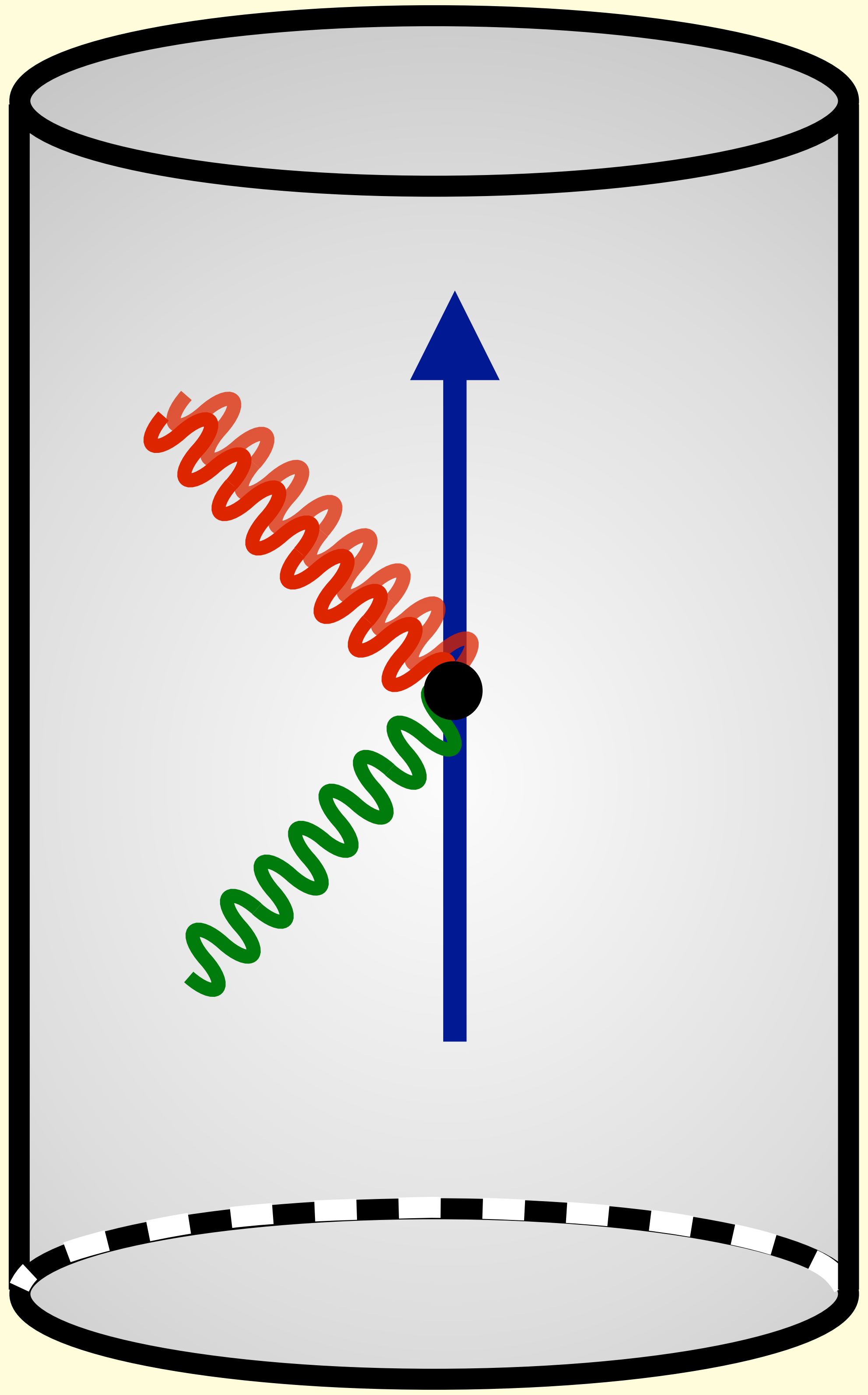
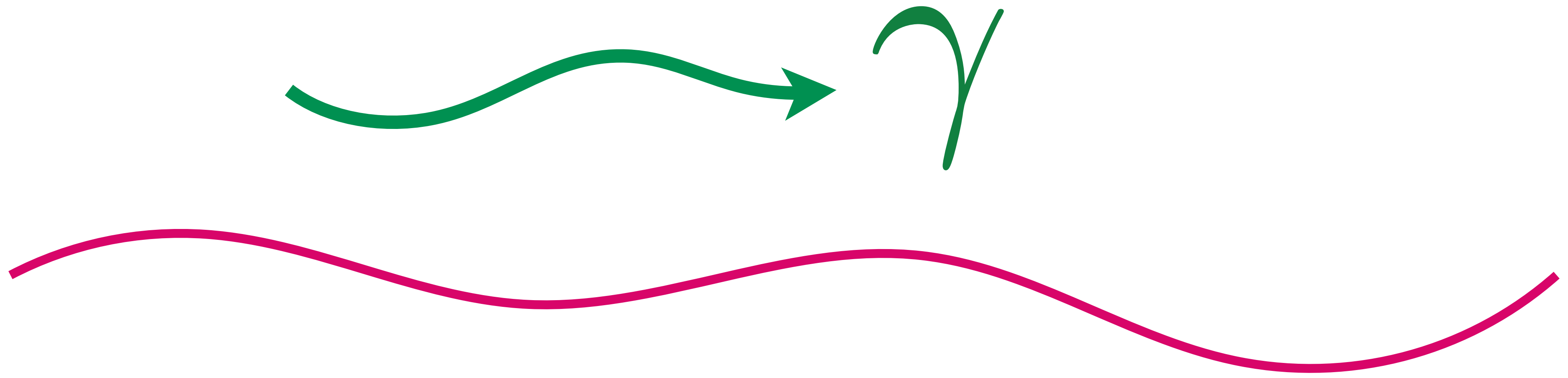
ACTE I

Light is the Ruler:
Static B-field Cavities



Interactions of Gravitational Waves *with light*



$$S_{\rm EM} = \int d^4x\sqrt{-g}\left(-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + g^{\mu\nu}J_\mu A_\nu\right)$$

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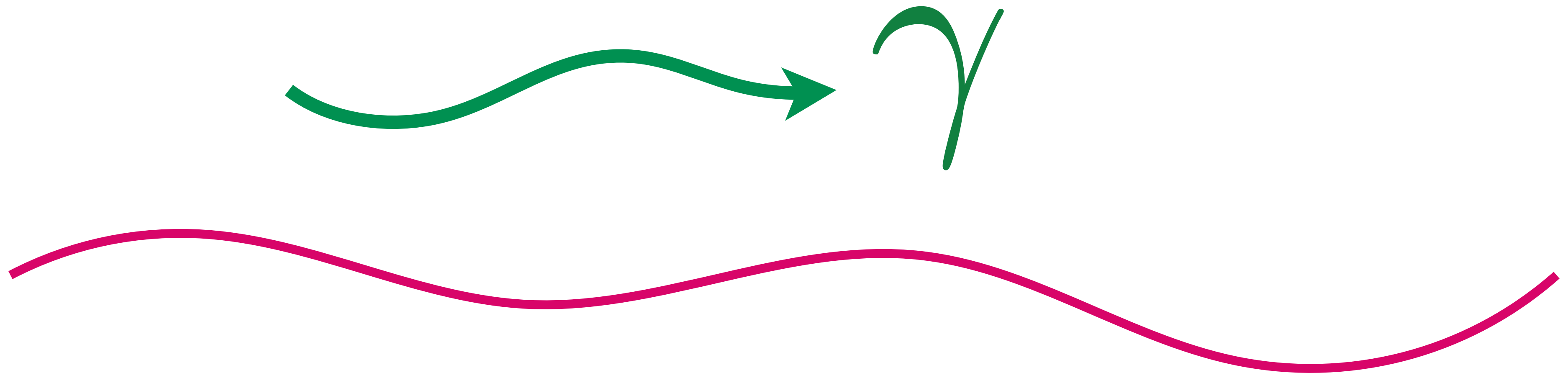

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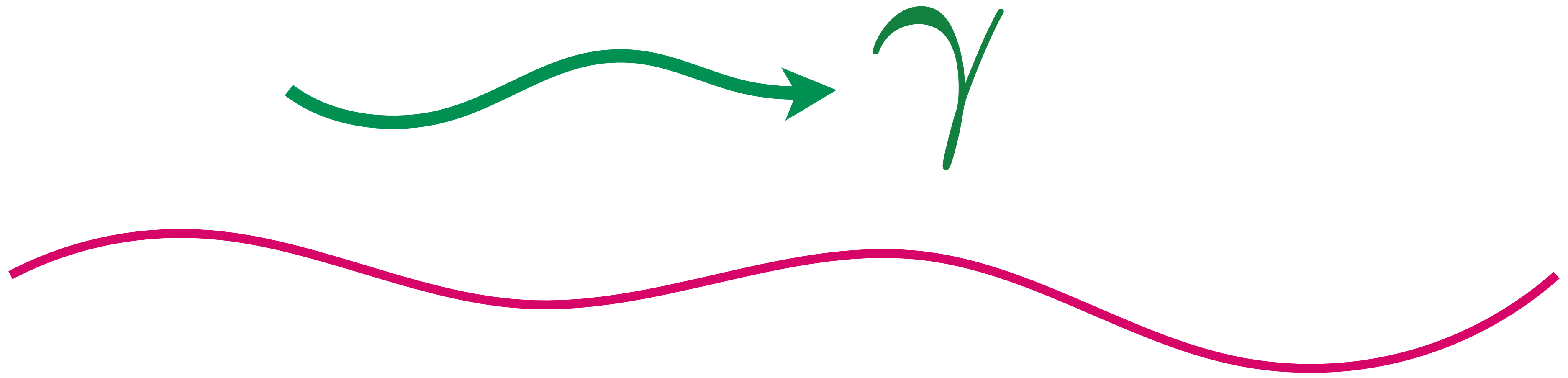

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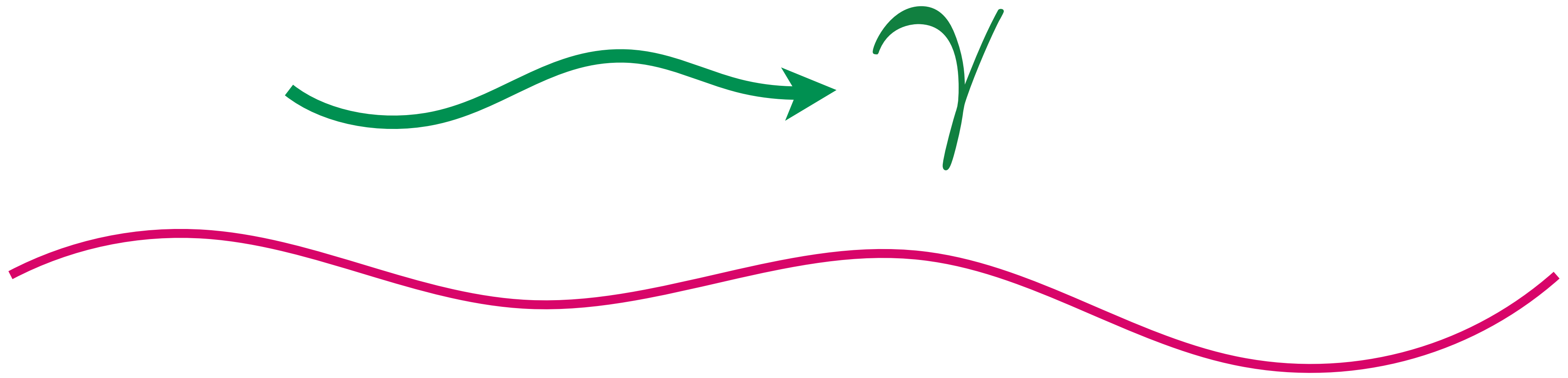

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Effective current from spatial or temporal variations of h or F

$$j^\mu_{\rm eff} \equiv \partial_\nu\left(\frac{1}{2}\,h\,F^{\mu\nu} + h^\nu{}_\alpha\,F^{\alpha\mu} - h^\mu{}_\alpha\,F^{\alpha\nu}\right)$$

Cur Cavis?* Part I: Electromagnetic Ruler

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Currents can excite cavity modes $\mathbf{E}_{\text{cav}}$ as long as η non-zero:

$$\eta \propto \int_V \mathbf{E}^*_{\text{cav}} \cdot \mathbf{J}_{\text{eff}}$$

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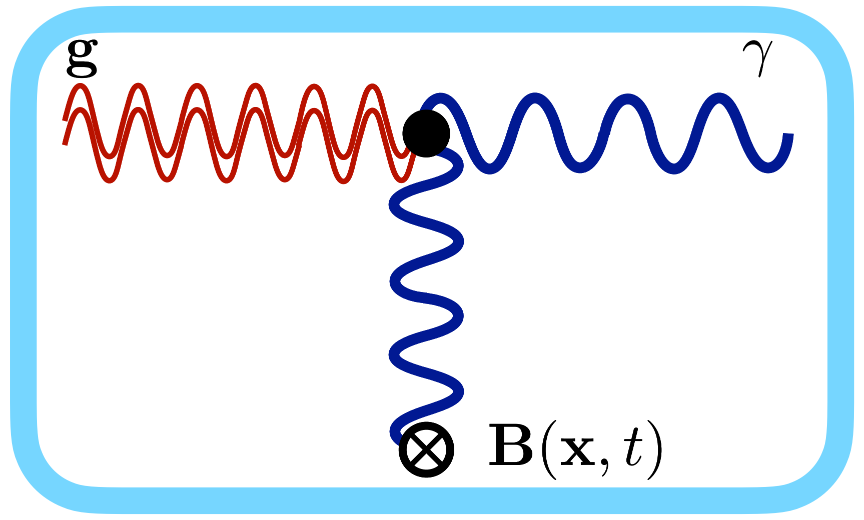

Gertsenshtein effect (1962)

Also Zeldovich (1973)

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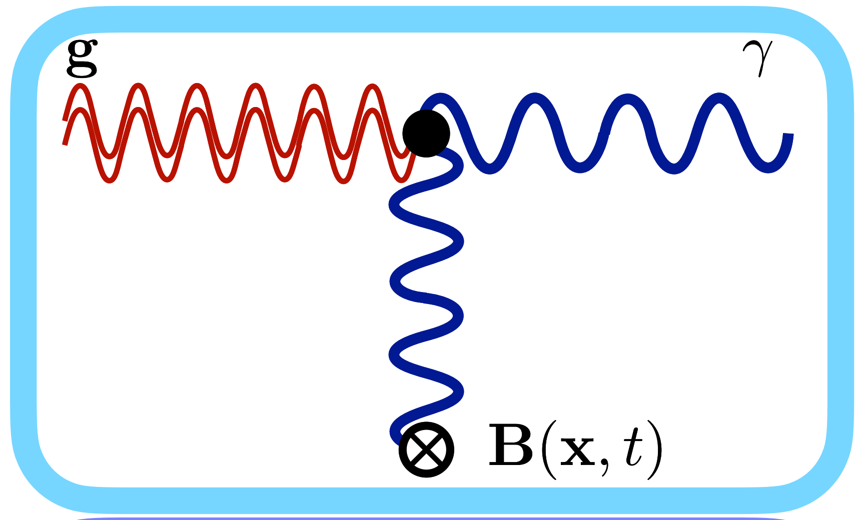

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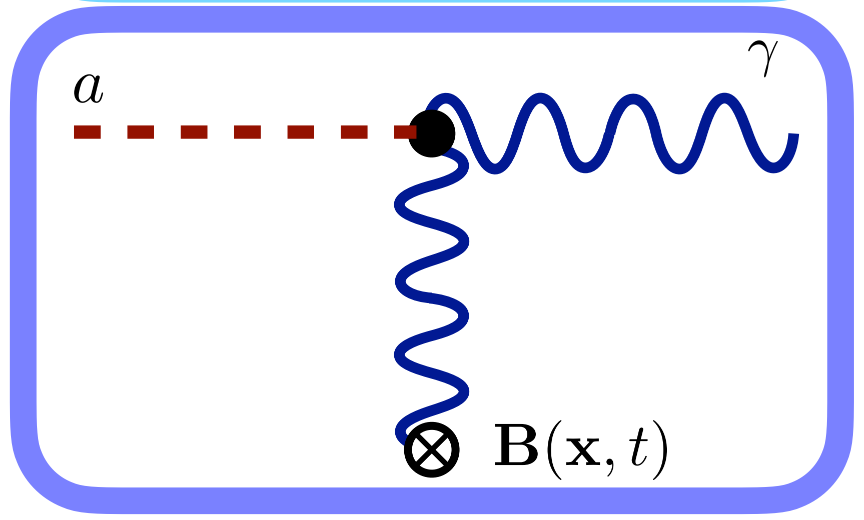

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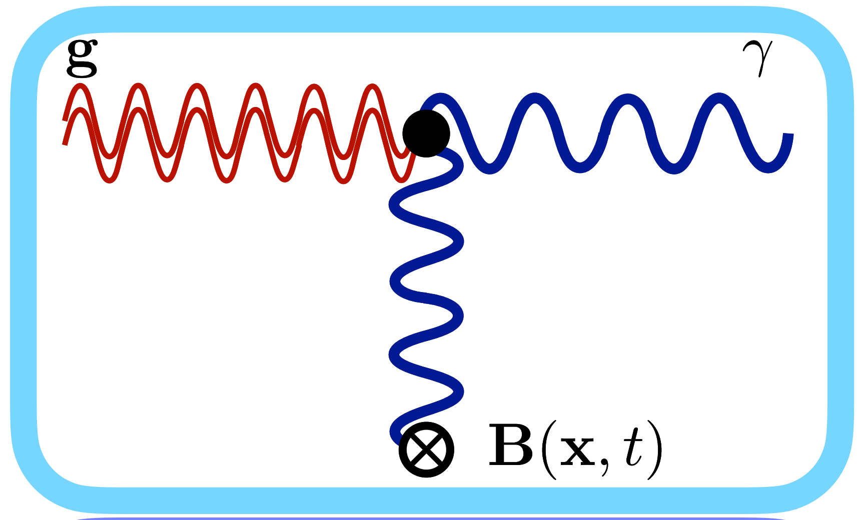

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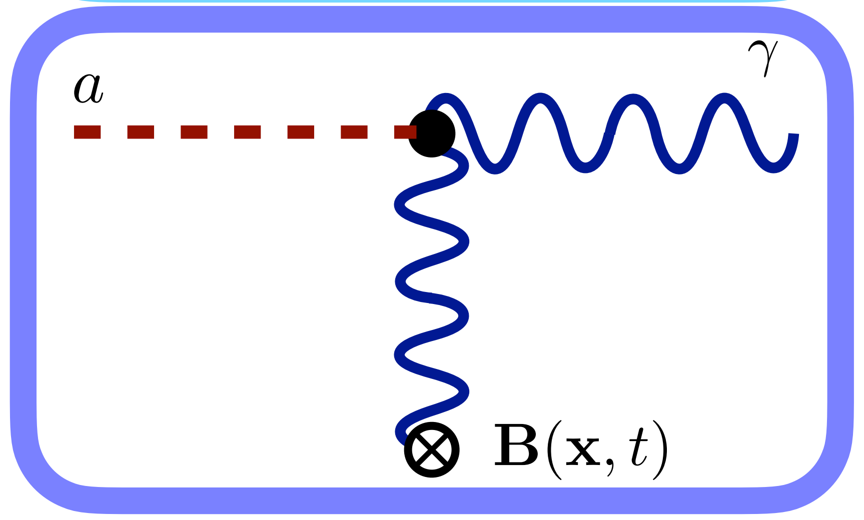

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Raffelt & Stodolsky (1988)

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Framing the Question

Crucial to work in appropriate reference frame!

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Detector in Local Inertial Frame (LIF)

$$\hat{\boldsymbol{n}} \times \boldsymbol{E} = 0 \qquad \hat{\boldsymbol{n}} \cdot \boldsymbol{B} = 0$$

Maxwell (19th century)

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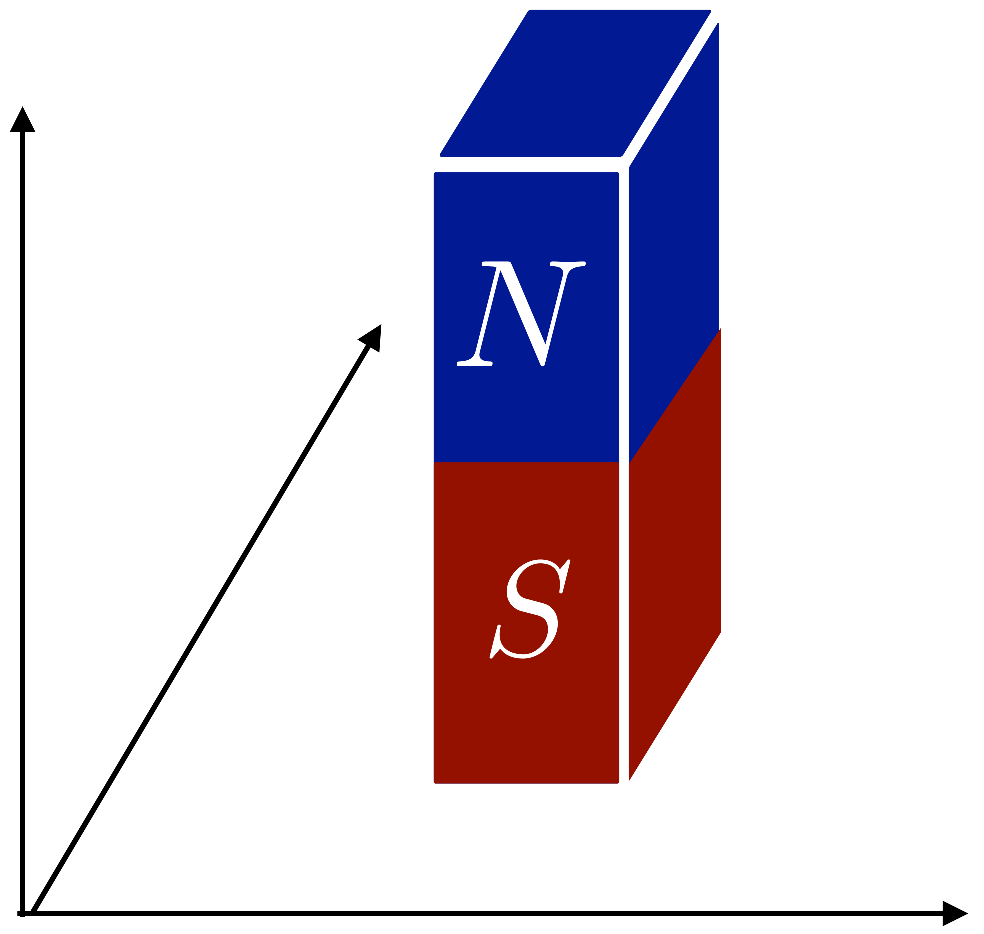
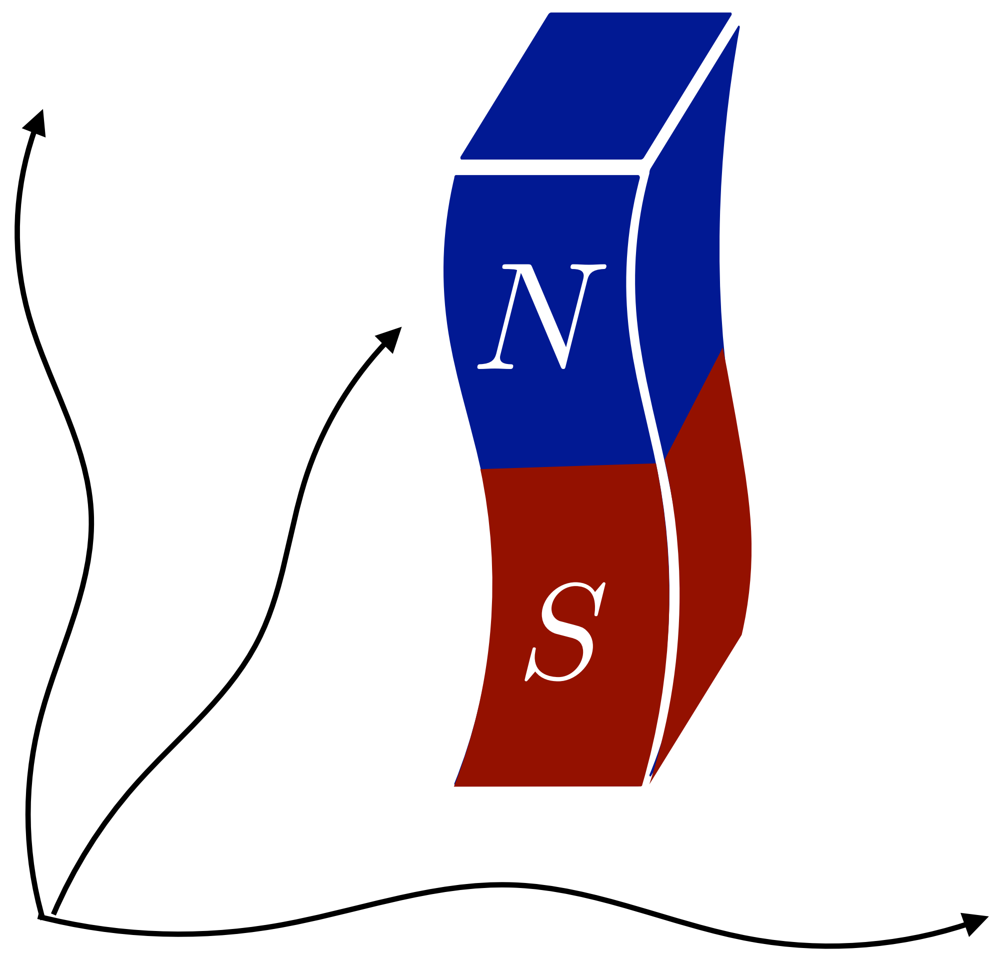


B-field in LIF $\neq$ B-field in TT



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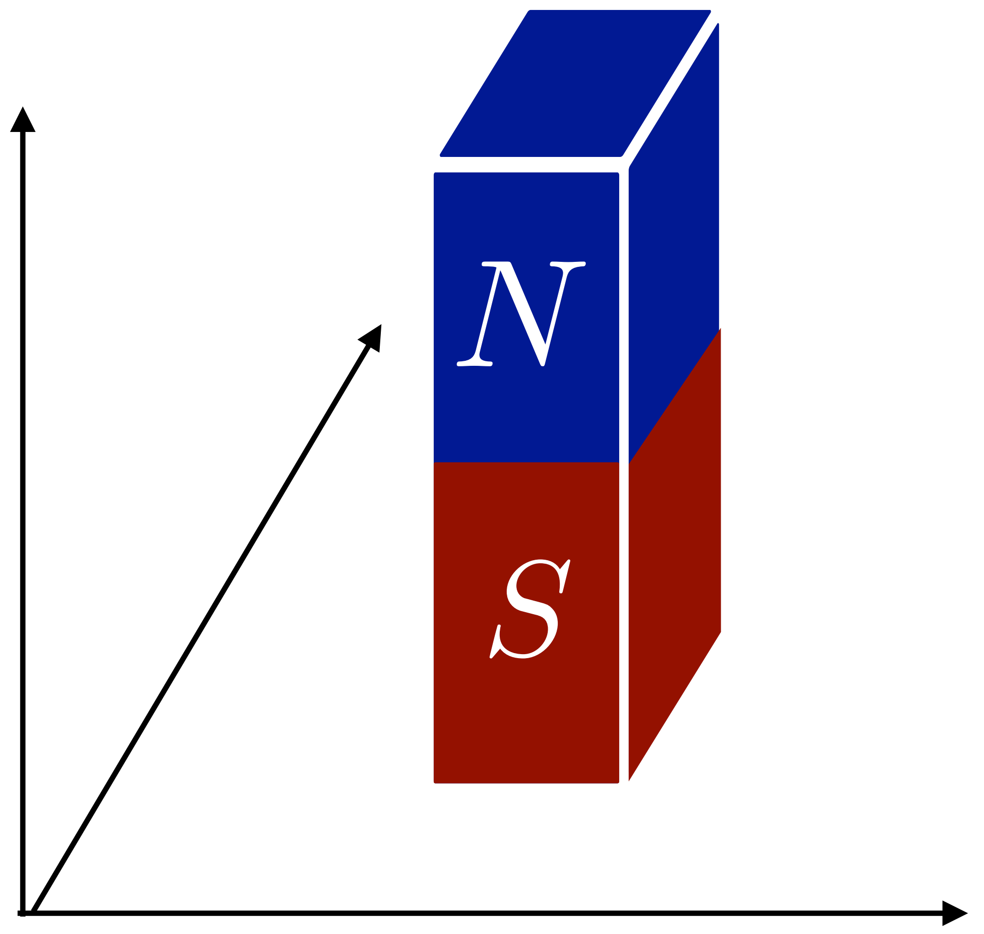

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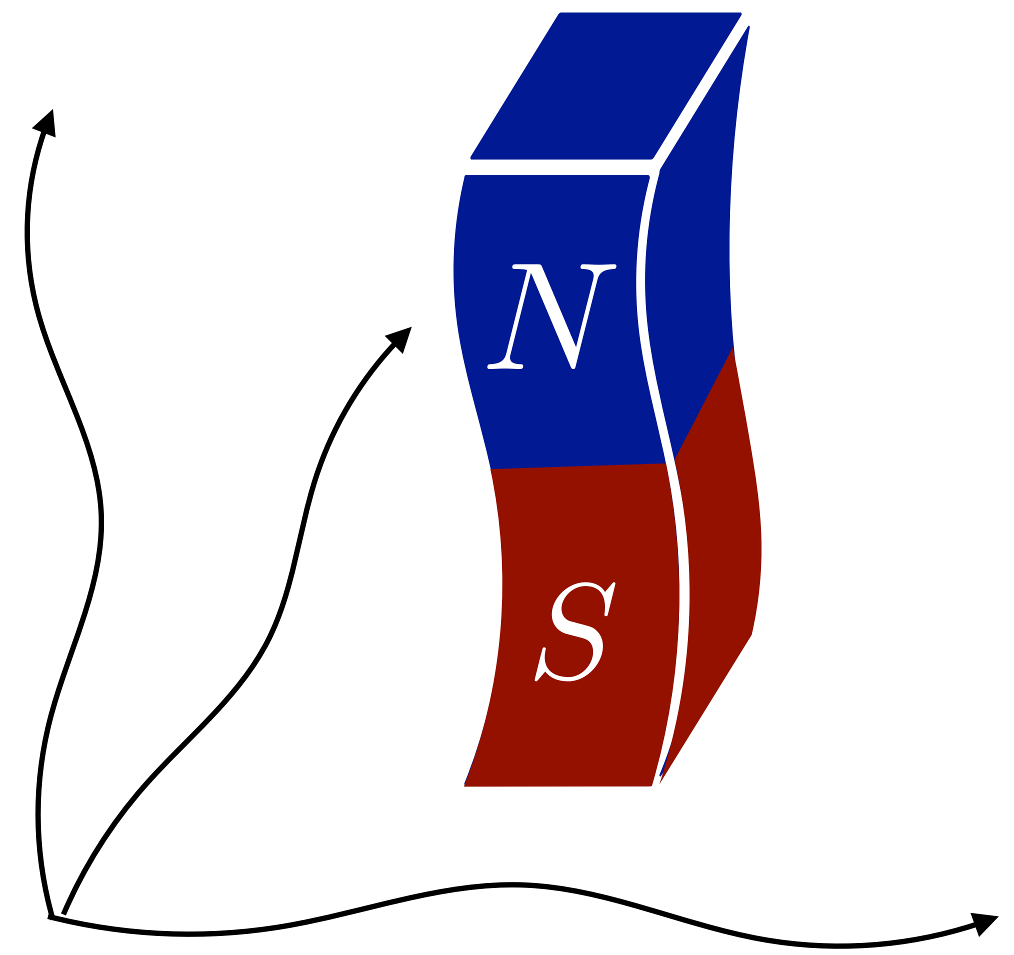
Which frame is the right one to use?



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Proper Detector Frame — complication

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e.g. Maggiore (2007)

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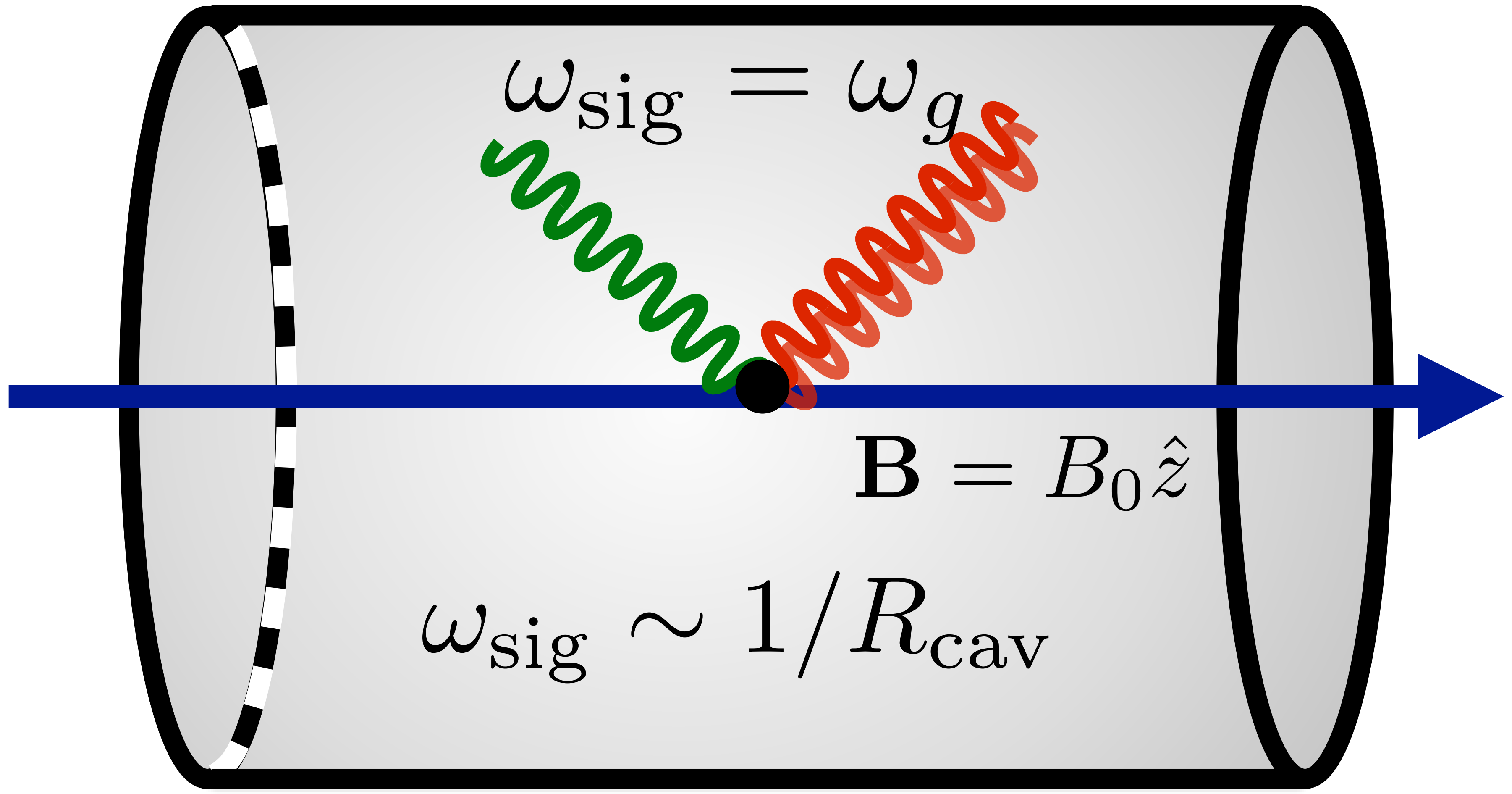

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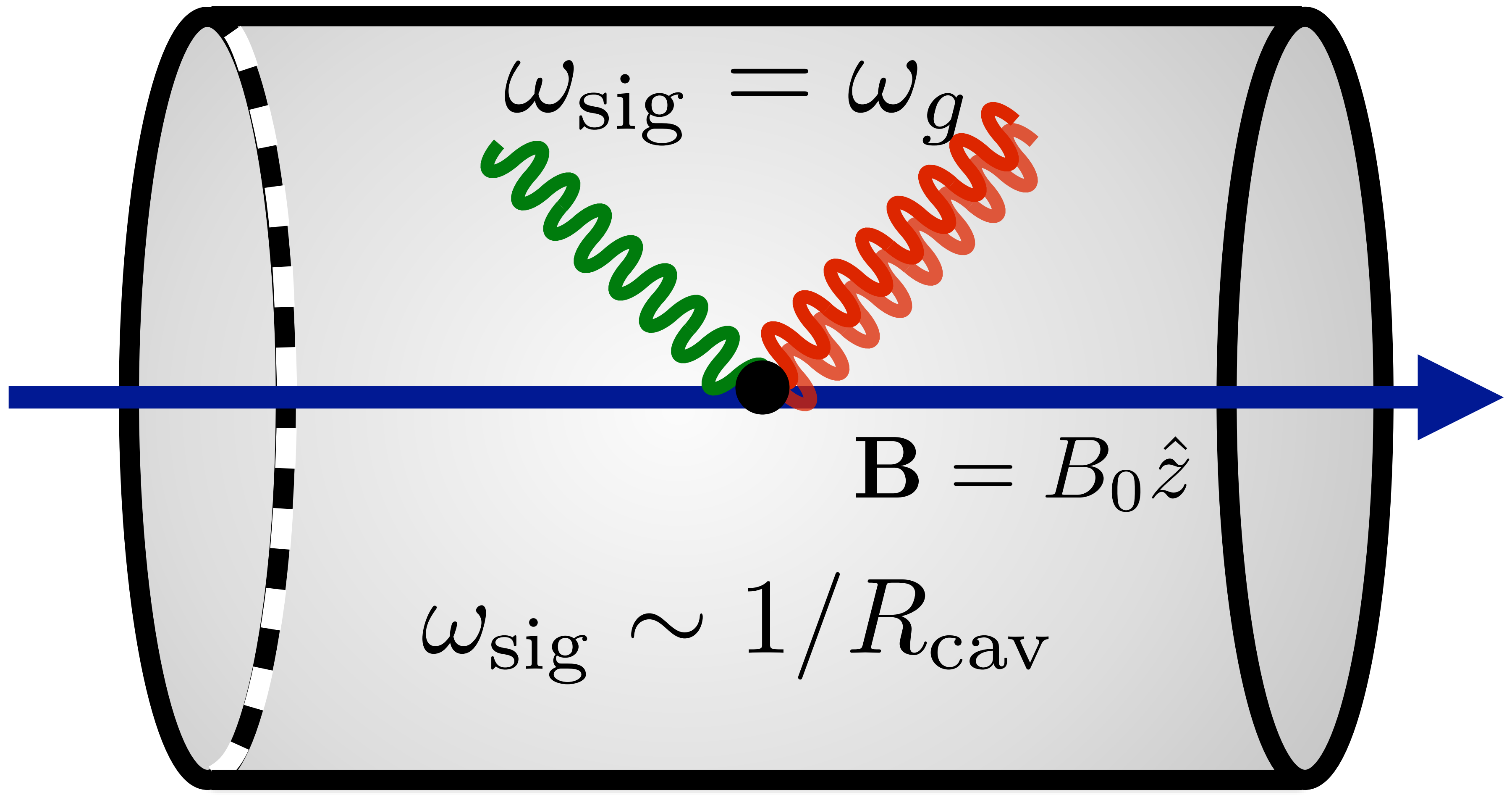
Resonant Cavity:



Framing the Question

Proper Detector Frame — complication

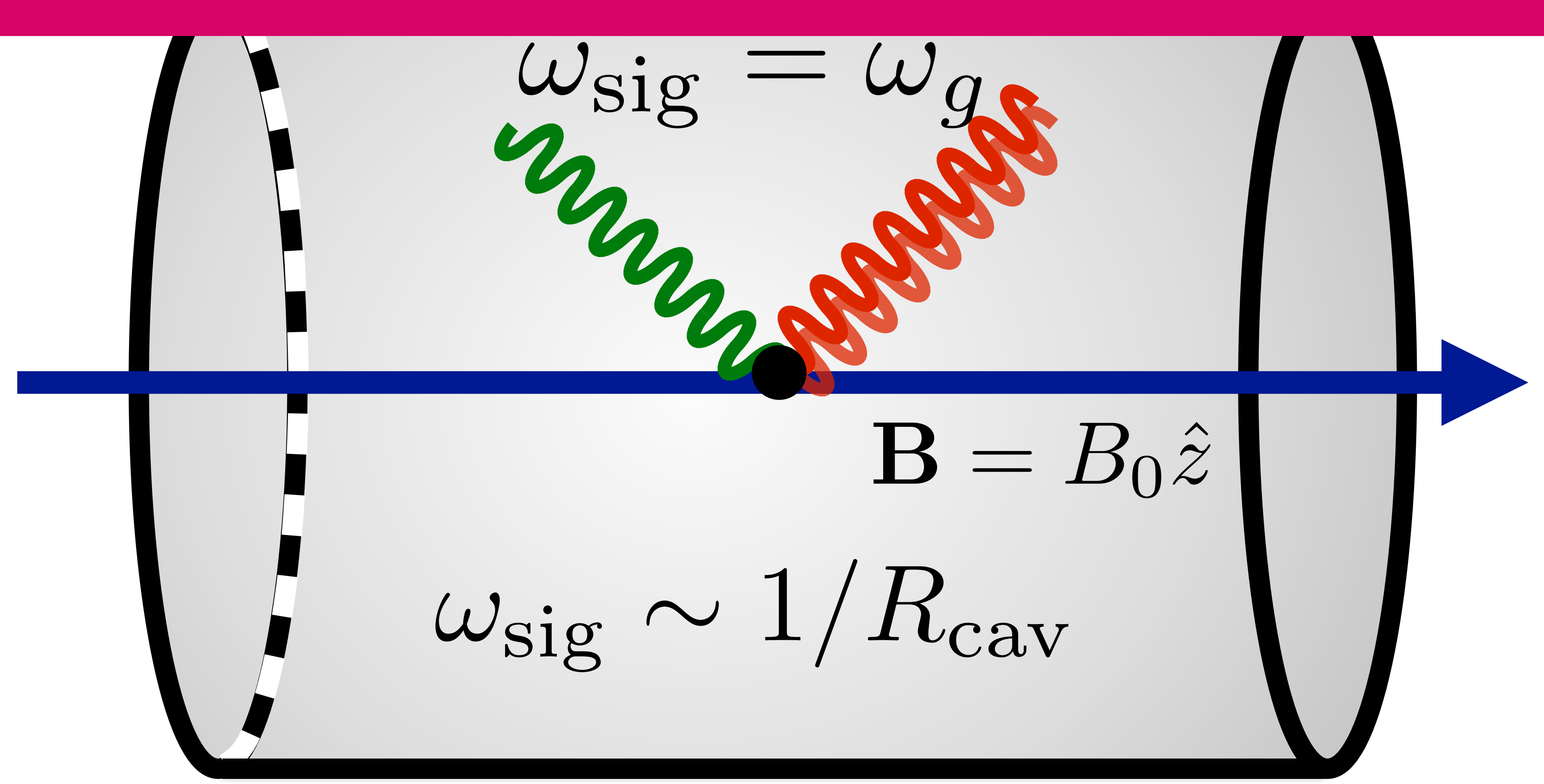
Textbooks give long-wavelength approximation $\omega_g R_{\text{cav}} \ll 1$

$$ds^2 \simeq -dt^2(1 + R_{0i0j}x^i x^j) - \frac{4}{3}\, dt\, dx^i \left(R_{0ijk}x^j x^k\right) + dx^i\, dx^j \left(\delta_{ij} - \frac{1}{3} R_{ikjl}x^k x^l\right)$$

e.g. Maggiore (2007)

Long-wavelength limit invalid!

Resonant Cavity:

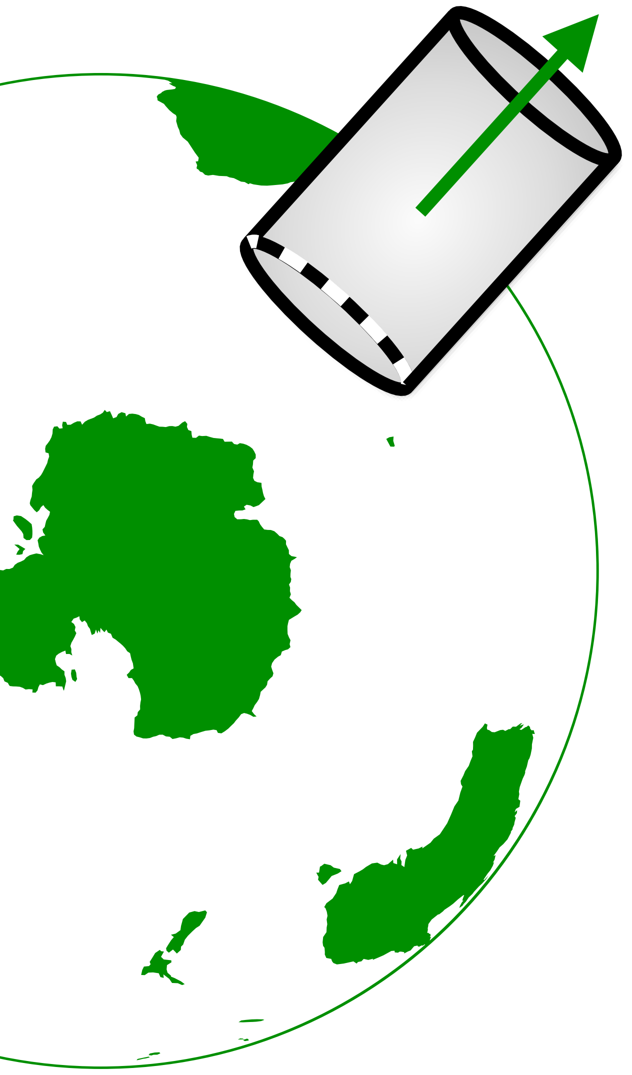


Framing the Question

Solution — GW as sum of plane waves

$$h \propto e^{i\omega_g(t-z)} \longrightarrow \partial_i h^{\mathrm{TT}}_{jk} \sim -\delta_{iz}\partial_t h^{\mathrm{TT}}_{jk}$$

$$x^{k_1}\dots x^{k_r} R_{\mu\nu\rho\sigma,k_1\dots k_r} = (-i\omega_g z)^r R_{\mu\nu\rho\sigma}$$



$$h_{00} = -2\sum_{r=0}^{\infty}\frac{r+3}{(r+3)!}R_{0n0n,k_1,\cdots k_r}x^m x^n x^{k_1}\cdots x^{k_r}$$

$$h_{0i} = -2\sum_{r=0}^{\infty}\frac{r+2}{(r+3)!}R_{0nin,k_1,\cdots k_r}x^m x^n x^{k_1}\cdots x^{k_r}$$

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Märzlin (1994)
Rakhmanov (2014)

Framing the Question

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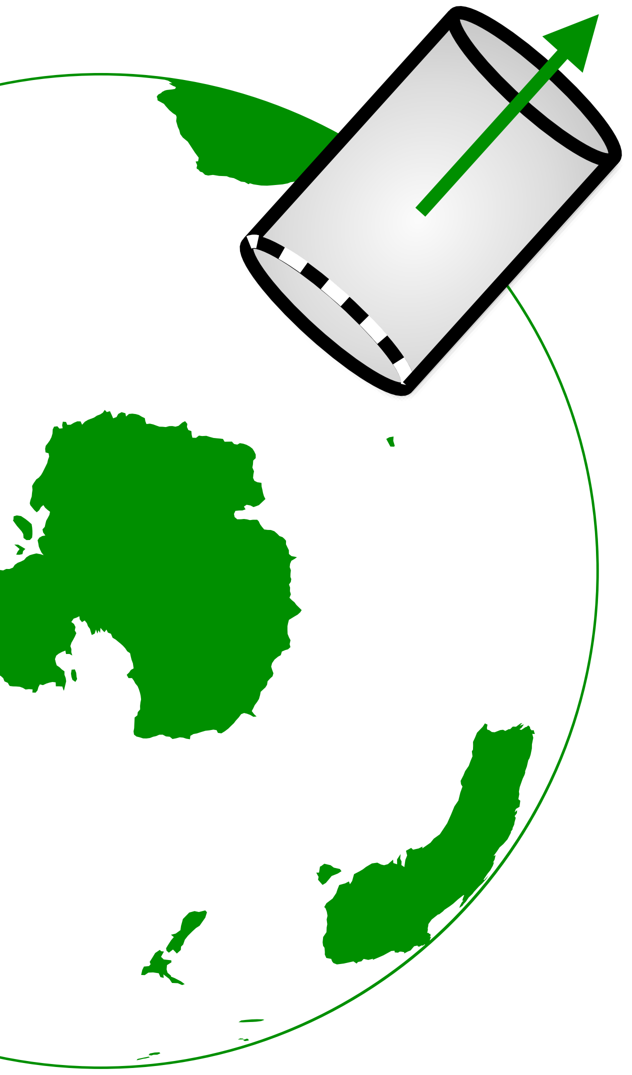

$$h_{00} = -2R_{0m0n}x^m x^n \left(-\frac{i}{\omega_g z} + \frac{1+e^{-i\omega_g z}}{(\omega_g z)^2}\right)$$

$$h_{0i} = -2R_{0min}x^m x^n \left(-\frac{i}{2\omega_g z} - \frac{e^{-i\omega_g z}}{(\omega_g z)^2} - i\frac{1-e^{-i\omega_g z}}{(\omega_g z)^3}\right)$$

$$h_{ij} = -2R_{imjn}x^m x^n \left(-\frac{1+e^{-i\omega_g z}}{(\omega_g z)^2} - 2i\frac{1-e^{-i\omega_g z}}{(\omega_g z)^3}\right)$$

Märzlin (1994)
Rakhmanov (2014)

Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel (PRD 2022)

Axion Cavity Modes Couple to GWs

$$\eta \propto \int_V \mathbf{E}^*_{\rm cav} \cdot \mathbf{J}_{\rm eff}$$

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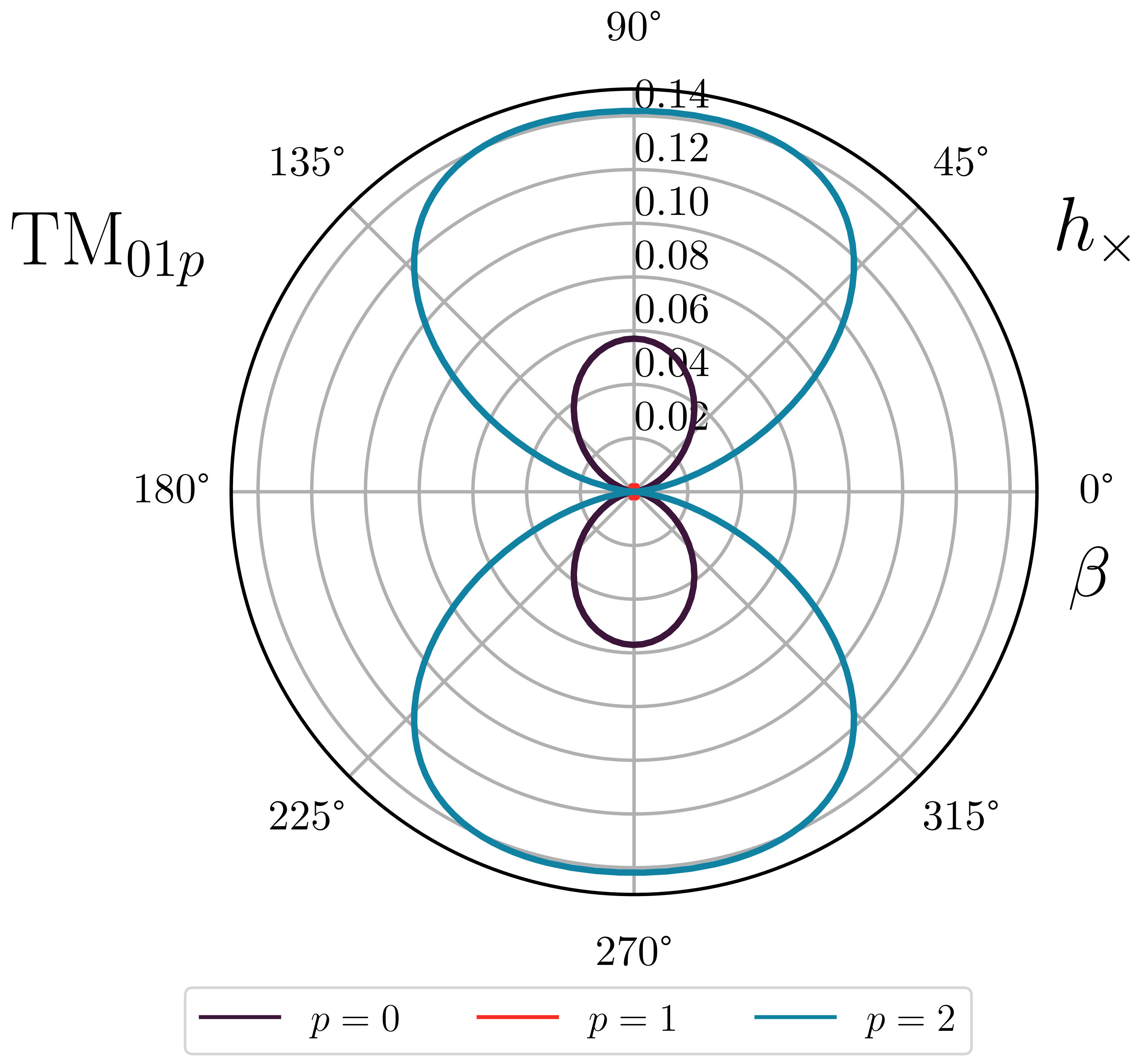

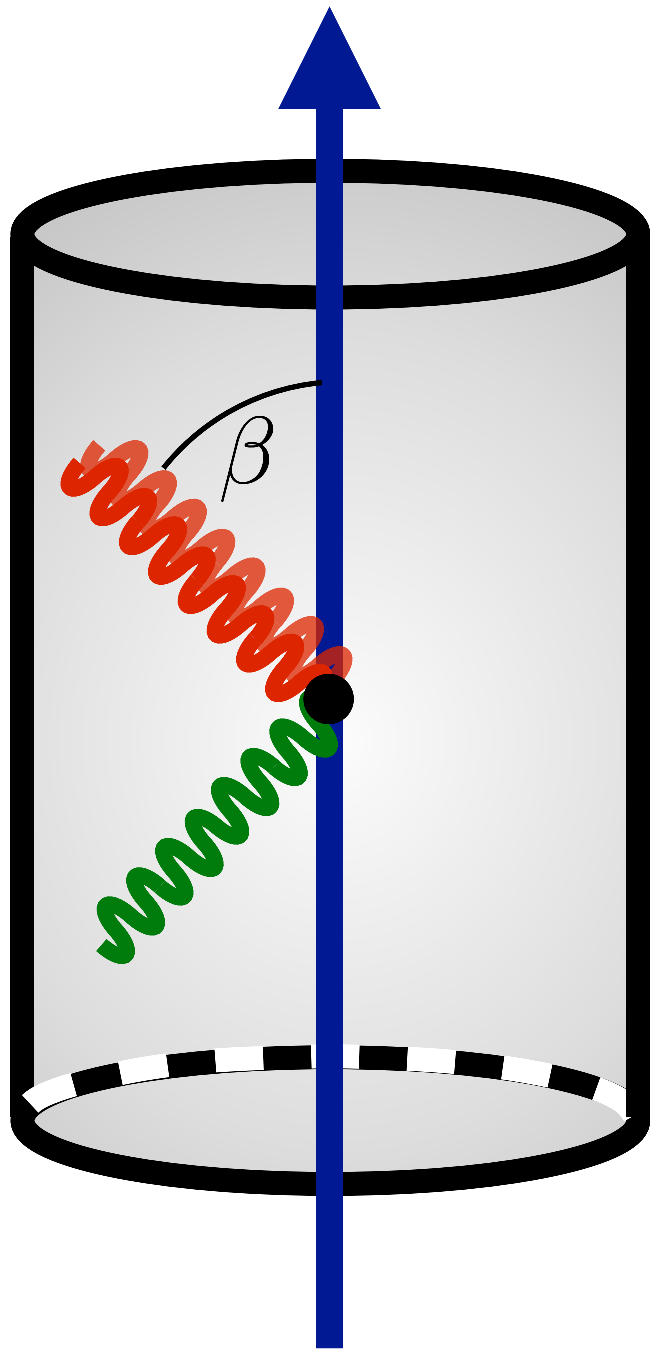

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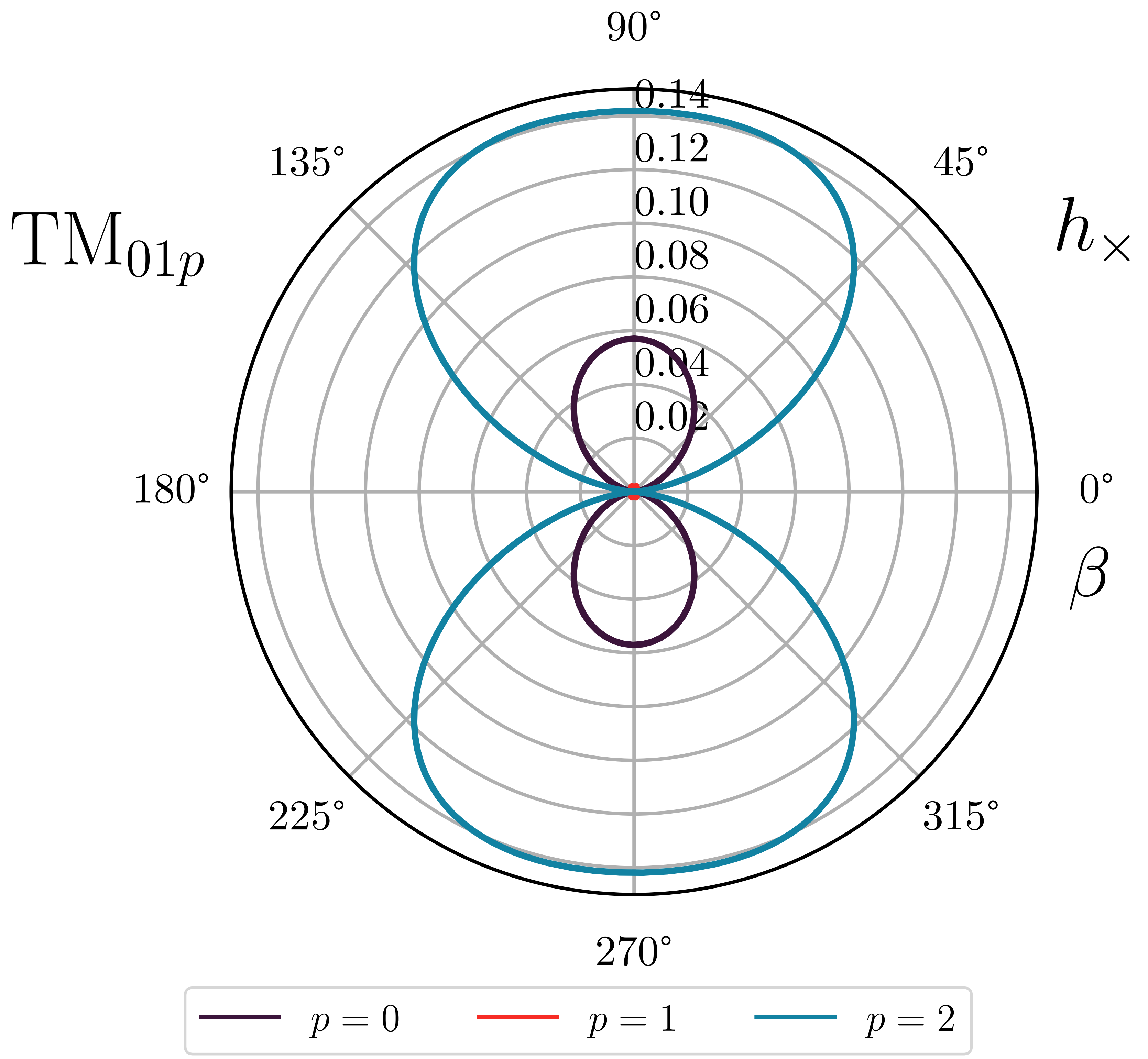
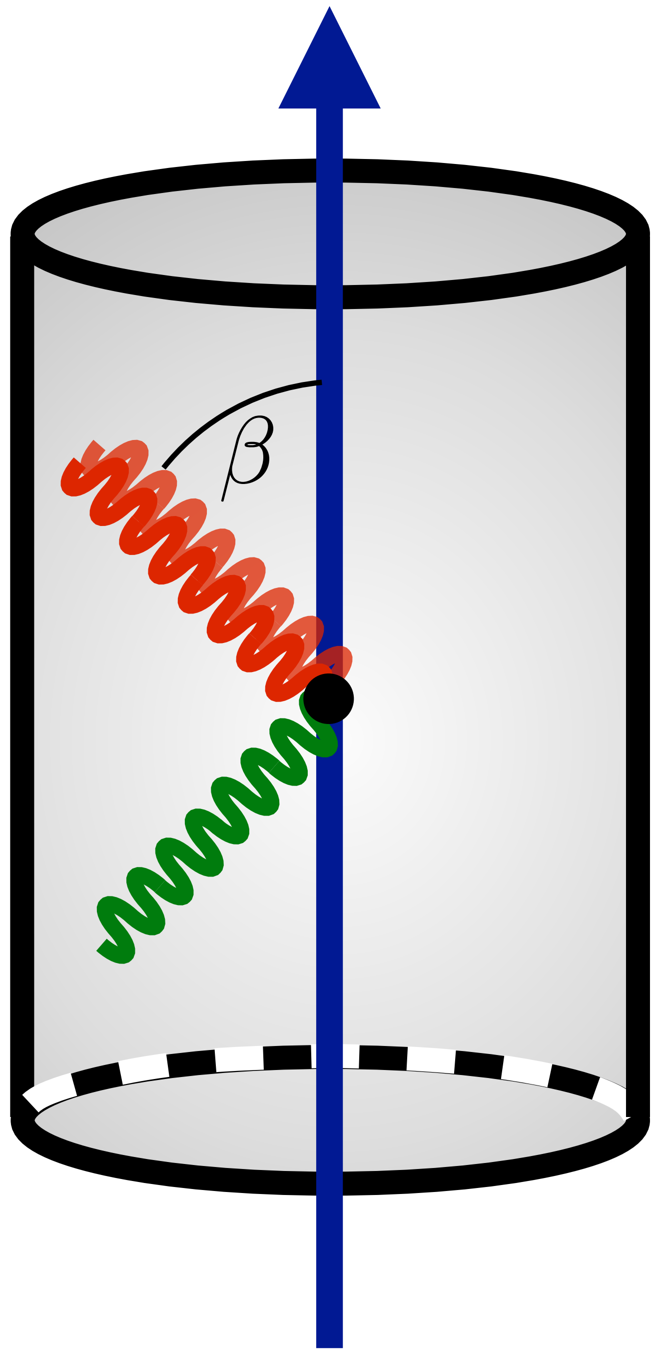
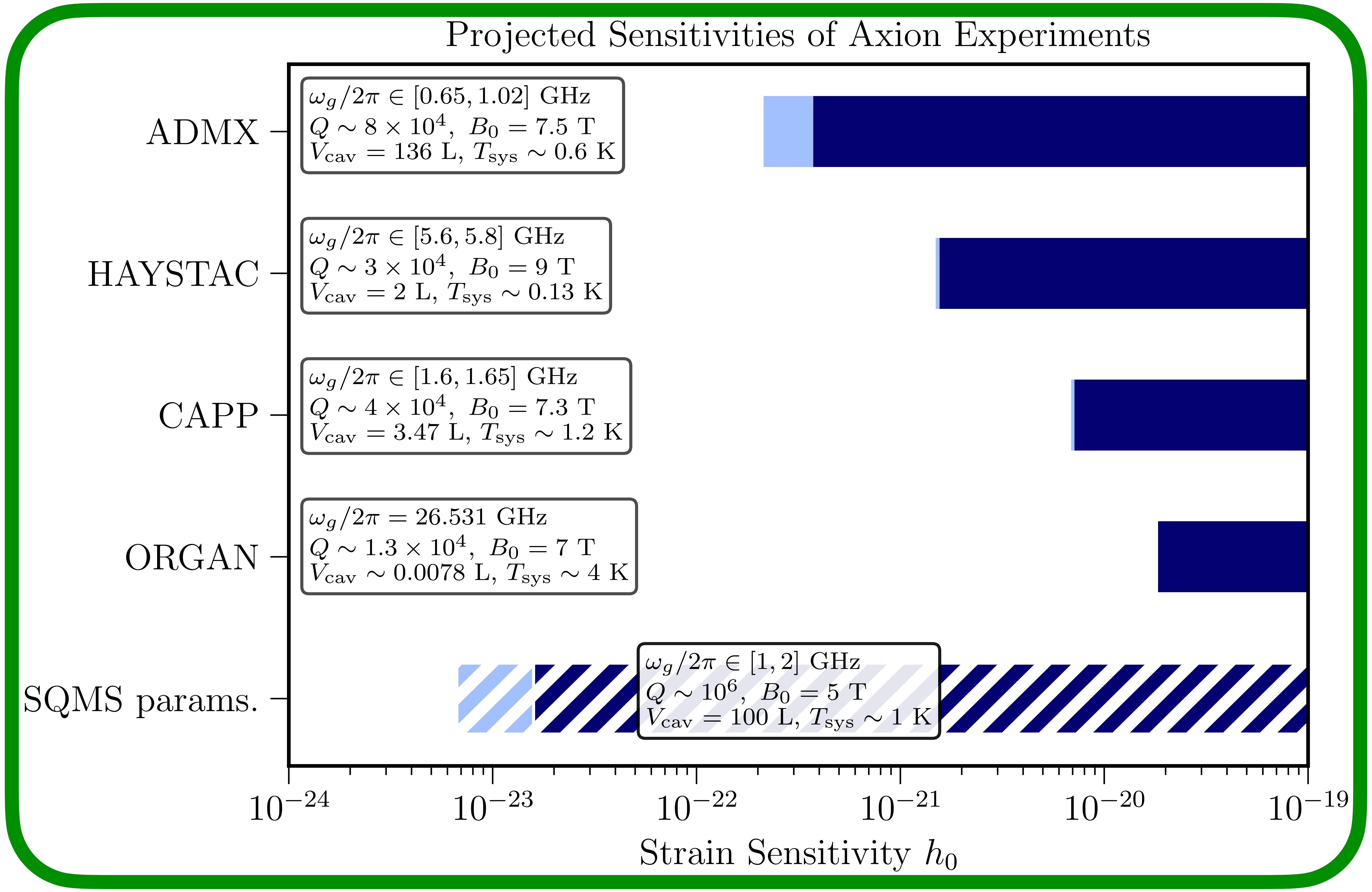


Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel (PRD 2022)

But TM modes not optimal...



Axion Cavity Sensitivity



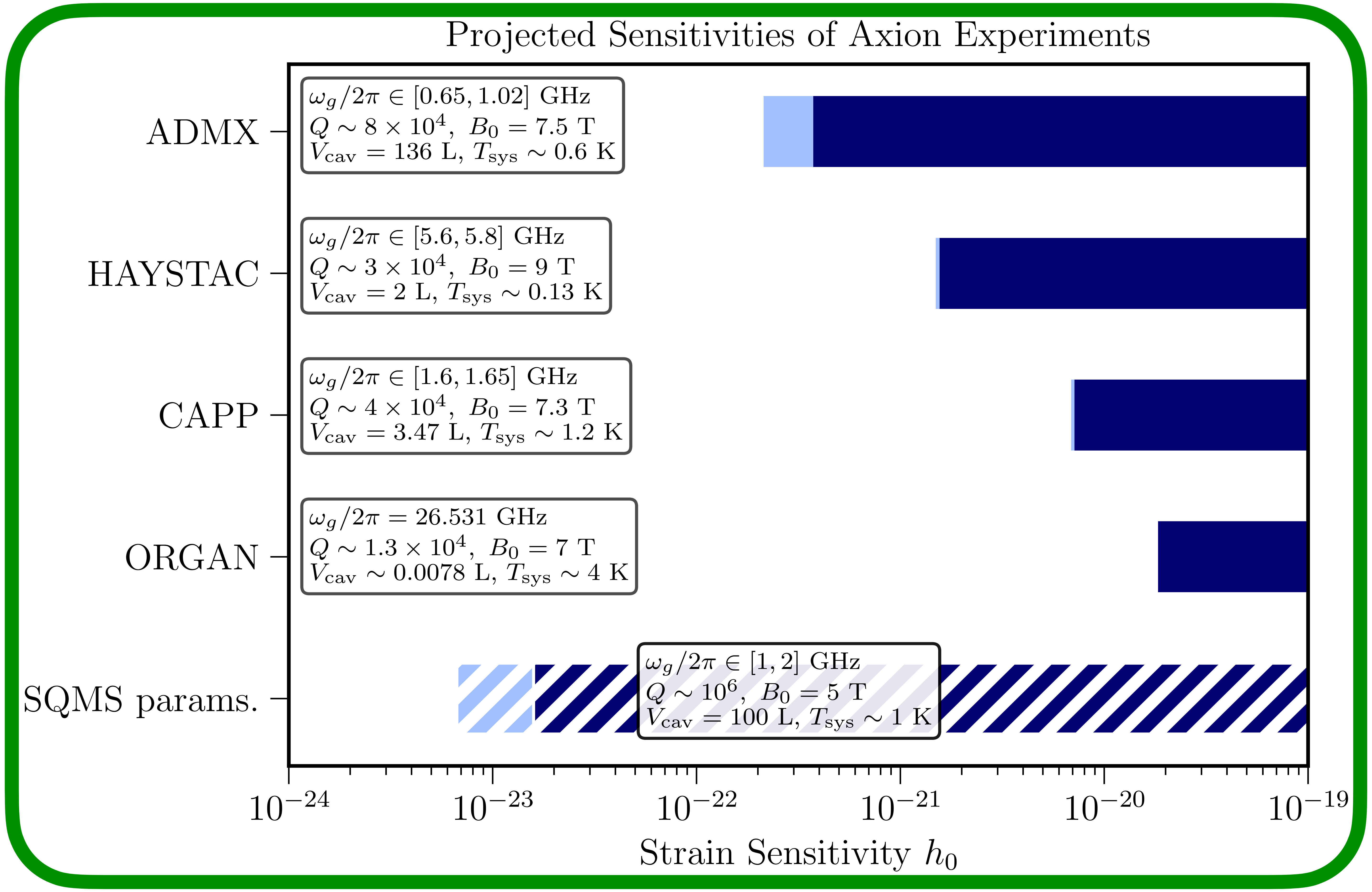
Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel (PRD 2022)

Coherent GW

$$P_{\rm sig} = \frac{1}{2} Q\, \omega_g^3\, V_{\rm cav}^{5/3} \left(\eta_n\, h_0\, B_0\right)^2$$

see Weds-Fri talks for more on sources

Axion Cavity Sensitivity



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Stochastic GWs

$$\mathrm{SNR} \sim Q\, \omega_g\, \eta_{\rm stoch}^2\, B_0^2\, V_{\rm cav}\, S_h(\omega_g)/T_{\rm sys}$$

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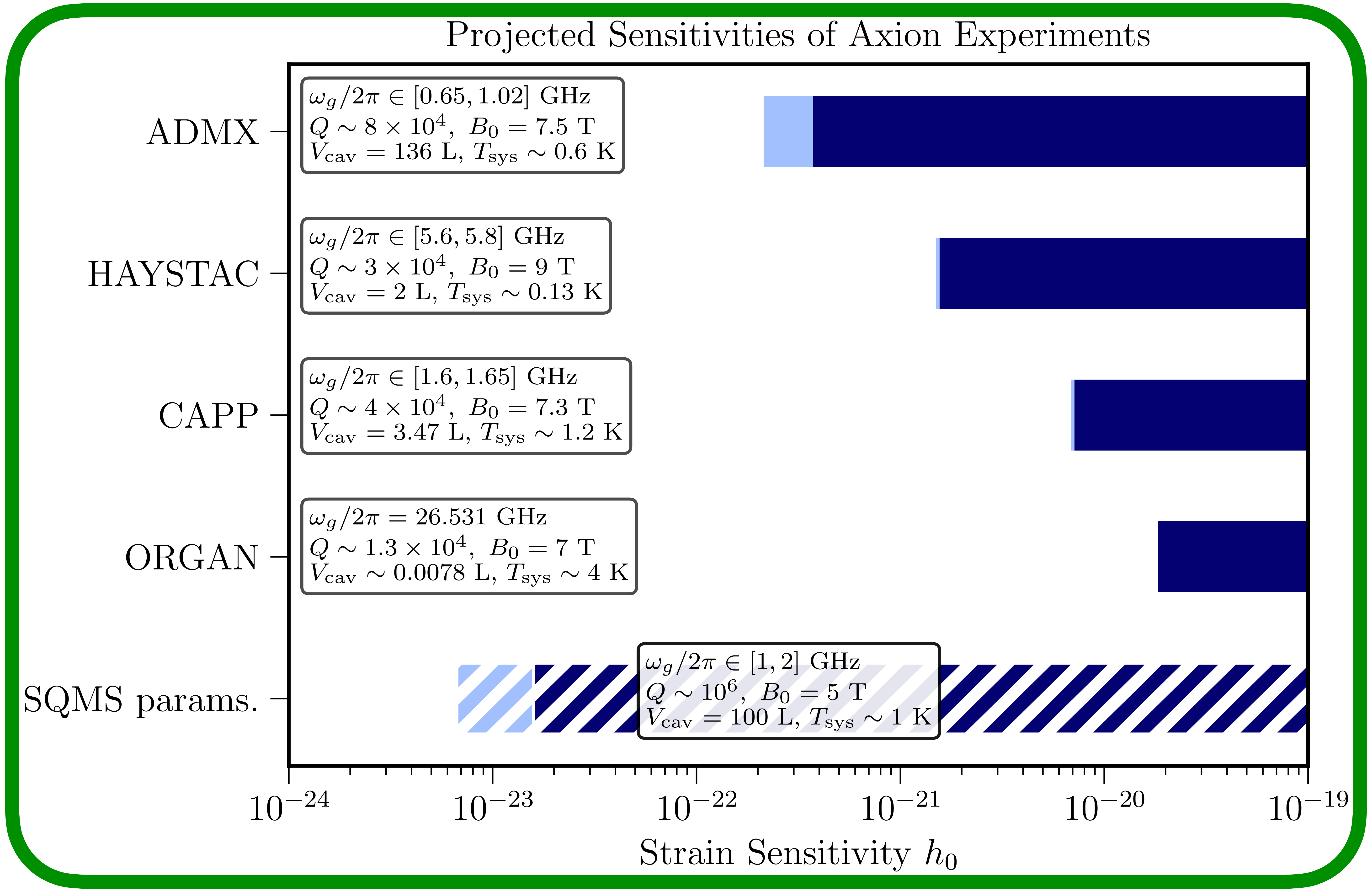

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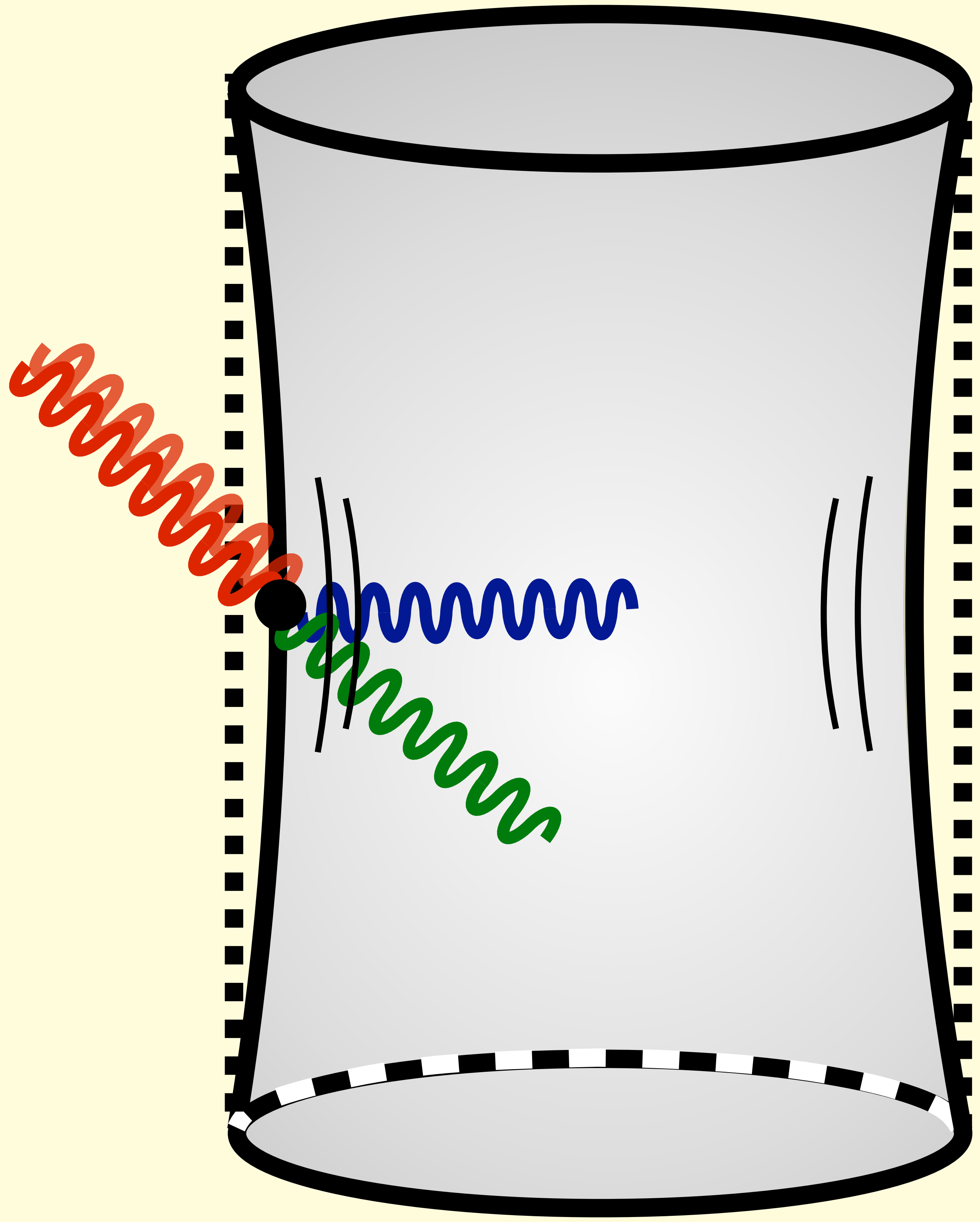
Not beyond BBN bound...

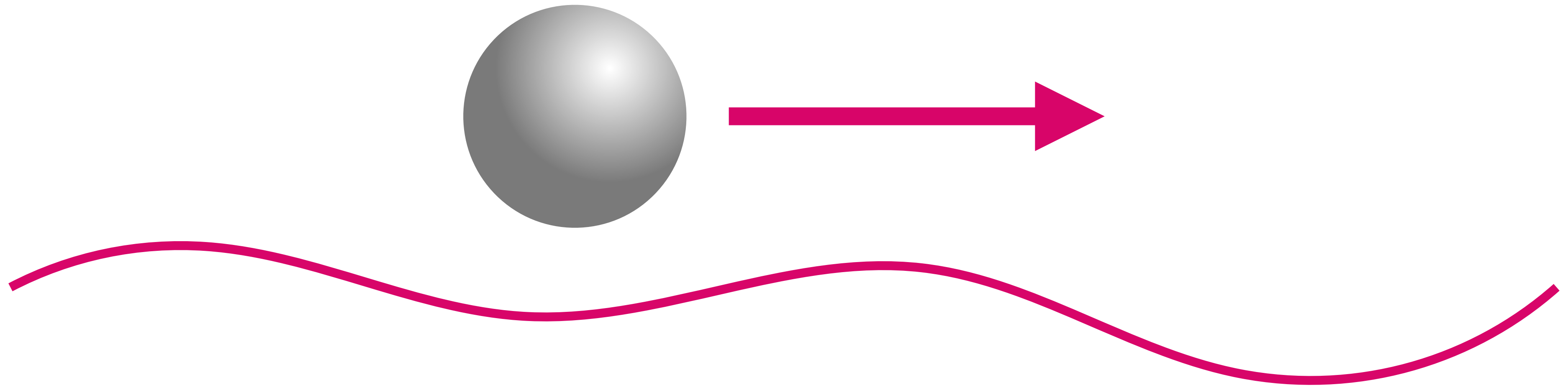
see Weds-Fri talks for more on sources

ACTE II

Matter is the Ruler: Excited Cavities As Weber Bars



Interactions of Gravitational Waves *with masses*



$$S = -\int dt\; m\sqrt{-g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$$

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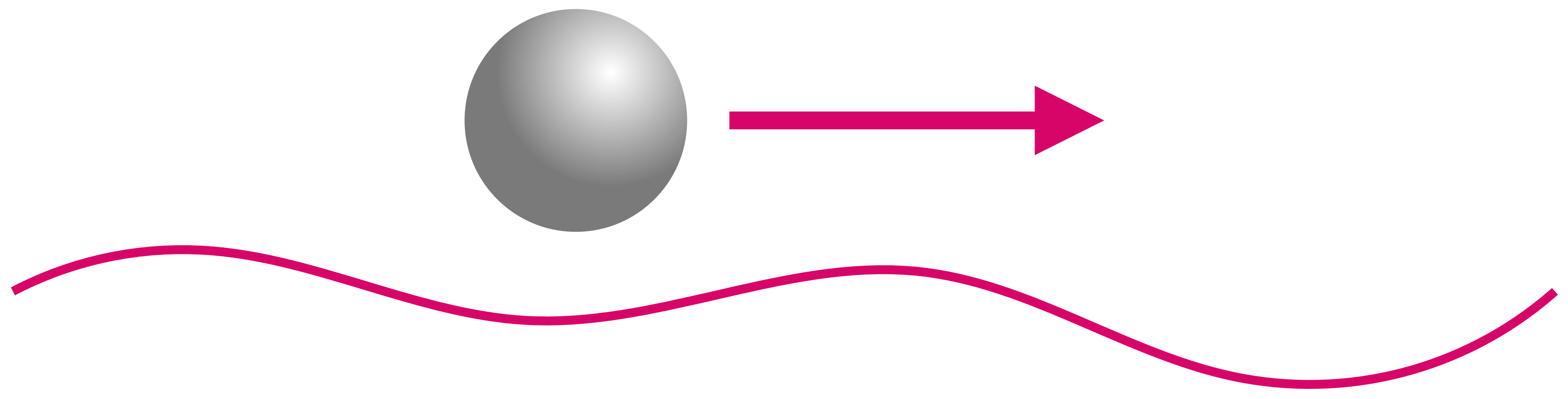

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Equation of motion: $$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho}(x)\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = 0$$

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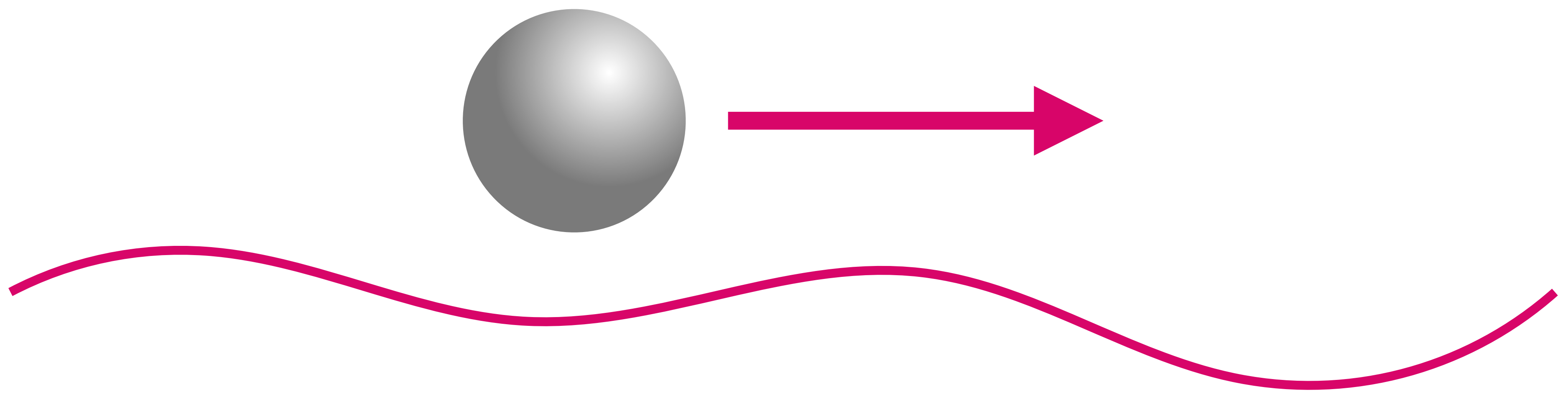

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Effect of GW encoded in Christoffel symbol $$\Gamma \propto \partial h$$

Encore: Framing the Question

Work in appropriate reference frame!

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Detector in Local Inertial Frame (LIF)

Encore: Framing the Question

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Detector in Local Inertial Frame (LIF)

Effect of GW in LIF is that of a Newtonian Force

Encore: Framing the Question

Work in appropriate reference frame!

Detector in Local Inertial Frame (LIF)

Effect of GW in LIF is that of a Newtonian Force

$$\frac{d^2 x_i}{d\tau^2} \simeq -\partial_i \Gamma^j_{00} x^i$$

Encore: Framing the Question

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Detector in Local Inertial Frame (LIF)

Effect of GW in LIF is that of a Newtonian Force

$$\frac{d^2 x_i}{d\tau^2} \simeq -\partial_i \Gamma^j_{00} x^i \longrightarrow \frac{d^2 x_i}{d\tau^2} \simeq -\frac{F_i}{m}$$

Encore: Framing the Question

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Effect of GW in LIF is that of a Newtonian Force

$$\frac{d^2 x_i}{d\tau^2} \simeq -\partial_i \Gamma^j_{00} x^i \longrightarrow \frac{d^2 x_i}{d\tau^2} \simeq -\frac{F_i}{m} \longrightarrow F_i \simeq \frac{m}{2} \ddot{h}^{\mathrm{TT}}_{ij} x^i$$

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Long-wavelength approximation valid because materials have $c_s \ll 1$

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e.g. Maggiore (2007)

Cur Cavis* Pt 2: Mechanical and EM Signals

* “Why Cavities?” in Latin

Cur Cavis* Pt 2: Mechanical and EM Signals

On the operation of a tunable electromagnetic detector for gravitational waves

F Pegoraro†, E Picasso‡ and L A Radicati‡§

†Scuola Normale Superiore, Pisa, Italy
‡CERN, Geneva, Switzerland

Received 6 December 1977, in final form 20 April 1978

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Microwave Apparatus for Gravitational Waves Observation

R. Ballantini, A. Chincarini, S. Cuneo, G. Gemme,* R. Parodi, A. Podestà, and R. Vaccarone
INFN and Università degli Studi di Genova, Genova, Italy

Ph. Bernard, S. Calatroni, E. Chiaveri, and R. Losito
CERN, Geneva, Switzerland

R.P. Croce, V. Galdi, V. Pierro, and I.M. Pinto
INFN, Napoli, and Università degli Studi del Sannio, Benevento, Italy

E. Picasso
INFN and Scuola Normale Superiore, Pisa, Italy and CERN, Geneva, Switzerland

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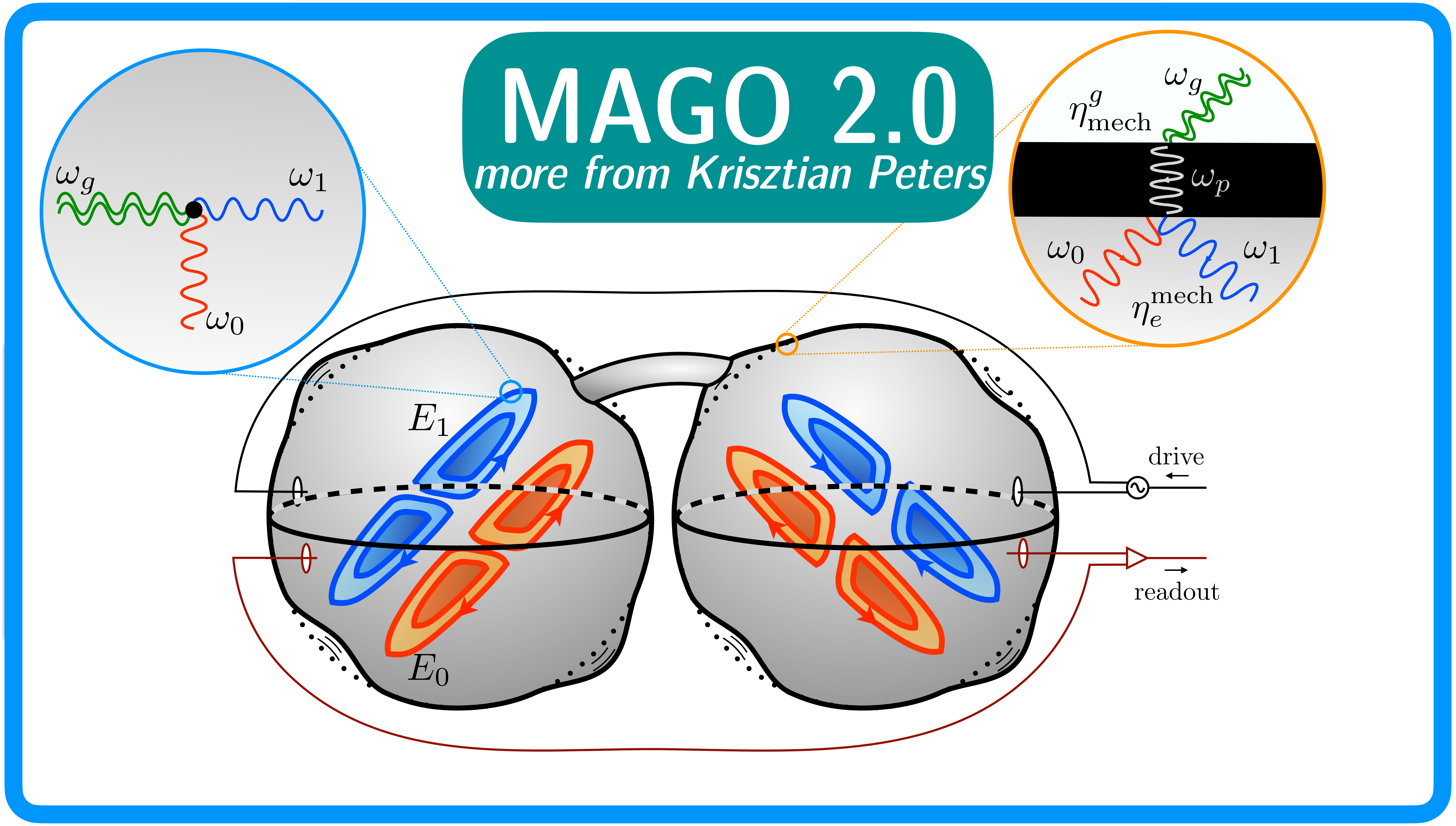
Cur Cavis* Pt 2: Mechanical and EM Signals



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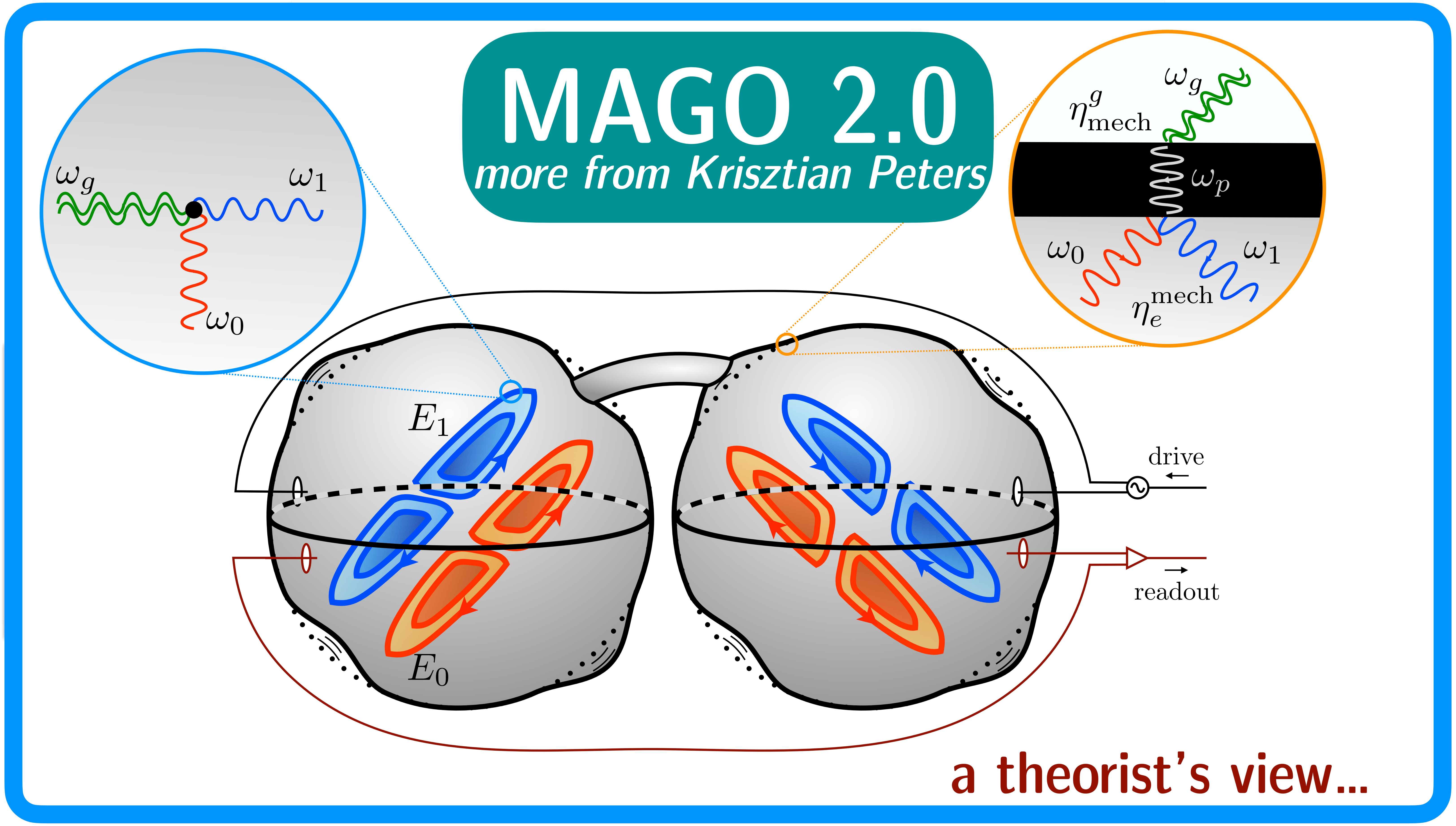
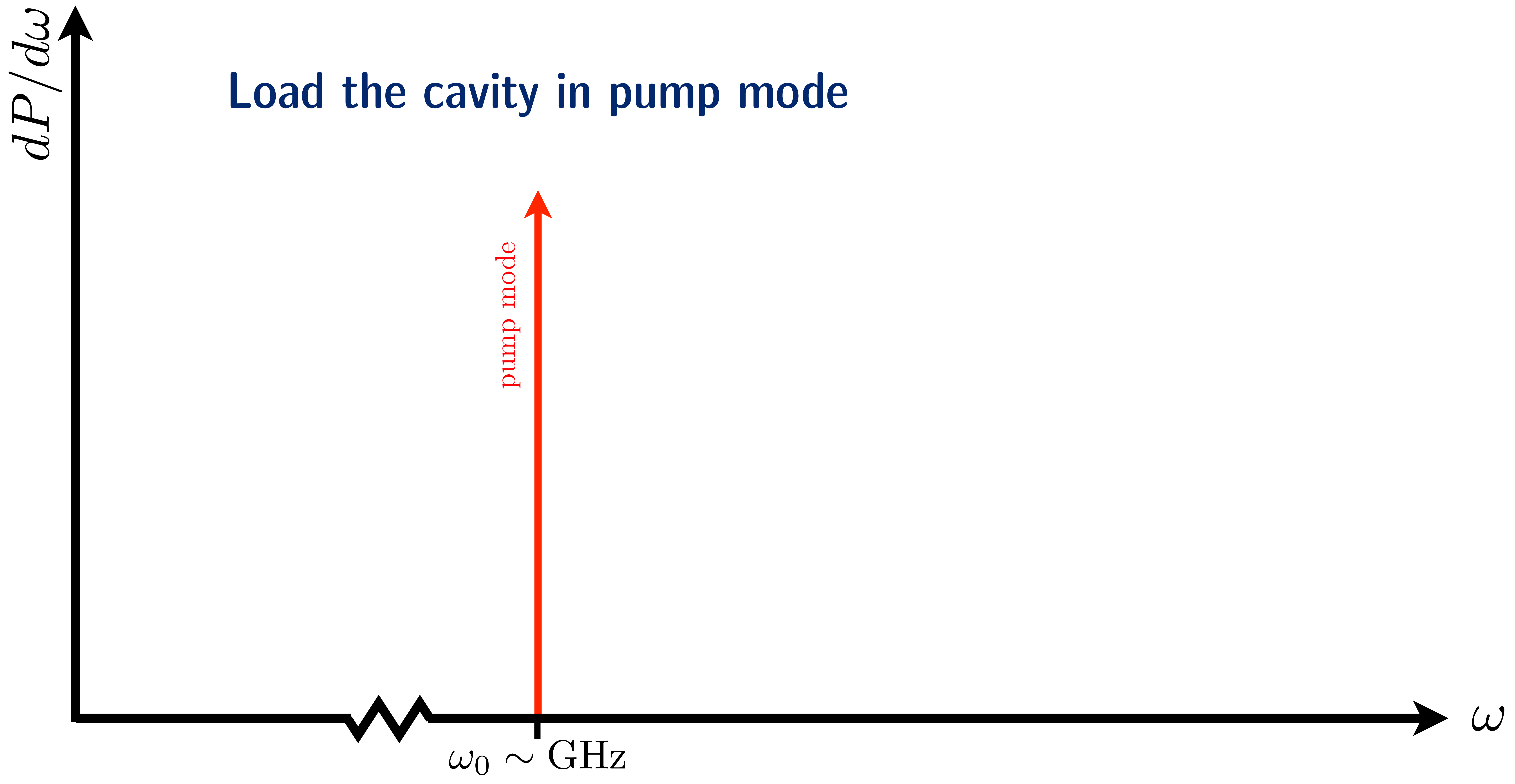


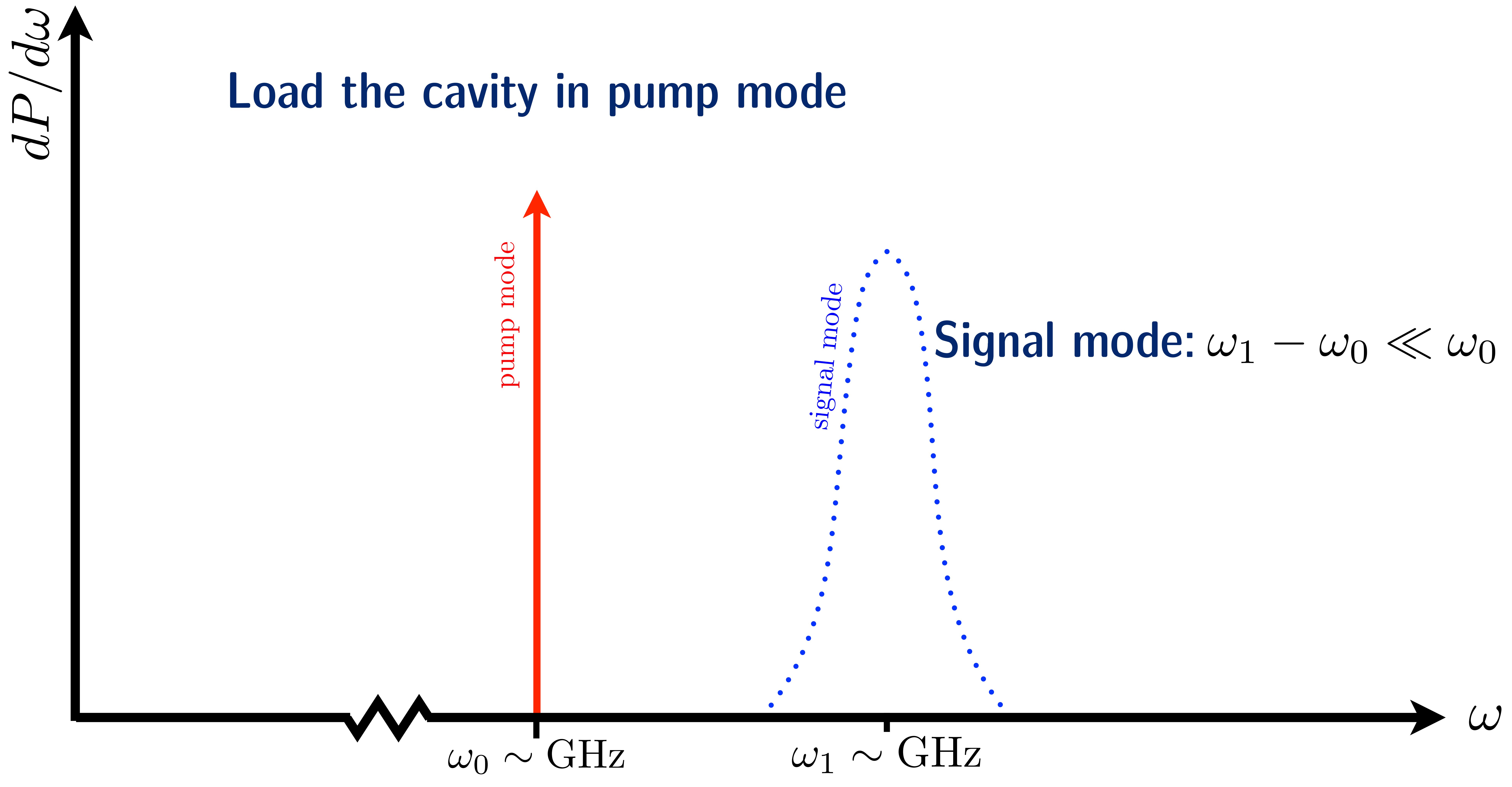
* "Why Cavities?" in Latin

MAGO 2.0



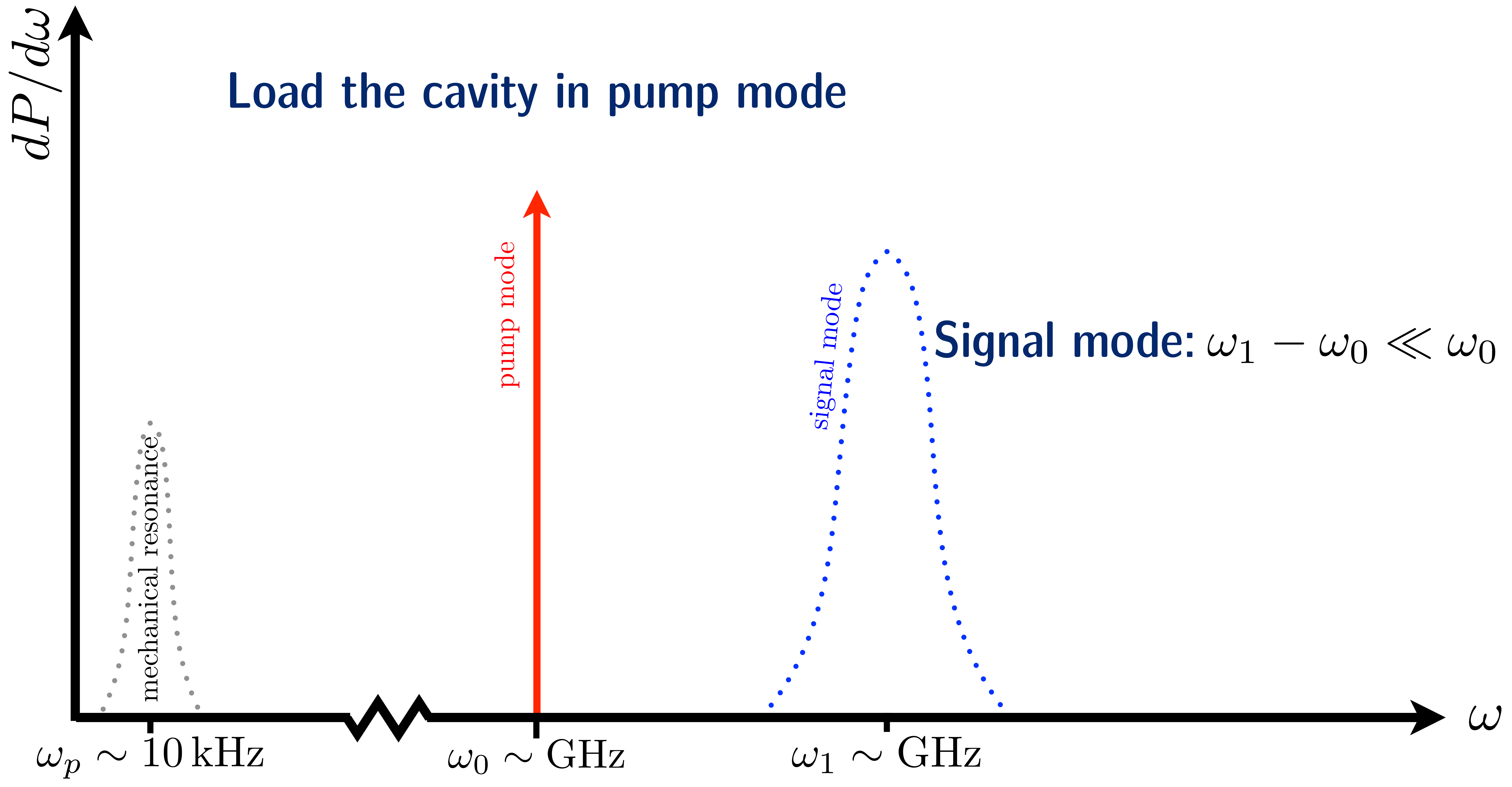
$dP/d\omega$
Load the cavity in pump mode
pump mode
$\omega_0 \sim \mathrm{GHz}$
ω

MAGO 2.0



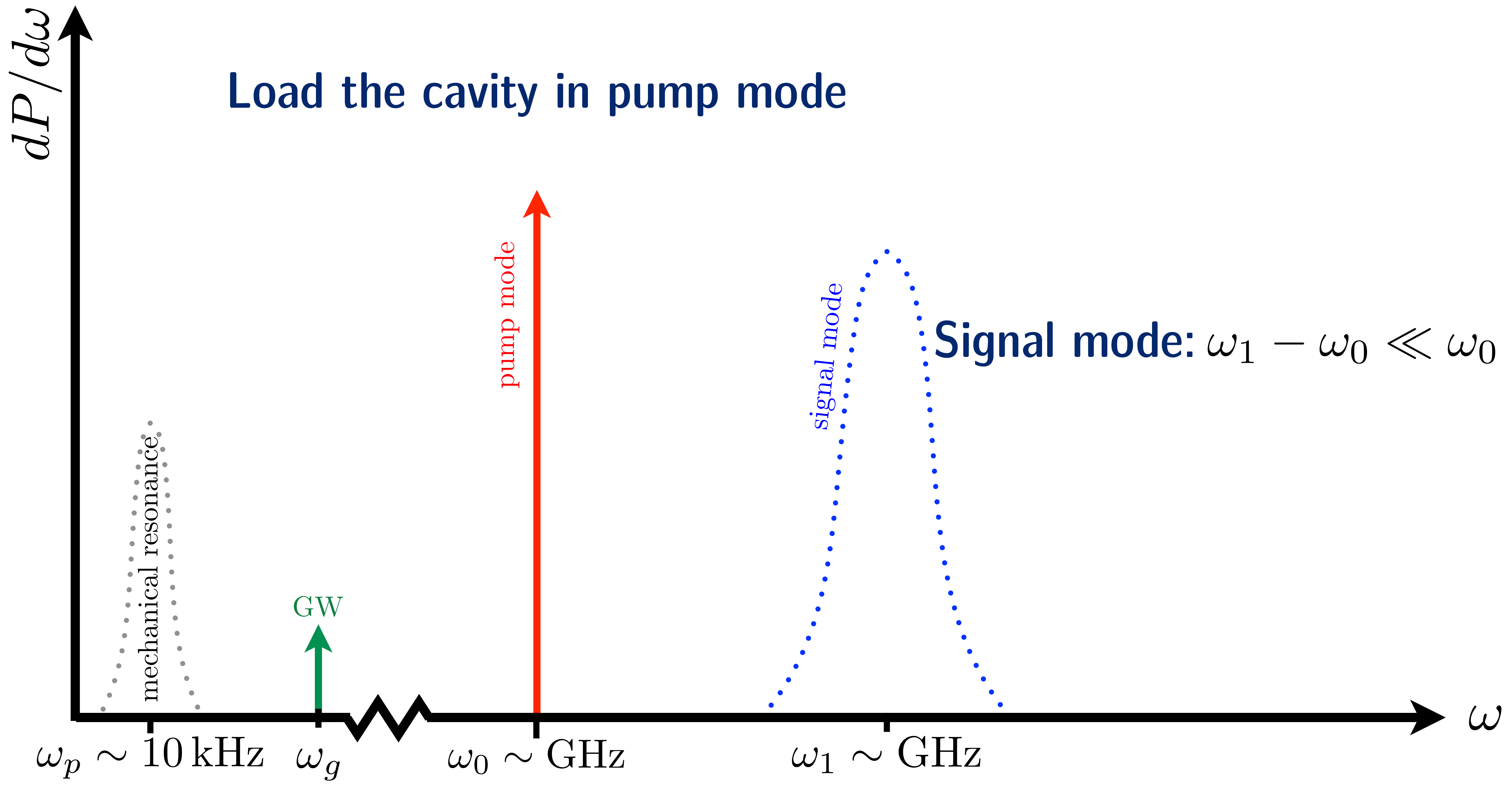
$dP/d\omega$
Load the cavity in pump mode
pump mode
signal mode
Signal mode: $\omega_1 - \omega_0 \ll \omega_0$
ω
$\omega_0 \sim \text{GHz}$
$\omega_1 \sim \text{GHz}$

MAGO 2.0



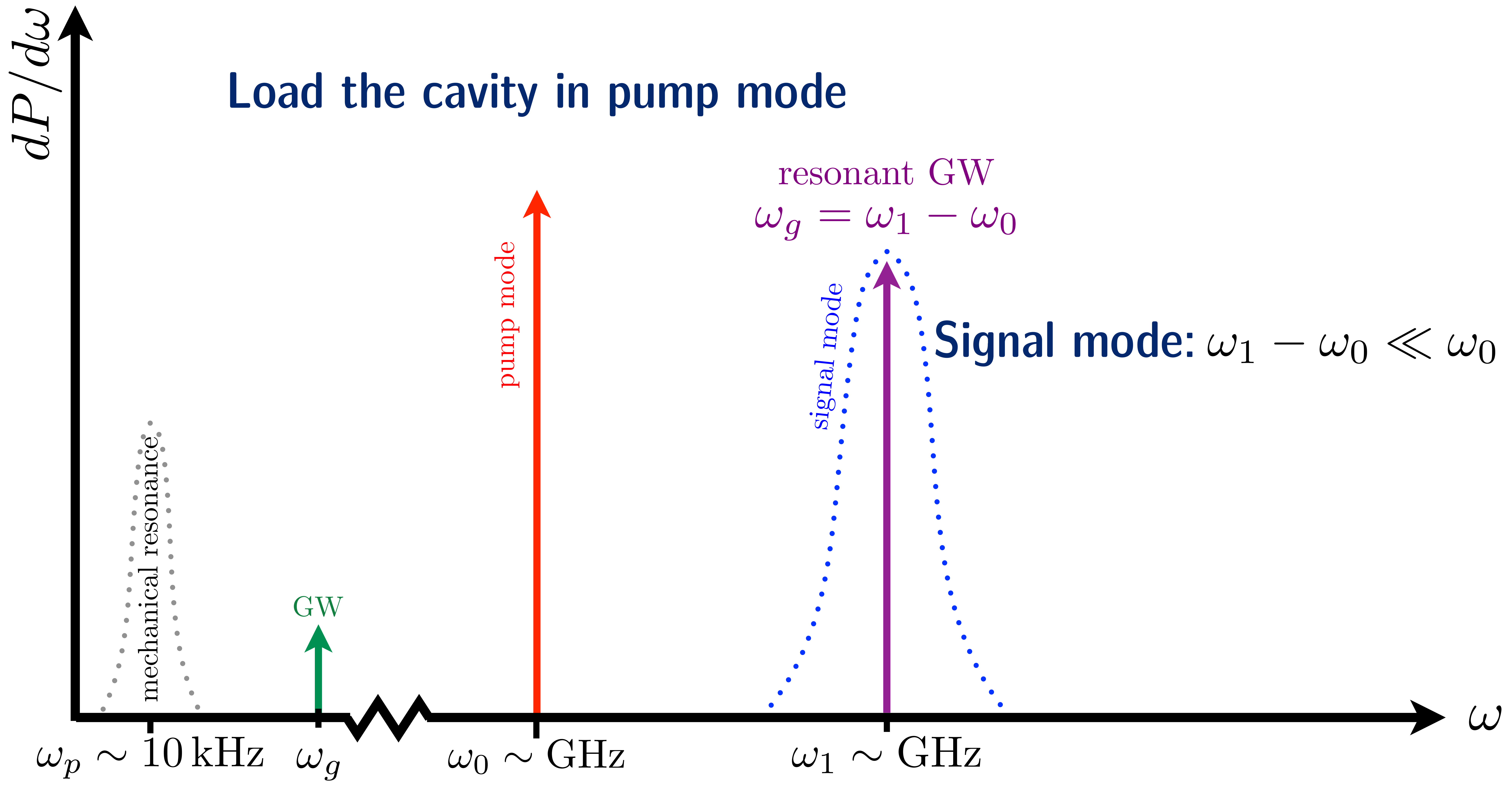
$dP/d\omega$
Load the cavity in pump mode
pump mode
signal mode
Signal mode: $\omega_1 - \omega_0 \ll \omega_0$
mechanical resonance
ω
$\omega_p \sim 10\,\mathrm{kHz}$
$\omega_0 \sim \mathrm{GHz}$
$\omega_1 \sim \mathrm{GHz}$

MAGO 2.0



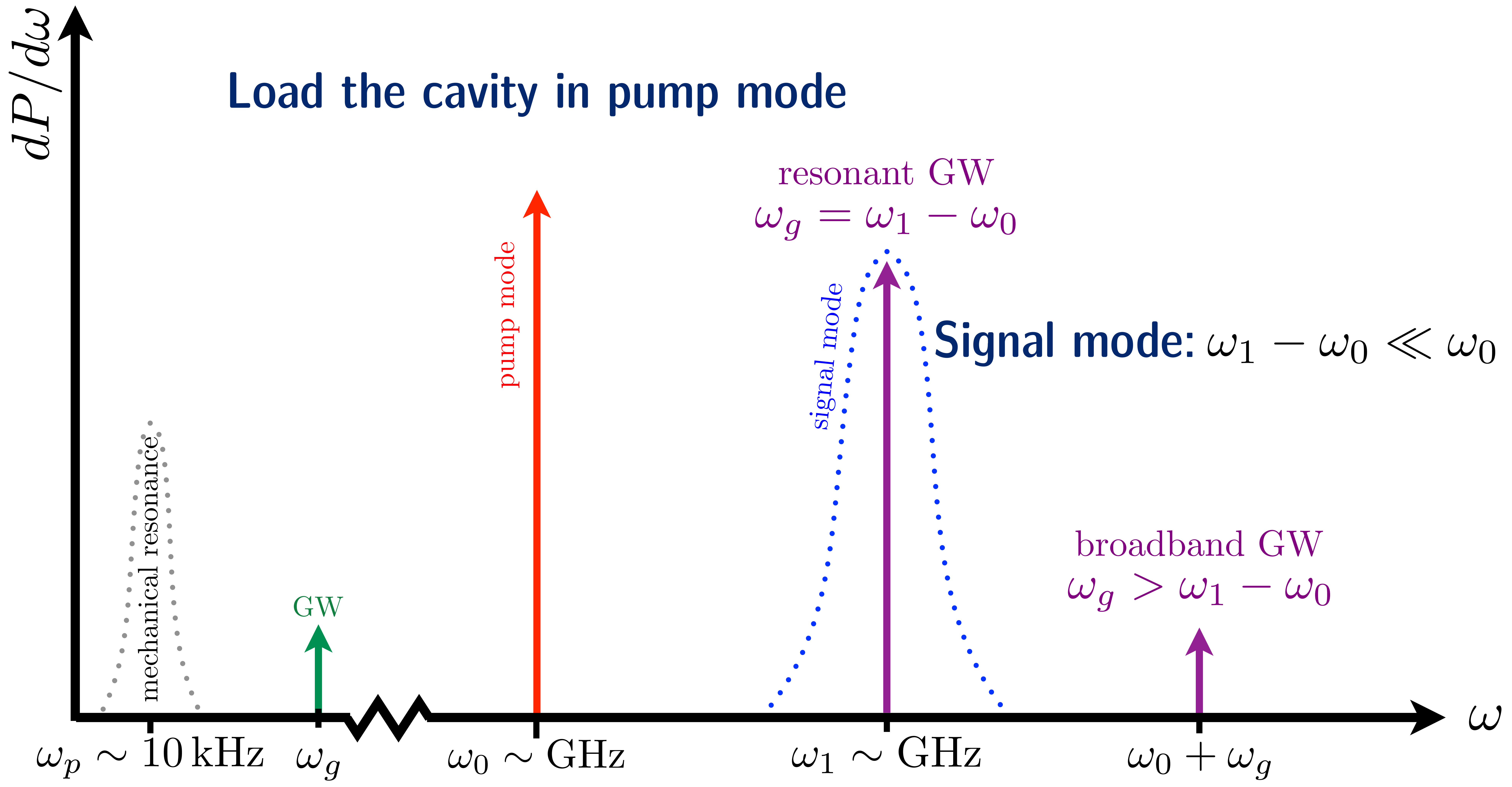
$dP/d\omega$
Load the cavity in pump mode
pump mode
signal mode
Signal mode: $\omega_1 - \omega_0 \ll \omega_0$
mechanical resonance
GW
ω
$\omega_p \sim 10\,\mathrm{kHz}$
ω_g
$\omega_0 \sim \mathrm{GHz}$
$\omega_1 \sim \mathrm{GHz}$

MAGO 2.0



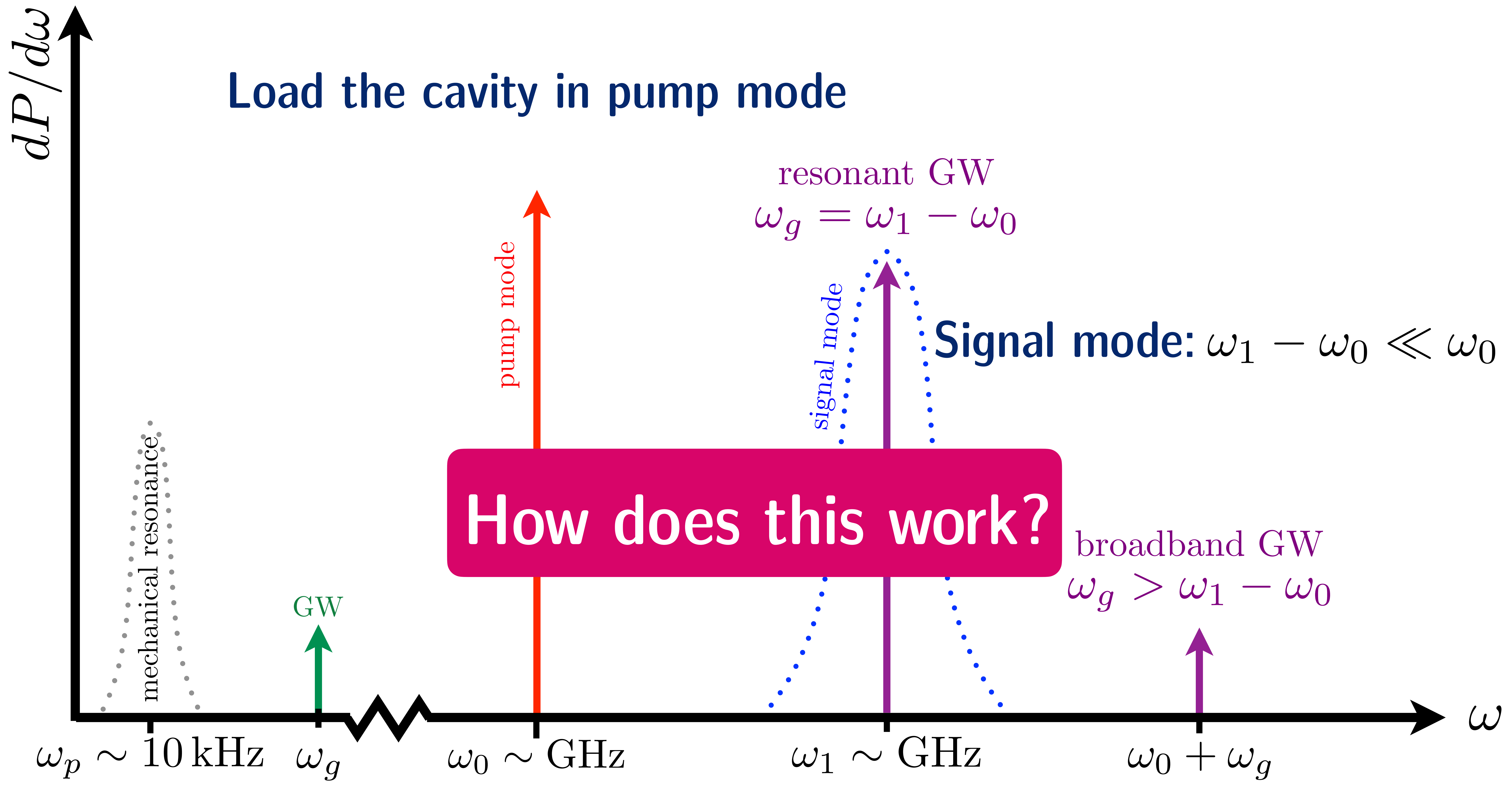
$dP/d\omega$
Load the cavity in pump mode
resonant GW
$\omega_g = \omega_1 - \omega_0$
pump mode
signal mode
Signal mode: $\omega_1 - \omega_0 \ll \omega_0$
mechanical resonance
GW
ω
$\omega_p \sim 10\,\mathrm{kHz}$
ω_g
$\omega_0 \sim \mathrm{GHz}$
$\omega_1 \sim \mathrm{GHz}$

MAGO 2.0



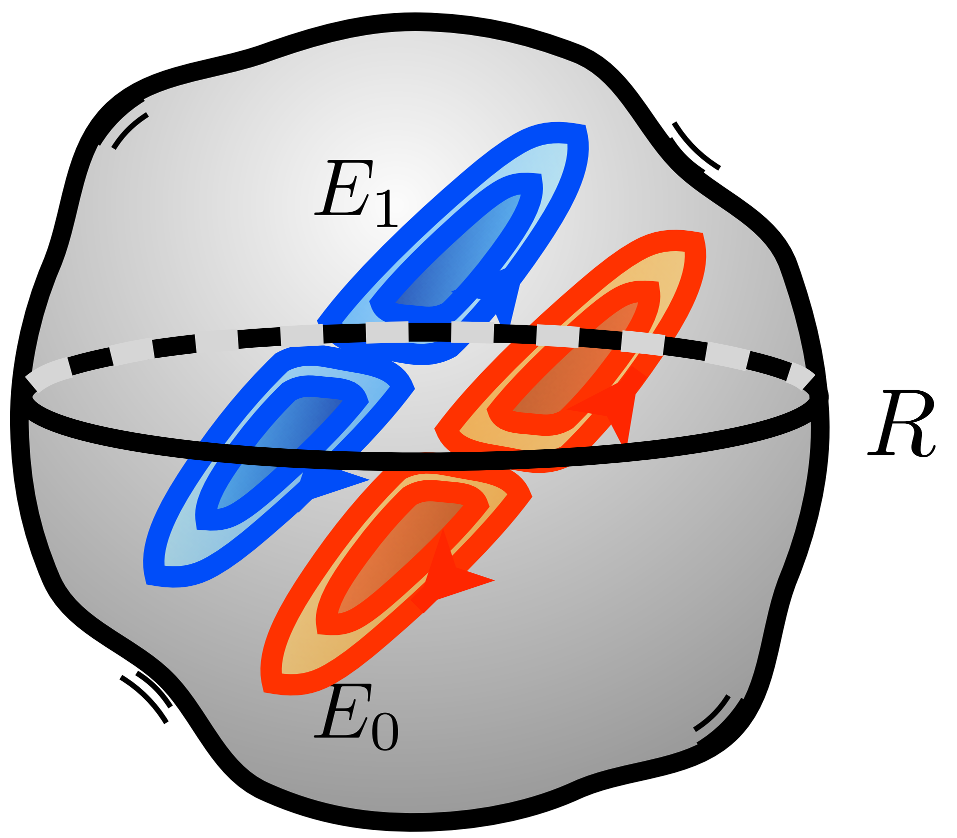
$dP/d\omega$
Load the cavity in pump mode
resonant GW
$\omega_g = \omega_1 - \omega_0$
pump mode
signal mode
Signal mode: $\omega_1 - \omega_0 \ll \omega_0$
mechanical resonance
broadband GW
$\omega_g > \omega_1 - \omega_0$
GW
ω
$\omega_p \sim 10\,\mathrm{kHz}$
ω_g
$\omega_0 \sim \mathrm{GHz}$
$\omega_1 \sim \mathrm{GHz}$
$\omega_0 + \omega_g$

MAGO 2.0


$dP/d\omega$
Load the cavity in pump mode
resonant GW
$\omega_g = \omega_1 - \omega_0$
pump mode
signal mode
Signal mode: $\omega_1 - \omega_0 \ll \omega_0$
How does this work?
broadband GW
$\omega_g > \omega_1 - \omega_0$
mechanical resonance
GW
ω
$\omega_p \sim 10\,\mathrm{kHz}$
ω_g
$\omega_0 \sim \mathrm{GHz}$
$\omega_1 \sim \mathrm{GHz}$
$\omega_0 + \omega_g$

MAGO 2.0

$\omega \sim \mathrm{GHz}$



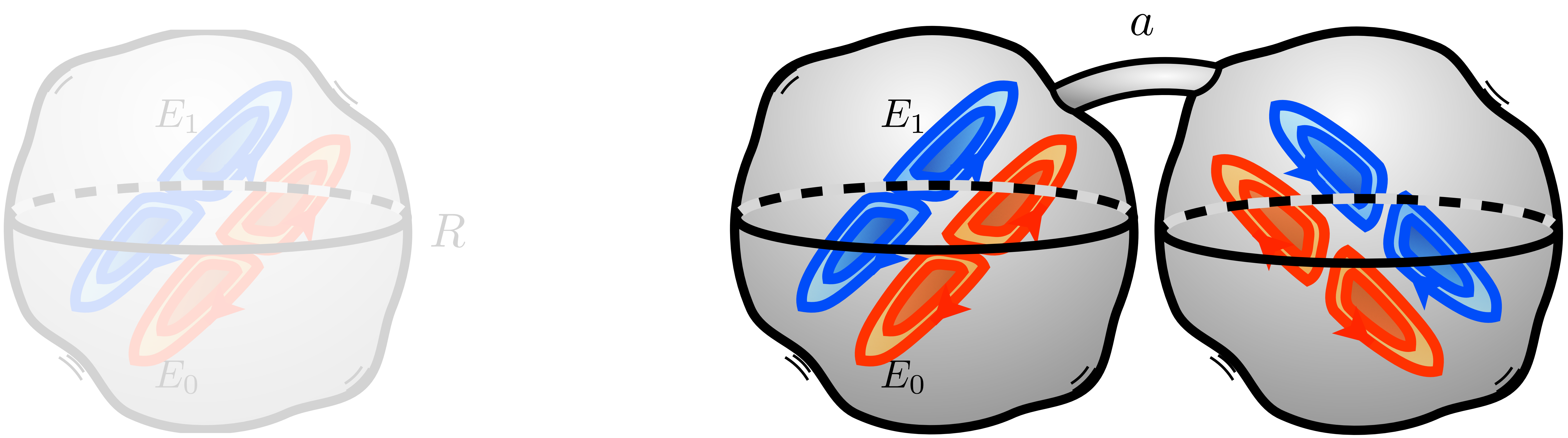
MAGO 2.0

$$\omega \sim \text{GHz}$$

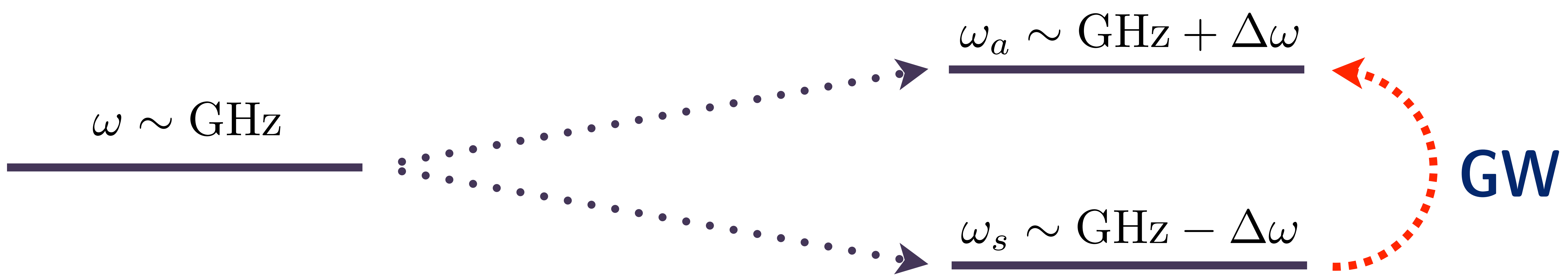
$$\omega_a \sim \text{GHz} + \Delta\omega$$

$$\omega_s \sim \text{GHz} - \Delta\omega$$

$$\Delta\omega \sim (a/R)^3$$



MAGO 2.0



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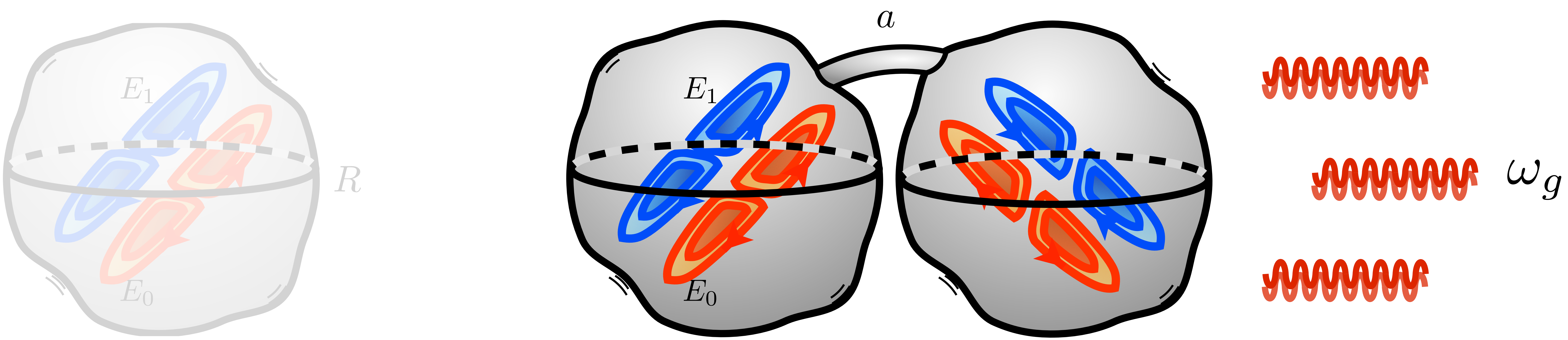
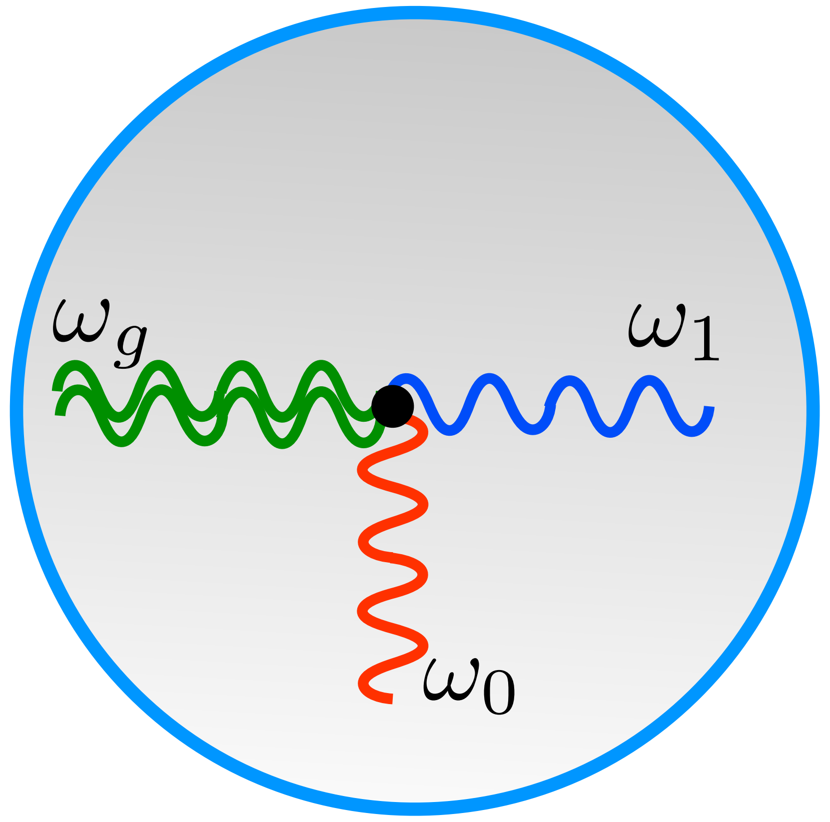


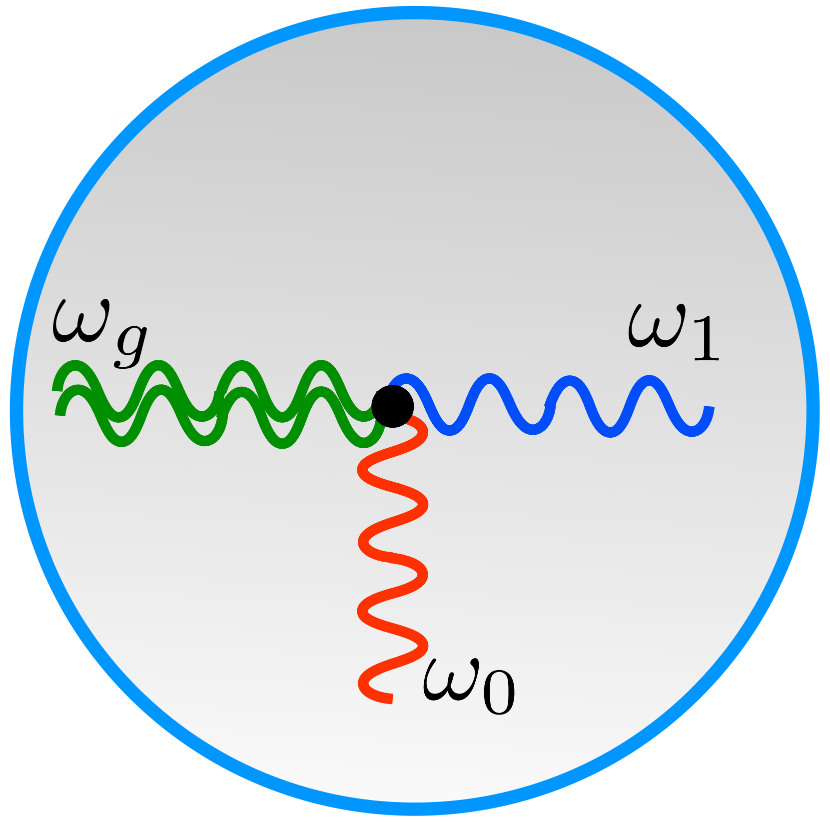
EM and Mechanical signals

EM and Mechanical signals



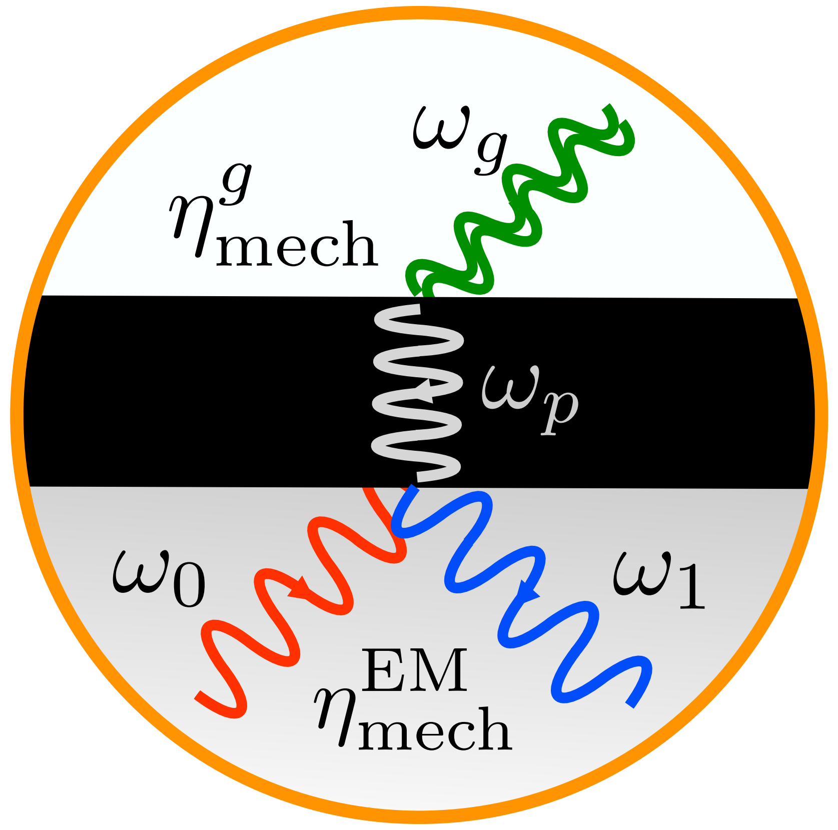
Parametrics of the EM signal: $E_{\rm sig}^{\rm (EM)} \sim Q_{\rm em}\,(\omega_g\,L_{\rm cav})^2\,h^{\rm TT}\,E_0$

EM and Mechanical signals

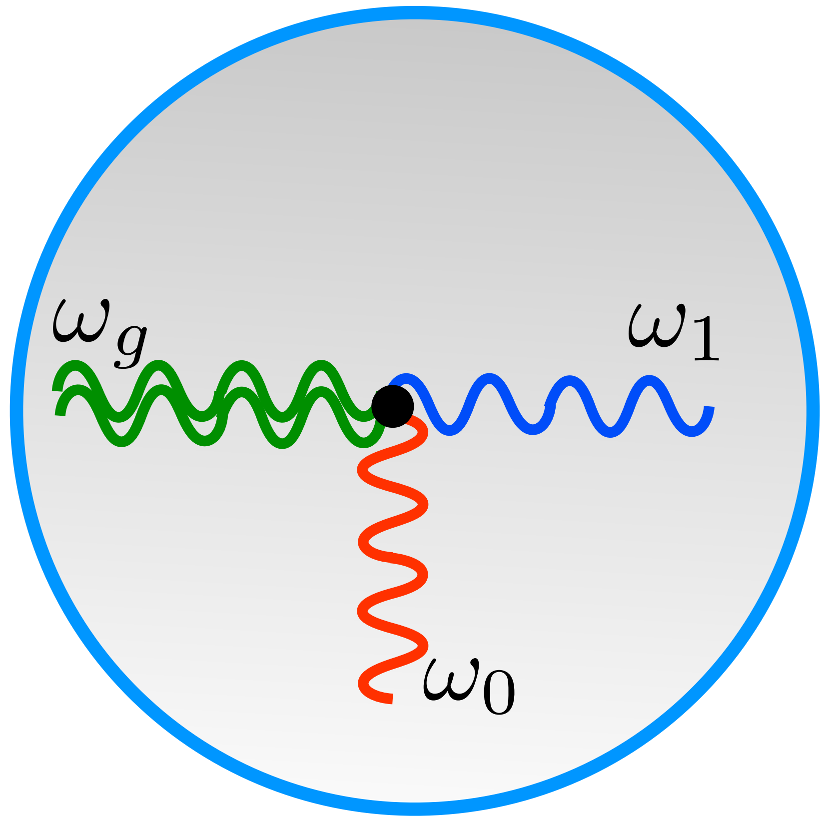


Parametrics of the EM signal: $E_{\rm sig}^{\rm (EM)} \sim Q_{\rm em}\,(\omega_g\,L_{\rm cav})^2\,h^{\rm TT}\,E_0$

Mechanical signal: $E_{\rm sig}^{\rm (mech)} \sim Q_{\rm em}\,h^{\rm TT}\,E_0\,\min\left(1\,,\frac{\omega_g\,L_{\rm cav}}{c_s}\right)^2$



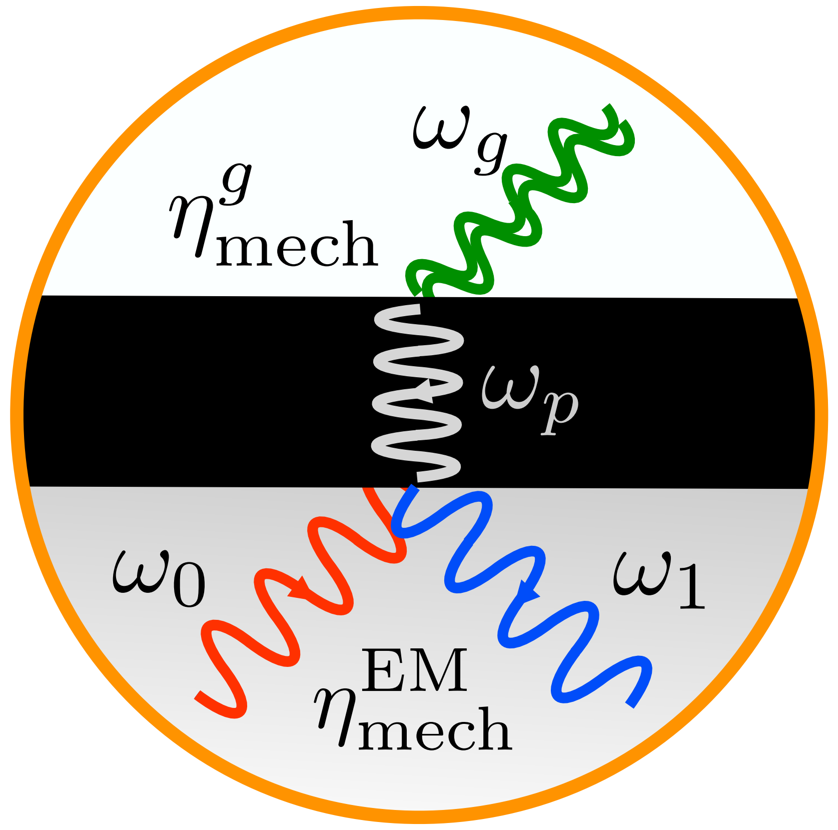
EM and Mechanical signals



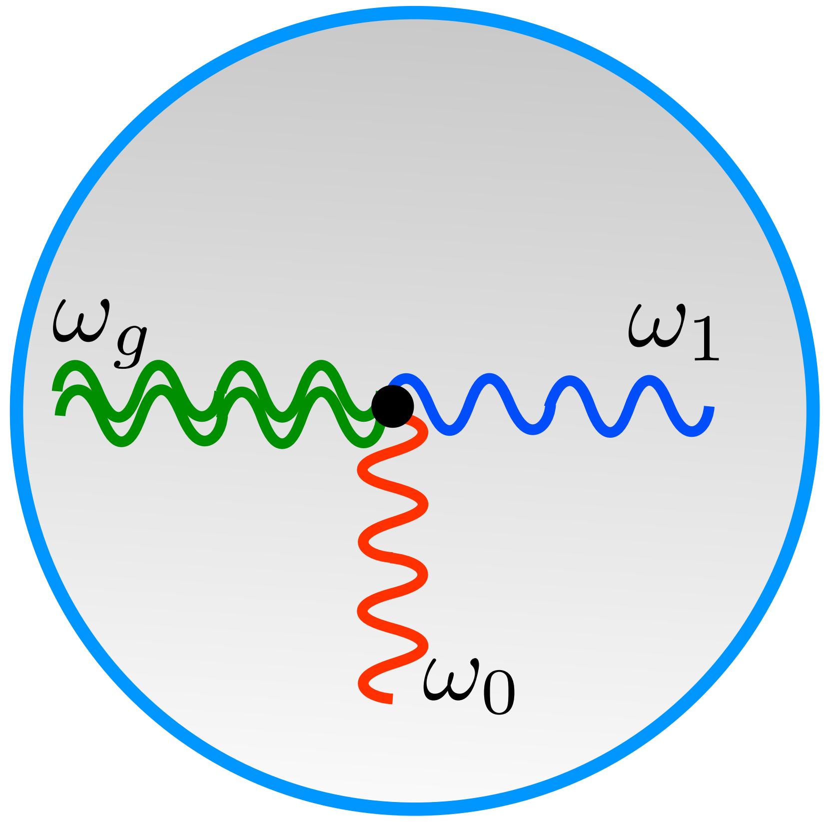
Parametrics of the EM signal: $E_{\text{sig}}^{(\text{EM})} \sim Q_{\text{em}} \, (\omega_g \, L_{\text{cav}})^2 \, h^{\text{TT}} \, E_0$

Enhanced by $1/c_s^2 \gg 1$ *(!)*

Mechanical signal: $E_{\text{sig}}^{(\text{mech})} \sim Q_{\text{em}} \, h^{\text{TT}} \, E_0 \, \min\left(1 \, , \frac{\omega_g \, L_{\text{cav}}}{c_s}\right)^2$



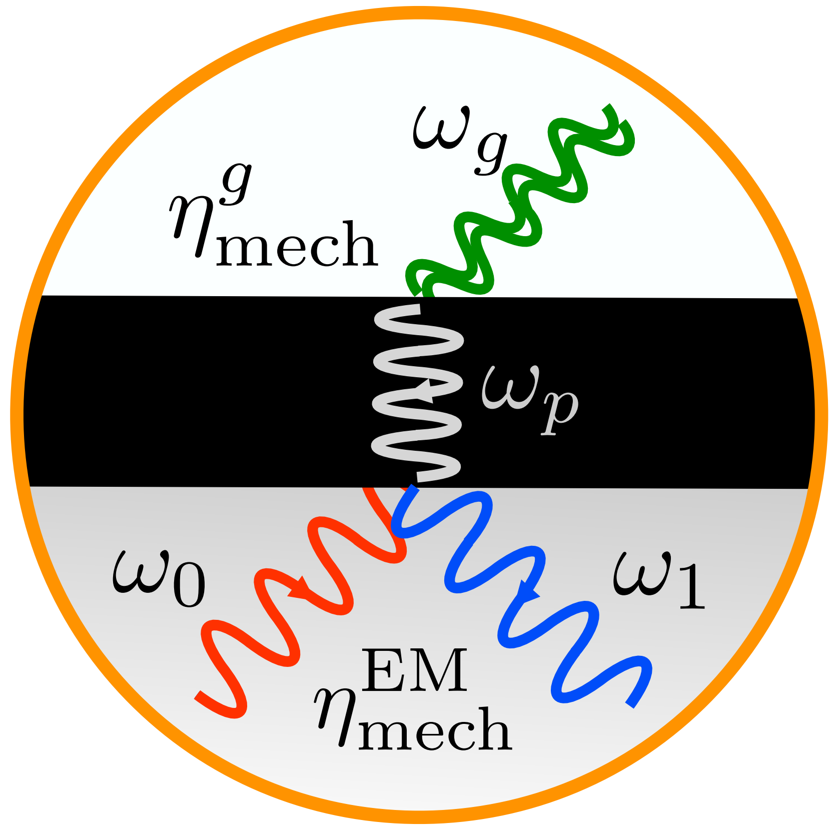
EM and Mechanical signals



Parametrics of the EM signal: $E_{\rm sig}^{\rm (EM)} \sim Q_{\rm em}\,(\omega_g\, L_{\rm cav})^2\, h^{\rm TT}\, E_0$

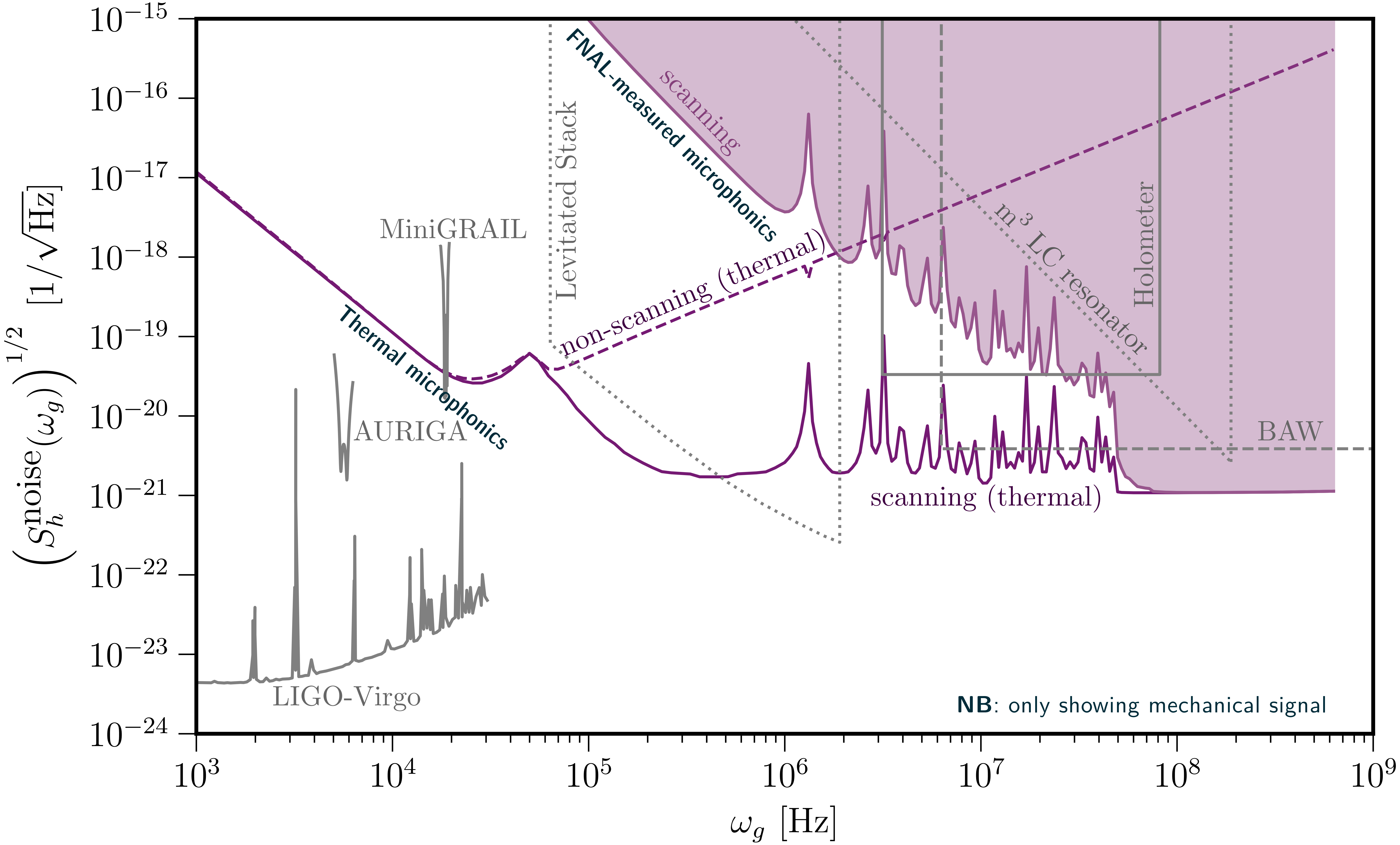
Enhanced by $1/c_s^2 \gg 1$ (!)

Mechanical signal: $E_{\rm sig}^{\rm (mech)} \sim Q_{\rm em}\, h^{\rm TT}\, E_0\, \min\left(1\,,\, \frac{\omega_g\, L_{\rm cav}}{c_s}\right)^2$



Mechanical modes less "rigid" than EM modes

Noise in MAGO 2.0



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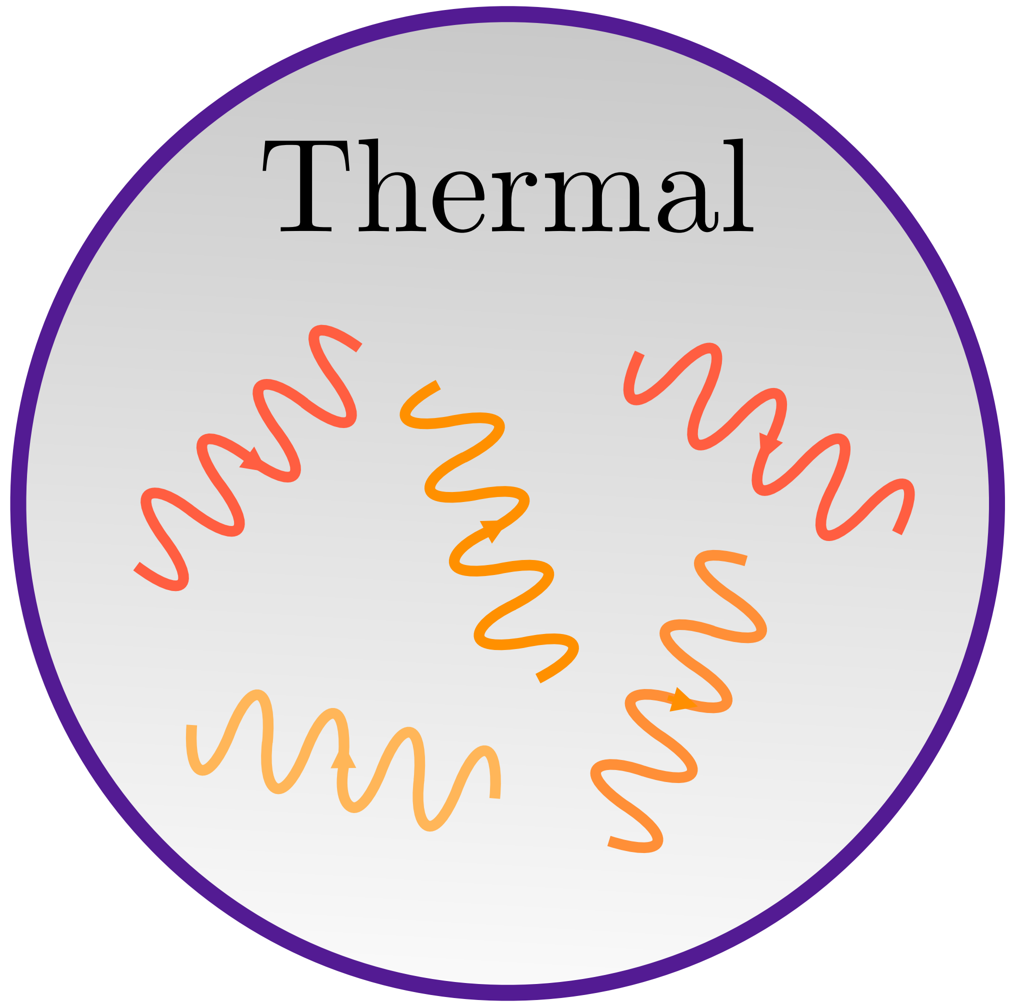

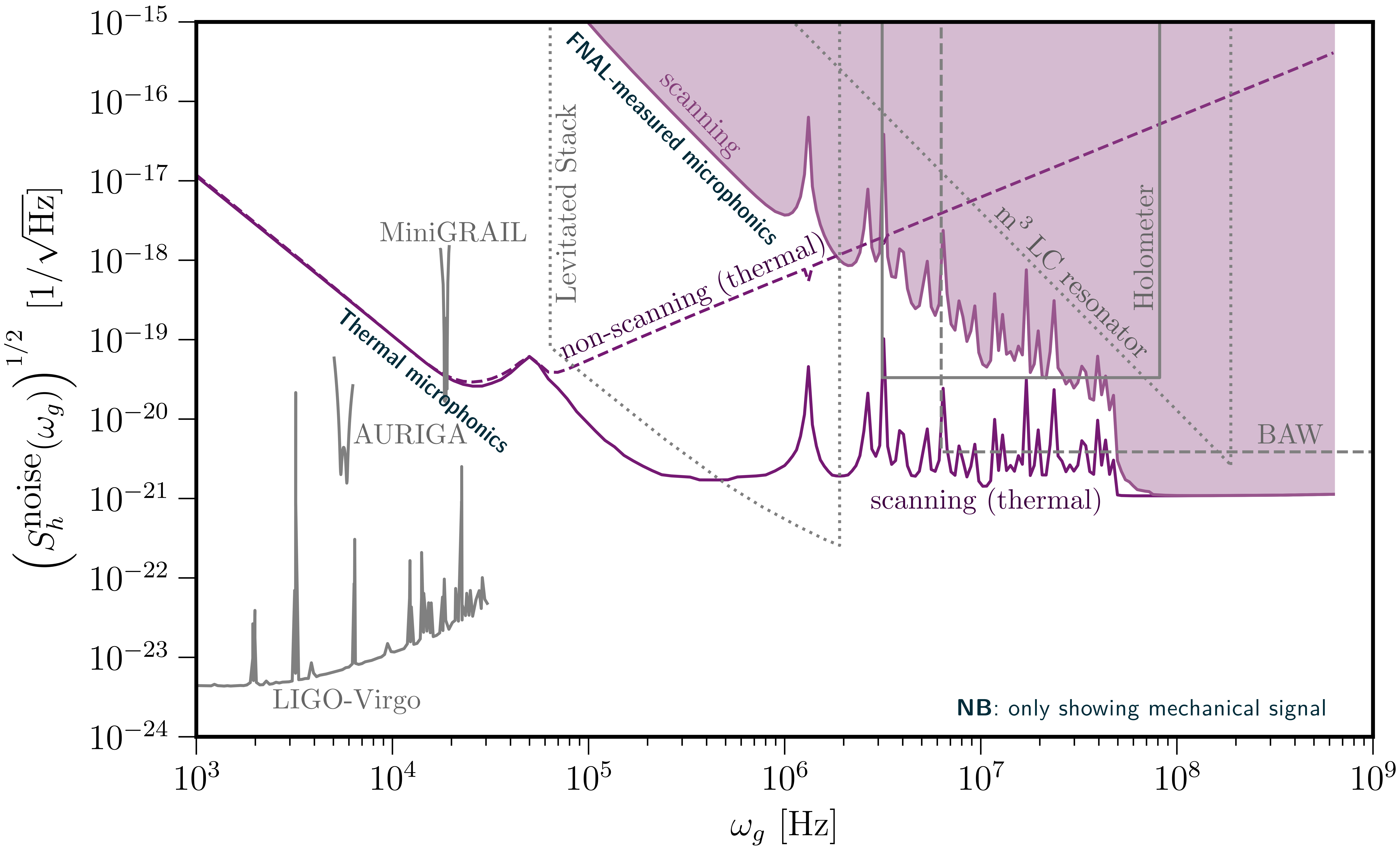

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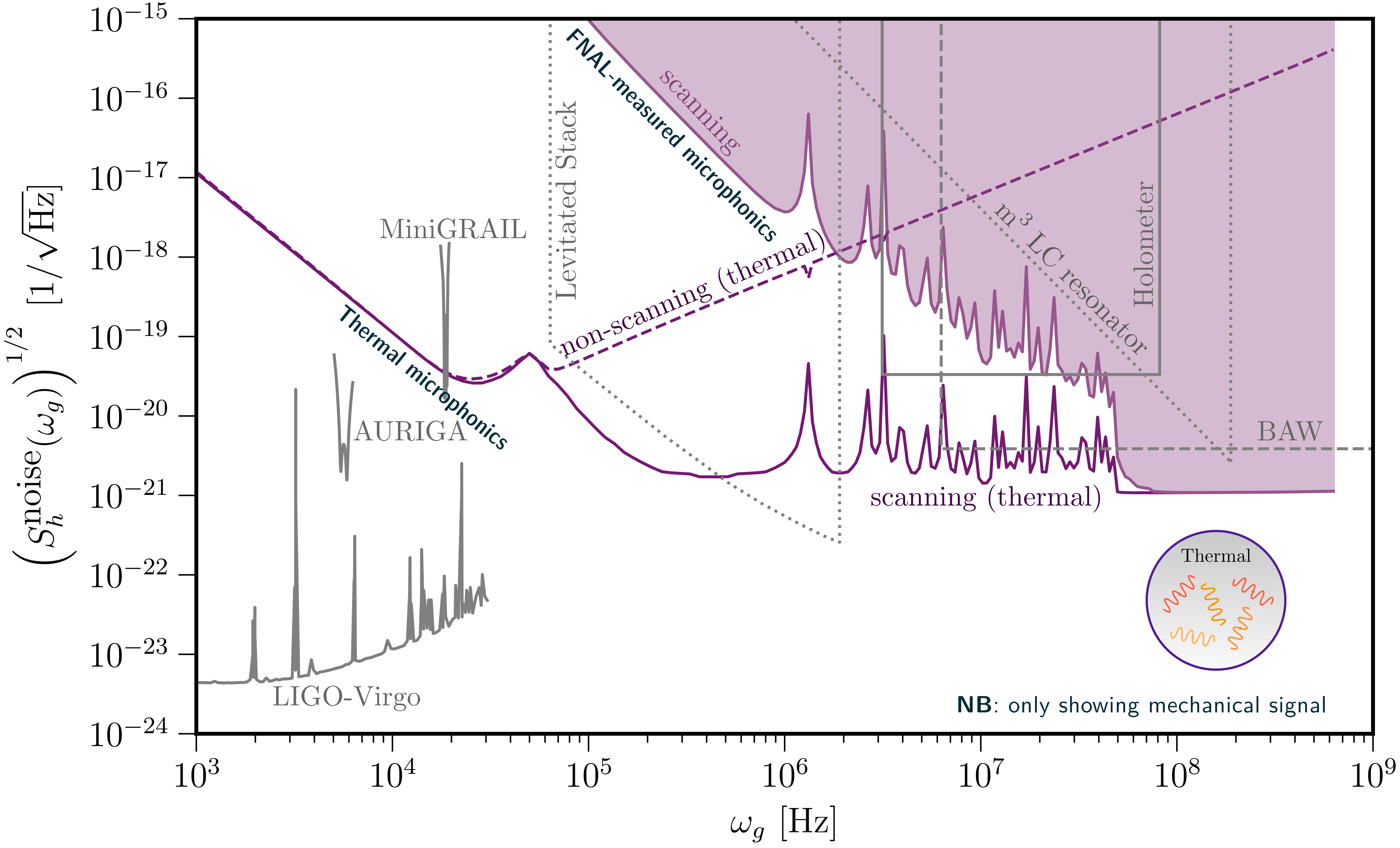

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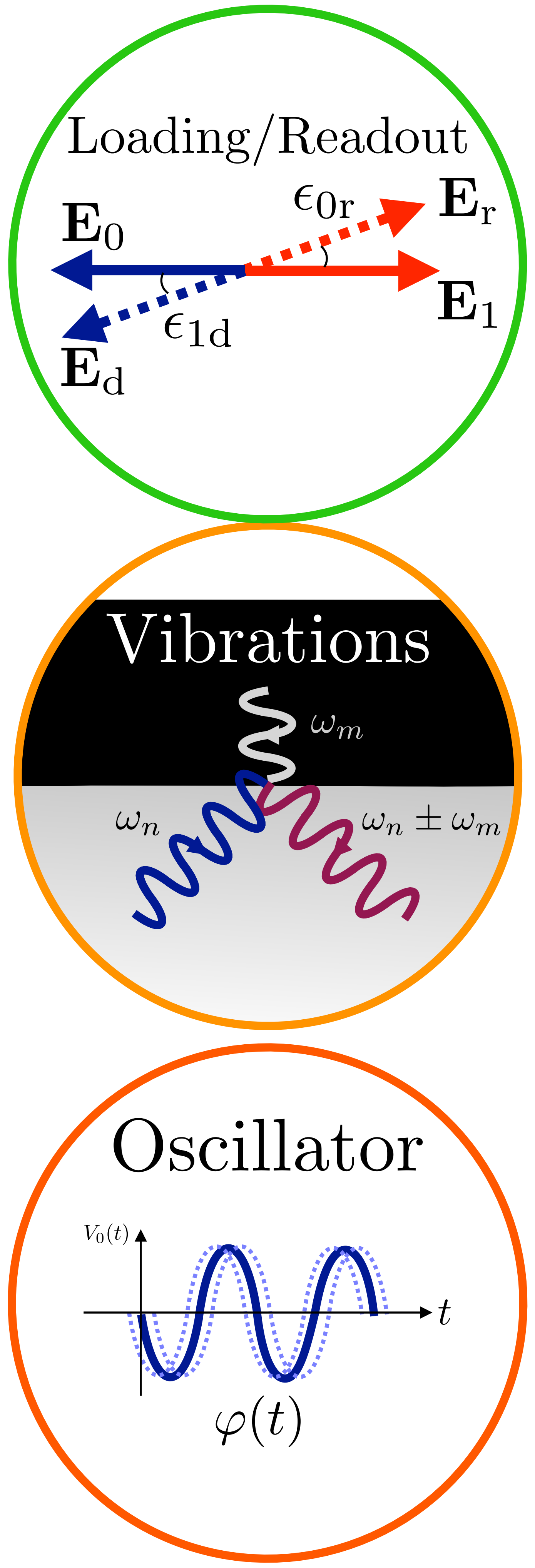

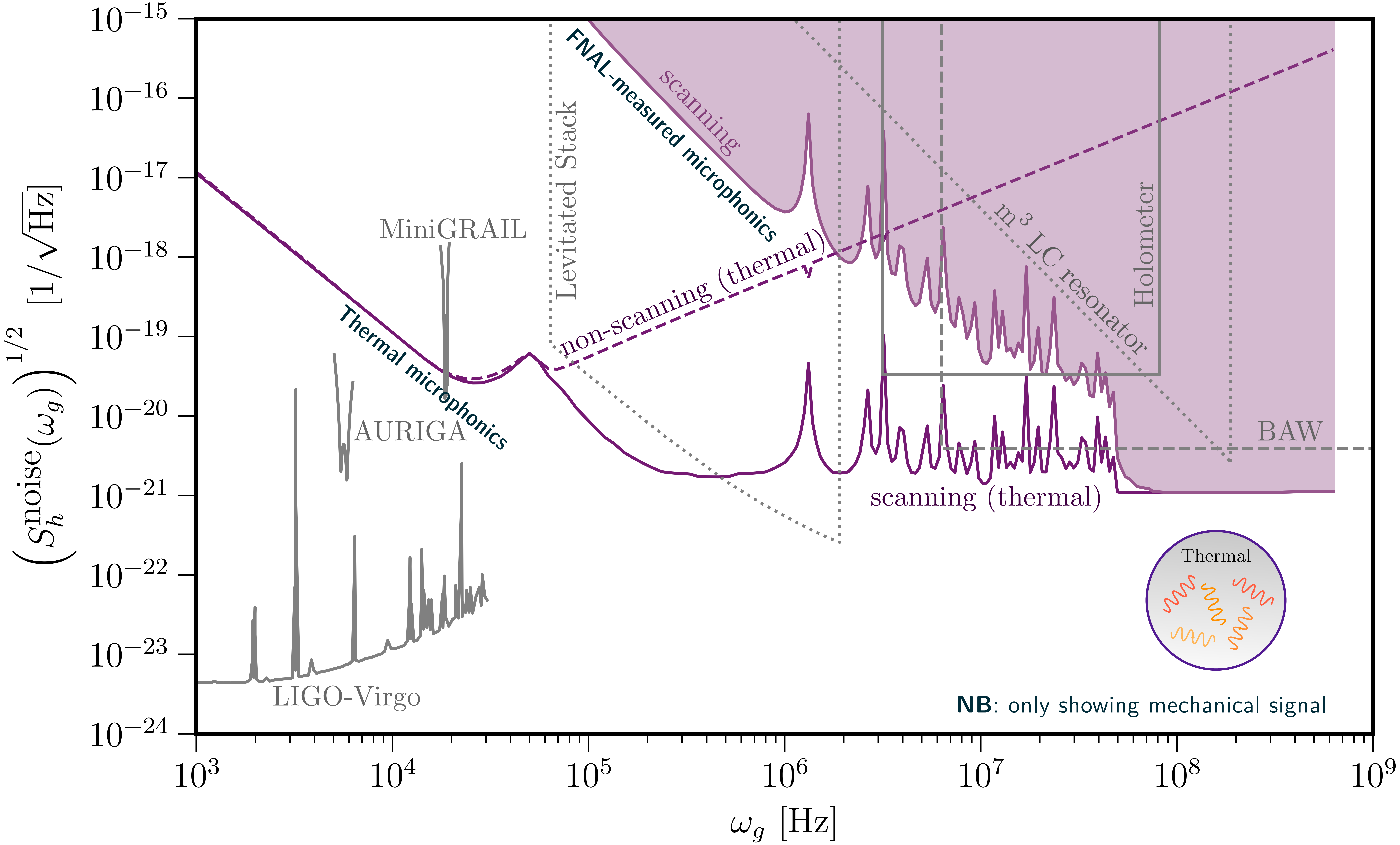

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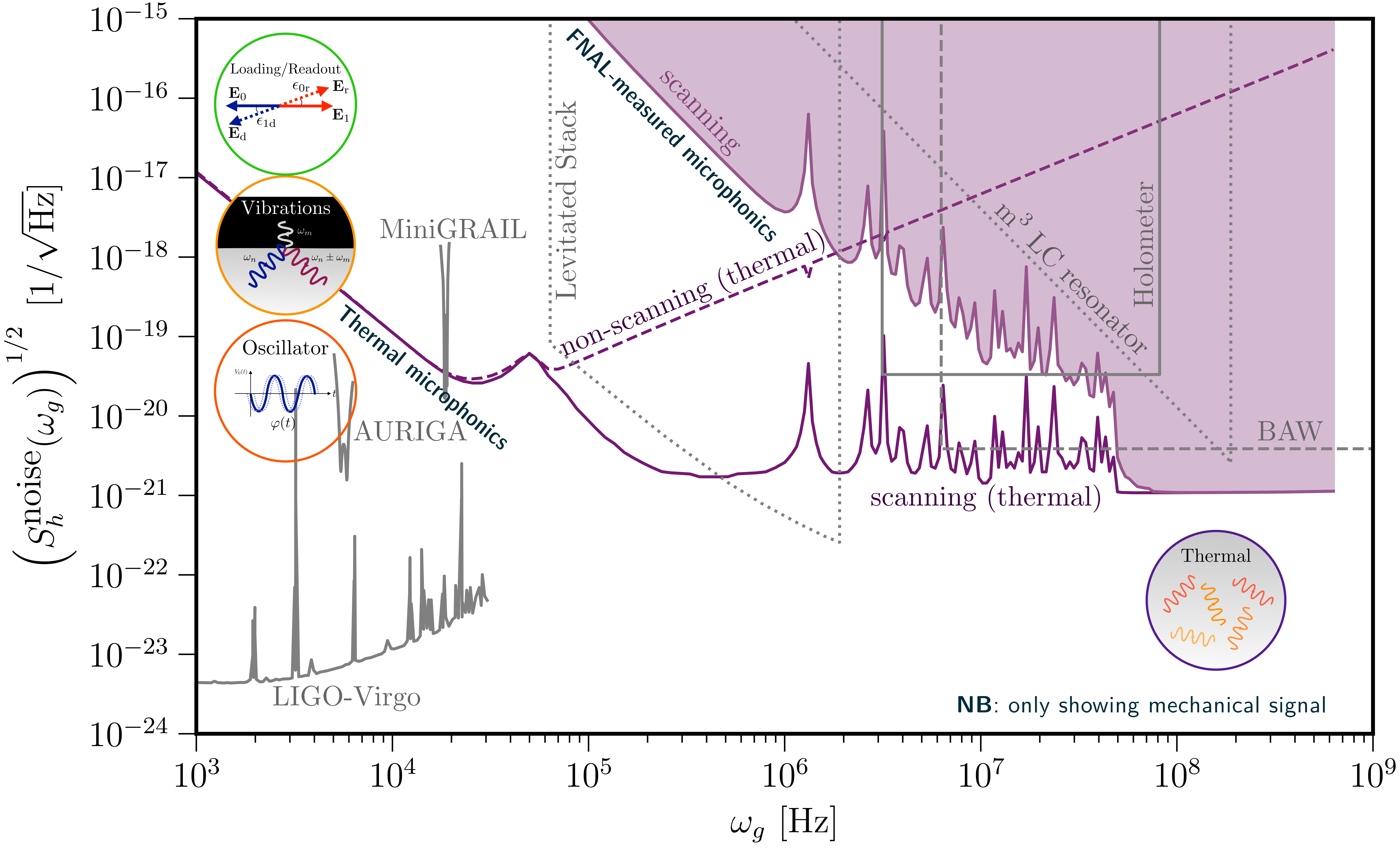

Why SRF Cavity as a Weber Bar?

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Thermal mechanical noise smaller for larger mass $S_{\text{noise}}^{(\text{mech.})} \propto \frac{4T\omega_p}{Q_p M}$

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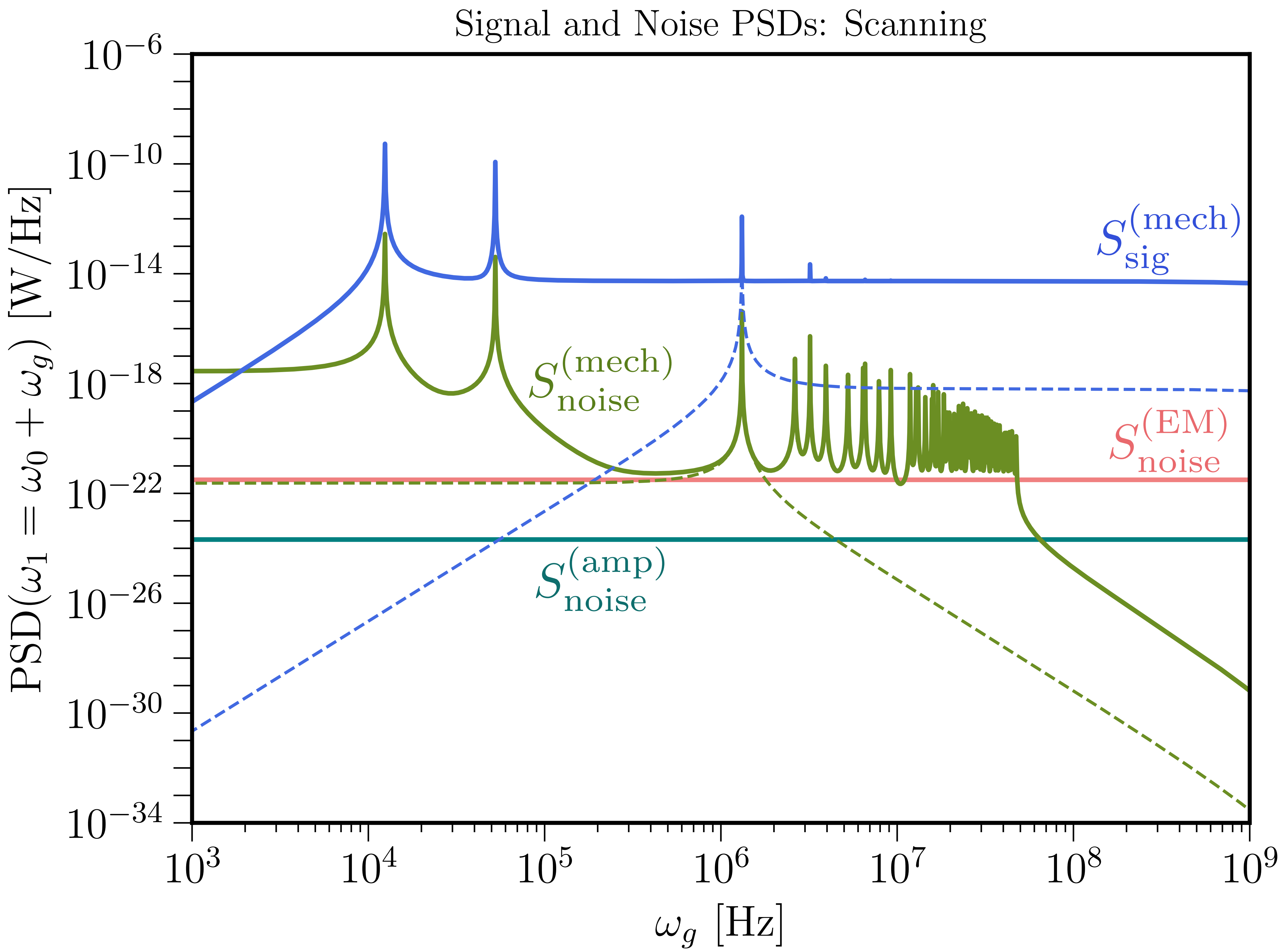
SRF Cavity w/ Q~10^{10} means better sensitivity

Noise in MAGO 2.0



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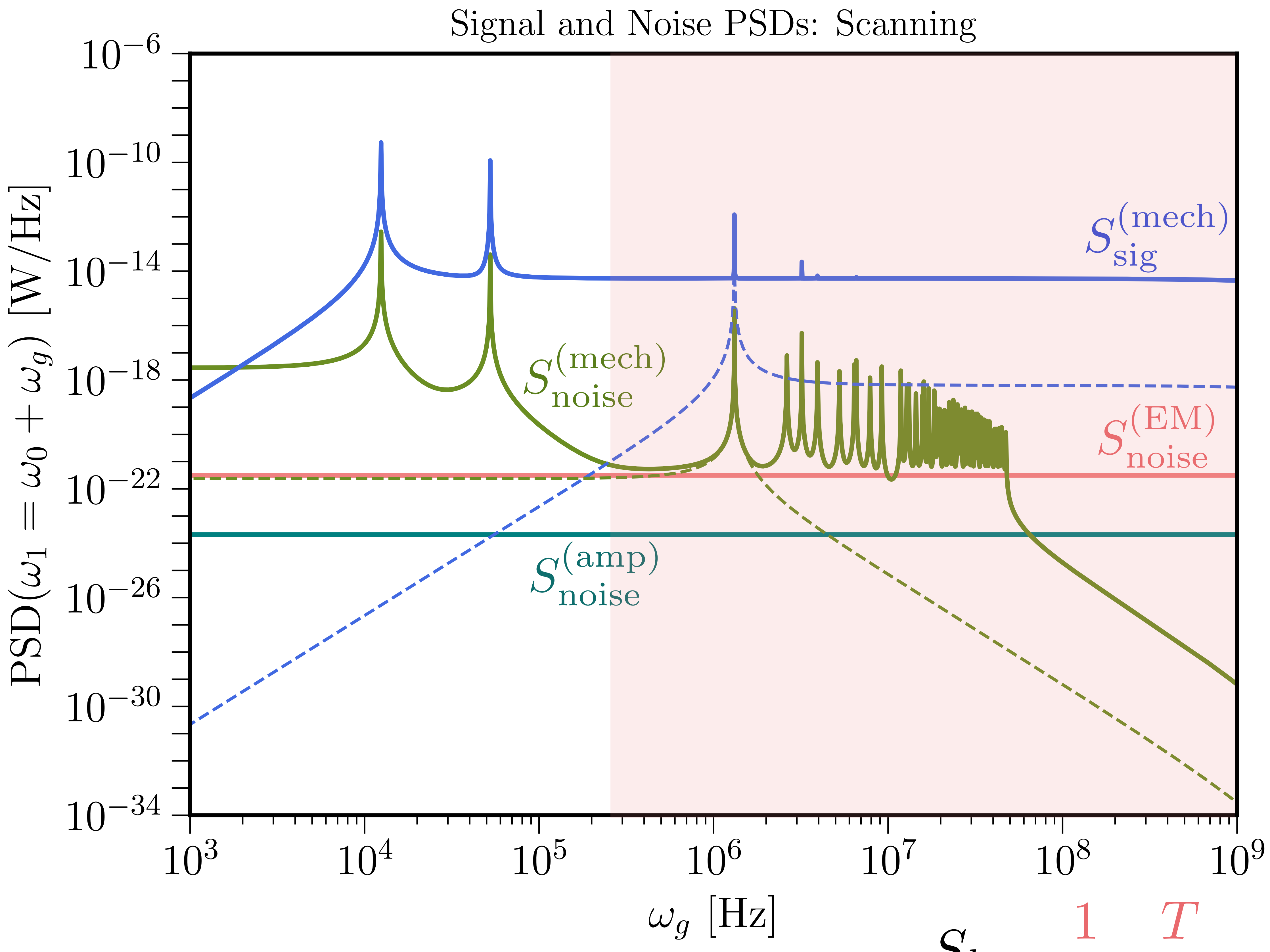
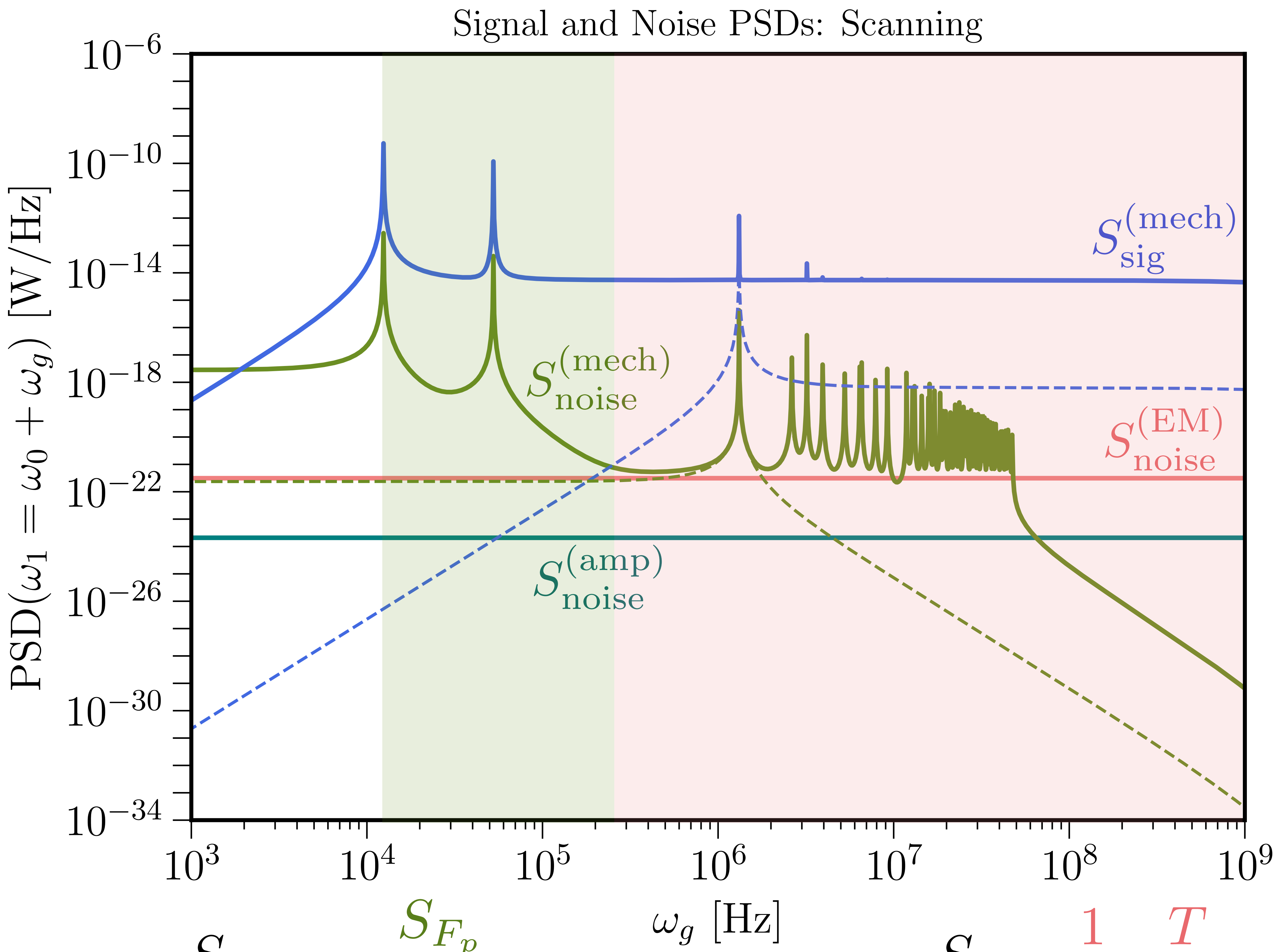


$$S_h \sim \frac{1}{Q}\frac{T}{\omega_0 U_0}$$

EM Th. > MHz

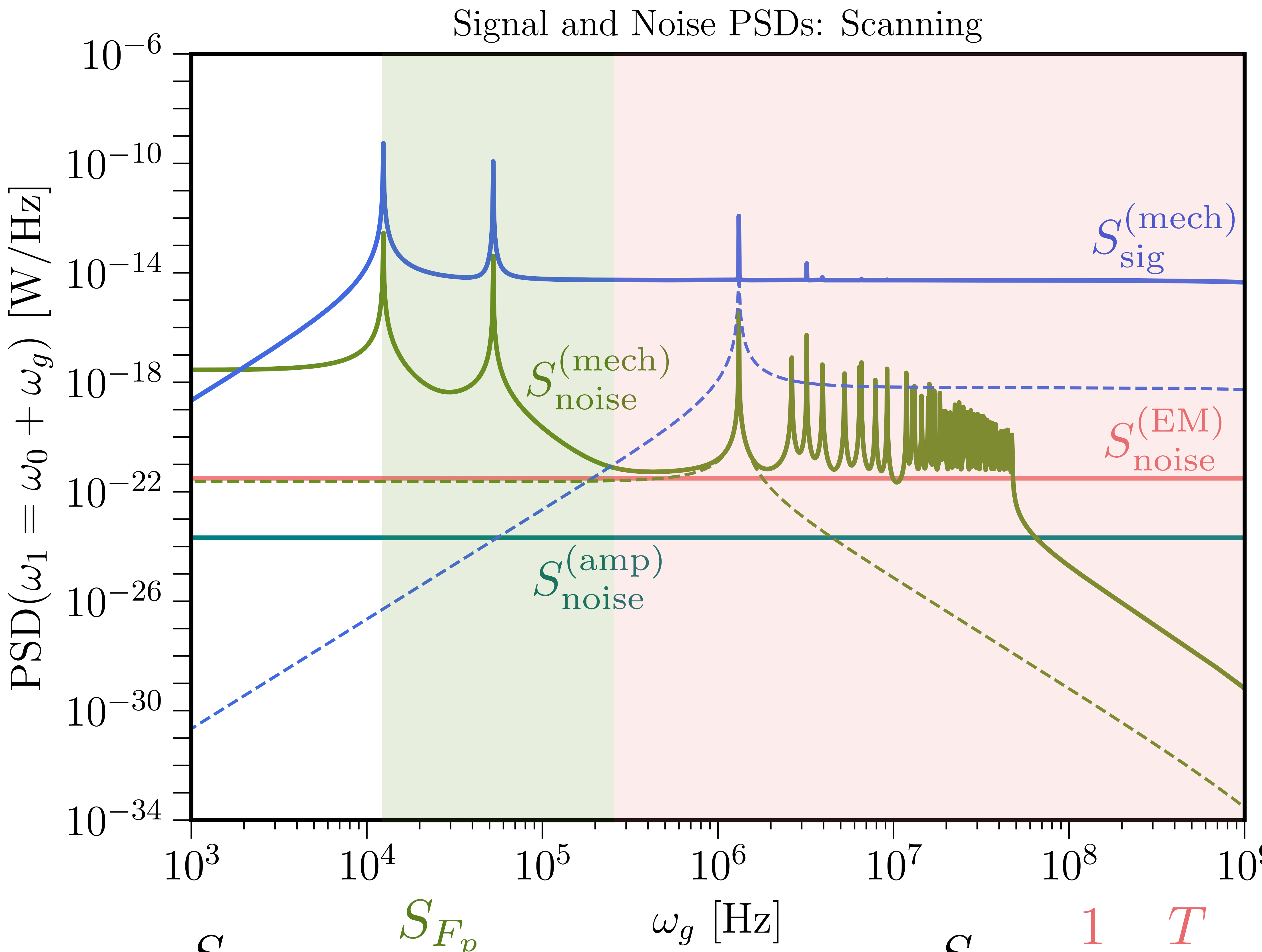
Noise in MAGO 2.0

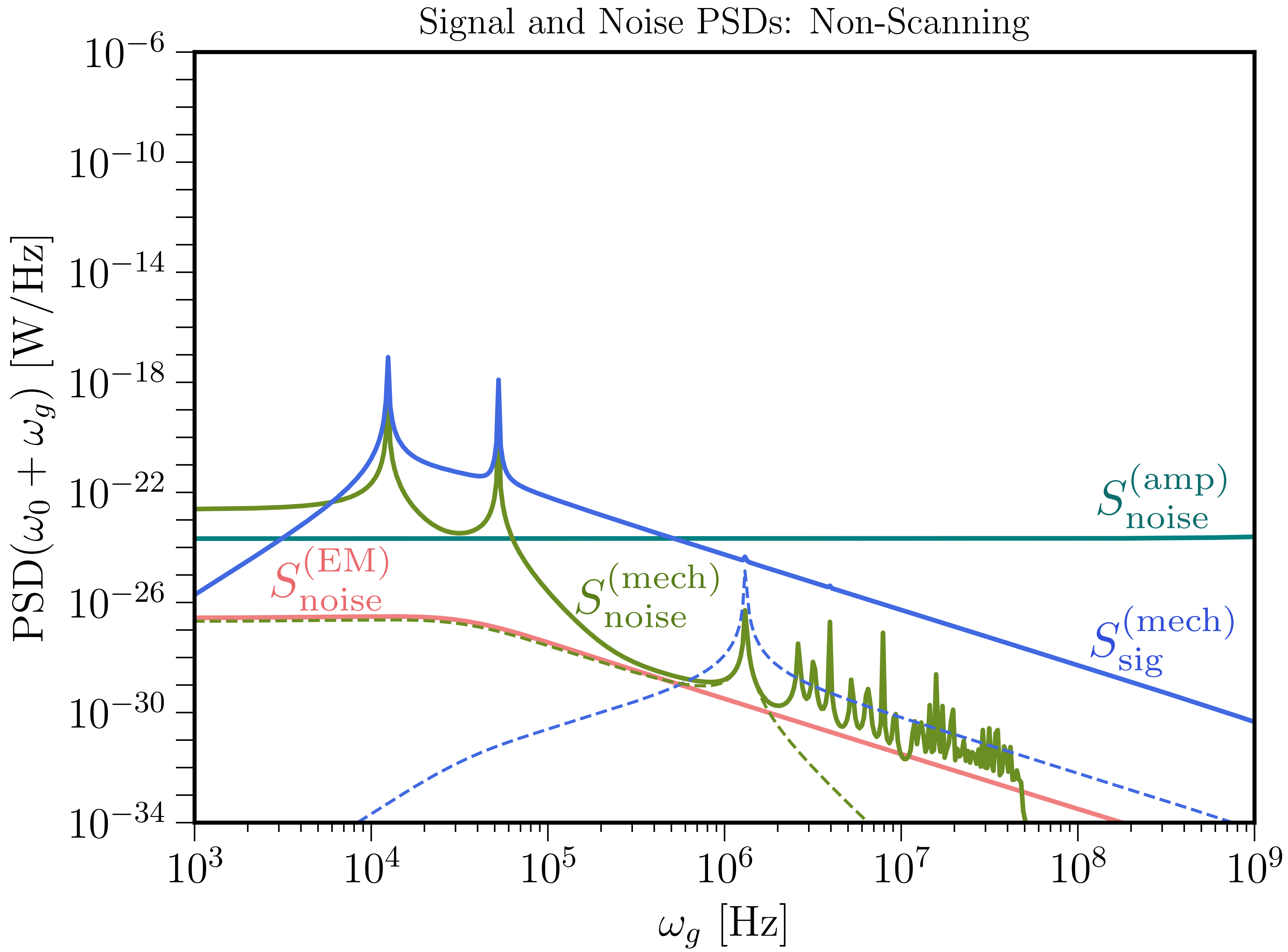


$$S_h \sim \frac{S_{F_p}}{\omega_g^4 (ML)^2} \qquad S_h \sim \frac{1}{Q}\frac{T}{\omega_0 U_0}$$

Mech $<$ MHz **EM Th. $>$ MHz**

Noise in MAGO 2.0





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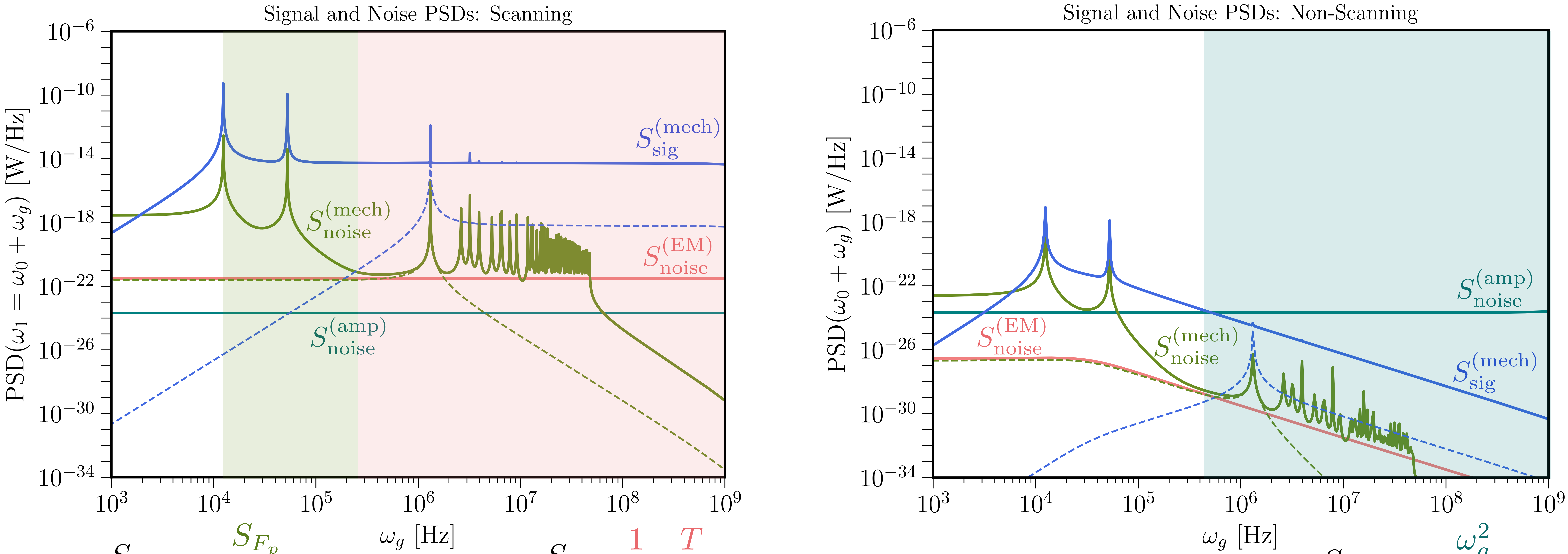

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Amp > MHz

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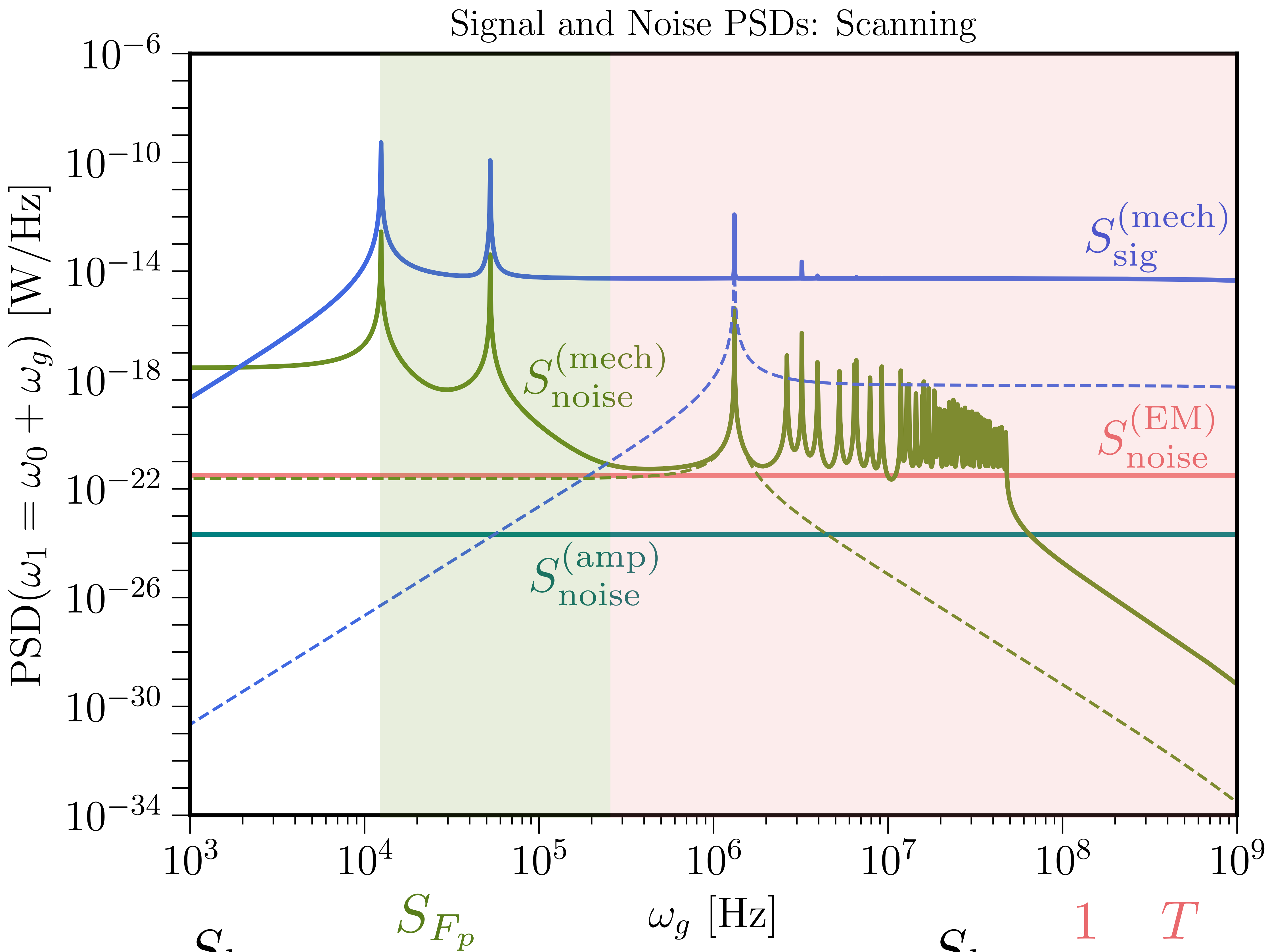

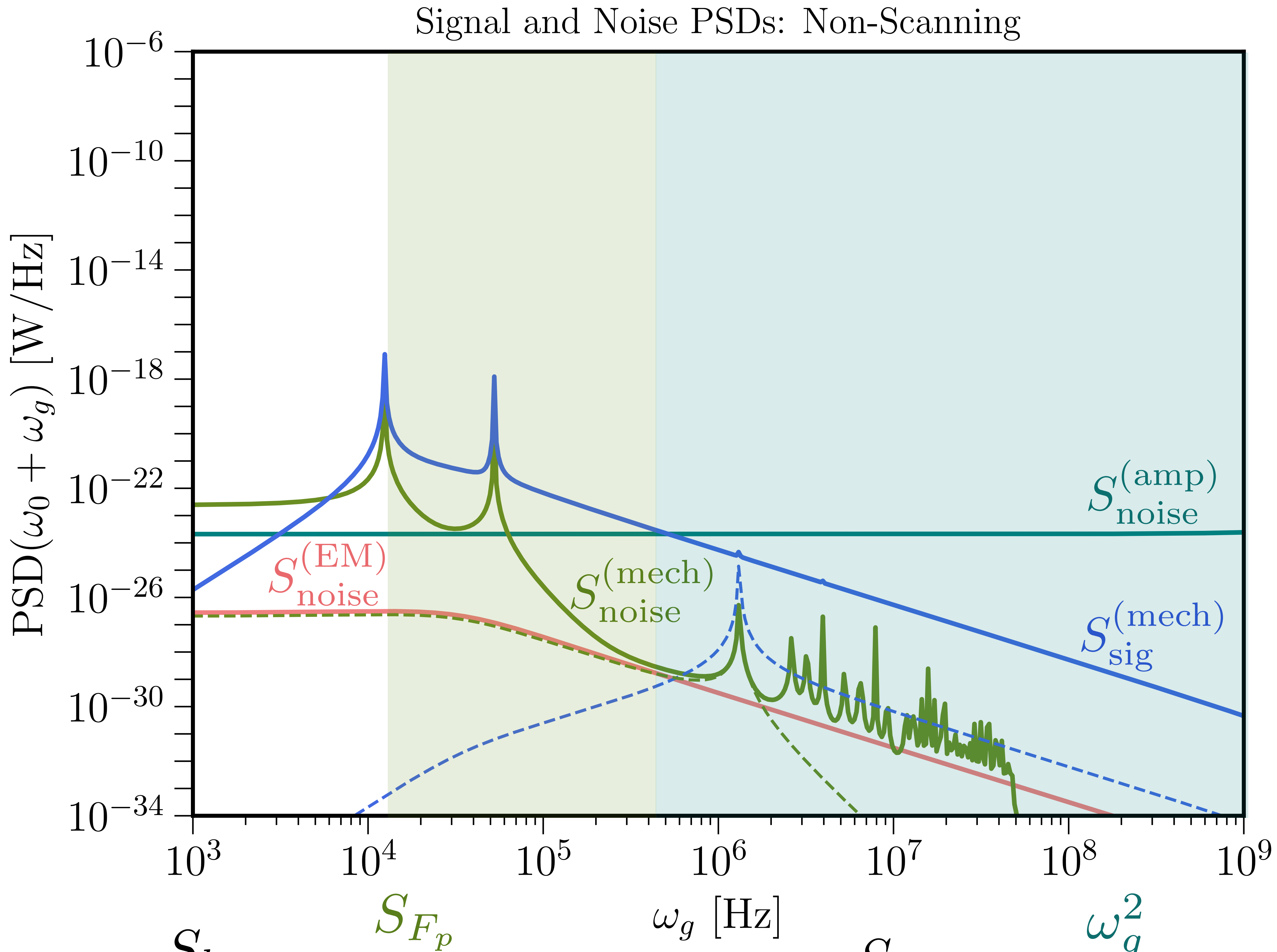

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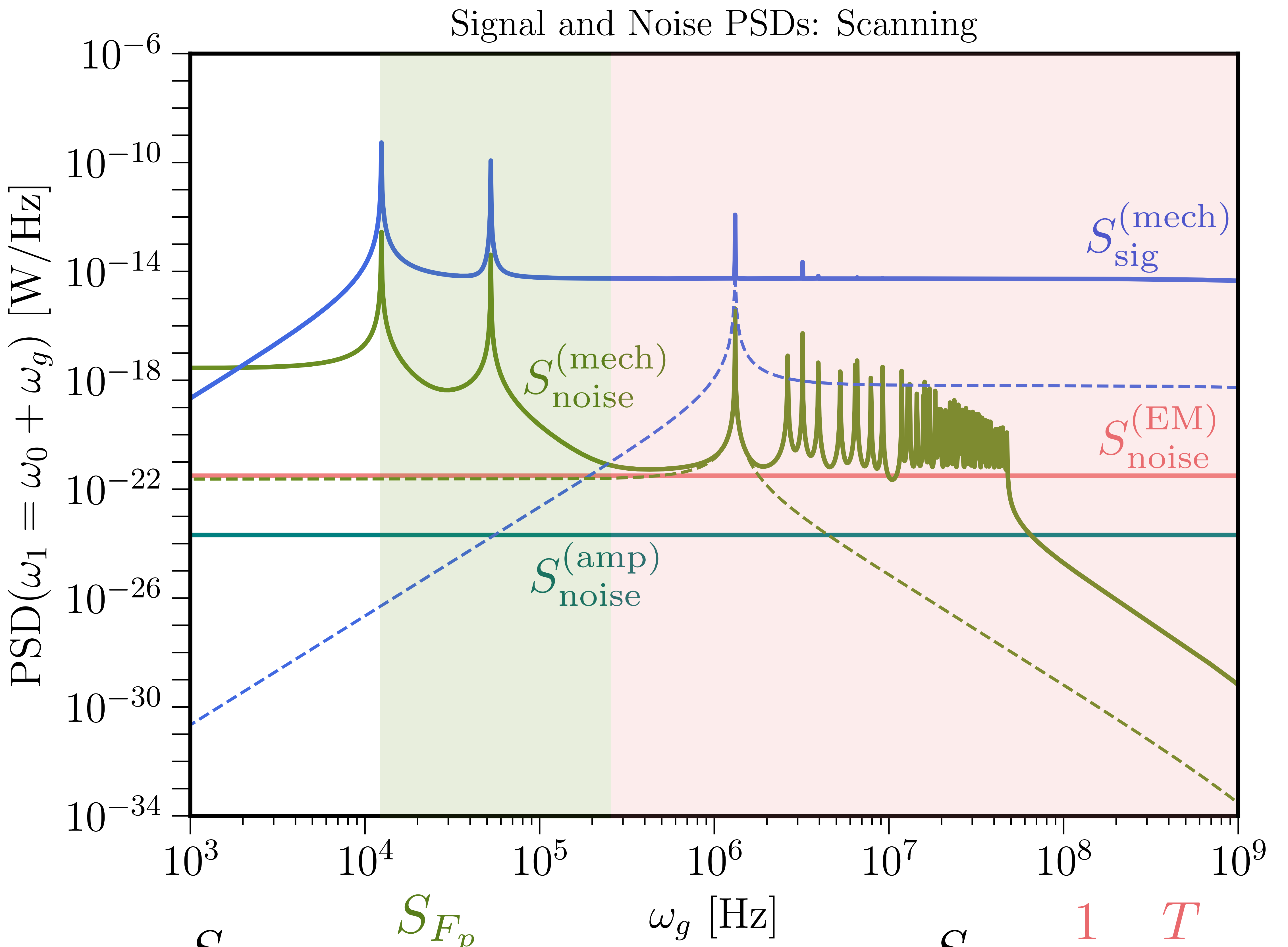

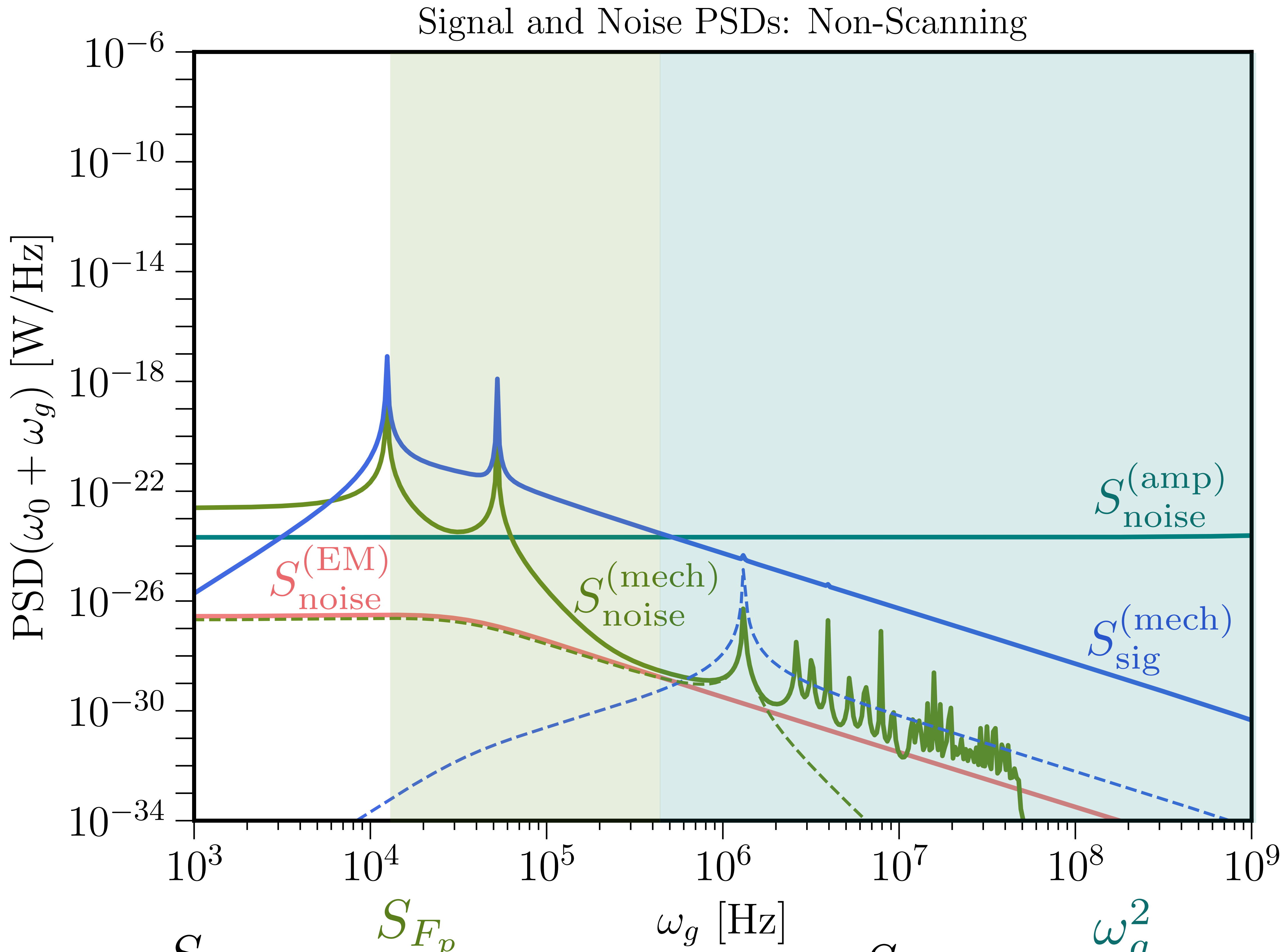

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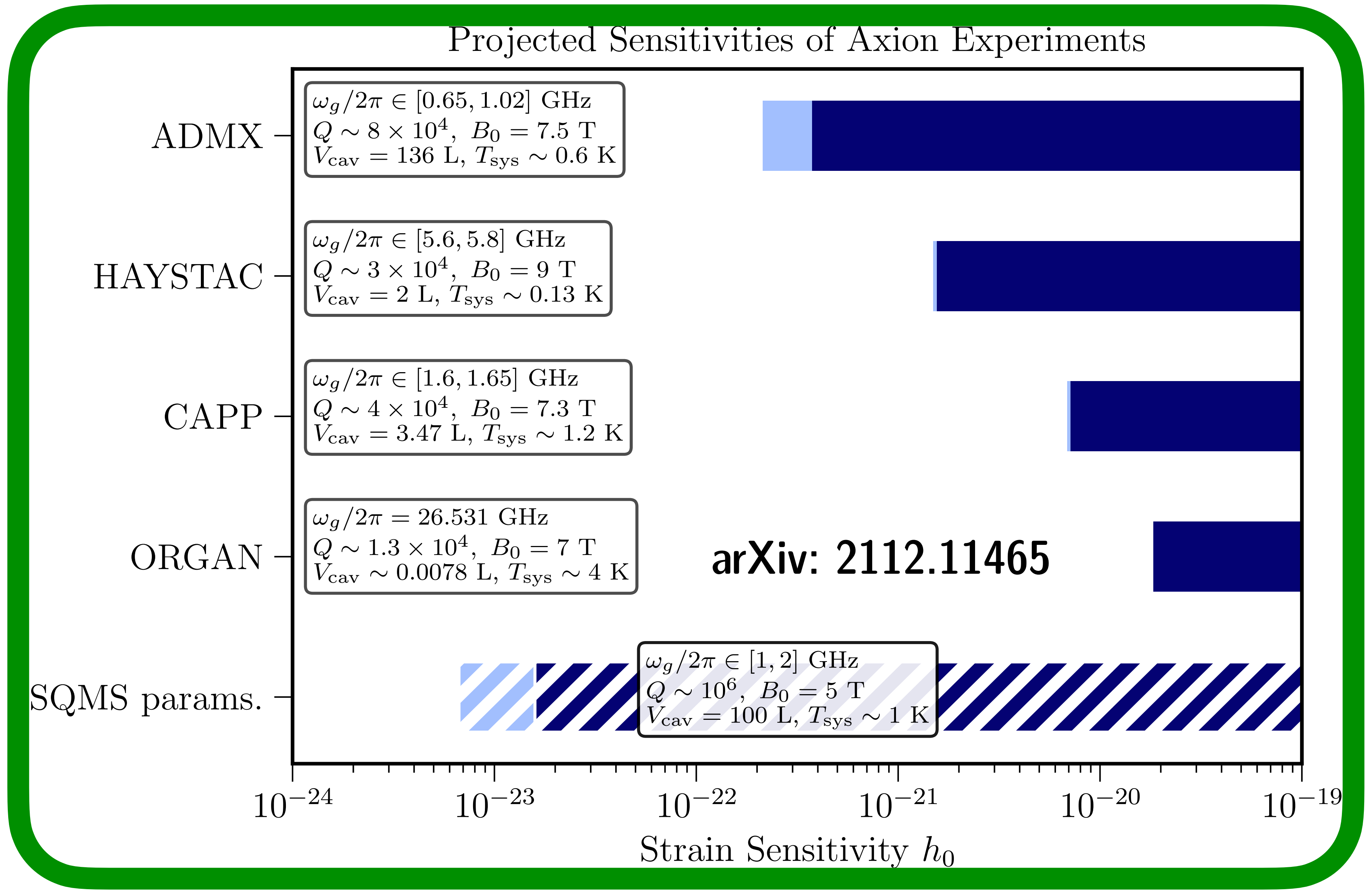
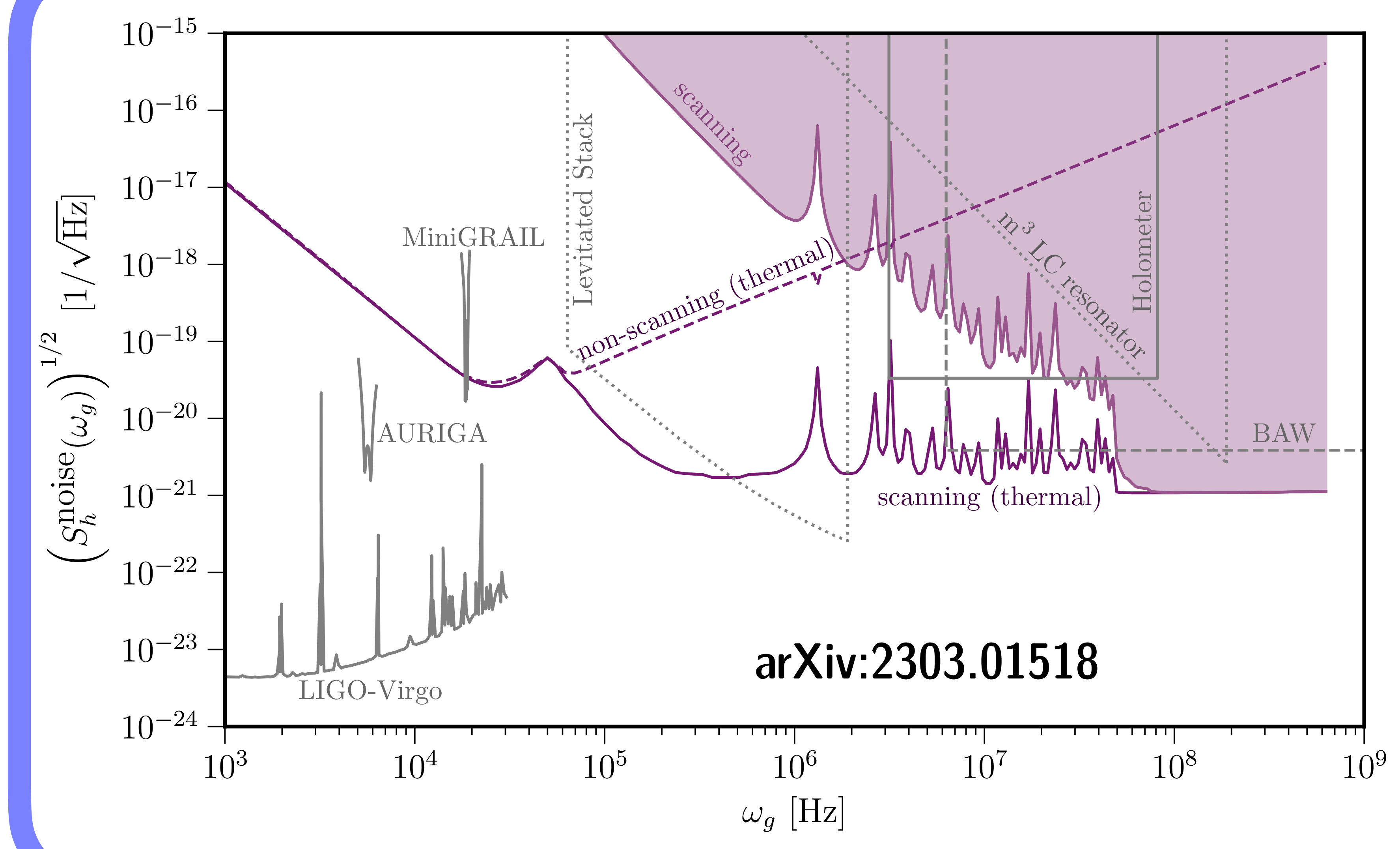
NB: missing radiation damping effect studied in *Löwenberg, Moortgat-Pick: 2307.14379*

Open questions

Resonant cavities optimal in the radio band?





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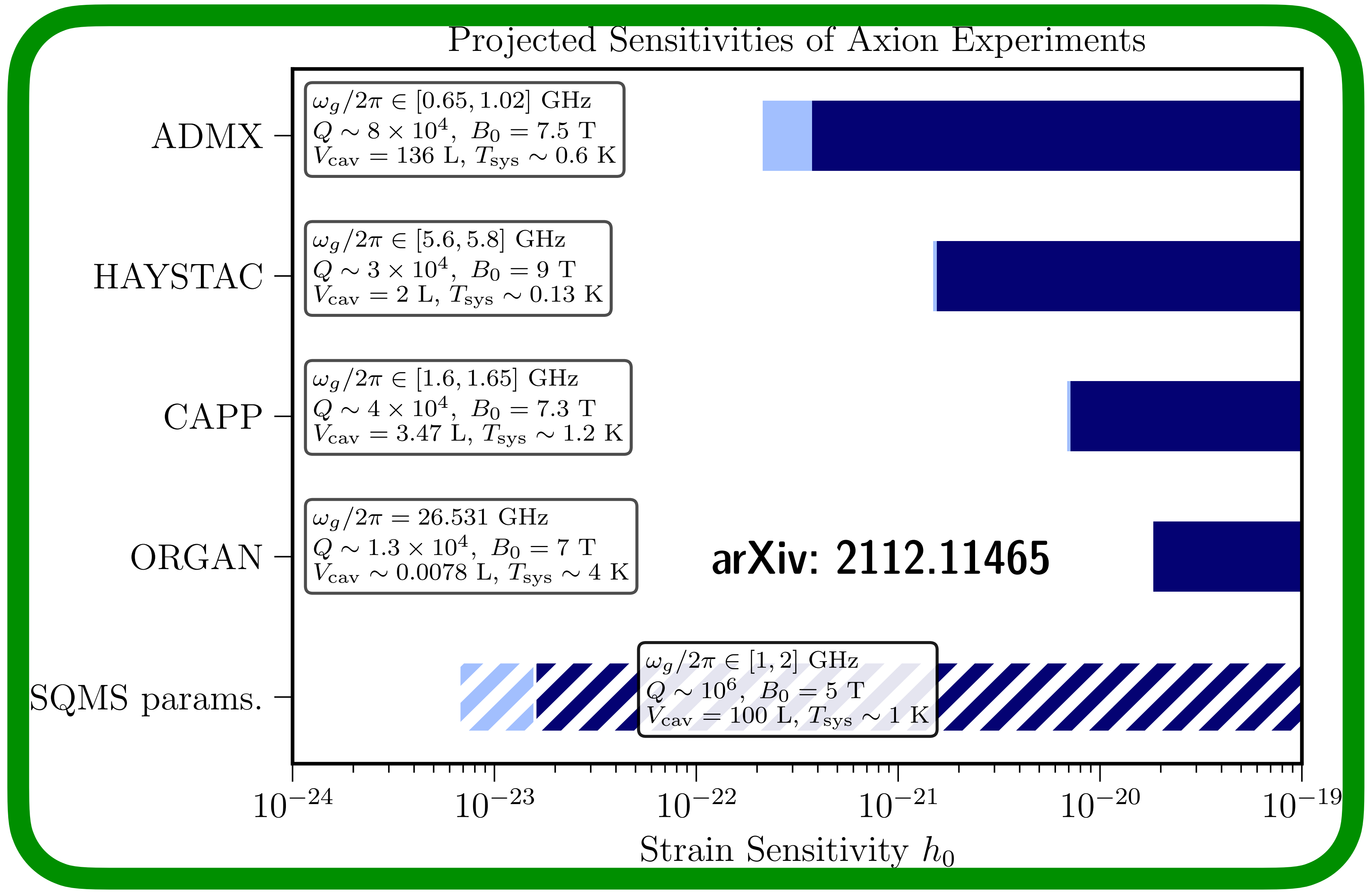

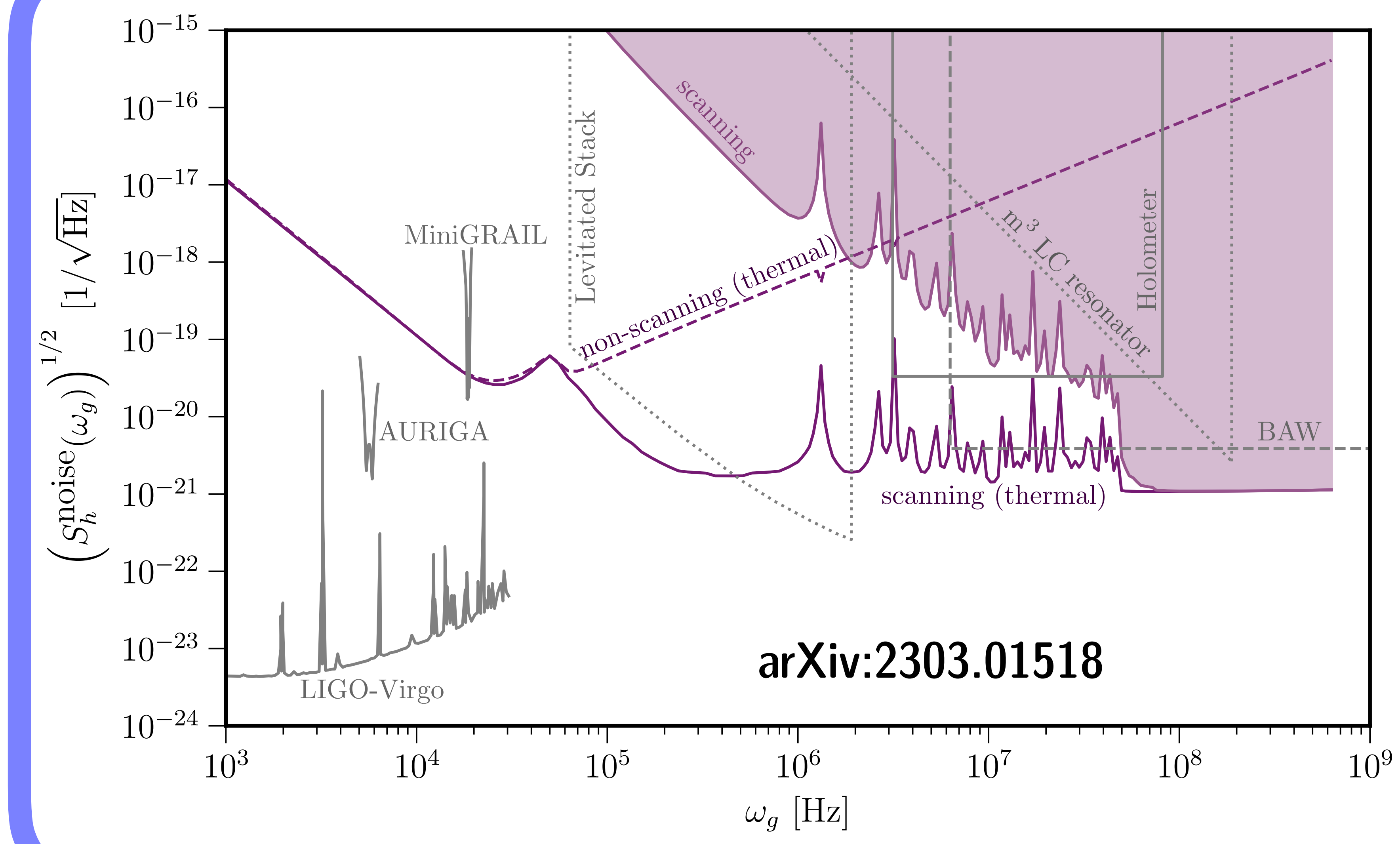

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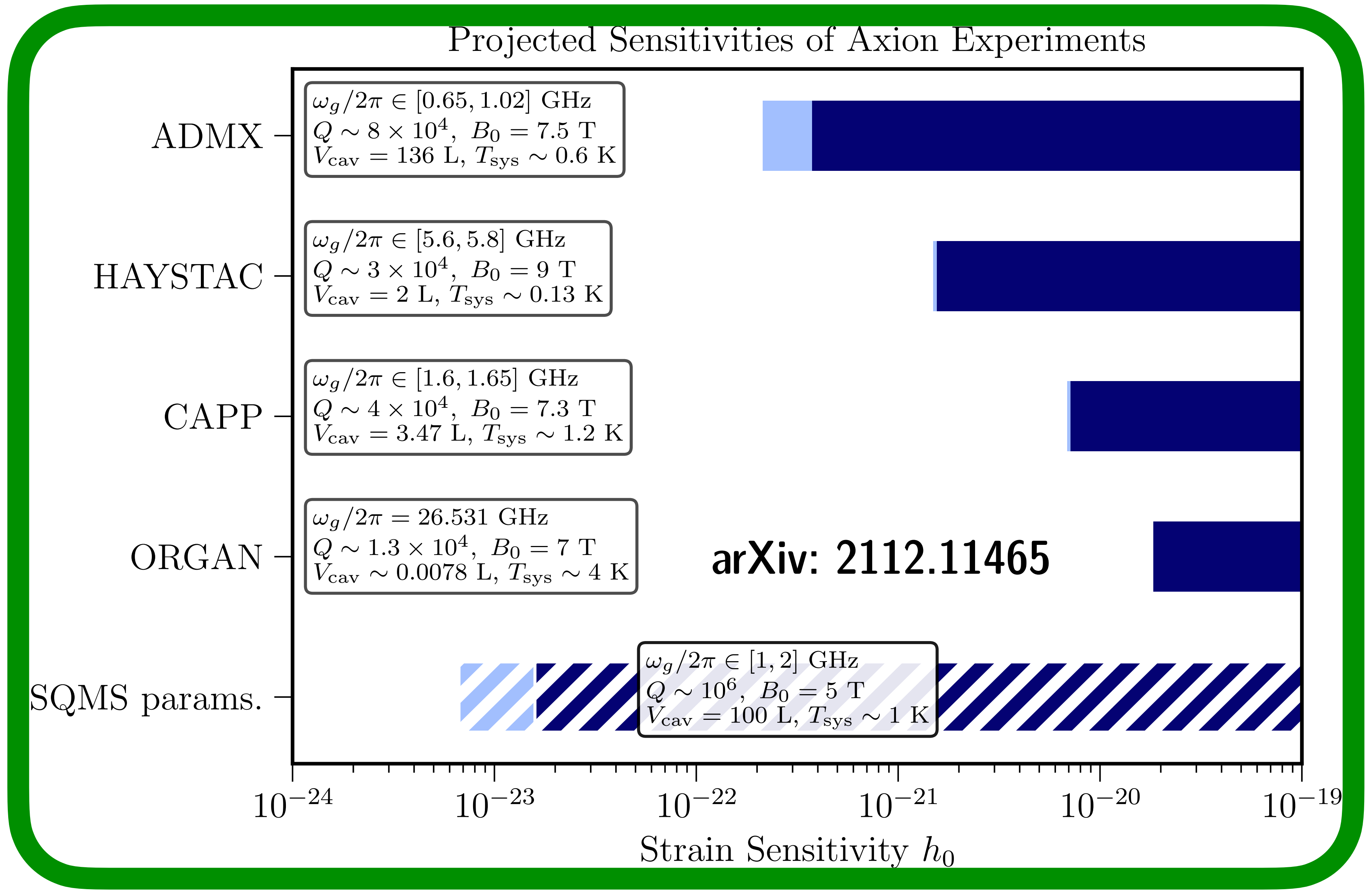

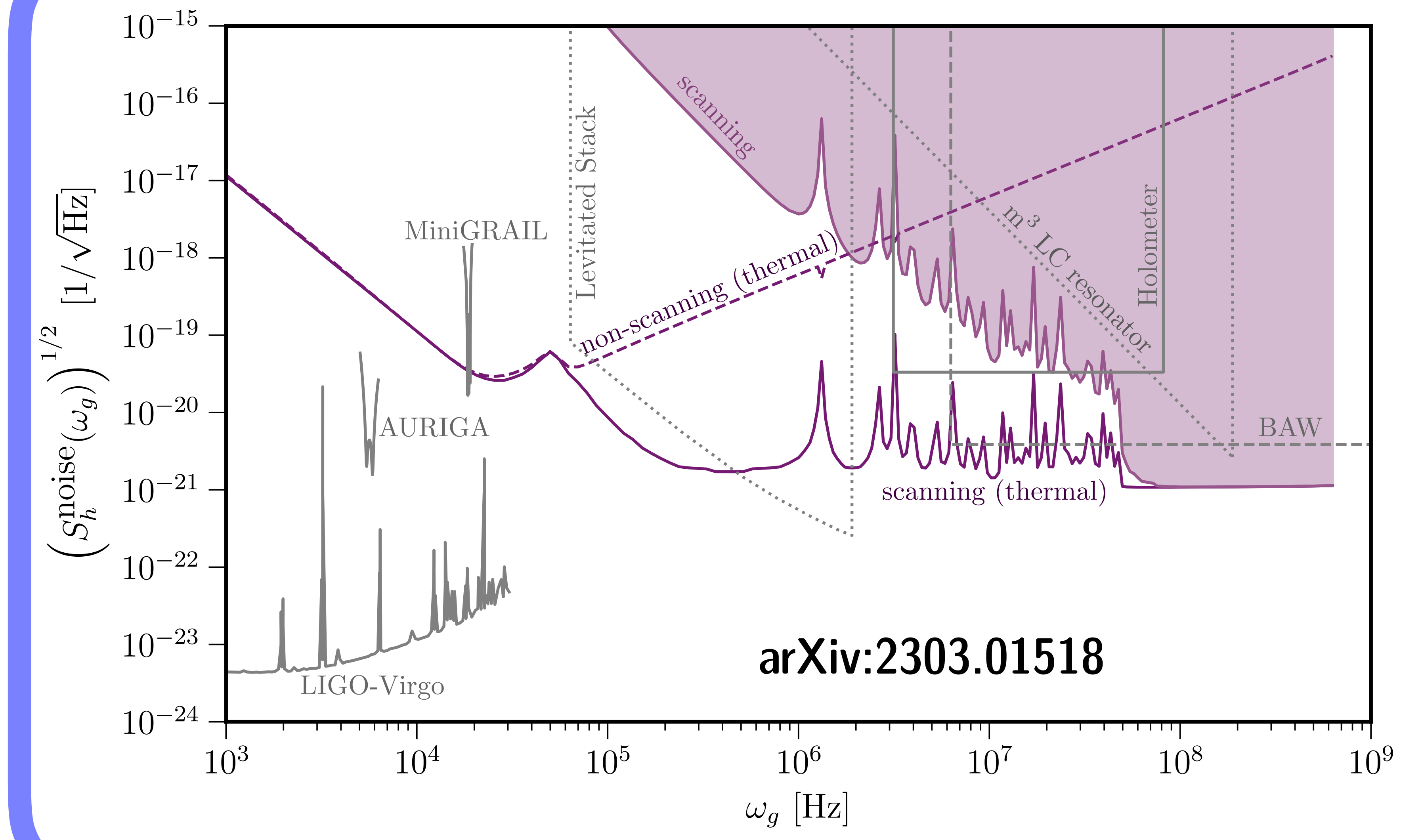

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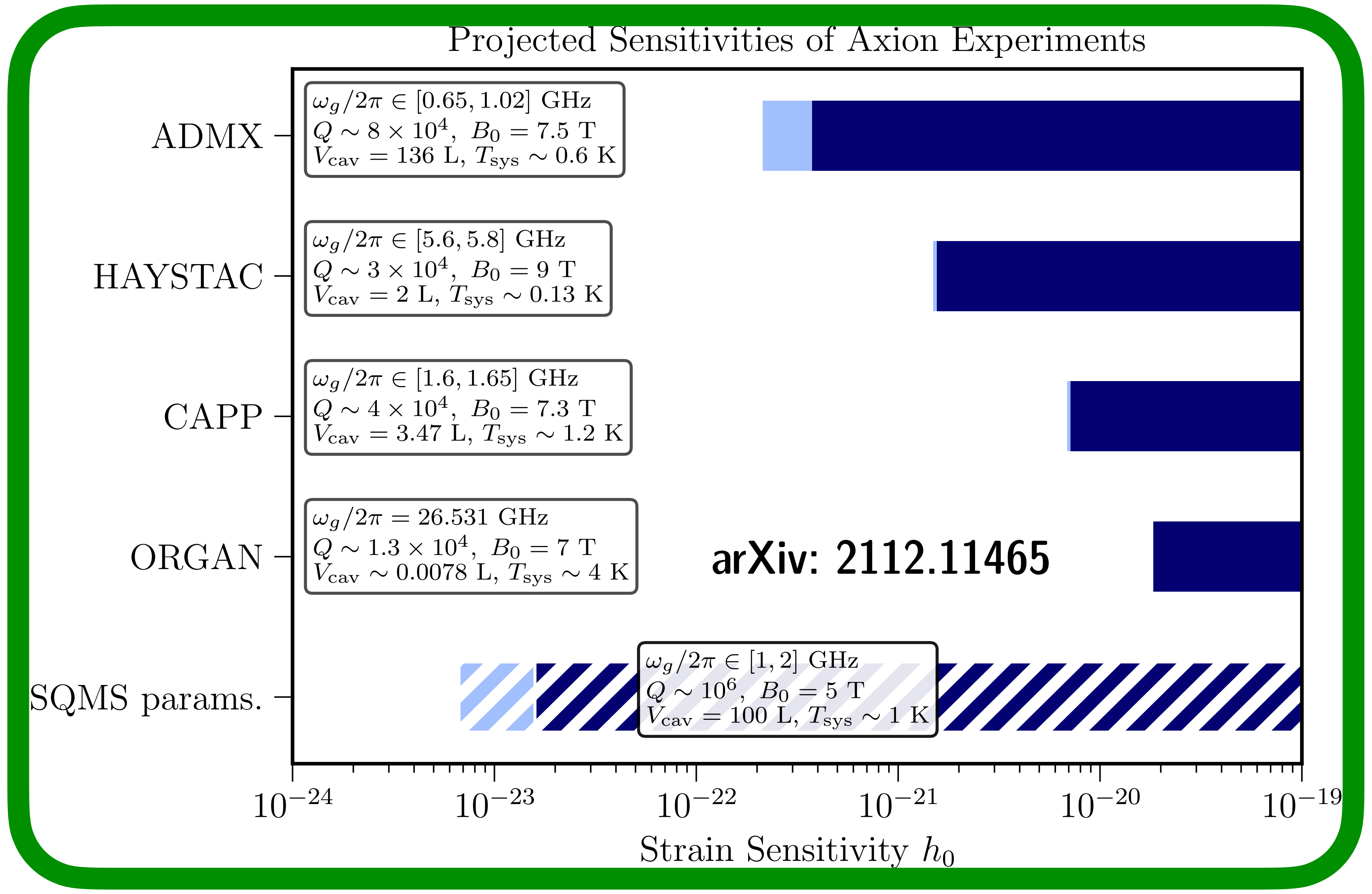
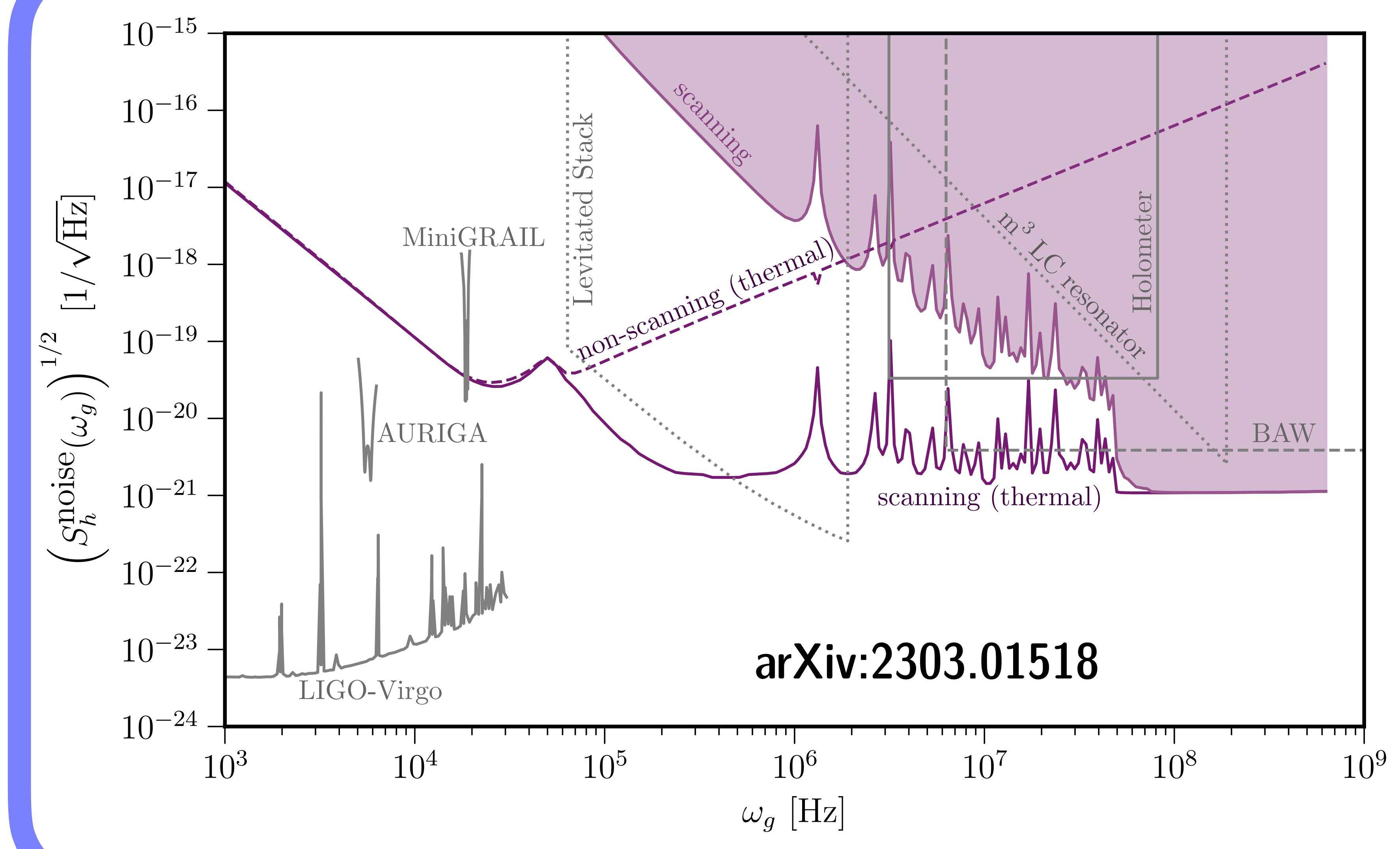
Optimisation of MAGO-style setup?

FNAL + DESY





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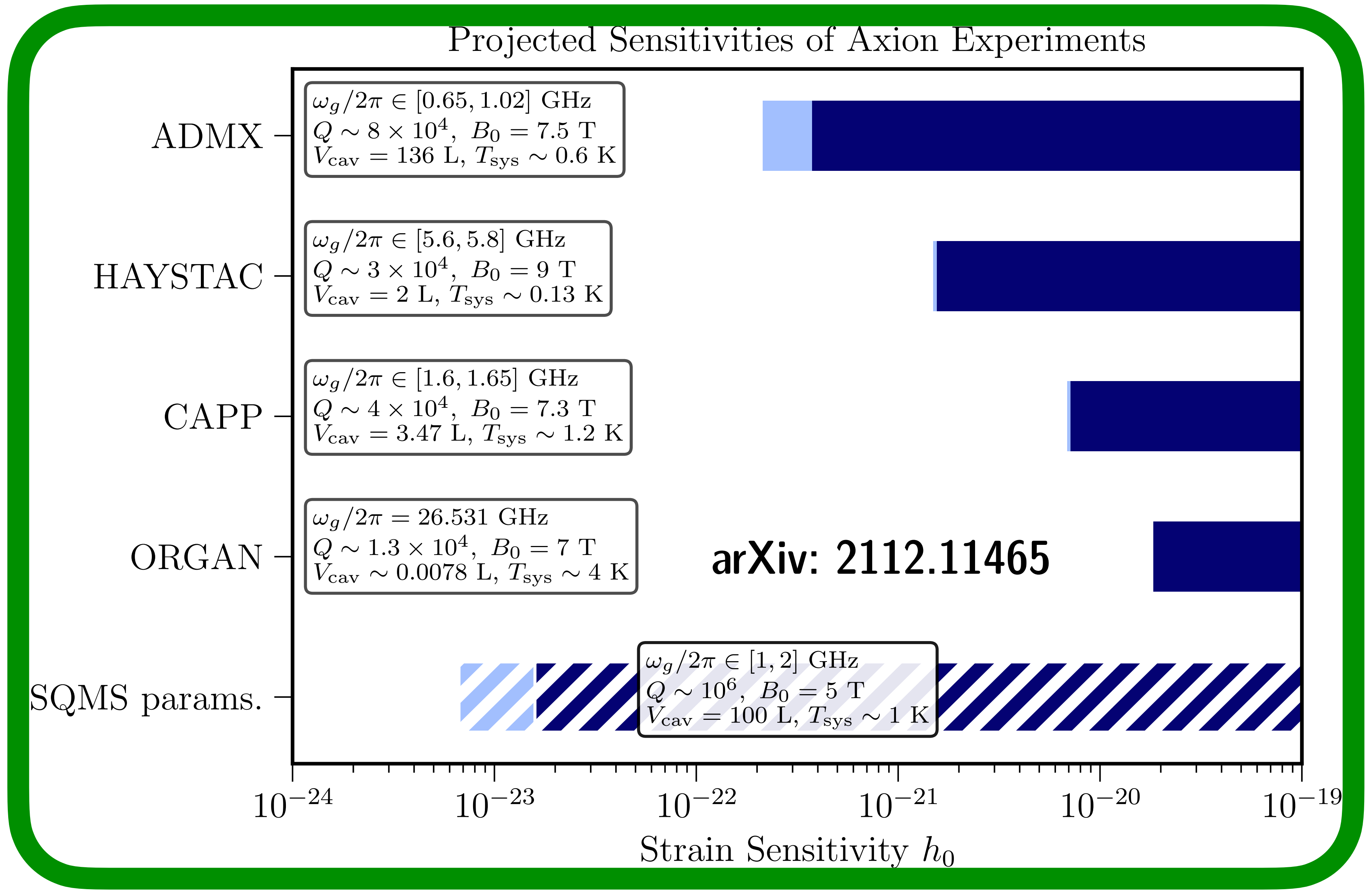

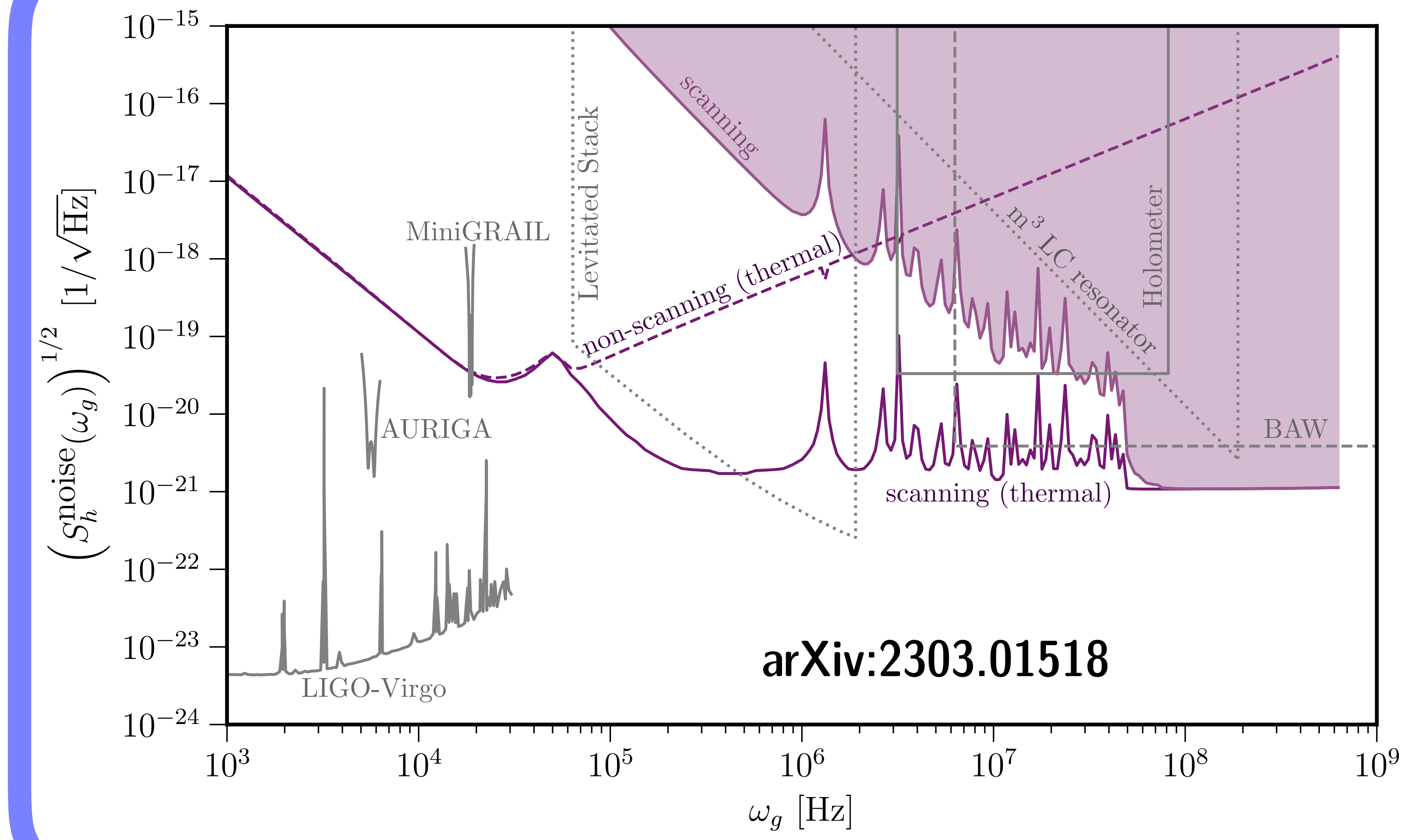

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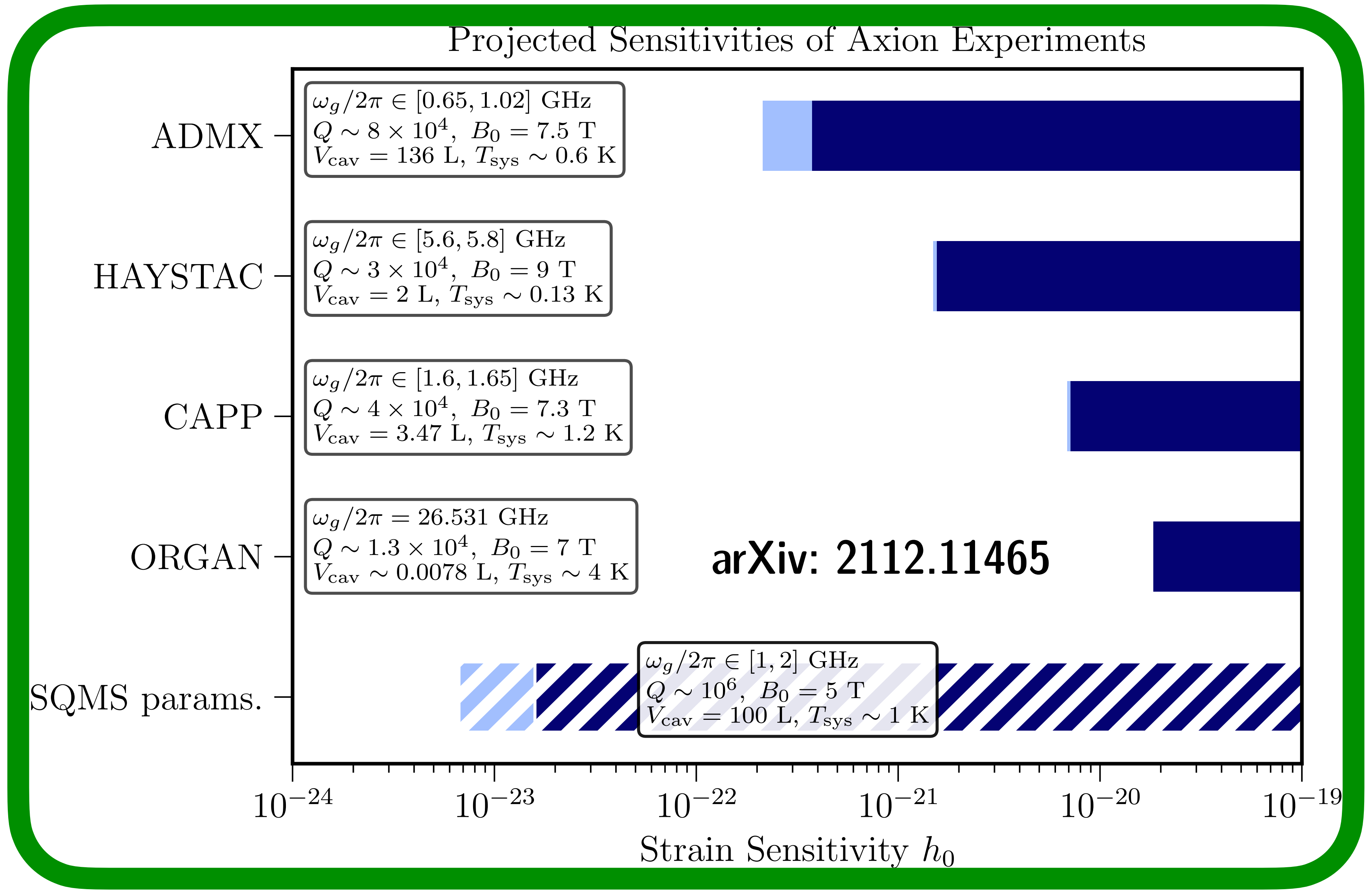
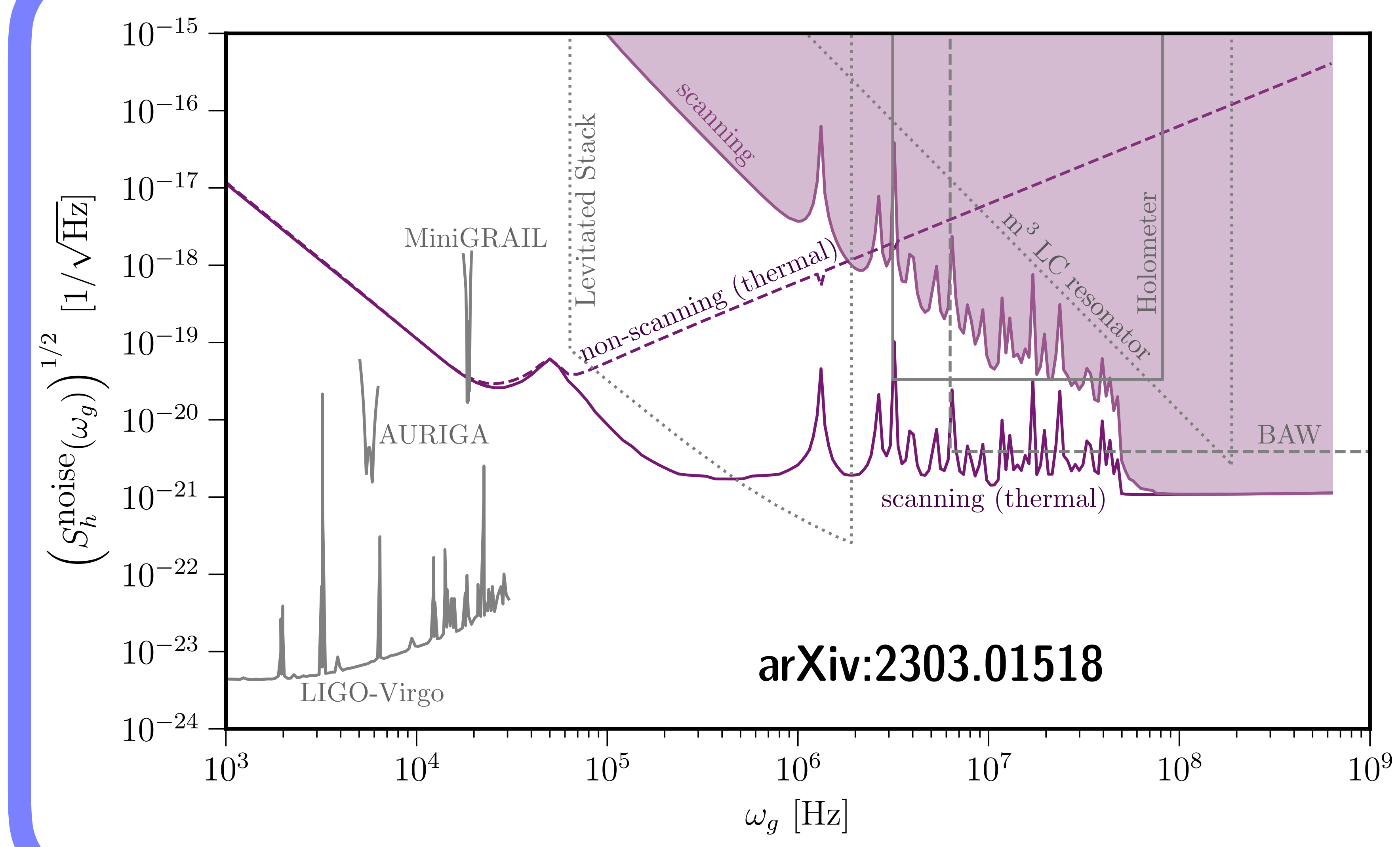
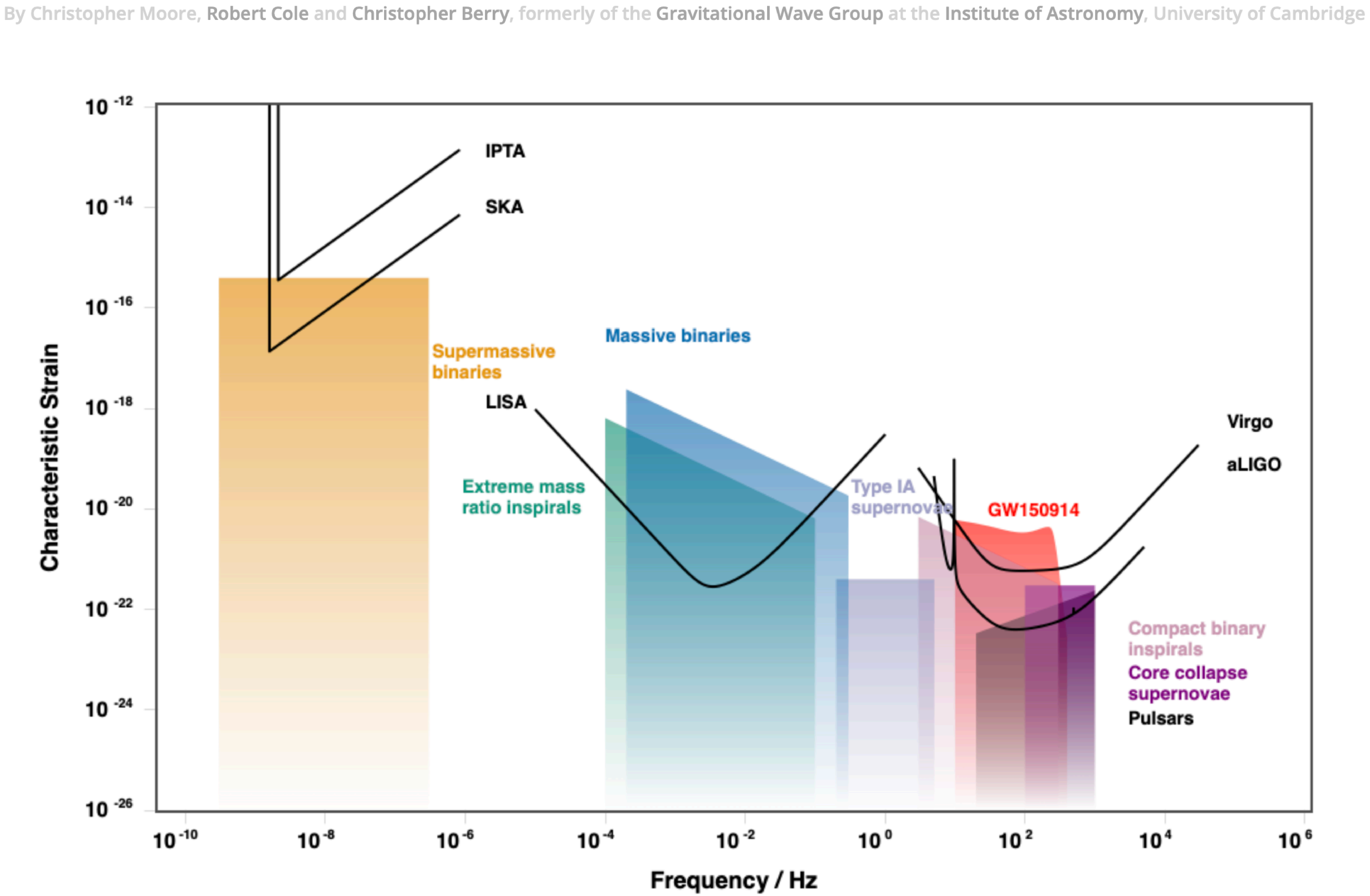




BACKUP

Gravitational Waves



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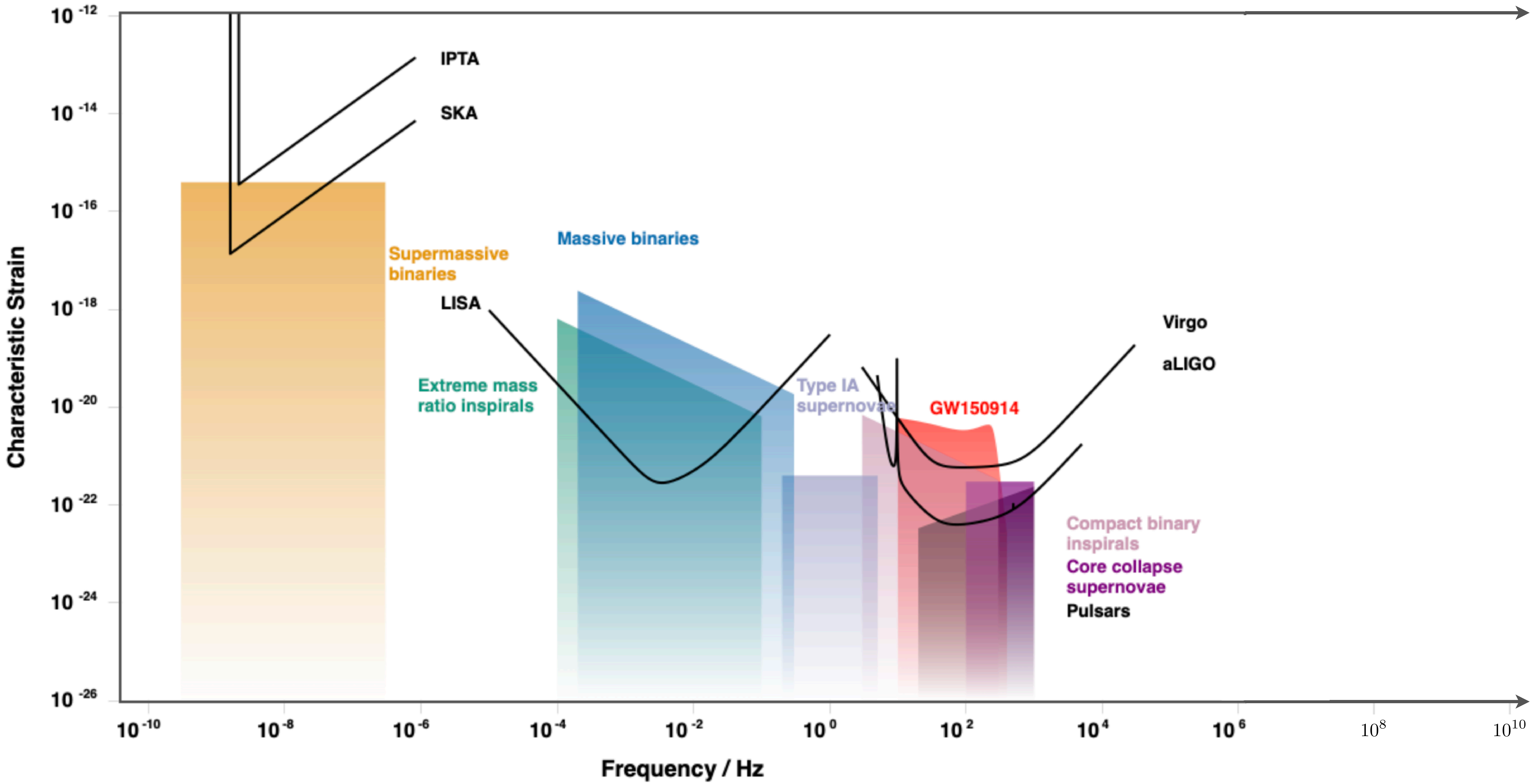
By Christopher Moore, Robert Cole and Christopher Berry, formerly of the Gravitational Wave Group at the Institute of Astronomy, University of Cambridge



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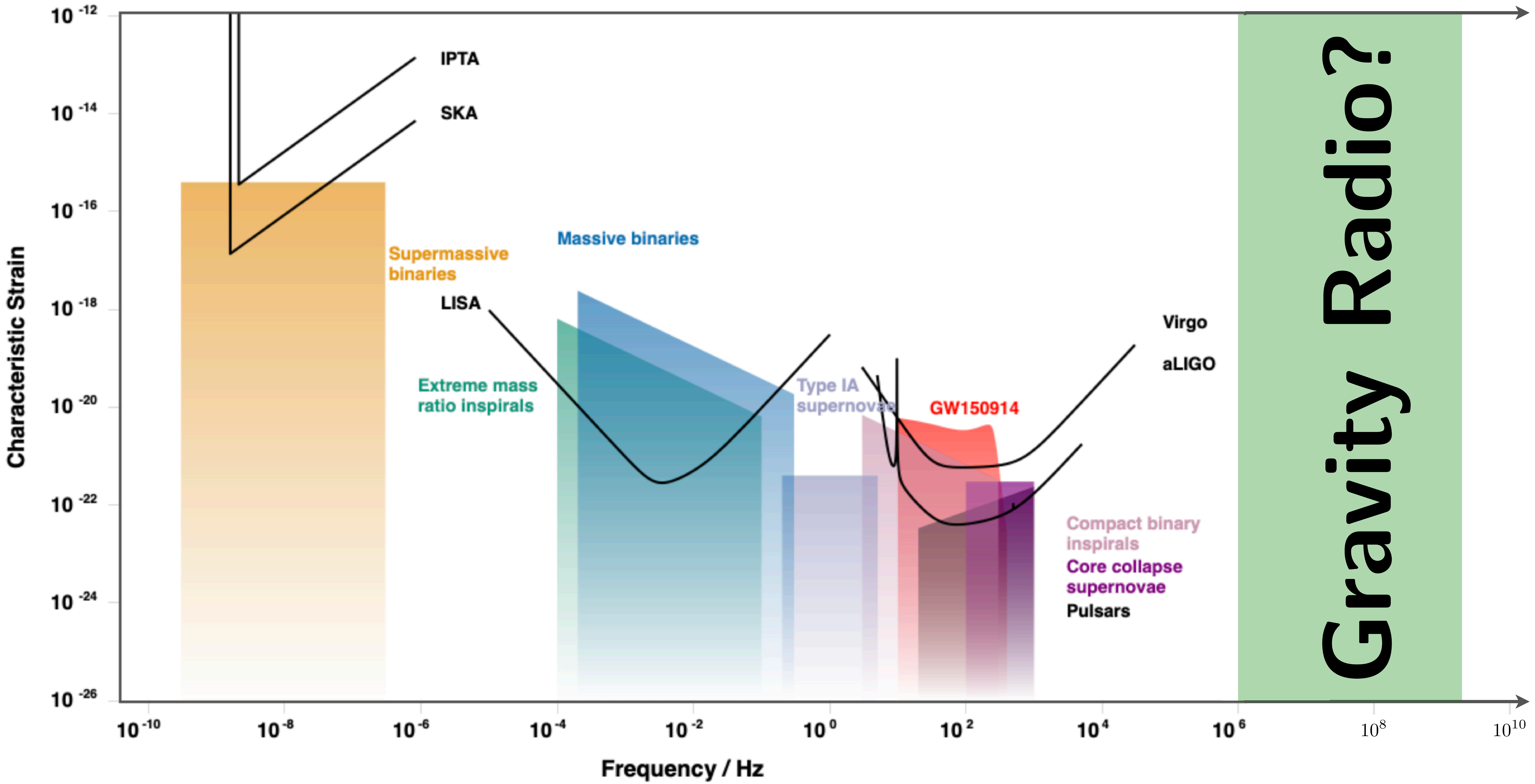
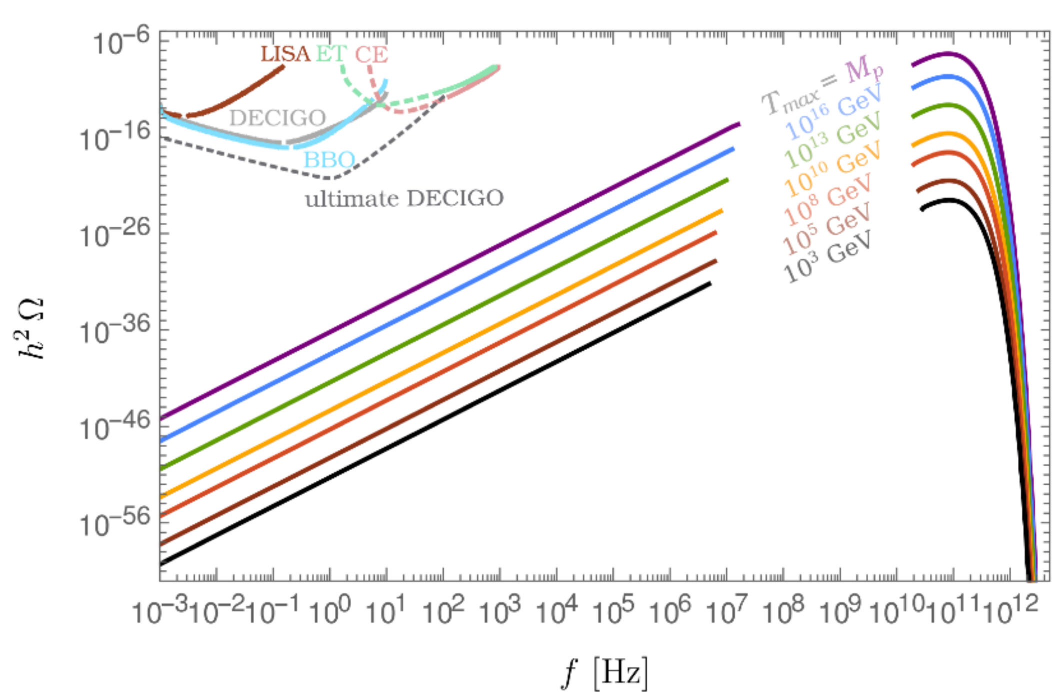


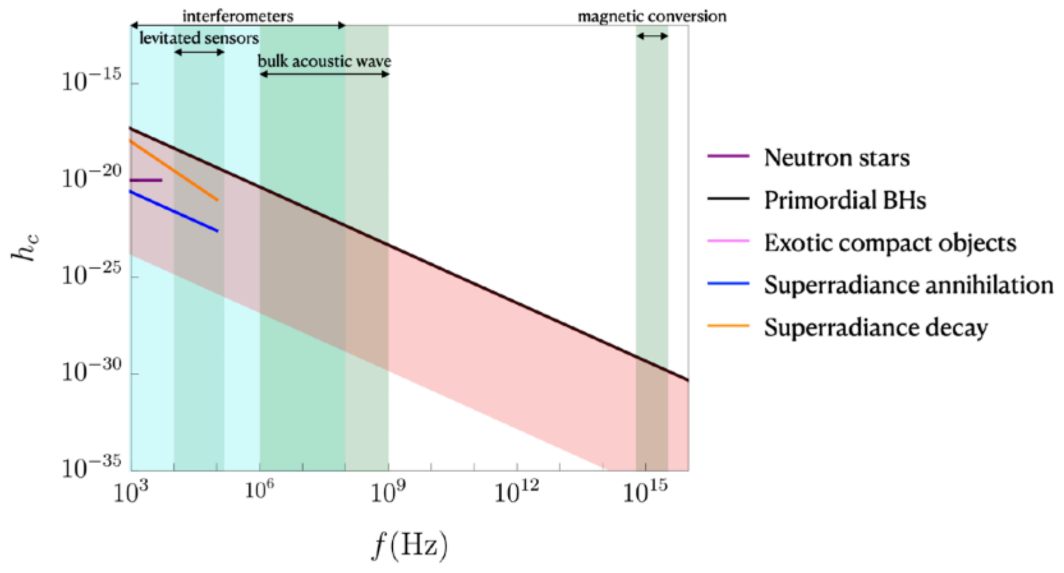
Sources of HFGWs

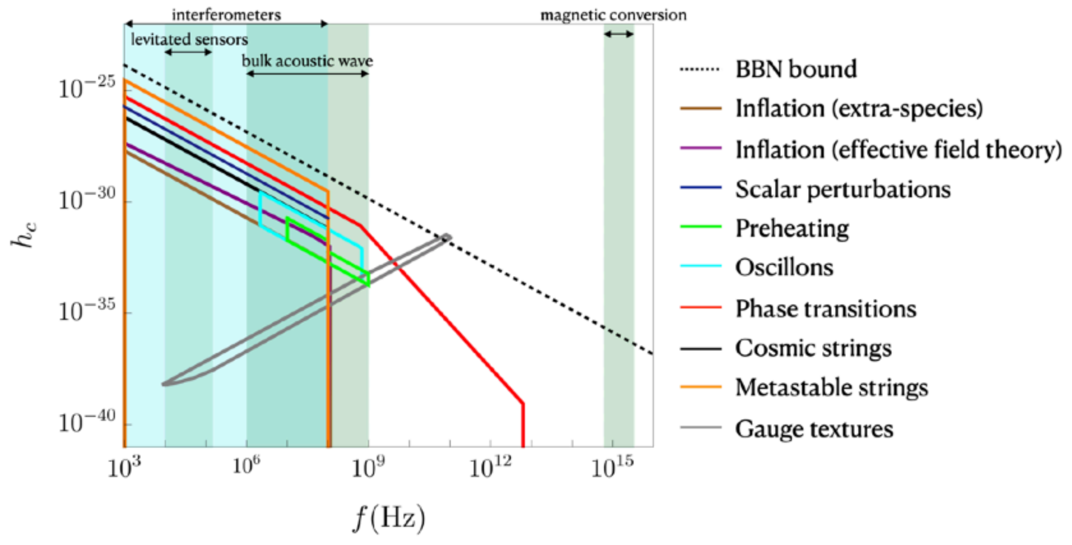
Sources discussed in detail Weds-Fri

	Stochastic	Coherent
Standard Model:		
BSM:		

Ringwald et al, 2011.04731







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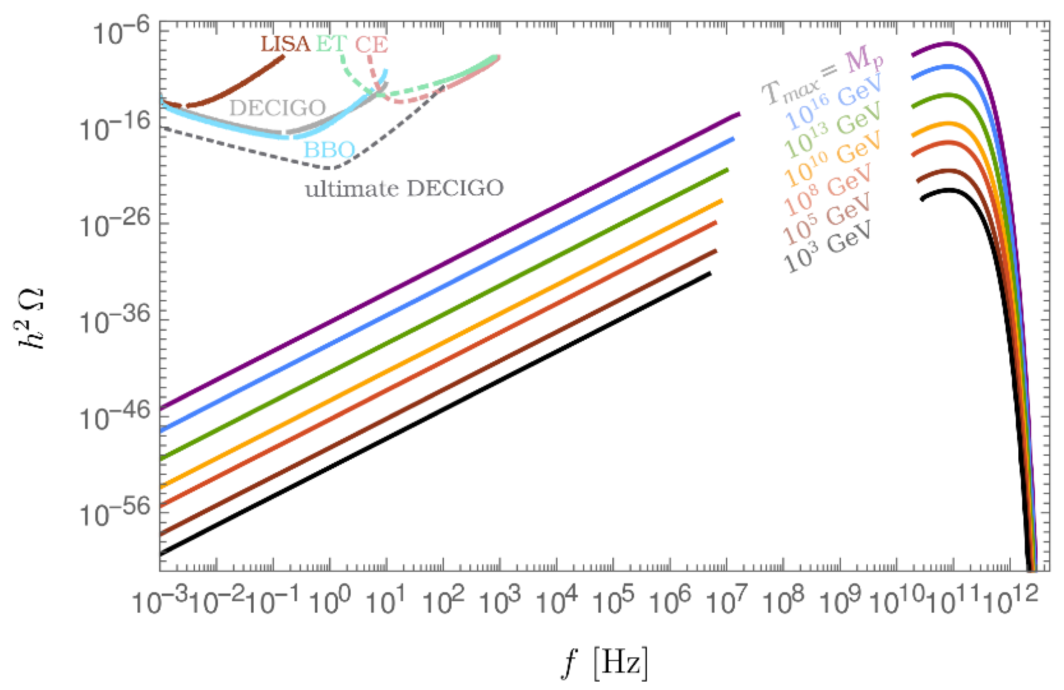

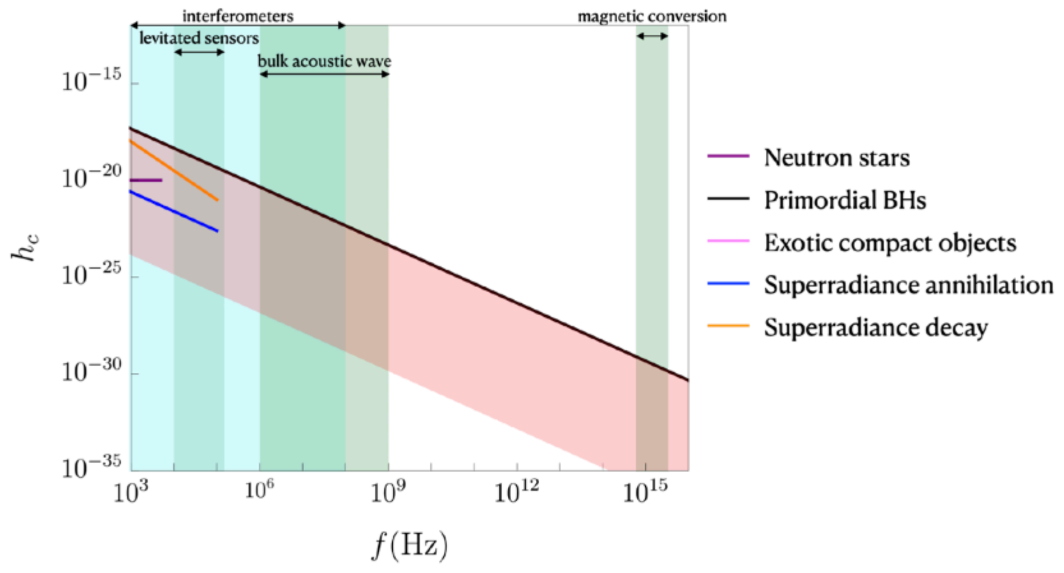

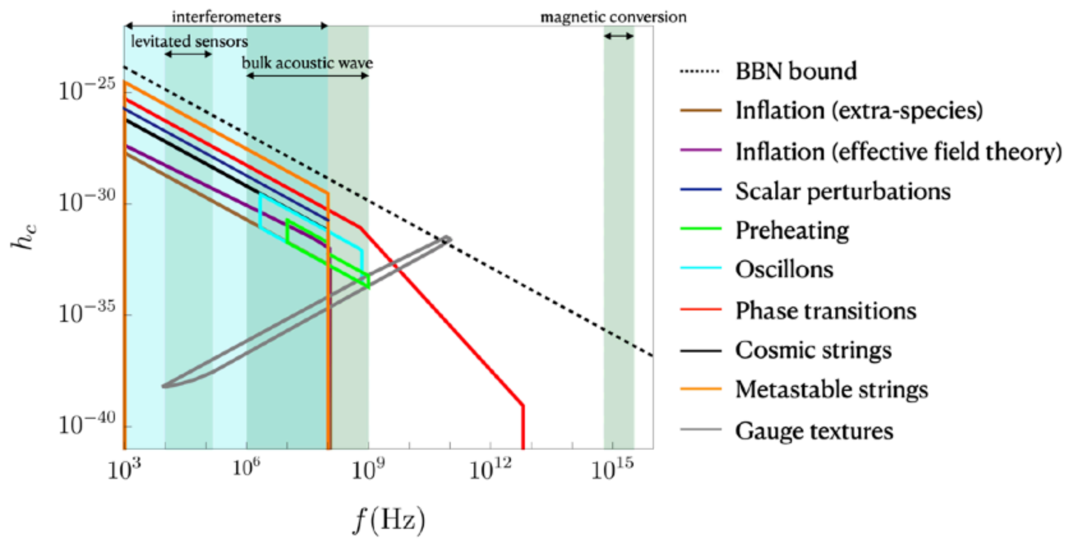

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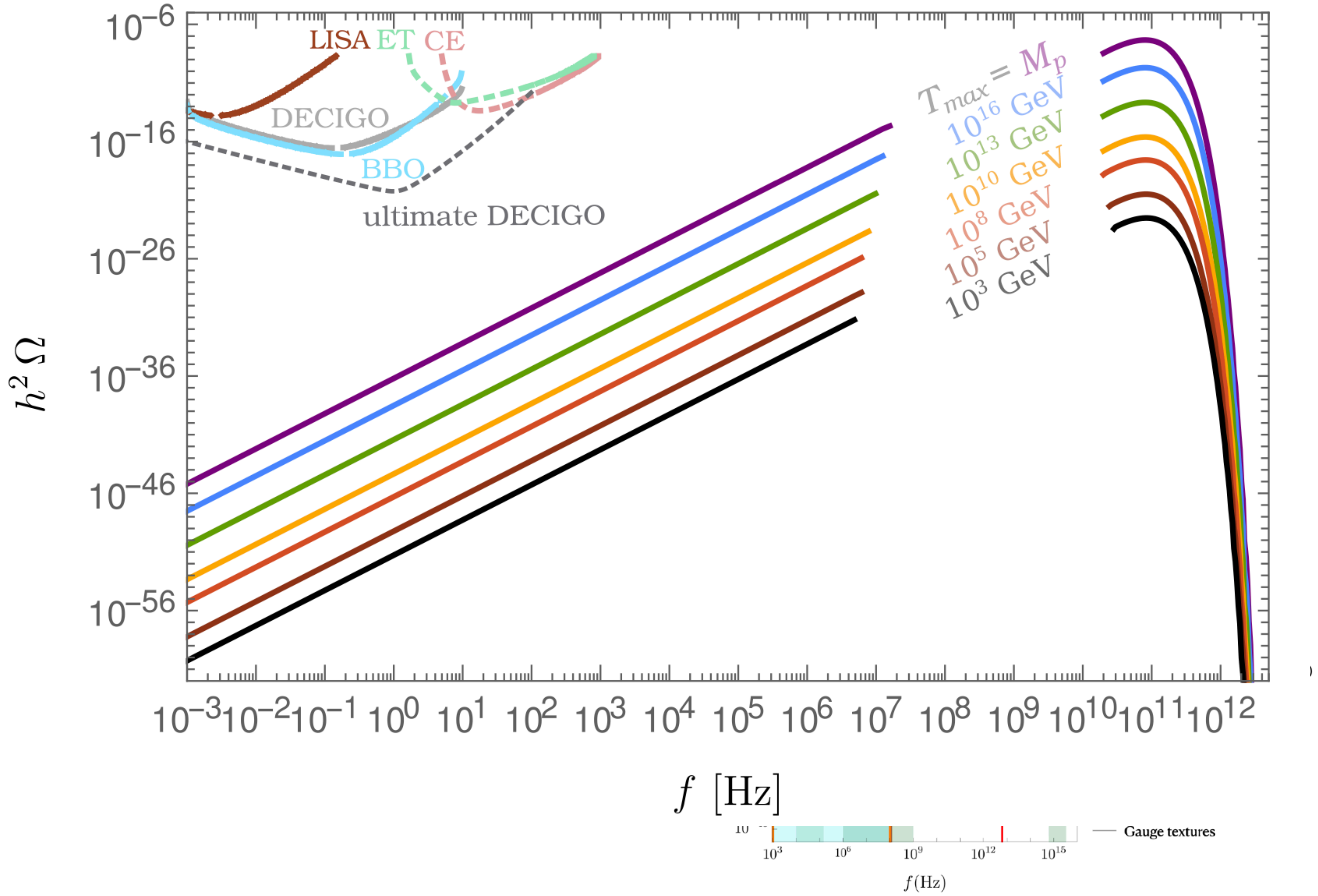

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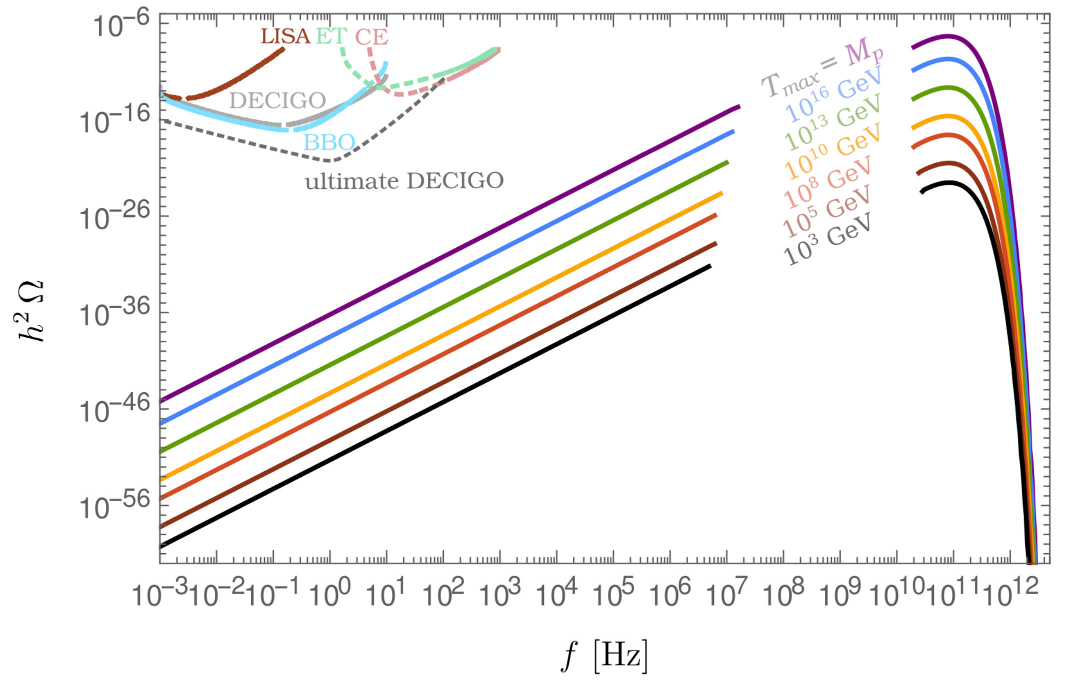

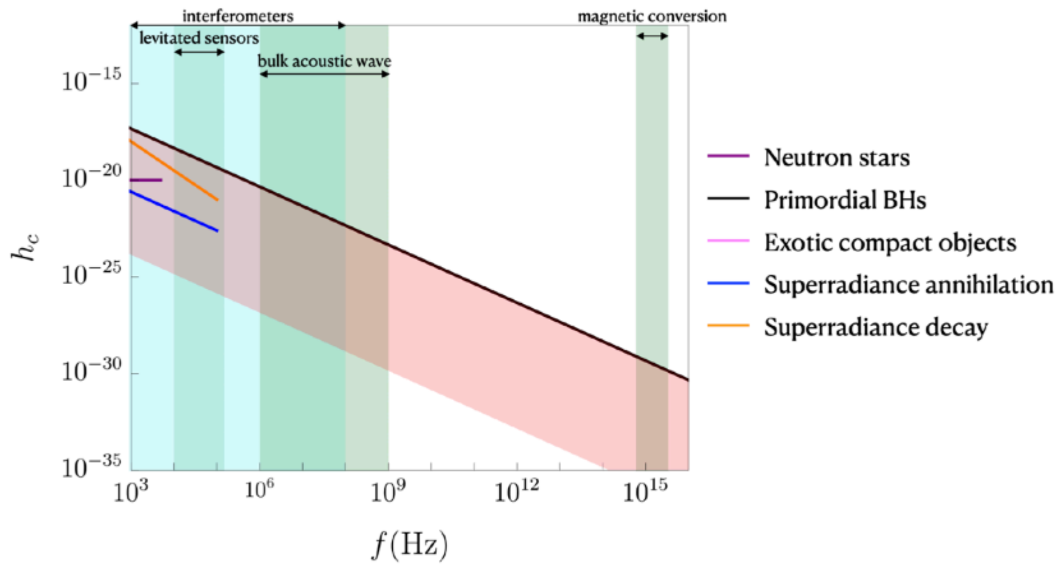

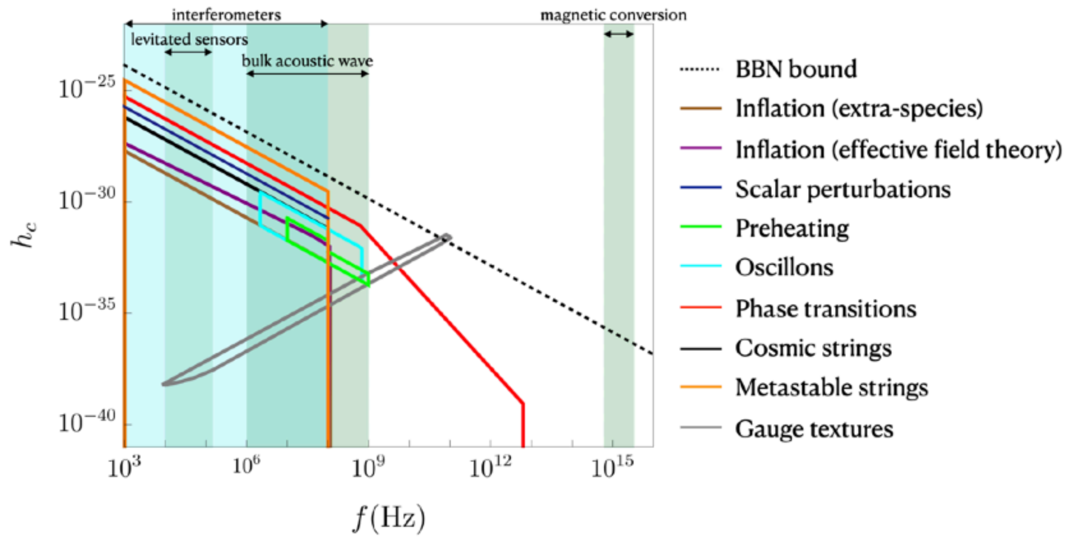

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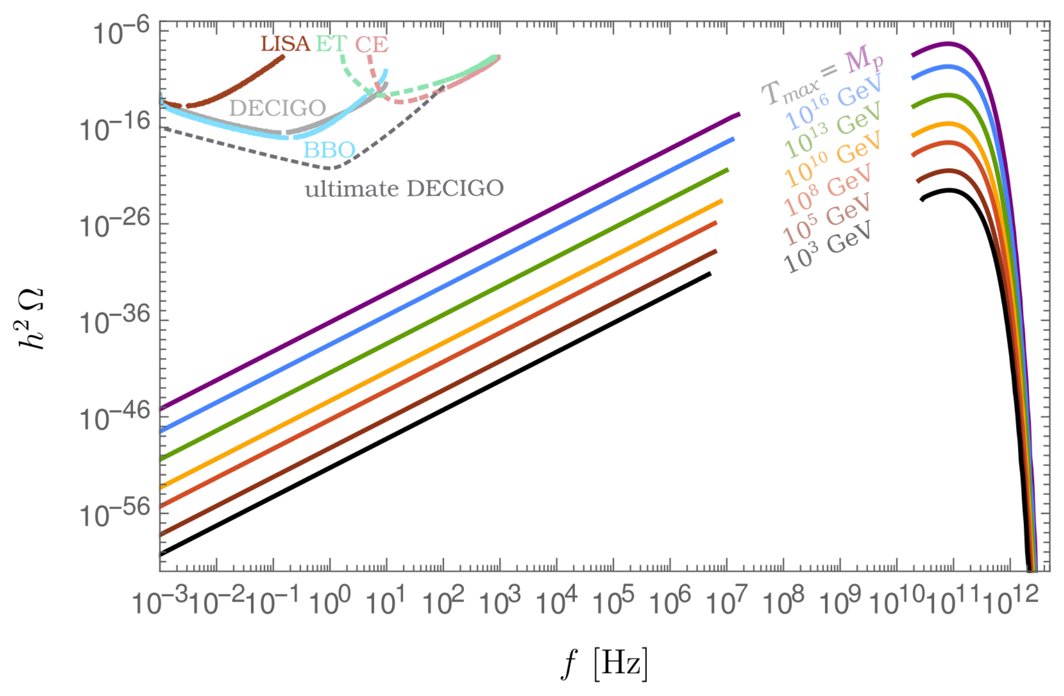

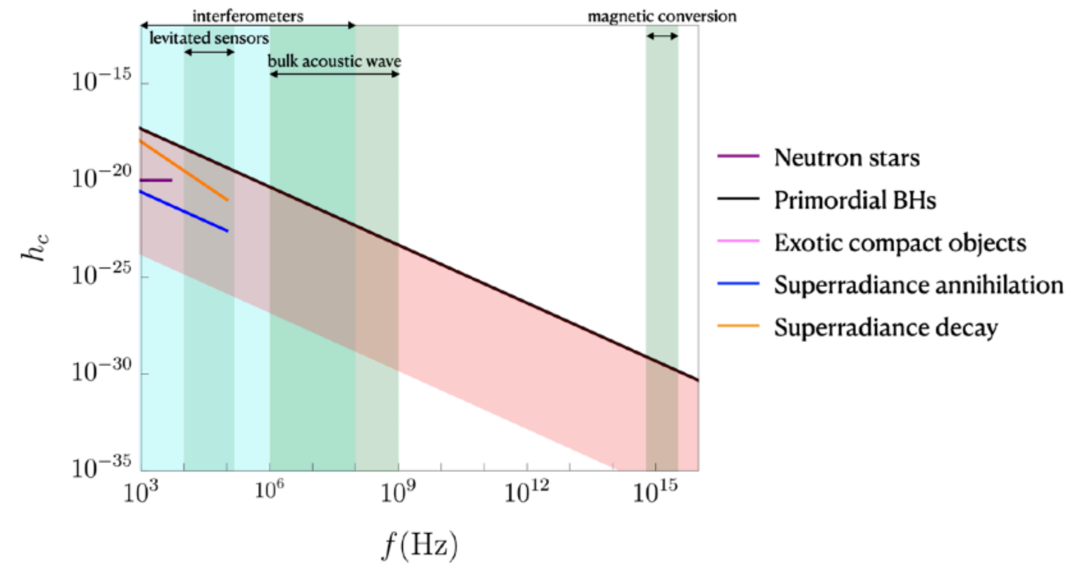

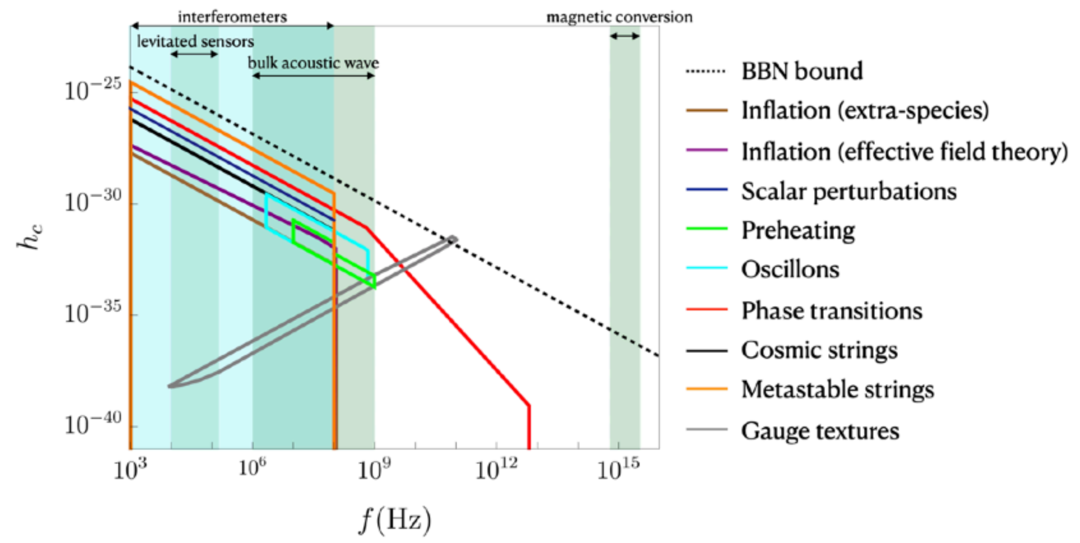

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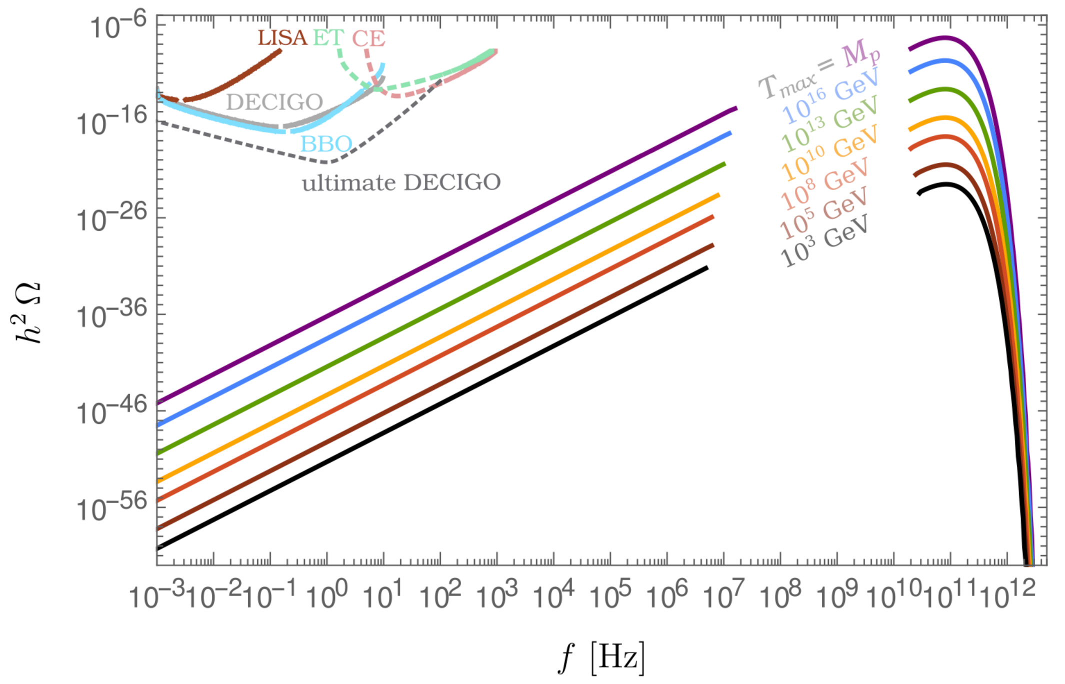

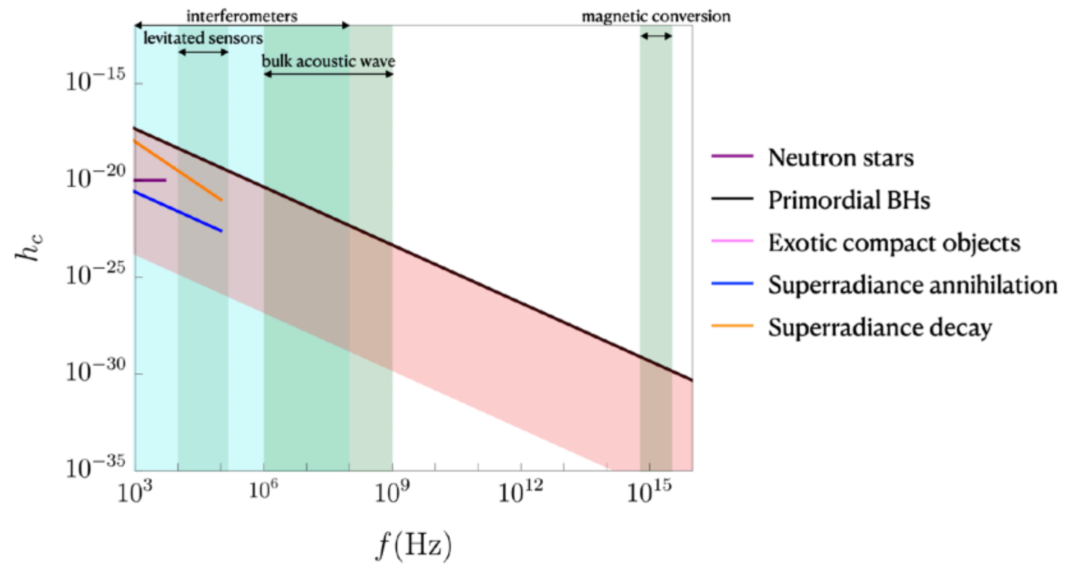

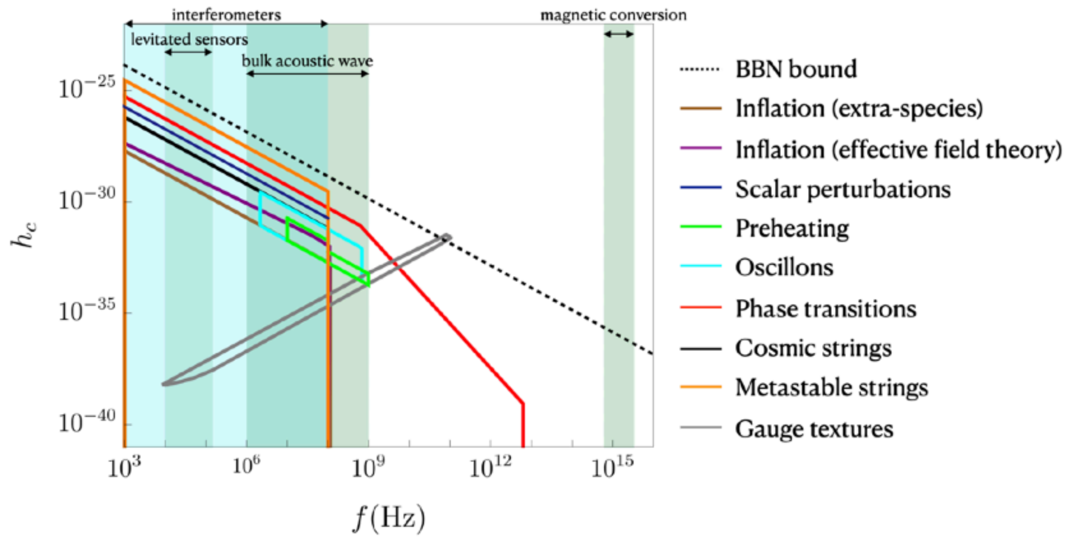

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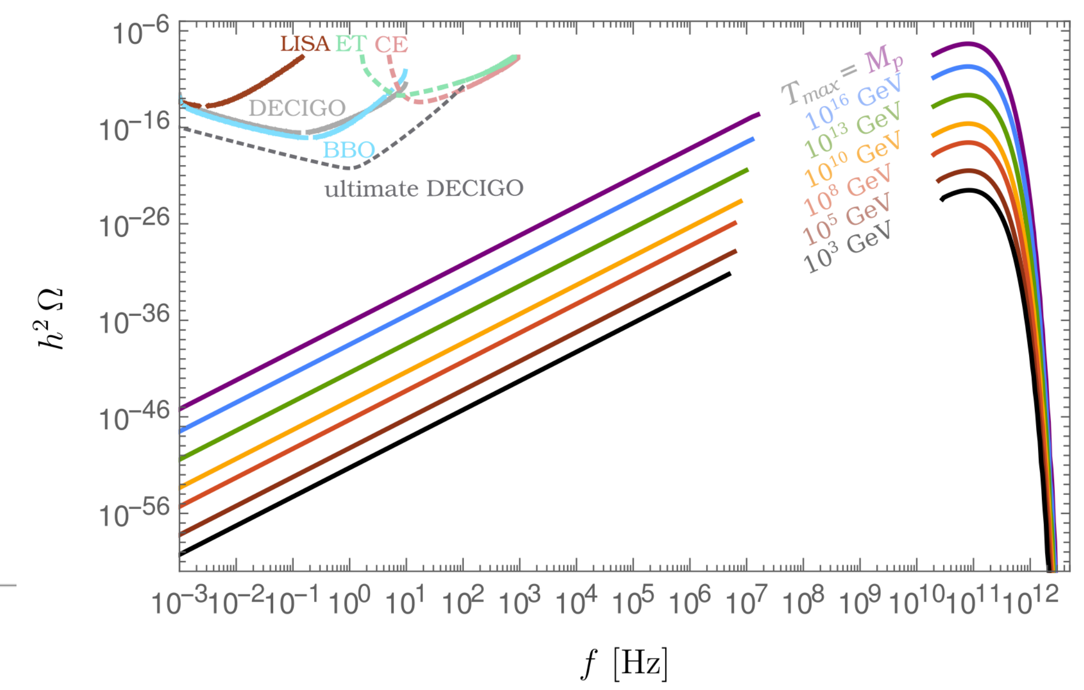

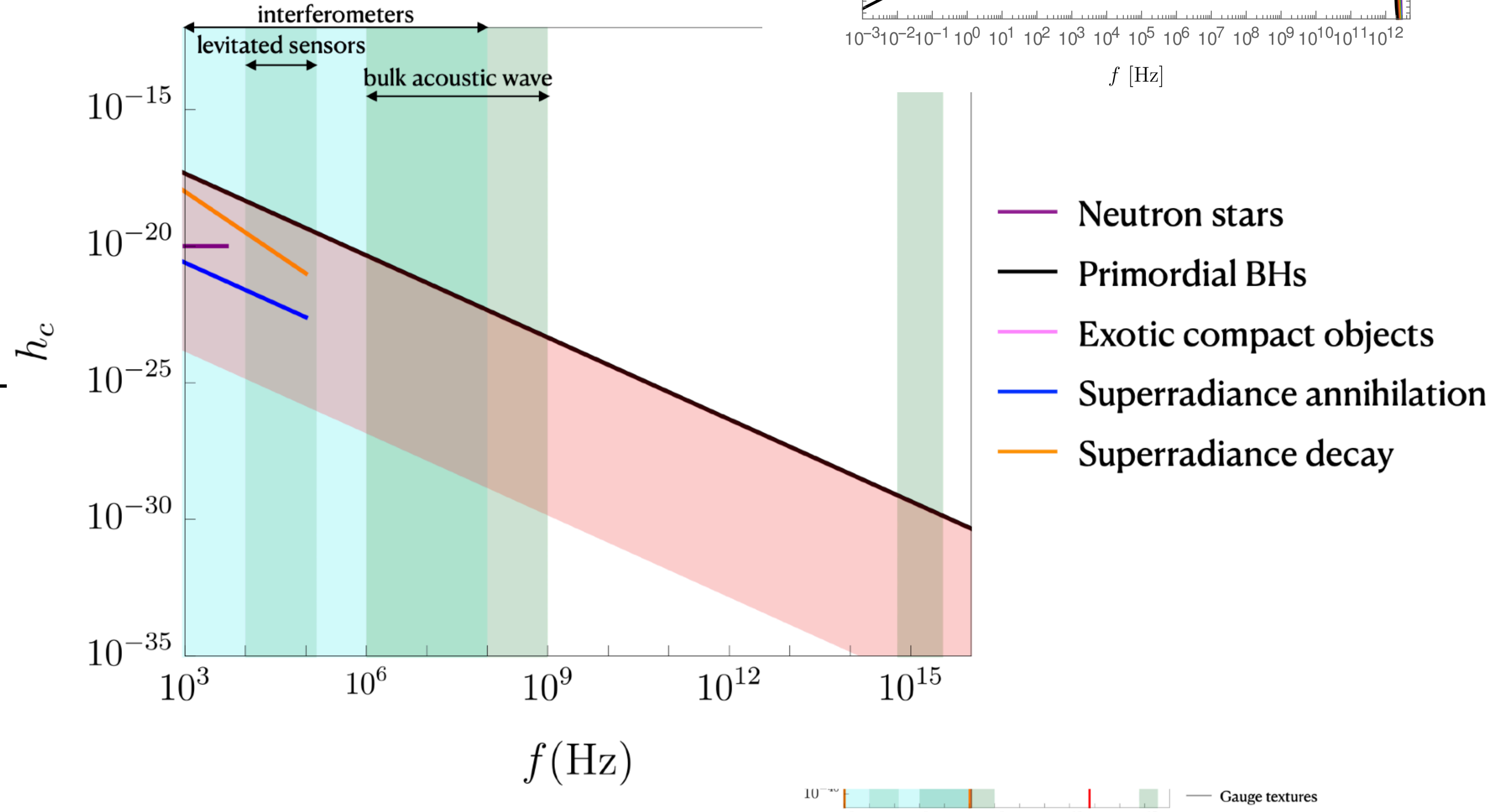

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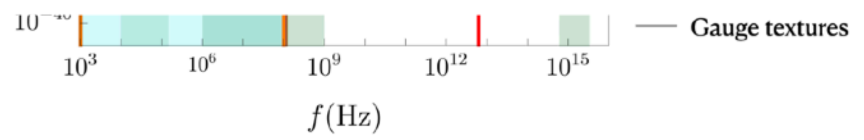

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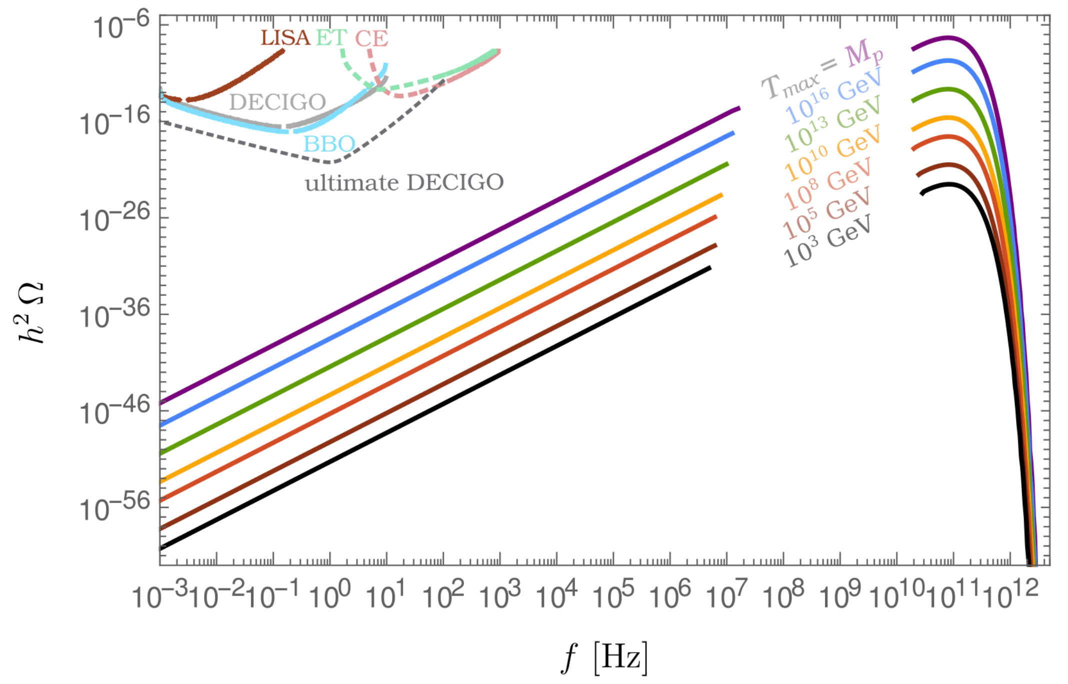

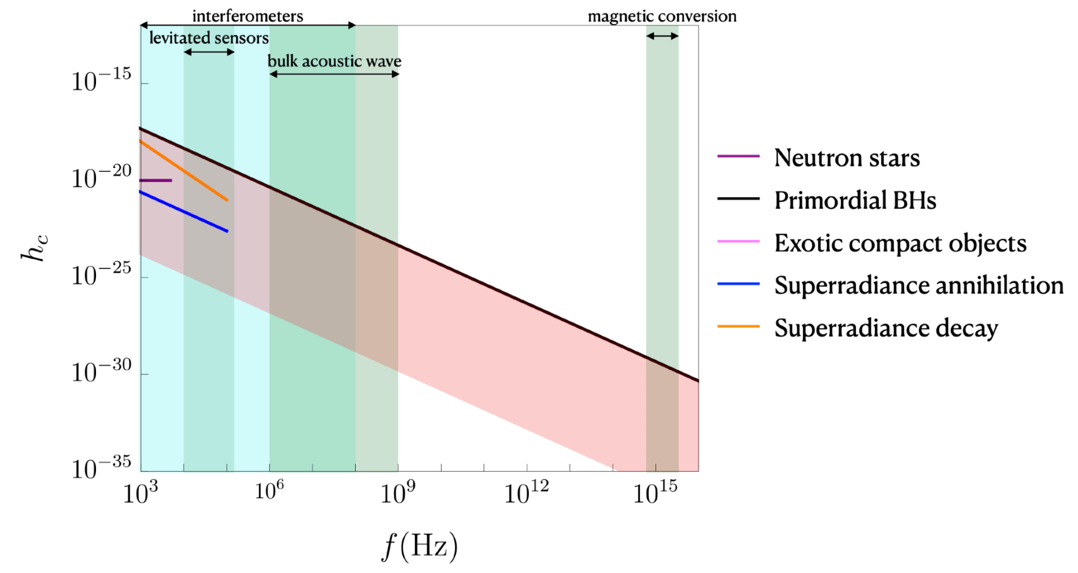

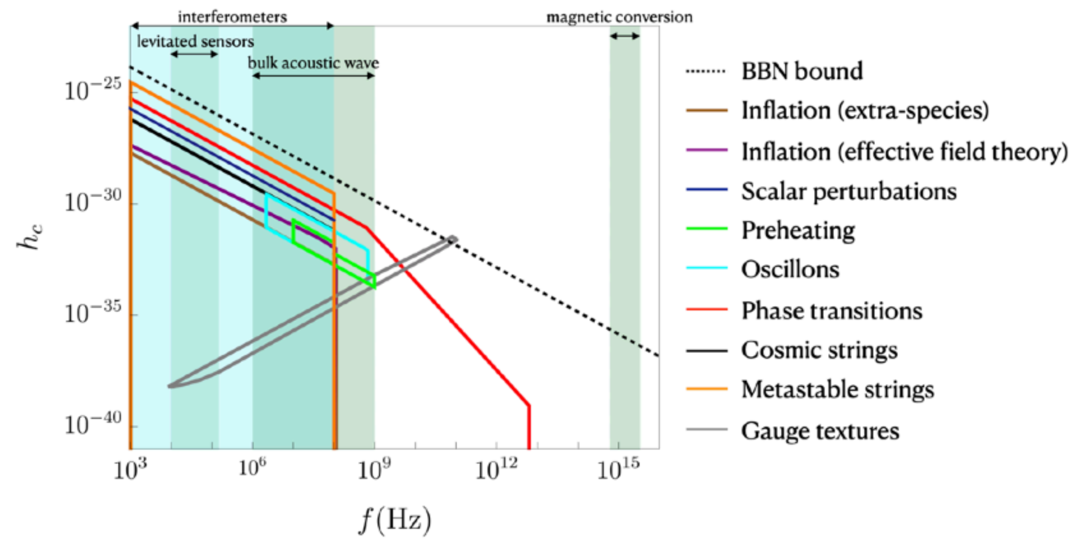

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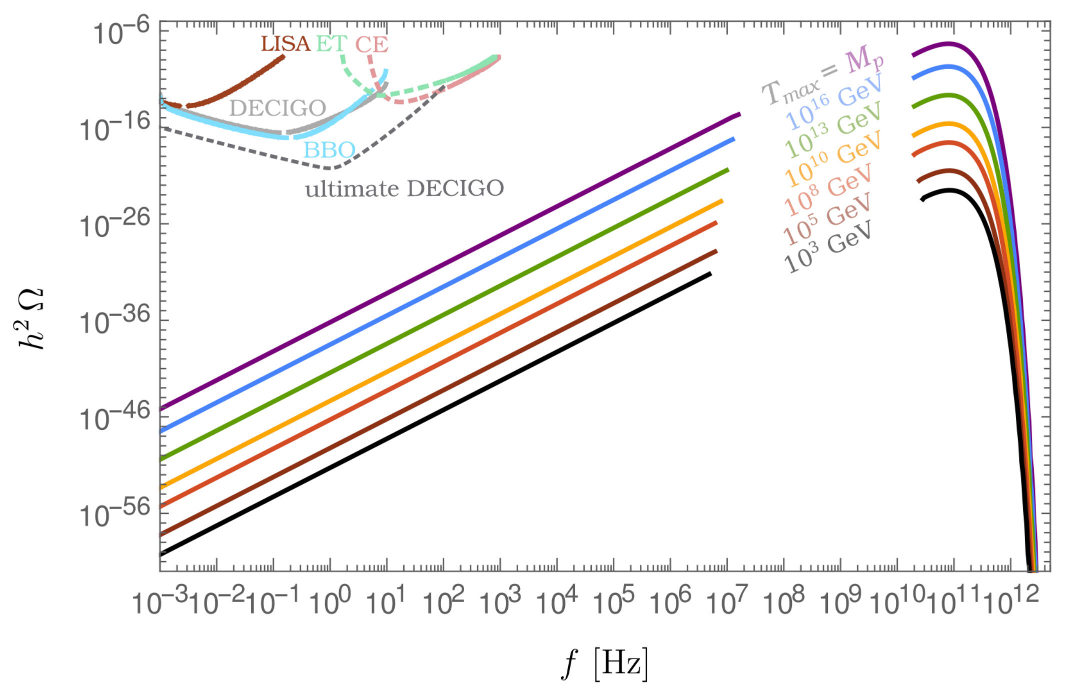
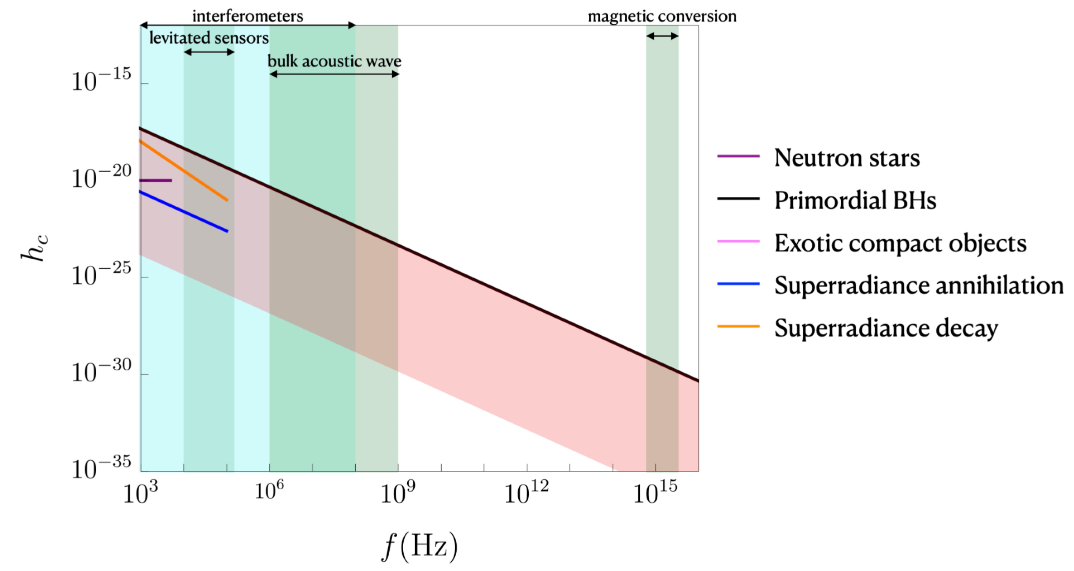
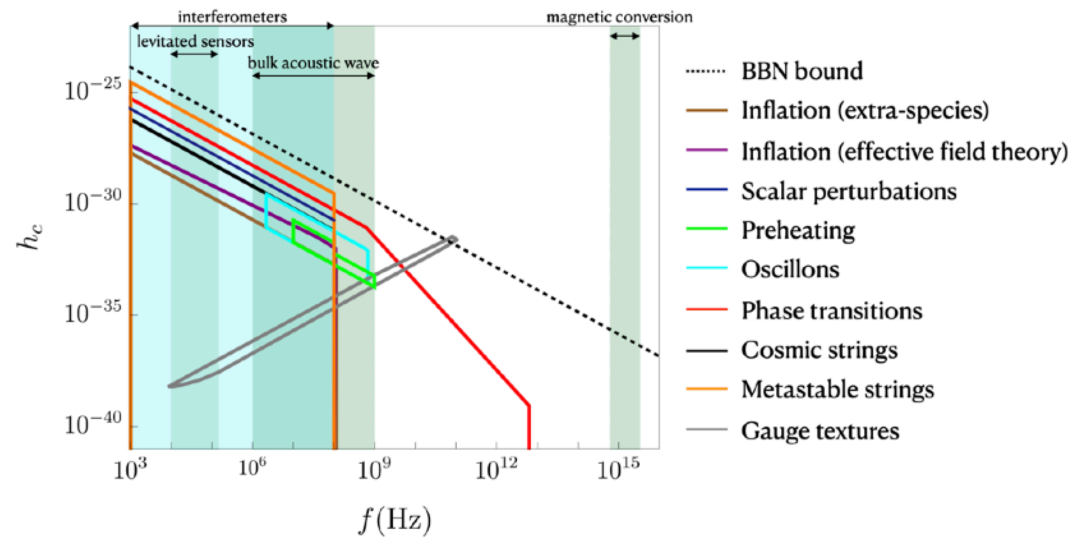
	Stochastic	Coherent
Standard Model:	✓ Thermal plasma fluctuations Ghiglieri & Laine (2015) Ghiglieri et al (2020) Ringwald et al (2020)	✗ ?
BSM:	✓ Inflation Phase transitions Cosmic Strings ...	✓ PBH inspirals Superradiance Exotic objects ...

Ringwald et al, 2011.04731







Aggarwal et al, 2011.12414

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Sources discussed in detail Weds-Fri

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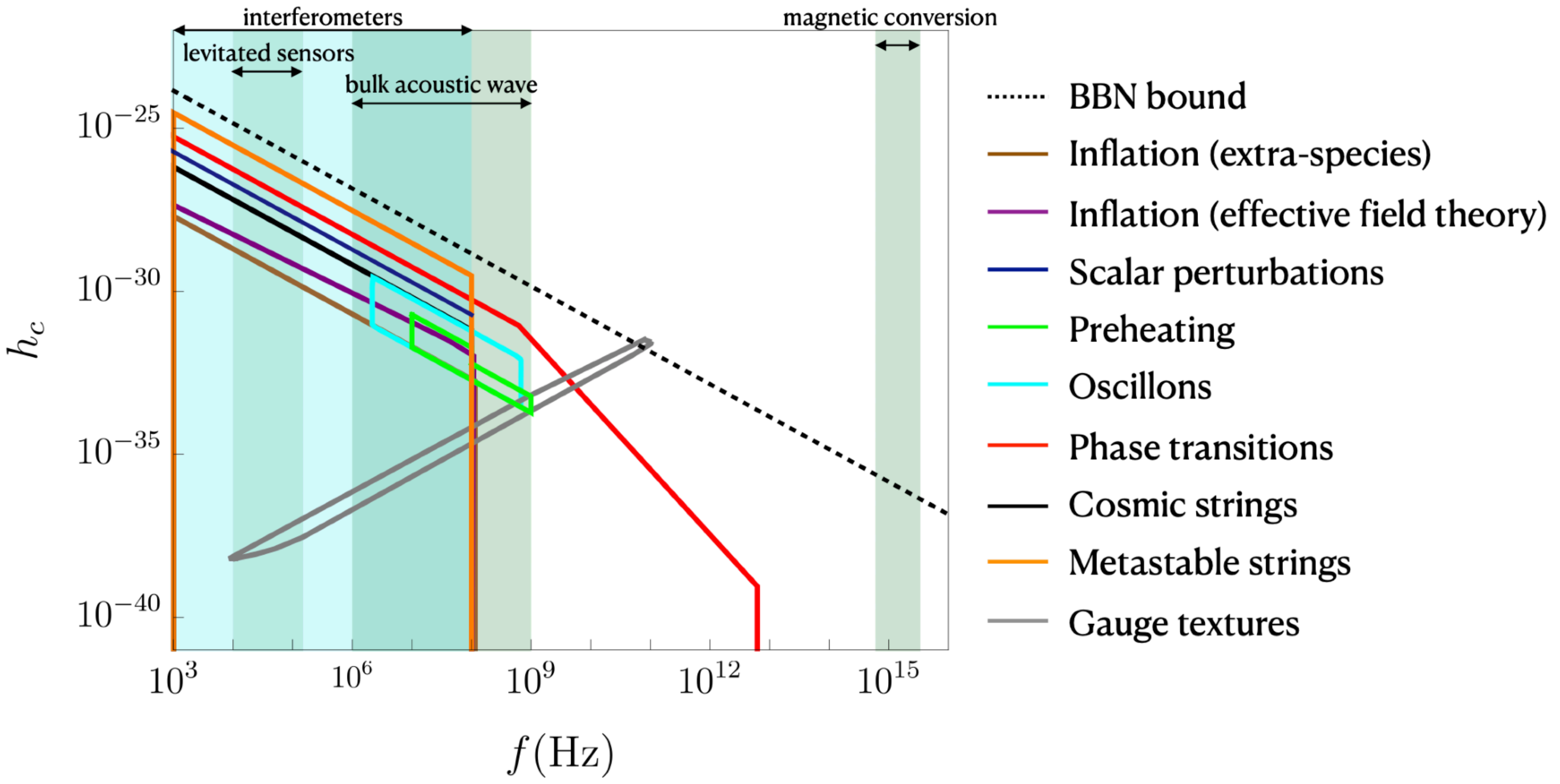

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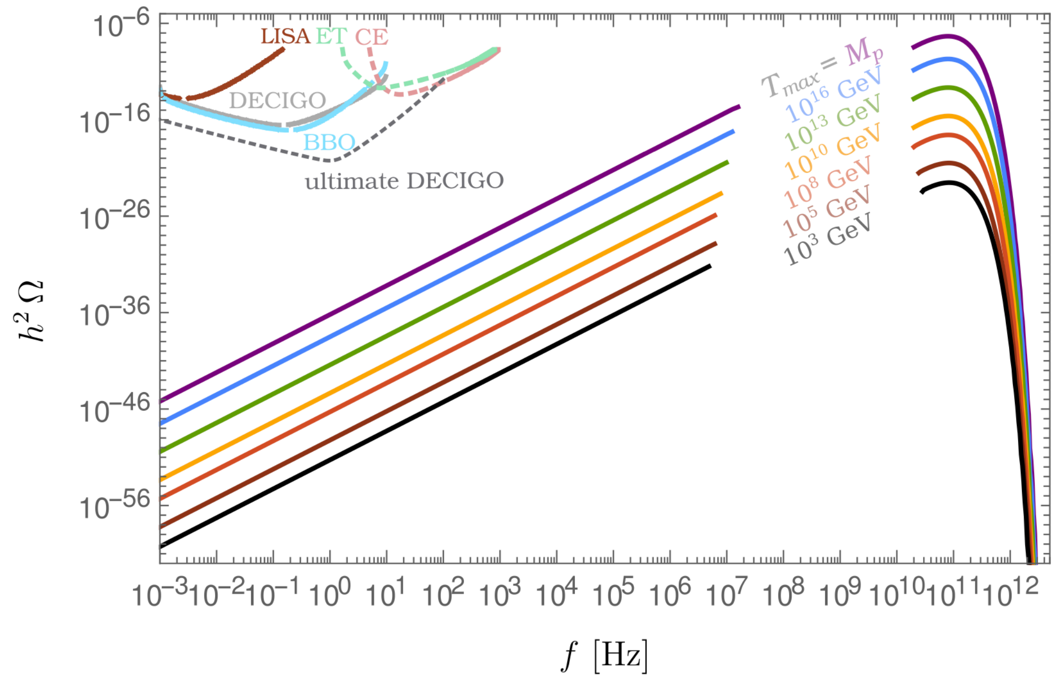

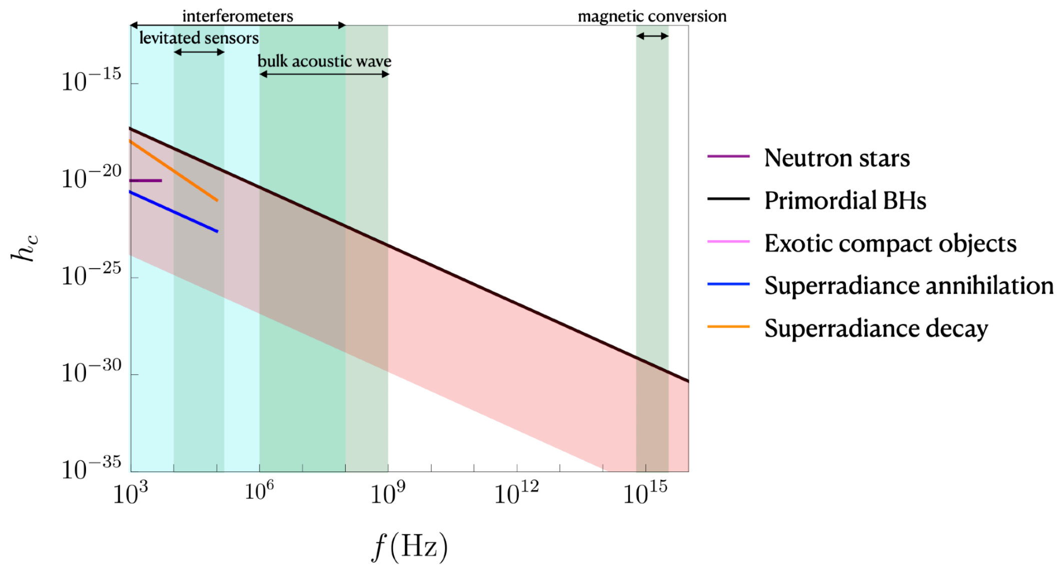

BSM:

Inflation	PBH inspirals
Phase transitions	Superradiance
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...	...

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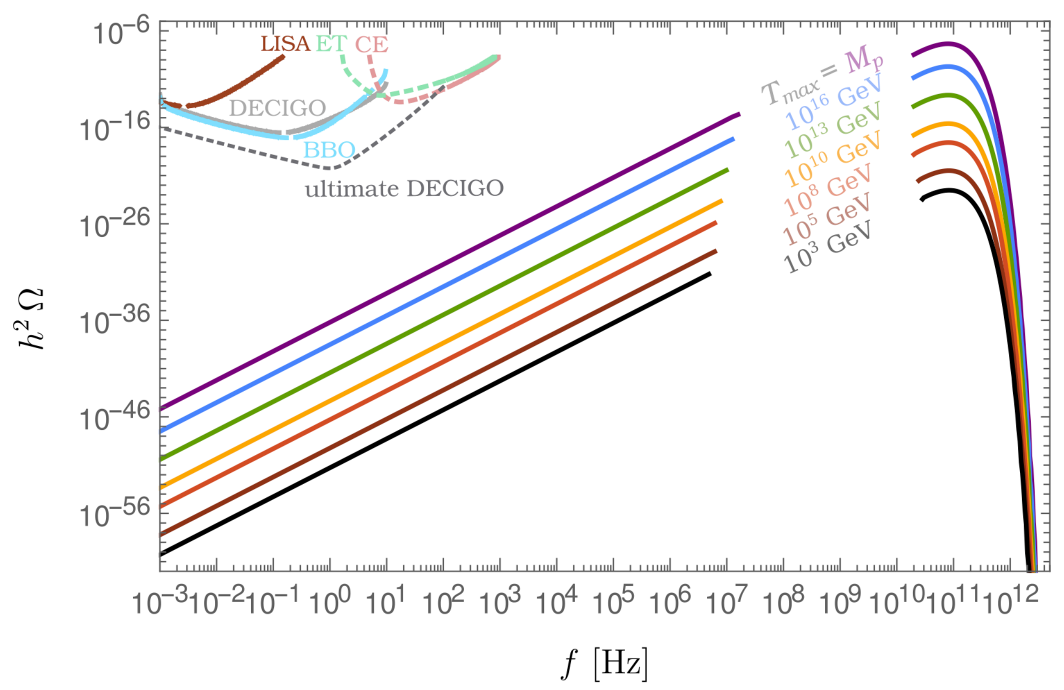

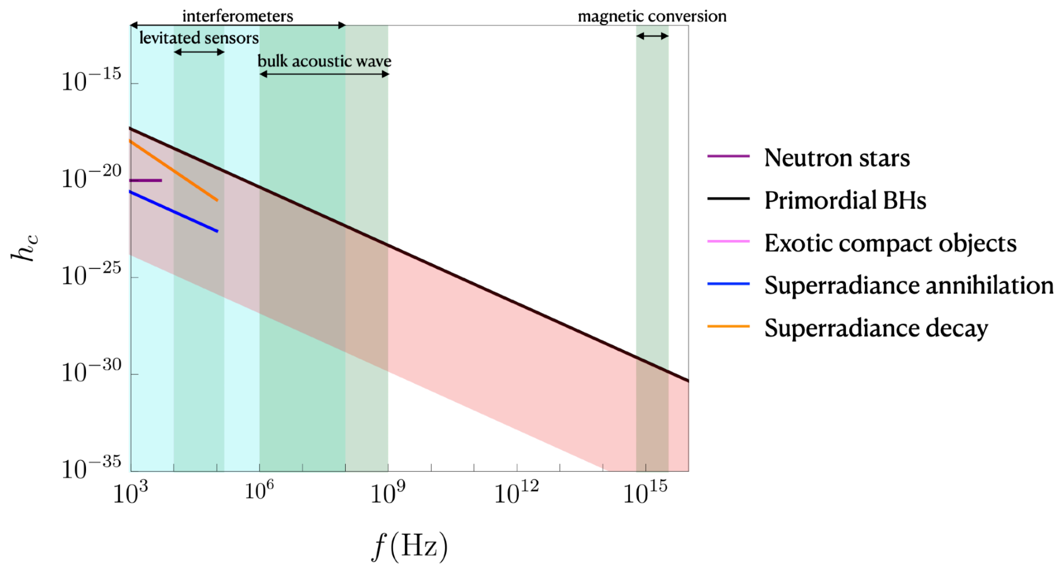

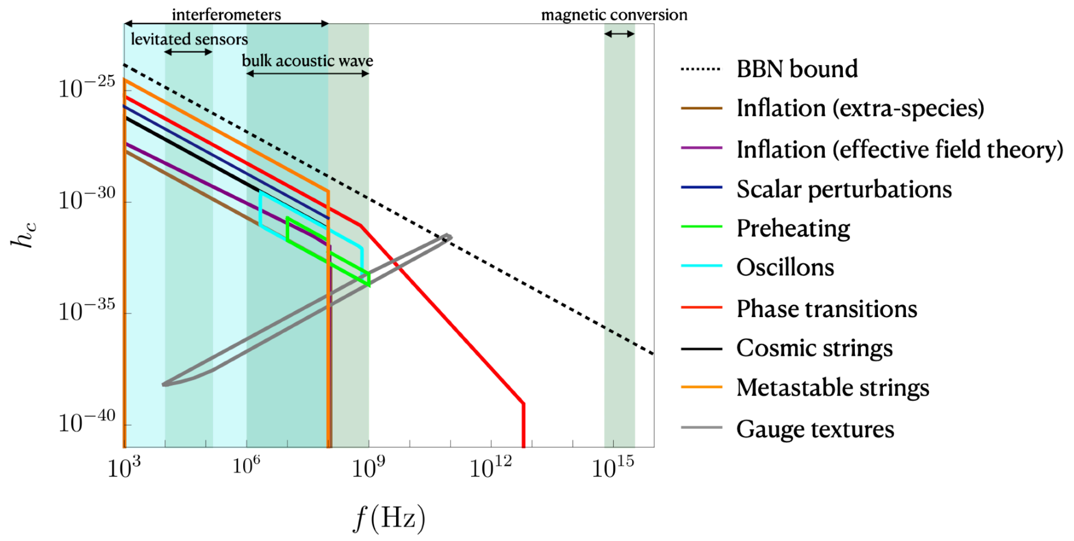

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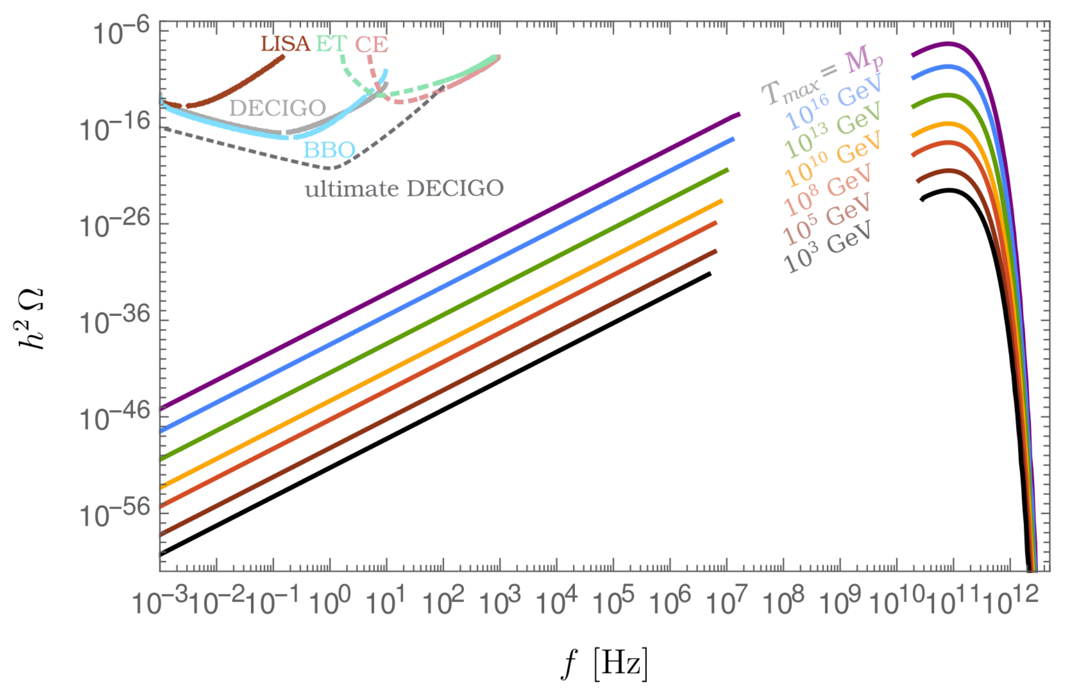
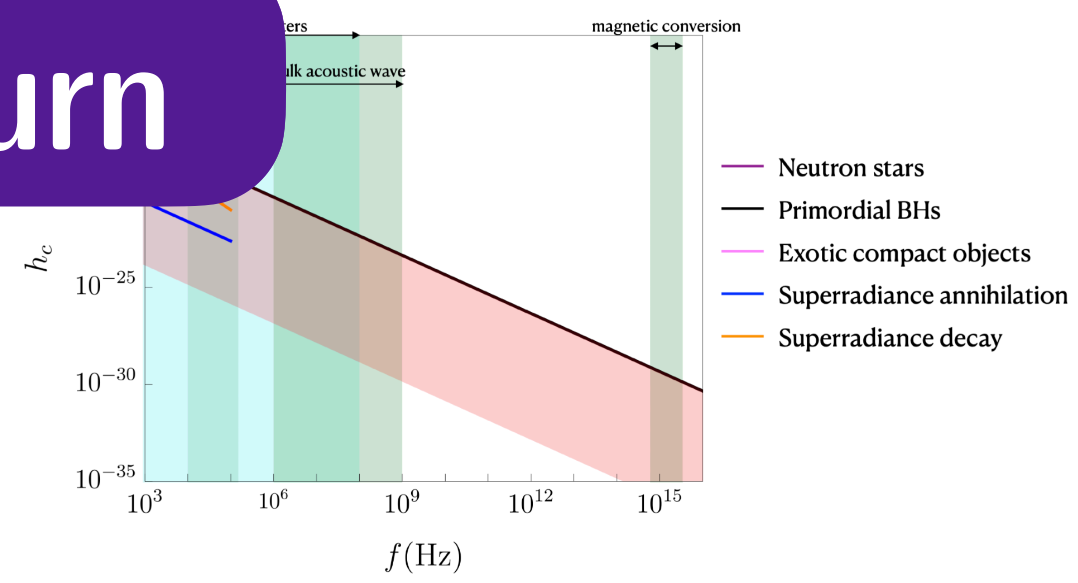
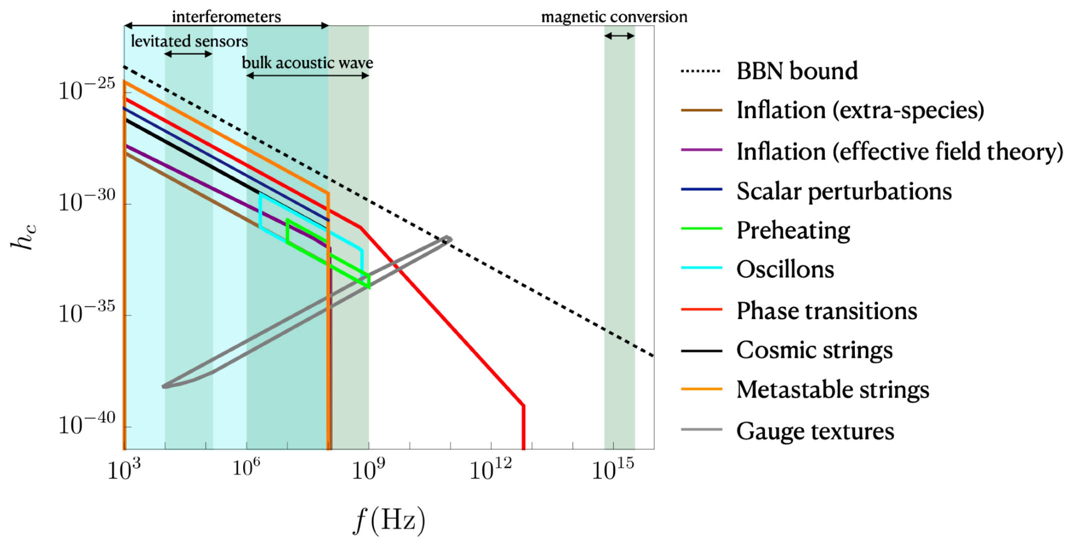
HFGW searches — high risk, high return

Ringwald et al, 2011.04731







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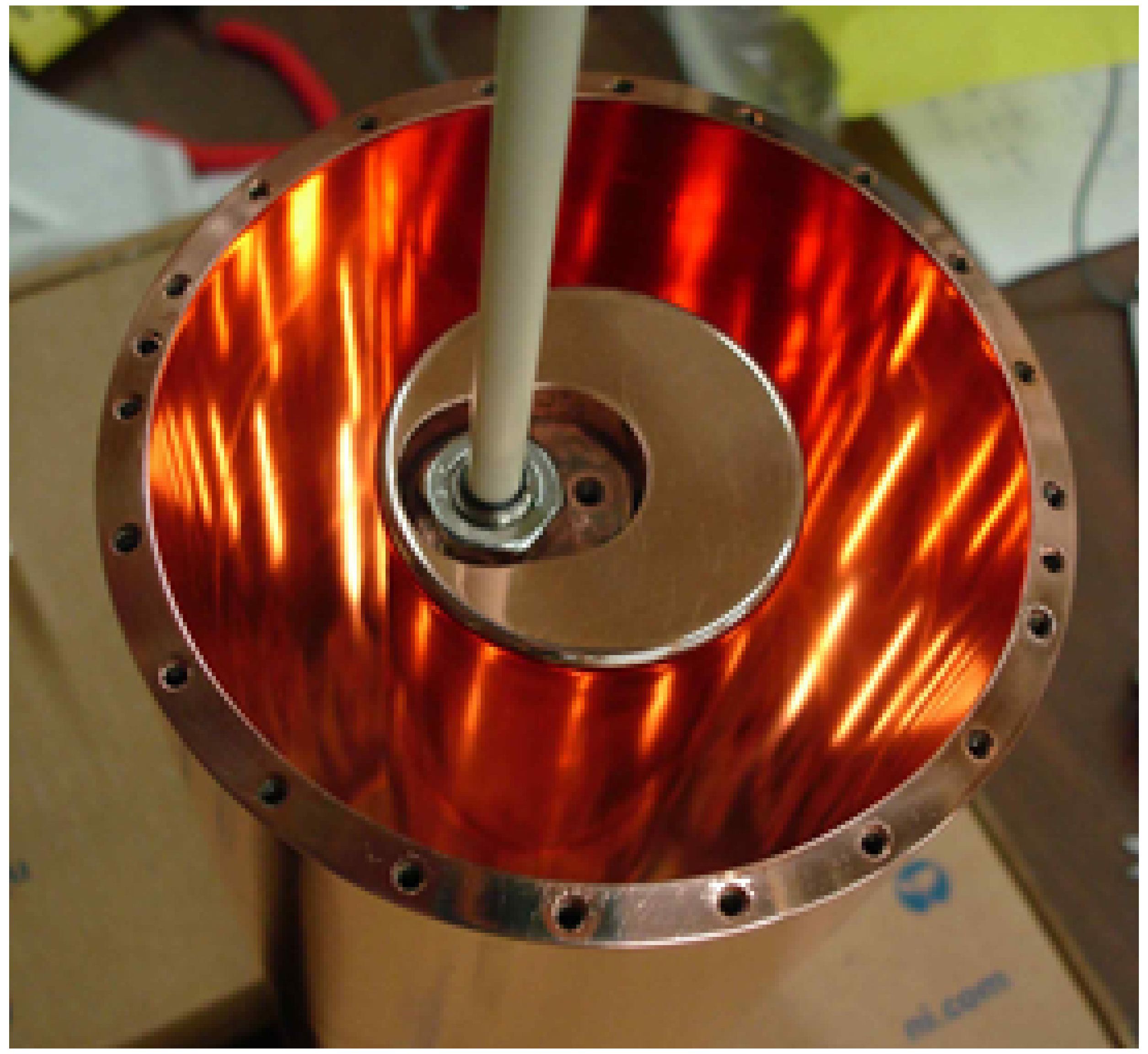
Resonant Cavities

Why?



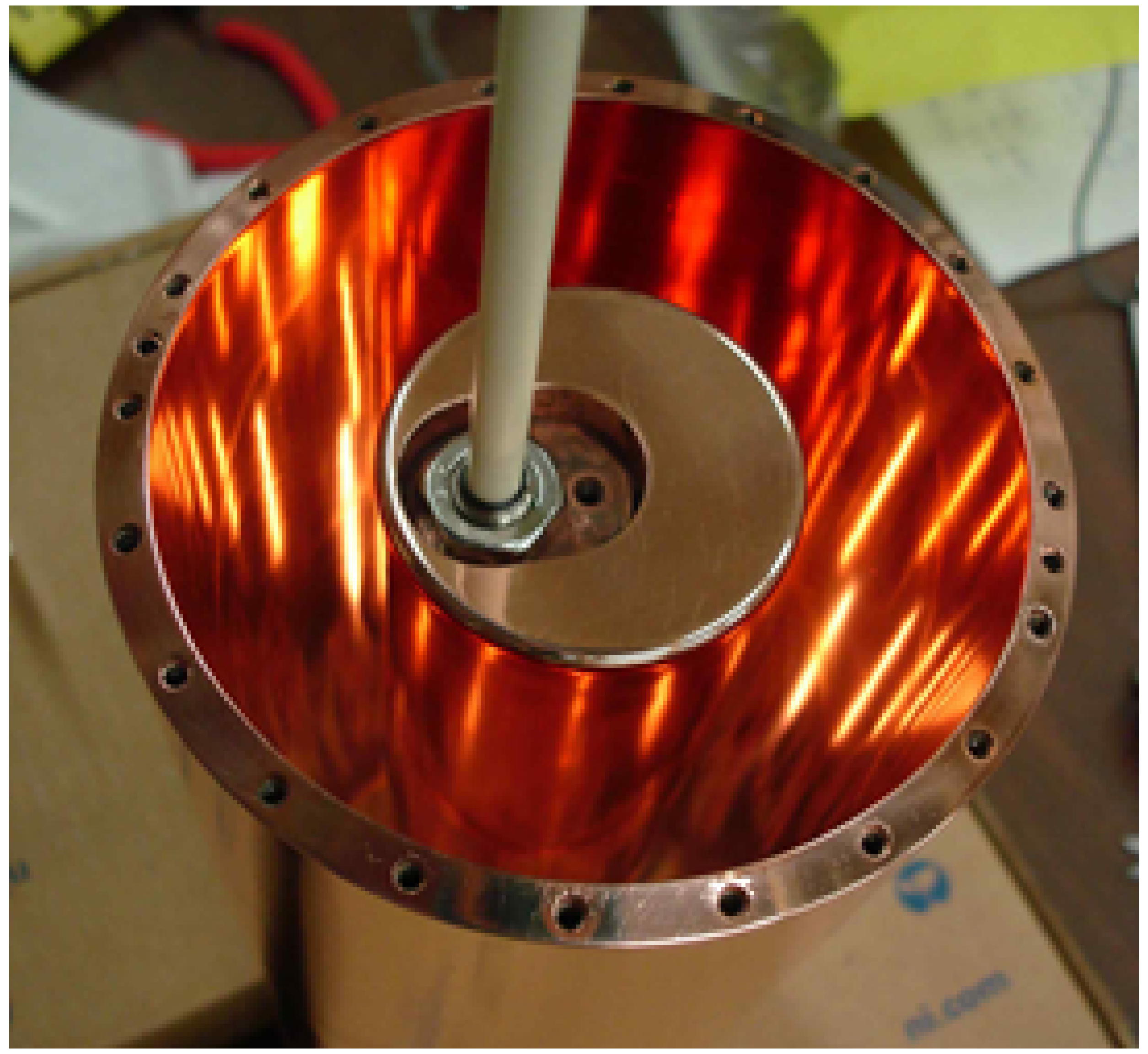
HAYSTAC

Resonant Cavities

Why?

Mature technology & constantly improving

Benefit from decades of development for accelerator use



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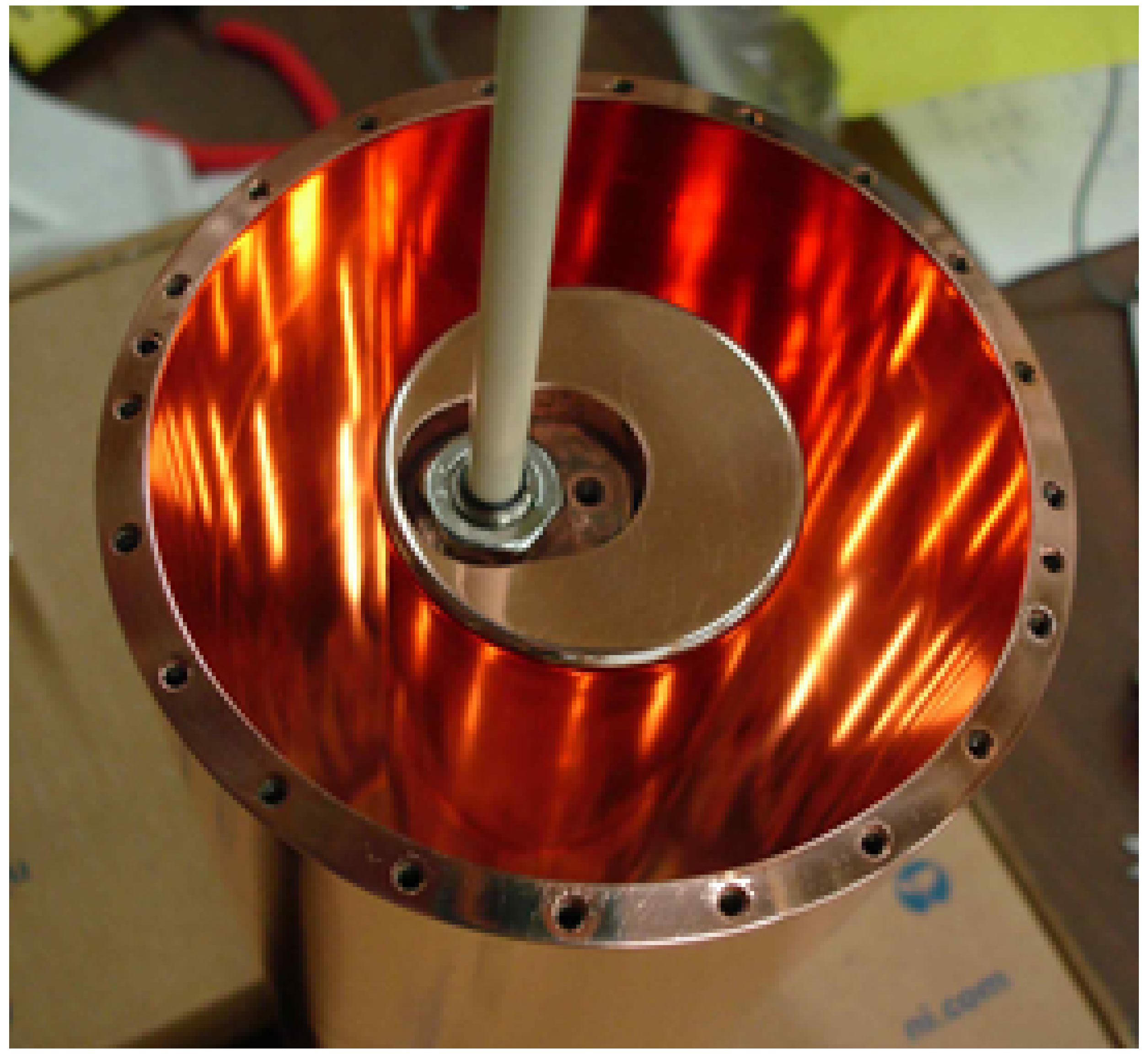

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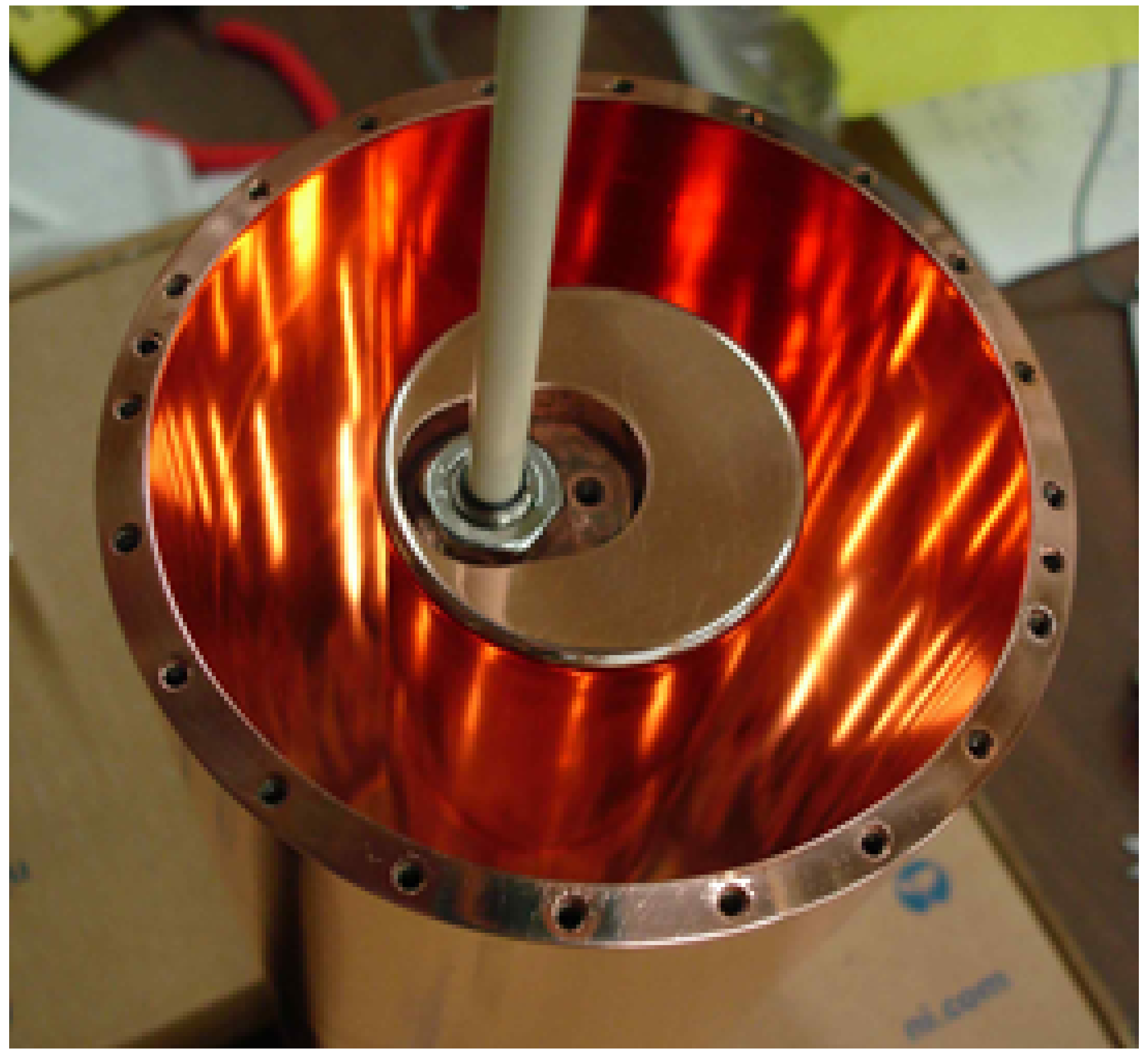


HAYSTAC

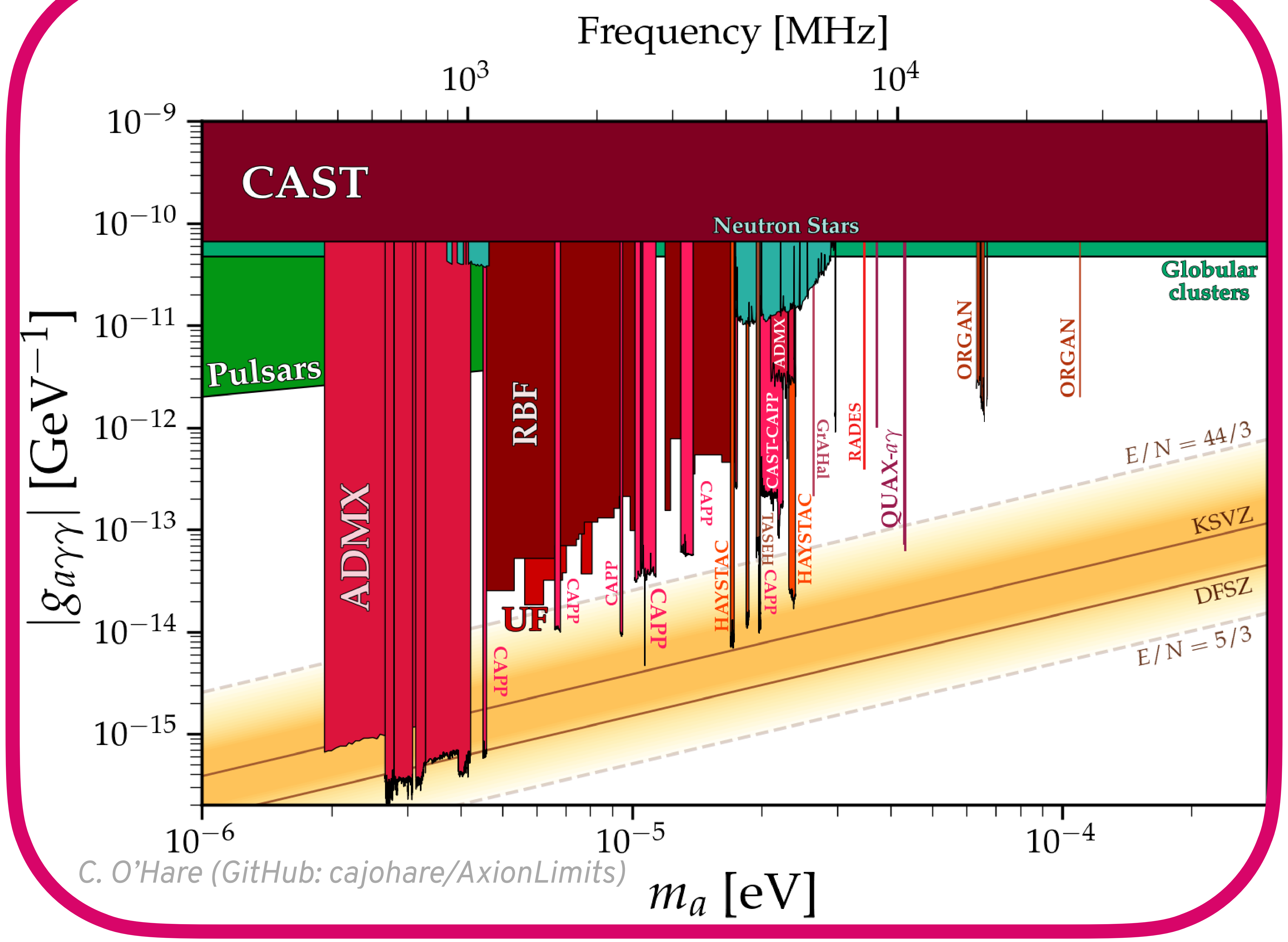
Domain where various subtleties arise...

Intuition for EM signal

Intuition for EM signal

Estimate sensitivity to GWs by comparing sizes of currents

Intuition for EM signal



C. O'Hare (GitHub: cajohare/AxionLimits)

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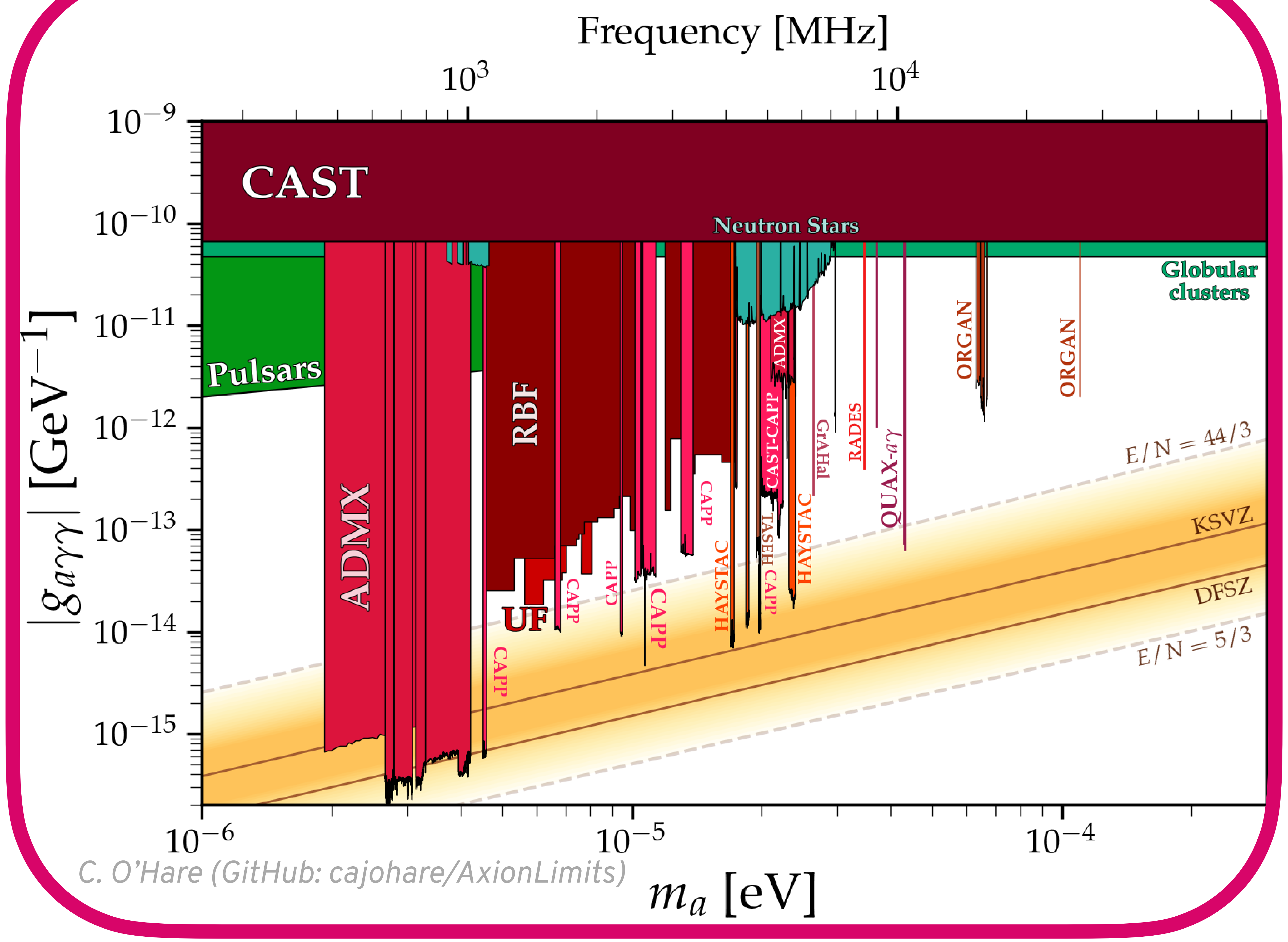

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Estimate sensitivity to GWs by comparing sizes of currents

$$j_{\rm eff}^{\rm axion} \sim g_{a\gamma\gamma}\partial_t(a\mathbf{B}) + \mathcal{O}(v)$$

$$j_{\rm eff}^{\rm axion} \lesssim 10^{-19}\,\mathrm{T/m}$$

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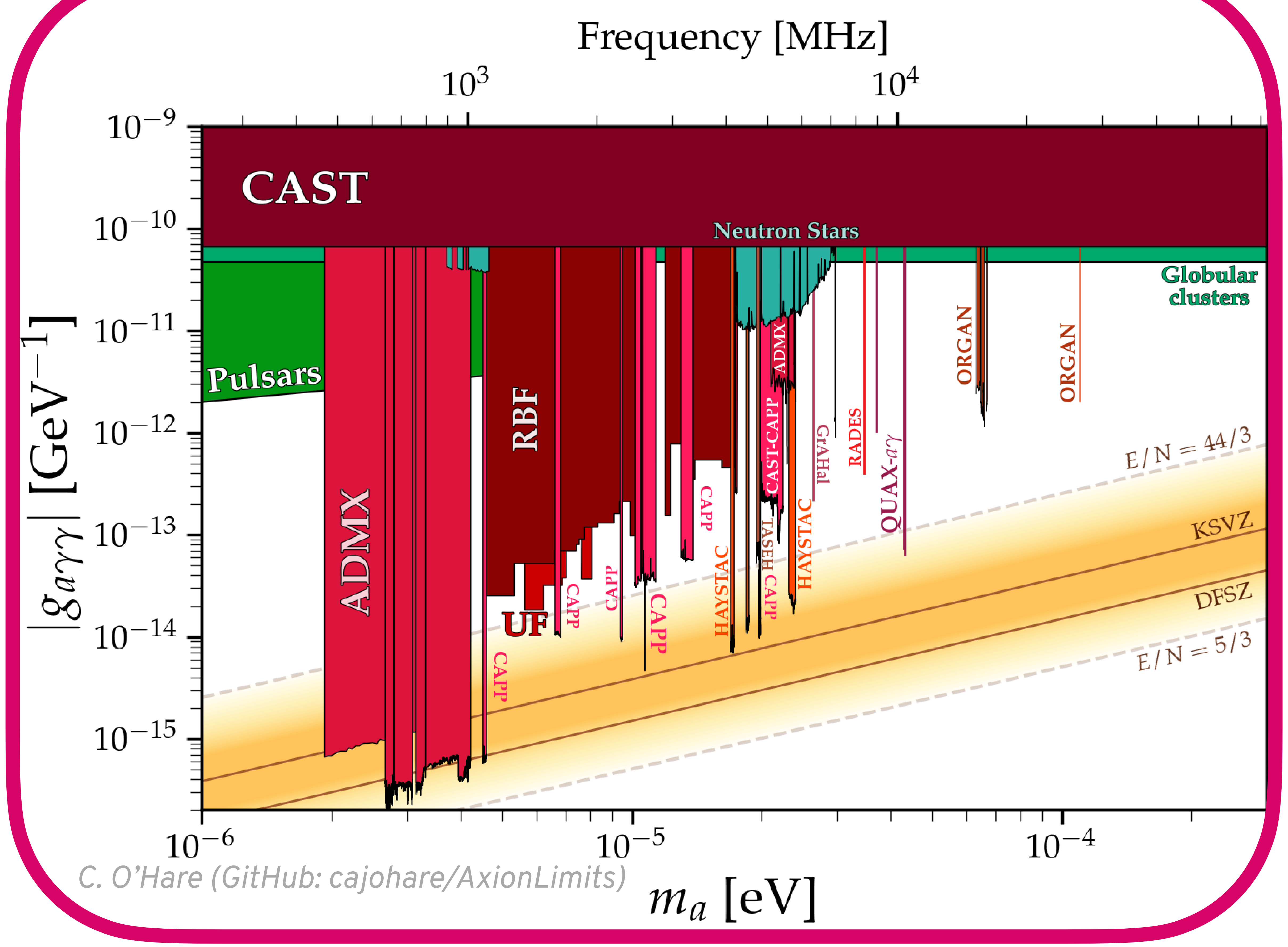

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$$\Downarrow$$

$$j_{\rm eff}^{\rm GW} \sim \partial_t(h\mathbf{B}) + \ldots$$

$$h \lesssim 10^{-21}$$

Framing the Question

A more detailed estimate requires some GR

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GW in TT gauge: $\partial_\mu h^{\mu\nu} = 0, \quad h_\mu{}^\mu = 0, \quad h_{00} = h_{0i} = 0$

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A more detailed estimate requires some GR

GW in TT gauge: $\partial_\mu h^{\mu\nu} = 0, \quad h_\mu{}^\mu = 0, \quad h_{00} = h_{0i} = 0$

Riemann tensor invariant at O(h):

$$R_{0i0j} = -\frac{1}{2}\partial_t^2 h_{ij}^{\mathrm{TT}},$$

$$R_{0ijk} = \frac{1}{2}\partial_t\left(\partial_k h_{ij}^{\mathrm{TT}} - \partial_j h_{ik}^{\mathrm{TT}}\right),$$

$$R_{ikjl} = \frac{1}{2}\left(\partial_k\partial_j h_{il}^{\mathrm{TT}} + \partial_i\partial_l h_{jk}^{\mathrm{TT}} - \partial_i\partial_j h_{kl}^{\mathrm{TT}} - \partial_k\partial_l h_{ij}^{\mathrm{TT}}\right)$$

MAGO 2.0

Revive an old idea from the 1970s, and a prototype from the 2000s

On the operation of a tunable electromagnetic detector for gravitational waves

F Pegoraro†, E Picasso‡ and L A Radicati‡§

†Scuola Normale Superiore, Pisa, Italy
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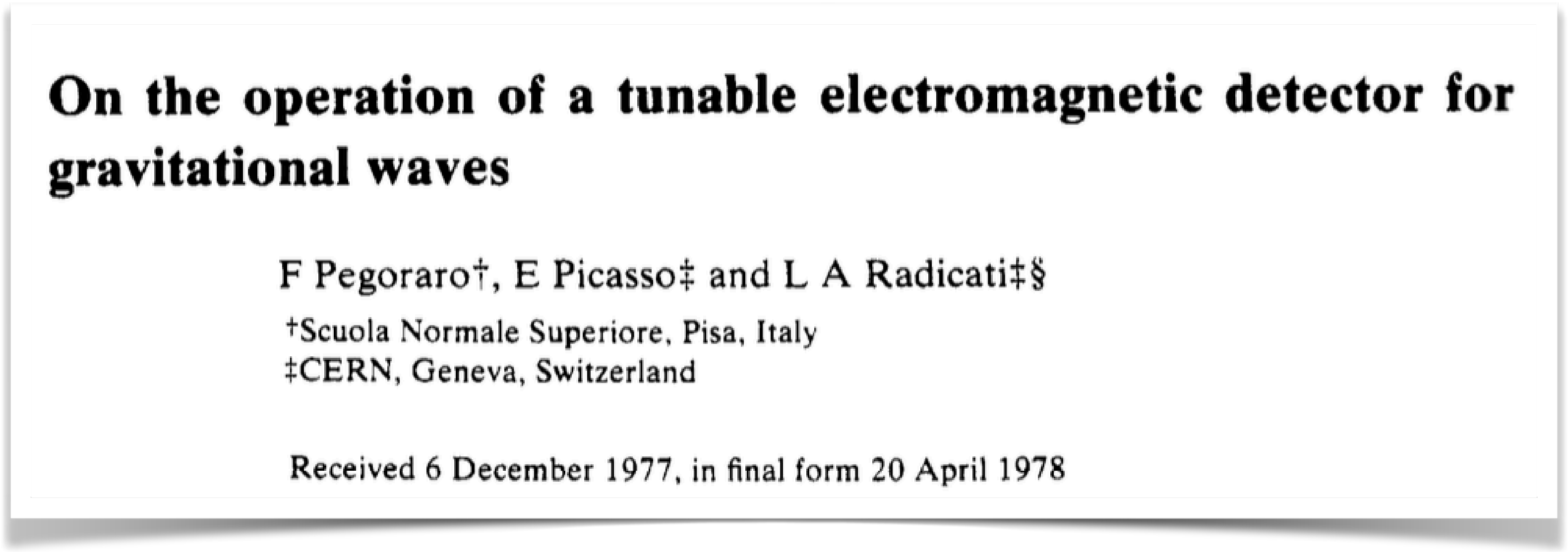

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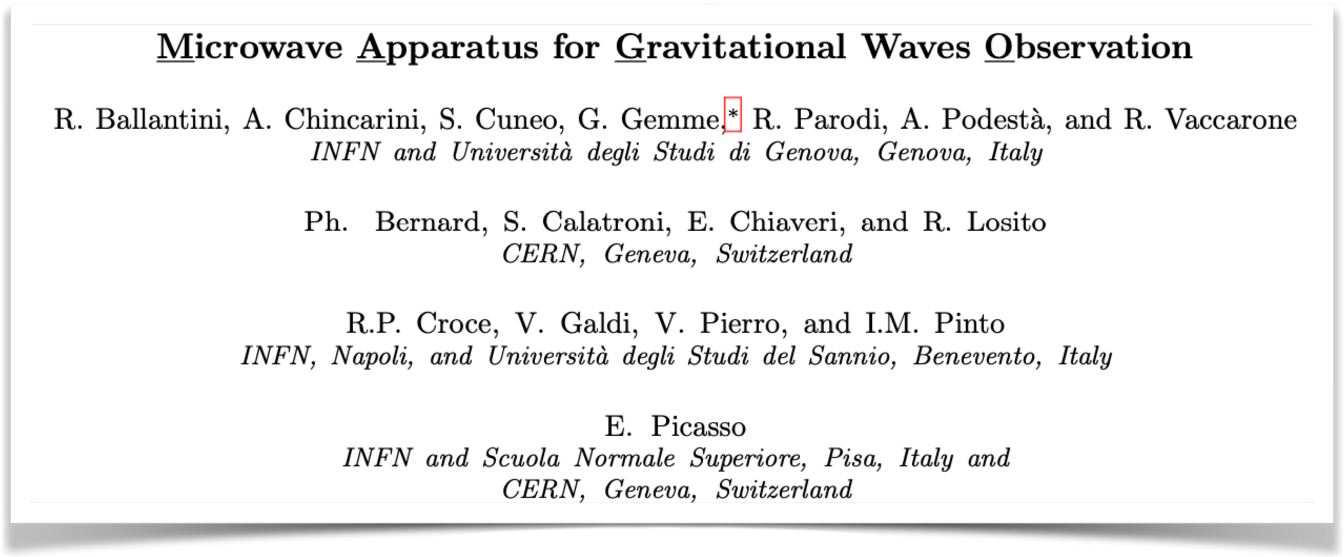

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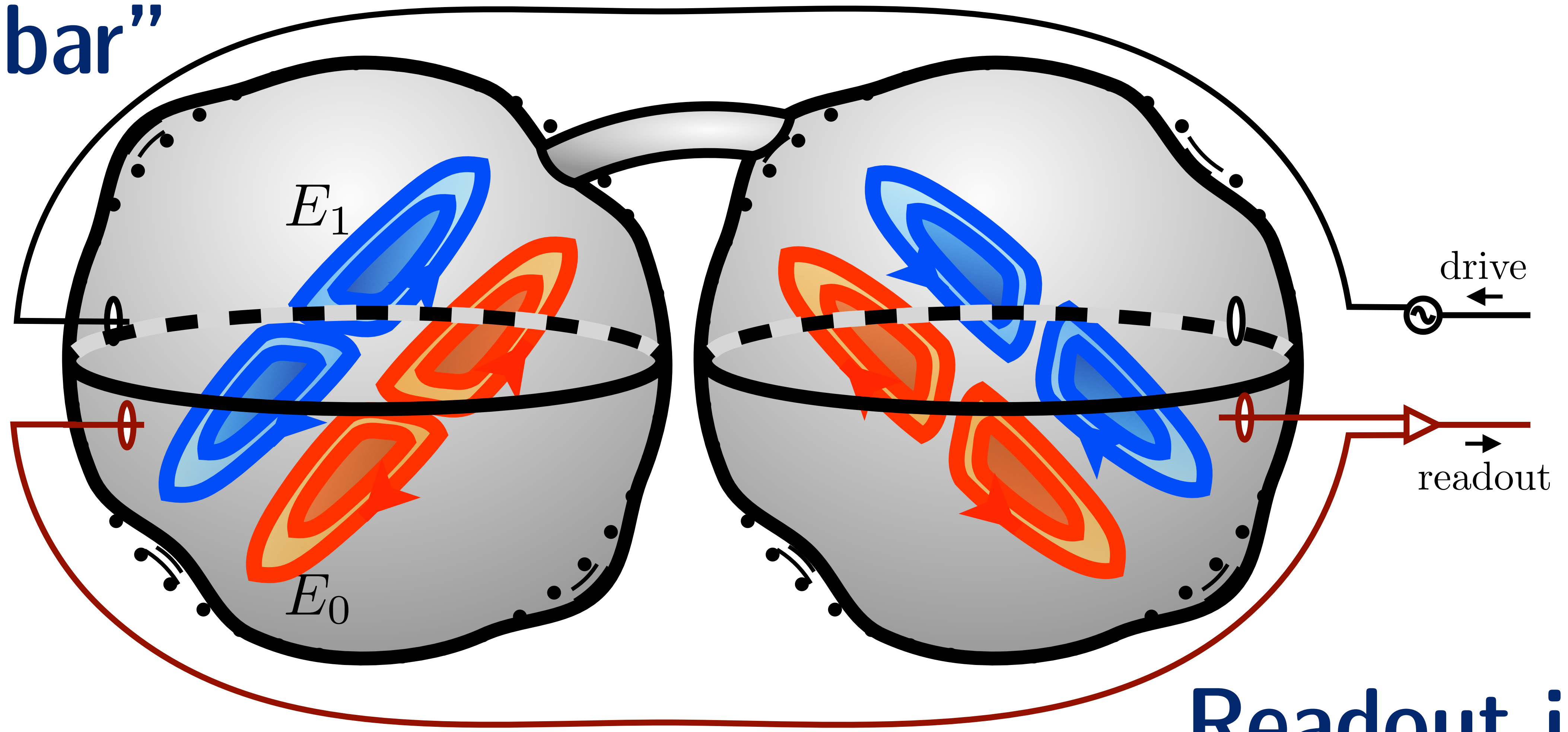
Cavity walls are a “Weber bar”



Readout in a quiet mode of cavity

Gravitational Wave and a Hollow Sphere

Gravitational Wave and a Hollow Sphere

Mechanical modes of a sphere

$$\mathbf{U}_{lmn} = \nabla\phi_L + i\,\nabla\times\mathbf{L}\phi_{T_1} + i\,\mathbf{L}\phi_{T_2}\ .$$

$$\mathbf{U}(\mathbf{x},t) = u_p(t)\,\mathbf{U}_p(\mathbf{x})$$

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Equation of motion

$$\ddot{u}_p + \frac{\omega_p}{Q_p}\,\dot{u}_p + \omega_p^2\,u_p \simeq -\frac{1}{2}\,\omega_g^2\,V_{\rm cav}^{1/3}\,\eta^g_{\rm mech}\,h_0\,e^{i\omega_g t}$$

$$\eta^g_{\rm mech} = \frac{\hat{h}^{TT}_{ij}}{V_{\rm cav}^{1/3}\,V_{\rm shell}}\int_{V_{\rm shell}} d^3\mathbf{x}\,U_p^{*i}\,x^j$$

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$$\langle \mathbf{U}_p \rangle \sim h_0\,V_{\mathrm{cav}}^{1/3}\,\eta^g_{\mathrm{mech}} \times \begin{cases} \frac{\omega_g^2}{\omega_g^2-\omega_p^2}\ , & |\omega_g - \omega_p| \gg \omega_p/Q_p \\ Q_p\ , & |\omega_g - \omega_p| \ll \omega_p/Q_p \end{cases}$$

Tiny displacement $\ll$ nm

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Cur Cavis?* pt. 2

MAGO 2.0

* “Why Cavities?” in Latin

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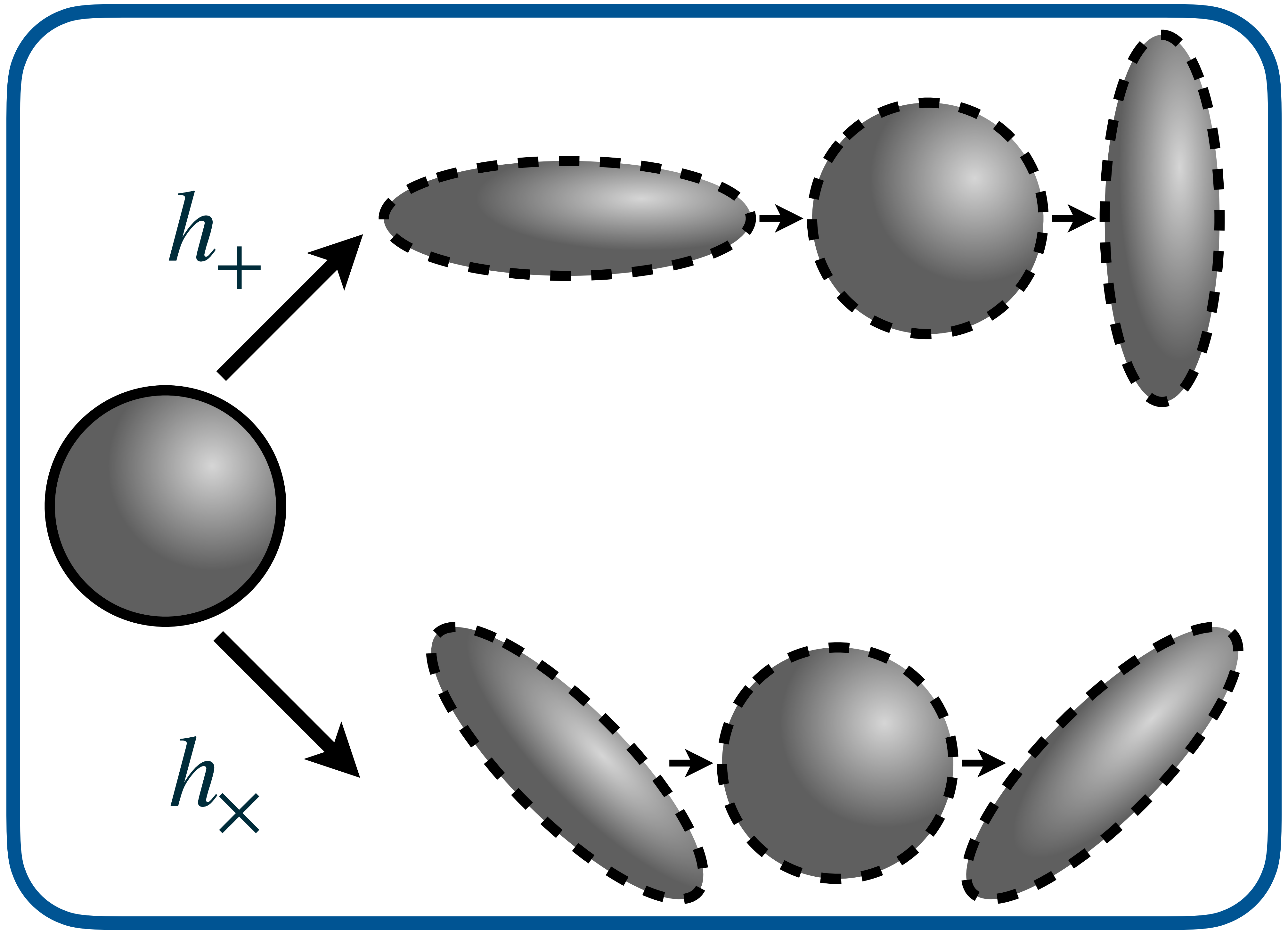
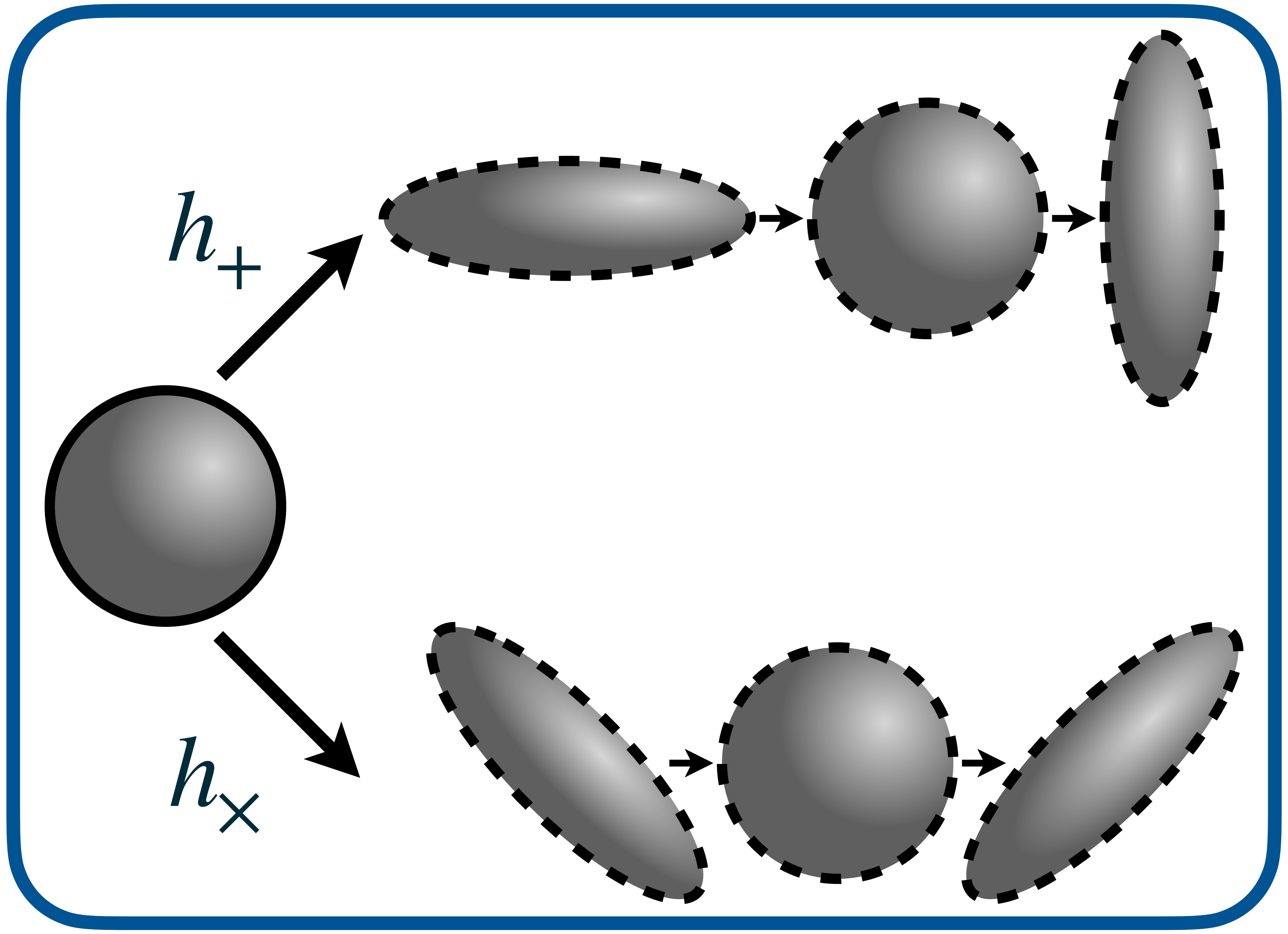


TT frame intuition

Gravitational Wave and a Hollow Sphere



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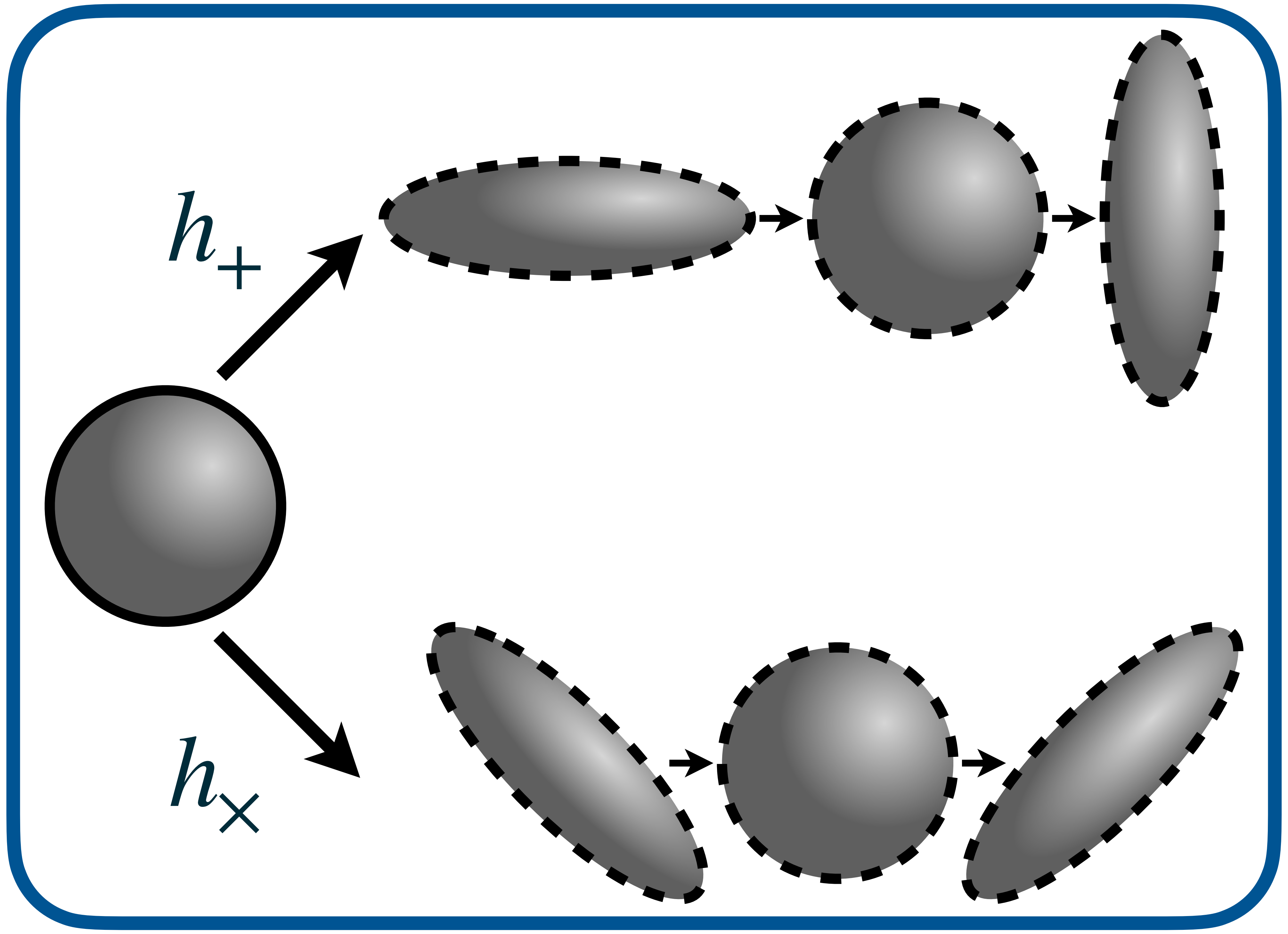

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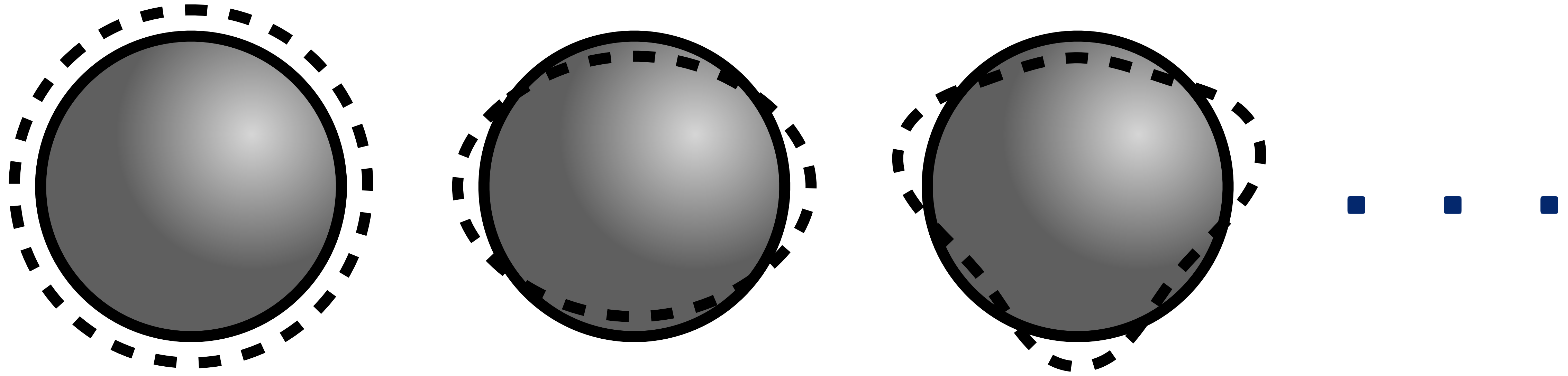

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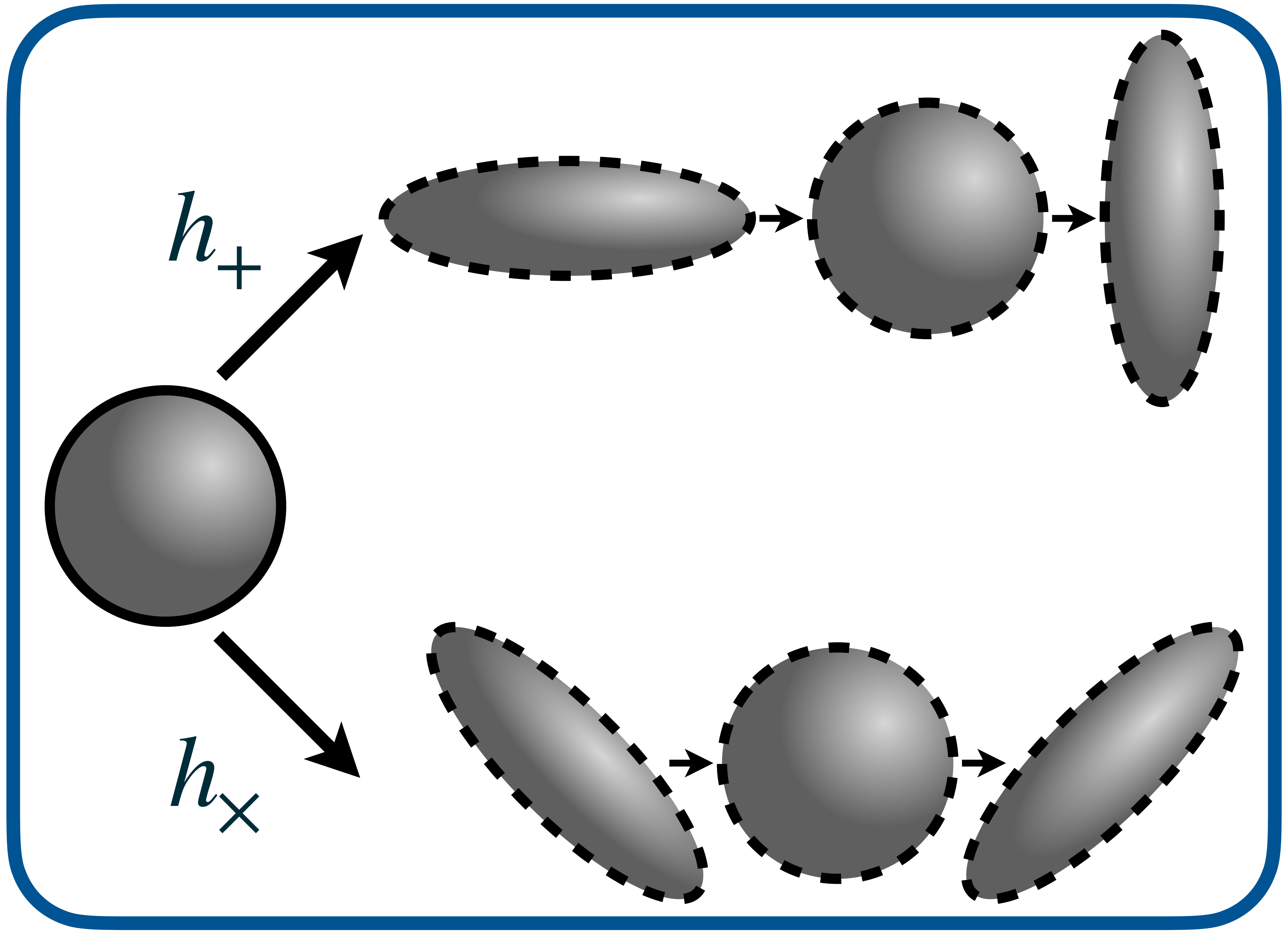

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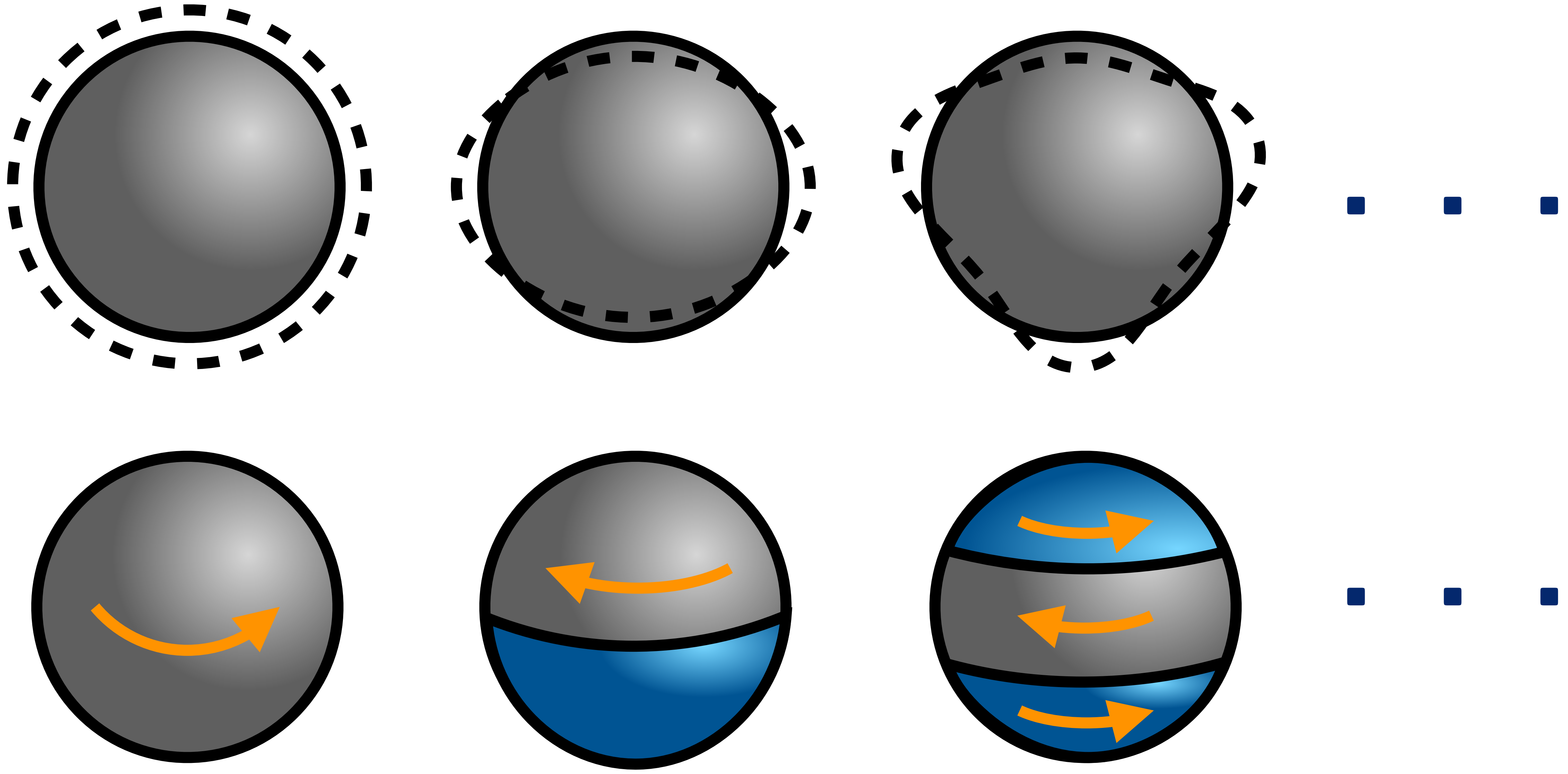
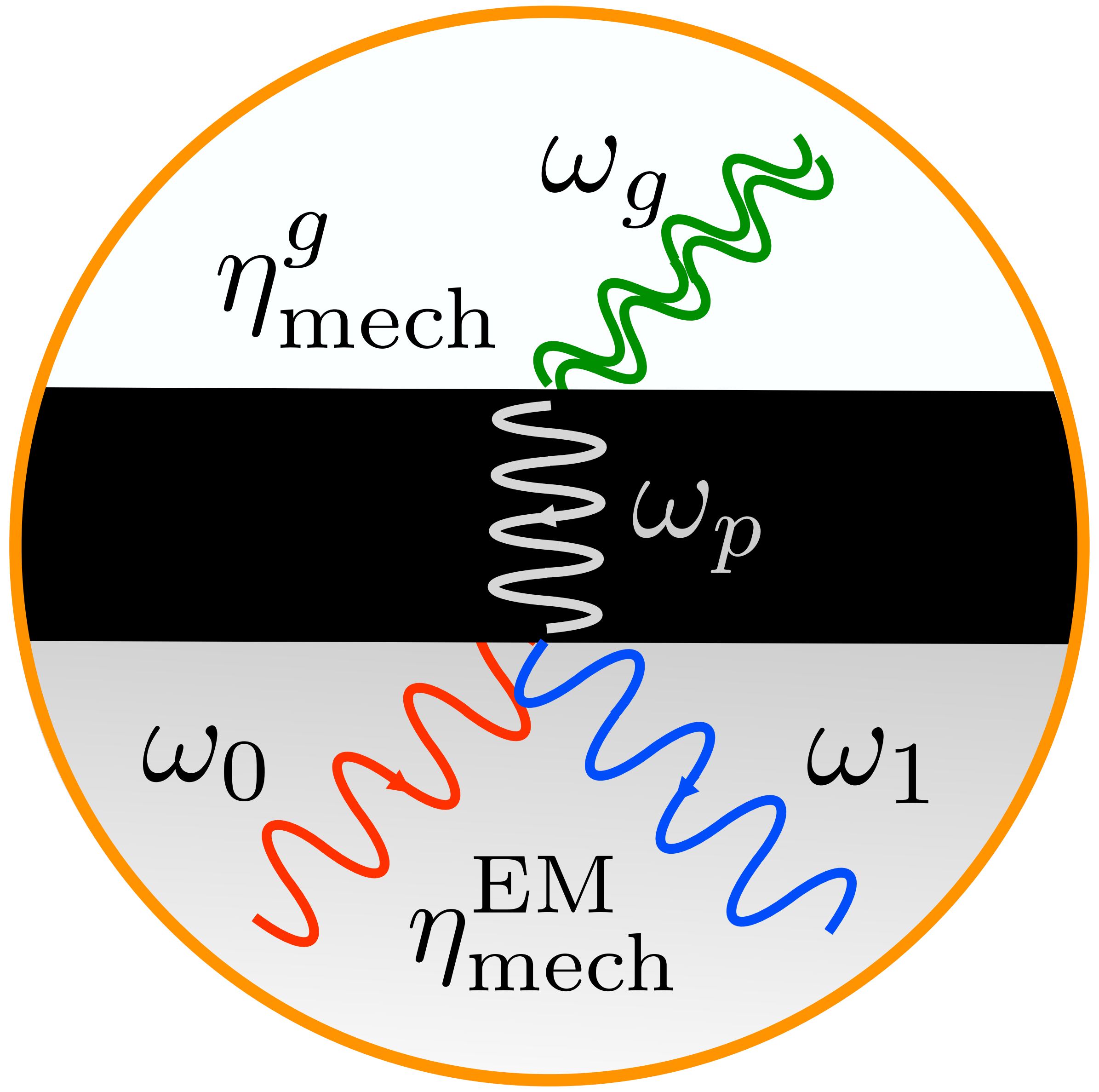


MAGO 2.0

Vibrations of cavity walls — modification of boundary conditions



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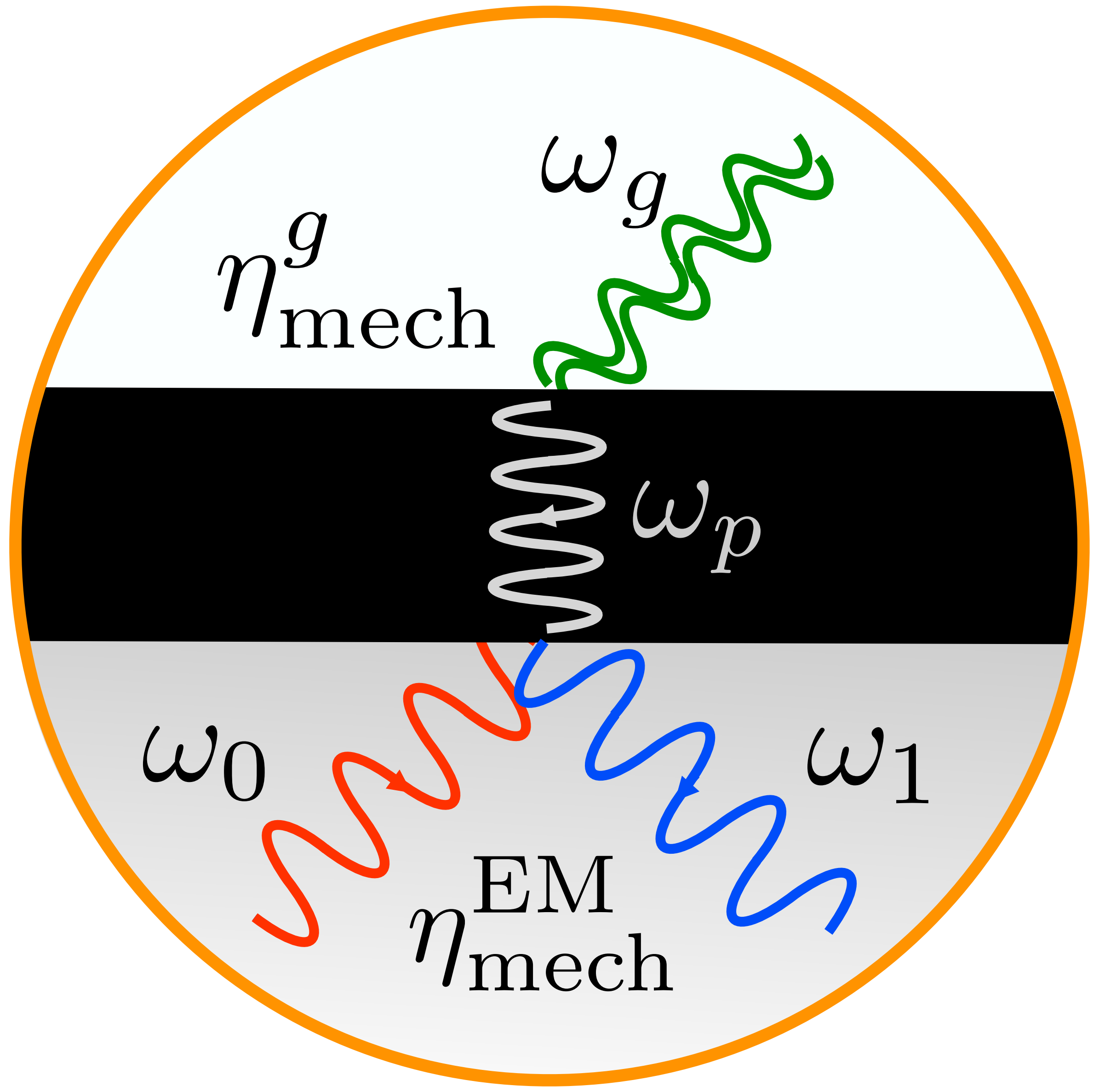

$$\eta^{\text{EM}}_{\text{mech}} = V_{\text{cav}}^{1/3} \, \frac{\int_{S_0} d\mathbf{A} \cdot \mathbf{U}_p \, \left(\mathbf{E}_0 \cdot \mathbf{E}_1^* - \mathbf{B}_0 \cdot \mathbf{B}_1^*\right)}{\int_{V_{\text{cav}}} d^3\mathbf{x} \, |\mathbf{E}_1|^2}$$

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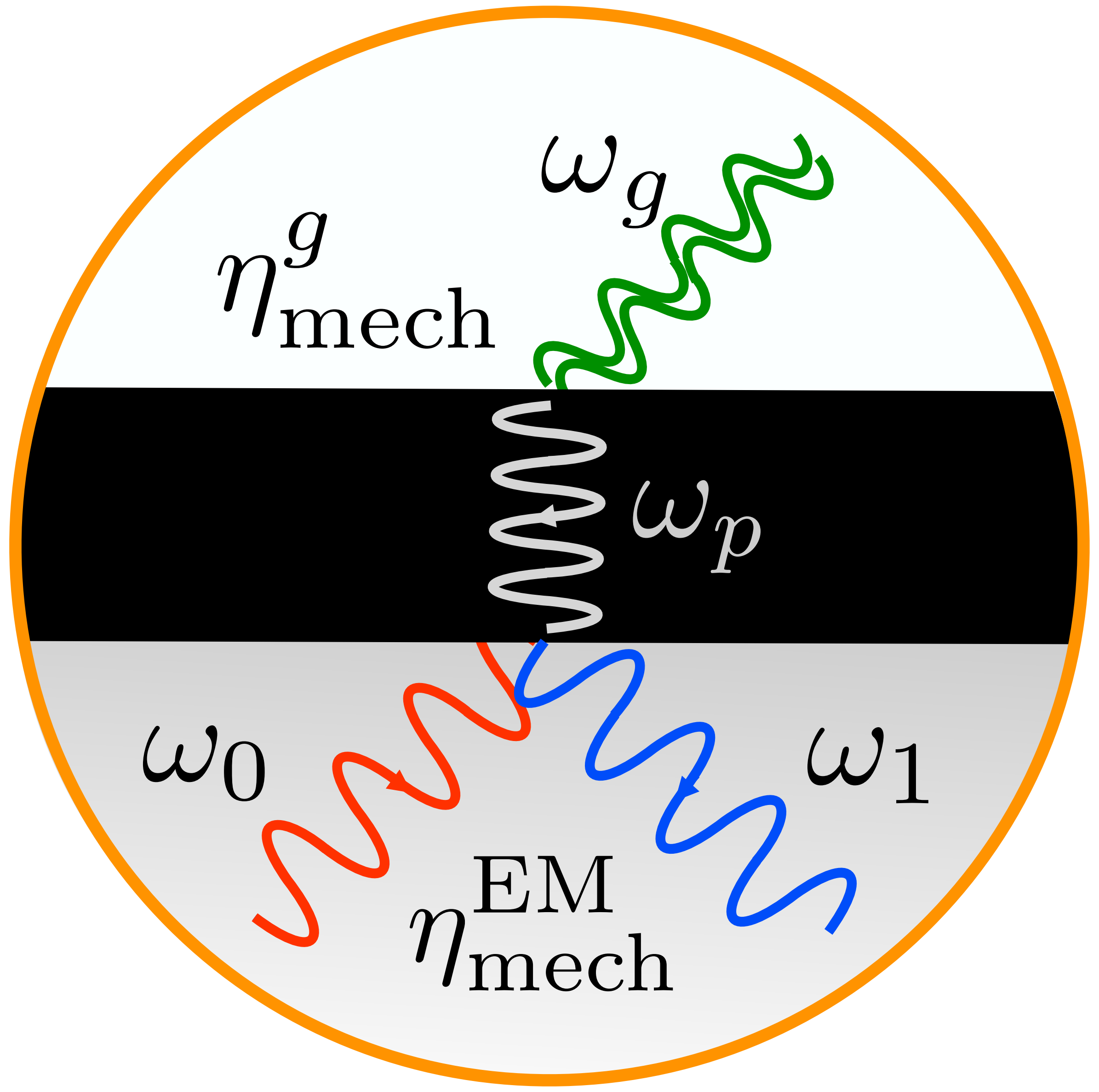

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$$\ddot{e}_1 + \frac{\omega_1}{Q_1} \, \dot{e}_1 + \omega_1^2 \, e_1 \simeq -2 \, \eta^{\rm EM}_{\rm mech} \, \omega_1^2 \, V_{\rm cav}^{-1/3} \, u_p \, e_0$$

$$\mathbf{E}(\mathbf{x}, t) = e_i(t) \, \mathbf{E}_i(\mathbf{x})$$

MAGO 2.0

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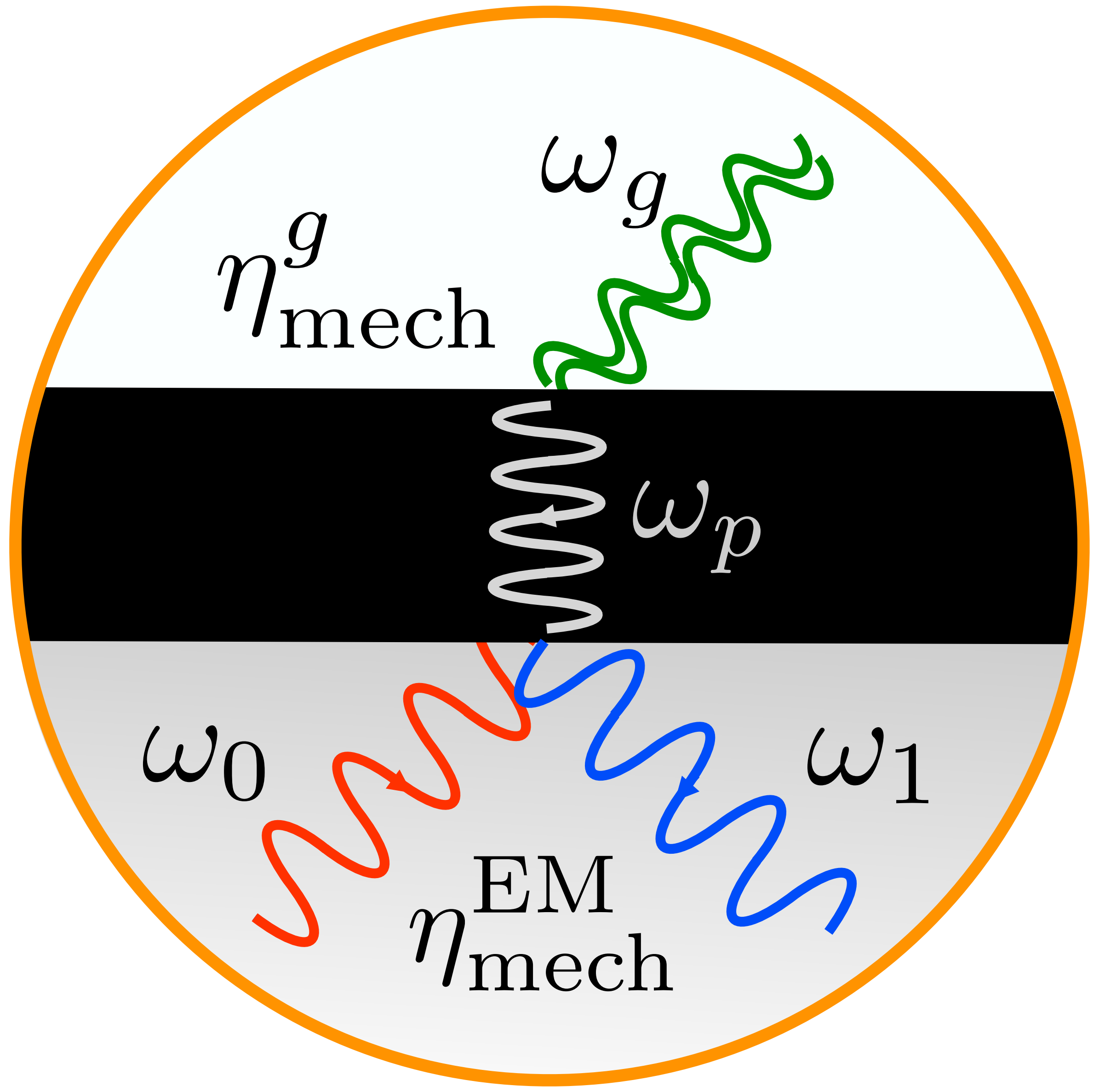

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Recall effect of GW force

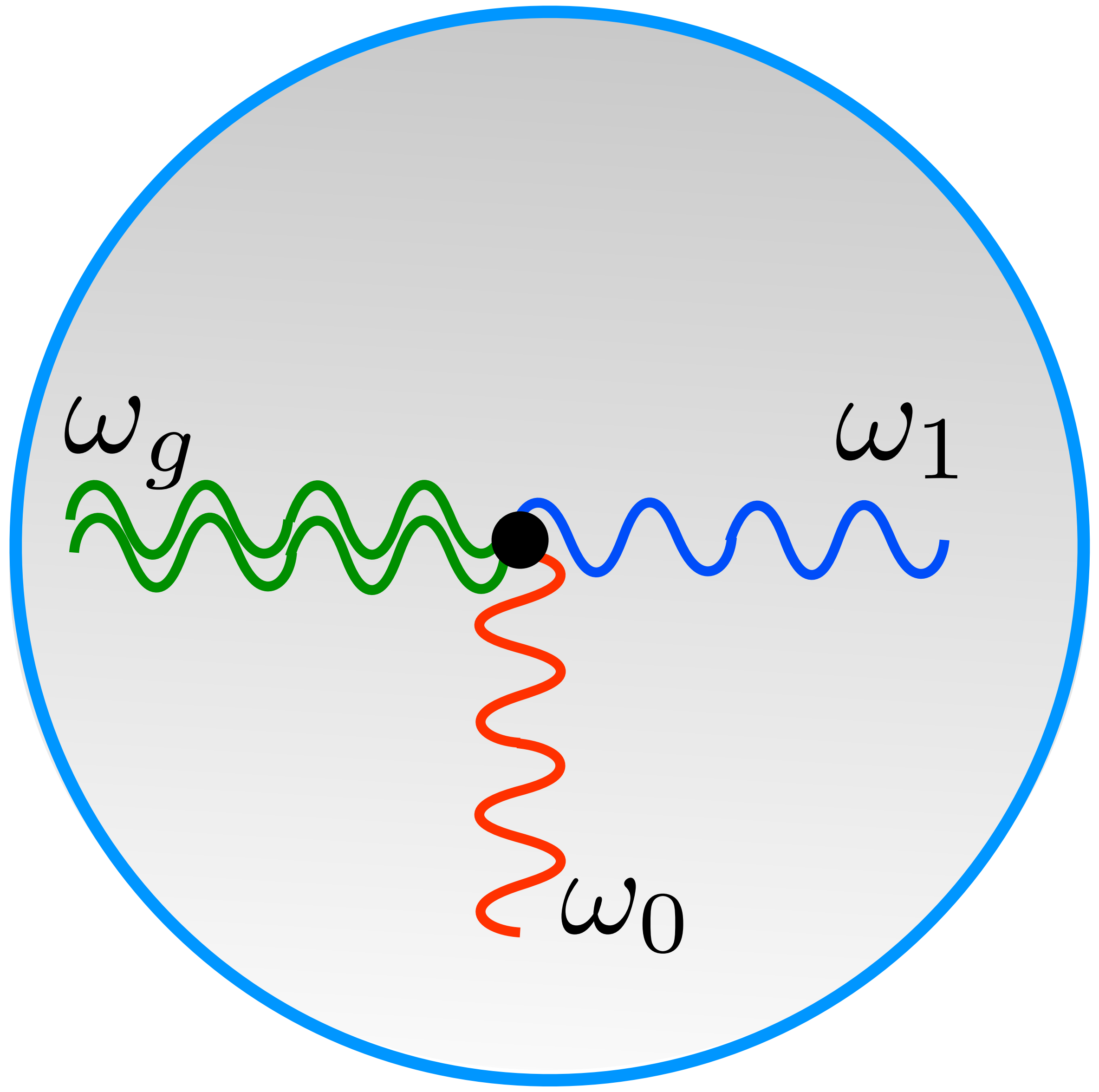
$u_p \propto h_0$

$$\ddot{e}_1 + \frac{\omega_1}{Q_1}\,\dot{e}_1 + \omega_1^2\,e_1 \simeq -2\,\eta^{\rm EM}_{\rm mech}\,\omega_1^2\,V_{\rm cav}^{-1/3}\,u_p\,e_0$$

$$\mathbf{E}(\mathbf{x},t) = e_i(t)\,\mathbf{E}_i(\mathbf{x})$$

MAGO 2.0

Inverse Gertsenshtein effect — direct coupling to EM waves



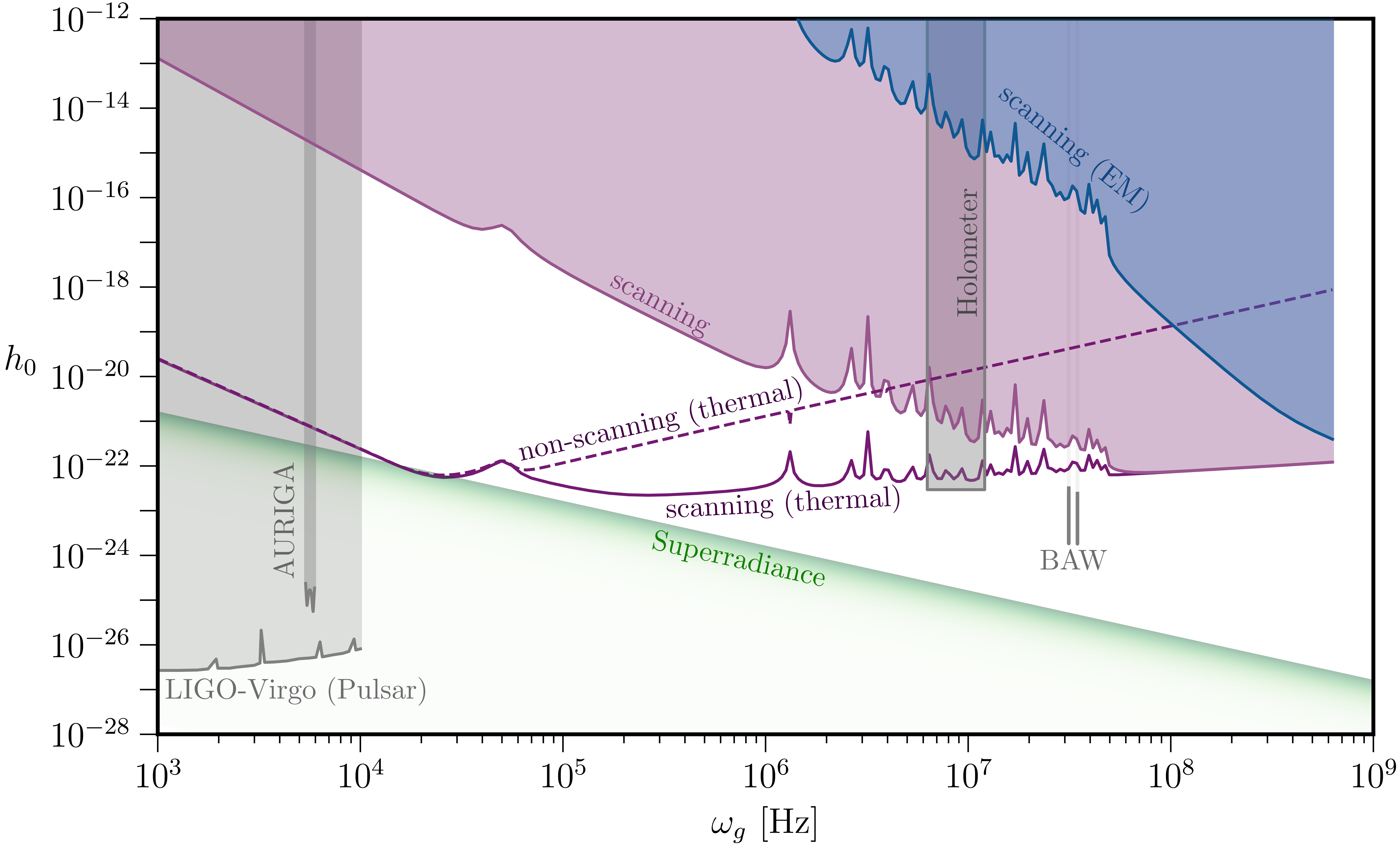
$$j^\mu_{\rm eff} \equiv \partial_\nu \left(\frac{1}{2} h\, F^{\mu\nu} + h^\nu{}_\alpha\, F^{\alpha\mu} - h^\mu{}_\alpha\, F^{\alpha\nu} \right)$$

$$P_{\rm sig} \sim \frac{Q_1^2 Q_{\rm int}}{Q_{\rm cpl}} |\eta^g_{\rm EM}|^2\, P_{\rm in}\, h_0^2\, (\omega_g V_{\rm cav}^{1/3})^4$$

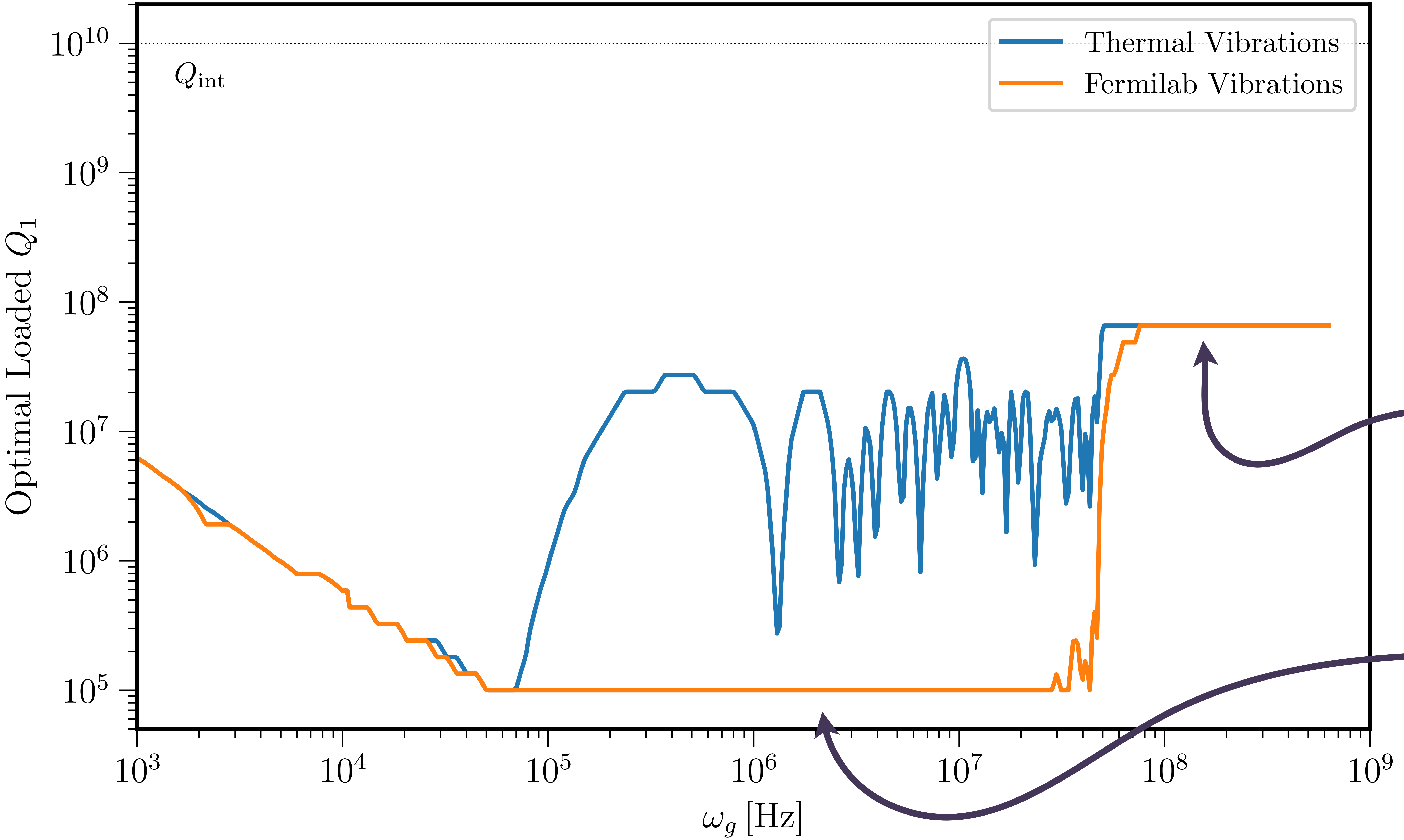
Recall discussion of rigid ruler

$$\eta^g_{\rm EM} = \frac{\int d^3\mathbf{x} \mathbf{E}_1^* \cdot \hat{\boldsymbol{j}}}{(V_{\rm cav} \int d^3\mathbf{x} |\mathbf{E}_1|^2)^{1/2}}$$

MAGO 2.0 sensitivity to coherent GWs



Optimal Scanning



Integration time:

$$t_{\text{int}} \sim t_e \min\left(\frac{\omega_1}{Q_1\,\omega_g}\,,\,1\right)$$

Thermal:

$$Q_1 \sim Q_{\text{int}}(\omega_1/T)$$

Vibrations:

$$Q_1 \sim Q_1^{\text{min}}$$