

Recent developments for the detection of gravitational waves in haloscopes

Ultra-high frequency gravitational waves: where to next?

Geneva, Switzerland

December 6, 2023



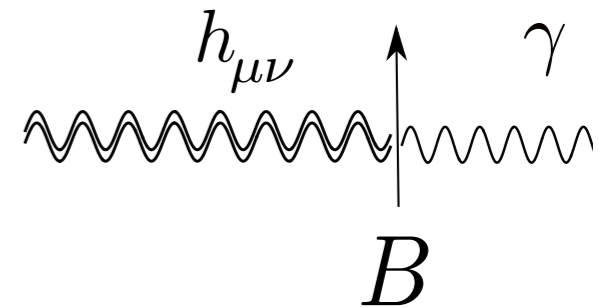
Camilo García Cely

Based on PRL 129, 041101 and hep-ph/2306.03125
In collaboration with Valerie Domcke, Sung Mook Lee and Nicholas L. Rodd.

Recent developments for the detection of gravitational waves in haloscopes

Outline

- Motivation
- Haloscopes based on lumped-element detectors
- Impact of the geometry and selection rules
- Novel effects
- Conclusions



Motivation

Revisiting Gertsenhstein's ideas

SOVIET PHYSICS JETP

VOLUME 14, NUMBER 1

JANUARY, 1962

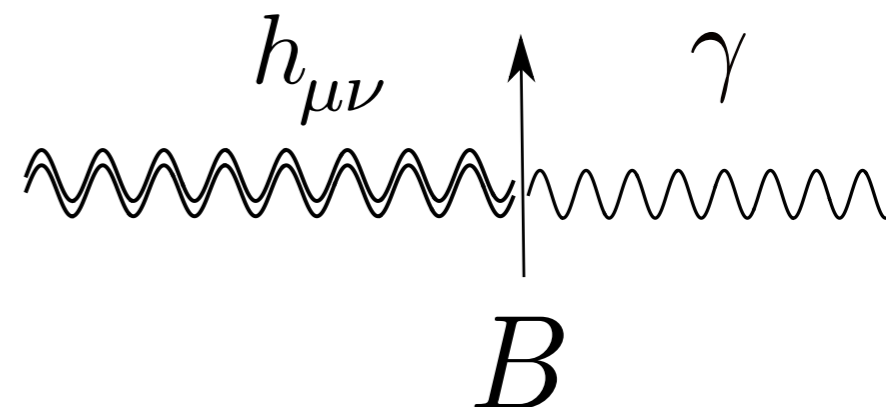
WAVE RESONANCE OF LIGHT AND GRAVITATIONAL WAVES

M. E. GERTSENSHTEĪN

Submitted to JETP editor July 29, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **41**, 113-114 (July, 1961)

The energy of gravitational waves excited during the propagation of light in a constant magnetic or electric field is estimated.



SOVIET PHYSICS JETP

VOLUME 16, NUMBER 2

FEBRUARY, 1963

ON THE DETECTION OF LOW FREQUENCY GRAVITATIONAL WAVES

M. E. GERTSENSHTEĪN and V. I. PUSTOVOĪT

Submitted to JETP editor March 3, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **43**, 605-607 (August, 1962)

It is shown that the sensitivity of the electromechanical experiments for detecting gravitational waves by means of piezocrystals is ten orders of magnitude worse than that estimated by Weber.^[1] In the low frequency range it should be possible to detect gravitational waves by the shift of the bands in an optical interferometer. The sensitivity of this method is investigated.

Terrestrial
interferometers



The (inverse) Gertsenhstein Effect

- The conversion of gravitational waves into electromagnetic waves is a classical process. Its rate does not involve \hbar

$$P \sim GB^2L^2$$

- Cosmological conversion

Potential of Radio Telescopes as High-Frequency Gravitational Wave Detectors

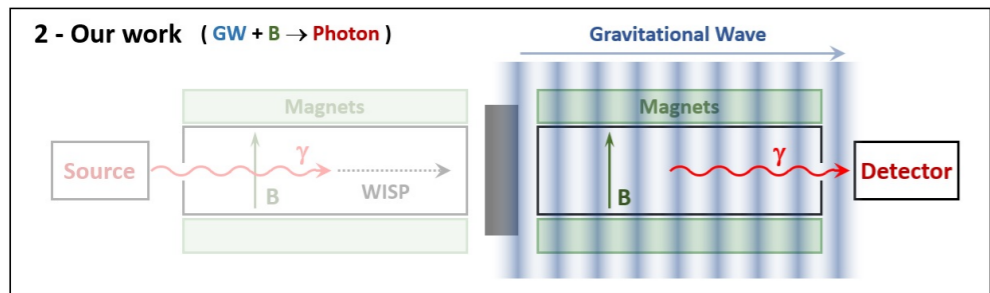
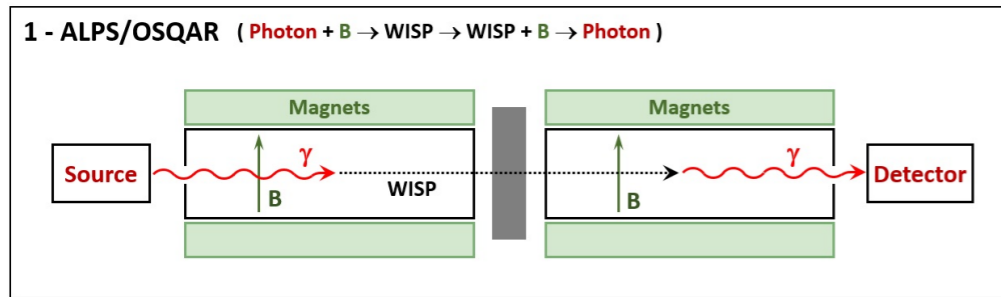
Valerie Domcke and Camilo Garcia-Cely
Phys. Rev. Lett. **126**, 021104 – Published 14 January 2021



- The process is strictly analogous to axion dark matter conversion.

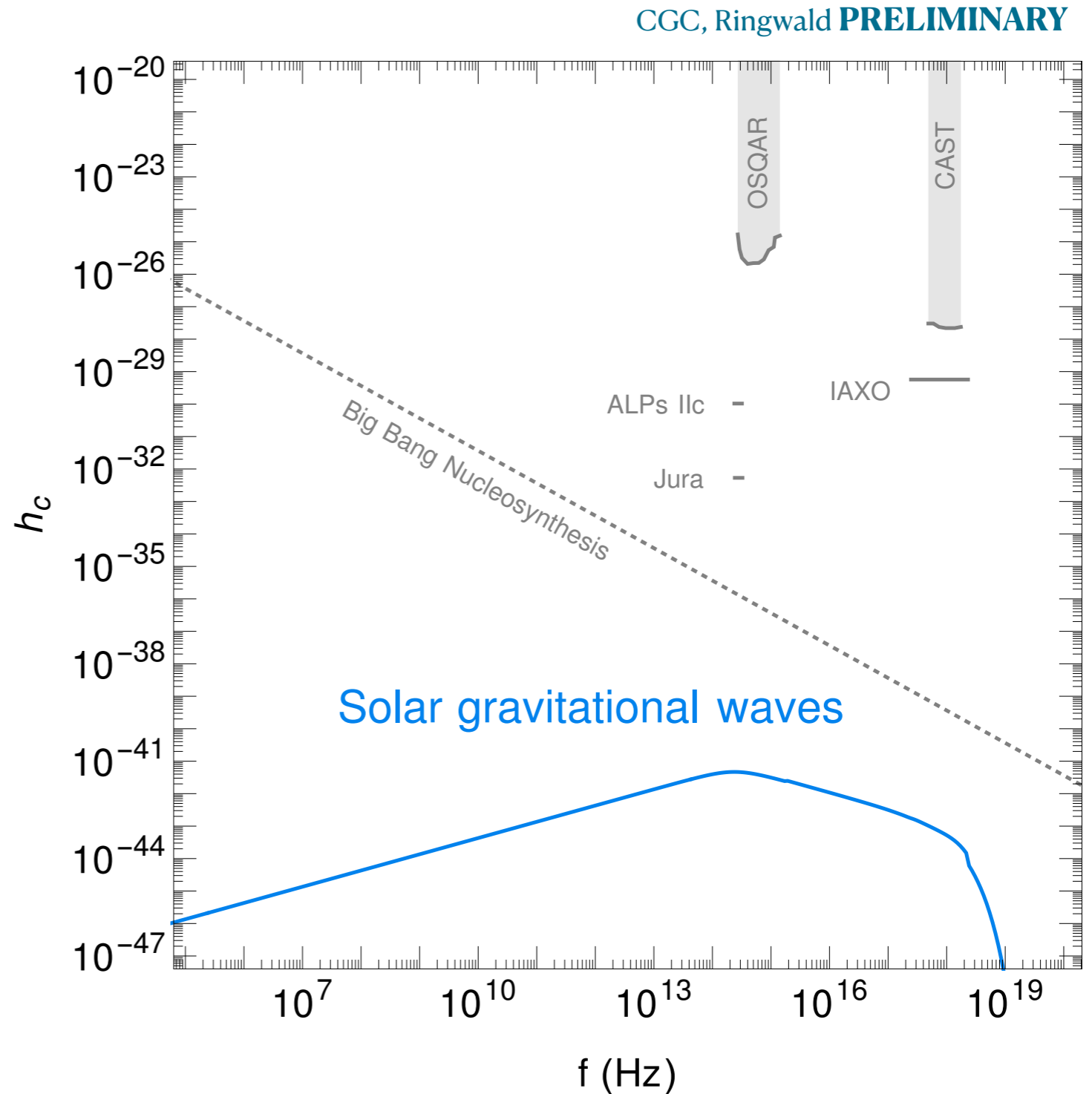
Raffelt, Stodolski'89

The (inverse) Gertsenhstein Effect



Ejlli et al, 2019

The solar spectrum is similar to that of sub-keV spin-2 particles



How does it work?

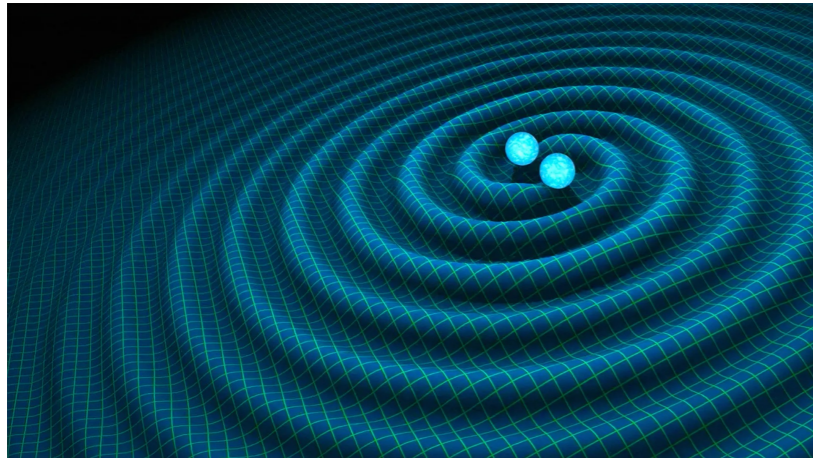
Axions act as a source term to Maxwell's equations, effectively inducing an electromagnetic current. [Sikivie, 1983](#)

$$j^0 = -g_{a\gamma\gamma} \nabla a \cdot \mathbf{B} \quad \mathbf{j} = g_{a\gamma\gamma} (\nabla a \times \mathbf{E} + \partial_t a \mathbf{B})$$

How does it work?

Gravitational waves act as a source term to Maxwell's equations, **effectively inducing an electromagnetic current.**

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad |h_{\mu\nu}| \ll 1$$

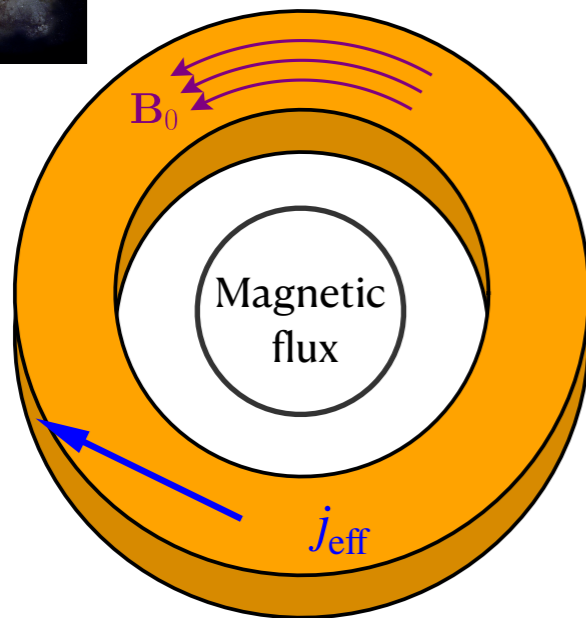


$$j_{\text{eff}}^{\mu} = \partial_{\nu} \left(-\frac{1}{2} h F^{\mu\nu} + F^{\mu\alpha} h^{\nu}_{\alpha} - F^{\nu\alpha} h^{\mu}_{\alpha} \right)$$

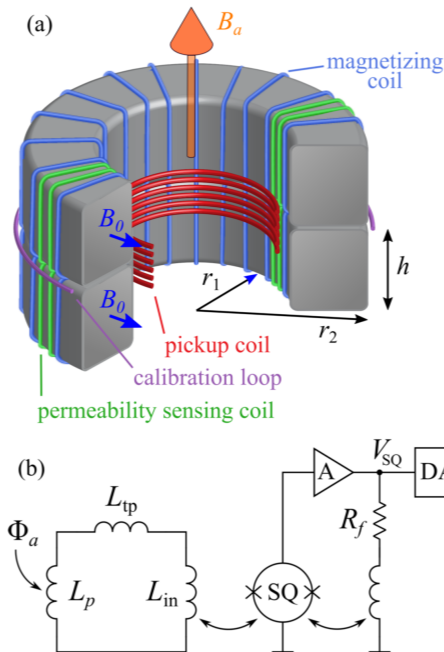
Discussion yesterday

Haloscopes based on lumped-element detectors

Haloscopes based on lumped-element detectors



$$\nabla \times \mathbf{B} - \partial_t \mathbf{E} = \underbrace{g_{a\gamma\gamma} \partial_t a \mathbf{B}_0}_{j_{\text{eff}}}$$



(c) SHAFT



PRL 117, 141801 (2016)

PHYSICAL REVIEW LETTERS

week ending
30 SEPTEMBER 2016

Broadband and Resonant Approaches to Axion Dark Matter Detection

Yonatan Kahn,^{1,*} Benjamin R. Safdi,^{2,†} and Jesse Thaler^{2,‡}

¹Department of Physics, Princeton University, Princeton, New Jersey 08544, USA

²Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

(Received 3 March 2016; published 30 September 2016)

physics

<https://doi.org/>

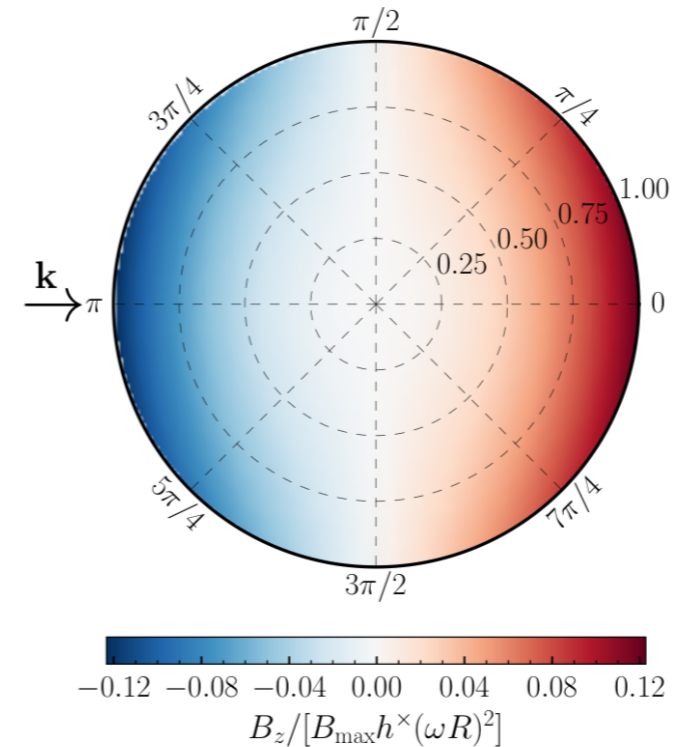
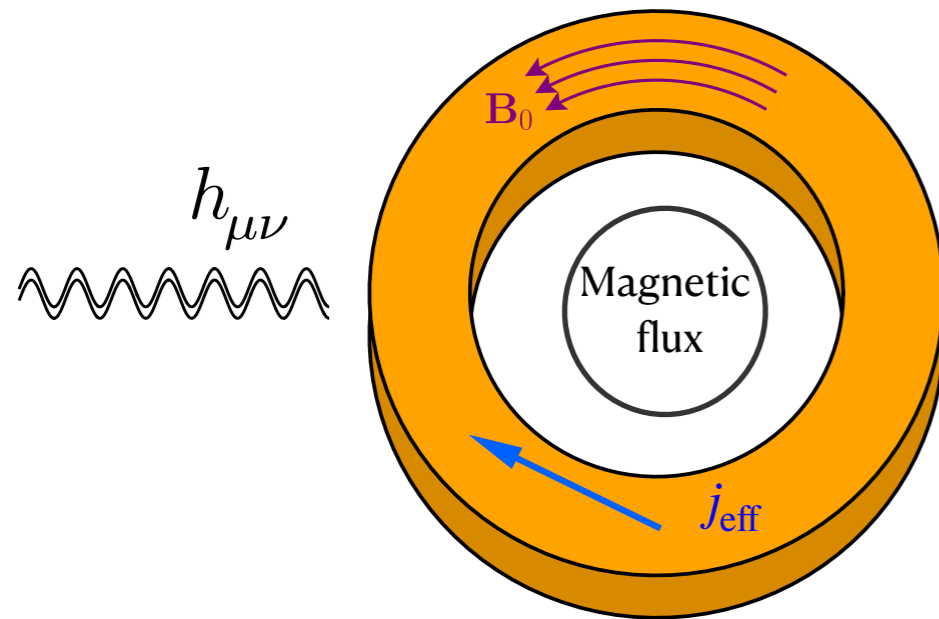
Search for axion-like dark matter with ferromagnets

Alexander V. Gramolin¹, Deniz Aybas^{1,2}, Dorian Johnson¹, Janos Adam¹ and Alexander O. Sushkov^{1,2,3}

The electromagnetic fields produced by the axion drive a current through a pickup coil

Haloscopes based on lumped-element detectors

Valerie Domcke, Camilo Garcia-Cely, and Nicholas L. Rodd
 Phys. Rev. Lett. **129**, 041101 – Published 20 July 2022



$$\Phi \approx \frac{i e^{-i\omega t}}{16\sqrt{2}} h^{\times} \omega^3 B_{\max} \pi r^2 R a (a + 2R) s_{\theta_h}^2$$

$$\Phi_{\text{axions}} \approx e^{-i\omega t} g_{a\gamma\gamma} \sqrt{2\rho_{\text{DM}}} B_{\max} \pi r^2 R$$

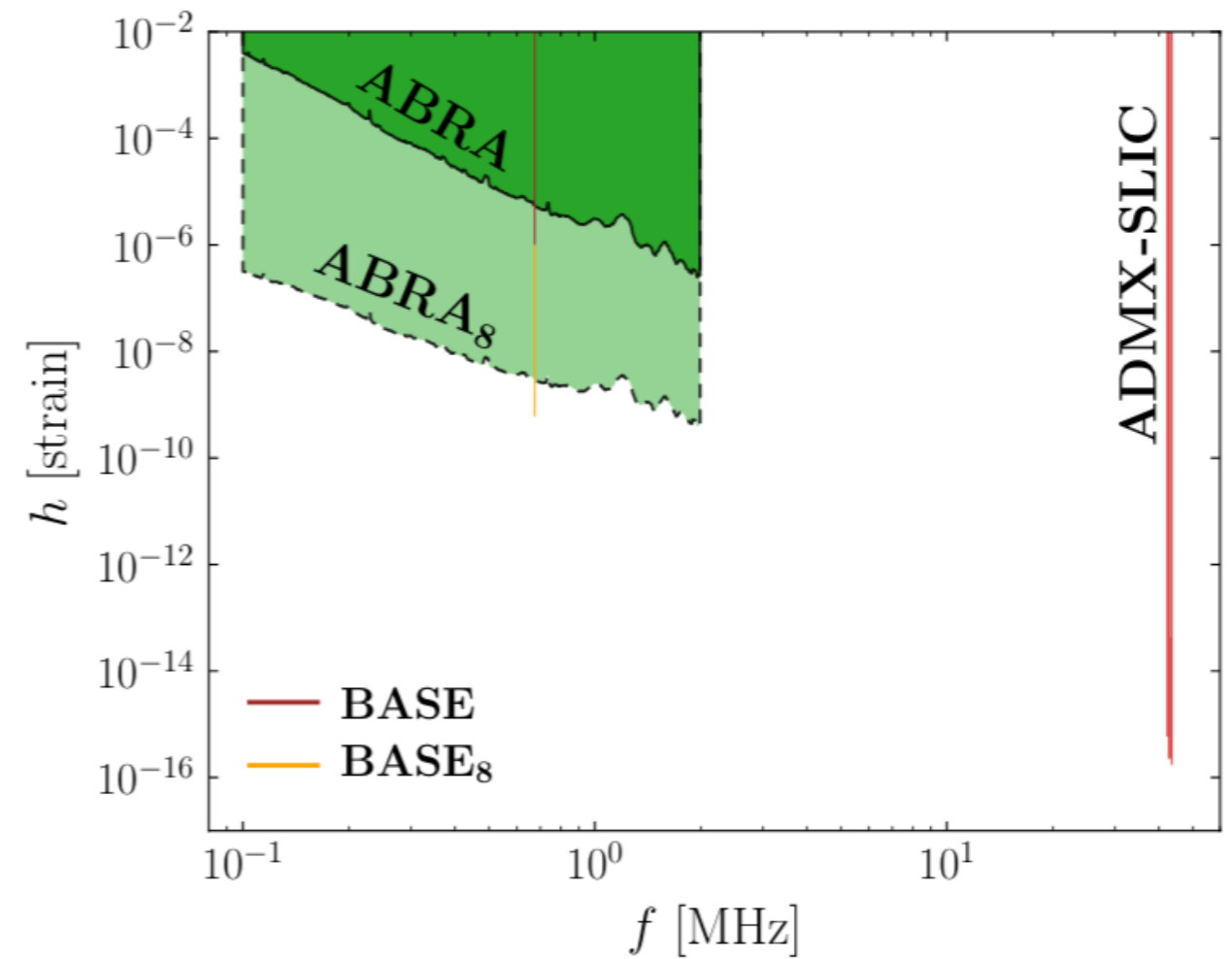
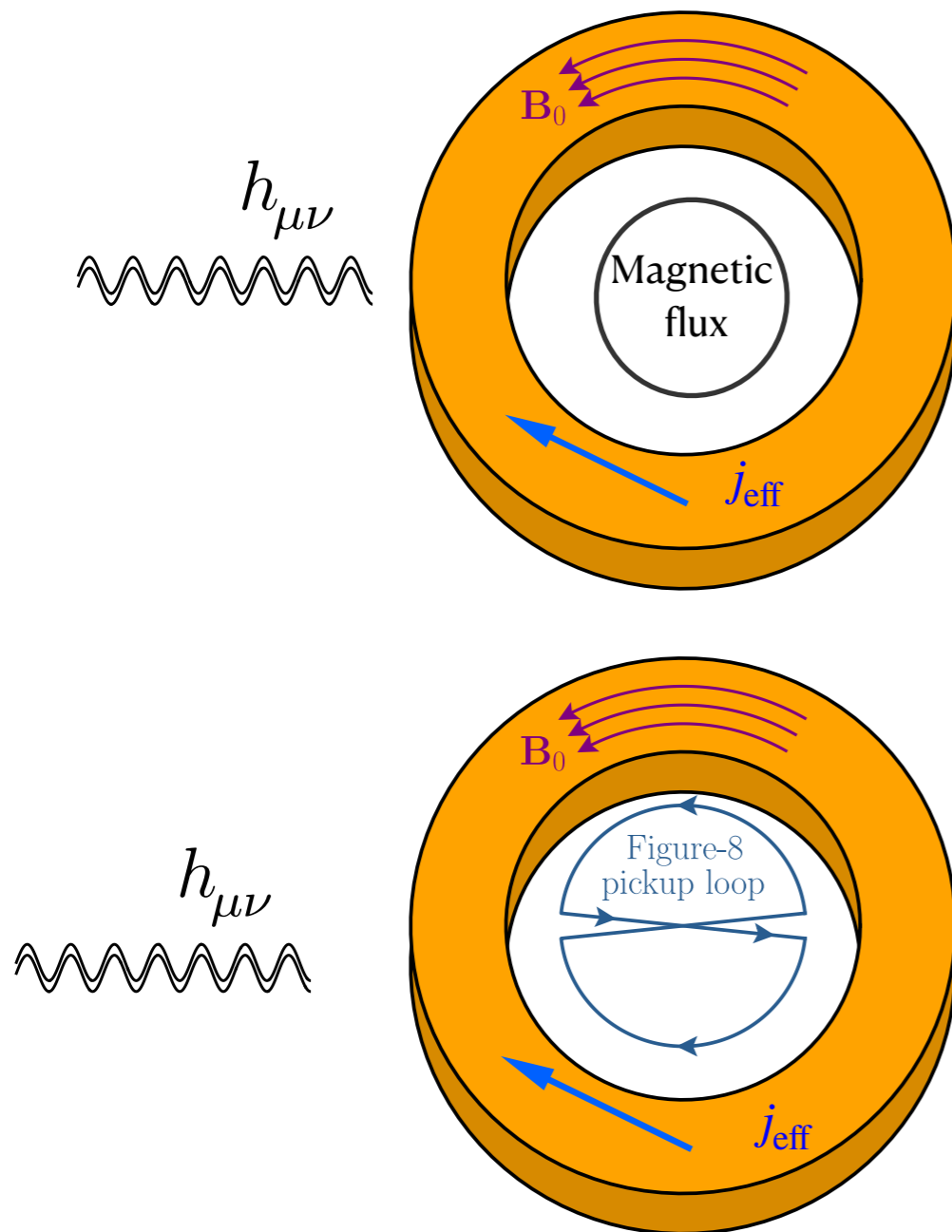
Only one polarization

Suppression at small frequencies

The sensitivity scaling with the volume is faster than for axions

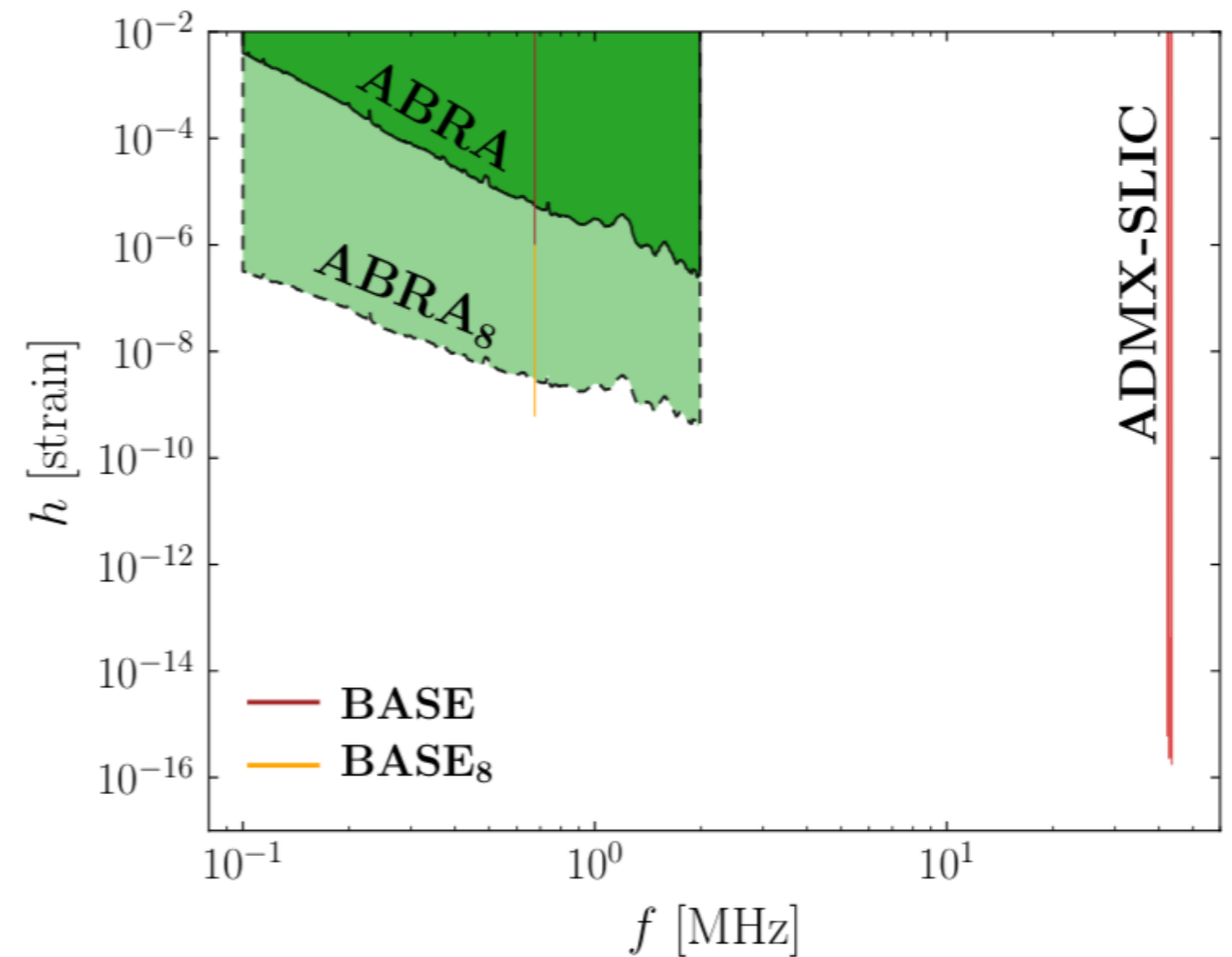
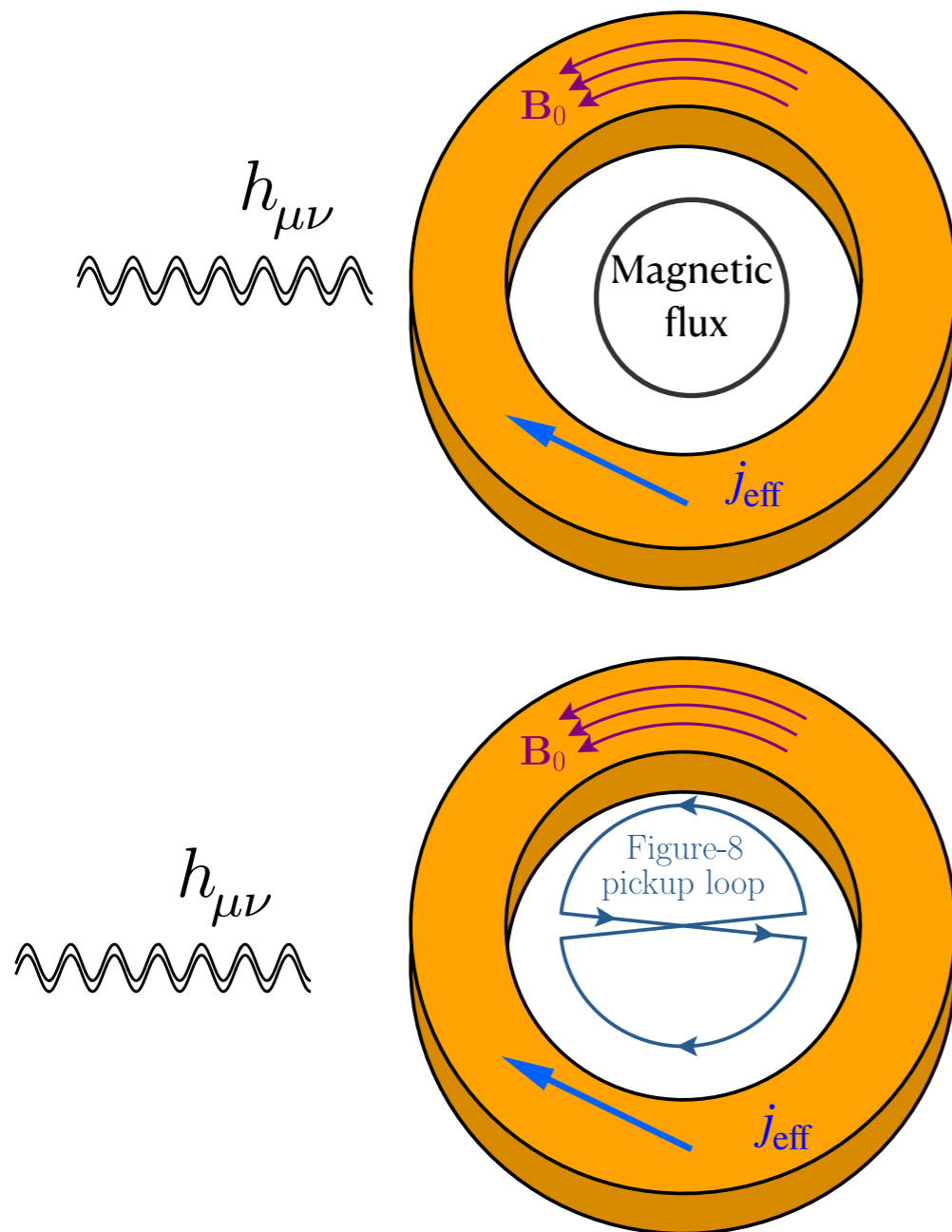
Toroidal magnetic fields

Valerie Domcke, Camilo Garcia-Cely, and Nicholas L. Rodd
 Phys. Rev. Lett. **129**, 041101 – Published 20 July 2022



Toroidal magnetic fields

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 Phys. Rev. Lett. **129**, 041101 – Published 20 July 2022



Talk by Kaliroe Pappas

Solenoidal configurations

Domcke, CGC, Lee, Rodd, 2023

ADMX SLIC: Results from a Superconducting LC Circuit Investigating Cold Axions

N. Crisosto, P. Sikivie, N. S. Sullivan, D. B. Tanner, J. Yang, and G. Rybka
Phys. Rev. Lett. **124**, 241101 – Published 17 June 2020

BASE

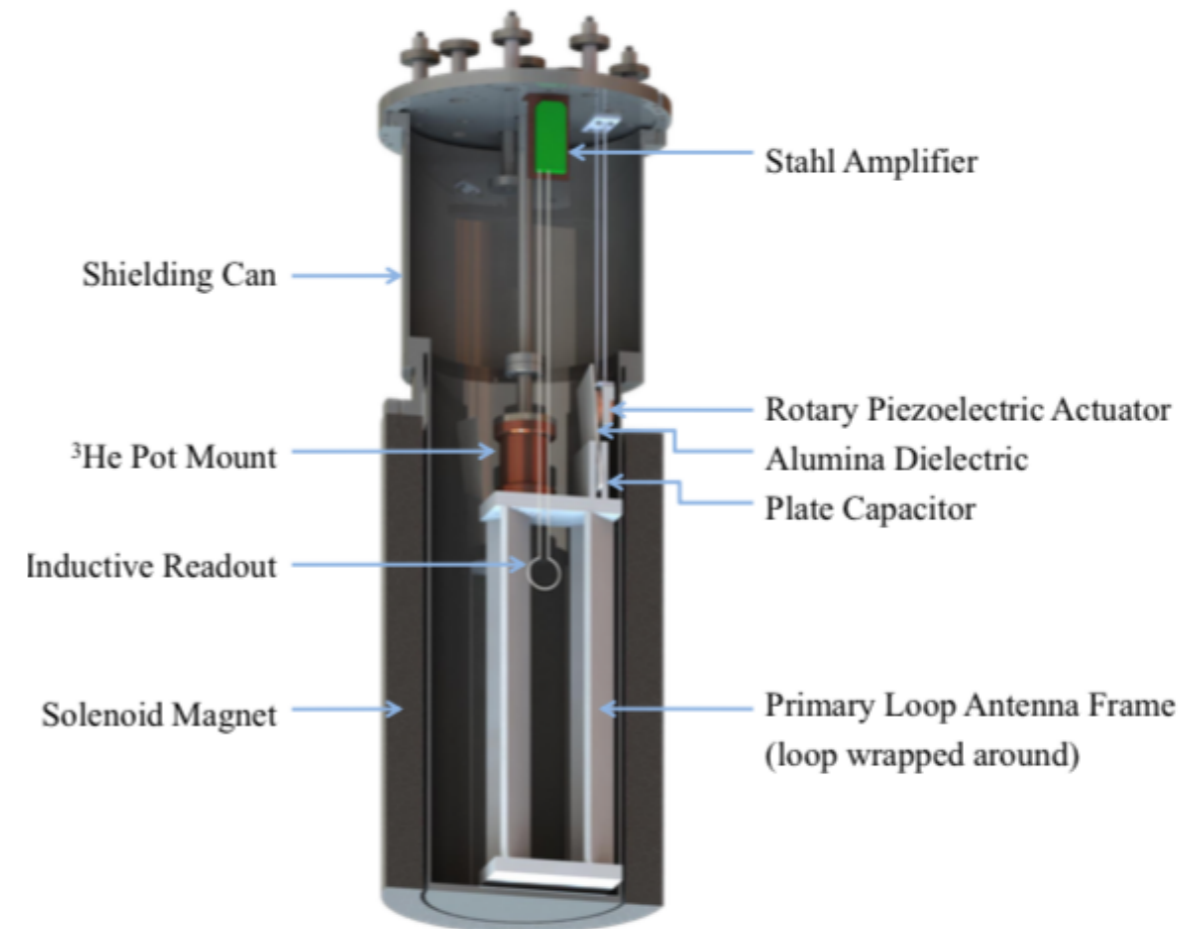
Constraints on the Coupling between Axionlike Dark Matter and Photons Using an Antiproton Superconducting Tuned Detection Circuit in a Cryogenic Penning Trap

Jack A. Devlin, Matthias J. Borchert, Stefan Erlewein, Markus Fleck, James A. Harrington, Barbara Latacz, Jan Warncke, Elise Wursten, Matthew A. Bohman, Andreas H. Mooser, Christian Smorra, Markus Wiesinger, Christian Will, Klaus Blaum, Yasuyuki Matsuda, Christian Ospelkaus, Wolfgang Quint, Jochen Walz, Yasunori Yamazaki, and Stefan Ulmer
Phys. Rev. Lett. **126**, 041301 – Published 25 January 2021

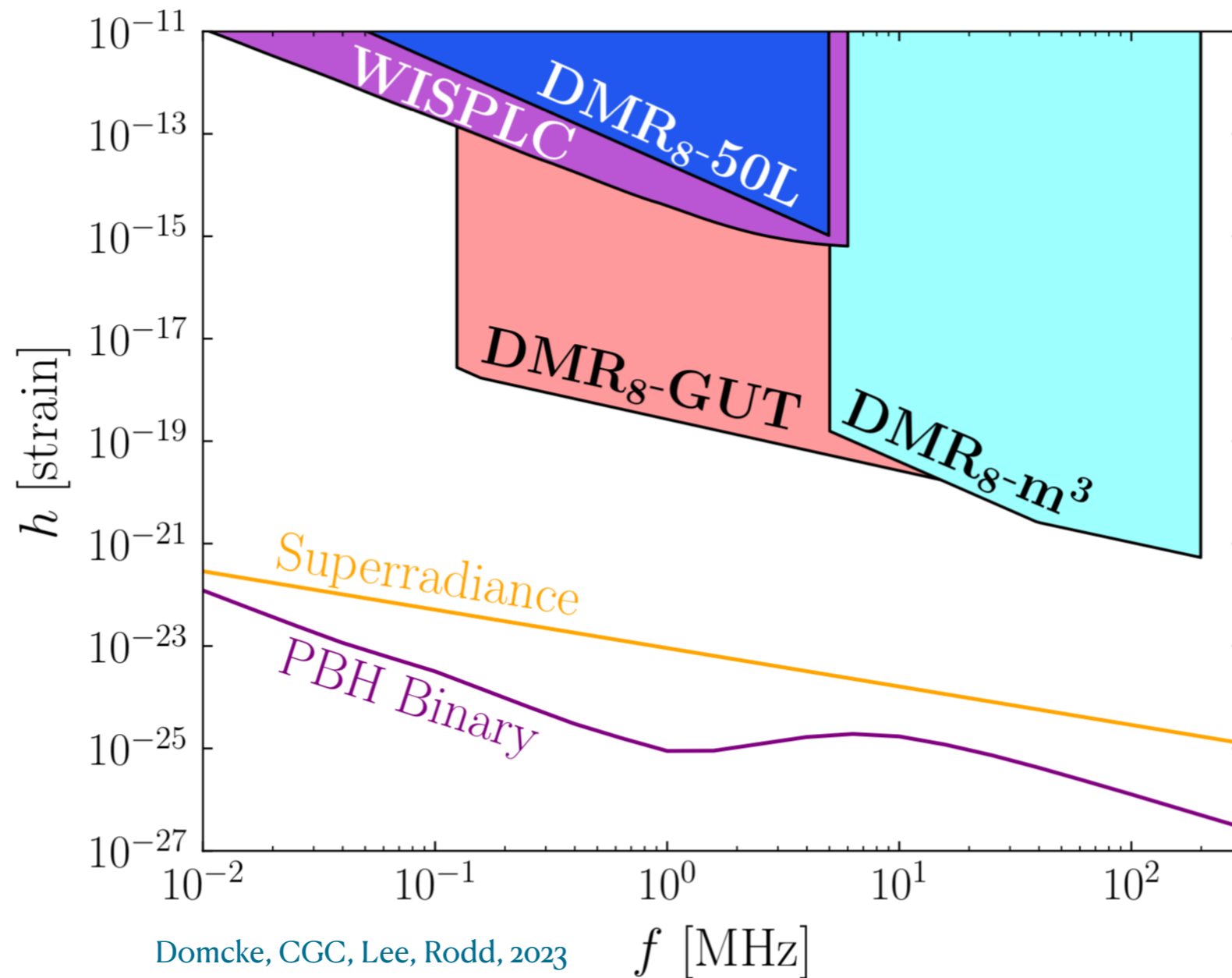
WISPLC

Search for dark matter with an *LC* circuit

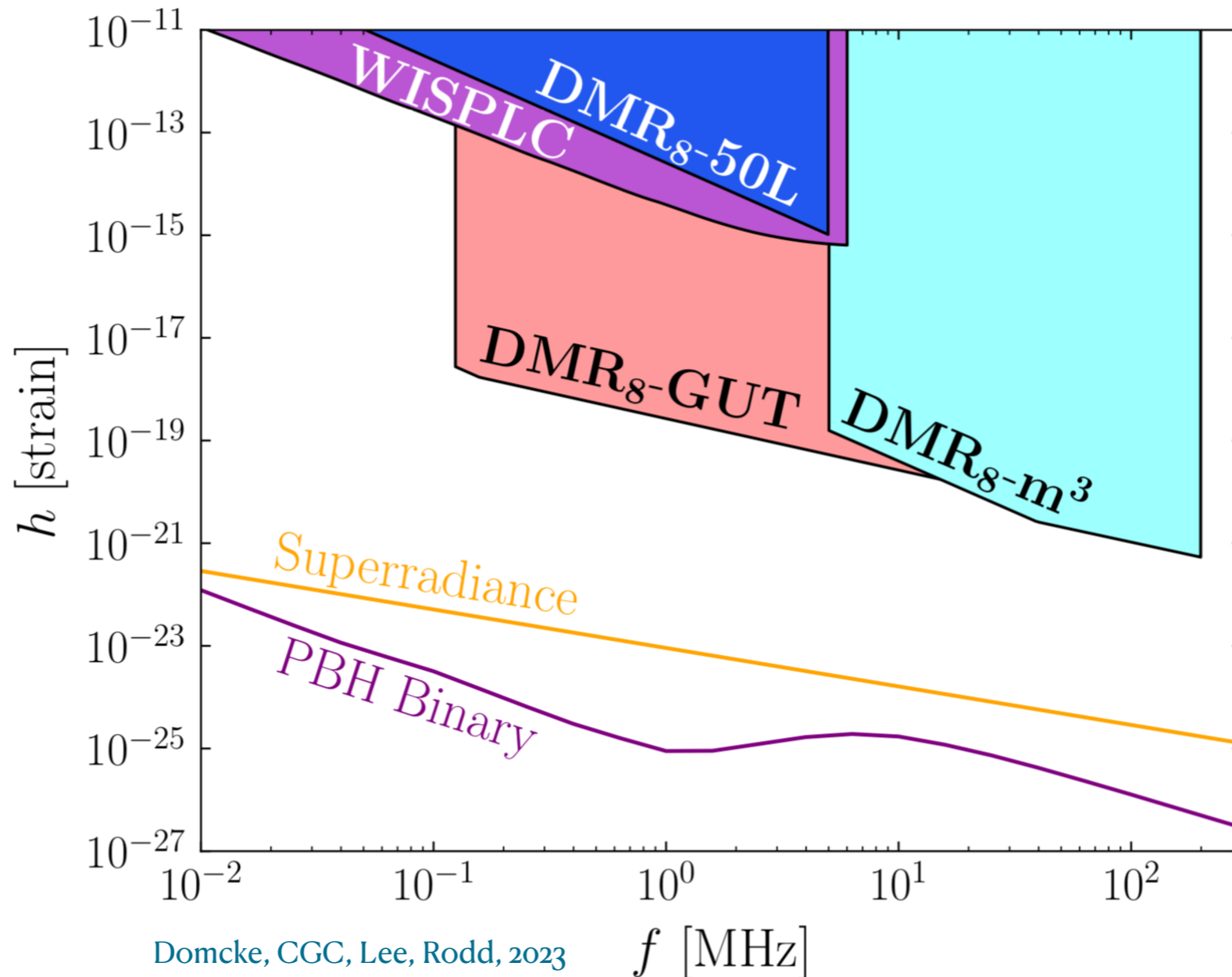
Zhongyue Zhang (张钟月), Dieter Horns, and Oindrila Ghosh
Phys. Rev. D **106**, 023003 – Published 5 July 2022



Haloscopes based on lumped-element detectors



How should we compare signals?



We propose a **coherence ratio** to recast axion searches taking into account the different time scales involved in the signals and detectors.

$$\Phi_h(h^+, h^\times; \phi_h, \theta_h) = \mathcal{R}_c \Phi_a(g_{a\gamma\gamma}),$$

Discussion on Thursday

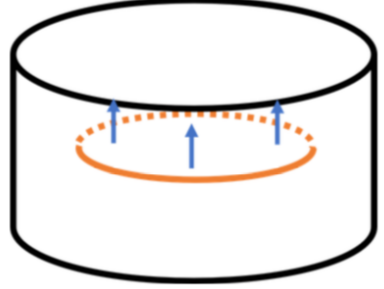
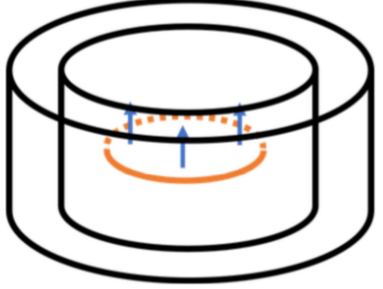
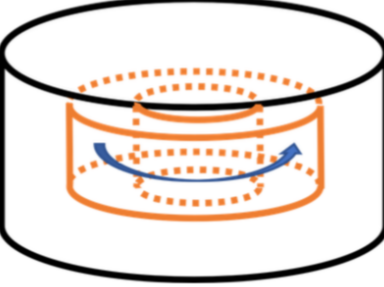
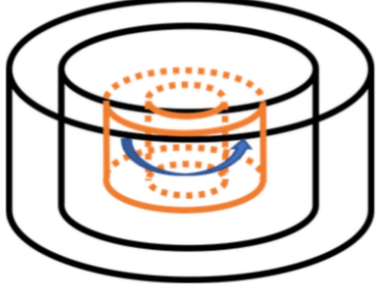
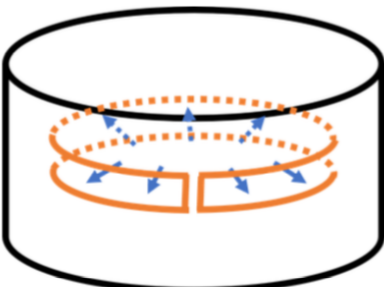
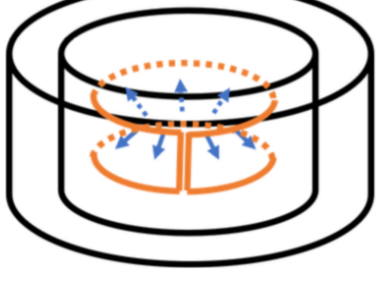
Impact of the geometry and selection rules

Impact of the geometry

Type of external field

Domcke, CGC, Lee, Rodd, 2023

Pickup loop orientation

	Solenoid: $\mathbf{B}_0 \propto \hat{\mathbf{e}}_z$	Toroid: $\mathbf{B}_0 \propto \hat{\mathbf{e}}_\phi$
$\hat{\mathbf{n}}' \propto \hat{\mathbf{e}}_z$	$h^+, n \text{ even} \Rightarrow \mathcal{O}[(\omega L)^2]$ $\Phi_h = \frac{e^{-i\omega t}}{48\sqrt{2}} h^+ \omega^2 B_0 s_{\theta_h}^2 \pi r^2 (11r^2 + 14R^2 + 16R^2 \ln \frac{R}{H})$ 	$h^\times, n \text{ odd} \Rightarrow \mathcal{O}[(\omega L)^3]$ $\Phi_h = \frac{ie^{-i\omega t}}{48\sqrt{2}} h^\times \omega^3 B_{\max} \pi r^2 a R (a + 2R) s_{\theta_h}^2$ 
$\hat{\mathbf{n}}' \propto \hat{\mathbf{e}}_\phi$	$h^\times, n \text{ odd} \Rightarrow \mathcal{O}[(\omega L)^3]$ $\Phi_h = \frac{ie^{-i\omega t}}{96\sqrt{2}} h^\times \omega^3 B_0 \pi r^2 l (12R^2 - 5r^2) s_{\theta_h}^2$ 	$h^+, n \text{ even} \Rightarrow \mathcal{O}[(\omega L)^2]$ $\Phi_h = \frac{3e^{-i\omega t}}{4\sqrt{2}} h^+ \omega^2 B_{\max} \frac{\pi r^2 a R l (a + 2R)}{H^2} s_{\theta_h}^2$ 
$\hat{\mathbf{n}}' \propto \hat{\mathbf{e}}_\rho$	$h^+, n \text{ odd} \Rightarrow \mathcal{O}[(\omega L)^3]$ $\Phi_h = \frac{ie^{-i\omega t}}{96\sqrt{2}} h^+ B_0 \omega^3 c_{\theta_h} s_{\theta_h}^2 \times \pi r^2 l (3l^2 - 22(r^2 + 2R^2) - 36R^2 \ln \frac{R}{H})$ 	$h^\times, n \text{ even} \Rightarrow \mathcal{O}[(\omega L)^4]$ $\Phi_h = \frac{e^{-i\omega t}}{32\sqrt{2}} h^\times \omega^4 B_{\max} \pi r^2 a R l (a + 2R) c_{\theta_h} s_{\theta_h}^2$ 

Selection rules

In the proper detector frame the coordinate system closely matches the intuitive description of an Earth-based laboratory

Domcke, CGC, Lee, Rodd, 2023

$$h_{00} = \omega^2 e^{-i\omega t} f(\mathbf{k} \cdot \mathbf{r}) r_m r_n \sum_{A=+, \times} h_A e_{mn}^A(\hat{\mathbf{k}}),$$

$$h_{0i} = \frac{1}{2} \omega^2 e^{-i\omega t} [f(\mathbf{k} \cdot \mathbf{r}) - i f'(\mathbf{k} \cdot \mathbf{r})] [\hat{\mathbf{k}} \cdot \mathbf{r} r_m \delta_{ni} - r_m r_n \hat{k}_i] \sum_{A=+, \times} h_A e_{mn}^A(\hat{\mathbf{k}}),$$

$$h_{ij} = -i \omega^2 e^{-i\omega t} f'(\mathbf{k} \cdot \mathbf{r}) [|\mathbf{r}|^2 \delta_{im} \delta_{jn} + r_m r_n \delta_{ij} - r_n r_j \delta_{im} - r_m r_j \delta_{in}] \sum_{A=+, \times} h_A e_{mn}^A(\hat{\mathbf{k}}),$$

$$f(\xi) = [e^{i\xi} - 1 - i\xi] / \xi^2$$

The ω^2 dependence is unavoidable

Selection rules

Domcke, CGC, Lee, Rodd, 2023

Write down the detector response matrix for a wave coming from an arbitrary direction, and impose **cylindrical symmetry** for both external magnetic field and loop:

Selection Rule 1: For an instrument with azimuthal symmetry, $\Phi_h \propto h^+$ at $\mathcal{O}[(\omega L)^2]$

Selection Rule 2: For an instrument with azimuthal symmetry, the flux is proportional to either h^+ or h^\times , but not both. This holds to all orders in (ωL) .

Selection Rule 3: For an instrument with full cylindrical symmetry, Φ_h will contain only even or odd powers of ω .

Novel effects

Effective magnetization and polarization

$$j_{\text{eff}}^{\mu} = \left(-\nabla \cdot \mathbf{P}, \nabla \times \mathbf{M} + \partial_t \mathbf{P} \right)$$

$$\mathbf{P} = g_{\alpha\gamma} a \mathbf{B}, \quad \mathbf{M} = g_{\alpha\gamma} a \mathbf{E}$$

$$P_i = -h_{ij} E_j + \frac{1}{2} h E_i + h_{00} E_i - \epsilon_{ijk} h_{0j} B_k$$
$$M_i = -h_{ij} B_j - \frac{1}{2} h B_i + h_{jj} B_i + \epsilon_{ijk} h_{0j} E_k$$

McAllister et al, 1803.07755

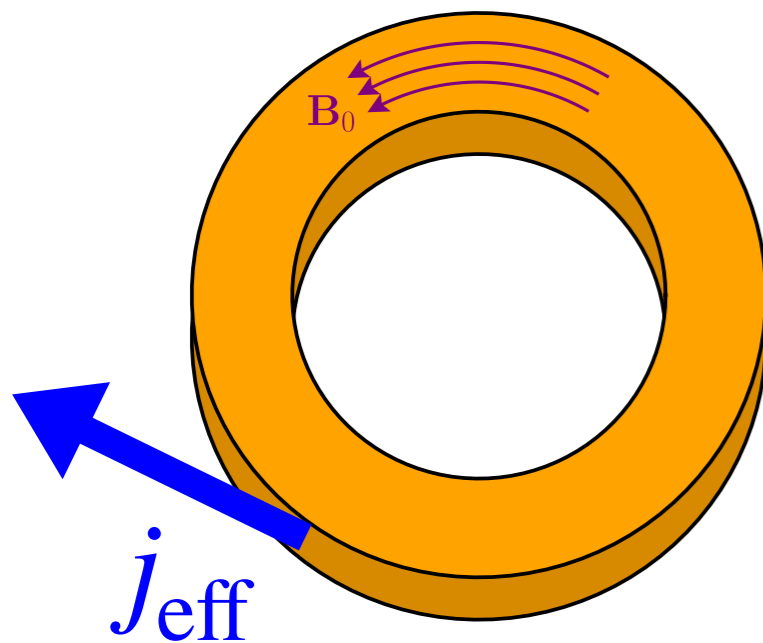
Tobar et al, 1809.01654

Ouellet et al, 1809.10709

Domcke, CGC, Rodd, 2202.00695

Non-zero *effective* surface currents

Domcke, CGC, Lee, Rodd, 2023



At the interface of two bodies with different values of the magnetisation vector M , Maxwell's equations predict a **surface current** proportional to $\mathbf{n} \times \Delta M$

**For axions this happens to vanish,
but that is not the case of GWs**

Sizeable effects. This should also be relevant for cavities

Excitation of mechanical modes

The proper detector frame closely matches the intuitive description of an Earth-based laboratory

Fermi, 1922

Manasse and Misner, 1963

Ni and Zimmermann, 1978

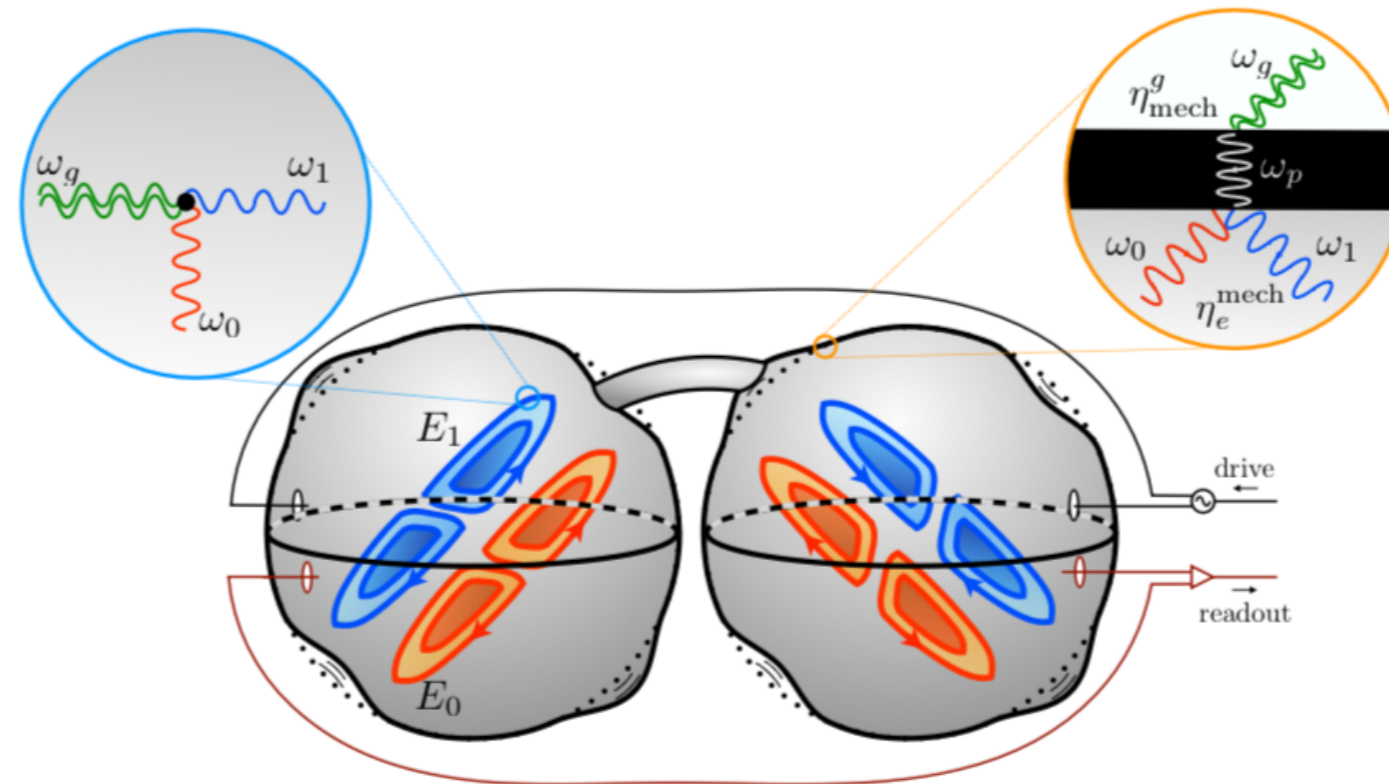
- Coordinates given by ideal rigid rulers

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{\mu\nu} dx^\mu dx^\nu \text{ for } dx^\mu = (0, dr \hat{\mathbf{r}})$$

- The gravitational wave acts as a Newtonian force.
If negligible, the static fields applied in experiments remain static in the presence of GWs.

Berlin et al 2022

Excitation of mechanical modes



- The gravitational wave acts as a Newtonian force. If not negligible, coupling of the mechanical modes can play an important role (this is certainly the case at frequencies above the first mechanical resonance)
- This can enhance the sensitivity

Berlin et al [2022](#)

Conclusions

The techniques developed for detecting **axion dark matter** could potentially be used to discover new sources of **gravitational waves**.

Different experimental proposals have coalesced on a **strain sensitivity of 10^{-22} for MHz GWs**, still orders of magnitude away from signals of the early Universe.

Lots of room for improvement because experiments are not optimized for gravitational wave searches.

Indeed, theoretical studies indicate that **selection rules** limit the detectability of gravitational waves in highly symmetric detectors.

Simple modifications of readout (such as the figure-8 pickup loop) can overcome this limitation