Atomic Precision Measurements for HFGW detection

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Ultra-High Frequency Gravitational Waves: Where To Next?
Workshop at CERN, December 6th, 2023
Quantum Sensors

i) Discrete, resolvable energy levels, typically 2-level system

|1⟩

ii) possible to initialize quantum system in known state & read it out

|0⟩

E = ħω₀

iii) quantum system can be coherently manipulated

Degen, Reinhard, Cappallaro '16
Quantum Sensors

i) Discrete, resolvable energy levels, typically 2-level system

ii) possible to initialize quantum system in known state & read it out

iii) quantum system can be coherently manipulated

iv) interaction with external field → energy shift or transition between levels

\[ E = \hbar \omega_0 \]

\[ \Gamma \]
Quantum Sensors

i) Discrete, resolvable energy levels, typically 2-level system

| 1⟩ $\rightarrow$ $E = \hbar \omega_0$

| 0⟩

ii) possible to initialize quantum system in known state & read it out

| 1⟩ $\rightarrow$ $\Gamma$

| 0⟩

iii) quantum system can be coherently manipulated

iv) interaction with external field $\rightarrow$ energy shift or transition between levels

e.g.: atoms, ions, Rydberg states, superconducting circuits, cavities, clocks, interferometers, …

& entanglement/squeezing $\rightarrow$ well suited for light DM/NP, GW, also for HEP detectors
The virtue of frequency measurements

“Never measure anything but frequency!”

Arthur Schawlow,
Nobel Prize in physics 1981
for the co-development of the laser

Use precise frequency measurements as a tool for GW searches
Atomic clocks as Quantum Sensor

\[ E_1 \quad |1\rangle \]
\[ E_0 \quad |0\rangle \]

atomic reference

Precision \(10^{-18}\) → \(10^{-20}\)
Atomic clocks as Quantum Sensor

Figure: M. Safronova

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J. Hall & T. Hänsch, Nobel Prize 2005
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Figure: M. Safronova
**Photon in gravitational field**

**Goal:** compare frequency of photon measured by S and D

Free-falling observer moving with 4-velocity $u^\mu$ measures at D

$$\omega_\gamma = -g_{\mu\nu}p^\mu u^\nu$$
Photon in gravitational field

**Goal:** compare frequency of photon measured by S and D

**Gravitational Wave:** perturbs metric

$$g_{\mu\nu} = \eta_{\mu\nu} - h_{\mu\nu}$$

$$p^\mu = (\omega_0, \omega_0, 0, 0) + \delta p^\mu$$

$$u^\mu = (1, 0, 0, 0) + \delta u^\mu ,$$

~h (GW strain)

Geodesic equation $\Rightarrow$ master formula for frequency change at O(h):

$$\frac{\omega_D^\gamma - \omega_S^\gamma}{\omega_D^\gamma} = - \frac{\omega_0}{2} \int_0^{\lambda_D} d\lambda' \partial_0 \left[ h_{00} + 2h_{10} + h_{11} \right]_{x^\mu = x^\mu_{\lambda', 0}} + \left[ \delta u^0 - \delta u^1 \right](\lambda_D) - \left[ \delta u^0 - \delta u^1 \right](\lambda_S).$$
**Photon in gravitational field**

**Goal:** compare frequency of photon measured by S and D

**Gravitational Wave:** perturbs metric

\[ g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu} \]

\[ p^\mu = (\omega_0, 0, 0, 0) + \delta p^\mu \]

\[ u^\mu = (1, 0, 0, 0) + \delta u^\mu, \quad \sim h \text{ (GW strain)} \]

Geodesic equation \( \to \) master formula for frequency change at \( O(h) \):

\[
\frac{\omega_D^\gamma - \omega_S^\gamma}{\omega_D^\gamma} = -\frac{\omega_0}{2} \int_0^{\lambda_D} d\chi' \partial_0 \left[ h_{00} + 2h_{10} + h_{11} \right] x^\mu = x_{\chi', 0}^\mu
\]

\[ + \left[ \delta u^0 - \delta u^1 \right](\lambda_D) - \left[ \delta u^0 - \delta u^1 \right](\lambda_S). \]

Varying gravitational field along photon trajectory
Photon in gravitational field

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Geodesic equation →…→ master formula for frequency change at O(h):

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A) Free-falling detectors – TT frame

- S and D in free fall (move freely at least in direction of photon propagation)
  → most convenient in transverse traceless (TT) gauge \( h^{TT}_{\mu0} = 0 \), \( \partial^i h^{TT}_{ij} = 0 \), \( \eta^{ij} h^{TT}_{ij} = 0 \)
  where observers at rest remain at rest

\[
h^{TT}_{11}(x^\mu) = h_+ s^2_\vartheta \cos [\omega_g(x^0 - c_\vartheta x^1 - s_\vartheta x^3) + \varphi_0]
\]

Plane wave

GW in x polarization do not alter photons in x, direction
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Plane wave

\[ \frac{\omega_\gamma^D - \omega_\gamma^S}{\omega_\gamma^D} = h_+ c_{\varphi/2} \left\{ \cos \varphi_0 - \cos[\omega_g L(1 - c_\varphi) + \varphi] \right\} , \]
A) Free-falling detectors – PD frame

Photon propagation

\[
\left. \frac{\omega^D_\gamma - \omega^S_\gamma}{\omega^D_\gamma} \right|_{\text{prop}} = -\frac{\omega_0}{2} \int_0^{\lambda^D} dx \partial_0 [h^{PD}_{00}] \big|_{x^\mu = (\lambda', \omega_0, \lambda' \omega_0, 0, 0)}
\]

\[
= h + c_\theta^2/2 \left\{ \cos \varphi_0 + t_\varphi^2 \cos[\omega_g L + \varphi_0] \right. \\
- \left. c_\varphi^2 \cos[\omega_g L (1 - c_\theta) + \varphi_0] \right\}
\]

\[
+ h \omega_g L \frac{s_\varphi t_\varphi}{2} \sin(\omega_g L + \varphi_0).
\]

Doppler shift

\[
\left. \frac{\omega^D_\gamma - \omega^S_\gamma}{\omega^D_\gamma} \right|_{\text{Doppler}} = \lim_{\epsilon \to 0} \frac{1}{2} \int_{-\infty}^{L} dt \left[ h^{PD}_{00,0} - h^{PD}_{00,1} \right] \big|_{x^\mu = (t, L, 0, 0)}
\]

\[
= h + c_\theta^2/2 \left\{ -\cos[\omega_g L + \varphi_0] \right. \\
+ \cos [\omega_g L (1 - c_\theta) + \varphi_0] \right\}
\]

\[
- h \omega_g L \frac{s_\varphi t_\varphi}{2} \sin(\omega_g L + \varphi_0).
\]

TT vs PD for free-falling:
Careful with initial conditions → gradually switch on GW

Sum recovers result of free-falling detector in TT frame

Note cancellation of terms $\propto \omega_g L$
B) Rigid ruler – Proper Detector frame

- **Proper-detector (PD) frame**: distances an observer with a rigid ruler would measure; detector moves due to GW force

\[
\frac{\omega_D^D - \omega_D^S}{\omega_D^D} = \frac{h_+}{2} \left\{ \cos \varphi_0 - \left( \frac{\omega_g L}{2} \right) \sin (\omega_g L + \varphi_0) \right. \\
+ \left. \left( \frac{1}{2} \omega_g^2 L^2 - 1 \right) \cos (\omega_g L + \varphi_0) \right\}
\]

Enhanced sensitivity for large \( \omega_g L \gg 1 \)?
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$$+ \left( \frac{1}{2} \omega_g^2 L^2 - 1 \right) \cos(\omega_g L + \varphi_0)$$

Enhanced sensitivity for large $\omega_g L \gg 1$?

- no material is perfectly rigid at high frequencies!
- generic implication for detector design:
  this equation is not directly applicable for $\omega_g L \gg v_s$
Detection: 1) Sidebands

- Naive expectation was: GW changes photon frequency
- Instead: tiny sidebands
  - separated from carrier (original photon frequency) by GW frequency $\omega_g$
  - suppressed by the GW amplitude $h^2 \sim 10^{-40}$

Advantage for high-frequency GWs

Still: tails from intense carrier line can hide the sidebands

How to make the sidebands detectable?
Detection: 1) Sidebands

Idea: design filter to cut out the intense carrier line → direct detection of sidebands → need to detect photons above background
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  $\Rightarrow$ modulate carrier line? (further investigation)
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- Interference term “only” suppressed by $h$, but overwhelmed by background from carrier → modulate carrier line? (further investigation)
- **Narrow filter** of bandwidth $\Delta \lambda$ and suppression of carrier $\alpha_T \ll 1$
  - Optical cavity tuned to sideband
    - PTB: finesse 500,000, $L=30\text{cm}$, $\Delta \lambda = \text{kHz}$
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  - Fiber Bragg Grating
Detection: 2) Optical clocks

Original setup:

Phase modulation is averaged out during GW period

Only sidebands, no net frequency shift of \( \gamma \)
Detection: 2) Optical rectifier

**Idea:** block the photon propagation during half of the GW period

Shift does not average out

Photon net frequency shift detectable by Ramsey spectroscopy
Rectifier: small $\omega L$

Pass if $\sin \varphi_0 = \sin \omega_g t > 0$

$$\langle \delta \omega_\gamma \rangle = \frac{h \omega_g L}{(2\pi)} \quad (\theta = \pi/2)$$

Orange sideband: effect of shutter

Not only sideband, but also frequency shift of photon carrier line
Rectifier: large $\omega L$

Pass if \[ \sin[\varphi_0 + \pi/2] > 0 \]

\[ \langle \delta \omega_\gamma \rangle = \hbar/\pi \]

optimal rectifier, $\omega L = (2n + 1)\pi \gg 1$

- $\omega_g = 0.09$
- $\omega_g L = 21\pi$
- $\omega_\gamma = 1$
- $h = 0.1$

FFT [a.u.]

angular frequency $\omega$ [A.U.]
Sensitivity

Assumptions in the limits:

\[ \tau = 1 \text{ s}, \quad L = 1 \text{ m}, \quad \omega_g^S/2\pi = 2 \times 10^{14} \text{ Hz} \]

Integration time optical

\[ P = \text{mW} \quad \text{Laser power: need high #photons} \]

- \( \alpha_T, \alpha_{th} \) = \{(10^{-10}, 10^{-15}), \quad \text{conservative} \}
- \{(10^{-15}, 10^{-17}), \quad \text{realistic} \}
- \{(10^{-20}, 10^{-19}), \quad \text{optimistic} \}
Impact of the integration time $\tau$
(l) Atomic spectroscopy

Does relative frequency precision of $10^{-20}$ translate into $h \sim 10^{-20}$?

- Record precision requires long integration times → waveform of GW cannot be resolved

- Typical atomic size $\sim 0.1\text{nm} << \text{mm} \sim \text{GW of } O(100 \text{ GHz})$ → GW signal suppressed by $(\omega_g L)$

- Try macroscopic systems
  Rydberg states: highly excited electron (large $n$) → $O(\mu m)$ still insufficient, competing with large Coulomb interaction
How not to build a HFGW detector (II)

(II) Muon (g-2)
- among the most precisely measured properties \(\rightarrow\) sensitive to GW?

Precession frequency \(\omega_s\)
cyclotron frequency \(\omega_c \sim B/m_\mu\)
difference \(\omega_a = \omega_s - \omega_c \sim 10^{-2}\omega_c\)

Leading effect: cyclotron frequency \(\mathcal{O}(hB/m_\mu)\) \(\rightarrow\) \(h \sim 10^{-8}\)

No obvious suppression, but numbers not sufficient to probe GWs.
Conclusions

- HFGW detection via optical frequency modulation
- Fundamental limitation: perfectly rigid detector impossible
- Sidebands, demodulation, atomic clocks investigated
- Challenging, but promising over broad MHz – GHz range

Thank you!
APPENDIX
No rigid ruler in limit of large $\omega L$

Model material by oscillator chain:
displacement $x^1$ from rest position $\xi$

\[
\ddot{\xi} - \frac{\omega_0^2 L^2}{\pi^2} \xi'' + \gamma \dot{\xi} = \frac{1}{2} x^1 \dot{h}_{11}
\]

Negligible for high GW frequency $\omega_g \gg \omega_0, \gamma$

$\rightarrow$ free-falling test mass in PD $\omega_0^2 / \omega_g^2$

up to corrections of

$\rightarrow$ not “rigid” behavior