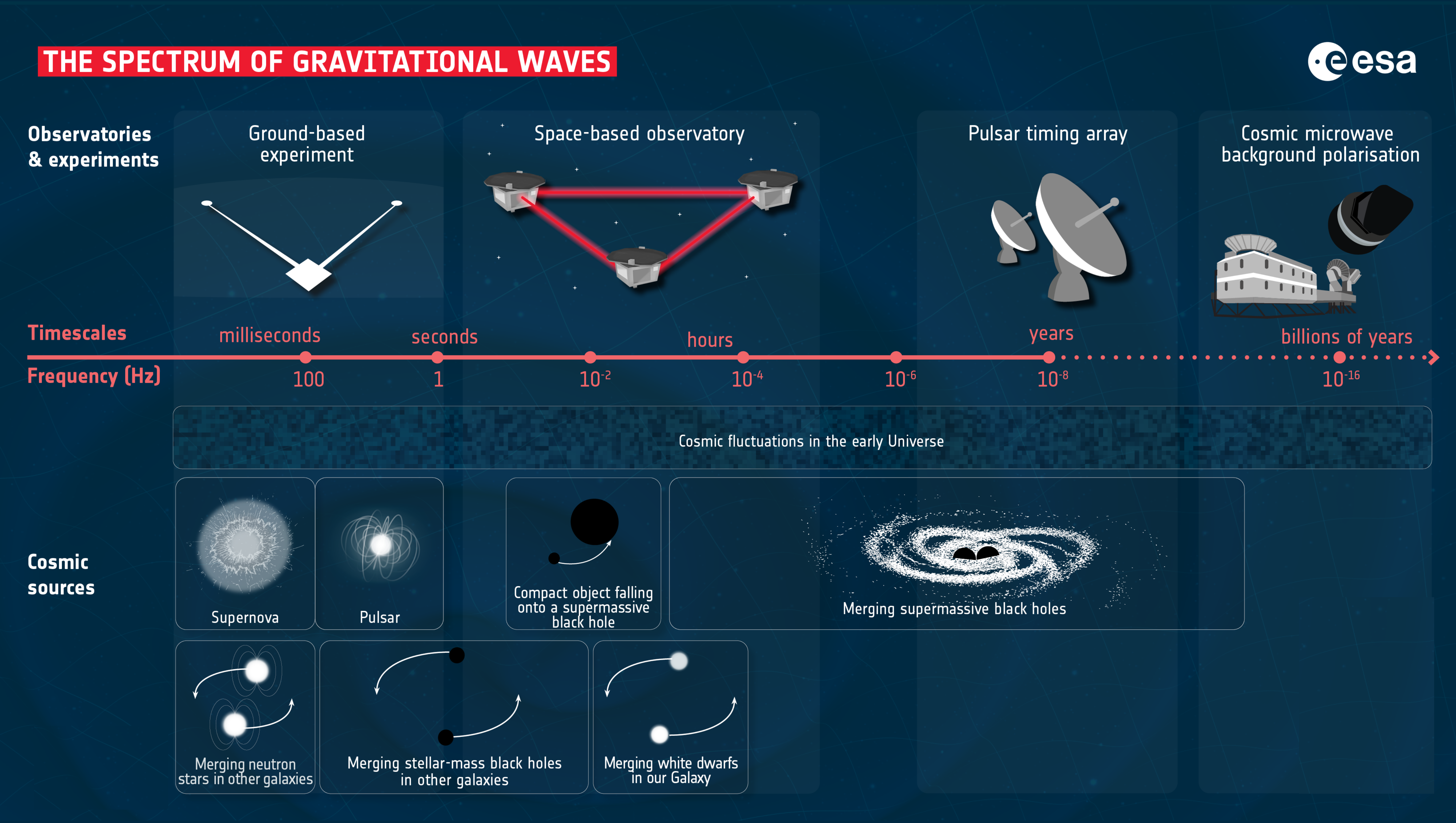
Gabriele Franciolini

Primordial black holes:

a dark matter candidate in the UHF-GW window

6-12-2023 - UHF-GW workshop - CERN TH Colloquium

The GW spectrum



THE SPECTRUM OF GRAVITATIONAL WAVES

esa

Observatories & experiments

Ground-based experiment

Space-based observatory

Pulsar timing array

Cosmic microwave background polarisation

Timescales

milliseconds

seconds

hours

years

billions of years

Frequency (Hz)

100

1

10^{-2}

10^{-4}

10^{-6}

10^{-8}

10^{-16}

Cosmic fluctuations in the early Universe

Cosmic sources

Supernova

Pulsar

Compact object falling onto a supermassive black hole

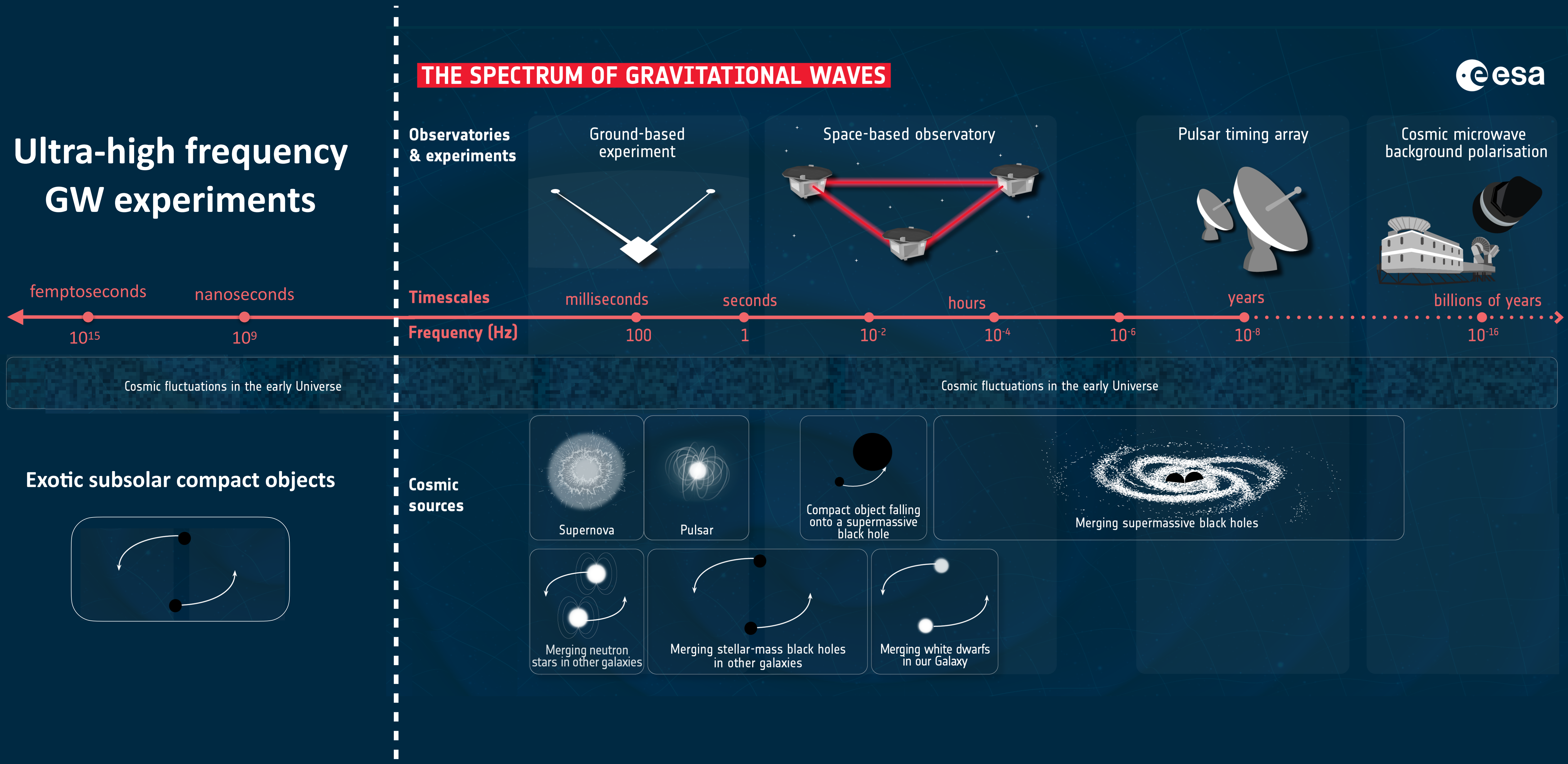
Merging supermassive black holes

Merging neutron stars in other galaxies

Merging stellar-mass black holes in other galaxies

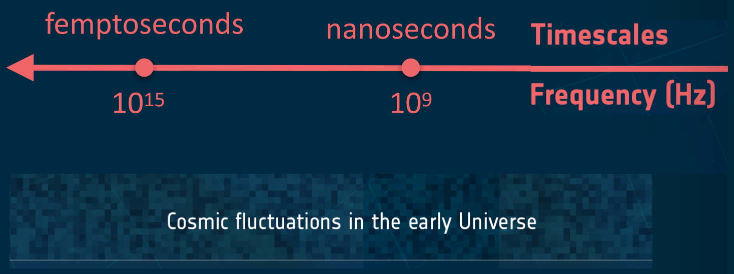
Merging white dwarfs in our Galaxy

The GW spectrum



Which kind of sources?

Ultra-high frequency GW experiments



$$\text{Hubble radius} \sim H^{-1}$$

Cosmological Backgrounds

Typical scale: Hubble crossing

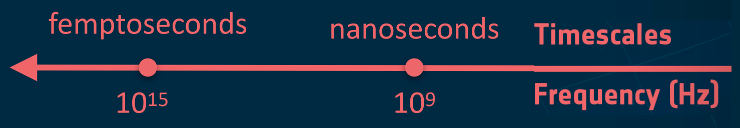
$$f = k/2\pi = aH/2\pi$$

frequency - temperature of the universe

$$f \sim 3 \cdot \mathrm{kHz} \left(\frac{T}{10^{11}\mathrm{GeV}} \right)$$

Which kind of sources?

Ultra-high frequency GW experiments



Exotic subsolar compact objects

From compact object collisions

Typical scale: Innermost stable circular orbit

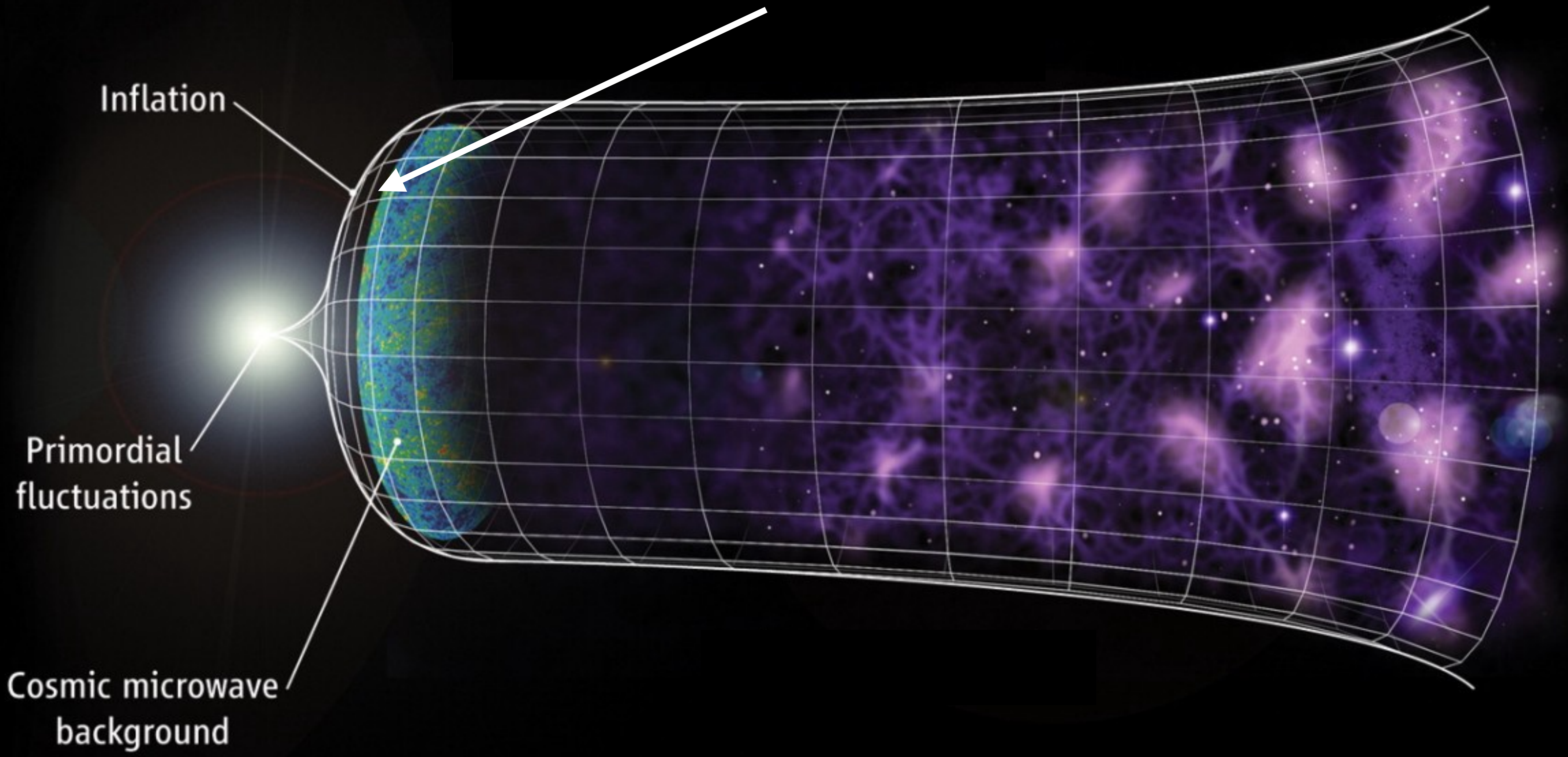
$$r_{\mathrm{ISCO}} = \frac{6Gm}{c^2}$$

frequency - binary mass

$$f_{\mathrm{ISCO}} \sim 4\mathrm{kHz}\left(\frac{M_\odot}{m}\right)$$

Compact objects below the Chandrasekhar limit

Compact objects below the Chandrasekhar limit



Primordial BHs

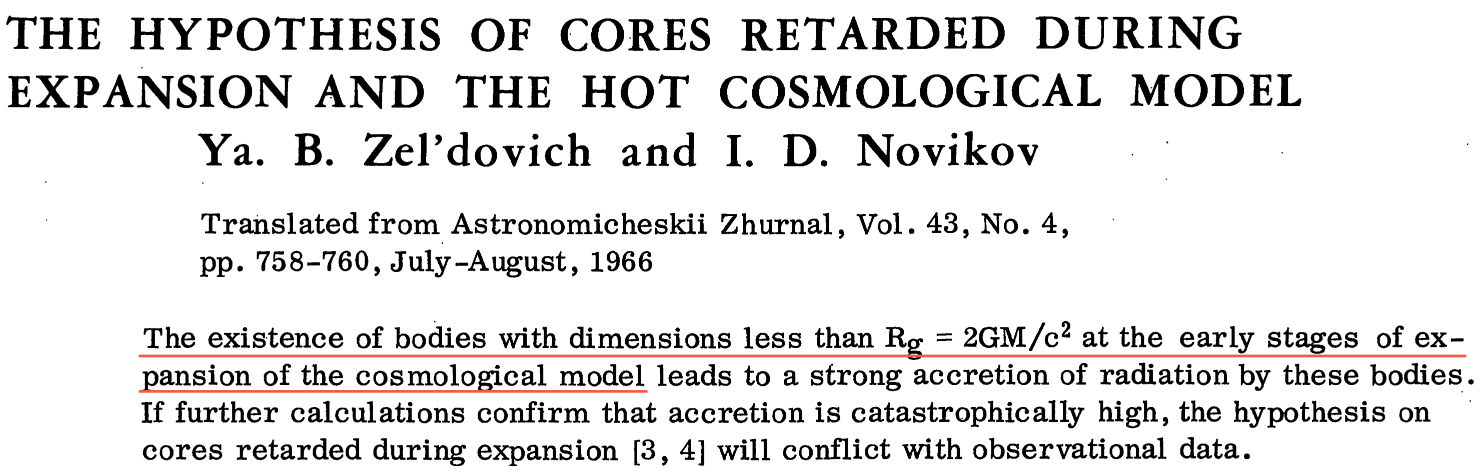
Outline

- Introduction to Primordial Black holes (PBHs)
- PBH binary formation and merger rates
- Signatures of PBH mergers in the UHF-GW window

Introduction

The “birth” of PBHs

- Zel’dovich-Novikov (1966)



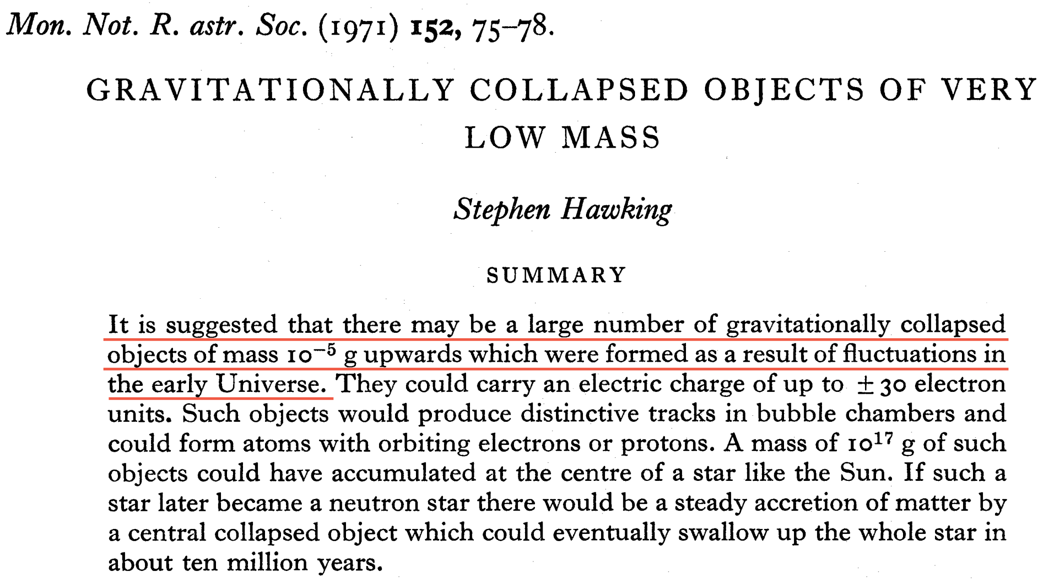
THE HYPOTHESIS OF CORES RETARDED DURING EXPANSION AND THE HOT COSMOLOGICAL MODEL

Ya. B. Zel’dovich and I. D. Novikov

Translated from Astronomicheskii Zhurnal, Vol. 43, No. 4, pp. 758-760, July-August, 1966

The existence of bodies with dimensions less than $R_g = 2GM/c^2$ at the early stages of expansion of the cosmological model leads to a strong accretion of radiation by these bodies. If further calculations confirm that accretion is catastrophically high, the hypothesis on cores retarded during expansion [3, 4] will conflict with observational data.

- Hawking (1971)



Mon. Not. R. astr. Soc. (1971) **152,** 75–78.

GRAVITATIONALLY COLLAPSED OBJECTS OF VERY LOW MASS

Stephen Hawking

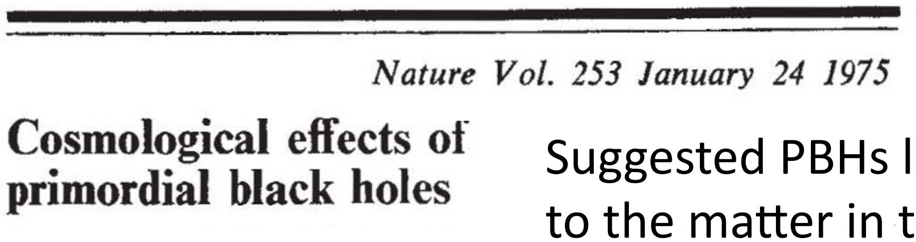
SUMMARY

It is suggested that there may be a large number of gravitationally collapsed objects of mass 10^{-5} g upwards which were formed as a result of fluctuations in the early Universe. They could carry an electric charge of up to ± 30 electron units. Such objects would produce distinctive tracks in bubble chambers and could form atoms with orbiting electrons or protons. A mass of 10^{17} g of such objects could have accumulated at the centre of a star like the Sun. If such a star later became a neutron star there would be a steady accretion of matter by a central collapsed object which could eventually swallow up the whole star in about ten million years.

- Carr, Carr-Hawking (1974)

Formalisation of the idea, computation of the mass distribution

- Chapline (1975)



Nature Vol. 253 January 24 1975

Cosmological effects of primordial black holes

Suggested PBHs large contribution to the matter in the universe

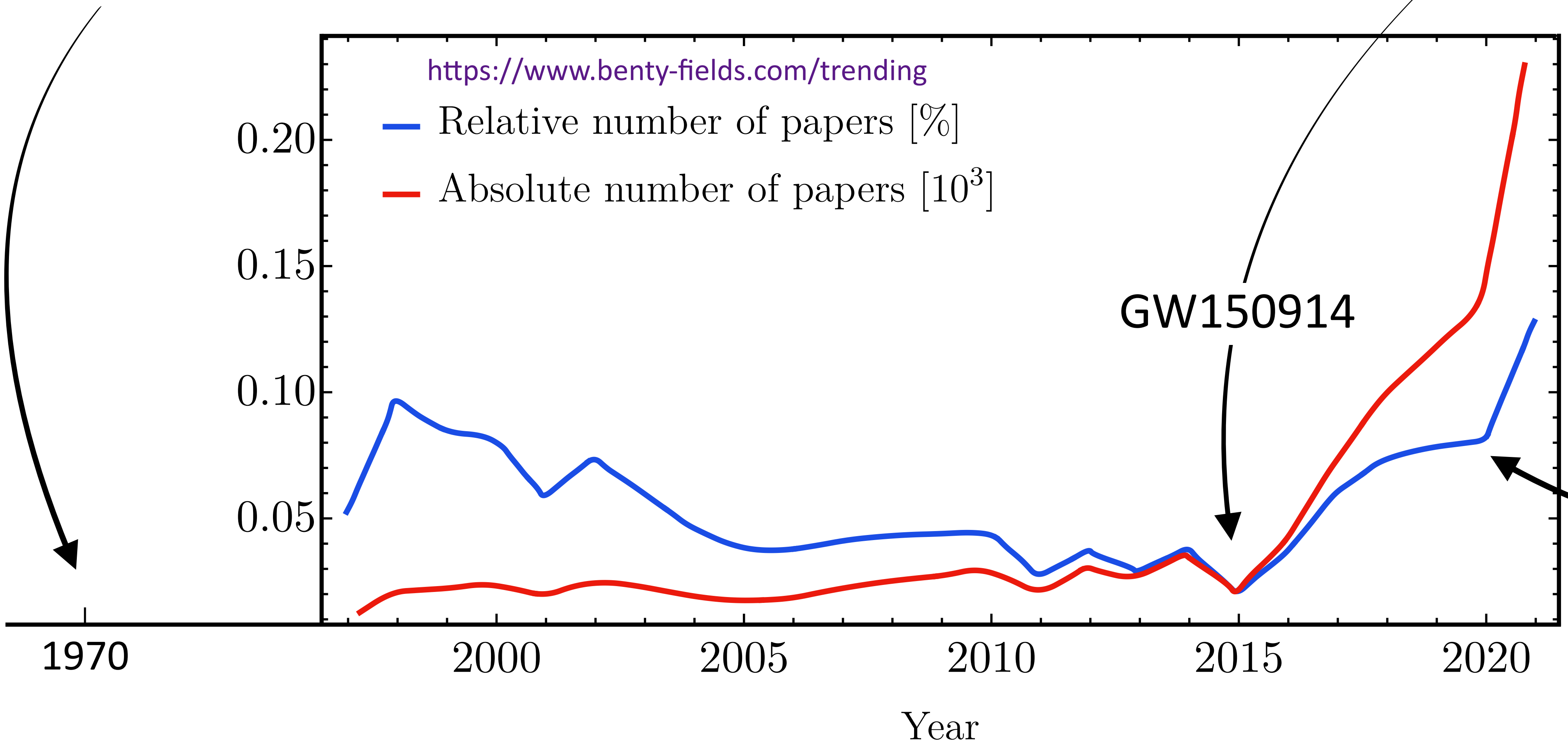
Recent attention devoted to PBHs

Y. B. Zel'dovich and I. D. Novikov, …, (1967)
S. W. Hawking, Nature 248, 30 (1974)
B. J. Carr and S. W. Hawking, … , (1974)
G. F. Chapline, Nature 253, 251 (1975)
B. J. Carr, Astrophys. J. 201, 1 (1975)

S. Bird *et al* Phys. Rev. Lett. **116**, 201301 (2016), [arXiv:1603.00464]
M. Sasaki, *et al* Phys. Rev. Lett. **117**, 061101 (2016), [arXiv:1603.08338]
S. Clesse and J. Garcìa-Bellido, Phys. Dark Univ. **15** (2017), 142-147 [arXiv:1603.05234]
….

Did LIGO detect dark matter?

Simeon Bird,* Ilias Cholis, Julian B. Muñoz, Yacine Ali-Haïmoud, Marc Kamionkowski, Ely D. Kovetz, Alvise Raccanelli, and Adam G. Riess[1]



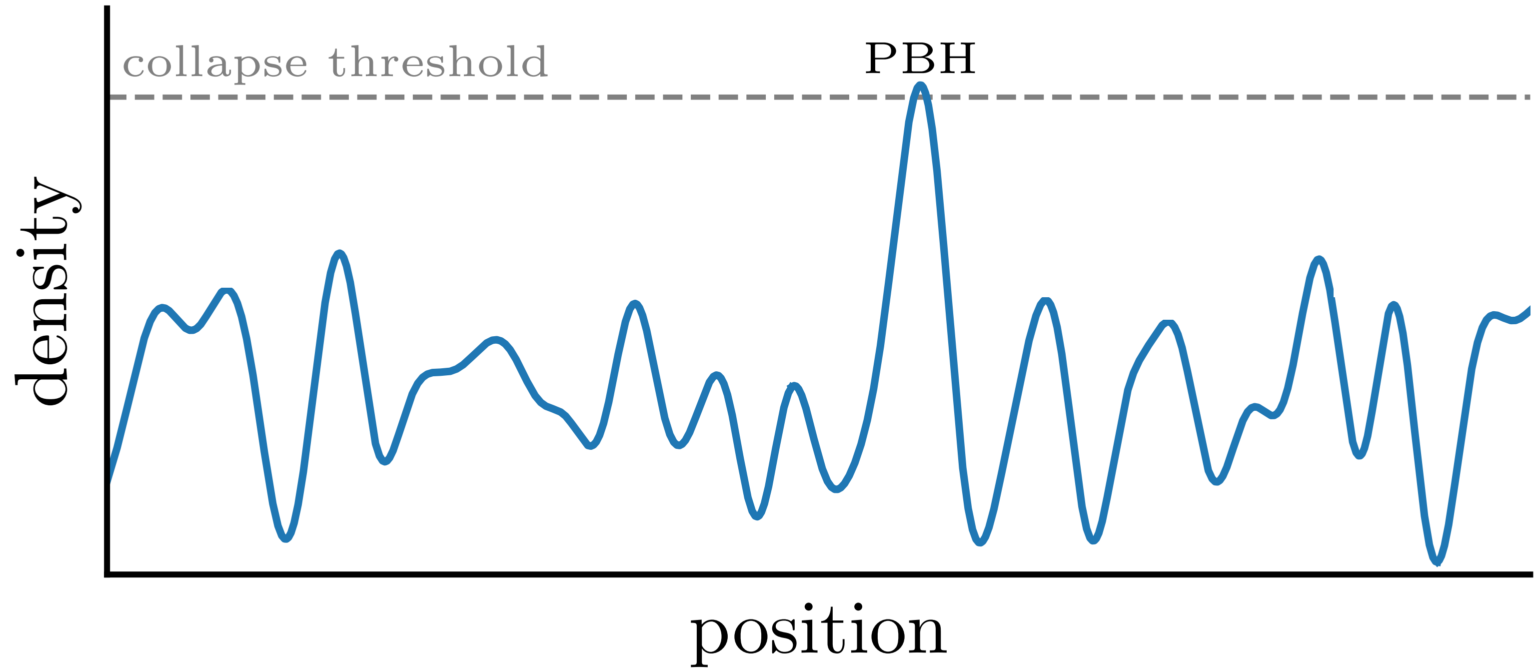
- Mass gap event GW190521
- PTA observations
- ….

Gravitational Wave observations will set strong bounds on PBHs,

or lead to unprecedented discoveries…

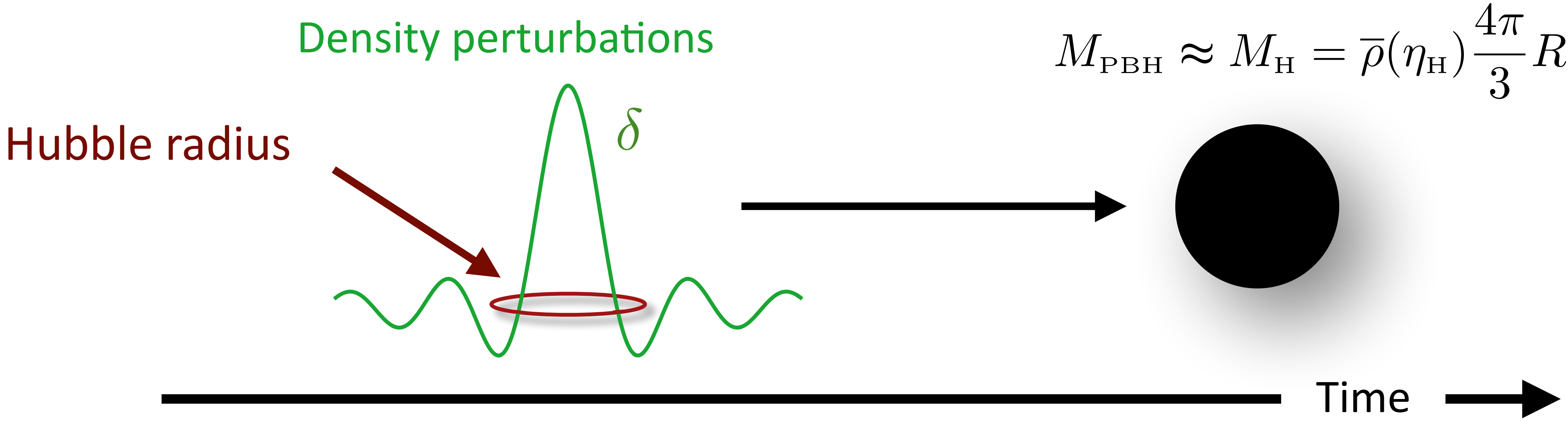
Formation of PBHs

Collapse of large over densities in the early universe



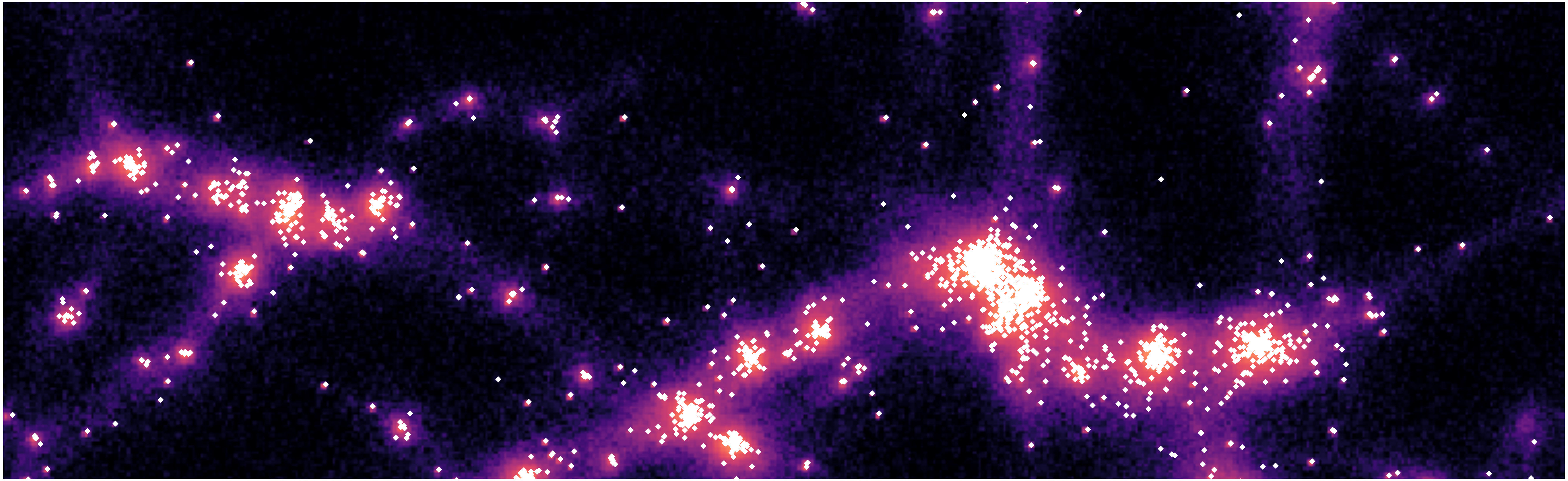
$$\delta_c = \frac{\rho_c - \rho_b}{\rho_b}$$

$$M_{\rm PBH} \approx M_{\rm H} = \overline{\rho}(\eta_{\rm H})\frac{4\pi}{3}R_{\rm H}^3$$



Typically in radiation domination

Primordial black hole dark matter



D. Inman and Y. Ali-Haïmoud, Phys. Rev. D **100**, no.8, 083528 (2019) [arXiv:1907.08129]

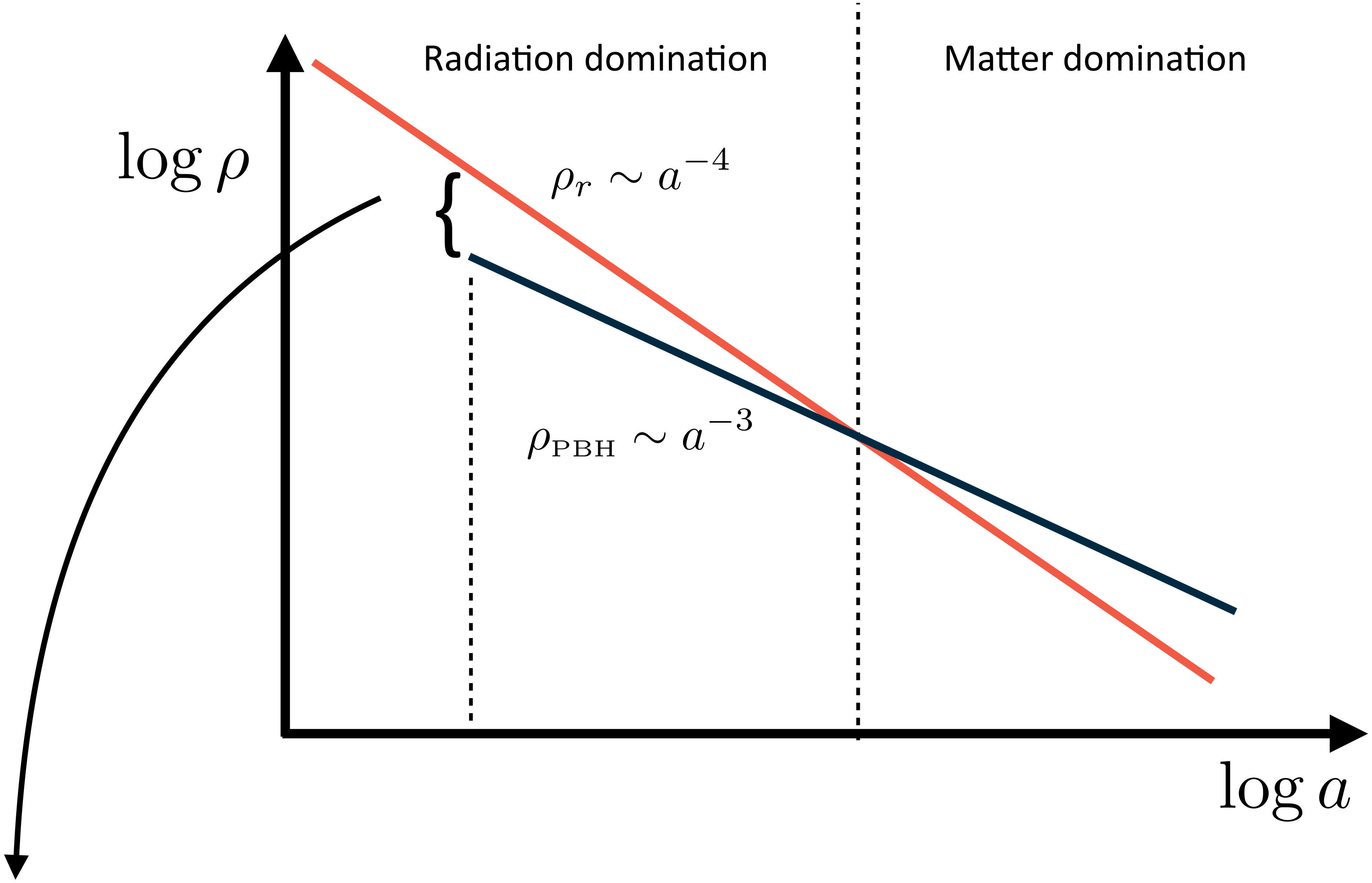
- PBHs on large scales behave as a cold and collisionless fluid
- PBH abundance expressed in terms of the dark matter

$$f_{\rm PBH} \equiv \Omega_{\rm PBH}/\Omega_{\rm DM}$$

(can be thought as a proxy for the average PBH number density)

$$n_{\rm PBH} \approx f_{\rm PBH}\rho_{\rm DM}/\langle m_{\rm PBH}\rangle$$

PBH formation needs to be rare



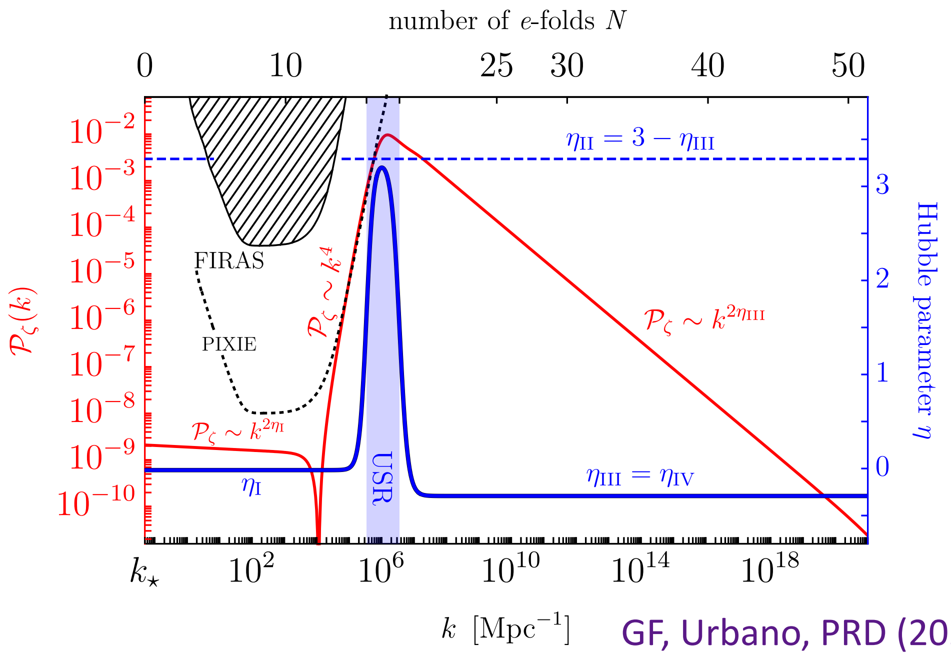
Mass fraction at formation = Fraction of Hubble patches which collapse to form a PBH

$$\beta \equiv \frac{\rho_{\rm PBH}}{\rho_r} \sim 4 \times 10^{-9} f_{\rm PBH} \left(\frac{m_{\rm PBH}}{M_\odot} \right)^{1/2}$$

Most studied scenarios of PBH formation

- Enhanced curvature perturbations at small scales:

P. Ivanov, P. Naselsky, and I. Novikov, PRD 50, 7173 (1994).
J. Garcia-Bellido, A. D. Linde, and D. Wands, PRD 54, 6040 (1996)
…

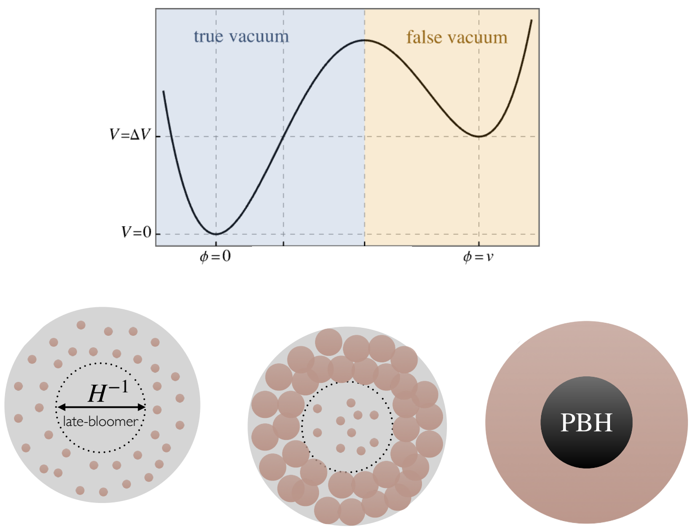


GF, Urbano, PRD (2022)

- Ultra-slow roll inflation
- Multi-field models
- Curvaton
- ….

- Cosmological 1st order phase transitions:

Hawking, Moss, Stewart, PRD 26 (1982)
Baker, Breitbach, Kopp and Mittnacht, [arXiv:2105.07481]
Liu *et al*, Phys. Rev. D **105** (2022) no.2, L021303 [arXiv:2106.05637]
Lewicki, Toczek, Vaskonen, 2305.04924
Gouttenoire and Volansky, [arXiv:2305.04942]
…

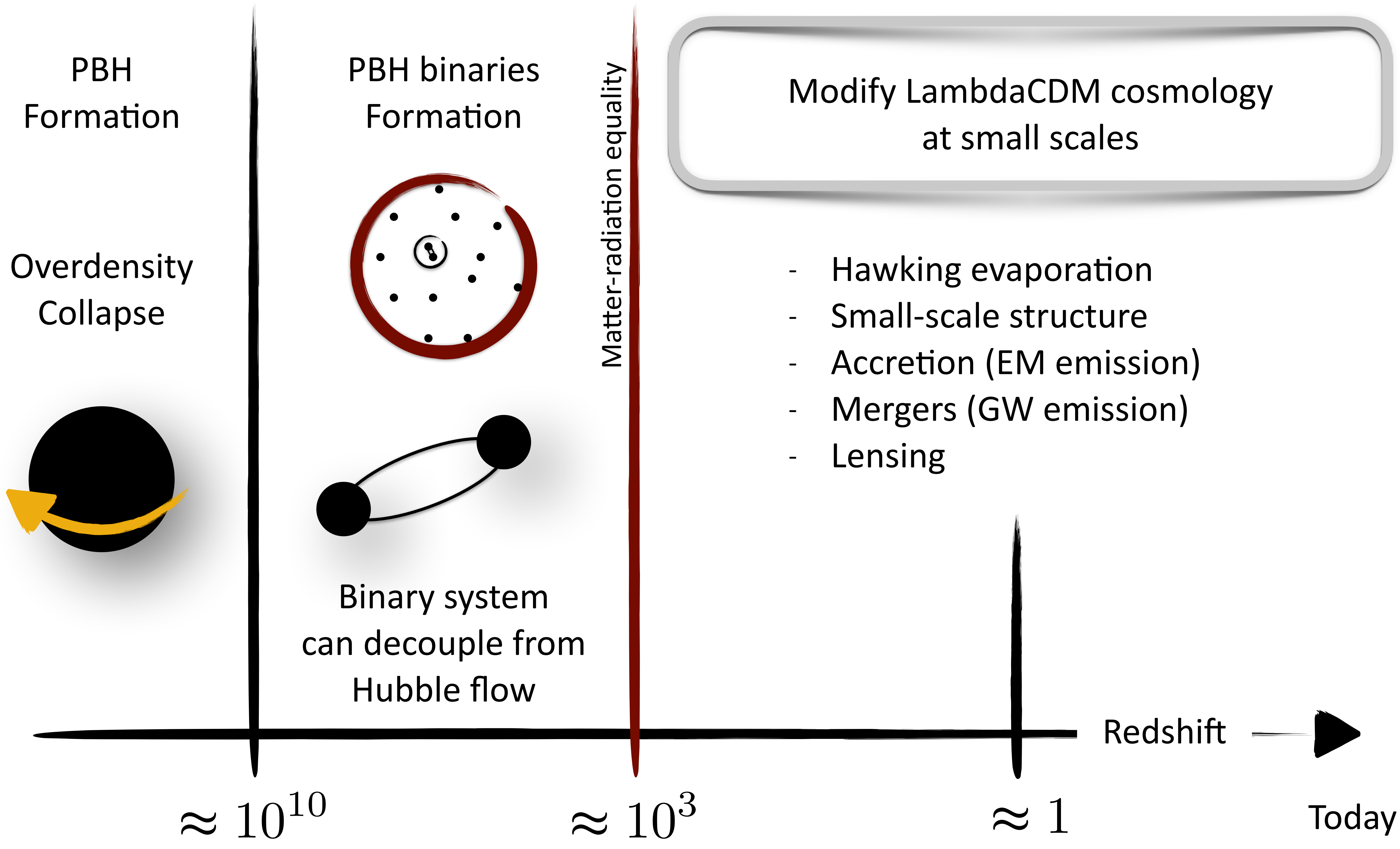


Gouttenoire and Volansky, [arXiv:2305.04942]

- Bubble wall collisions
- Collapse of the last false vacuum remnants in a first-order phase transition
- …

- ….

Primordial black hole cosmology



PBH
Formation

Overdensity
Collapse

PBH binaries
Formation

Binary system
can decouple from
Hubble flow

Matter-radiation equality

Modify LambdaCDM cosmology
at small scales

- Hawking evaporation
- Small-scale structure
- Accretion (EM emission)
- Mergers (GW emission)
- Lensing

Redshift

$\approx 10^{10}$

$\approx 10^{3}$

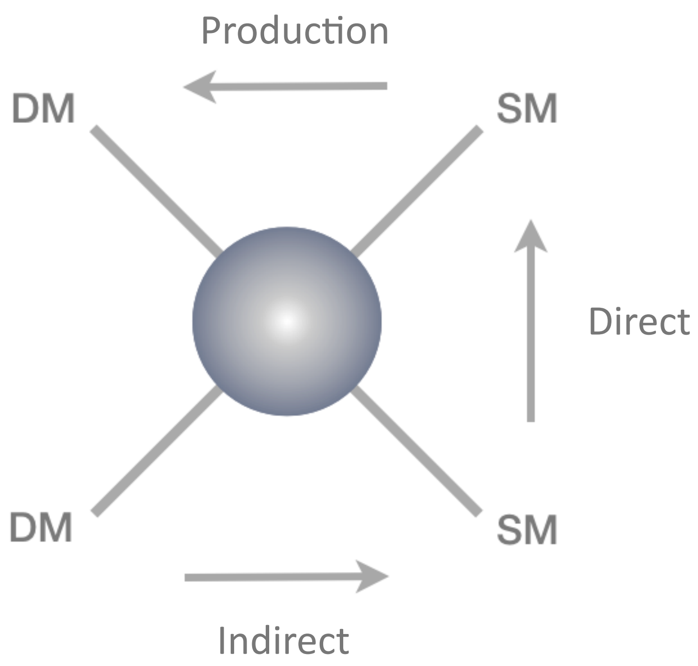
≈ 1

Today

Primordial black hole dark matter searches

Nice analogy put forward in: E. D. Kovetz, PRL (2017) [arXiv:1705.09182]

Particle dark matter search strategies:



Production

DM

SM

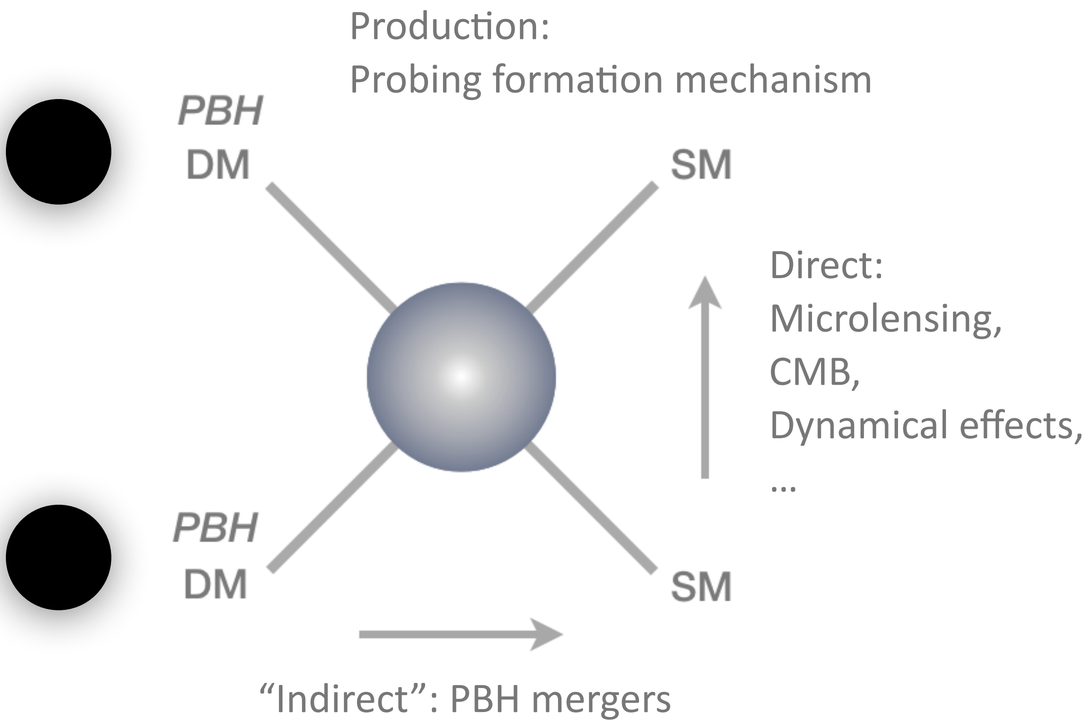
Direct

DM

SM

Indirect

PBH dark matter search strategies:



Production:
Probing formation mechanism

PBH
DM

SM

Direct:
Microlensing,
CMB,
Dynamical effects,
…

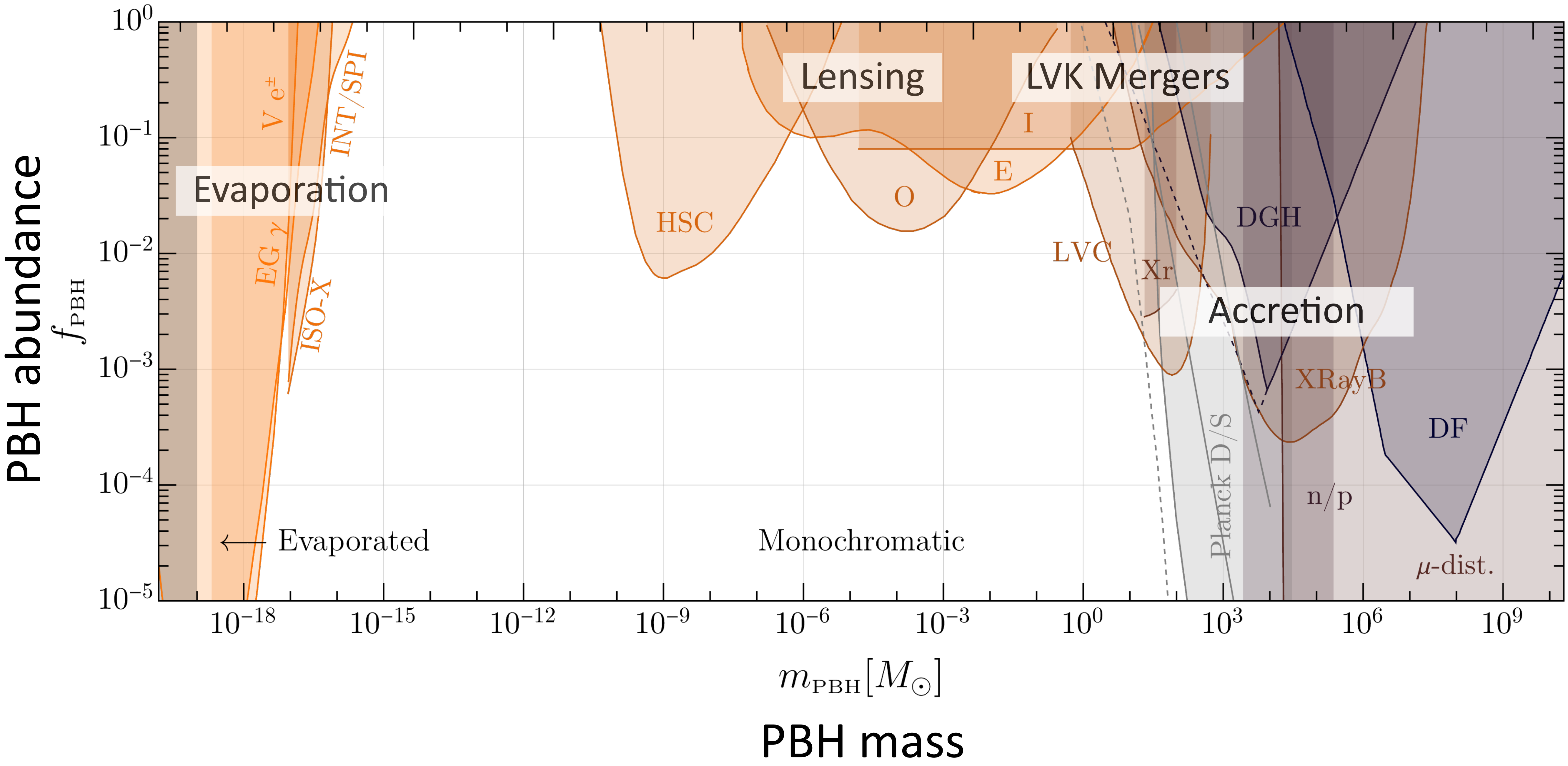
PBH
DM

SM

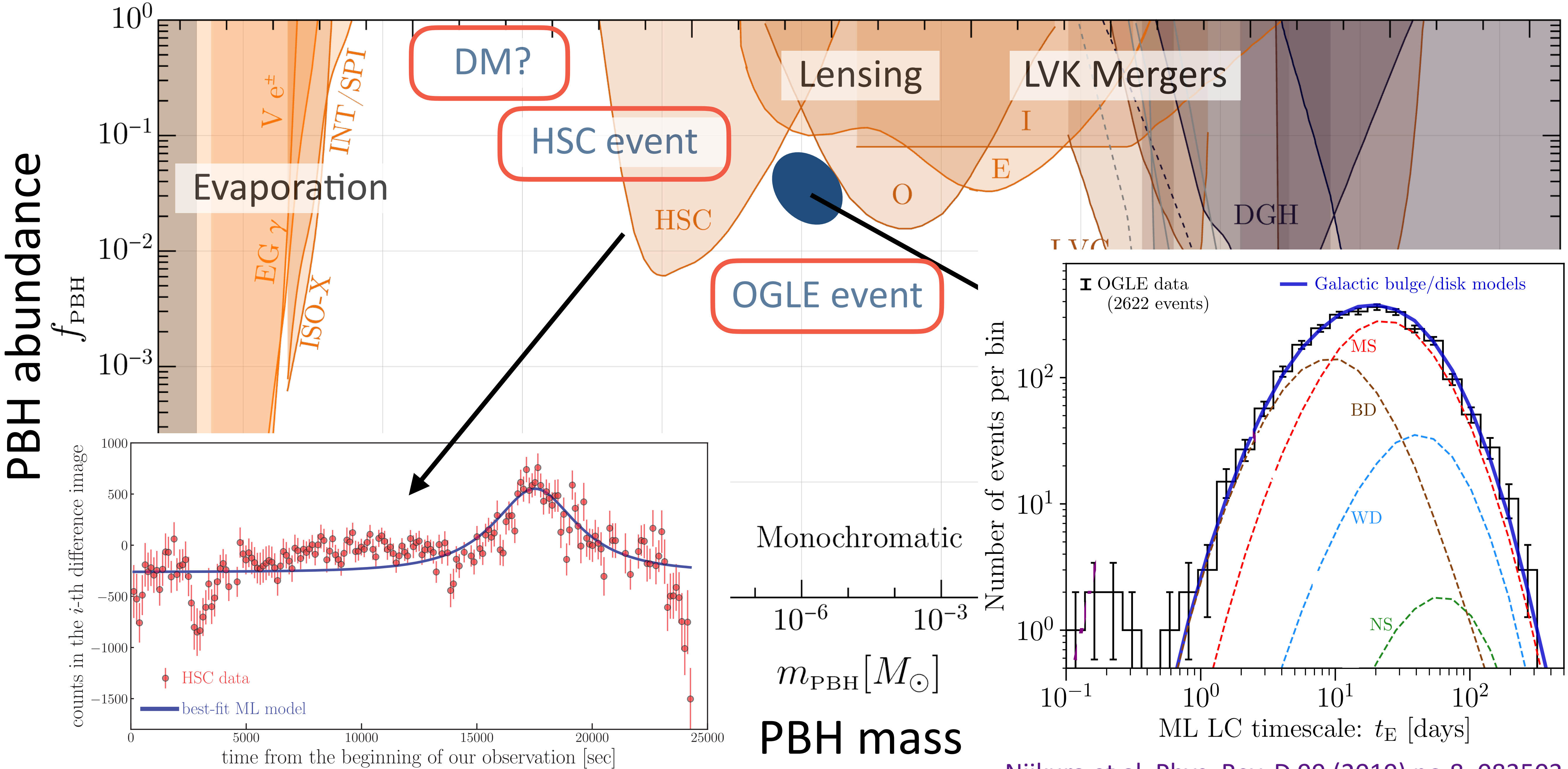
“Indirect”: PBH mergers

Current status of PBH constraints

Review: B. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, Rept. Prog. Phys. **84**, no.11, 116902 (2021) [arXiv:2002.12778]



Current status of PBH constraints



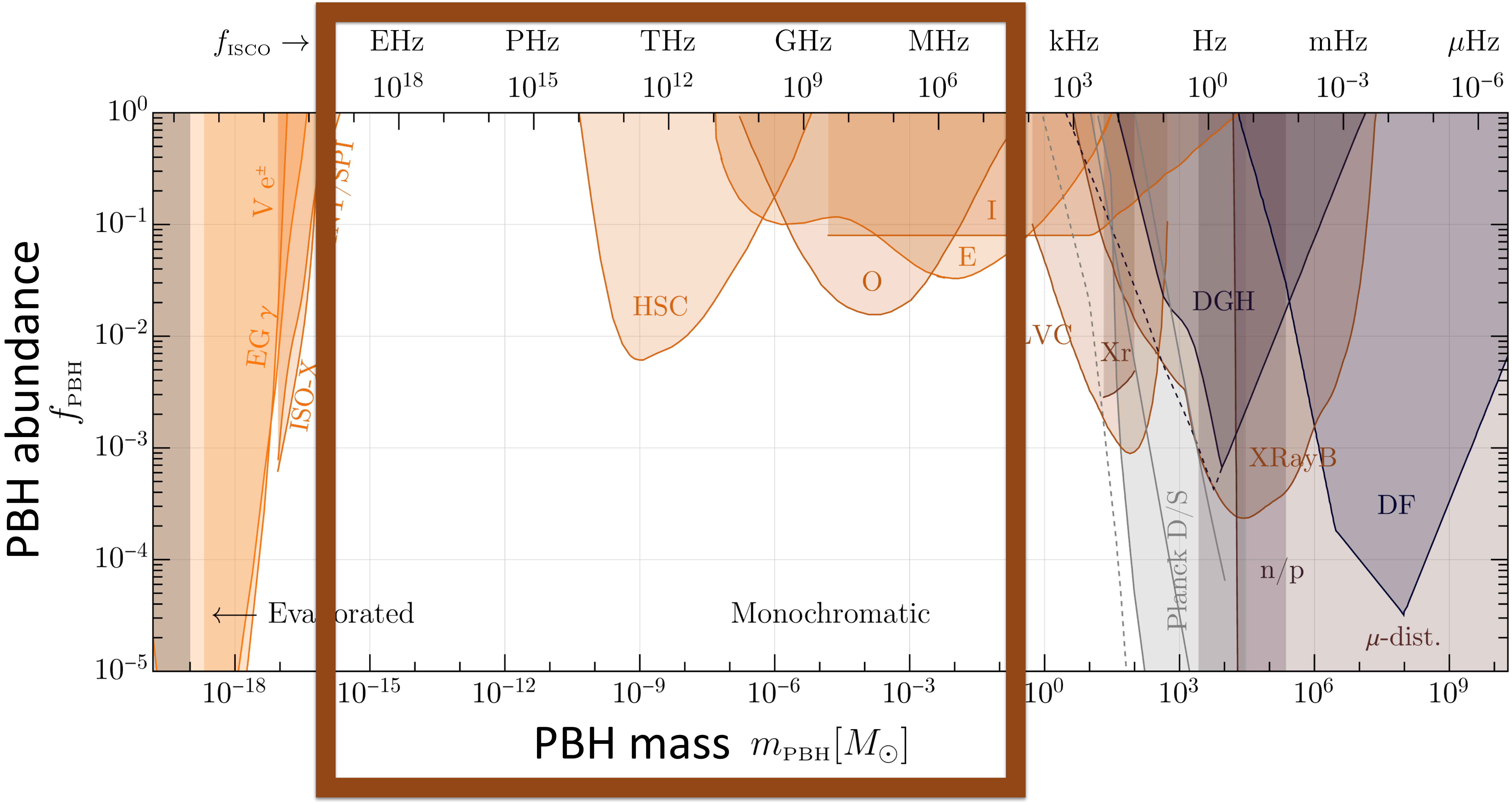
H. Niikura et al. Nature Astron. 3 (2019) no.6, 524-534

Niikura et al. Phys. Rev. D 99 (2019) no.8, 083503

Ultra-high frequency Gravitational waves

Review: B. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, Rept. Prog. Phys. **84**, no.11, 116902 (2021) [arXiv:2002.12778]

ISCO frequency of PBH binary at each mass



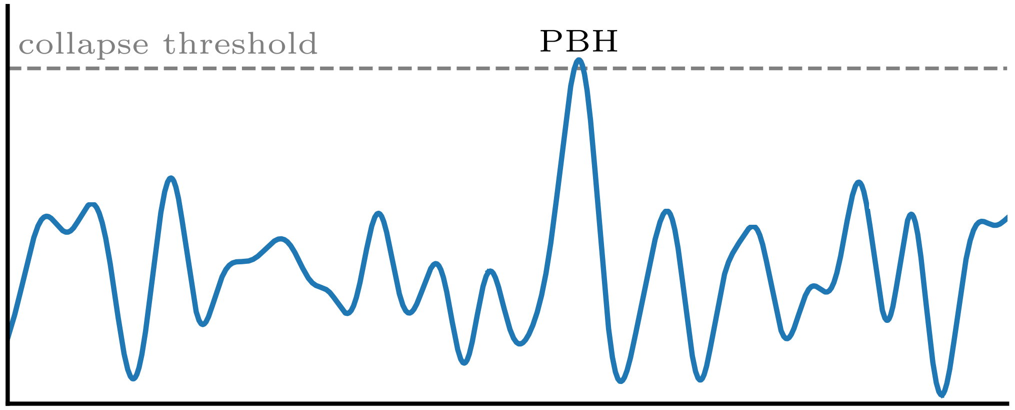
PBH binary formation and merger rates

PBH Initial conditions

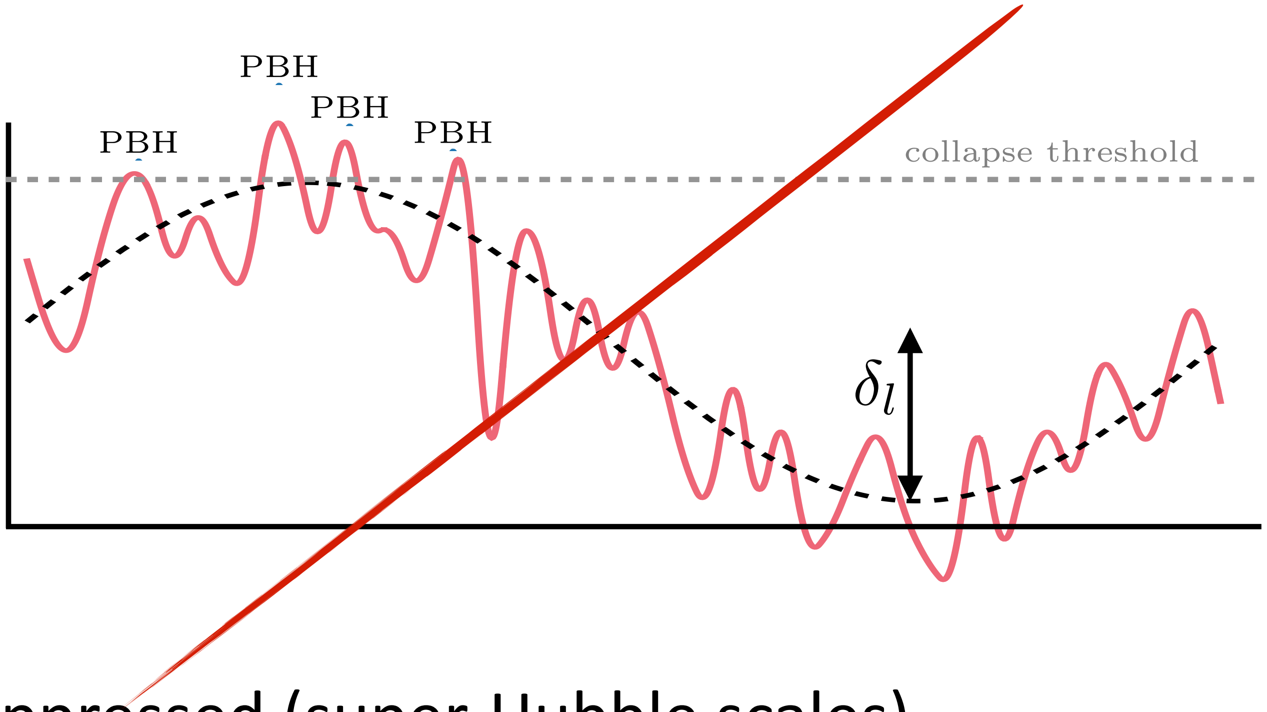
In the standard scenario, PBHs are not clustered at formation

$$\left\langle \frac{\delta\rho_{\rm PBH}(\vec{x},z)}{\bar{\rho}_{\rm DM}}\frac{\delta\rho_{\rm PBH}(0,z)}{\bar{\rho}_{\rm DM}}\right\rangle = \frac{f^2_{\rm PBH}}{n_{\rm PBH}}\delta_{\rm D}(\vec{x}) + \cancel{\xi(\vec{x},z)}.$$

Y. Ali-Haïmoud, Phys. Rev. Lett. **121**, 081304 (2018), [arXiv:1805.05912]
V. Desjacques and A. Riotto, Phys. Rev. D **98**, 123533 (2018), [arXiv:1806.10414]
G. Ballesteros, P. D. Serpico, and M. Taoso, JCAP **10**, 043 (2018), [arXiv:1807.02084]
A. Moradinezhad Dizgah, G. Franciolini and A. Riotto, JCAP **11**, 001 (2019) [arXiv:1906.08978]
….



vs



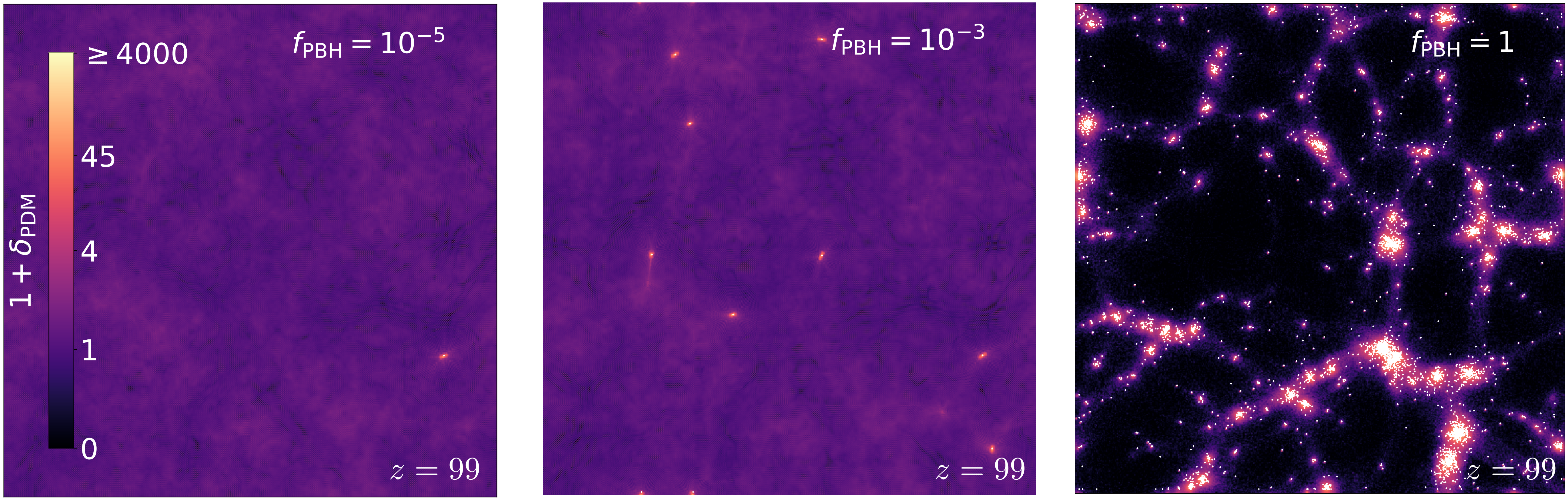
Bias induced by long modes suppressed (super-Hubble scales)

$$\delta_l \approx \frac{\nu}{\sigma}\times\left(\frac{k_l}{k_s}\right)^2 \zeta(k_l) \ll \delta_s$$

(*unless specific local non-Gaussianities are introduced in the model)

PBH clustering evolution

Cosmological N-body simulation



D. Inman and Y. Ali-Haïmoud, Phys. Rev. D **100**, no.8, 083528 (2019) [arXiv:1907.08129]

- Shot noise induce early small structures depending on the abundance $f_{\rm PBH}$
- PBHs isolated if $f_{\rm PBH} \lesssim z \times 10^{-4}$

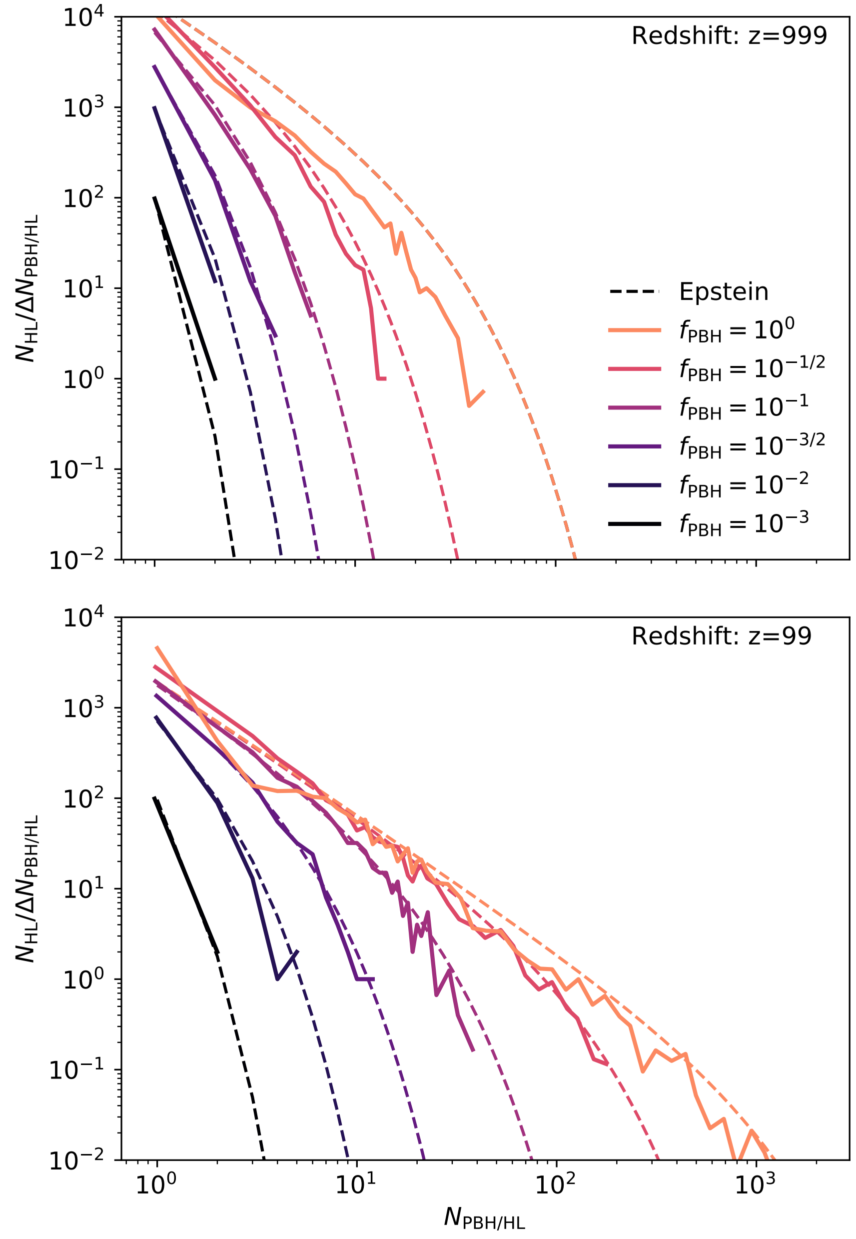
see also:
M. Raidal, C. Spethmann, V. Vaskonen and H. Veermäe, JCAP **02**, 018 (2019) [arXiv:1812.01930]
M. Trashorras, J. García-Bellido, and S. Nesseris, Universe 7, 18 (2021), [arXiv:2006.15018]
K. Jedamzik, JCAP **09**, 022 (2020), [arXiv:2006.11172]

Analytical description matching numerical simulations + extrapolation at lower redshift

V. De Luca, V. Desjacques, G. Franciolini and A. Riotto, JCAP **11**, 028 (2020) [arXiv:2009.04731]

Halo mass function in PBH cosmologies

- Press-Schecter description: in the matter-dominated era density perturbations grow with scale factor and collapse to form virtualised halos (of N PBHs) when reaching the "linear" threshold $\delta_c \simeq 1.68$



- Characteristic density variance: $\sigma(N, z_{\rm eq}) \simeq 1/\sqrt{N}$

- Abundance of halos with N PBHs:

$$\frac{\mathrm{d}n_{\rm cl}(N,t)}{\mathrm{d}N} = \frac{\overline{n}}{\sqrt{\pi}} \left[\frac{N}{N_*(t)}\right]^{-\frac{1}{2}} \frac{e^{-N/N_*(t)}}{N^2}$$

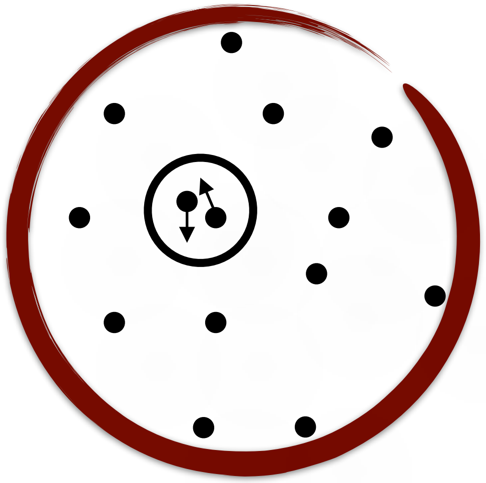
$$N_*(t) \simeq f_{\rm PBH}^2 \left(\frac{2600}{1+z}\right)^2$$

- Halo "evaporation": 2b interactions expel objects from clusters

$$t_{\rm ev} \simeq 140 t_{\rm rlx} \qquad t_{\rm rlx} \simeq \frac{1}{10}\frac{N}{\ln N}\left(\frac{R}{\sigma_v}\right)$$

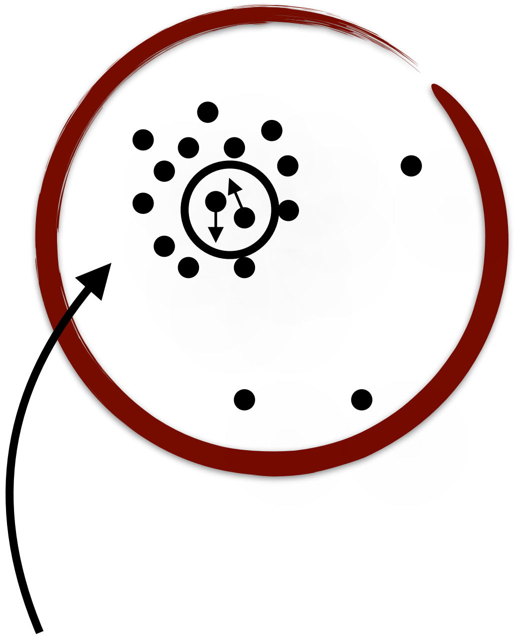
PBH binary formation

Early universe binaries

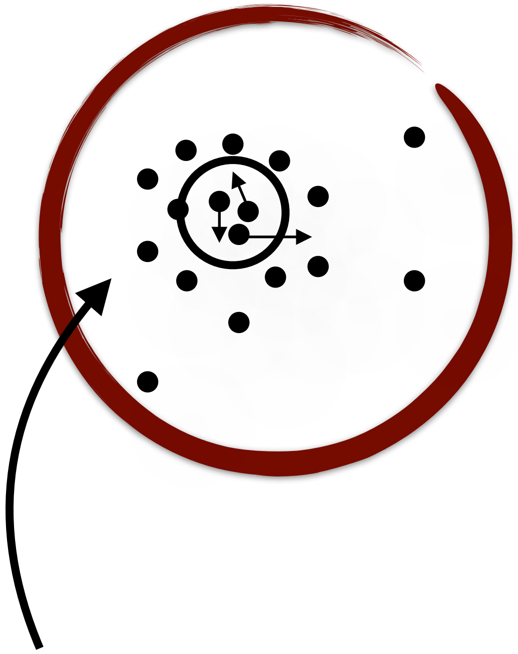


Dominant contribution
even with suppressions

Dynamical 2b capture
(adopted in Bird et al.)



3b dynamical interaction



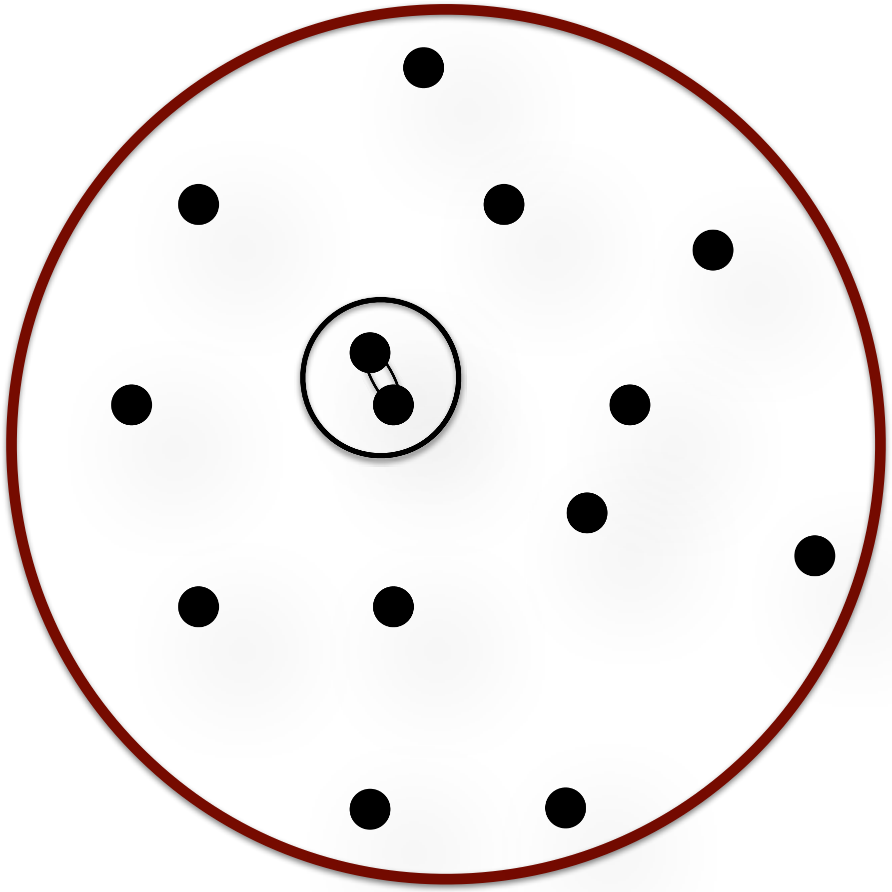
Within clusters seeded by Poisson-induced clustering

S. Bird et al Phys. Rev. Lett. 116, 201301 (2016), [arXiv:1603.00464]
M. Sasaki, et al Phys. Rev. Lett. 117, 061101 (2016), [arXiv:1603.08338]
S. Clesse and J. Garcìa-Bellido, Phys. Dark Univ. 15 (2017), 142-147 [arXiv:1603.05234]
Ali-Haïmoud,Kovetz,M.Kamionkowski, Phys. Rev. D 96 (2017) no.12, 123523 [arXiv:1709.06576]
M. Raidal, C. Spethmann, V. Vaskonen and H. Veermäe, JCAP **02**, 018 (2019) [arXiv:1812.01930]
G. Franciolini, K. Kritos, E. Berti and J. Silk, Phys. Rev. D **106** (2022) no.8, 083529 [arXiv:2205.15340]
…

Early-universe binary formation

- Initial spatial Poisson distribution
- Random decoupling of binary systems from the Hubble flow

Nakamura (1997), …



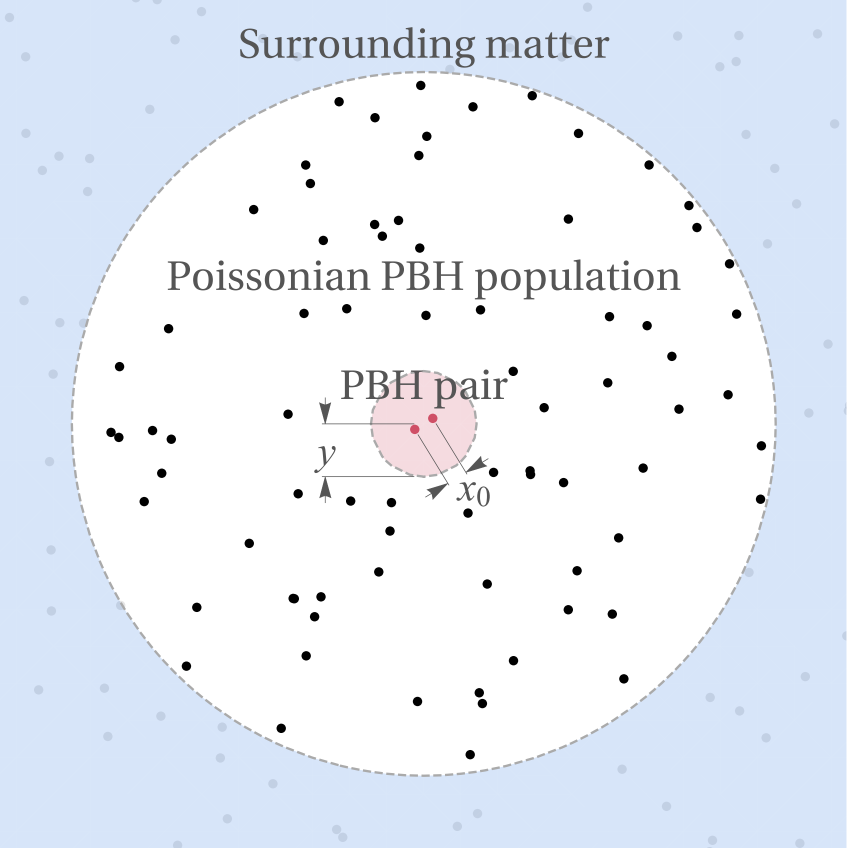
$$\ddot{r} - (\dot{H} + H^2) r + \frac{2M}{r^2} \frac{r}{|r|} = 0$$

- Initial binary angular momentum induced by nearest PBH
- Binary formation happening before matter-radiation equality
- The distribution of initial semi-major axis a and eccentricity e determines the merger time (Peters' time)

$$\tau = \frac{3}{170} \frac{c^5 a^4}{(Gm)^3} \left(1 - e^2\right)^{7/2}$$

Merger rate suppression from dynamics in the early universe

M. Raidal, C. Spethmann, V. Vaskonen and H. Veermäe, JCAP **02**, 018 (2019) [arXiv:1812.01930]



$$\mathrm{d}R = \int \mathrm{d}n_{\rm b}\mathrm{d}j \frac{\mathrm{d}P}{\mathrm{d}j}\delta\left(\tau - \frac{3}{85}\frac{a^4}{\eta M^3}j^7\right)$$

i) Exponential suppression requiring initially the binary is a 2-body system (otherwise likely disrupted)

$$\mathrm{d}n_{\rm b} = \frac{1}{2}e^{-\bar{N}(y)}\mathrm{d}n(m_1)\mathrm{d}n(m_2)\mathrm{d}V(x_0)$$

ii) Surrounding PBHs and dark matter overdensities modifies the distribution of angular momentum j

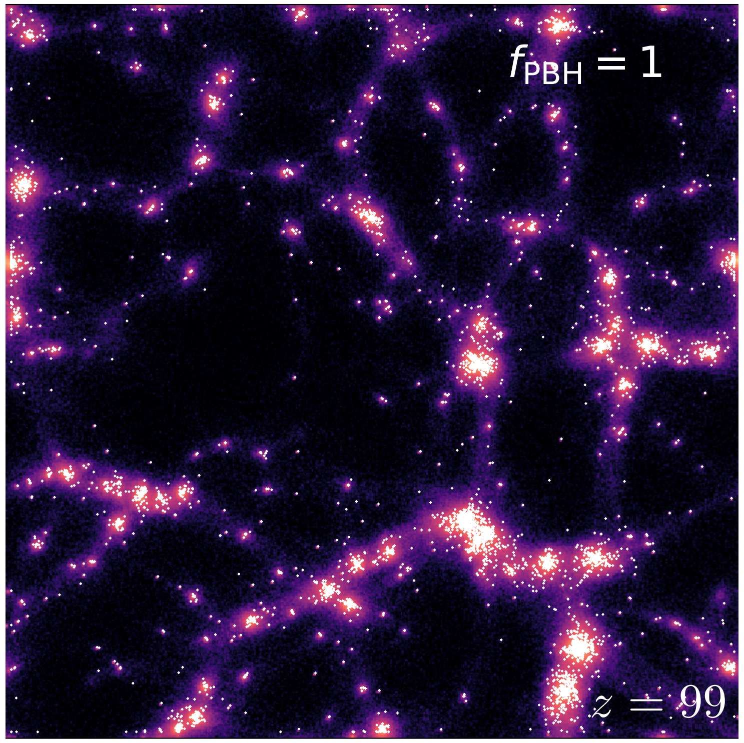
$$\vec{J} = \vec{J}_{\rm PBH} + \vec{J}_{\rm M}$$

Tested with N-body simulations for narrow/broad mass distributions

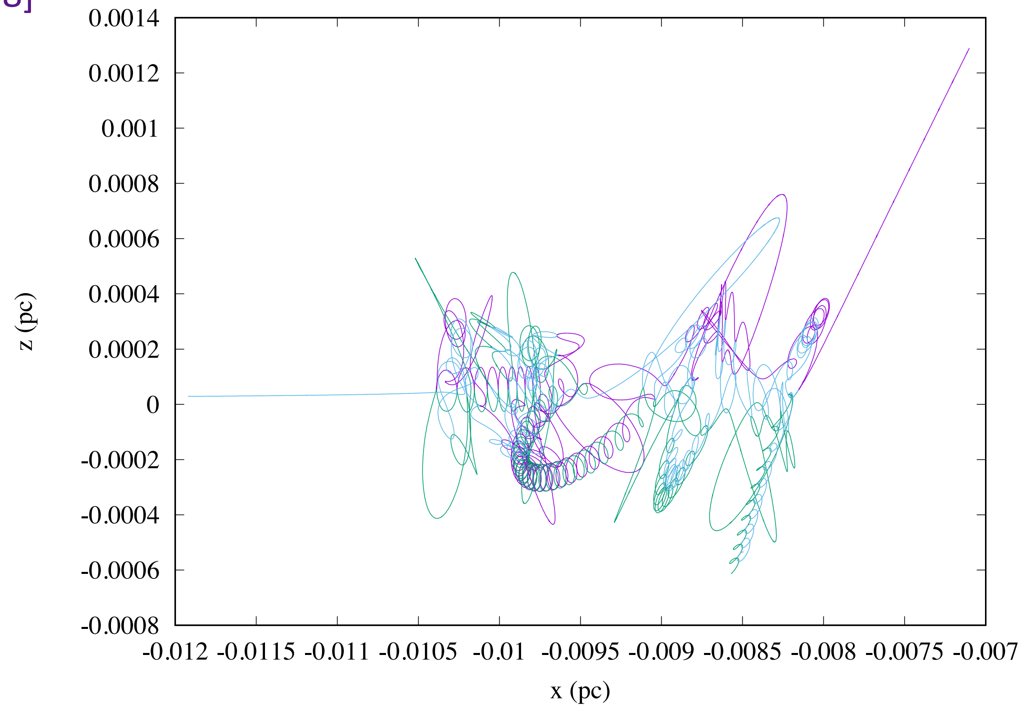
$$S_{\rm early} \approx 1.42\left[\frac{\langle m^2\rangle/\langle m\rangle^2}{\bar{N}(y)+C} + \frac{\sigma_{\rm M}^2}{f_{\rm PBH}^2}\right]^{-21/74}\exp\left[-\bar{N}(y)\right] \qquad \text{with} \qquad \bar{N}(y) \equiv \frac{M}{\langle m\rangle}\left(\frac{f_{\rm PBH}}{f_{\rm PBH}+\sigma_{\rm M}}\right)$$

Merger rate suppression in clusters

K. Jedamzik, JCAP **09**, 022 (2020), [arXiv:2006.11172]
M. Trashorras, J. Garcìa-Bellido and S. Nesseris, Universe 7, no.1, 18 (2021) [arXiv:2006.15018]



PBH-PBH interactions in clusters



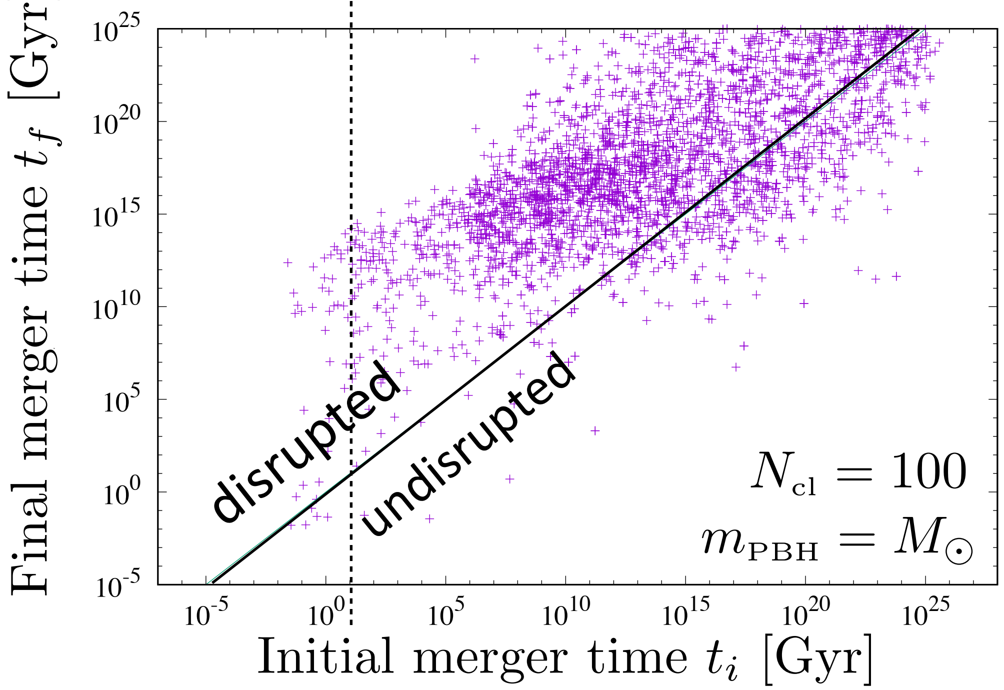
Heggie-Hills law: Hard binaries becomes harder after interaction with third objects

D. C. Heggie, MNRAS 173, 729 (1975)
J. G. Hills and L. W. Fullerton, AJ 85, 1281 (1980).

Hard binaries: binding E > average PBH E

But large loss of eccentricity: longer merger times

K. Jedamzik, JCAP **09**, 022 (2020), [arXiv:2006.11172]



Merger rate suppression in clusters - Lower bound

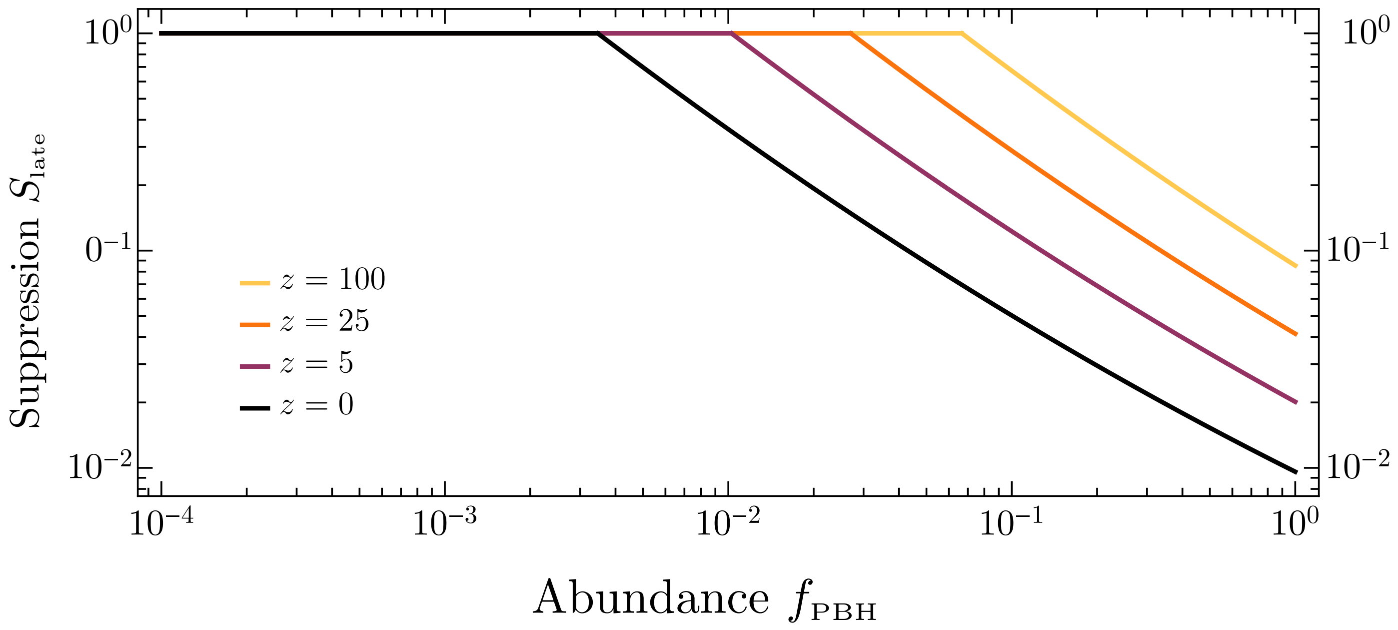
Residual fraction of mergers reside outside dense clusters and it is not disrupted

Conservatively neglect all binaries in environment with binary-single interaction rates which are larger than

Heggie and Rasio, Mon. Not. Roy. Astron. Soc. 282 (1996), 1064 [arXiv:9506082]

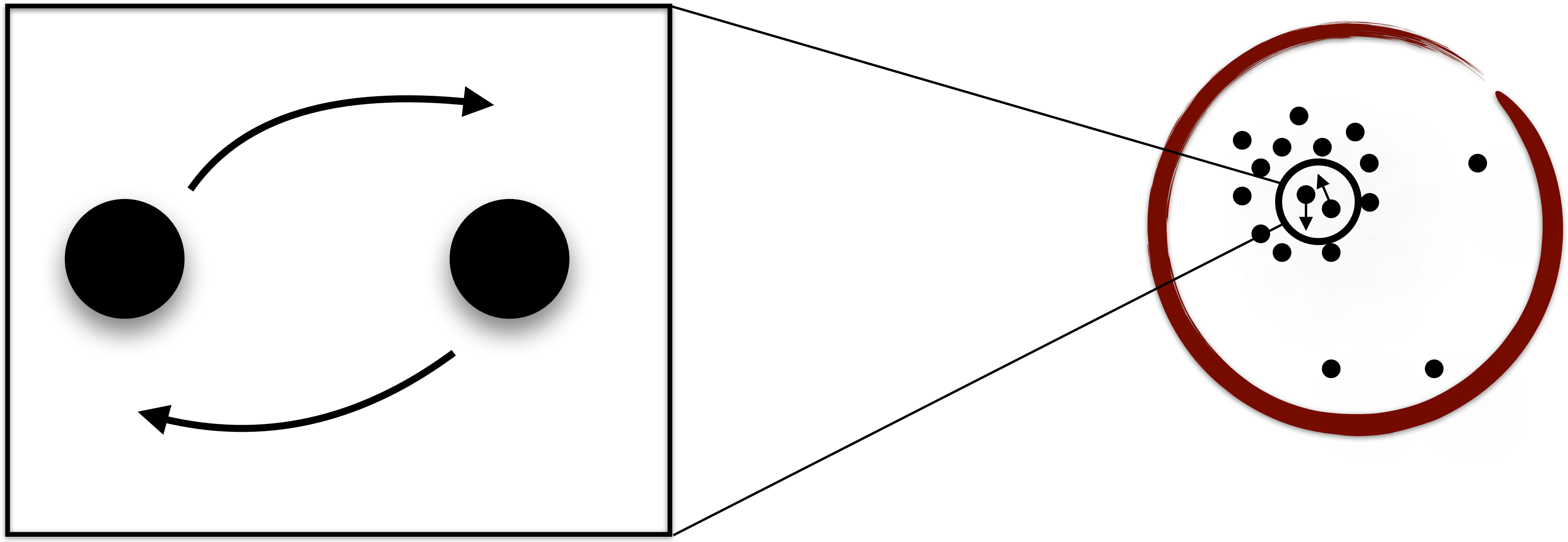
$$1/t_{\rm p} = n_{\rm loc} \langle \sigma_{\Delta j > j_\tau} v \rangle$$

$$P_{\rm np}(z) \gtrsim 1 - \sum_{N=3}^{N_c(z)} \bar{p}_N(z_c) - \sum_{N' > N_c(z)} \left[\sum_{N=3}^{N_c(z)} \tilde{p}_N(z_c) \right] \bar{p}_{N'}(z_c)\,, \qquad N_{\rm cl} \lesssim 5000 \quad \textbf{\textit{for}} \quad f_{\rm PBH} = 1$$



V. Vaskonen and H. Veermäe, Phys. Rev. D **101**, 043015 (2020), [arXiv:1908.09752]
V. De Luca, V. Desjacques, G. Franciolini and A. Riotto, JCAP **11**, 028 (2020) [arXiv:2009.04731]
G. Hütsi, M. Raidal, V. Vaskonen and H. Veermäe, JCAP **03** (2021), 068 [arXiv:2012.02786]
M. Martinelli *et al.* JCAP **08** (2022) no.08, 006 [arXiv:2205.02639]

PBH capture



- Close encounters within cluster can lead to copious emission of GW and formation of a bounded system

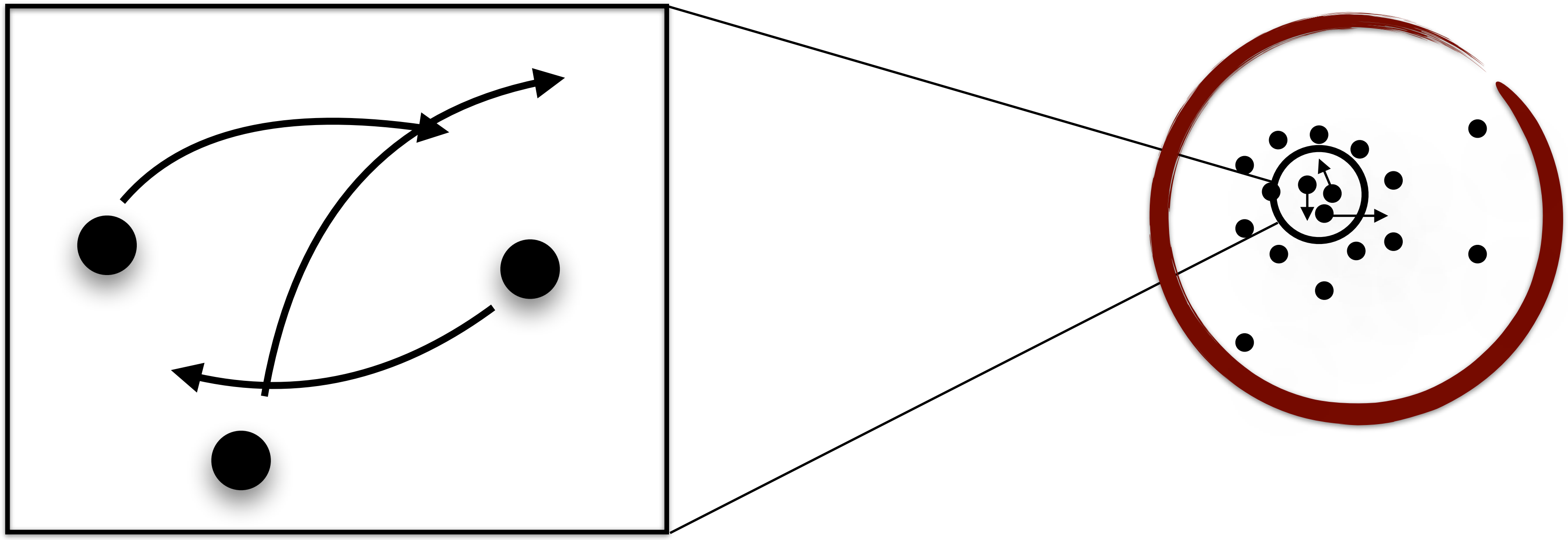
G. D. Quinlan and S. L. Shapiro, ApJ 343, 725 (1989)

$$\sigma_{\rm gw}(v) = 4\pi\left(\frac{85\pi}{3}\right)^{2/7}\frac{M^2}{v^{18/7}}$$

- Merger time for these binaries is fast, as they form with small a (i.e we neglect time-delays)
- Integrate over environment times the PBH-induced halo mass function

$$\Gamma = \frac{1}{2}\int d^3r\,\frac{\rho(\boldsymbol{r})^2}{M^2}\langle v\sigma_{\rm gw}\rangle \longrightarrow \mathcal{R}^{\rm cap}_{\rm BPBH}(z) = \sum_{N=N_{\rm ev}(z)}^{N_*(z)}\Gamma_{\rm cap}(N)\frac{dn^{\rm ev}_{\rm cl}(N,z)}{dN}$$

PBH from three-body dynamics



- Much more distant encounter can form a binary if third object drains energy
- Rate density of 3b interaction C. L. Rodriguez et al. Astrophys. J. Supp. 258 (2022) no.2, 22 [arXiv:2106.02643]

$$\gamma_{3\mathrm{b}}(\eta \geq \eta_{\min}) = \frac{3^{9/2}\pi^{13/2}}{2^{25/2}}\eta_{\min}^{-\frac{11}{2}}(1+2\eta_{\min})(1+3\eta_{\min}) \times \frac{n^3(Gm)^5}{\sigma_v^9}$$

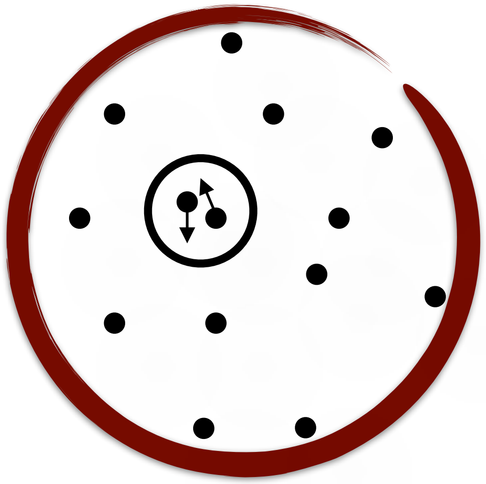
- Binary formation for large hardness ratio $\eta \equiv \dfrac{Gm}{a\sigma_v^2} \gtrsim 5$ S. J. Aarseth and D. C. Heggie, A&A 53, 259 (1976)
- Uncertainties between "thermal" P(e) = 2 e vs super-thermal eccentricity distribution

$$\mathcal{R}^{3\mathrm{b}}_{\mathrm{BPBH}}(z) = \sum_{N=N_{\min}}^{N_*(z)} \left[\Gamma_{3\mathrm{b}}(N) \times \int_{t_{\min}}^{t(z)} \mathrm{d}t' \underline{Q(N, t(z)-t')} \frac{\mathrm{d}n^{\mathrm{ev}}_{\mathrm{cl}}(N,t')}{\mathrm{d}N}\right]$$

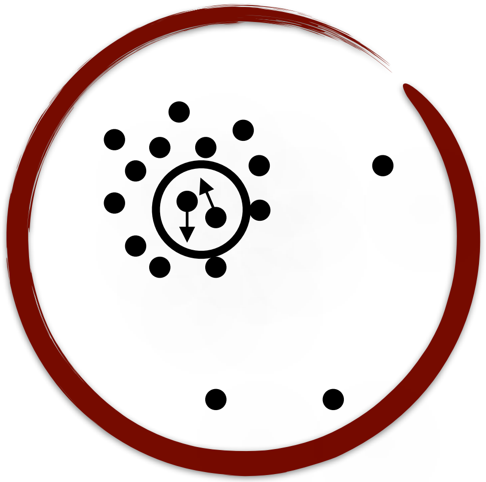
important time-delays: wider binaries at formation

PBH merger channels

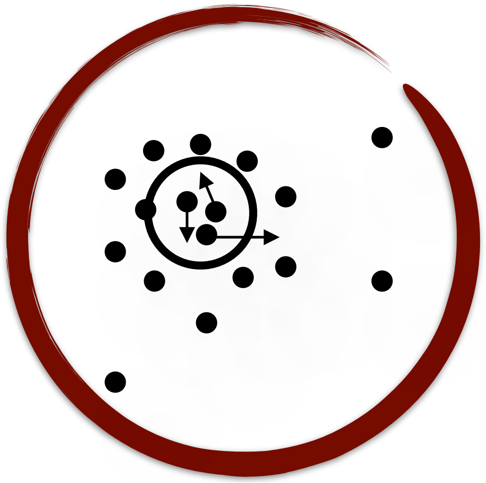
Early universe binaries



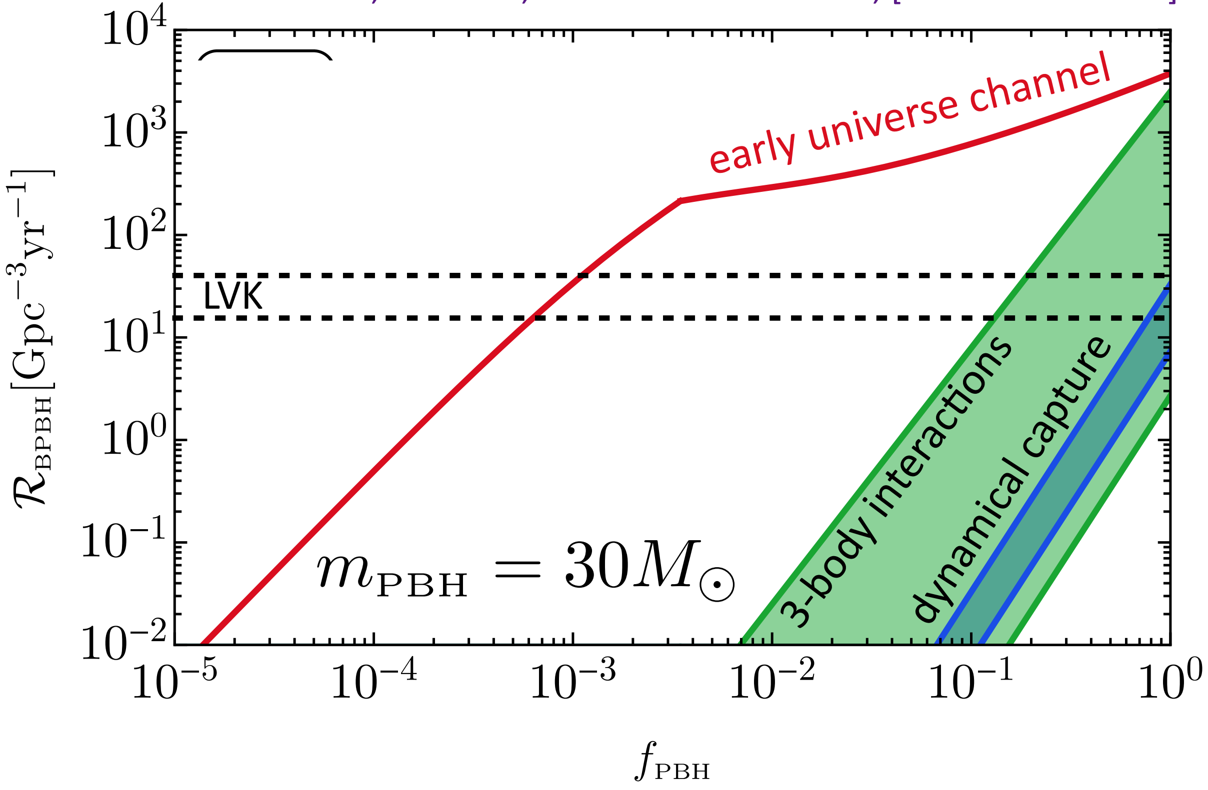
Dynamical capture



3-b dynamical interaction



GF, K. Kritos, E. Berti and J. Silk PRD, [arXiv:2205.15340]

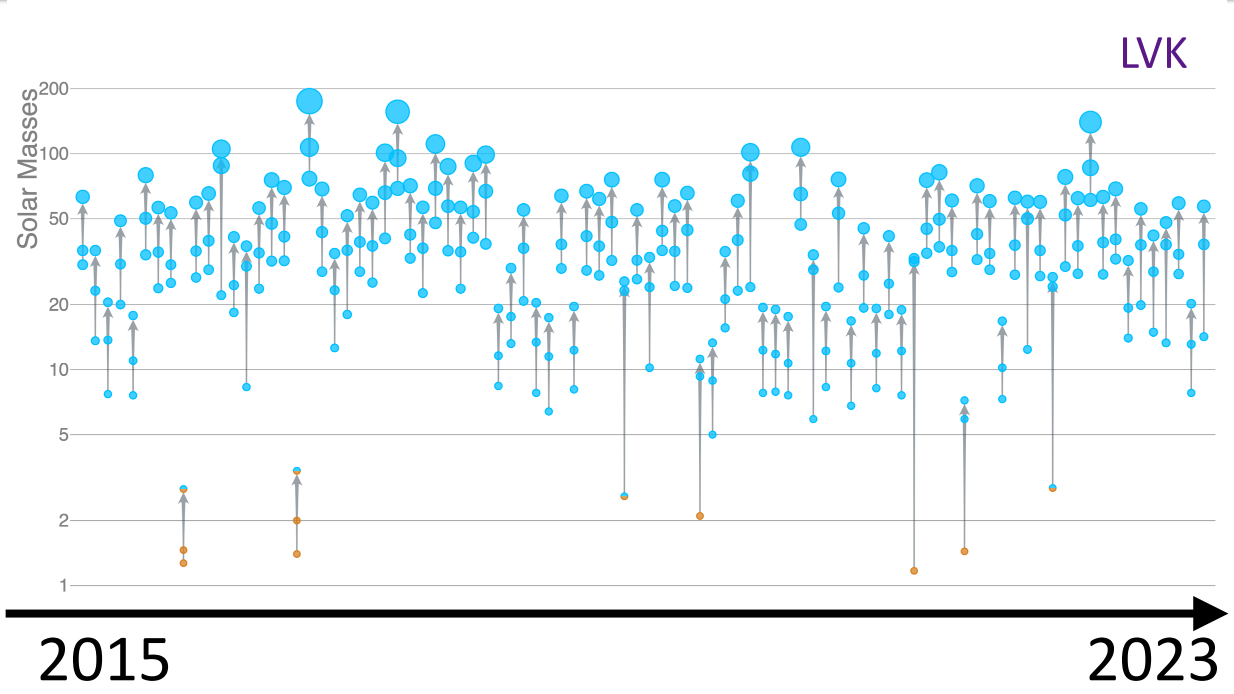


- Dynamical channels largely subdominant in the standard scenario
- Rate in the ballpark of LVK already with sub-percent DM abundance of PBHs

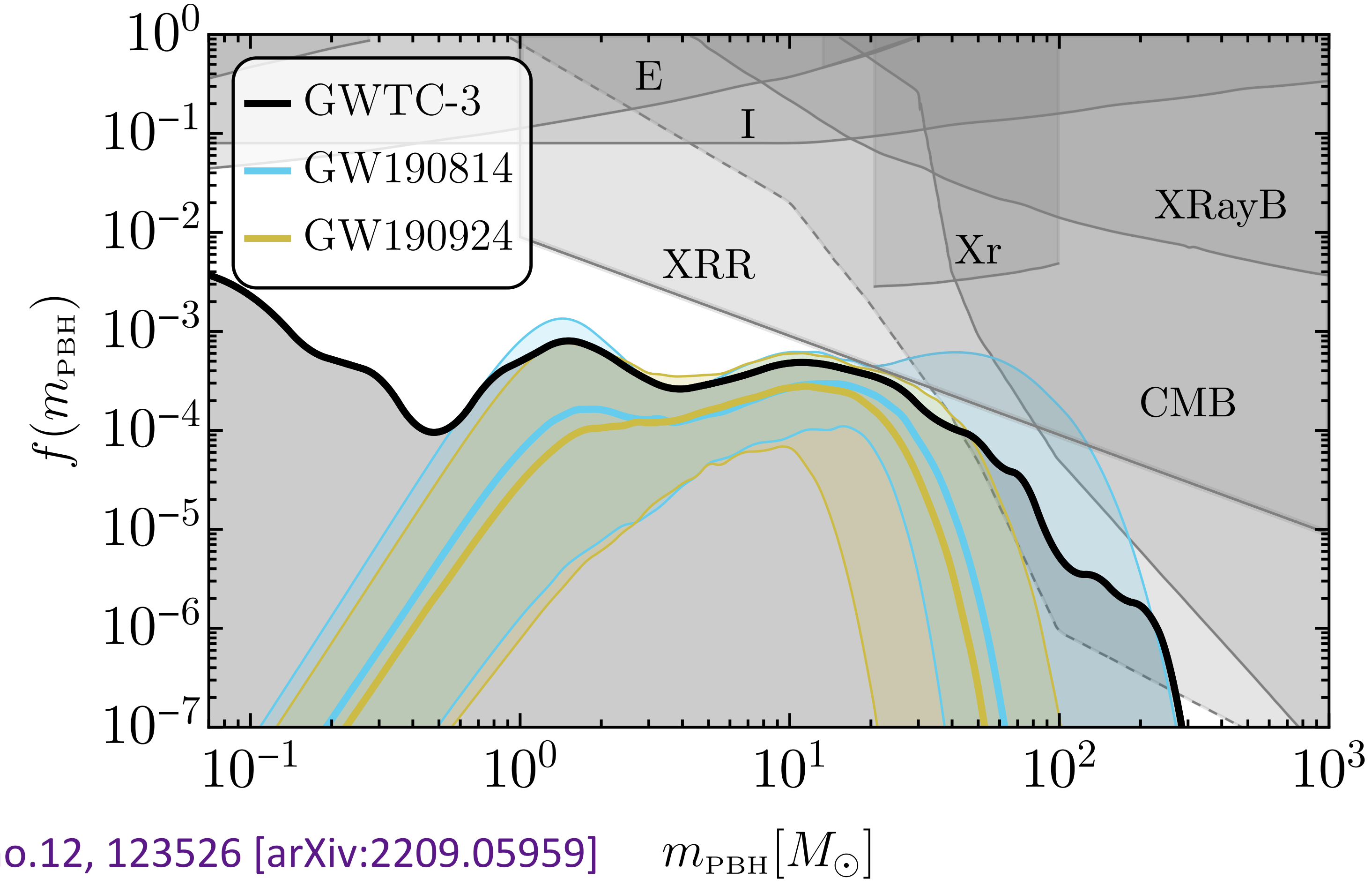
$$\frac{{\rm d}R}{{\rm d}m_1{\rm d}m_2} = \frac{1.6\times 10^6}{{\rm Gpc}^3\,{\rm yr}} f_{\rm PBH}^{\frac{53}{37}} \eta^{-\frac{34}{37}} \left(\frac{t}{t_0}\right)^{-\frac{34}{37}} \left(\frac{M_{\rm tot}}{M_\odot}\right)^{-\frac{32}{37}} S\left(M_{\rm tot}, f_{\rm PBH}\right) \psi(m_1)\psi(m_2)$$

PBH mergers bounded by LVK in stellar mass range

- Around 90 observed events
- O4 observing run ongoing (expected to double this number)



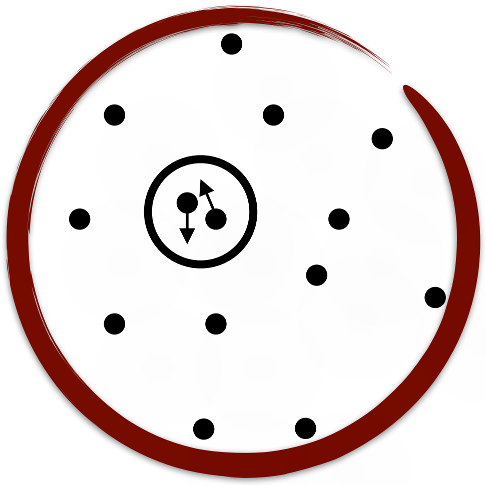
- Constraints on the merger rate giving stringent bounds on PBHs
- Some contribution from PBH mergers to the catalog allowed



GF, Musco, Pani, Urbano, Phys. Rev. D **106** (2022) no.12, 123526 [arXiv:2209.05959]

Scaling in the low-mass range

Early universe binaries



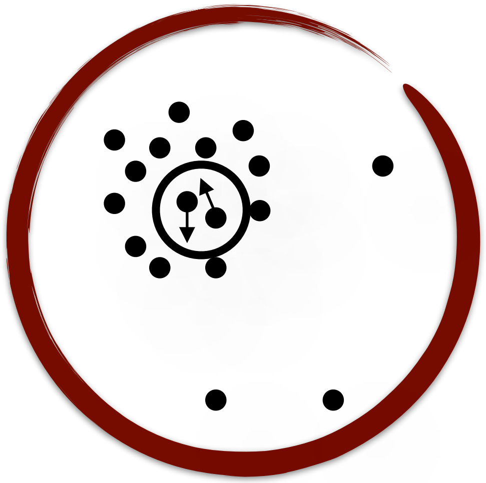
Dominant contribution
even with suppressions

$$\mathcal{R}^{\rm EU}_{\rm BPBH} \sim \bar{n} \times \frac{\mathrm{d}P}{\mathrm{d}j}$$

$$\propto m^{-32/37}$$

Dynamical capture

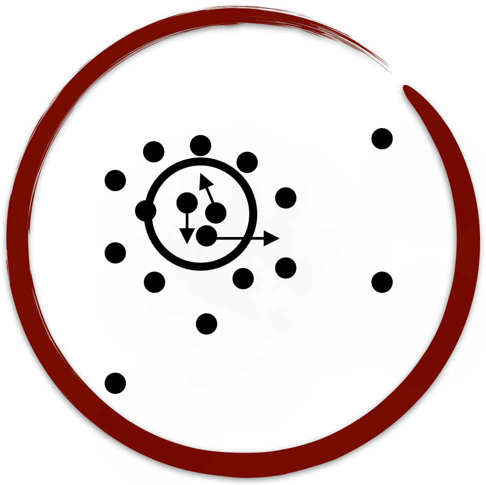
(adopted in Bird et al.)



$$\mathcal{R}^{\rm cap}_{\rm BPBH}(z) \propto R^3 \bar{n} \times \frac{m^2 n^2}{\sigma_v^{11/7}} \propto m_{\rm PBH}^{-11/21}$$

$$\propto m_{\rm PBH}^{-11/21}$$

3-b dynamical interaction



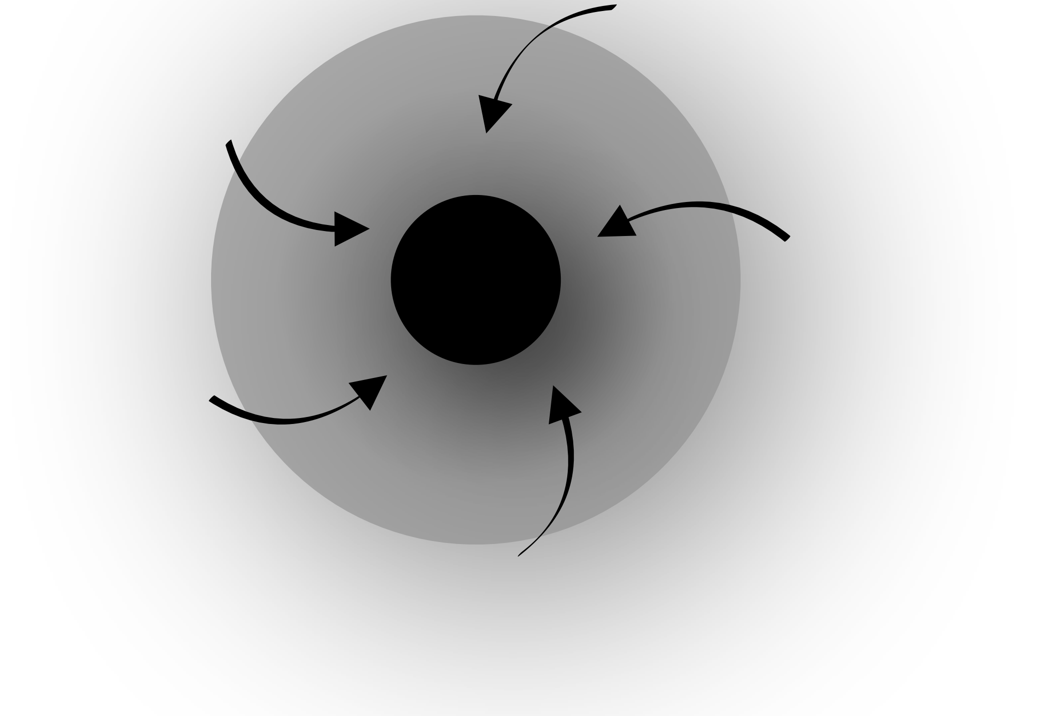
$$\mathcal{R}^{\rm 3b}_{\rm BPBH} \propto Q \times R^3 \bar{n} \times t_{\rm ev} \frac{m^5 n^3}{\sigma_v^9}$$

$$\propto m^{-1+5(1+\gamma)/21}$$

Effect of accretion on the binary evolution - rate

Accretion timescale >> binary period: mass vary adiabatically

$$I_\phi = \frac{1}{2\pi}\int_0^{2\pi} p_\phi \mathrm{d}\phi = L_z \simeq \mathrm{const.}$$
$$I_r = \frac{1}{2\pi}\int_{r_{\min}}^{r_{\max}} p_r \mathrm{d}r = -L_z + \sqrt{M_{\mathrm{tot}}\mu^2 a} \simeq \mathrm{const}$$



Can derive effect of mass accretion on the binary evolution

$$\frac{\dot a}{a} + 3\frac{\dot m_{\mathrm{PBH}}}{m_{\mathrm{PBH}}} = 0$$

Accretion hardens binary (speed up mergers)

Accretion can only enhance the merger rate if sizeable mass growth

$$\dot M_{\mathrm{bin}} = 4\pi\lambda\rho_{\mathrm{gas}} v_{\mathrm{eff}}^{-3} M_{\mathrm{tot}}^2$$

$$R_{\mathrm{PBH}} \propto \left(1 + \int \mathrm{d}t \frac{\dot m_{\mathrm{PBH}}}{m_{\mathrm{PBH}}}\right)^{9/37} \exp\left[\frac{36}{37}\int \mathrm{d}t \frac{\dot m_{\mathrm{PBH}}}{m_{\mathrm{PBH}}}\right]$$

with

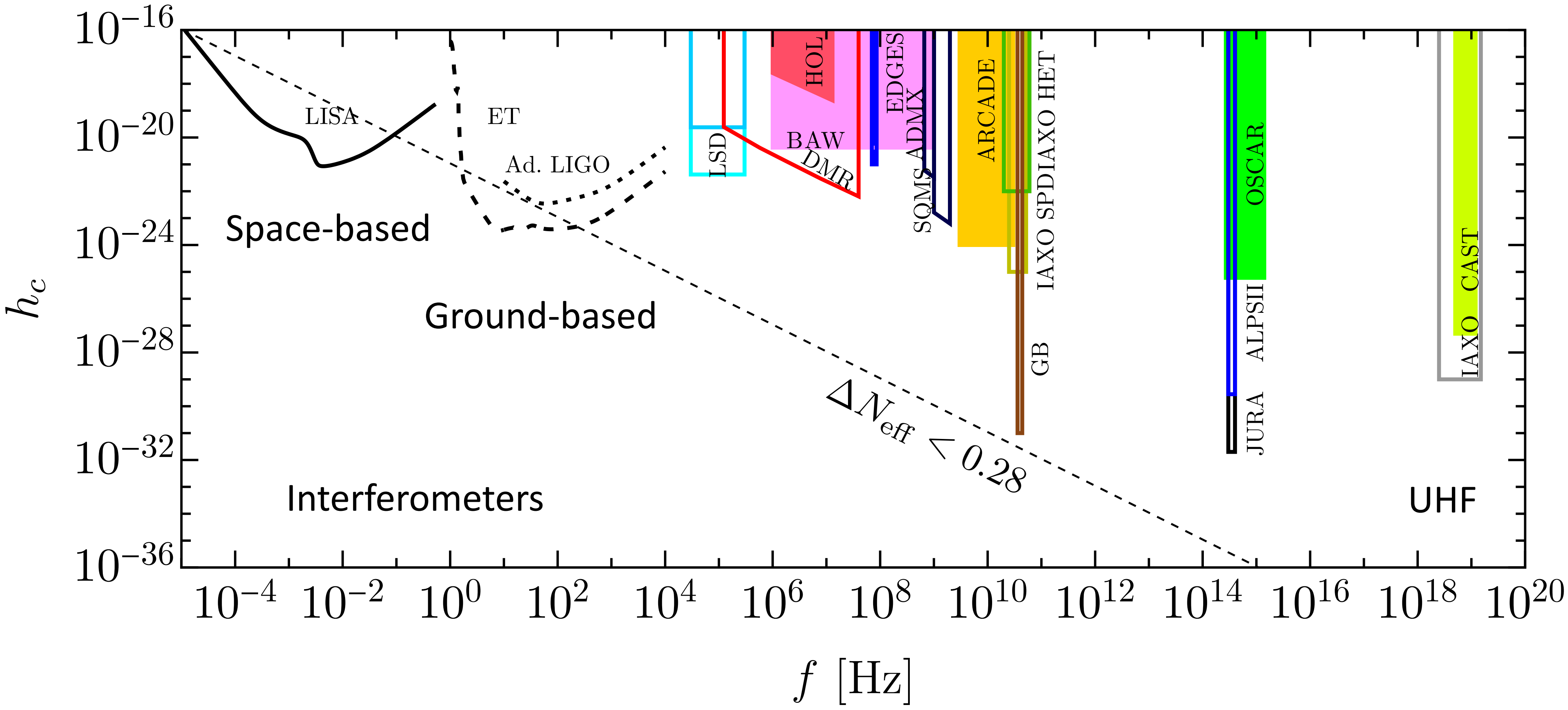
$$\int dt \frac{\dot m_{\mathrm{PBH}}}{m_{\mathrm{PBH}}} \sim 3 \times 10^{-4} \left(\frac{m_{\mathrm{PBH}}}{M_\odot}\right)$$

Ali-Haïmoud, Kovetz and Kamionkowski, Phys. Rev. D 96 (2017) no.12, 123523 [arXiv:1709.06576]
De Luca, **GF**, Pani and Riotto, JCAP 06 (2020), 044 [arXiv:2005.05641]

Signatures of PBH mergers in the UHF-GW window

Landscape of UHF-GW detectors

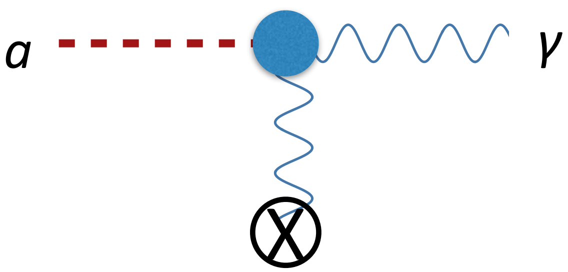
N. Aggarwal, et al. Living Rev. Rel. 24 (2021) no.1, 4 [arXiv:2011.12414]

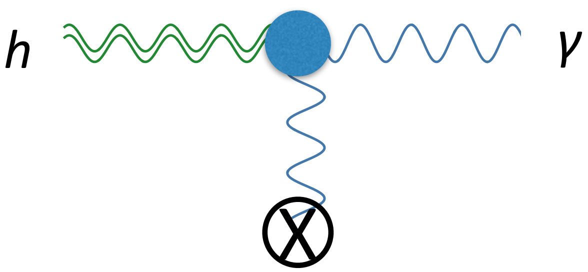


- *Inverse* Gertsenshtein effect

$$\mathcal{L} = -\frac{g_{a\gamma}}{4} a F_{\mu\nu}\tilde{F}^{\mu\nu}$$

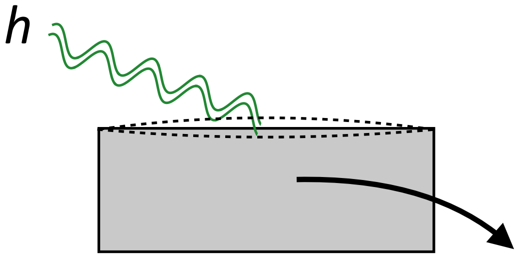
$$\mathcal{L} = -\frac{1}{4}\, g^{\mu\alpha}\, g^{\nu\beta}\, F_{\mu\nu}\, F_{\alpha\beta}$$





- *Mechanical resonators*

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho}(x)\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = 0 \qquad \Gamma \propto \partial h$$



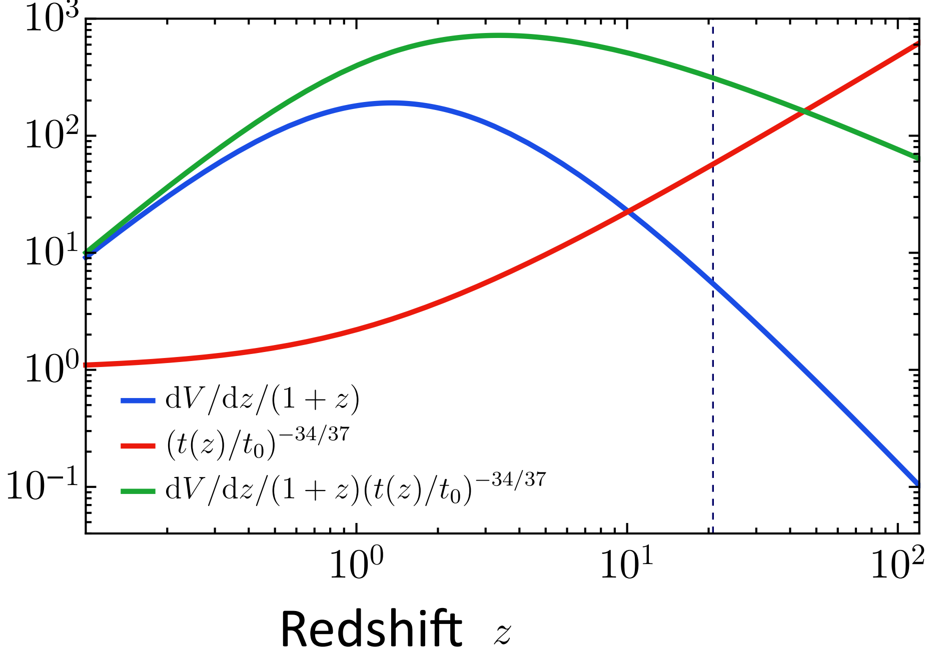
Readout deformation's effect

Stochastic Gravitational Wave Background

$$\Omega_{\mathrm{GW}}(\nu) = \frac{\nu}{\rho_0}\iint \mathrm{d}m_1\mathrm{d}m_2 \int_0^{\nu_{\mathrm{cut}}/\nu-1} \frac{\mathrm{d}z}{(1+z)H(z)}\,\frac{\mathrm{d}R_{\mathrm{PBH}}}{\mathrm{d}m_1\mathrm{d}m_2}\,\frac{\mathrm{d}E_{\mathrm{GW}}(\nu_s)}{\mathrm{d}\nu_s}$$

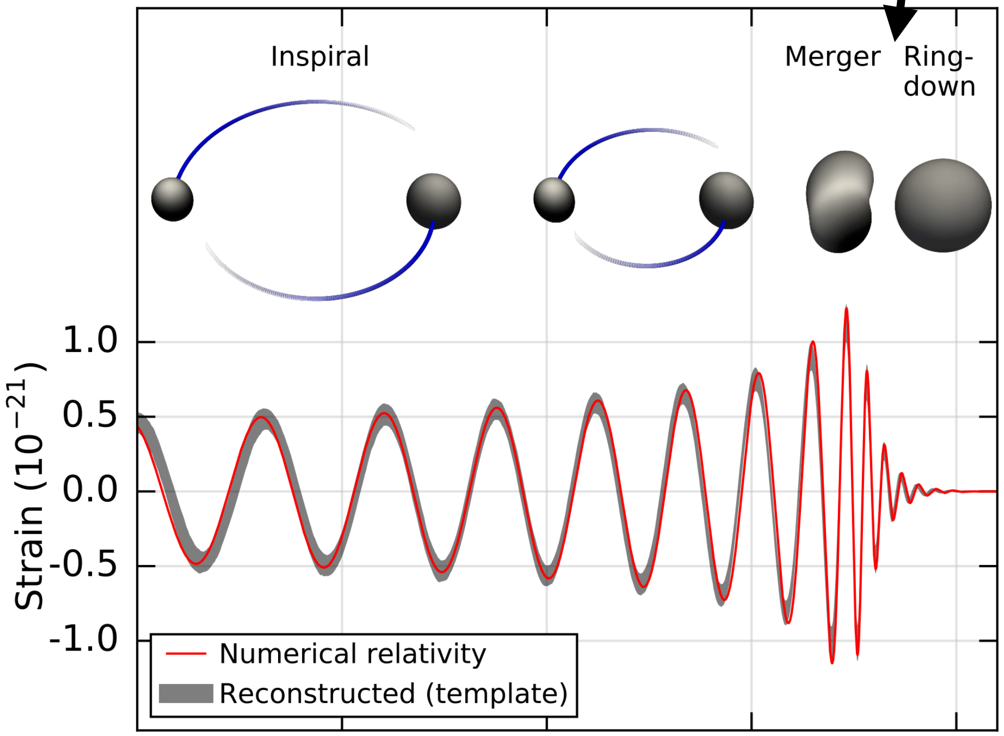
Integrate over PBH mass distribution

Merger rate evolution



This SGWB not subject to CMB bound

Emitted energy spectrum in frequency

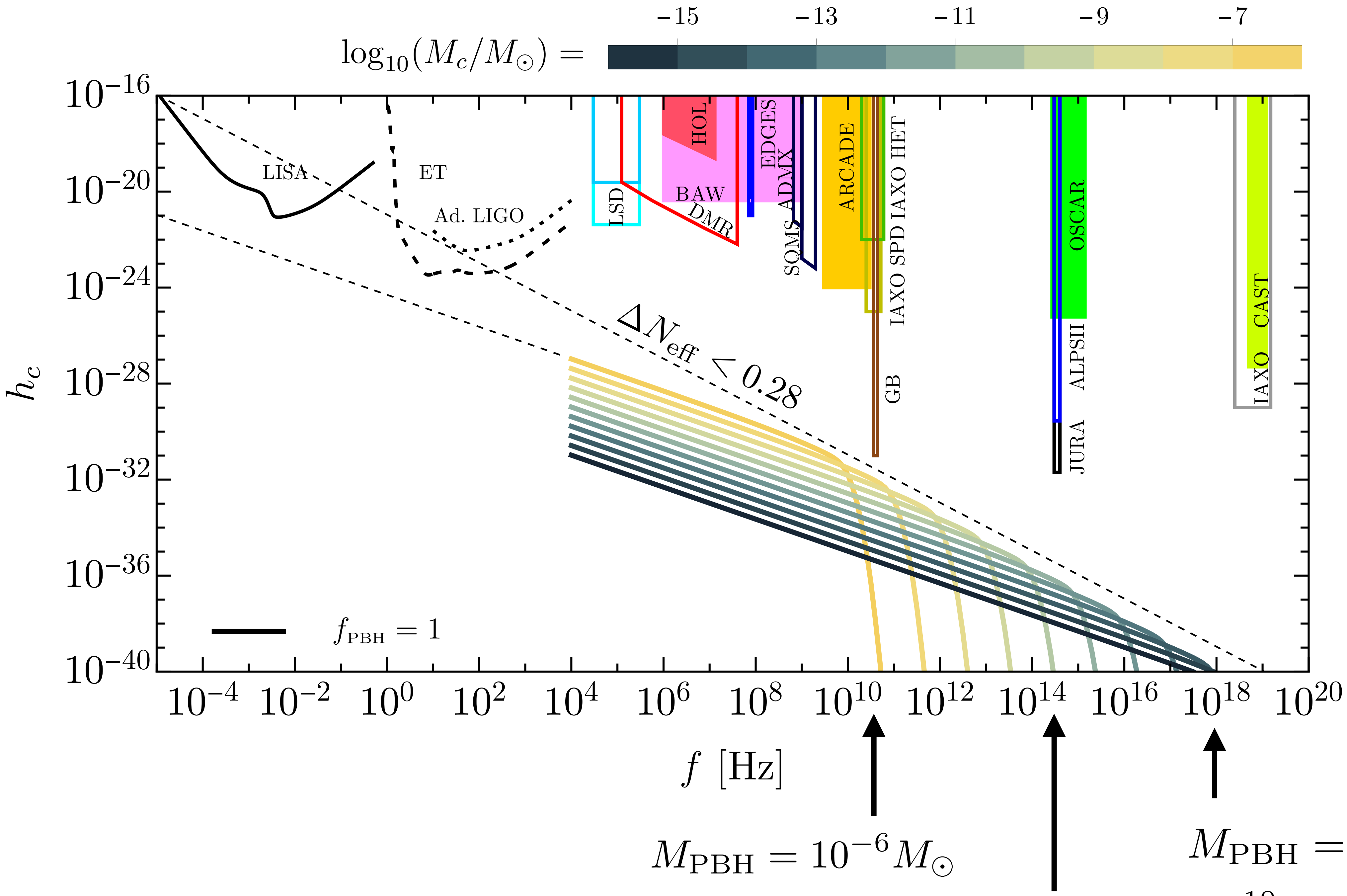


$$\frac{\mathrm{d}E}{\mathrm{d}\nu} = \frac{(G\pi)^{2/3}\mathcal{M}^{5/3}}{3}\begin{cases} \nu^{-1/3} f_1^2 & \nu < \nu_{\mathrm{merger}}, \\ \omega_1 \nu^{2/3} f_2^2 & \nu_{\mathrm{merger}} \le \nu < \nu_{\mathrm{ringdown}}, \\ \omega_2 f_3^2 & \nu_{\mathrm{ringdown}} \le \nu < \nu_{\mathrm{cut}}. \end{cases}$$

Ajith, P., Hannam, M., Husa, S., et al. 2011, Phys. Rev. Lett., 106, 241101

Stochastic Gravitational Wave Background

- Characteristic strain: $h_c(f) \approx \left[\frac{3}{4\pi^2}\left(\frac{H_0^2}{f^2}\right)\Omega_{\rm GW}(f)\right]^{1/2}$



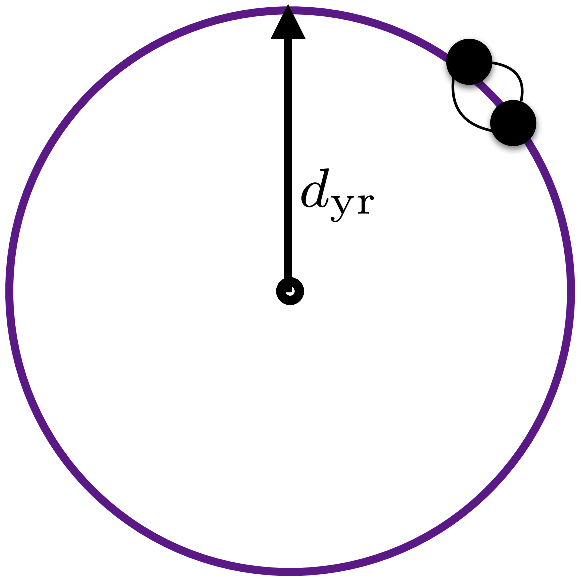
$M_{\rm PBH} = 10^{-6} M_\odot$

$M_{\rm PBH} = 10^{-10} M_\odot$

$M_{\rm PBH} = 10^{-14} M_\odot$

G.Franciolini, A.Maharana and F.Muia, Phys. Rev. D 106 (2022) no.10, 103520 [arXiv:2205.02153]

Individual events: typical source distance



- Light PBH binaries: we assume they follow the dark matter distribution

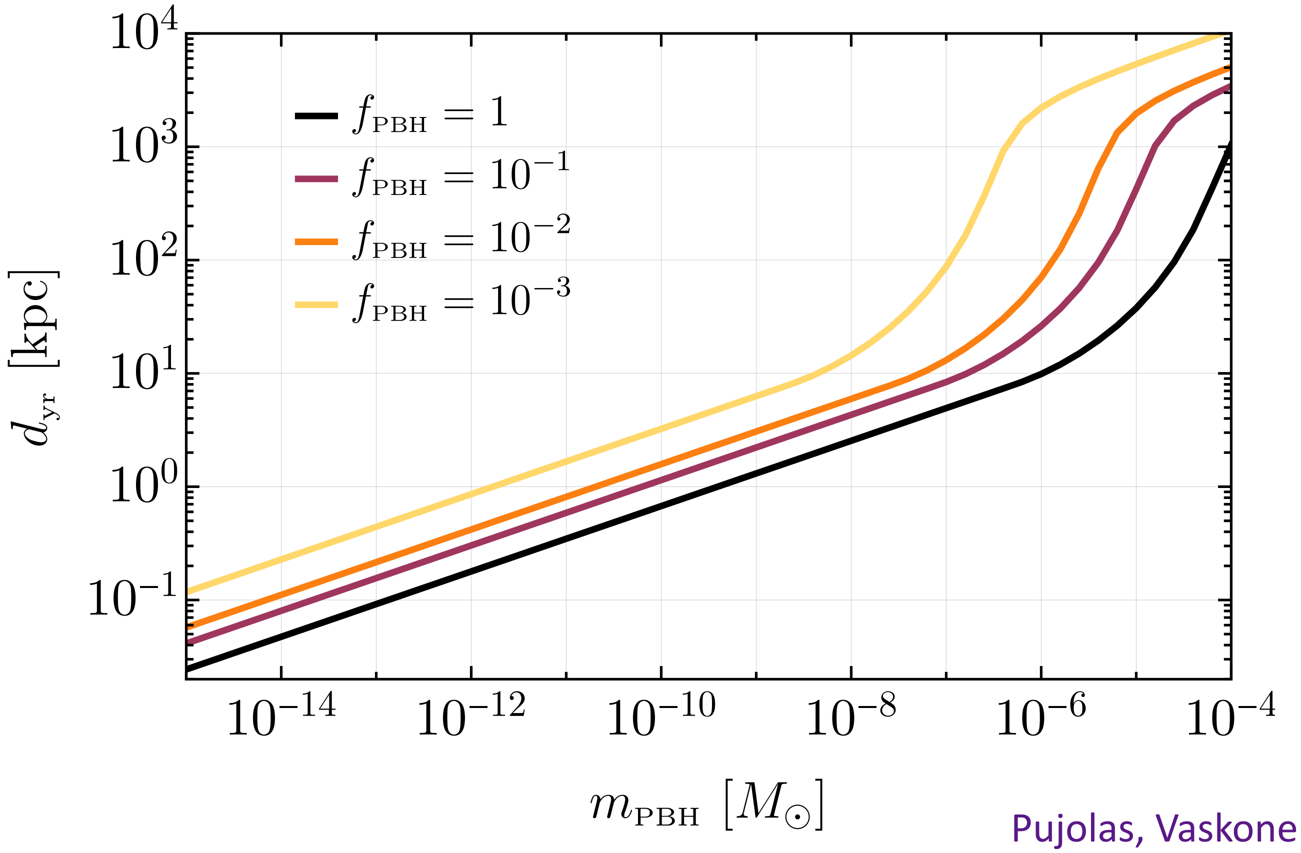
$$n_{\rm PBH} \approx f_{\rm PBH}\rho_{\rm DM}/\langle m_{\rm PBH}\rangle$$

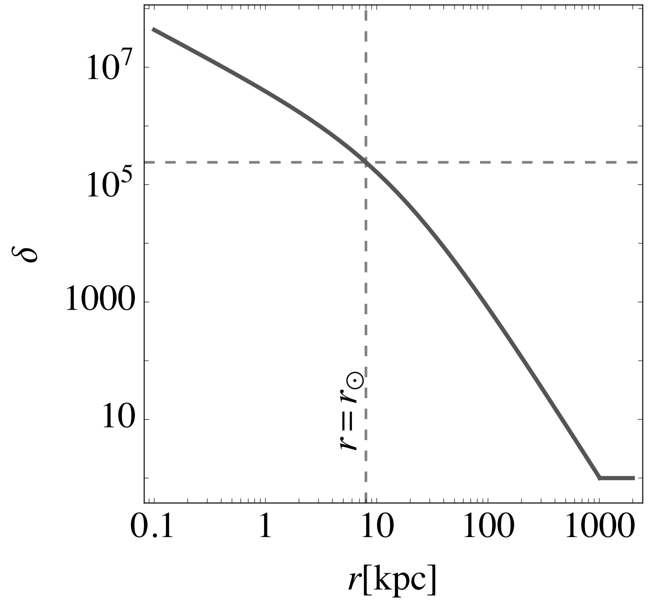
- Typical distance of events with 1/yr rate:

$$N_{\rm yr} \equiv \Delta t \int_0^{d_{\rm yr}} {\rm d}r 4\pi r^2 R_{\rm PBH}^{\rm local}(r)$$

$$R_{\rm PBH}^{\rm local}(r) = \delta(r) R_{\rm PBH}$$

$$\rho_{\rm DM}(r) = \frac{\rho_0}{\frac{r}{r_0}\left(1+\frac{r}{r_0}\right)^2}$$

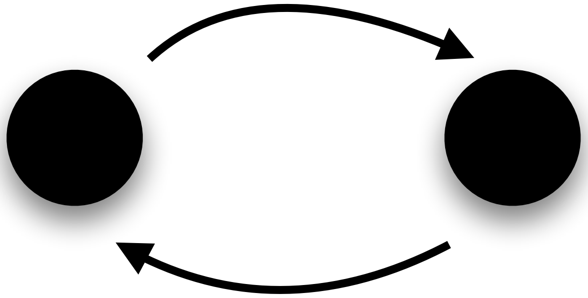




Pujolas, Vaskonen and Veermäe, Phys. Rev. D 104(2021) no.8, 083521 [arXiv:2107.03379]
Domcke, Garcia-Cely and Rodd, Phys. Rev. Lett. 129 (2022) no.4, 041101 [arXiv:2202.00695]
G.Franciolini, A.Maharana and F.Muia, Phys. Rev. D 106 (2022) no.10, 103520 [arXiv:2205.02153]

Limiting factor: signal duration

- Binary evolution dominated by gravitational wave emission



$$h_{+,\times}(t) = h_0\, F_{+,\times}(\theta)\, G_{+,\times}(t)$$

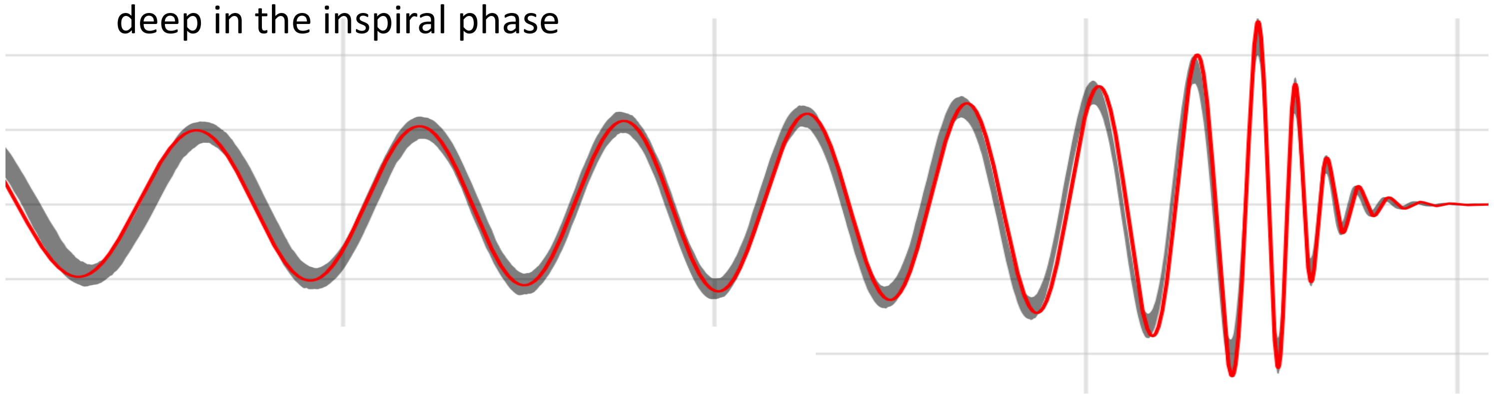
- Amplitude as a function of the system masses, source distance and frequency:

$$h_0 \quad \simeq 9.77 \times 10^{-34} \left(\frac{f}{1\,\mathrm{GHz}}\right)^{2/3} \left(\frac{m_{\mathrm{PBH}}}{10^{-12}\,M_\odot}\right)^{5/3} \left(\frac{d_L}{1\,\mathrm{kpc}}\right)^{-1}$$

- GW emission dictates the time evolution of the binary. Time to coalescence:

$$\tau(f) \approx 83\,\mathrm{sec} \left(\frac{m_{\mathrm{PBH}}}{10^{-12} M_\odot}\right)^{-5/3} \left(\frac{f}{\mathrm{GHz}}\right)^{-8/3}$$

← Almost monochromatic only deep in the inspiral phase



At maximum frequency

$$\Delta t \sim \mathcal{O}(1) \times \frac{1}{f_{\mathrm{ISCO}}}$$

Computing the detector sensitivities to isolated mergers

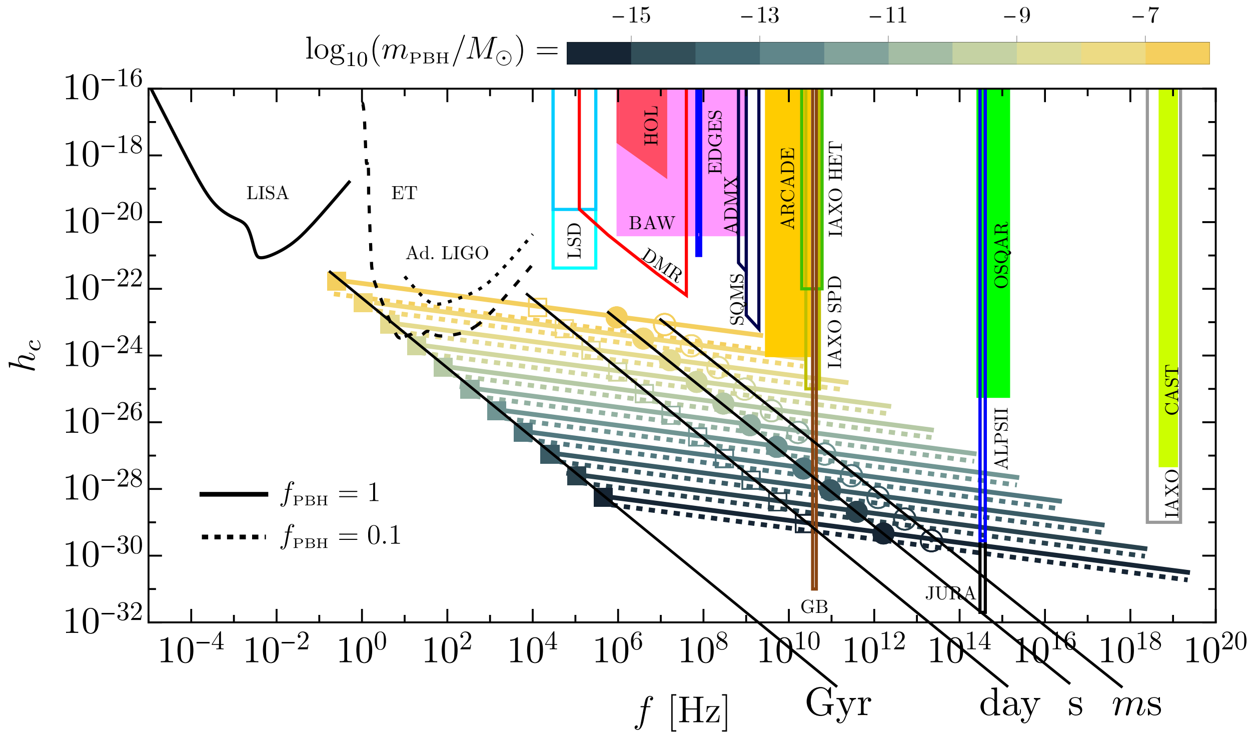
- Typical timescale for the frequency evolution: $t_{\mathrm{f}} = f/\dot{f}$
- Effective number of cycles per frequencies: $N_{\text{cycles}} \sim f^2/\dot{f}$
- Observation time: either dictated by the **detector** or by the time it takes to the **binary** to span the bandwidth

---

Events with 1/yr rate and PBH abundance = totality of dark matter (optimistic)

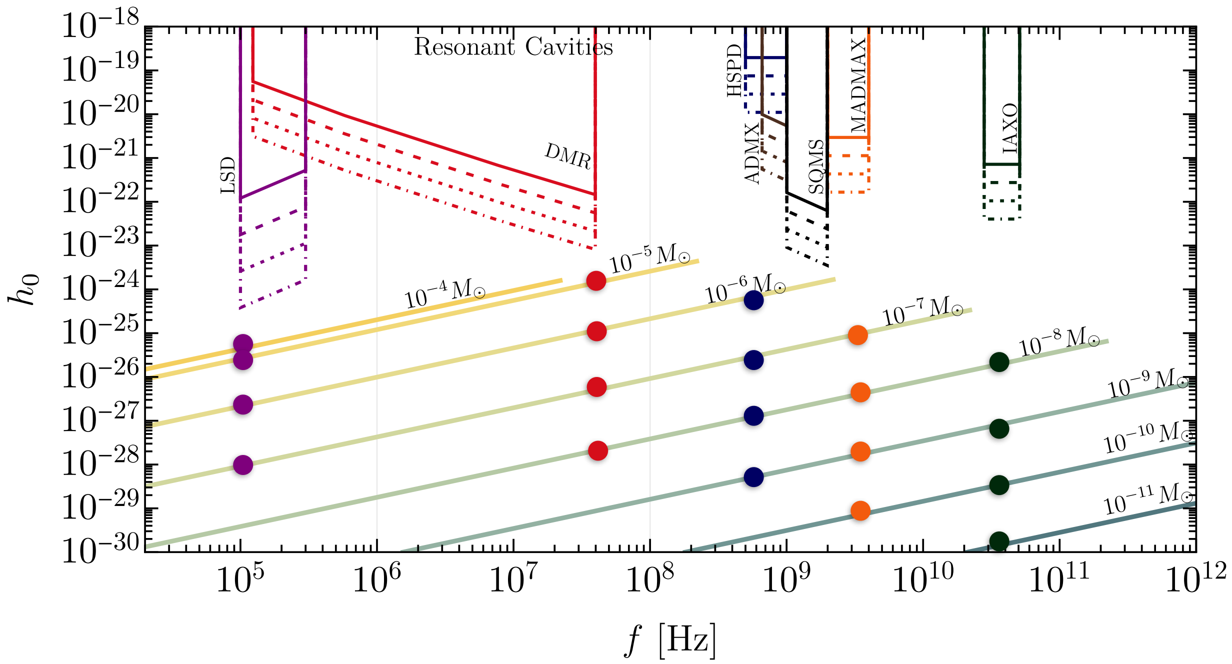
“Naive” comparison to sensitivities

Characteristic strain: $h_c(f) \simeq \sqrt{N_{\text{cycles}}}\, h_0$



Sensitivities accounting for binary evolution

GW amplitude: h_0



G.Franciolini, A.Maharana and F.Muia, Phys. Rev. D 106 (2022) no.10, 103520 [arXiv:2205.02153]

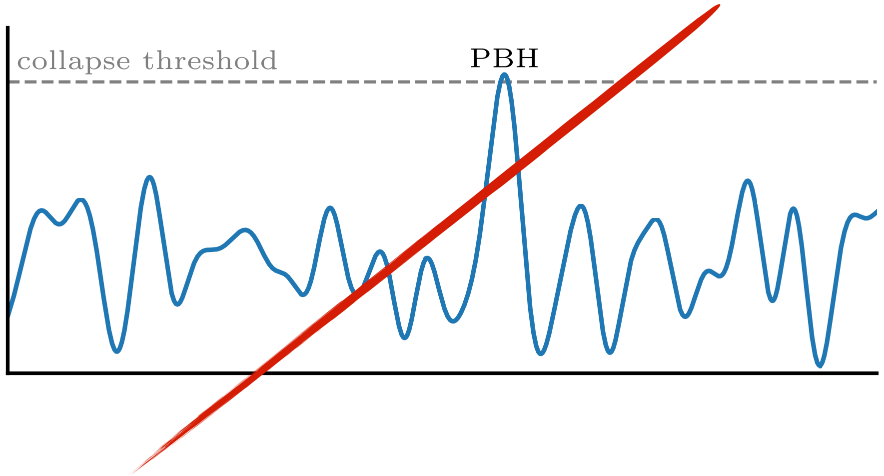
Maximum theoretical merger rate (?)

M. Raidal, V. Vaskonen and H. Veermäe, JCAP 09 (2017), 037 [arXiv:1707.01480]
T. Bringmann, P. F. Depta, V. Domcke and K. Schmidt-Hoberg, Phys. Rev. D 99 (2019) no.6, 063532 [arXiv:1808.05910]
V. De Luca, G. Franciolini, P. Pani and A. Riotto, JCAP 11 (2021), 039 [arXiv:2106.13769]

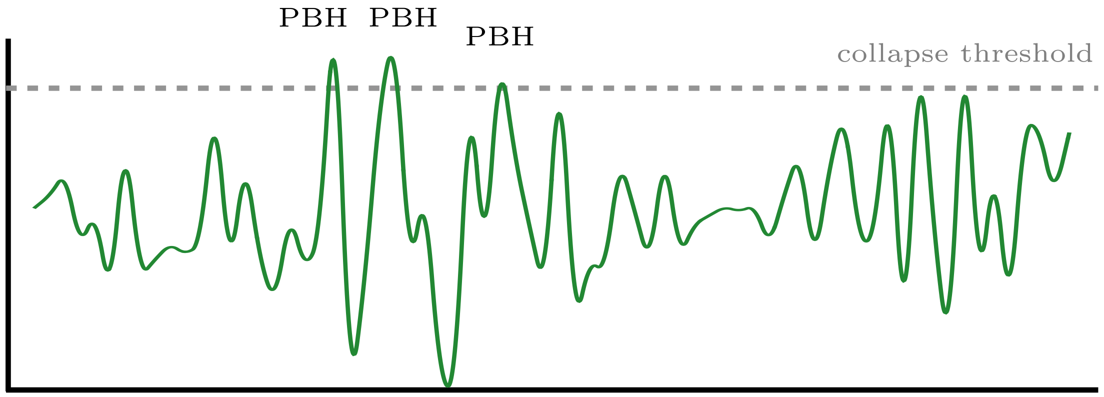
Modify PBH initial conditions, including initial PBH clustering (e.g. local NGs)

$$\left\langle \frac{\delta\rho_{\rm PBH}(\vec{x},z)}{\bar{\rho}_{\rm DM}}\,\frac{\delta\rho_{\rm PBH}(0,z)}{\bar{\rho}_{\rm DM}}\right\rangle = \xcancel{\frac{f^2_{\rm PBH}}{n_{\rm PBH}}\delta_{\rm D}(\vec{x})} + \xi(x,z).$$

$$\delta_{\rm dc} \approx 1 + \xi_{\rm PBH}(x)$$



vs

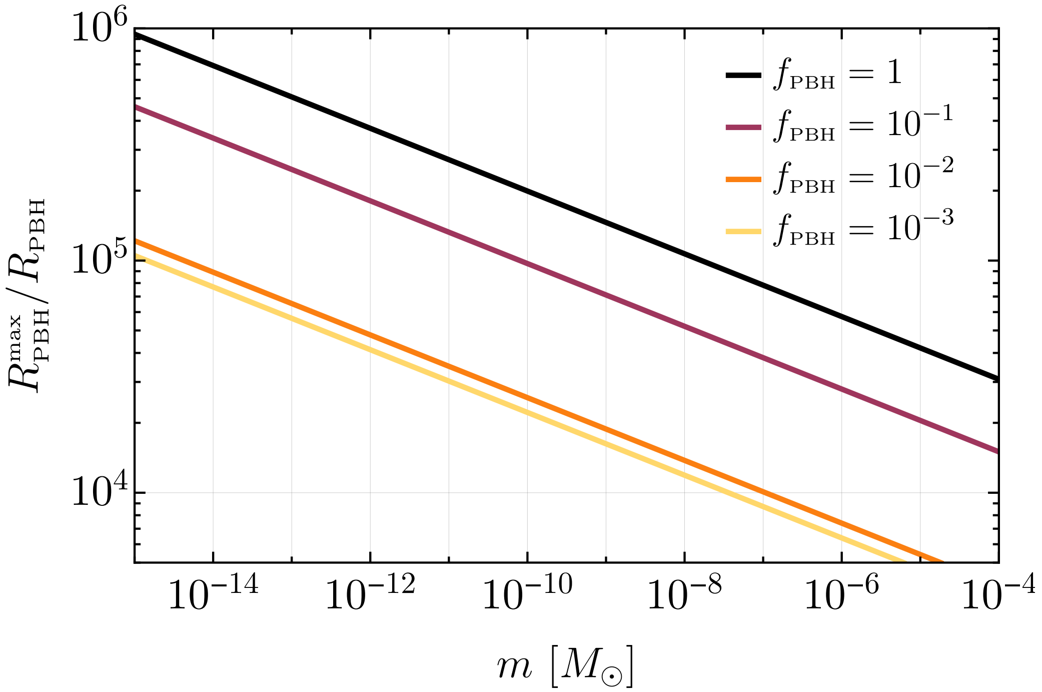


Neglecting binary suppression (strong assumption!), maximise the merger rate of early binaries

$$\mathrm{d}n_{\rm b} = \frac{1}{2}e^{-\bar{N}(y)}\mathrm{d}n(m_1)\mathrm{d}n(m_2)\mathrm{d}V(x_0)$$

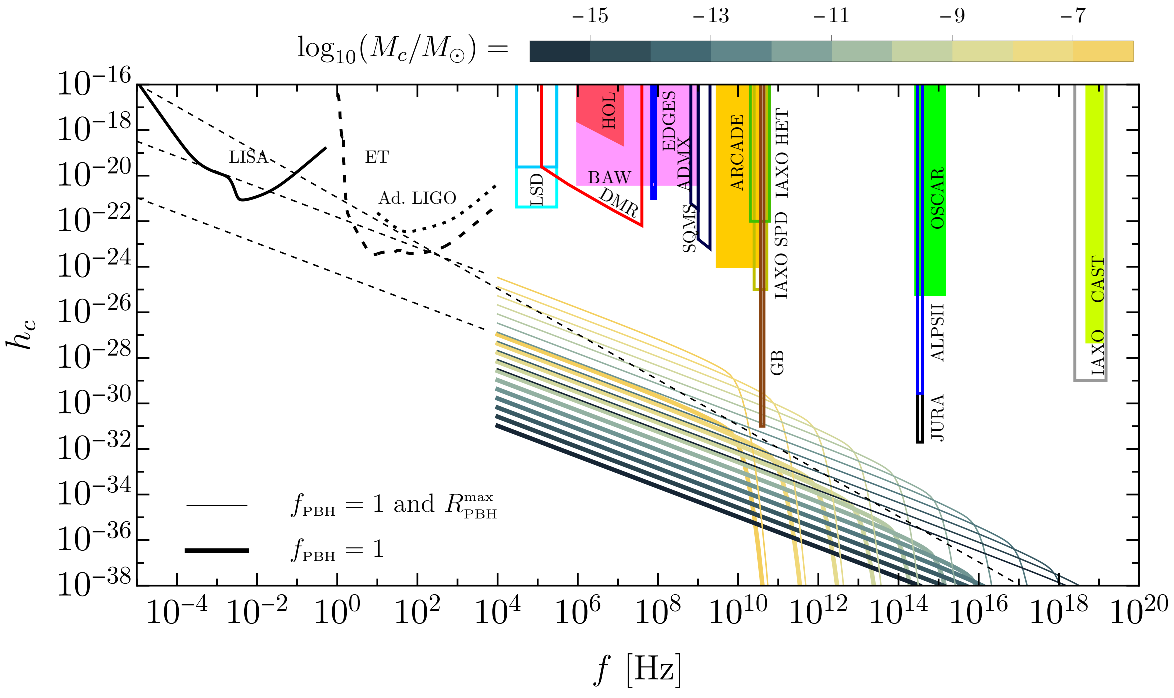
SGWB: $h_c \sim \Omega_{\rm GW}^{1/2} \sim R_{\rm PBH}^{1/2}$

Individual mergers: $h_0 \sim 1/d_{\rm yr} \sim R_{\rm PBH}^{1/3}$

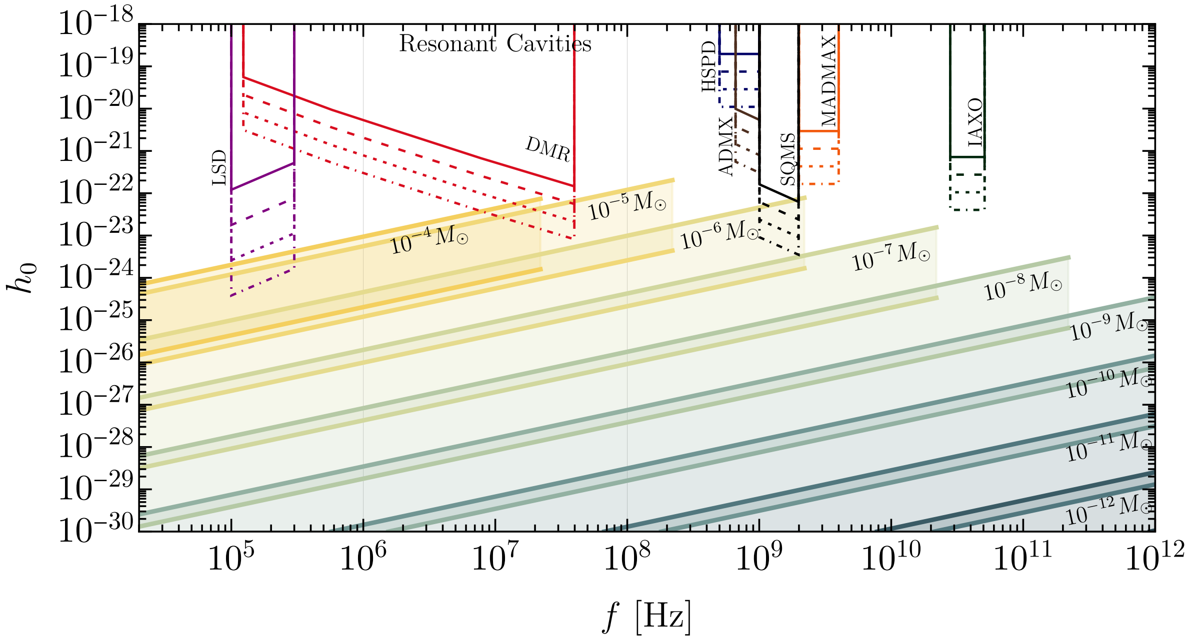


Conclusions

Stochastic gravitational wave background

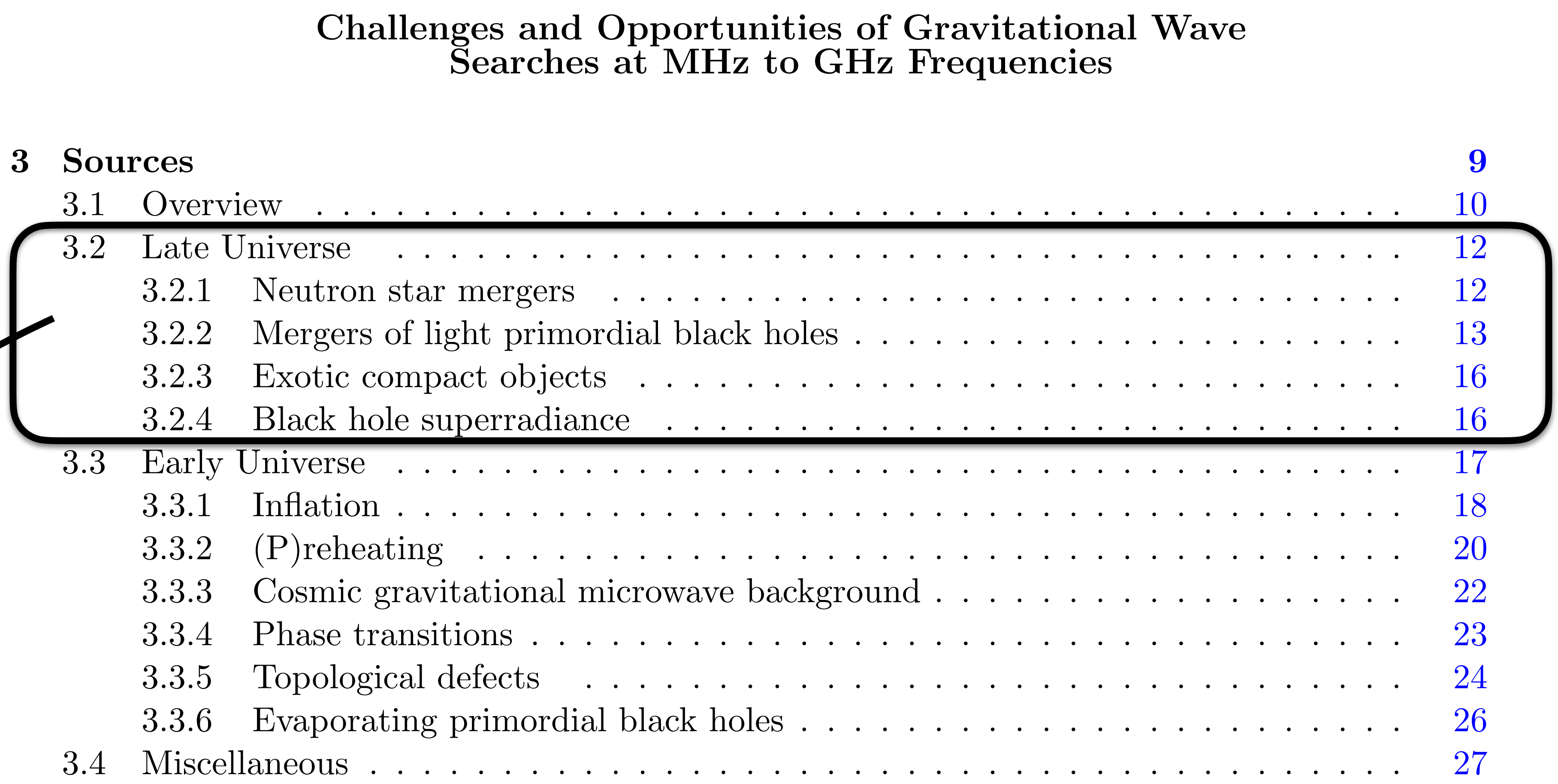


Individual mergers



- Primordial Black holes represent a special dark matter candidate, constrained only in a portion of masses
- Much of the interesting parameter space corresponds to mergers showing up in the UHF window
- Reaching competitive sensitivity is hard, interesting synergies with experimental searches for axions

Update of the Living Review



Challenges and Opportunities of Gravitational Wave Searches at MHz to GHz Frequencies

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- Group of people gathered and discussion started
- Include suggestions/new ideas in the google doc
- Email: gabriele.franciolini@cern.ch

Gabriele Franciolini

Thanks!

6-12-2023 - UHF-GW workshop - CERN TH Colloquium

Gabriele Franciolini

Backup

6-12-2023 - UHF-GW workshop - CERN TH Colloquium

Signatures of evaporated PBHs in the UHF GW window

- Emission of GW from the formation mechanism (e.g. enhanced scalar perturbations)

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} \approx \mathcal{S}_{ij}\left(\zeta\zeta\right)$$

K. Tomita, Prog. Theor. Phys. 54, 730 (1975).
S. Matarrese, O. Pantano, and D. Saez, Phys. Rev. Lett. 72, 320 (1994), [arXiv:9310036].
V. Acquaviva, *et al.* Nucl. Phys. B 667, 119 (2003), [arXiv:0209156].
S. Mollerach, D. Harari, and S. Matarrese, Phys. Rev. D 69, 063002 (2004), [arXiv:0310711].
K. N. Ananda, C. Clarkson, and D. Wands, Phys. Rev. D 75, 123518 (2007), [arXiv:0612013].
...

- Typical frequency related to the mass:

$$f \simeq 5\,\mathrm{kHz}\left(\frac{m_{\mathrm{H}}}{10^{-24}M_{\odot}}\right)^{-1/2}$$

A. D. Dolgov and D. Ejlli, Phys. Rev. D 84 (2011) 024028

- SGWB from emission of gravitons through hawking evaporation

$$T_{\mathrm{PBH}} = \frac{m_{\mathrm{Pl}}^2}{8\pi m_{\mathrm{PBH}}} \simeq 2\times 10^{13}\,\mathrm{GHz}\left(\frac{m_{\mathrm{PBH}}}{3\times 10^{-19}M_{\odot}}\right)^{-1}$$

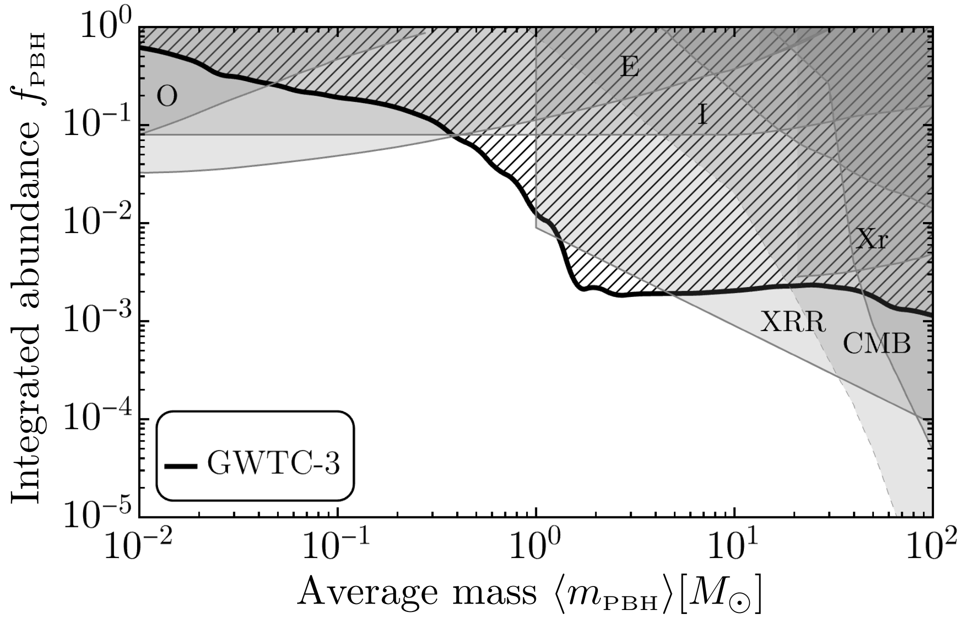
- UHF GW could probe the formation of evaporated PBHs

- Early universe emission of GW, subject to Delta-Neff bound

Current LVK bounds on PBH mergers

G. Franciolini, I. Musco, P. Pani and A. Urbano, Phys. Rev. D **106** (2022) no.12, 123526 [arXiv:2209.05959]

- Multi-population analysis of GWTC-3 including both astro+primordial mergers
- PBH mass distribution from primordial curvature spectrum and QCD effects
- Stellar mass PBHs forced to be $f_{\rm PBH} \lesssim 10^{-3}$
- Current data allow for a PBH contribution to the catalog (currently very difficult to confirm…)


Stringent bounds from LVKC

PBH merger smoking gun signatures:

- Subsolar BBH masses: no confident detections

A. H. Nitz and Y. F. Wang, Phys. Rev. D **106** (2022) no.2, 023024 [arXiv:2202.11024]

R. Abbott *et al,* [LIGO Scientific, VIRGO and KAGRA], [arXiv:2212.01477]

- High redshift mergers: only accessible by next generation of detectors (e.g. M. Branchesi, M. Maggiore, *et al.* [arXiv:2303.15923])

Population studies, subject to large uncertainties:

- Search for mass-spin correlations induced by PBH accretion

G. Franciolini and P. Pani, Phys. Rev. D **105** (2022) no.12, 123024 [arXiv:2201.13098]

- Full multi-pop inference with astro population synthesis models

M. Zevin *et al*, Astrophys. J. **910** (2021) no.2, 152 [arXiv:2011.10057]

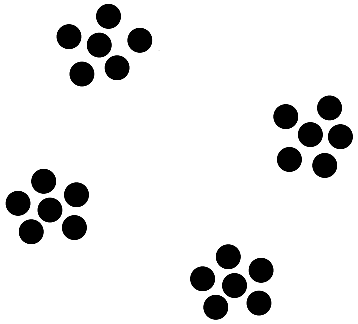
G. Franciolini *et al*, Phys. Rev. D **105** (2022) no.8, 083526 [arXiv:2105.03349]

…

No initial clustering for stellar masses

V. De Luca, G. Franciolini, A. Riotto and H. Veermäe, Phys. Rev. Lett. **129** (2022) no.19, 191302 [arXiv:2208.01683]

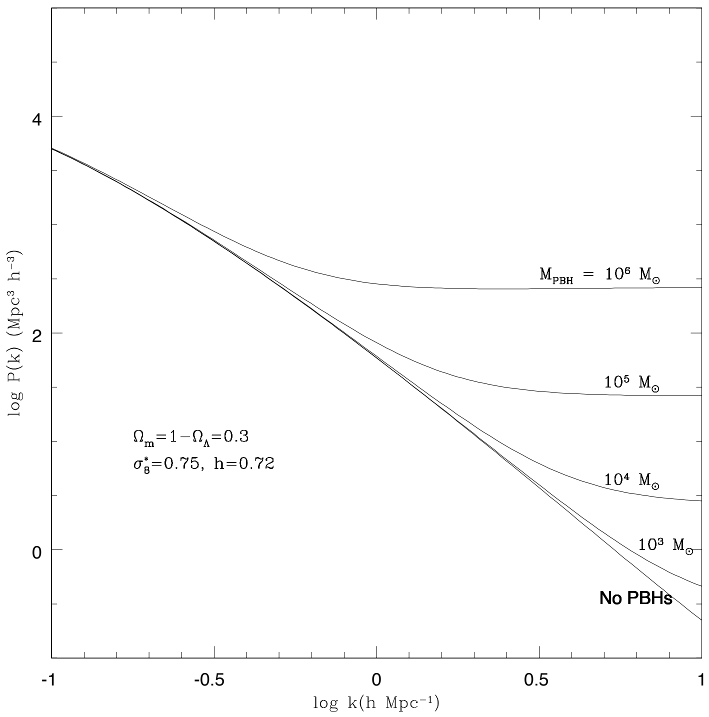
Current GW bounds based on the merger rate
computation that assume Poisson initial conditions



Inducing initial clustering beyond Poisson could suppress the rate

S. Young and C. T. Byrnes, JCAP **03** (2020), 004 [arXiv:1910.06077]
V. Vaskonen and H. Veermäe, Phys. Rev. D **101** (2020) no.4, 043015 [arXiv:1908.09752]
V. Atal, A Sanglas and N. Triantafyllou, JCAP \textbf{11} (2020), 036 [arXiv:2007.07212]
V. De Luca, V. Desjacques, G. Franciolini and A. Riotto, JCAP **11** (2020), 028 [arXiv:2009.04731]

Initial clustering (beyond Poisson) cannot be invoked to evade lensing/GW bounds



Clustered initial conditions for PBH DM would generate too large isocurvature perturbations which are ruled out by Lyman-alpha data

N. Afshordi, P. McDonald and D. N. Spergel, Astrophys. J. Lett. **594** (2003), L71-L74 [arXiv:astro-ph/0302035]
R. Murgia, G. Scelfo, M. Viel and A. Raccanelli, Phys. Rev. Lett. 123 (2019) no.7, 071102 [arXiv:1903.10509]

$$f_{\rm PBH} M_{\rm cl} = f_{\rm PBH} N_{\rm cl} M_{\rm PBH} \lesssim 60\, M_\odot$$