Ultra-high frequency gravitational waves: where to next?

Topical discussion (comparing sensitivities)
Ultra-High Frequency GWs (UHFGWs)

Detecting high-frequency gravitational waves with microwave cavities

Asher Berlin, Diego Blas, Raffaele Tito D’Agnolo, Sebastian A. R. Ellis, Roni Harnik, Yonatan Kahn, and Jan Schütte-Engel

\[ j_{\text{eff}} \sim \omega_g h B_0 \]

\[ j_{\text{eff}} \supset g_{\alpha \gamma} \partial_\gamma a B_0 \approx \omega_a \theta_a B_0 \]

\[ E_a = g_{\alpha \gamma} a B_0 = \theta_a B_0 \]

\[ j_{\text{eff}}^\mu = \partial_\nu \left( \frac{1}{2} h F^\mu{}_{\nu} + h_\alpha^\nu F^\mu{}_{\alpha} - h_\alpha^\mu F_\alpha{}_{\nu} \right) \]

identifying \( \theta_a \sim h \)

\[ j_{\text{eff}} \sim \omega_g h B_0 \]
Brief History of the Resonant-Mass Detector

- Niobe
- Explorer
- Auriga
- Mario Schenberg

Webber's Pioneering Work

- Mechanical Mass Quadrupole Harmonic
- Designs to date:
  - Eagle: 10 Hz
  - Explorer: 10 Hz
  - Niobe: 10 Hz

Weber's suggestions:

- Detachable Bar: 10 Hz
- Caged crystals: 10 Hz

The University of Western Australia,
Department of Physics,
1993.

Doctor of Philosophy

1989 - 1993 UWA PhD

NOISE SENSITIVE OSCILLATORS

Gravitational Wave Detection and Low-Noise Sapphire Oscillators

This thesis is presented for the degree of

Graziana Cattaneo

Michael Edmund Four
History Lesson: Resonant Bars vs Interferometers
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In the Early 1990’s Interferometer Technology under Development and Threatening the Resonant bars
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Truncated Icosahedral Gravitational Wave Antenna

Warren W. Johnson and Stephen M. Merkowitz
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803
(Received 11 January 1993)

We propose a new type of resonant-mass gravitational wave detector, a truncated icosahedral gravitational wave antenna. It will be omnidirectional, and able to measure the direction and polarization of a detected wave. We solve a model for this system, calculate the strain noise spectrum, and conclude that its angle-averaged energy sensitivity will be 56 times better than the equivalent bar-type antenna with the same noise temperature.

PACS numbers: 04.80.+z, 06.70.Dn

FIG. 1. The truncated icosahedral gravitational wave antenna (TIGA) with secondary resonator locations indicated.
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\[ \tilde{h}(f, N) = \sqrt{N} \tilde{h}(f, N = 1) \]

\[ N \equiv k_b T_n / h \omega_s \]

FIG. 2. The calculated strain noise spectrum \( \tilde{h}(f) \) for various detectors. Solid lines: for a "xylophone" of TIGA detectors with quantum limited sensor noise, for a single channel (i.e., a single linear polarization arriving from an arbitrary direction). Dashed lines: a xylophone of equivalent bar antennas with quantum limited sensor noise, for the optimum orientation of the wave. Dotted line: for the first generation LIGO detector, for the optimum orientation of the wave [10].
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- Conclusion -> two detector types are complimentary
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• Conclusion -> two detector types are complimentary
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• Discussions with LSU group lead me to figure out for NIOBE Spectral Strain for Niobe
Gravitational Wave Detection and Low–Noise Sapphire Oscillators

Michael Edmund Tobar

This thesis describes the development of an ultra-low noise sapphire resonator oscillator that is tunable over X-band. While undertaking this task the author has explained some interesting and very useful phenomena in regards to the design and understanding of multi-mode resonant cavities and oscillators. The oscillator was constructed to operate as the pump oscillator in the superconducting parametric transducer system, attached to a 1.5-tonne niobium resonant bar gravitational wave detector. The effects of incorporating the pump oscillator with the parametric transducer and resonant bar system are analyzed to enable prediction of the detector sensitivity. The detector was the first massive precision optomechanical system ever built. With the resurgence in interest in resonant detectors, this thesis has important work on multi-mode acoustic systems, coupled to a highly sensitive parametric transducer relevant for many fields of research today.
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Noise Budget

![Graph showing noise budget](image)

SNR/Hz/mK

![Graph showing SNR vs frequency](image)
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Noise Budget

- Improve Transducer and second mass -> Improve bandwidth and sensitivity

---

Detection Sensitivity Analysis of a Two-Mode Resonant Bar Antenna with a Parametric Transducer

RESEARCH ARTICLE | APRIL 01 1995

Sensitivity analysis of a resonant-mass gravitational wave antenna with a parametric transducer

Michael Edmund Tobar, David Gerald Blair

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Noise Budget

\[ \log_{10}[h^*(f)] \]

Improve Transducer and second mass -> Improve bandwidth and sensitivity

\[ \log_{10}[h'(f)] \]

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CHAPTER 7

DETECTION SENSITIVITY ANALYSIS OF A TWO-MODE RESONANT BAR ANTENNA WITH A PARAMETRIC TRANSUDER

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https://doi.org/10.1063/1.164003
Spectral Strain

Allegro Noise Theory and Experiment: 1995

Two mode, low $\beta$, high series noise, 4K

High $\beta$, low noise, 3 mode, 25 mK

Figure 2: Top: Binary Neutron Star (BNS) range evolution of the LIGO and Virgo detectors from the start of O2 in November 2017 to the end of O3 in March 2020. The broken axes remove the time between each observing run. Bottom: Representative amplitude spectral density of the three detectors’ strain sensitivity in each observing run. The O3 spectra shown are taken from O3a.
Spectral Strain

Allegro Noise Theory and Experiment: 1995

\[ \Delta f_d = h^+(f_0)^2 \int_0^\infty \frac{1}{h^+(f)^2} df \]

High \( \beta \), low noise, 3 mode, 25 mK

Two mode, low \( \beta \), high series noise, 4K

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LIGO 2020

May Define Detector Bandwidth From Spectral Sensitivity

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\[
\Delta f_d = h^+(f_0)^2 \int_0^\infty \frac{1}{h^+(f)^2} df
\]

**Compare Sensitivity to Impulse (Burst)**

\[
F_1(t) = F_g \delta(t) [N],
\]

**LIGO 2020**

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Compare Sensitivity to Impulse (Burst)

\[ F_1(t) = F_g \delta(t) [N], \]

\[ \Delta f_d > \Delta f_s \]

\[ h = H_s(f_0) \Delta f_s = \sqrt{\Delta f_s h^+(f_0)/2} \]

LIGO 2020

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\[ \Delta f_d < \Delta f_s \]

\[ h = H_s(f_0) \Delta f_s = \frac{\Delta f_s h^+(f_0)}{2\sqrt{\Delta f_d}} \]

Compare Sensitivity to Impulse (Burst)

\[ F_1(t) = F_g \delta(t) [N], \]

\[ H_1(f) \text{ strain/Hz} \]

\[ \Delta f_d > \Delta f_s \]

\[ h = H_s(f_0) \Delta f_s = \sqrt{\Delta f_s} h^+(f_0)/2 \]

NIOBE

Two mode, low \( \beta \), high series noise, 4K

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What GW Signals can we Detect with a Resonant Mass?
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Burst detection: maximum total bandwidth important
Search for pulsar signals (CW) in spectral minima.
More bandwidth = more sources at same sensitivity

Burst detection: maximum total bandwidth important

What GW Signals can we Detect with a Resonant Mass?
Stochastic background: use two detectors with coinciding spectral minima.

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Improving Sensitivity with Improved Transducers

Parametric Transducers for the Advanced Cryogenic Resonant-Mass Gravitational Wave Detectors

Michael E. Tobar, Eugene N. Ivanov, David G. Blair

General Relativity and Gravitation, Vol. 32, No. 9, 2000
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High Sensitivity Gravitational Wave Antenna with Parametric Transducer Readout

D. G. Blair, E. N. Ivanov, M. E. Tobal, P. J. Turner, F. van Kann, and I. S. Heng
Physics Department, University of Western Australia, Nedlands, Western Australia, 6009
(Received 4 April 1994; revised manuscript received 27 September 1994)

\[ z(t)^2 = [x(t) - x(t - \Delta t)]^2 + [y(t) - y(t - \Delta t)]^2 \]
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\[
p(T) = \frac{1}{T_{mz}} \exp\left(-\frac{T}{T_{mz}}\right)
\]

\[
r(t) = x^2(t) + y^2(t)
\]

Cold damped \(T/T_{m} = 13\)

Figure 3.5. Mode temperature histogram for the minus guide on day 60, 1997. The mode temperature is 367 mK.

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\[ p(T) = \frac{1}{T_m \lambda} \exp \left( -\frac{T}{T_m \lambda} \right) \]

\[ r(t) = x^2(t) + y^2(t) \]

Cold damped \( T/T_m = 13 \)

Figure 3.5b: Mode temperature histogram for the minus mode on day 60, 1997. The mode temperature is 367 mK.

\[ z(t)^2 = [x(t) - x(t - \Delta t)]^2 + [y(t) - y(t - \Delta t)]^2 \]

\[ Q_{\text{colddamped}} \approx 10^6 \]
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\[ p(T) = \frac{1}{T_{m\Delta}} \exp\left(-\frac{T}{T_{m\Delta}}\right) \]

\[ r(t) = x^2(t) + y^2(t) \]

Cold damped \( T/T_{in} = 13 \)

\[ z(t)^2 = [x(t) - x(t - \Delta t)]^2 + [y(t) - y(t - \Delta t)]^2 \]

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![Mode temperature histogram for the minus mode on day 60, 1997. The mode temperatures is 367 mK.](image)

\[ z(t)^2 = [x(t) - x(t - \Delta t)]^2 + [y(t) - y(t - \Delta t)]^2 \]

\[ Q_{\text{cold-damped}} \approx 10^6 \]

![Effective energy histogram for day 234, 1994, using a ZOP filter. The histograms show expected Gaussian behavior with no more than 60 non-Gaussian "events" in the high energy tail. (b) The effective temperature dependence on integration bandwidth for the two antenna normal modes. The data are sampled at 10 Hz, and then correctly filtered, decimated, and plotted a histogram to obtain each data point.](image)
Why is $T_n < T_m$

\[ X^2(f) = m^{-2} \omega_0^4 G^2(f) S_F(f) \begin{bmatrix} \frac{m^2}{Hz} \end{bmatrix} G^2(f) = \left( \left[ 1 - \frac{f^2}{f_0^2} \right]^2 + \left[ f/Q_0 \right]^2 \right) \]

\[ t_{\text{meas}} > \tau \]

\[ \langle x^2 \rangle = \langle x \rangle^2 + (\Delta x)^2 \]

\[ \langle x^2 \rangle = \frac{2}{m^2 \omega_0^4} \int_0^\infty G^2(f) \, df \]

\[ (\Delta x)^2 = \langle x^2 \rangle = \frac{S_F \tau}{4 m^2 \omega_0^2} \]

$t_{\text{meas}}$ (~1 second) < $\tau$

\[ (\Delta x)^2 = \frac{\langle x^2 \rangle t_{\text{meas}}}{\tau} = \frac{S_F t_{\text{meas}}}{4 m^2 \omega_0^2} \]

Not in Equilibrium with bath
Filtered or decoupled

\[ S_{\text{Fnyqi}}(f) = 4 k_B m_i \omega_i \frac{T}{Q_i} \left[ \frac{N^2}{Hz} \right], \]

\[ \tau = \frac{Q}{\pi f_0} \]

\[ f_0 \sim 700 \text{ Hz} \]

\[ Q \sim 2 \times 10^8 \]

\[ \tau \sim 450 \text{s} \]

\[ Q_{\text{colldamped}} \sim 10^6 \]

\[ \frac{t_{\text{meas}}}{\tau} \sim \frac{1}{100} \sim \frac{T_n}{T_m} \]

\[ \frac{Q_m}{T_m} = \text{Constant} \]
Why is $T_n < T_m$?

$X^2(f) = m^{-2} \omega_0^{-4} G^2(f) S_F(f) \left[ \frac{m^2}{Hz} \right] G^2(f) = \left( \left[ 1 - f^2/f_0^2 \right]^2 + \left[ f/Q_0 \right]^2 \right)$

$t_{meas} > \tau$

$\langle x^2 \rangle = \langle x \rangle^2 + (\Delta x)^2$

$\langle x^2 \rangle = 2 \int_0^\infty X^2(f) df = \frac{2S_F}{m^2} \int_0^\infty G^2(f) df$

$(\Delta x)^2 = \langle x^2 \rangle = \frac{S_F \tau}{4m^2 \omega_0^2}$

$t_{meas} \sim 1 \text{ second} < \tau$

$\langle x^2 \rangle = \frac{S_F t_{meas}}{m^2 \omega_0^2}$

Not in Equilibrium with bath
Filtered or decoupled

$S_{Fnyq}(f) = 4k_B m_i \omega_i \frac{T}{Q_i} [N^2/Hz]$, $T_m/T_{eff} \sim 412$

Accurate calibration technique for a resonant-mass gravitational wave detector.

$Q_{coldamped} \sim 10^6$

$\frac{t_{meas}}{\tau} \sim \frac{1}{100} \sim \frac{T_n}{T_m}$

$Q_m/T_m = \text{Constant}$

FIG. 6. Histogram for day 60 in 1997 between 1300 and 2000 UTC.
Another way: High-Q Also Allows Better Cooling

Test-Mass and Transducer Parameters

\[ Q_1 = 10^6; \]
\[ Q_1 = \Omega_t; \]
\[ f_1 = 3.2 \times 10^3; \]
\[ m_1 = 288; \]
\[ w_1 = 2 \pi f_1; \]
\[ t_{2i1} = \frac{w_1}{Q_1}; \]
\[ k_b = \frac{1.38}{10^{23}}; \]
\[ \text{Temp} = 5; \]
\[ \text{dfdx} = 5 \times 10^{18}; \]
\[ \text{be} = 0.8; \]
\[ \text{Pinc} = 10^{-10}; \]
\[ \text{fe} = 10^{10}; \]
\[ Q_e = 1.5 \times 10^6 \]

\[ Q_m/T_m = \text{Constant} \]

\[ S_x = \sqrt{\frac{4 k_B T_m Q_m}{m \omega^3}} \]

\[ x_{rms} = \sqrt{\frac{k_B T_m}{m \omega^2}} \]

[Graphs and data plots are shown, illustrating the relationships between various parameters and their effects on cooling and stability.]
Can we get to the SQL

\[ t_{\text{meas}} < \tau \quad T_{\text{eff}} = T\left(\frac{t_{\text{meas}}}{\tau}\right) \]

\[ (\Delta x)^2_{\text{SQL}} = \frac{\hbar}{2m\omega_o} \]

\[ t_{\text{meas}} > \tau \quad T_{\text{eff}} \sim T \]

For all measurement times

\[ T_n \approx \frac{\hbar\omega}{k_B} \]

1 kHz resonance  \(\rightarrow\) Thermal regime when \( T_{\text{eff}} > 0.05 \ \mu K \)

\(\rightarrow\) Quantum regime when \( T_{\text{eff}} < 0.05 \ \mu K. \)

High-Q \(\rightarrow\) Lets you get to the SQL at a higher bath temperature
Can we get to the SQL

\[ t_{meas} < \tau \quad T_{\text{eff}} = T \left( \frac{t_{meas}}{\tau} \right) \]
\[ t_{meas} > \tau \quad T_{\text{eff}} \sim T \]

For all measurement times

\[ (\Delta x)^2_{\text{SQL}} = \frac{\hbar}{2m \omega_o} \]

\[ T_{\text{bath}} = 5K \quad N_{ph} \sim 10^8 \]

\[ T_n \approx \frac{h\omega}{k_B} \]

1 kHz resonance \(\rightarrow\) Thermal regime when \(T_{\text{eff}} > 0.05 \ \mu K\)
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\[ (\Delta x)^2_{SQL} = \frac{\hbar}{2m\omega_o} \]

\[ T_{bath} = 5K \quad N_{ph} \sim 10^8 \]

Parametric Cold Damping

\[ T_n \approx \frac{\hbar \omega}{k_B} \]

1 kHz resonance -> Thermal regime when \( T_{eff} > 0.05 \mu K \)

-> Quantum regime when \( T_{eff} < 0.05 \mu K \).

High-Q -> Lets you get to the SQL at a higher bath temperature
Can we get to the SQL

\[ T_{\text{meas}} < \tau \quad \Rightarrow \quad T_{\text{eff}} = T \left( \frac{t_{\text{meas}}}{\tau} \right) \]

\[ T_{\text{meas}} > \tau \quad \Rightarrow \quad T_{\text{eff}} \sim T \]

For all measurement times

\[ (\Delta x)^2_{\text{SQL}} = \frac{\hbar}{2m\omega_o} \]

\[ T_{\text{bath}} = 5K \quad N_{ph} \sim 10^8 \]

\[ T_{\text{mode}} = 370mK \quad N_{ph} \sim 8 \times 10^6 \]

\[ T_n \approx \frac{\hbar \omega}{k_B} \]

1 kHz resonance  \( \rightarrow \) Thermal regime when \( T_{\text{eff}} > 0.05 \, \mu K \)
\( \rightarrow \) Quantum regime when \( T_{\text{eff}} < 0.05 \, \mu K. \)

High-Q  \( \rightarrow \) Lets you get to the SQL at a higher bath temperature
Can we get to the SQL

\[ t_{\text{meas}} < \tau \quad \Rightarrow \quad T_{\text{eff}} = T \left( \frac{t_{\text{meas}}}{\tau} \right) \]

\[ t_{\text{meas}} > \tau \quad \Rightarrow \quad T_{\text{eff}} \sim T \]

\[ (\Delta x)^2_{\text{SQL}} = \frac{\hbar}{2m\omega_o} \]

For all measurement times

\[ T_n \approx \frac{\hbar \omega}{k_B} \]

1 kHz resonance \( \rightarrow \) Thermal regime when \( T_{\text{eff}} > 0.05 \, \mu K \)
\( \rightarrow \) Quantum regime when \( T_{\text{eff}} < 0.05 \, \mu K. \)

\[ T_{\text{bath}} = 5K \quad N_{ph} \sim 10^8 \]

Parametric Cold Damping

\[ T_{\text{mode}} = 370mK \quad N_{ph} \sim 8 \times 10^6 \]

Filtering due to high-Q

High-Q \( \rightarrow \) Lets you get to the SQL at a higher bath temperature
Can we get to the SQL

\[ T_{\text{meas}} < \tau \quad T_{\text{eff}} = T \left( \frac{t_{\text{meas}}}{\tau} \right) \]

\[ T_{\text{meas}} > \tau \quad T_{\text{eff}} \sim T \]

For all measurement times

\[ T_n \approx \frac{\hbar \omega}{k_B} \]

1 kHz resonance  -> Thermal regime when \( T_{\text{eff}} > 0.05 \ \mu K \)

-> Quantum regime when \( T_{\text{eff}} < 0.05 \ \mu K \).

\[ (\Delta x)_{\text{SQL}}^2 = \frac{\hbar}{2m \omega_0} \]

- \( T_{\text{bath}} = 5K \)  \( N_{\text{ph}} \sim 10^8 \)
- \( T_{\text{mode}} = 370mK \)  \( N_{\text{ph}} \sim 8 \times 10^6 \)
- \( T_n = 890\mu K \)  \( \Delta N_{\text{ph}} \sim 10^4 \)

Parametric Cold Damping
Filtering due to high-Q

High-Q  ->  Lets you get to the SQL at a higher bath temperature
Rare Events Detected with a Bulk Acoustic Wave High Frequency Gravitational Wave Antenna

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The multi-mode acoustic gravitational wave experiment: MAGE

William M. Campbell, Maxim Goryachev & Michael E. Tobar

Scientific Reports 13, Article number: 10638 (2023) | Cite this article

FIG. 1. Experimental setup showing BAW cavity connected to SQUID amplifier and shielding arrangement. Note that 4 and 50 K shields as well as stainless still vacuum chamber not shown.

FIG. 3. Top figure displays averaged amplitude spectral density (ASD) of each output channel of lock-ins for longest continuous data taking run; here, each mode has been demodulated from the carrier. Bottom figure shows corresponding spectral strain sensitivity determined for each trace, as well as current best sensitivity in region given by Holometer experiment [6], which uses the cross spectral density (CSD) of two identical interferometers to search for HFGWs.
Searching for low-mass axions using resonant upconversion

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Anyon

Accepted in PRL

Search for ultralight axions with twisted cavity resonators of anyon rotational symmetry with bulk modes of nonzero helicity

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Accepted in PRD
Dark matter axion haloscopes -> via axion-photon chiral anomaly -> Sensitive to GWs
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Magnetic conversion (Inverse Gertsenshtein effect)

- Gravitational-wave propagating in magnetic fields convert into photons.
• Dark matter axion haloscopes -> via axion-photon chiral anomaly -> Sensitive to GWs

Magnetic conversion (Inverse Gertsenshtein effect)

- Gravitational-wave propagating in magnetic fields convert into photons.  
Axion Haloscopes and UHFGW Detectors

Signal for virialized axion in the Halo
Approximate as a narrow-band noise source

\[ \frac{\Delta f_a}{f_a} \sim 10^{-6} \]
Axion Haloscopes and UHFGW Detectors

- Shown $h_g \sim \theta_a$ where $\theta_a = g_{a\gamma\gamma} a$, $a$ is axion scalar field, $g_{a\gamma\gamma}$ is the 2-photon coupling

Signal for virialized axion in the Halo
Approximate as a narrow-band noise source

$$S_{\theta}(f) \sim 10^{-6}$$

$$\frac{\Delta f_a}{f_a}$$

$$[\theta_a^2/\text{Hz}]$$
Axion Haloscopes and UHFGW Detectors

- Shown $h_g \sim \theta_a$ where $\theta_a = g_{a\gamma\gamma} a$, $a$ is axion scalar field, $g_{a\gamma\gamma}$ is the 2-photon coupling
- Unlikely that a gravitational wave and an axion signal will be of the same form

Signal for virialized axion in the Halo
Approximate as a narrow-band noise source

\[
S_\theta(f) \approx \frac{\Delta f_a}{f_a} \sim 10^{-6}
\]
Axion Haloscopes and UHFGW Detectors

- Shown $h_g \sim \theta_a$ where $\theta_a = g_{a\gamma\gamma} a$, $a$ is axion scalar field, $g_{a\gamma\gamma}$ is the 2-photon coupling.

- Unlikely that a gravitational wave and an axion signal will be of the same form.

- Calculate Spectral Sensitivity in terms of $\theta_a$.

Signal for virialized axion in the Halo
Approximate as a narrow-band noise source

\[
S_{\theta}(f) \sim \frac{\Delta f_a}{f_a} \sim 10^{-6}
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Axion Haloscopes and UHFGW Detectors

- Shown $h_{g} \sim \theta_{a}$ where $\theta_{a} = g_{a\gamma\gamma}a$, $a$ is axion scalar field, $g_{a\gamma\gamma}$ is the 2-photon coupling
- Unlikely that a gravitational wave and an axion signal will be of the same form
- Calculate Spectral Sensitivity in terms of $\theta_{a}$

Signal for virialized axion in the Halo
Approximate as a narrow-band noise source

Cold flows are more coherent
Spectral Sensitivity of Axion Detectors

* Signal as a narrow band noise spectral density

\[ \langle \theta_0^2 \rangle = g_{a\gamma}^2 \langle a_0^2 \rangle \]

\[ \langle \theta_0^2 \rangle = \int_{f_1}^{f_2} S_{\theta_{\text{Sig}}} \, df = g_{a\gamma} \int_{f_1}^{f_2} S_A \, df \]

\[ S_{\theta_{\text{Sig}}} (f) \quad [1/\text{Hz}] \]

\[ \sqrt{S_{\theta_{\text{Sig}}} (f)} \quad [1/\sqrt{\text{Hz}}] \]
Spectral Sensitivity of Axion Detectors

* Signal as a narrow band noise spectral density

\[
\langle \theta_0^2 \rangle = g_{a\gamma}^2 \langle a_0^2 \rangle \quad \langle \theta_0^2 \rangle = \int_{f_1}^{f_2} S_{\theta_{\text{Sig}}} df = g_{a\gamma} \int_{f_1}^{f_2} S_A df \quad S_{\theta_{\text{Sig}}} (f) \quad [1/\text{Hz}] \quad \sqrt{S_{\theta_{\text{Sig}}} (f)} \quad [1/\sqrt{\text{Hz}}]
\]

Experiment: Maximize Signal wrt an Observable

\[
\mathcal{O} = \mathcal{K} \langle \theta_0 \rangle \quad \langle \theta_0 \rangle = g_{a\gamma} \sqrt{\rho_{DM} c^3} \quad \langle a_0 \rangle = \sqrt{\rho_{DM} c^3} \quad \frac{\omega_a}{\omega_a}
\]

\[
S_{\theta}(f) \quad \frac{\Delta f_a}{f_a} \sim 10^{-6}
\]

\[
[\theta_a^2/\text{Hz}]
\]

\[
[0, f_a]
\]
Spectral Sensitivity of Axion Detectors

* Signal as a narrow band noise spectral density

\[
\langle \theta_0^2 \rangle = g_{ar\gamma}^2 \langle a_0^2 \rangle \quad \langle \theta_0^2 \rangle = \int_{f_1}^{f_2} S_{\theta_{\text{Sig}}} df = g_{ar\gamma} \int_{f_1}^{f_2} S_A df \quad S_{\theta_{\text{Sig}}} (f) \ [1/\text{Hz}] \quad \sqrt{S_{\theta_{\text{Sig}}} (f)} \ [1/\sqrt{\text{Hz}}]
\]

Experiment: Maximize Signal wrt an Observable

\[
\mathcal{O} = \mathcal{K} \langle \theta_0 \rangle \quad \langle \theta_0 \rangle = g_{ar\gamma} \frac{\sqrt{\rho_{DM} c^3}}{\omega_a} \quad \langle a_0 \rangle = \frac{\sqrt{\rho_{DM} c^3}}{\omega_a}
\]

\( \mathcal{K} \) : Maximise Experiment conversion ratio from rms axion-photon theta angle \( \langle \theta_0 \rangle \) to the observable \( \mathcal{O} \)
Spectral Sensitivity of Axion Detectors

* Signal as a narrow band noise spectral density

\[ \left\langle \theta_0^2 \right\rangle = g_{a\gamma}^2 \left\langle a_0^2 \right\rangle \]
\[ \left\langle \theta_0^2 \right\rangle = \int_{f_1}^{f_2} S_{\theta \text{Sig}} \, df = g_{a\gamma} \int_{f_1}^{f_2} S_A \, df \]
\[ S_{\theta \text{Sig}}(f) \quad [1/\text{Hz}] \]
\[ \sqrt{S_{\theta \text{Sig}}(f)} \quad [1/\sqrt{\text{Hz}}] \]

Experiment: Maximize Signal wrt an Observable

\[ \mathcal{H} = \mathcal{K} \left\langle \theta_0 \right\rangle \]
\[ \left\langle \theta_0 \right\rangle = g_{a\gamma} \frac{\sqrt{\rho_{\text{DM}c^3}}}{\omega_a} \]
\[ \left\langle a_0 \right\rangle = \frac{\sqrt{\rho_{\text{DM}c^3}}}{\omega_a} \]

\( \mathcal{H} \): Maximise Experiment conversion ratio from rms axion-photon theta angle \( \left\langle \theta_0 \right\rangle \) to the observable \( \mathcal{H} \)

\[ S_{\mathcal{H} \text{Sig}} = \mathcal{H}^2 S_{\theta \text{Sig}} \]
Spectral Sensitivity of Axion Detectors

* Signal as a narrow band noise spectral density

\[
\langle \theta_0^2 \rangle = g_{a\gamma}^2 \langle a_0^2 \rangle \quad \langle \theta_0^2 \rangle = \int_{f_1}^{f_2} S_{\theta \text{sig}} df = g_{a\gamma} \int_{f_1}^{f_2} S_A df \quad S_{\theta \text{sig}}(f) \quad [1/\text{Hz}] \quad \sqrt{S_{\theta \text{sig}}(f)} \quad [1/\sqrt{\text{Hz}}]
\]

Experiment: Maximize Signal wrt an Observable

\[
\mathcal{O} = \mathcal{K} \langle \theta_0 \rangle \quad \langle \theta_0 \rangle = g_{a\gamma} \frac{\sqrt{\rho_{DM} c^3}}{\omega_a} \quad \langle a_0 \rangle = \frac{\sqrt{\rho_{DM} c^3}}{\omega_a}
\]

\( \mathcal{K} \) : Maximise Experiment conversion ratio from rms axion-photon theta angle \( \langle \theta_0 \rangle \) to the observable \( \mathcal{O} \)

\[
S_{\mathcal{O} \text{sig}} = \mathcal{K}^2 S_{\theta \text{sig}} \quad S_{\theta \text{sig}} = g_{a\gamma}^2 S_A
\]

\[
\frac{\Delta f_a}{f_a} \sim 10^{-6}
\]

\[
S_{\theta}(f) \quad [\theta_a^2/\text{Hz}]
\]
Spectral Sensitivity of Axion Detectors

* Signal as a narrow band noise spectral density

\[
\langle \theta_0^2 \rangle = g_{a\gamma}^2 \langle a_0^2 \rangle \\
\langle \theta_0^2 \rangle = \int_{f_1}^{f_2} S_{\theta_{\text{Sig}}} df = g_{a\gamma} \int_{f_1}^{f_2} S_A df \\
S_{\theta_{\text{Sig}}} (f) \quad [1/\text{Hz}] \\
\sqrt{S_{\theta_{\text{Sig}}} (f)} \quad [1/\sqrt{\text{Hz}}]
\]

Experiment: Maximize Signal wrt an Observable

\[
\mathcal{O} = \mathcal{K} \langle \theta_0 \rangle \\
\langle \theta_0 \rangle = g_{a\gamma} \frac{\sqrt{\rho_{DM} c^3}}{\omega_a} \\
\langle a_0 \rangle = \frac{\sqrt{\rho_{DM} c^3}}{\omega_a}
\]

\(\mathcal{K} : \) Maximise Experiment conversion ratio from rms axion-photon theta angle \(\langle \theta_0 \rangle\) to the observable \(\mathcal{O}\)

\[
S_{\theta_{\text{Sig}}} = \mathcal{K}^2 S_{\theta_{\text{Sig}}} \\
S_{\theta_{\text{Sig}}} = g_{a\gamma}^2 S_A
\]

* Experimentally design the largest possible \(\mathcal{K}\) value
Noise

Spectral Noise Density of Axion Detectors
Spectral Noise Density of Axion Detectors

Experiment: Minimise Noise Sources wrt an Observable

\[ \mathcal{O} = \mathcal{H} \langle \theta_0 \rangle \]
Noise

Spectral Noise Density of Axion Detectors

\[ \mathcal{O} = \mathcal{K} \left\langle \theta_0 \right\rangle \]

* Refer the noise within the detector with respect to the mean square of the axion-photon theta angle noise

\[ S_{\theta_N} = \frac{S_{\mathcal{O}_N}}{\mathcal{K}^2} \]

\[ \sqrt{S_{\theta_N}} = \frac{\sqrt{S_{\mathcal{O}_N}}}{|\mathcal{K}|} \]
Noise

Spectral Noise Density of Axion Detectors

Experiment: Minimise Noise Sources wrt an Observable

\[ \mathcal{O} = \mathcal{H} \langle \theta_0 \rangle \]

* Refer the noise within the detector with respect to the mean square of the axion-photon theta angle noise

\[ S_{\theta \mathcal{N}} = \frac{S_{\mathcal{O} \mathcal{N}}}{\mathcal{H}^2} \]

\[ \sqrt{S_{\theta \mathcal{N}}} = \frac{\sqrt{S_{\mathcal{O} \mathcal{N}}}}{|\mathcal{H}|} \]

* Measurement time \( t < \frac{t}{\Delta f_a} \) coherent signal integrates

\[ SNR \sim \frac{\mathcal{H} \langle \theta_0 \rangle}{\sqrt{S_{\mathcal{O} \mathcal{N}}}} t^\frac{1}{2} = \frac{\langle \theta_0 \rangle}{\sqrt{S_{\theta \mathcal{N}}}} t^\frac{1}{2} \]
Noise

Spectral Noise Density of Axion Detectors

Experiment: Minimise Noise Sources wrt an Observable

\[ \mathcal{O} = \mathcal{H} \langle \theta_0 \rangle \]

* Refer the noise within the detector with respect to the mean square of the axion-photon theta angle noise

\[ S_{\theta N} = \frac{S_{\mathcal{O} N}}{\mathcal{H}^2} \]

\[ \sqrt{S_{\theta N}} = \frac{\sqrt{S_{\mathcal{O} N}}}{|\mathcal{H}|} \]

* Measurement time \( t < \frac{t}{\Delta f_a} \) coherent signal integrates

\[ \text{SNR} \sim \frac{\mathcal{H} \langle \theta_0 \rangle}{\sqrt{S_{\mathcal{O} N}}} \left( \frac{t}{t^2} \right)^{\frac{1}{2}} = \frac{\langle \theta_0 \rangle}{\sqrt{S_{\theta N}}} \left( \frac{t}{\Delta f_a} \right)^{\frac{1}{4}} \]

* Measurement time \( t > \frac{t}{\Delta f_a} \)

\[ \text{SNR} \sim \frac{\mathcal{H} \langle \theta_0 \rangle}{\sqrt{S_{\mathcal{O} N}}} \left( \frac{t}{\Delta f_a} \right)^{\frac{1}{4}} = \frac{\langle \theta_0 \rangle}{\sqrt{S_{\theta N}}} \left( \frac{t}{\Delta f_a} \right)^{\frac{1}{4}} \]
**Noise**

**Spectral Noise Density of Axion Detectors**

Experiment: Minimise Noise Sources wrt an Observable

\[ \mathcal{O} = \mathcal{H} \langle \theta_0 \rangle \]

* Refer the noise within the detector with respect to the mean square of the axion-photon theta angle noise

\[ S_{\theta N} = \frac{S_{\phi N}}{\mathcal{H}^2} \quad \sqrt{S_{\theta N}} = \frac{\sqrt{S_{\phi N}}}{|\mathcal{H}|} \]

* Measurement time \( t < \frac{t}{\Delta f_a} \) coherent signal integrates

\[ SNR \sim \frac{\mathcal{H} \langle \theta_0 \rangle}{\sqrt{S_{\phi N}} t^{1/2}} = \frac{\langle \theta_0 \rangle}{\sqrt{S_{\theta N}} t^{1/2}} \]

* Measurement time \( t > \frac{t}{\Delta f_a} \)

\[ SNR \sim \frac{\mathcal{H} \langle \theta_0 \rangle}{\sqrt{S_{\phi N}} \left( \frac{t}{\Delta f} \right)^{1/4}} = \frac{\langle \theta_0 \rangle}{\sqrt{S_{\theta N}}} \left( \frac{t}{\Delta f} \right)^{1/4} \]

\[ \sqrt{S_{\theta N}} = \frac{\sqrt{S_{\phi N}}}{|\mathcal{H}|} \]
Noise

**Spectral Noise Density of Axion Detectors**

实验：最小化噪声源
wrt一个可观察的量

\[ \mathcal{O} = \mathcal{H} \langle \theta_0 \rangle \]

* 将噪声与探测器内部的噪声与平均平方的轴子-光子角度噪声的均方根。

\[ S_{\theta_N} = \frac{S_{\mathcal{O}_N}}{\mathcal{H}^2} \]

\[ \sqrt{S_{\theta_N}} = \frac{S_{\mathcal{O}_N}}{|\mathcal{H}|} \]

* 测量时间 \( t < \frac{t}{\Delta f_a} \) 时，相干信号积分

\[ SNR \sim \frac{\mathcal{H} \langle \theta_0 \rangle}{\sqrt[4]{S_{\theta_N}}} t^\frac{1}{2} = \frac{\langle \theta_0 \rangle}{\sqrt[4]{S_{\theta_N}}} t^\frac{1}{2} \]

* 测量时间 \( t > \frac{t}{\Delta f_a} \) 时

\[ SNR \sim \frac{\mathcal{H} \langle \theta_0 \rangle}{\sqrt[4]{S_{\theta_N}}} \left( \frac{t}{\Delta f} \right)^\frac{1}{4} = \frac{\langle \theta_0 \rangle}{\sqrt[4]{S_{\theta_N}}} \left( \frac{t}{\Delta f} \right)^\frac{1}{4} \]

* 探测器光谱灵敏度 [角度/根赫兹] 

\[ \sqrt{S_{\theta_N}} = \frac{S_{\mathcal{O}_N}}{|\mathcal{H}|} \]
Noise

Spectral Noise Density of Axion Detectors

Experiment: Minimise Noise Sources wrt an Observable

\[ \mathcal{O} = \mathcal{H} \langle \theta_0 \rangle \]

* Refer the noise within the detector with respect to the mean square of the axion-photon theta angle noise

\[ S_{\theta_N} = \frac{S_{\mathcal{O}_N}}{\mathcal{H}^2} \]

\[ \sqrt{S_{\theta_N}} = \frac{\sqrt{S_{\mathcal{O}_N}}}{|\mathcal{H}|} \]

* Measurement time \( t < \frac{t}{\Delta f_a} \) coherent signal integrates

\[ SNR \sim \frac{\mathcal{H} \langle \theta_0 \rangle}{\sqrt{S_{\mathcal{O}_N}}} \frac{t^{1/2}}{\sqrt{S_{\theta_N}}} = \frac{\langle \theta_0 \rangle}{\sqrt{S_{\theta_N}}} \left( \frac{t}{\Delta f_a} \right)^{1/2} \]

* Measurement time \( t > \frac{t}{\Delta f_a} \)

\[ SNR \sim \frac{\mathcal{H} \langle \theta_0 \rangle}{\sqrt{S_{\mathcal{O}_N}}} \left( \frac{t}{\Delta f} \right)^{1/4} = \frac{\langle \theta_0 \rangle}{\sqrt{S_{\theta_N}}} \left( \frac{t}{\Delta f} \right)^{1/4} \]

* Detector spectral sensitivity [Theta angle per root Hz]

* Assumes nothing about the signal

\[ \sqrt{S_{\theta_N}} = \frac{\sqrt{S_{\mathcal{O}_N}}}{|\mathcal{H}|} \]
**Noise**

**Spectral Noise Density of Axion Detectors**

* Refer the noise within the detector with respect to the mean square of the axion-photon theta angle noise

\[ \mathcal{O} = \mathcal{H} \langle \theta_0 \rangle \]

**Experiment: Minimise Noise Sources wrt an Observable**

\[ S_{\theta N} = \frac{S_{\mathcal{O} N}}{\mathcal{H}^2} \quad \sqrt{S_{\theta N}} = \frac{\sqrt{S_{\mathcal{O} N}}}{|\mathcal{H}|} \]

* Measurement time \( t < \frac{t}{\Delta f_a} \) coherent signal integrates

\[ SNR \sim \frac{\mathcal{H} \langle \theta_0 \rangle}{\sqrt{S_{\mathcal{O} N}}} t^{1/2} = \frac{\langle \theta_0 \rangle}{\sqrt{S_{\theta N}}} t^{1/2} \]

* Measurement time \( t > \frac{t}{\Delta f_a} \)

\[ SNR \sim \frac{\mathcal{H} \langle \theta_0 \rangle}{\sqrt{S_{\mathcal{O} N}}} \left( \frac{t}{\Delta f} \right)^{1/4} = \frac{\langle \theta_0 \rangle}{\sqrt{S_{\theta N}}} \left( \frac{t}{\Delta f} \right)^{1/4} \]

* Detector spectral sensitivity [Theta angle per root Hz]
* Assumes nothing about the signal
* Only considers the conversion efficiency of detection and the noise in the detector itself
Experiment: Minimise Noise Sources wrt an Observable

\[ \mathcal{O} = \mathcal{H} \langle \theta_0 \rangle \]

* Refer the noise within the detector with respect to the mean square of the axion-photon theta angle noise

\[ S_{\theta N} = \frac{S_{\mathcal{O}N}}{\mathcal{H}^2} \]

\[ \sqrt{S_{\theta N}} = \frac{\sqrt{S_{\mathcal{O}N}}}{|\mathcal{H}|} \]

* Measurement time \( t < \frac{t}{\Delta f_a} \) coherent signal integrates

\[ \text{SNR} \sim \frac{\mathcal{H} \langle \theta_0 \rangle}{\sqrt{S_{\mathcal{O}N}}} t^{\frac{1}{2}} = \frac{\langle \theta_0 \rangle}{\sqrt{S_{\theta N}}} t^{\frac{1}{2}} \]

* Measurement time \( t > \frac{t}{\Delta f_a} \)

\[ \text{SNR} \sim \frac{\mathcal{H} \langle \theta_0 \rangle}{\sqrt{S_{\mathcal{O}N}}} \left( \frac{t}{\Delta f} \right)^{\frac{1}{4}} = \frac{\langle \theta_0 \rangle}{\sqrt{S_{\theta N}}} \left( \frac{t}{\Delta f} \right)^{\frac{1}{4}} \]

* Detector spectral sensitivity [Theta angle per root Hz]
* Assumes nothing about the signal
* Only considers the conversion efficiency of detection and the noise in the detector itself
* Like in GWs good to compare detectors, without considering the signal
\[ |\Phi_a|^2 = \frac{\omega_a^2}{c^2} V^2 \mathcal{G}_V^2 B_{\text{max}}^2 \langle \theta_0 \rangle^2 \]

\[ \rho_{DM} = \frac{\omega_a^2}{c^3} \langle a_0 \rangle^2 \quad \sqrt{\rho_{DM}} = \frac{\omega_a \langle \theta_0 \rangle}{g_{\alpha\gamma} \sqrt{c^3}} \]

\[ S_{\bar{\theta}_{\text{Sig}}} = \mathcal{H}^2 S_{\bar{\theta}_{\text{Sig}}} \]

\[ \mathcal{H}_{\text{ABRA}} = \frac{\omega_a}{c} V \mathcal{G}_V B_{\text{max}} \frac{M_{\text{in}}}{L_T} \]

\[ \langle \theta_0 \rangle = g_{\alpha\gamma} \frac{\sqrt{\rho_{DM} c^3}}{\omega_a} \]

\[ \sqrt{S_{\theta_\mathcal{N}}} = \sqrt{S_{\theta_\mathcal{N}}} = \frac{c L_T \sqrt{S_{\phi\phi}}}{\omega_a V \mathcal{G}_V B_{\text{max}} M_{\text{in}}} \]

\[ |\Phi_a|^2 = g_{\alpha\gamma} \rho_{DM} c V^2 \mathcal{G}_V^2 B_{\text{max}}^2 \]

**TABLE I. Summary of the ABRACADABRA-10 cm detector design parameters.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pickup loop radius</td>
<td>20.1 mm</td>
</tr>
<tr>
<td>Pickup loop wire diameter</td>
<td>1.0 mm</td>
</tr>
<tr>
<td>Magnet inner radius</td>
<td>30 mm</td>
</tr>
<tr>
<td>Magnet outer radius</td>
<td>60 mm</td>
</tr>
<tr>
<td>Magnet height</td>
<td>120 mm</td>
</tr>
<tr>
<td>Magnet max field</td>
<td>1.0 T</td>
</tr>
<tr>
<td>Geometric factor</td>
<td>0.027</td>
</tr>
<tr>
<td>Pickup loop inductance</td>
<td>95.5 nH</td>
</tr>
<tr>
<td>SQUID input inductance</td>
<td>150 nH</td>
</tr>
<tr>
<td>SQUID inductive coupling</td>
<td>2.5 nH</td>
</tr>
</tbody>
</table>

**Diagram:**

- A diagram showing the components of the ABRACADABRA detector, including inductors and capacitors.
- A graph showing a frequency spectrum with labeled axes and a plot area.

**Legend:**

- G: Amplifier
- FFT: Fast Fourier Transform
- Power: Measured power
- Frequency: Measured frequency
- J_{eff}: Effective current
- \Phi_a: Flux quantum
- L_{wires}: Inductor wires
- L_p: Pickup loop inductance
- L_in: SQUID input inductance
- M_{in}: SQUID inductive coupling
- M_f: Magnet field
- R_f: Feedback resistor

**Equations:**

- Lorentz force equation
- Magnetic field intensity
- Electric field intensity
- Energy conservation law
- Faraday's law of induction
- Ohm's law
- Kirchhoff's voltage law
- Power dissipation
- Energy dissipation
- Impedance calculation
- Voltage transfer function
- Current transfer function
- Impedance matching
- Feedback network design
\[
\sqrt{S_{\theta_N}} = \frac{\sqrt{S_{\phi \phi}}}{|H|} = \frac{cL_T \sqrt{S_{\phi \phi}}}{\omega_a V G V B_{\text{max}} M_{\text{in}}}
\]

**Figure 4.** Estimated spectral sensitivity for the broadband haloscopes ABRACADABRA (black) and SHAFT (green) based on data in its latest experimental runs [51,76].
\[ \langle \Phi_a \rangle = \mathcal{K}_{\text{shaft}} \langle \theta_0 \rangle \]

\[ \mathcal{K}_{\text{shaft}} = \frac{N_p M_{\text{in}}}{L_p + L_{tp} + L_{in}} \frac{\omega_a}{c} B_0 V_{\text{eff}} \]

\[ \Phi_a = \frac{N_p M_{\text{in}}}{L_p + L_{tp} + L_{in}} \frac{\omega_a}{c} g_{ar} a_0 \cos (\omega_a t) B_0 V_{\text{eff}} \]

\[ \sqrt{S_{\theta_N}} = \frac{\sqrt{S_{\phi\phi}}}{|\mathcal{K}|} = \frac{c}{\omega_a V_{\text{eff}} B_0} \]

\[ \Phi_{SQ} = \frac{N_p M_{\text{in}}}{L_p + L_{tp} + L_{in}} \Phi \]

\[ N_p = 6 \]

\[ x_0 = \frac{R_f}{M_f} \frac{N_p M_{\text{in}}}{L_p + L_{tp} + L_{in}} \approx 6.3 \times 10^{12} \text{ V/Wb} \]

\[ \Phi_{SQ} = \frac{M_f V_{SQ}}{R_f} \]

**TABLE S4:** Parameters for \( x(t) \) obtained using the central calibration loop.

<table>
<thead>
<tr>
<th>Detection channel</th>
<th>( x_0 ) (( \times10^{12} \text{ V/Wb} ))</th>
<th>( v_0 ) (( \times10^{6} \text{ Hz} ))</th>
<th>( Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.13 ( \pm ) 0.15</td>
<td>1.98 ( \pm ) 0.04</td>
<td>0.52 ( \pm ) 0.02</td>
</tr>
<tr>
<td>B</td>
<td>5.94 ( \pm ) 0.06</td>
<td>1.92 ( \pm ) 0.03</td>
<td>0.61 ( \pm ) 0.03</td>
</tr>
</tbody>
</table>
Resonant Haloscopes

\[ \mathcal{O} = \mathcal{K} \langle \theta_0 \rangle \]

\[ \sqrt{P_a} = \frac{\sqrt{\beta_p}}{\sqrt{1 + \beta_p \sqrt{1 + 4Q_p^2 \left( \frac{\omega_a}{\omega} \right)^2}}} \sqrt{\omega_a \epsilon_0 V_1 C_1 c B_0 \langle \theta_0 \rangle} \]

\[ \mathcal{K}_{adm} = \frac{\sqrt{\beta_p}}{(1 + \beta_p) \sqrt{1 + 4Q_p^2 \left( \frac{\omega_a}{\omega} \right)^2}} \sqrt{\omega_a \epsilon_0 V_1 C_1 c B_0} \]

\[ \text{SNR}^2 = \frac{P_a}{k_B T_{sys}/\epsilon} \sqrt{\frac{t}{\Delta f}} \]

\[ \sqrt{S_{\mathcal{O}_N}} = \sqrt{k_B T_{sys}/\epsilon} \]

\[ h^+(f) = \sqrt{\sum_i h_i^+(f)^2} \]
SNR = \frac{P_a}{k_B T_{\text{sys}}/e} \sqrt{\frac{t}{\Delta f}}

\begin{equation}
E_{\text{stored}} = \frac{Q_t}{\omega} \frac{4\beta_1}{(1 + \beta_1 + \beta_2)^2} \frac{1}{1 + 4Q_t^2 \frac{(\omega - \omega_{\text{DEF}})}{\omega_{\text{DEF}}}}.
\end{equation}

\begin{align*}
N_{\text{RF}} &= \frac{k_B}{2} \left( T^C_0 |\mathcal{T}|^2 + \frac{T^A_{\text{eff}}}{K_{A1}} \right) K_{A1} K_{A2} \\
\mathcal{T} &= \frac{2\sqrt{\beta}}{(1 + \beta) \left( 1 + 2iQ_{\text{DET}} \frac{\omega - \omega_{\text{DEF}}}{\omega_{\text{DEF}}} \right)}
\end{align*}

Cryogenic resonant microwave cavity searches for hidden sector photons

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Resonant Haloscopes

\[ P_N \sim \frac{4\beta_1}{(\beta_1 + 1)^2} \left( 1 + 4Q_{L1}^2 \left( \frac{\omega_a - \omega_1}{\omega_a} \right)^2 \right) \frac{k_B T_1}{2} + \frac{k_B T_{\text{amp}}}{2}. \]
Figure 2: Top: Binary Neutron Star (BNS) range evolution of the LIGO and Virgo detectors from the start of O2 in November 2017 to the end of O3 in March 2020. The broken axes remove the time between each observing run. 
Bottom: Representative amplitude spectral density of the three detectors’ strain sensitivity in each observing run. The O3 spectra shown are taken from O3a.
Comparing Instrument Spectral Sensitivity of Dissimilar Electromagnetic Haloscopes to Axion Dark Matter and High Frequency Gravitational Waves

Michael E. Tobar *, Catriona A. Thomson, William M. Campbell, Aaron Quiskamp, Jeremy F. Bourhill, Benjamin T. McAllister, Eugene N. Ivanov and Maxim Goryachev

Special Issue
The Dark Universe: The Harbinger of a Major Discovery
Edited by
Prof. Konstantin Zioutas

ADMX and ORGAN (purple) with current tuning locus (blue); 0.6-1.2 GHz for ADMX and 15.2 to 16.2 GHz for ORGAN