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Scaling behaviour of dN/dy in high energy collisions

GÁBOR KASZA, TAMÁS CSÖRGŐ

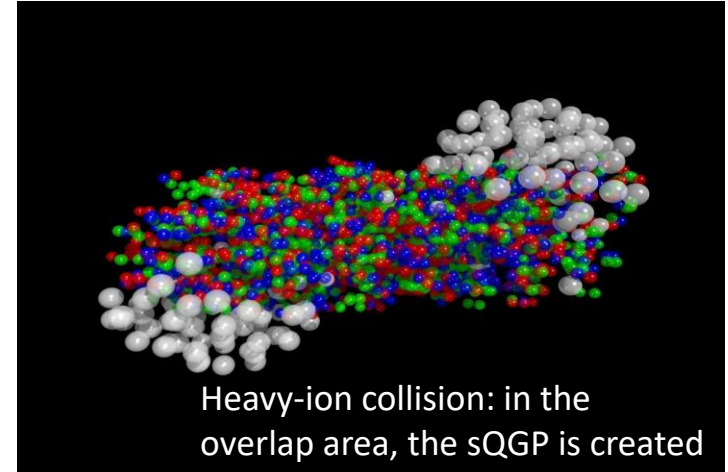
52ND INTERNATIONAL SYMPOSIUM ON MULTIPARTICLE DYNAMICS

GYÖNGYÖS, 08/24/2023



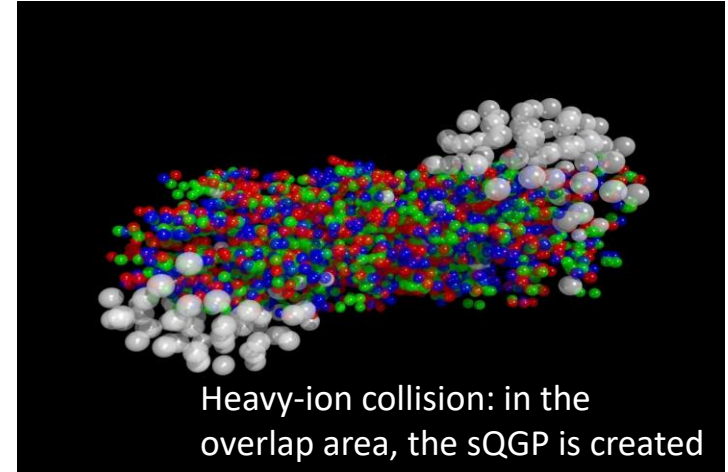
Various application of hydrodynamics

- Why is hydrodynamics so effective?
- Works well on ...
 - ... *microscopic scales*
 - ... *macroscopic scales*
 - ... *cosmic scales*
- Different systems in many aspects



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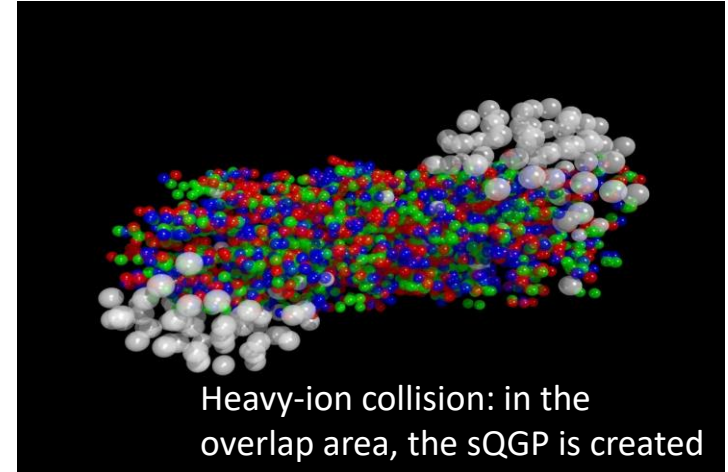
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Hydrodynamics has no internal scale!

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Hydrodynamics has no internal scale!

- Today's presentation: ***a new example of hydrodynamic scaling behaviour in dN/dy data***

Relativistic perfect fluid solution with accelerating velocity field

- Equation of state: $\varepsilon = \kappa_0 p \quad (\mu=0)$
- Rindler coordinates: $\tau = \sqrt{t^2 - r_z^2} \quad \eta_z = \frac{1}{2} \ln \left(\frac{t + r_z}{t - r_z} \right)$
- Velocity field: $u^\mu = \begin{pmatrix} \cosh(\Omega) \\ \sinh(\Omega) \end{pmatrix} \quad \Omega \equiv \Omega(\eta_z)$
- 1+1 dimensional, parametric, almost self-similar, finite solution

λ : rate of acceleration
(Hwa-Bjorken: $\lambda=1$)

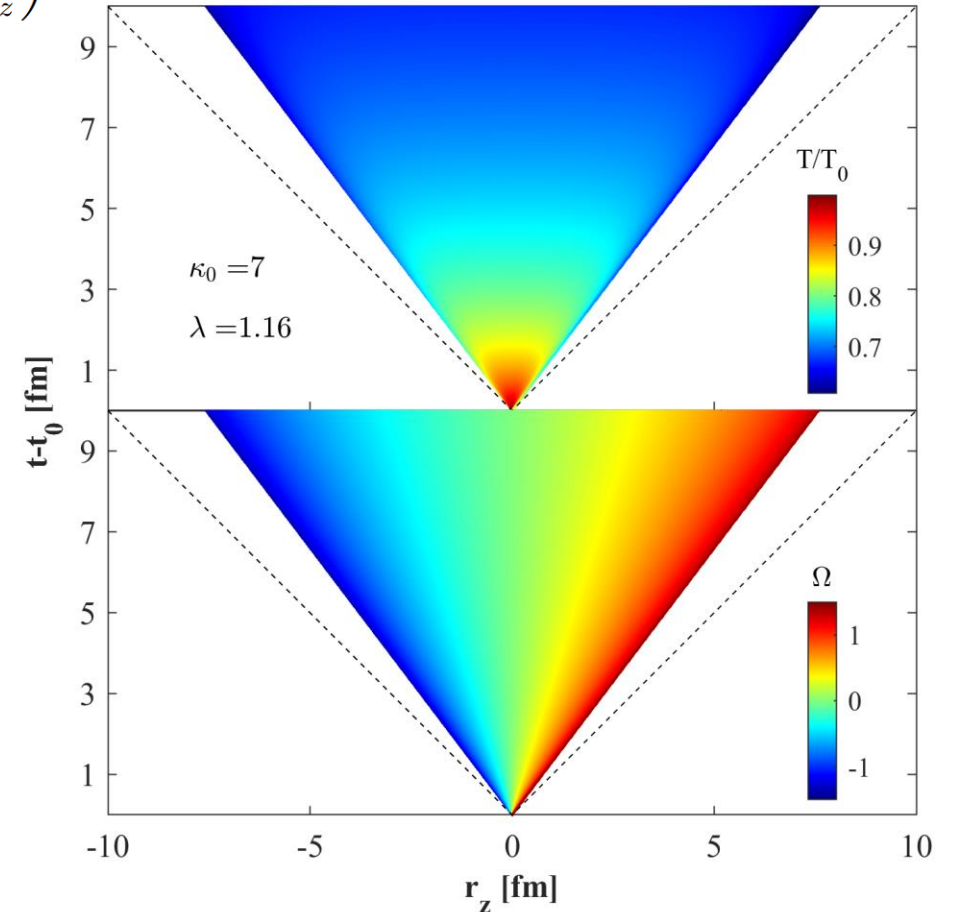


accelerating expansion



realistic $dN/d\eta_p$

T. Csörgő, G. Kasza, M. Csanád and Z-F. Jiang:
Universe 4 (2018) 69



Pseudorapidity distribution

- Starting from the rapidity distribution, we calculated the pseudorapidity distribution
- Parametric curve:

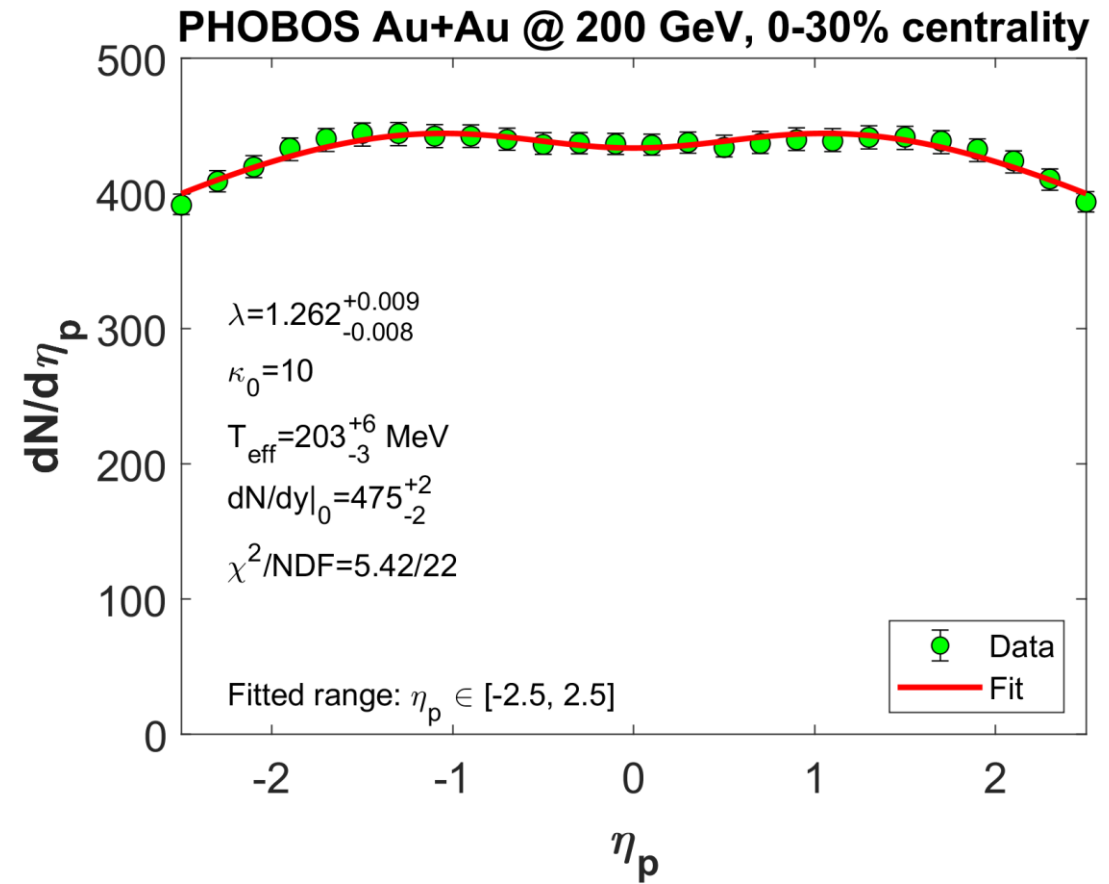
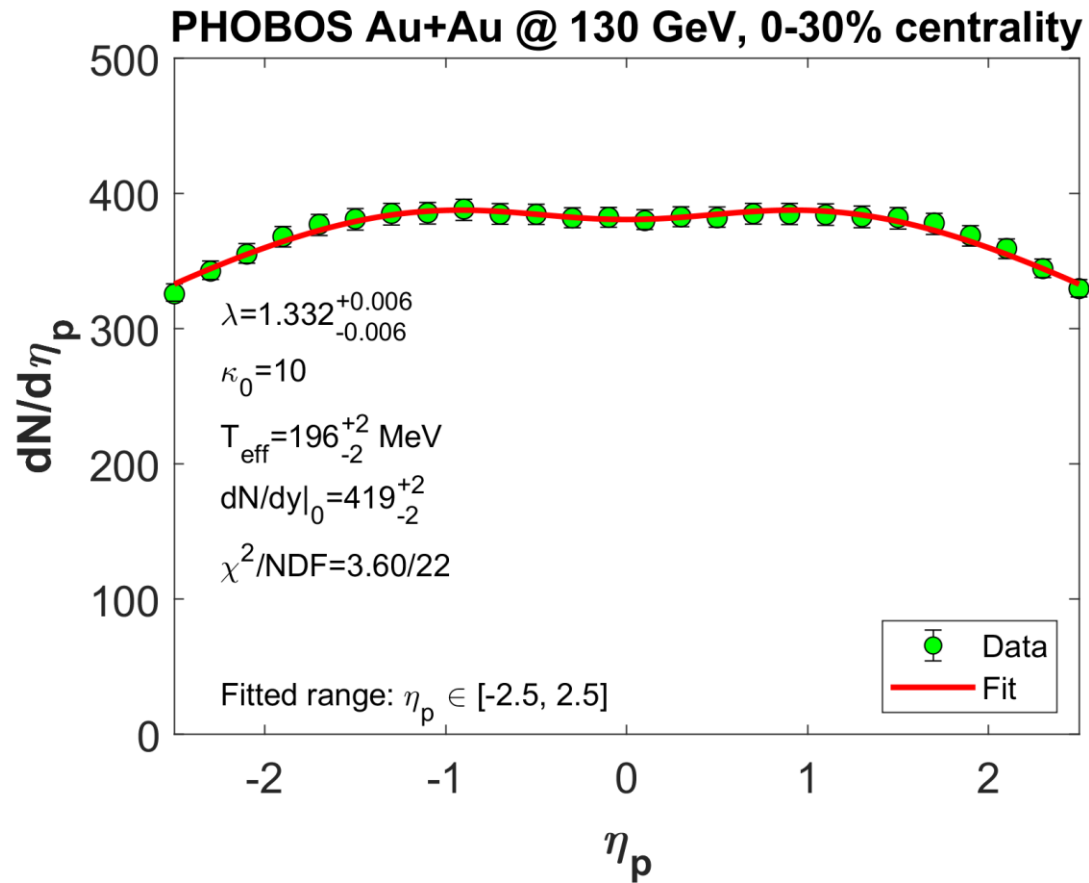
$$\left(\eta_p(y), \frac{dN}{d\eta_p}(y) \right) = \left(\frac{1}{2} \ln \left[\frac{\langle |p(y)| \rangle + \langle p_z(y) \rangle}{\langle |p(y)| \rangle - \langle p_z(y) \rangle} \right], \frac{\langle |p(y)| \rangle}{\langle E(y) \rangle} \frac{dN}{dy} \right)$$

- We compared this curve with experimental data :
 - PHOBOS Au+Au 130 GeV, 200 GeV
 - ALICE Pb+Pb 5.02 TeV
 - CMS p+p 7 TeV, 8 TeV, 13 TeV
 - CMS Xe+Xe 5.44 TeV

*G. Kasza, T. Csörgő:
Int.J.Mod.Phys. A34 no.26, 1950147 (2019)*

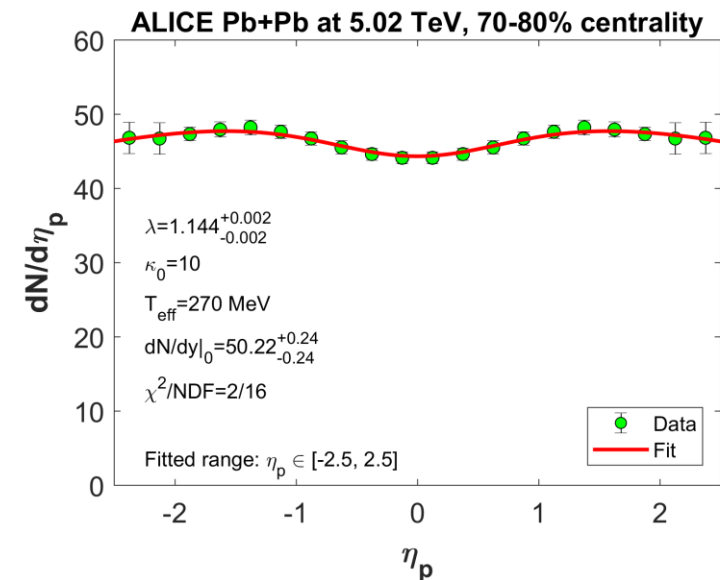
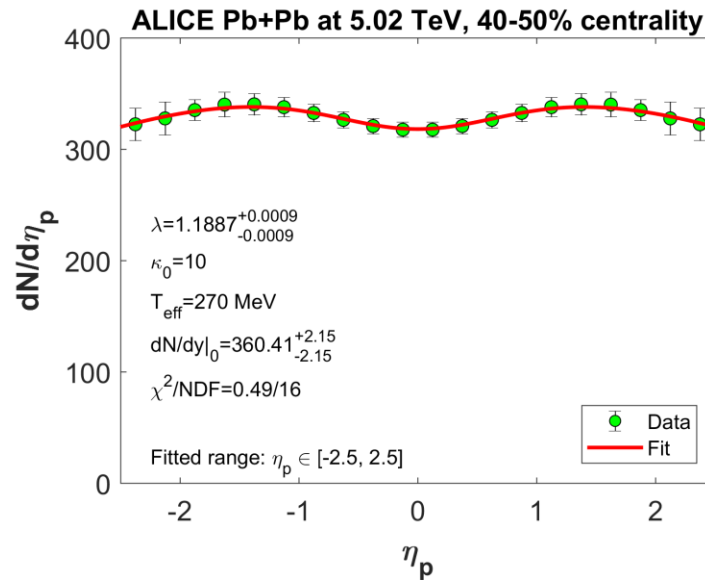
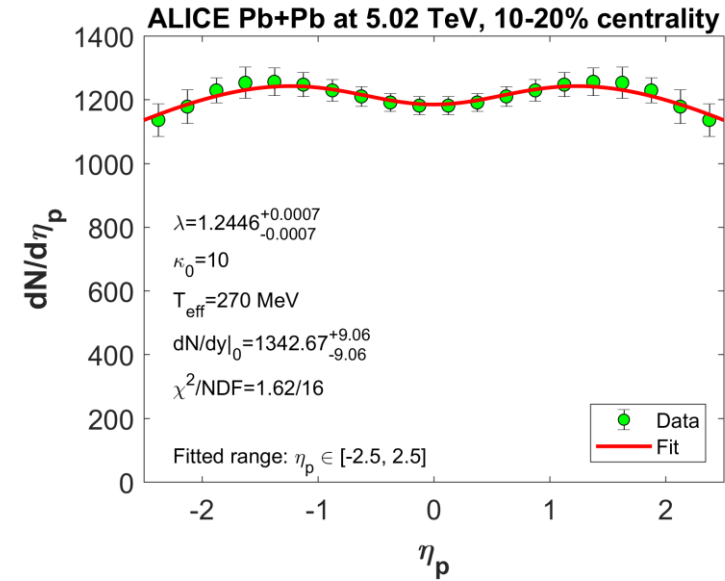
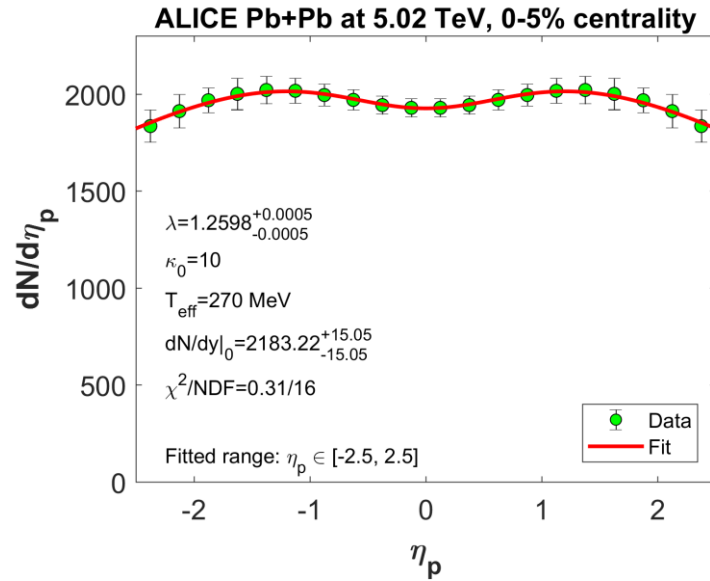
- Different sizes (p+p, A+A), different centralities, different collision energies

Fits to PHOBOS data (Au+Au)

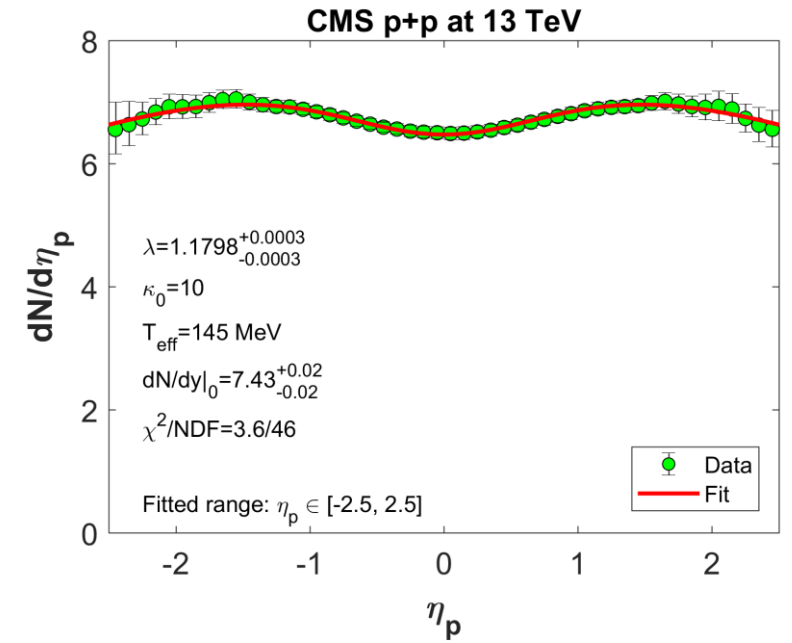
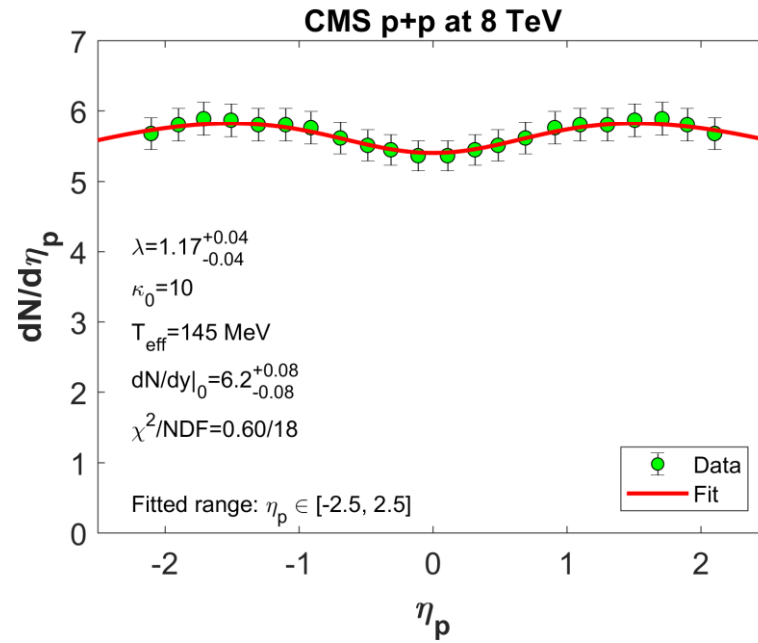
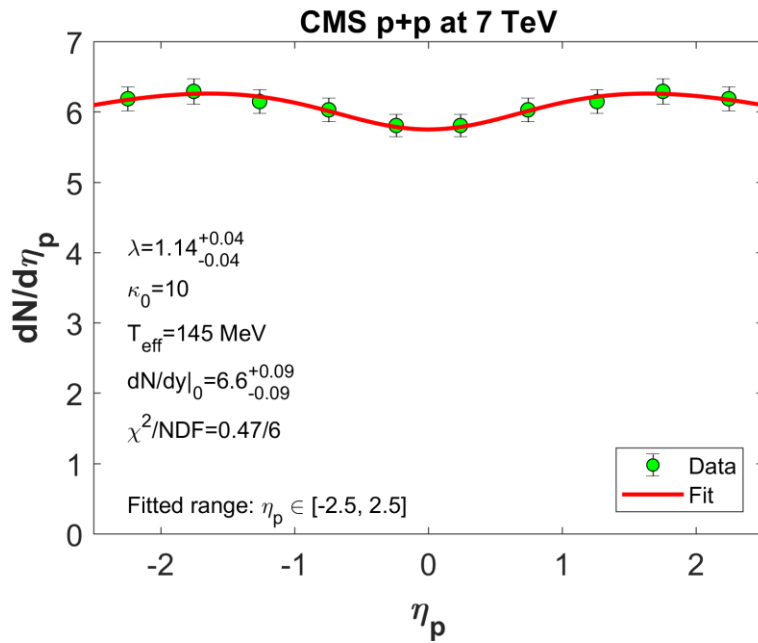


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Fits to ALICE data (centrality dependence)



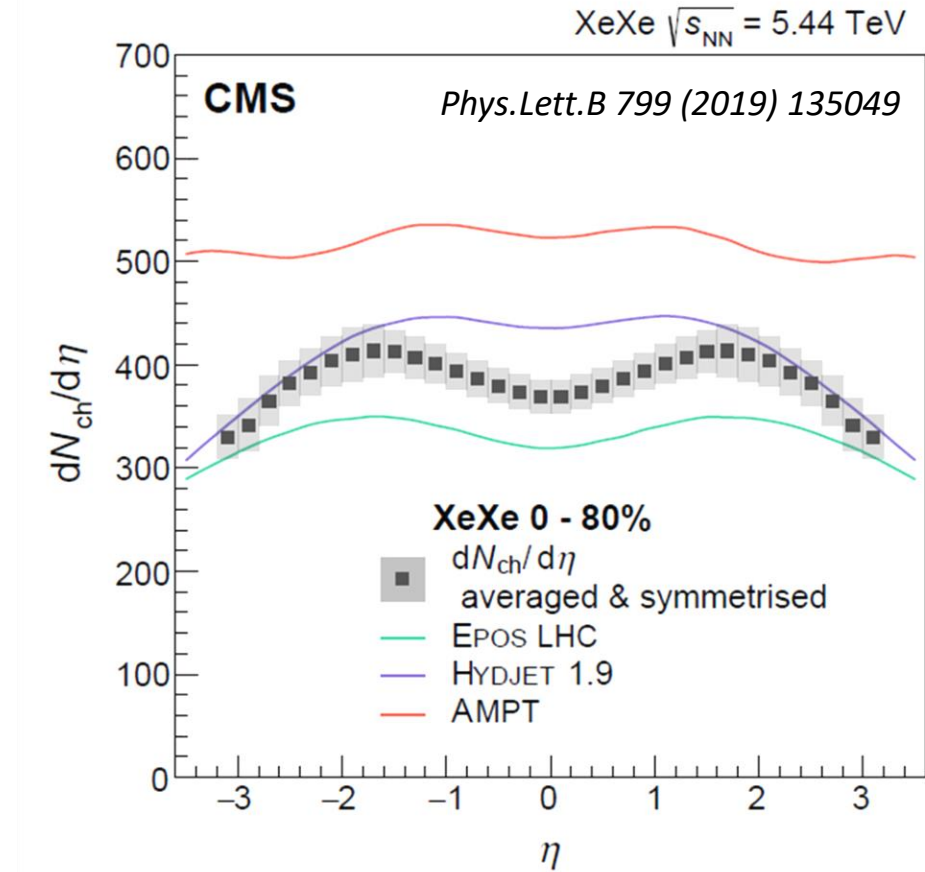
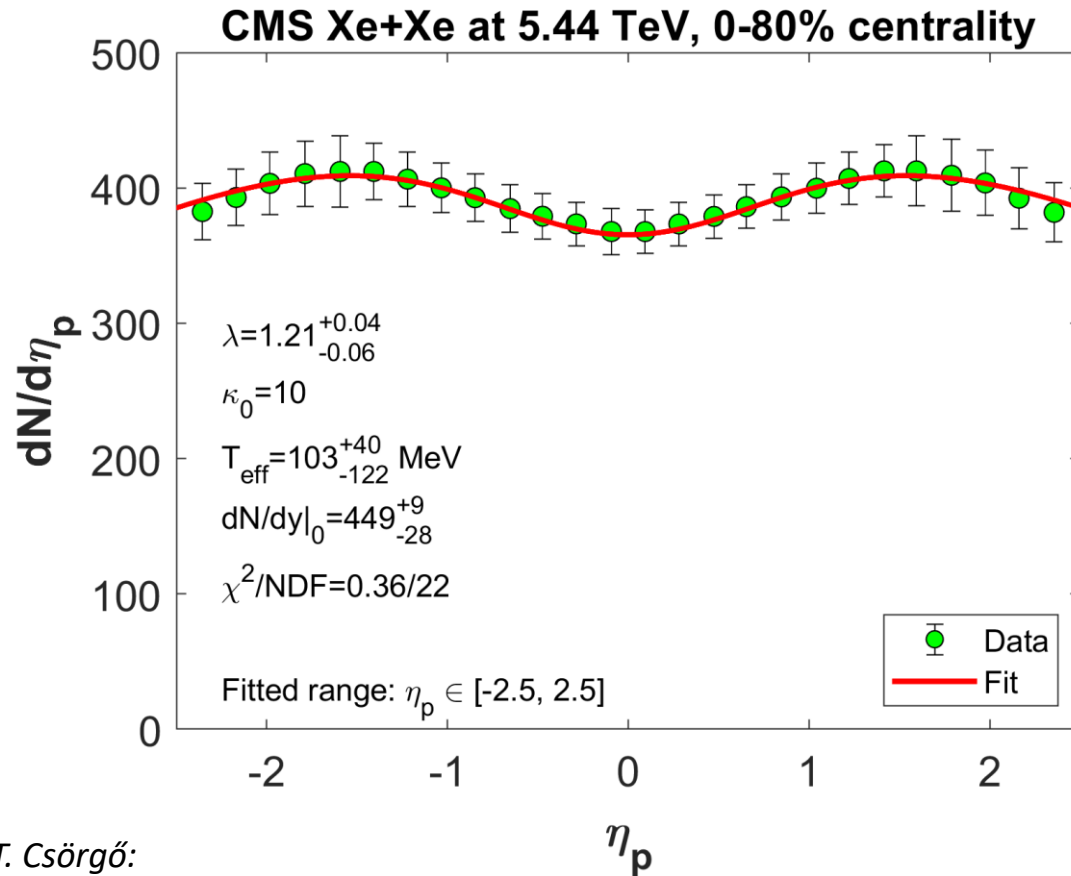
Fits to CMS data (p+p)



Collectivity in p+p collisions?

Fits to CMS data (Xe+Xe)

- The parametric formula works well in such cases where other models fail:



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A reasonable approximation of the rapidity distribution

- If $|y| \ll 2+1/(\lambda-1)$, then the rapidity distribution becomes Gaussian:

$$\frac{dN}{dy} \approx \frac{\langle N \rangle}{(2\pi\Delta y^2)^{1/2}} \exp\left(-\frac{y^2}{2\Delta y^2}\right)$$

- Manifest of the hydrodynamic scaling behaviour:

$$\frac{1}{\Delta y^2} = (\lambda - 1)^2 \left[1 + \left(1 + \frac{1}{\kappa_0} \right) \left(\frac{1}{2} + \frac{m}{T_{\text{eff}}} \right) \right]$$

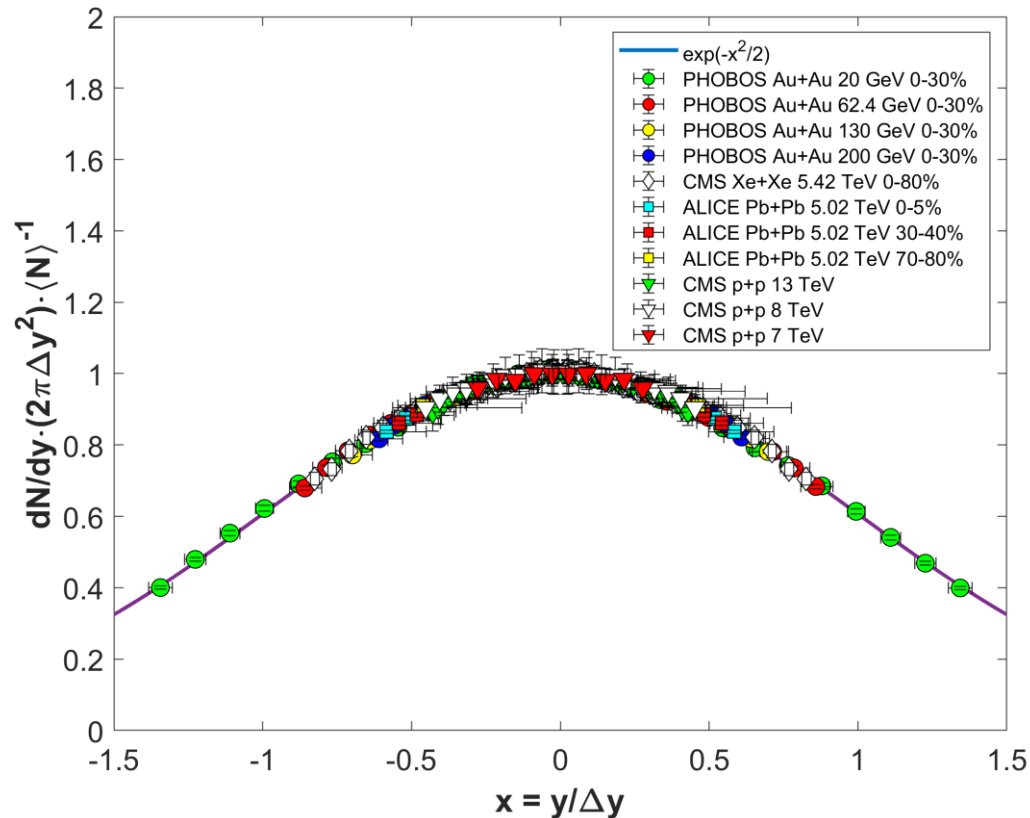
$$\langle N \rangle = (2\pi\Delta y^2)^{1/2} \left. \frac{dN}{dy} \right|_{y=0}$$

- The physical properties of different collisions (\sqrt{s} , centrality, size) are scaled out:

$$\frac{dN}{dy} = \left. \frac{dN}{dy} \right|_{y=0} \exp\left(-\frac{y^2}{2\Delta y^2}\right) \longrightarrow f(x) = \exp\left(-\frac{x^2}{2}\right)$$

Data collapsing

- Pseudorapidity distributions were transformed into rapidity distributions \rightarrow fits to the dN/dy data series



For each dataset, the fit range satisfies:
 $|y| \ll 2+1/(\lambda-1)$

This condition is the strictest in the case of
PHOBOS Au+Au@20 GeV $\rightarrow |y| \ll 3, |x| \ll 1.8$

Another prediction for Gaussian dN/dy from Landau hydrodynamics:
C-Y. Wong: Phys. Rev. C 78, 054902
C-Y. Wong et al.: Phys.Rev.C 90 (2014) 6, 064907
A. Sen et al.: J.Phys.Conf.Ser. 630 (2015) 1, 012042

- Data collapsing on the $f(x)=exp(-x^2/2)$ curve of the scale function

In conclusion...

- p+p collisions can be described as collective systems
- Our fits indicate low c_s value (≈ 0.35)
- Low c_s value indicate the presence of fluid, so the presence of QGP
- p+p and A+A collisions: self-similar systems

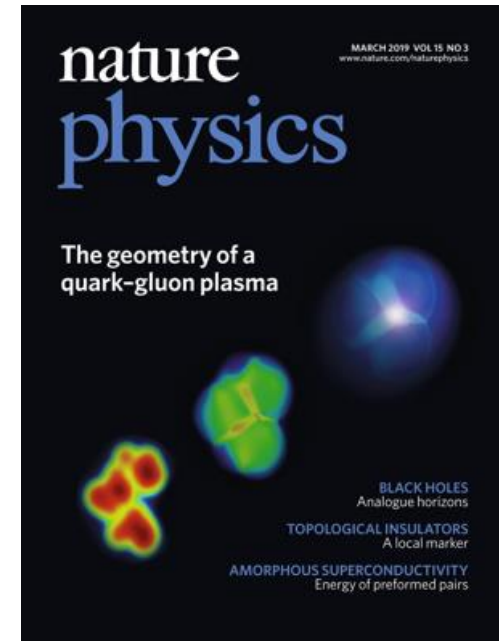
Thank you for your attention!

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Is the hydrodynamic description well-accepted?

- A+A collisions: become a major trend since 2005
- p+A, d+A and He+A collisions: accepted since 2019
- p+p collisions: not widely accepted yet



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Is the hydrodynamic description well-accepted?

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- p+p collisions: not widely accepted yet
- However, describing H+H systems by hydro is not a recent idea

arXiv:hep-ex/9711009v2 19 Dec 1997

Nijmegen preprint
HEN-405
Dec. 97

ESTIMATION OF HYDRODYNAMICAL MODEL PARAMETERS FROM
THE INVARIANT SPECTRUM AND THE BOSE-EINSTEIN CORRELATIONS OF
 π^- MESONS PRODUCED IN $(\pi^+/K^+)p$ INTERACTIONS AT 250 GeV/c

EHS/NA22 Collaboration

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Abstract: The invariant spectra of π^- mesons produced in $(\pi^+/K^+)p$ interactions at 250 GeV/c are analysed in the framework of the hydrodynamical model of three-dimensionally expanding cylindrical symmetric finite systems. A satisfactory description of experimental data is achieved. The data favour the pattern according to which the hadron matter undergoes predominantly longitudinal expansion and non-relativistic transverse expansion with mean transverse velocity $\langle u_x \rangle = 0.20 \pm 0.07$, and is characterized by a large temperature inhomogeneity in the transverse direction: the extracted freeze-out temperature at the center of the tube and at the transverse rms radius are 140 ± 3 MeV and 82 ± 7 MeV, respectively. The width of the (longitudinal) space-time rapidity distribution of the pion source is found to be $\Delta\eta = 1.36 \pm 0.02$. Combining this estimate with results of the Bose-Einstein correlation analysis in the same experiment, one extracts a mean freeze-out time of the source of $\langle \tau_f \rangle = 1.4 \pm 0.1$ fm/c and its transverse geometrical rms radius, $R_G(\text{rms}) = 1.2 \pm 0.2$ fm.

Rapidity distribution

- We applied the Cooper-Frye formula
- Temperature is determined on the freeze-out hypersurface
- Integrals were calculated by saddle-point approximation
- Fluid rapidity could be well approximated by a linear function: $\Omega \approx \lambda \eta_z$
- The 1+1 dimensional rapidity distribution was embedded in 1+3 dimension

$$\frac{dN}{dy} \approx \left. \frac{dN}{dy} \right|_{y=0} \cosh^{-\frac{\alpha(\kappa_0)}{2}-1} \left(\frac{y}{\alpha} \right) \exp \left(-\frac{m}{T_{\text{eff}}} \left[\cosh^{\alpha(\kappa_0)} \left(\frac{y}{\alpha} \right) - 1 \right] \right)$$


G. Kasza, T. Csörgő:

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- Starting from the rapidity distribution, we calculated the pseudorapidity distribution
- Parametric curve:

$$\left(\eta_{\text{p}}(y), \frac{dN}{d\eta_{\text{p}}}(y) \right) = \left(\frac{1}{2} \ln \left[\frac{\langle |p(y)| \rangle + \langle p_z(y) \rangle}{\langle |p(y)| \rangle - \langle p_z(y) \rangle} \right], \frac{\langle |p(y)| \rangle}{\langle E(y) \rangle} \frac{dN}{dy} \right)$$


$$\eta_{\text{p}}(y) = \tanh^{-1} (\mathcal{J}^{-1} \tanh (y)) = \tanh^{-1} \left(\frac{\tanh (y)}{\sqrt{1 - \frac{m^2}{\langle m_{\text{T}}(y) \rangle^2 \cosh^2 (y)}}} \right)$$

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$$\frac{dN}{d\eta_{\text{p}}}(y) \approx \frac{dN}{dy} \Big|_{y=0} \sqrt{1 - \frac{m^2}{\langle m_{\text{T}}(y) \rangle^2 \cosh^2(y)}} \cosh^{-\frac{\alpha(\kappa_0)}{2} - 1} \left(\frac{y}{\alpha} \right) \exp \left(-\frac{m}{T_{\text{eff}}} \left[\cosh^{\alpha(\kappa_0)} \left(\frac{y}{\alpha} \right) - 1 \right] \right)$$

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