Scaling behaviour of dN/dy in high energy collisions

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52nd International Symposium on Multiparticle Dynamics

GYÖNGYÖS, 08/24/2023
Various application of hydrodynamics

- Why is hydrodynamics so effective?
- Works well on ...
  - ... microscopic scales
  - ... macroscopic scales
  - ... cosmic scales
- Different systems in many aspects
Various application of hydrodynamics

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*Hydrodynamics has no internal scale!*

- Today’s presentation: a new example of hydodynamic scaling behaviour in dN/dy data
Relativistic perfect fluid solution with accelerating velocity field

- **Equation of state:** \( \varepsilon = \kappa_0 p \quad (\mu=0) \)
- **Rindler coordinates:**
  \[ \tau = \sqrt{t^2 - r_z^2} \]
  \[ \eta_z = \frac{1}{2} \ln \left( \frac{t + r_z}{t - r_z} \right) \]
- **Velocity field:**
  \[ u^\mu = \left( \begin{array}{c} \cosh (\Omega) \\ \sinh (\Omega) \end{array} \right) \]
  \[ \Omega \equiv \Omega (\eta_z) \]
- **1+1 dimensional, parametric, almost self-similar, finite solution**

\( \lambda: \) rate of acceleration
\( \text{(Hwa-Bjorken: } \lambda=1) \)

\( \text{accelerating expansion} \)

\( \text{realistic } dN/dn_p \)

Pseudorapidity distribution

- Starting from the rapidity distribution, we calculated the pseudorapidity distribution

- Parametric curve:

\[
\left( \eta_p(y), \frac{dN}{d\eta_p(y)} \right) = \left( \frac{1}{2} \ln \left[ \frac{\langle |p(y)| \rangle + \langle p_z(y) \rangle}{\langle |p(y)| \rangle - \langle p_z(y) \rangle} \right], \frac{\langle |p(y)| \rangle \frac{dN}{dy}}{\langle E(y) \rangle} \right)
\]

- We compared this curve with experimental data:

  - PHOBOS Au+Au 130 GeV, 200 GeV
  - ALICE Pb+Pb 5.02 TeV
  - CMS p+p 7 TeV, 8 TeV, 13 TeV
  - CMS Xe+Xe 5.44 TeV

- Different sizes (p+p, A+A), different centralities, different collision energies

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Fits to PHOBOS data (Au+Au)

G. Kasza, T. Csörgő:
Fits to ALICE data (centrality dependence)
Fits to CMS data (p+p)

Collectivity in p+p collisions?
Fits to CMS data (Xe+Xe)

The parametric formula works well in such cases where other models fail:

\[ \lambda = 1.21^{+0.04}_{-0.06} \]
\[ \kappa_0 = 10 \]
\[ T_{eff} = 103^{+40}_{-122} \text{ MeV} \]
\[ dN/dy|_0 = 449^{+9}_{-28} \]
\[ \chi^2/\text{NDF} = 0.36/22 \]

Fitted range: \( \eta_p \in [-2.5, 2.5] \)

G. Kasza, T. Csörgő: 
*Int.J.Mod.Phys.* A34 no.26, 1950147 (2019)
A reasonable approximation of the rapidity distribution

- If $|y| \ll 2 + 1/(\lambda - 1)$, then the rapidity distribution becomes Gaussian:

$$\frac{dN}{dy} \approx \frac{\langle N \rangle}{(2\pi \Delta y^2)^{1/2}} \exp \left( - \frac{y^2}{2\Delta y^2} \right)$$

- Manifest of the hydrodynamic scaling behaviour:

$$\frac{1}{\Delta y^2} = (\lambda - 1)^2 \left[ 1 + \left( 1 + \frac{1}{\kappa_0} \right) \left( \frac{1}{2} + \frac{m}{T_{\text{eff}}} \right) \right]$$

$$\langle N \rangle = (2\pi \Delta y^2)^{1/2} \left. \frac{dN}{dy} \right|_{y=0}$$

- The physical properties of different collisions ($\sqrt{s}$, centrality, size) are scaled out:

$$\left. \frac{dN}{dy} \right|_{y=0} \exp \left( - \frac{y^2}{2\Delta y^2} \right) \rightarrow f(x) = \exp \left( - \frac{x^2}{2} \right)$$
Data collapsing

- Pseudorapidity distributions were transformed into rapidity distributions → fits to the $dN/dy$ data series

For each dataset, the fit range satisfies: $|y| < 2 + 1/\lambda$.

This condition is the strictest in the case of PHOBOS Au+Au@20 GeV $\rightarrow |y| < 3$, $|x| < 1.8$

Another prediction for Gaussian $dN/dy$ from Landau hydrodynamics:
C-Y. Wong: Phys. Rev. C 78, 054902

- Data collapsing on the $f(x) = \exp(-x^2/2)$ curve of the scale function
In conclusion...

- p+p collisions can be described as collective systems
- Our fits indicate low $c_s$ value ($\approx 0.35$)
- Low $c_s$ value indicate the presence of fluid, so the presence of QGP
- p+p and A+A collisions: *self-similar* systems

Thank you for your attention!
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*Is the hydrodynamic description well-accepted?*

- A+A collisions: become a major trend since 2005
- p+A, d+A and He+A collisions: accepted since 2019
- p+p collisions: not widely accepted yet
In conclusion...

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- p+p collisions: not widely accepted yet
- However, describing H+H systems by hydro is *not* a recent idea
Rapidity distribution

- We applied the Cooper-Frye formula
- Temperature is determined on the freeze-out hypersurface
- Integrals were calculated by saddle-point approximation
- Fluid rapidity could be well approximated by a linear function: \( \Omega \approx \lambda \eta_z \)
- The 1+1 dimensional rapidity distribution was embedded in 1+3 dimension

\[
\frac{dN}{dy} \approx \left. \frac{dN}{dy} \right|_{y=0} \cosh \left( -\frac{\alpha(\kappa_0)}{2} \right) - 1 \left( \frac{y}{\alpha} \right) \exp \left( -\frac{m}{T_{\text{eff}}} \left[ \cosh \left( \alpha(\kappa_0) \left( \frac{y}{\alpha} \right) \right) - 1 \right] \right)
\]

G. Kasza, T. Csörgő: 
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\]

\[
\eta_p(y) = \tanh^{-1} \left( \mathcal{J}^{-1} \tanh(y) \right) = \tanh^{-1} \left( \frac{\tanh(y)}{\sqrt{1 - \frac{m^2}{\langle m_T(y) \rangle^2 \cosh^2(y)}}} \right)
\]

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\]

\[
\frac{dN}{d\eta_p(y)} \approx \frac{dN}{dy} \bigg|_{y=0} \sqrt{1 - \frac{m^2}{\langle m_T(y) \rangle^2 \cosh^2(y)}} \cosh^{-\frac{\alpha(\kappa_0)}{2} - 1} \left( \frac{y}{\alpha} \right) \exp \left( -\frac{m}{T_{\text{eff}}} \left[ \cosh^{\alpha(\kappa_0)} \left( \frac{y}{\alpha} \right) - 1 \right] \right)
\]

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