QCD in the Early Universe

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2) Based on the review article A. Addazi, T. Lundberg, A. Marcianò, R. Pasechnik and M. Šumbera, *Cosmology from Strong Interactions*, Universe **8**, no.9, 451 (2022), [arXiv:2204.02950 [hep-ph]].

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Introduction: Equation of State of the Early Universe

Standard Cosmological Model

Einstein equations of general relativity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}(R - 2\Lambda) = -8\pi G \mathcal{T}_{\mu\nu} \tag{1}$$

$g^{\mu\nu}$ – metric tensor, $R_{\mu\nu} = f(g_{\mu\nu}, \partial_\lambda g_{\mu\nu}, \partial^2_{\lambda,\kappa} g_{\mu\nu})$ – Ricci tensor,
$R = R_{\mu\nu} g^{\mu\nu}$ – scalar curvature, Λ, G – cosmological, gravitational constants,
$\mathcal{T}_{\mu\nu}$ – energy-momentum tensor.

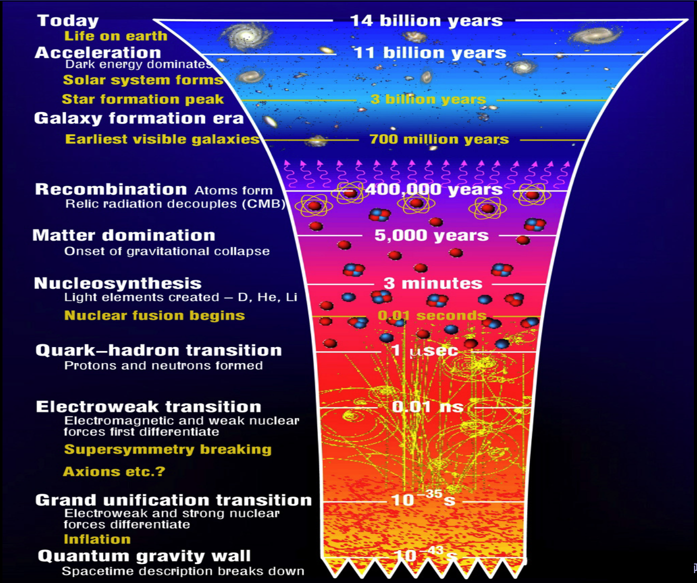
Cosmological principle: the Universe is Homogenous and Isotropic

Solution of (1) preserving space homogeneity and isotropy under its time evolution is spacetime of constant curvature $k = \{+1, 0, -1\}$ with FLRW metrics

$$ds^2 = g^{FLRW}_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \tag{2}$$

$a(t)$ – scale factor of the Universe connects co-moving (Lagrange) and physical (Euler) coordinates $\hat{r}(t) = a(t)r$.

A Brief History of the Universe



Early Universe Made Simple

Friedman equation: $g_{\mu\nu}^{FLRW} \rightarrow$ Eq. (1) with $\mu = \nu = 0$

$$H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\epsilon - \frac{k}{a^2} + \frac{\Lambda}{3} \qquad (3)$$

Perfect fluid: $\mathcal{T}_{\nu}^{\mu} = diag(\epsilon, -p, -p, -p) \Rightarrow$ Fluid equation

$$\dot{\epsilon} + 3(\epsilon + p)H(t) = 0 \qquad (4)$$

- Ornik,Weiner:1987 Early Universe: $\epsilon \gtrsim 1\text{GeV fm}^{-3} \Rightarrow$ neglect k and Λ in (3) :

$$-\frac{d\epsilon}{3\sqrt{\epsilon}(\epsilon + p)} = \sqrt{\frac{8\pi G}{3}}dt \quad \Rightarrow \quad \dot{\epsilon} + \sqrt{\frac{24\pi G}{3}}(\sqrt{\epsilon}(\epsilon + p(\epsilon)) = 0 \qquad (5)$$

- Integration of (5) using barotropic form of EoS $p(\epsilon)$ yields $\epsilon(t)$.
- **Example:** Time-independent speed of sound $c_s^2 = dp/d\epsilon$ [Sanches et al. 2014]

$$\epsilon(t) = \frac{1}{6\pi G(1 + c_s^2)^2 t^2} \Rightarrow H(t) \sim \frac{1}{t}, \; \dot{a} \sim t^{-\alpha}, \; \alpha = \frac{1 + 3c_s^2}{3(1 + c_s^2)} \qquad (6)$$

Evolution of the Early Universe

Cooling of the Universe: The Insight from the Asymptotic Freedom

Universe cooled down via series of first- or second-order phase transitions (PT) associated with the various spontaneous symmetry breakings (SSBs) of the basic non-Abelian gauge fields, see e.g. the classical textbooks [Linde:1978], [Bailin and Love:2004], [Boyanovsky et al:2006].

Standard Model Predicts Two Phase Transitions

1. The electroweak (EW) PT at $T \sim m_H$ provides masses to elementary particles. LQFT calculations show that for $m_H \geq 67$ GeV this PT is an analytic crossover [Kajantie et al.:1996], [Csikor et al.:1999].

2. At $T < 200$ MeV the SSB of the chiral symmetry of the $SU(3)_c$ color group, the QCD, takes place. In a strong first-order PT scenario, the de-confined matter supercools before bubbles of hadron gas are formed. $\Rightarrow$ Produced inhomogeneities in this phase could have a strong effect on the nucleosynthesis epoch.

Standard Model Couplings in the Hot Big Bang Era

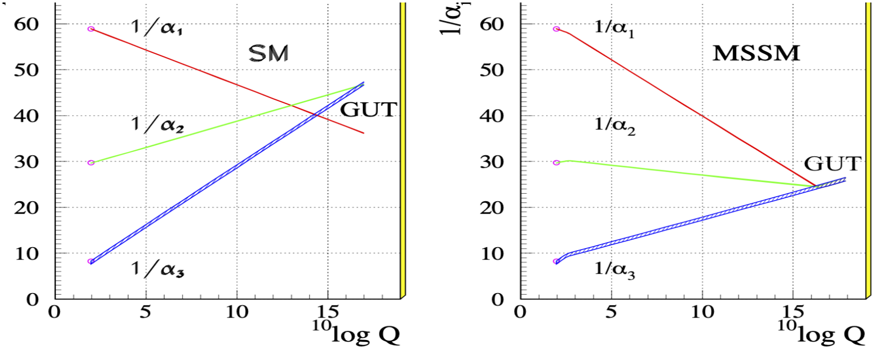


Figure 1: Evolution of the inverse of the three coupling constants $\alpha_1 = \alpha_{\rm EM}, \alpha_2 = \alpha_{\rm W}, \alpha_3 = \alpha_{\rm S}$ in the Standard Model $\rm U(1)_Y \times SU(2)_L \times SU(3)_C$ (left) and in its supersymmetric extension MSSM (right).

- Thermodynamics is applicable if the Universe is in global equilibrium.
- Hydrodynamical description needs only local thermal equilibrium (LTE).
- For $10^{15} - 10^{17}$ GeV $\gtrsim T \gtrsim T_c^{\rm QCD} \approx 160$ MeV LTE in expanding fluid persists, see Slide 27.

What is Changing: Effective Degrees of Freedom

$$g_{\text{eff}}(T) \equiv \frac{\epsilon(T)}{\epsilon_0(T)}, \quad \epsilon_0(T) = \frac{\pi^2}{30}T^4 \tag{7}$$

$$h_{\text{eff}}(T) \equiv \frac{s(T)}{s_0(T)}, \quad s_0(T) = \frac{2\pi^2}{45}T^3 \tag{8}$$

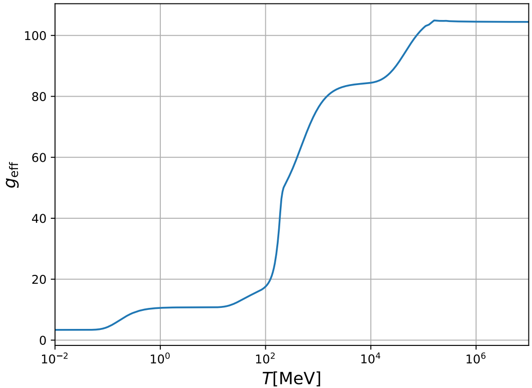
$$g_{\text{eff}}^{id}(T) = h_{\text{eff}}^{id}(T) = \frac{7}{8}4N_{\text{F}} + 3N_{\text{V}} + 2N_{\text{V0}} + N_{\text{S}} \tag{9}$$

- For an adiabatic process

$$s(T) = \frac{\epsilon(T) + p(T)}{T} \tag{10}$$

- EoS in cosmology $p = w\epsilon$

$$w(T) = \frac{sT}{\epsilon} - 1 = \frac{4h_{\text{eff}}(T)}{3g_{\text{eff}}(T)} - 1 \tag{11}$$



$g_{\text{eff}}(T)$ in the SM taking into account interactions between particles, obtained with both perturbative and lattice methods. [Hindmarsh et al.:2020].

Energy Density of Ideal Massless Gas in Early Universe

- Simultaneous presence of the EW and the QCD matter in thermal equilibrium is one of the remarkable differences between the QGP produced in accelerator experiments and the deconfined QCD matter in the early universe.
- Including only the particles which at $T \lesssim T_{\mathrm{EW}} = 160 GeV$ considered as massless: $g_{\mathrm{eff}}^{\mathrm{EW}} = \frac{7}{8}(12+6)+2 = 17.75$ and $g_{\mathrm{eff}}^{\mathrm{QCD}} = 2\times 8 + \frac{7}{8}(3\times N_F \times 2 \times 2)$.
- At $T \lesssim T_{\mathrm{EW}}$ and for N_F active quark flavors, the QGP contains a factor of $g_{\mathrm{eff}}^{\mathrm{QCD}}/g_{\mathrm{eff}}^{\mathrm{EW}} \simeq 2 \div 4$ more energy and pressure than the EW matter.
- Even for $T \gg T_c^{\mathrm{EW}}$ wih $g_{\mathrm{eff}}^{\mathrm{QCD}} = 79$ and $g_{\mathrm{eff}}^{\mathrm{EW}} = 26.75$ the QCD matter has a factor of $g_{\mathrm{eff}}^{\mathrm{QCD}}/g_{\mathrm{eff}}^{\mathrm{EW}} \simeq 3$ larger energy density and pressure than the EW matter.
- QCD matter represents the densest form of matter filling the early universe during both the QCD and EW epochs and beyond.

Cosmological Parametrization of the EoS

- In cosmology the EoS is parametrized as $p = w\epsilon$

$$s(T) = \frac{dp}{dT} = \frac{dp}{d\epsilon}\frac{d\epsilon}{dT} \tag{12}$$

$$c_s^2(T) = \frac{dp}{d\epsilon} = T\frac{ds}{d\epsilon} + s\frac{dT}{d\epsilon} - 1 = \frac{4}{3}\left[\frac{4h_{\text{eff}}(T) + Th'_{\text{eff}}(T)}{4g_{\text{eff}}(T) + Tg'_{\text{eff}}(T)}\right] - 1 \tag{13}$$

where the prime indicates differentiation with respect to T.

- Causality: $$c_s \leq 1 \quad \Rightarrow \quad \frac{h_{\text{eff}}(T)}{g_{\text{eff}}(T)} \leq \frac{3}{2}, \tag{14}$$

  $\Rightarrow$ upper bound at $w = 1$, i.e. $p = \epsilon$, corresponds to absolutely stiff fluid: baryons interacting via massive vector particles [Zeldovich:1961] or free scalar field.

- The observation that the stiff fluid saturates the holographic covariant entropy bound was used to describe a cosmology of the very early universe [Banks:2001] .

- Can the stiff fluid represent some form of QCD matter?

EoS Based on the Fundamental Theory: the SM and GUT

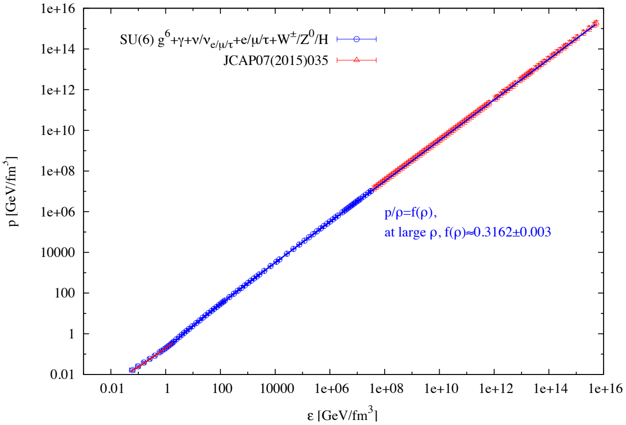


Figure 2: Combined EoS $p(\epsilon), \epsilon \equiv \rho$ of QCD and EW matter, using lattice results of [Borsanyi et al:2016] extended to include other DoFs such as γ, neutrinos, leptons, EW, and Higgs bosons as well as perturbative results of [Laine,Mayer:2015]. Adapted from [Tawfik,Mishustin:2019].

- For details see page 28

The EoS parametrization

- GUT EoS (△ in Fig. 2) $p_{\rm GUT} = (0.330 \pm 0.024)\epsilon$ valid for $10^8 \lesssim \epsilon \leq 10^{16}$ GeV·fm^{-3} is already in the ideal gas limit.
- Combined EoS for QCD and EW eras has two independent contributions $p_1(\epsilon)$ and $p_2(\epsilon)$.

$$p_{\rm SM} = p_1(\epsilon) + p_2(\epsilon)\,, \quad p_1(\epsilon) = b\epsilon\,, \quad p_2(\epsilon) = a + c\epsilon^d \tag{15}$$

$$a = 0.048 \pm 0.016,\ b = 0.316 \pm 0.031,\ c = -0.21 \pm 0.014,\ d = -0.576 \pm 0.034$$

- $p_1(\epsilon) \approx \frac{1}{3}\epsilon$ – ideal gas of massles particles EoS.
- $p_2(\epsilon) = -\mathcal{B}(\epsilon) = a + c\epsilon^d$ – density-dependent bag function (instanton liquid?)
- The sound velocities of both components are positive:

$$c^2_{s,1}(\epsilon) = \frac{dp_1}{d\epsilon} = b > 0\,, \qquad c^2_{s,2}(\epsilon) = \frac{dp_2}{d\epsilon} = c \cdot d \cdot \epsilon^{d-1} = \frac{d \cdot (p_2 - a)}{\epsilon} > 0 \tag{16}$$

- $a \approx 0 \Rightarrow$ EoS $p_2(\epsilon) \approx c\epsilon^d$, $c < 0, d < 0 \Rightarrow$ must have $p_2 < 0$! Coincides with generalized Chaplygin EoS often applied as a model of DE, see e.g. [Kamenshchik et al:2001].

Saturated QCD matter in the Early Universe

Evolution of the Strong Coupling Constant $\alpha_s(Q)$

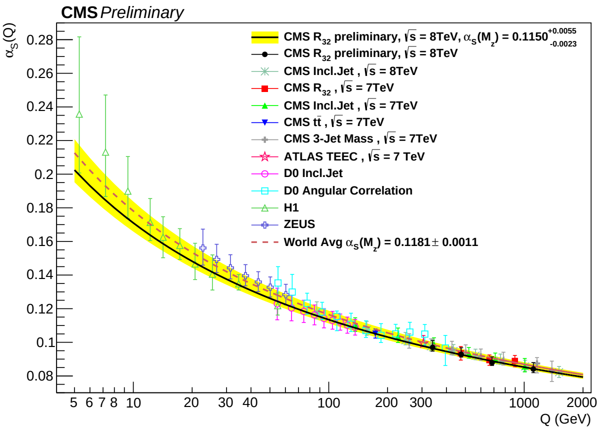


Figure 3: $\alpha_S(Q)$ obtained from MSTW2008 NLO PDF set. [CMS:2017].

- For $0.2 \lesssim \alpha_S \lesssim 0.08$ "temperatures" $T \approx Q/(2\pi) \in (1, 200)$ GeV are reachable?.

The Weakly Interacting QCD: DGLAP and BFKL

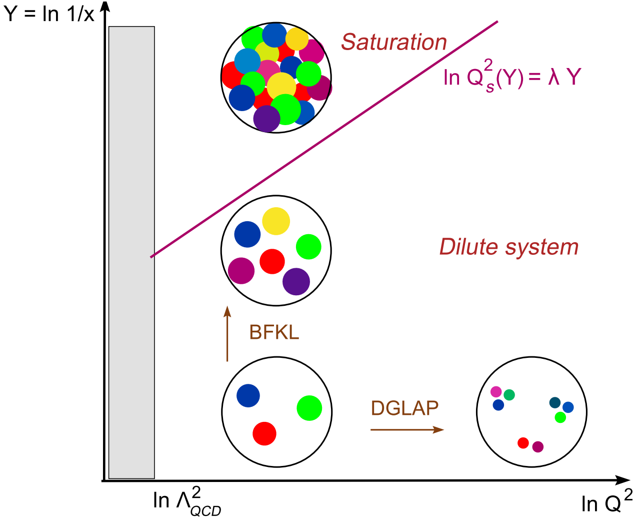


Figure 4: Parton density and size as a function $Y = \ln(1/x)$ and $\ln Q^2$. From [Gelis et al:2010].

QCD at High Parton Densities and Saturation

[Gribov jr.,Levin,Ryskin:1983] , [McLerran,Venugopalan:1993]

- Partons "overlap" when $\sigma_{gg} \sim \alpha_{\rm S}/Q^2$ times $xG_A(x, Q^2)$ – the probability to find at fixed Q a parton carrying a fraction x of the parent parton momentum – becomes comparable to the geometrical cross section πR_A^2 of the object A occupied by the gluons.

$$Q_s^2(x) = \frac{\alpha_{\rm S}(Q_s)}{2(N_c^2-1)} \frac{xG_A(x, Q_s^2)}{\pi R_A^2} \sim \frac{1}{x^\lambda} \Rightarrow \ \ln Q_s^2(x) = \lambda Y \qquad (17)$$

- $Q_s(x)$ – Fixed point of the PDF evolution in x or, equivalently, the emergent "close packing" scale

- Repulsive gg interactions $\Rightarrow$ occupation number f_g (# of gluons with a given x times the area each gluon fills up divided by the transverse size of the object) saturates at $f_g \sim 1/\alpha_{\rm S}$.
- The same scaling as for the Higgs condensate, superconductivity or the inflaton field.
- Saturated gluonic matter is weakly coupled. $\Rightarrow$ weakly interacting means semi-classical. See page 29.

Glass properties of the Glasma and CGC–Black Hole correspondence

- In condensed matter physics glass is a non-equilibrium, disordered state of matter acting like solids on short time scales but liquids on long time scales [Mauro:2014,Sethna:2021].
- Two scales of glasma: $$\tau_{\rm wee} = \frac{1}{k^-} = \frac{2k^+}{k_\perp^2} = \frac{2xP^+}{k_\perp^2} \ll \frac{2P^+}{k_\perp^2} \approx \tau_{\rm valence}\,. \quad (18)$$
  $\Rightarrow$ Valence modes are static over the time scales of wee modes [Berges:2020].
- Glasses are formed when liquids are cooled too fast to form the crystalline equilibrium state. This leads to an enormous # of possible configurations $N_{\rm gl}(T)$ into which the glasses can freeze $\Rightarrow$ large entropy $S = \ln N_{\rm gl}(T)$, such that $S(T=0) > 0$, [Sethna:1988].
- Correspondence between Black Holes as highly occupied condensates of N weakly interacting gravitons and CGCs as highly occupied gluon states [Dvali, Venugopalan:2021].
- Both BH and CGC attain a maximal entropy S_{max} permitted by unitarity when the occupation number f and the coupling α of the respective constituents (gravitons, gluons) satisfy $f = 1/\alpha(Q_s)$, where Q_s represents the point of optimal balance between the kinetic energies of the individual constituents and their potential energies.

Finite Temperature Glasma in Early Universe

- Consider partons at fixed temperature $T \cong Q/2\pi$.
- The QCD saturation scale $R_S = 1/Q_S$ is now given by thermal de Broglie wavelength of massless gluons $\lambda_S = \pi^{2/3}/T_S$, where T_S is the saturation temperature.
- Consider a period of cosmological evolution when $T \gtrsim T_S \gg \Lambda_{\mathrm{QCD}}/(2\pi)$. For $T_S \gtrsim 1$ GeV the running coupling $\alpha_S(2\pi T_S) \lesssim 0.2$.
- At $T \approx T_{EW}$ for YM bosons $g_{\mathrm{eff}}^{\mathrm{QCD}}/g_{\mathrm{eff}}^{\mathrm{EW}} \simeq 8/3 \Rightarrow$
  Glasma might have been prevalent form of matter also during EW era.
- $T \gg T_{EW}$: the gluon exchange between quarks (antiquarks) becomes surpassed by the exchange of EW massless gauge bosons $W^{\pm}, W^0, B^0, \ldots$
  ... GUT YM bosons can also form the classical condensate.

Yang-Mills Field EoS in Cosmology

- Abelian vector field can not be isotropic $\Rightarrow$ one needs at least triplet of vector fields to ensure the isotropy.
- Early studies: [Galtsov, Volkov:1991]: coupled Einstein-Yang-Mills equations with gauge group SU(2) in the FLRW universes.
- Basic features of the Einstein-Yang-Mills homogeneous and isotropic cosmological solutions can be attributed to the conformal nature of the YM field.
- Non-linear nature of YM field configurations makes field-supported *radiation-dominated* universes (with zero temperature) possible. It may reconcile the hot Big Bang geometry with the non-thermal matter which arises quite naturally within the context of grand unified theories.

Q: How can we change this EoS?

A: By breaking the conformal invariance (needs effective action).

- N.B. The breakdown of translation invariance is due to the extended geometry of the object.

Modified Bag Model EoS

$$\epsilon(T) = \sigma T^4 - CT^2 + \mathcal{B}\,, \qquad p(T) = \frac{\sigma}{3} T^4 - DT^2 - \mathcal{B}\,. \tag{19}$$

$$\sigma = \frac{\pi^2}{30} g_{\rm eff}^{\rm QCD}\,, \quad g_{\rm eff}^{\rm QCD} = 2 \times 8 + \tfrac{7}{8}(3 \times N_F \times 2 \times 2) \tag{20}$$

$$\sigma(N_F = 0,\ 2,\ 3,\ 4,\ 5) \approx 5,\ 12,\ 16,\ 18,\ 23\,; \quad \mathcal{B}^{1/4} \approx 220 \text{ MeV}\,.$$

1. $C = D > 0$: LQCD motivated "fuzzy" bag model EoS [Pisarski:2006, Megias:2007].
2. $C = -D < 0$: Gluonic q-particle EoS with $\mathcal{B}(T) = -CT^2 + \mathcal{B}$ [Schneider:2001].

- In pure gauge theory up to $T \approx (2-5)T_c$, the dominant power-like correction to pQCD behavior is $\mathcal{O}(T^{-2})$ rather than $\mathcal{O}(T^{-4})$.
- Quadratic thermal terms in the deconfined phase can also be obtained from gauge/string duality [Zuo:2014vga].

Stiff EoS from Modified Bag Model EoS?

- The trace anomaly for $T \geq 1$ GeV

$$\Theta(T) = \frac{\epsilon - 3p}{T^4} = \frac{4\mathcal{B}}{T^4} + \frac{3D - C}{T^2} \approx \frac{3D - C}{T^2} \equiv \frac{A}{T^2} \tag{21}$$

- The barotropic form of the EoS

$$p(\epsilon) = \frac{1}{3}\left(\epsilon - \Theta(T)T^4\right) = \frac{\epsilon}{3} - \frac{A}{3}T^2(\epsilon)\,, \quad T^2(\epsilon) \equiv \frac{C + \sqrt{C^2 + 4\sigma\epsilon}}{2\sigma} > 0\,. \tag{22}$$

- Speed of sound: $$c_s^2(\epsilon) = \frac{dp(\epsilon)}{d\epsilon} = \frac{1}{3}\left(1 - \frac{2A\sigma}{\sqrt{C^2 + 4\sigma\epsilon}}\right) \geq 0 \tag{23}$$

- Recall: $h_{\text{eff}} = s/s_0$ # eff.d.o.f. in entropy , $g_{\text{eff}} = \epsilon/\epsilon_0$ # eff.d.o.f. in energy

$$w(T) = \frac{p(T)}{\epsilon(T)} = \frac{4h_{\text{eff}}}{3g_{\text{eff}}} - 1 = \frac{1}{3} - \frac{A}{3}\frac{T^2(\epsilon)}{\epsilon} \tag{24}$$

- Stiff EoS: $p = \epsilon$: $$w(T) = 1 \;\Rightarrow A = 3D - C < 0\,, \quad \epsilon_s = \frac{3C^2 - 12CD + 9D^2}{4\sigma} \tag{25}$$

Stiff EoS Scenario for the Early Universe

- The fast cooling of the universe has stiffened its EoS $w = 1/3 \rightarrow w = 1$.
- Saturation energy density $\epsilon_s(T_s) = \sigma T_s^4 - CT_s^2$

- For $T_s = 1GeV$, $\epsilon_s(T_s = 1GeV) = \sigma - C$
- Solutions satisfying conditions $A = 3D - C < 0 \quad \wedge \quad C > 0$
  1. $D < 0$:
     Works for $C = -D < 0$: Gluonic q-particle EoS [Schneider:2001]. In this case

$$\epsilon_s = \frac{6C^2}{\sigma} = \sigma - C \quad \Rightarrow \quad C = \frac{\sigma}{3}. \tag{26}$$

  2. $D > 0 \quad \wedge \quad x = C/D > 3$:
     Does not work for "fuzzy" bag model EoS [Pisarski:2006, Megias:2007] with $x = 1$.

$$\epsilon_s = \frac{C^2}{4\sigma}\phi(x) = \sigma - C \ , \quad \phi(x) = (9x^2 - 12x + 3) \tag{27}$$

$$C = 2\sigma\frac{\sqrt{1+\phi(x)}}{\phi(x)} \approx \frac{1}{7} - \frac{(x-3)}{15} + \mathcal{O}((x-3)^2) \tag{28}$$

Summary

Take-Home Message

- QCD represents a fruitful theory when applied to the early history of our Universe.
- It predicts the state of saturated QCD matter with large entropy which existed immediately before the creation of the QGP. It is likely that this classical Yang-Mills matter has played an essential role in the evolution of our Universe over a broad range of temperatures $T_P \gtrsim T \gtrsim T_{QGP}$.
- Power-like correction $\mathcal{O}(T^{-2})$ to conformal invariance of the classical YM theory describe the stiffening of the EoS of QCD matter during the expansion of the universe.
- (Not included due to time limitation): Breaking of the Mirror symmetry of the QCD vacuum by gravitational interactions induces non-vanishing leading order contribution to the QCD ground state energy compatible with the observed value of the cosmological constant Λ.

Thank you for your attention!

Backup Slides

The Early Universe in Local Thermal Equilibrium

How do we know that the Early Universe was in the state of LTE? [Mukhanov:2005]

- Collision time among the constituents $t_c = 1/(\sigma n v)$
- LTE $\Leftrightarrow$ Local equilibrium must be reached well before expansion becomes relevant.

$\Rightarrow$ At the cosmic time $t_H \sim 1/H(t)$: $t_c \ll t_H$.

- At $T > T_{\mathrm{EW}}$ all (most of) particles of the SM are ultra-relativistic $(k^2 \gg m^2)$ and gauge bosons are massless. $\Rightarrow \sigma \approx \mathcal{O}(1)\alpha^2\lambda^2 \sim \alpha^2/k^2 \sim \alpha^2/E^2 \sim \alpha^2/T^2$,
- For $n \sim T^3, v = 1$ and $\alpha \simeq 10^{-1} - 10^{-2}, \alpha = \alpha_{\mathrm{EM}}, \alpha_{\mathrm{W}}, \alpha_{\mathrm{S}}$:

$$t_c \sim \frac{1}{\alpha^2 T} \ll t_H \sim \frac{1}{H} \sim \frac{1}{\sqrt{\epsilon}} \sim \frac{1}{T^2}\,.$$

$\Rightarrow$ For $10^{15} - 10^{17}$ GeV $\gtrsim T \gtrsim T_{\mathrm{EW}}$ LTE in expanding fluid persists.

- For $T_{\mathrm{EW}} > T > T_c^{\mathrm{QCD}} \approx 160$ MeV the LTE continues due to large effective cross-section among the particles forming the QGP medium.

Equation of State Based on the Fundamental Theory

- In the SM $$p_B(T) = \lim_{V\to\infty} \frac{T}{V} \ln \mathcal{Z}(T,V)$$

$$\mathcal{Z}(T,V) = \exp\left[\frac{p_B(T)V}{T}\right], \quad p_B(T) = p_E(T) + p_M(T) + p_G(T) \tag{29}$$

- p_B is the "bare" result related to the physical (renormalized) pressure as $p(T) = p_B(T) - p_B(0)$.
- $p_E(T), p_M(T), p_G(T)$ collect the contributions from the momentum scales $k \sim \pi T$, $k \sim gT$, and $k \sim g^2 T/\pi$, respectively.
- Couplings of SM are $g \in \{h_t, g_1, g_2, g_3\}$, where h_t is the Yukawa coupling between the top quark and the Higgs boson, and g_1, g_2, g_3 are related to $U_Y(1)$, $\mathrm{SU_L}(2)$ and $\mathrm{SU}_c(3)$ gauge groups, respectively.
- Calculations of the dimensionless function $p(T)/T^4$ and of the trace anomaly $\Theta(T)$ up to $\mathcal{O}(g^5)$ were performed in [Laine,Mayer:2015].

The Glasma as a (Semi-)Classical Matter

QCD in the classical regime [Kharzeev:2002]

- Introduce coupling-independent field tensors

$$A_\mu^a \to \mathcal{A}_\mu^a \equiv g_{\mathrm{S}} A_\mu^a, \quad g_{\mathrm{S}}^2 = 4\pi\alpha_{\mathrm{S}}$$

$$F_{\mu\nu}^a \to g_{\mathrm{S}} F_{\mu\nu}^a \equiv \mathcal{F}_{\mu\nu}^a = \partial_\mu \mathcal{A}_\nu^a - \partial_\nu \mathcal{A}_\mu^a + f^{abc} \mathcal{A}_\mu^b \mathcal{A}_\nu^c \,, \tag{30}$$

- Calculate action of the gluon field

$$S_g = -\frac{1}{4} \int F_{\mu\nu}^a F^{\mu\nu,a} d^4x = -\frac{1}{4g_{\mathrm{S}}^2} \int \mathcal{F}^{\mu\nu,a} \mathcal{F}_{\mu\nu}^a d^4x \tag{31}$$

- Gluon occupation number

$$f_g \sim \frac{S_g}{\hbar} = \frac{1}{\hbar g_{\mathrm{S}}^2} \rho_4 V_4 \tag{32}$$

where $\rho_4 \sim \langle \mathcal{F}^{\mu\nu,a} \mathcal{F}_{\mu\nu}^a \rangle$ is four-dimensional gluon condensate density.

- Saturated gluon matter is weakly coupled.
- The limits $g_{\mathrm{S}}^2 \to 0$ and $\hbar \to 0$ are equivalent! $\Rightarrow$ weakly interacting means semi-classical.