Collision energy dependence of source sizes for primary and secondary pions at NICA energies

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- Introduction: Meaning of the two-particle correlation function
- Source sizes and correlation strength
- Core - halo picture: direct vs. resonance decay pions
- UrQMD + CRAB afterburner with and without momentum resolution smearing
- Conclusions and outlook
The two-particle correlation function

\[ C_2(p_1, p_2) = \frac{N_2(p_1, p_2)}{N_1(p_1)N_1(p_2)} \]

where

\[ N_1(p_1), \ N_1(p_2) \text{ and } N_2(p_1, p_2) \]

are the one- and two-particle invariant momentum distributions as functions of the single particle four-momenta \( p_1 \) and \( p_2 \).

The two-particle correlation analysis is performed as a function of the relative four-momentum \( q \) for a fixed average four-momentum \( K \)

\[ q = p_1 - p_2 = (q_0, \vec{q}), \quad K = \frac{p_1 + p_2}{2} = (k_0, \vec{k}) \]
Origin of two-particle correlations

- Mainly Bose-Einstein quantum statistics for identical particles
- Conservation laws
- Collective flow
- Jets
- Resonance decays
- ...
Bose-Einstein quantum statistics for identical particles
Two-pion correlation and space-time source

Neglecting dynamical correlations, the pair momentum distribution is related to the particle emitting source $S(x, p)$ by

$$N_2(p_1, p_2) = \int d^4x_1 d^4x_2 S(x_1, p_1) S(x_2, p_2) |\psi_{p_1, p_2}(x_1, x_2)|^2$$

where $\psi_{p_1, p_2}(x_1, x_2)$ is the symmetrized pair wave function.

$$C_2(p_1, p_2) = 1 + \text{Re} \left[ \frac{\tilde{S}(q, p_1) \tilde{S}^*(q, p_2)}{\tilde{S}(0, p_1) \tilde{S}^*(0, p_2)} \right]$$

where $\tilde{S}$ is the Fourier Transform of $S$. 
Two-pion correlation and space-time source

For typical sources and kinematical domains found in heavy-ion collisions the function $S(q, p)$ **does not change much as a function of $p$**. It is thus customary to use the approximation $p_1 \approx p_2 \approx K$ to get

\[
C_2(q, K) = 1 + \frac{\left| \tilde{S}(q, K) \right|^2}{\left| \tilde{S}(0, K) \right|^2}
\]

The space region of particle emission is sometimes parametrized in a Gaussian form

\[
S(R, r) = \frac{1}{(2\pi)^3} \int d^3q e^{i\vec{q} \cdot \vec{r}} e^{-\frac{1}{2} |\vec{q}|^2 R^2} |\vec{q}|
\]
Source size

- $R^2$ is the **matrix of homogeneity lengths or femtoscopy radii**
- For a spherically symmetric source the radius is given by the width of the correlation function

\[
R \sim \frac{1}{q}
\]

- This is a direct consequence of Heisenberg’s uncertainty relation
  \[(\Delta R)(\Delta p) \sim 1\]
- Correlated pairs are emitted predominantly in the same direction.
Correlation width
Correlation strength

- Neglecting final state (Coulomb, strong) interactions, the correlation function $C_2(0, K) = 2$
- Experimentally, limitations on two track resolution prevents correlation measurements at $q = 0$
- The correlation function is measured at $q \neq 0$ and then extrapolated to $q = 0$
- The extrapolated value can in general be different from the exact value at $q = 0$
- To quantify this value, define

$$\lambda(K) = \lim_{q \to 0} C_2(q, K) - 1$$

$\lambda(K)$ is also known as the chaoticity parameter.
Resolution

- For a totally chaotic source, $\lambda = 1$
- Partially coherent sources (possible contributions of a Bose-Einstein condensate) produce $\lambda < 1$

Even if the source is completely chaotic, since $R \sim 1/q$
maximum radius that can be resolved $R_{\text{max}} \sim 1/q_{\text{min}} \sim 25-30$ fm
Resolution
Core halo picture
Correlation strength

- Physical assumption: the phase space emitting source is made of two components

$$S = S_{\text{core}} + S_{\text{halo}}$$

- If the pions are correlated and their correlation function is resolved, **both pions need to come from the core**

- Each component has a Fourier Transform

$$\tilde{S}_{\text{core}}(q, K) \equiv \int d^4x \ e^{iqx} S_{\text{core}}(x, K)$$

$$\tilde{S}_{\text{halo}}(q, K) \equiv \int d^4x \ e^{iqx} S_{\text{halo}}(x, K)$$
Correlation strength

Thus, the correlation function can be expressed as

\[ C_2(q, K) = 1 + \left( \frac{N_{\text{core}}(K)}{N_{\text{core}}(K) + N_{\text{halo}}(K)} \right)^2 \left| \frac{\tilde{S}_{\text{core}}(q, K)}{\tilde{S}_{\text{core}}(0, K)} \right|^2 \]

Therefore, in the core-halo picture

\[ \lambda = \left( \frac{N_{\text{core}}(K)}{N_{\text{core}}(K) + N_{\text{halo}}(K)} \right)^2 \]

\( \lambda \) carries indirect information on the decay of long-lived resonances (\( \eta, \eta', \omega, K^0_s \)).
Possible effects on $\lambda$ measurements

- Single-track momentum resolution produces smearing of the correlation function.
- Track miss-identification decreases the maximum of the correlation.
- Track merging produces a lack of data at low $q$ and has a strong effect for $k_T > 0.6$ GeV/c.
- Two-track effects, such as track splitting, can be corrected by increasing the number of hits for track selection.
- To measure $\lambda$ for $k_T > 0.6$ GeV/c we need to increase the statistics to have a similar number of pairs at low $q$. 
Monte Carlo simulations to compute two-pion correlation functions. However, generators do not usually include the quantum statistical effects. We use the formalism of the correlation after-burner (CRAB)*, which boosts produced particles from a given MC generator to the pair center of mass system and computes their squared wave function, which is then used as a weight for the numerator.

The momentum distributions in the numerator are computed using the event mixing technique.

* S. Pratt et al., Nucl. Phys. A 566, 103c (1994)
We simulated 100,000 central Bi+Bi UrQMD central events (impact parameter between 0-1 fm) in the cascade mode at two energies: $\sqrt{s_{NN}} = 4.0, 9.2$ GeV.

CRAB was then used to include the correlation of charged pions.

We separated primary and secondary pions. We consider that secondaries were produced in the decay of resonances (mainly $\rho$, $\Delta$, $\omega$, $N(1440)$, $\rho(1700)$ and $K^*$ decays). Primary are the rest of the pions.
Two-pion correlation function, $\sqrt{s_{NN}} = 4.0$ GeV

All pions

Lorentzian: $C_2(q_{inv}) = 1 + \lambda \frac{R^2}{q_{inv}^2 + R^2}$

$R = 10.405 \pm 0.329$ fm, $\lambda = 1.002 \pm 0.024$

Gaussian $C_2(q_{inv}) = 1 + \lambda \exp \left( -\frac{q_{inv}R^2}{2} \right)$

$R = 9.268 \pm 0.18$ fm, $\lambda = 0.87 \pm 0.016$
Two-pion correlation function, $\sqrt{s_{NN}} = 9.2$ GeV

All pions

Lorentzian: $C_2(q_{inv}) = 1 + \lambda \frac{R^2}{q_{inv}^2 + R^2}$

$R = 11.737 \pm 0.404$ fm, $\lambda = 1.002 \pm 0.027$

Gaussian $C_2(q_{inv}) = 1 + \lambda \exp \left(-\frac{q_{inv} R^2}{2}\right)$

$R = 10.426 \pm 0.261$ fm, $\lambda = 0.867 \pm 0.02$
Comparison at the two different energies

Bi + Bi, b = 0 – 1fm
Separation of primaries and secondaries

$\sqrt{s_{NN}} = 4.0$ GeV

• Gaussian fit

- $R_{\text{prim},4.0\text{GeV}} = 4.905 \pm 0.046$ fm
- $\lambda_{\text{prim},4.0\text{GeV}} = 0.957 \pm 0.008$

- $R_{\text{second},4.0\text{GeV}} = 9.198 \pm 0.229$ fm
- $\lambda_{\text{second},4.0\text{GeV}} = 0.849 \pm 0.02$
Separation of primaries and secondaries

\( \sqrt{S_{NN}} = 9.2 \text{ GeV} \)

- Gaussian fit
  - \( R_{\text{prim,9.2GeV}} = 5.112 \pm 0.085 \text{ fm} \)
  - \( \lambda_{\text{prim,9.2GeV}} = 0.988 \pm 0.015 \)
  - \( R_{\text{second,9.2GeV}} = 10.551 \pm 0.206 \text{ fm} \)
  - \( \lambda_{\text{second,9.2GeV}} = 0.859 \pm 0.016 \)
MPD finite resolution

• The NICA-MPD has a momentum resolution of 1.5 % for particles with total momentum of about 0.2 GeV


• The nominal design relative momentum resolution is about 5 MeV

• This can be included in our calculations by fixing the smearing parameter of CRAB at 5 MeV
Finite resolution effects on the separation at $\sqrt{s_{NN}} = 4.0$ GeV

<table>
<thead>
<tr>
<th>Primary</th>
<th>Before smearing</th>
<th>After smearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>4.905 ± 0.046 fm</td>
<td>4.95 ± 0.068 fm</td>
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<tr>
<td>$\lambda$</td>
<td>0.957 ± 0.008</td>
<td>0.959 ± 0.012</td>
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<table>
<thead>
<tr>
<th>Secondary</th>
<th>Before smearing</th>
<th>After smearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>9.198 ± 0.229 fm</td>
<td>9.211 ± 0.283 fm</td>
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<tr>
<td>$\lambda$</td>
<td>0.849 ± 0.02</td>
<td>0.835 ± 0.024</td>
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Finite resolution effects on the separation at $\sqrt{s_{NN}} = 9.2$ GeV

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<tbody>
<tr>
<td>Primary</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>$5.112 \pm 0.085$ fm</td>
<td>$4.808 \pm 0.048$ fm</td>
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<tr>
<td>$\lambda$</td>
<td>$0.988 \pm 0.015$</td>
<td>$0.892 \pm 0.008$</td>
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<tr>
<td>Secondary</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>$10.551 \pm 0.206$ fm</td>
<td>$10.307 \pm 0.32$ fm</td>
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<tr>
<td>$\lambda$</td>
<td>$0.859 \pm 0.016$</td>
<td>$0.773 \pm 0.022$</td>
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Conclusions

- The source radii for primary particles is significantly smaller than for secondary particles.
- The radii grow as the energy increases.
- The finite resolution of the MPD will not affect significantly the estimation of the sizes the sources, provided the best case resolution scenario design is achieved.
- More studies needed, including less optimistic smearing and different total pair momenta.
- Implement Coulomb and other possible final state effects.
Thank you!
Backup
Bertsch-Pratt system

Bertsch-Pratt Momentum Coordinates

\[ C(q_{\text{out}}, q_{\text{side}}, q_{\text{long}}) = 1 + \lambda \cdot \exp(-R_{\text{out}}^2 \cdot q_{\text{out}}^2 - R_{\text{side}}^2 \cdot q_{\text{side}}^2 - R_{\text{long}}^2 \cdot q_{\text{long}}^2 - 2R_{\text{ol}}^2 \cdot q_{\text{out}} \cdot q_{\text{long}}) \]

![Diagram showing Bertsch-Pratt system with vectors and equations]

\[ \vec{K}_T = \frac{1}{2} (\vec{p}_{T1} + \vec{p}_{T2}) \]

beam direction (long)

beam direction (out, side)

STAR

November 19, 2003

John G. Cramer

November 19, 2003
In general $C_2$ depends on the two four-momenta $p_1$ and $p_2$. However $q \cdot K = q_0 K_0 - \vec{q} \cdot \vec{K} = 0$

This implies $q_0 = \frac{\vec{q} \cdot \vec{K}}{K_0}$

We may then transform the $q$-dependence into a dependence on $\vec{q}$

Moreover, if the pair is of similar energy then $K$ is approximately on-shell and the correlation function becomes a function of $\vec{q}$ and $\vec{K}$

$$C_2(q, K) \rightarrow C_2(\vec{q}, \vec{K})$$