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Collision energy dependence of source sizes for primary and secondary pions at NICA energies

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The two-particle correlation function

$$C_2(p_1, p_2) = \frac{N_2(p_1,p_2)}{N_1(p_1)N_1(p_2)}$$

where

$$N_1(p_1),\ N_1(p_2) \text{ and } N_2(p_1, p_2)$$

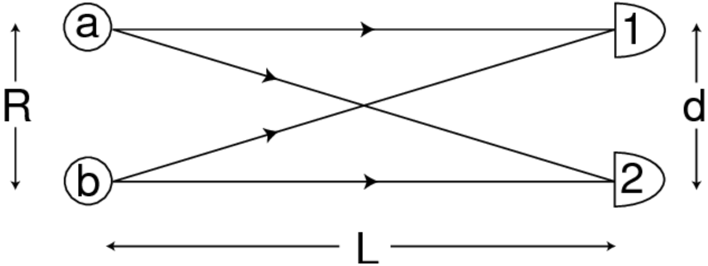
are the one- and two-particle invariant momentum distributions as functions of the single particle four-momenta p_1 and p_2.
The two-particle correlation analysis is performed as a function of the **relative four-momentum** q for a fixed **average four-momentum** K

$$q = p_1 - p_2 = (q_o, \vec{q}), \quad K = \frac{p_1+p_2}{2} = (k_0, \vec{k})$$

Origin of two-particle correlations

- **Mainly Bose-Einstein quantum statistics for identical particles**
- Conservation laws
- Collective flow
- Jets
- Resonance decays
- ...

Bose-Einstein quantum statistics for identical particles



Two-pion correlation and space-time source

Neglecting dynamical correlations, the pair momentum distribution is related to the particle emitting source $S(x, p)$ by

$$N_2(p_1, p_2) = \int d^4x_1 d^4x_2 S(x_1, p_1) S(x_2, p_2) \left|\Psi_{p_1,p_2}(x_1, x_2)\right|^2$$

where $\Psi_{p_1,p_2}(x_1, x_2)$ is the **symmetrized pair wave function.**

$$C_2(p_1, p_2) = 1 + \mathrm{Re}\left[\frac{\widetilde{S}(q, p_1)\widetilde{S}^*(q, p_2)}{\widetilde{S}(0, p_1)\widetilde{S}^*(0, p_2)}\right]$$

where $\widetilde{S}$ is the Fourier Transform of S.

Two-pion correlation and space-time source

For typical sources and kinematical domains found in heavy-ion collisions the function $S(q,p)$ **does not change much as a function of p.** It is thus customary to use the approximation $p_1 \simeq p_2 \simeq K$ to get

$$C_2(q,K) = 1 + \frac{\left|\widetilde{S}(q,K)\right|^2}{\left|\widetilde{S}(0,K)\right|^2}$$

The space region of particle emission is sometimes parametrized in a Gaussian form

$$S(R,r) = \frac{1}{(2\pi)^3}\int d^3q e^{i\vec{q}\cdot\vec{r}} e^{-\frac{1}{2}\left|\vec{q}\mathbf{R}^2\vec{q}\right|}$$

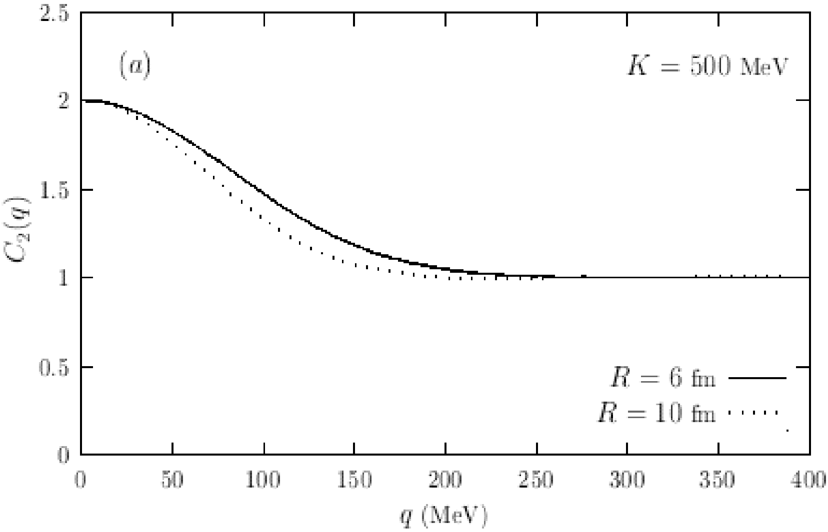
Source size

- $\mathbf{R}^2$ is the **matrix of homogeneity lengths or femtoscopy radii**
- For a spherically symmetric source the radius is given by the width of the correlation function

$$R \sim \frac{1}{q}$$

  - This is a direct consequence of Heisenberg's uncertainty relation
$$(\Delta R)(\Delta p) \sim 1$$
- Correlated pairs are emitted predominantly in the same direction.

Correlation width



Correlation strength

- Neglecting final state (Coulomb, strong) interactions, the correlation function $C_2(0, K) = 2$
- Experimentally, limitations on two track resolution prevents correlation measurements at $q = 0$
- The correlation function is measured at $q \neq 0$ and then **extrapolated** to $q = 0$
- The extrapolated value can in general be different from the exact value at $q = 0$
- To quantify this value, define

$$\lambda(K) = \lim_{q \to 0} C_2(q, K) - 1$$

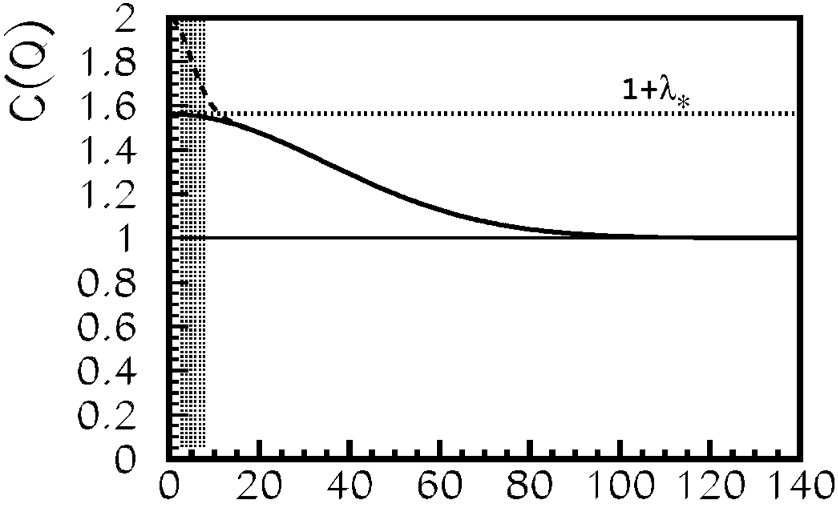
$\lambda(K)$ is also known as the **chaoticity parameter**

Resolution

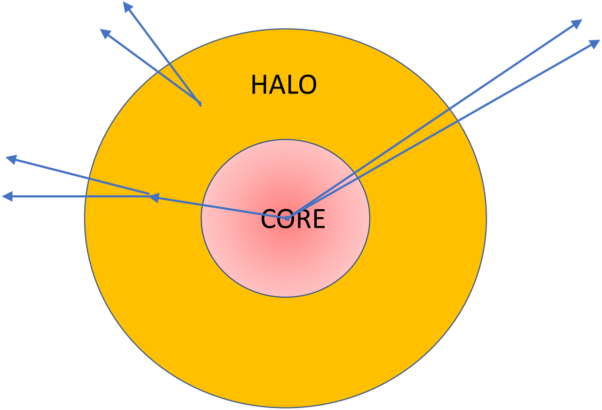
- For a totally chaotic source, $\lambda = 1$
- Partially coherent sources (possible contributions of a Bose-Einstein condensate) produce $\lambda < 1$

Even if the source is completely chaotic, since $R \sim 1/q$
maximum radius that can be resolved $R_{\text{max}} \sim 1/q_{\text{min}} \sim$ 25-30 fm

Resolution



Core halo picture



Correlation strength

- Physical assumption: the phase space emitting source is made of two components

$$S = S_{\text{core}} + S_{\text{halo}}$$

- If the pions are correlated and their correlation function is resolved, **both pions need to come from the core**
- Each component has a Fourier Transform

$$\begin{aligned}\widetilde{S}_{\text{core}}(q,K) &\equiv \int d^4x\, e^{iqx} S_{\text{core}}(x,K) \\ \widetilde{S}_{\text{halo}}(q,K) &\equiv \int d^4x\, e^{iqx} S_{\text{halo}}(x,K)\end{aligned}$$

Correlation strength

Thus, the correlation function can be expressed as

$$C_2(q,K) = 1 + \left(\frac{N_{\text{core}}(K)}{N_{\text{core}}(K)+N_{\text{halo}}(K)}\right)^2 \frac{\left|\widetilde{S}_{\text{core}}(q,K)\right|^2}{\left|\widetilde{S}_{\text{core}}(0,K)\right|^2}$$

Therefore, **in the core-halo picture**

$$\lambda = \left(\frac{N_{\text{core}}(K)}{N_{\text{core}}(K)+N_{\text{halo}}(K)}\right)^2$$

λ carries indirect information on the decay of long-lived resonances (η, η', ω, K_s^0).

Possible effects on λ measurements

- Single-track momentum resolution produces smearing of the correlation function
- Track miss-identification decreases the maximum of the correlation.
- Track merging produces a lack of data at low q and has a strong effect for $k_T > 0.6$ GeV/c.
- Two-track effects, such as track splitting, can be corrected by increasing the number of hits for track selection.
- To measure λ for $k_T > 0.6$ GeV/c we need to increase the statistics to have a similar number of pairs at low q

MC two-pion correlation functions

- Monte Carlo simulations to compute two-pion correlation functions.
- However, generators do not usually include the quantum statistical effects.
- We use the formalism of the **correlation after-burner (CRAB)***, which boosts produced particles from a given MC generator to the pair center of mass system and computes their squared wave function, which is then used as a weight for the numerator.
- The momentum distributions in the numerator are computed using the event mixing technique.

* S. Pratt *et al.*, Nucl. Phys. **A 566**, 103c (1994)

UrQMD simulation and core - halo separation

- We simulated 100,000 central Bi+Bi UrQMD central events (impact parameter between 0-1 fm) in the cascade mode at two energies: $\sqrt{s_{NN}}$ =4.0, 9.2 GeV.
- CRAB was then used to include the correlation of charged pions.
- We separated primary and secondary pions. We consider that secondaries were produced in the decay of resonances (mainly ρ, Δ, ω, $N(1440)$, $\rho(1700)$ and K^* decays). Primary are the rest of the pions.

Two-pion correlation function, $\sqrt{s_{NN}} = 4.0$ GeV

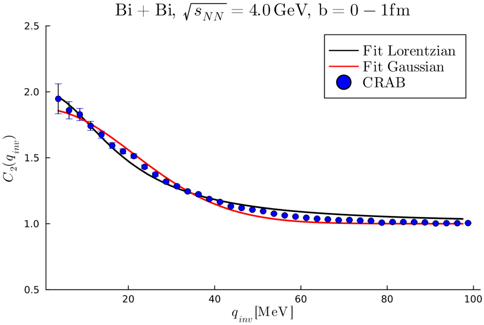
All pions

Lorentzian: $C_2(q_{inv}) = 1 + \lambda \frac{R^2}{q_{inv}^2 + R^2}$

$R = 10.405 \pm 0.329$ fm, $\lambda = 1.002 \pm 0.024$

Gaussian $C_2\,(q_{inv}) = 1 + \lambda \exp\left(-\frac{q_{inv} R^2}{2}\right)$

$R = 9.268 \pm 0.18$ fm , $\lambda = 0.87 \pm 0.016$



Two-pion correlation function, $\sqrt{s_{NN}} = 9.2$ GeV

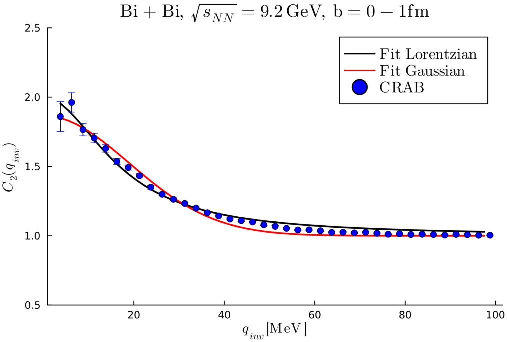
All pions

Lorentzian: $C_2(q_{inv}) = 1 + \lambda \frac{R^2}{q_{inv}^2 + R^2}$

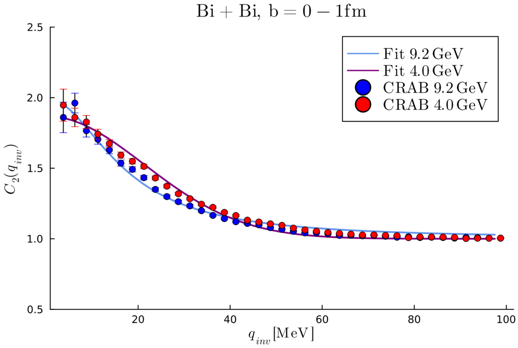
$R = 11.737 \pm 0.404$ fm, $\lambda = 1.002 \pm 0.027$

Gaussian $C_2\,(q_{inv}) = 1 + \lambda \exp\left(-\frac{q_{inv} R^2}{2}\right)$

$R = 10.426 \pm 0.261$ fm, $\lambda = 0.867 \pm 0.02$



Comparison at the two different energies

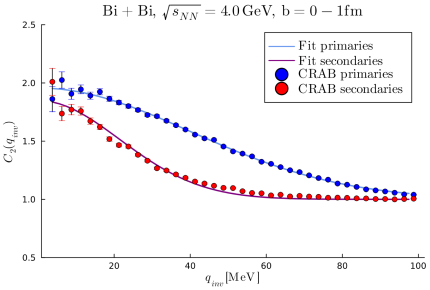


Separation of primaries and secondaries $\sqrt{s_{NN}} = 4.0$ GeV

- Gaussian fit

- $R_{prim,4.0GeV} = 4.905 \pm 0.046$ fm
- $\lambda_{prim,4.0GeV} = 0.957 \pm 0.008$

- $R_{second,4.0GeV} = 9.198 \pm 0.229$ fm
- $\lambda_{second,4.0GeV} = 0.849 \pm 0.02$

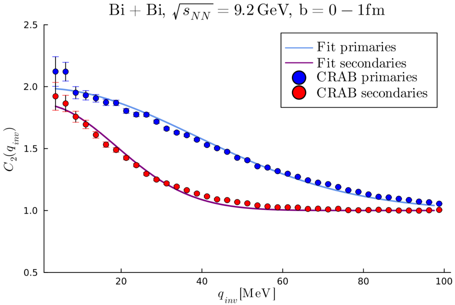


Separation of primaries and secondaries
$\sqrt{s_{NN}} = 9.2$ GeV

- Gaussian fit

- $R_{prim,9.2GeV} = 5.112 \pm 0.085$ fm
- $\lambda_{prim,9.2GeV} = 0.988 \pm 0.015$

- $R_{second,9.2GeV} = 10.551 \pm 0.206$ fm
- $\lambda_{second,9.2GeV} = 0.859 \pm 0.016$



MPD finite resolution

- The NICA-MPD has a momentum resolution of 1.5 % for particles with total momentum of about 0.2 GeV

  A. Maevskiy, *et al.*, *Eur. Phys. J. C* **81** 599 (2021)

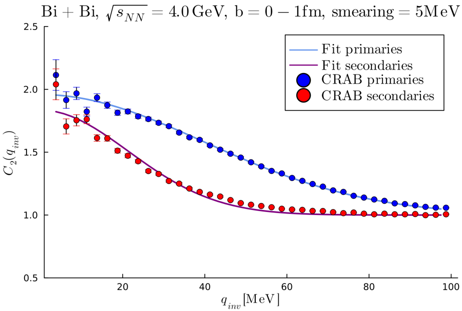
- The nominal design relative momentum resolution is about 5 MeV

- This can be included in our calculations by fixing the smearing parameter of CRAB at 5 MeV

Finite resolution effects on the separation at $\sqrt{s_{NN}} = 4.0$ GeV

Primary	Before smearing	After smearing
R	4.905 ± 0.046 fm	4.95 ± 0.068 fm
λ	0.957 ± 0.008	0.959 ± 0.012

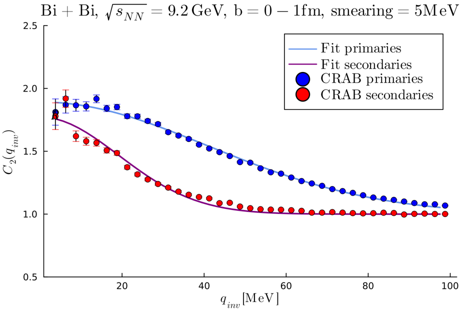
Secondary	Before smearing	After smearing
R	9.198 ± 0.229 fm	9.211 ± 0.283 fm
λ	0.849 ± 0.02	0.835 ± 0.024



Finite resolution effects on the separation at $\sqrt{s_{NN}} = 9.2$ GeV

Primary	Before smearing	After smearing
R	5.112 ± 0.085 fm	4.808 ± 0.048 fm
λ	0.988 ± 0.015	0.892 ± 0.008

Secondary	Before smearing	After smearing
R	10.551 ± 0.206 fm	10.307 ± 0.32 fm
λ	0.859 ± 0.016	0.773 ± 0.022



Conclusions

- The source radii for primary particles is significantly smaller than for secondary particles
- The radii grow as the energy increases
- The finite resolution of the MPD will not affect significantly the estimation of the sizes the sources, provided the best case resolution scenario design is achieved
- More studies needed, including less optimist smearing and different total pair momenta
- Implement Coulomb and other possible final state effects

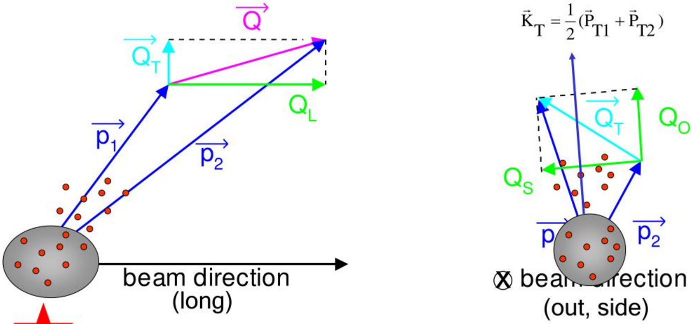
Thank you!

Backup

Bertsch-Pratt systmem

Bertsch-Pratt Momentum Coordinates

$$C(q_{out}, q_{side}, q_{long}) =$$
$$1 + \lambda \cdot \exp(-R_{out}^2 \cdot q_{out}^2 - R_{side}^2 \cdot q_{side}^2 - R_{long}^2 \cdot q_{long}^2 - 2R_{ol}^2 \cdot q_{out} \cdot q_{long})$$



STAR November 19, 2003 14 John G. Cramer

Variable dependence

- In general C_2 depends on the two four-momenta p_1 and p_2.
- However $q \cdot K = q_0 K_0 - \vec{q} \cdot \vec{K} = 0$
- This implies $q_0 = \frac{\vec{q} \cdot \vec{K}}{K_0}$
- We may then transform the q-dependence into a dependence on $\vec{q}$
- Moreover, if the pair is of similar energy then K is approximately on-shell and the correlation function becomes a function of $\vec{q}$ and $\vec{K}$

$$C_2(q, K) \to C_2(\vec{q}, \vec{K})$$