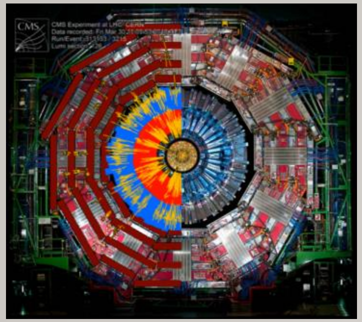
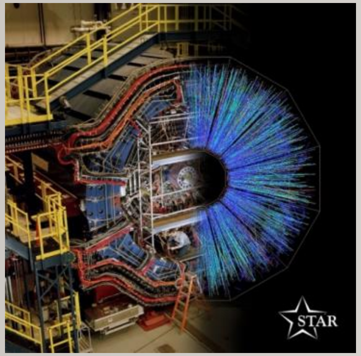
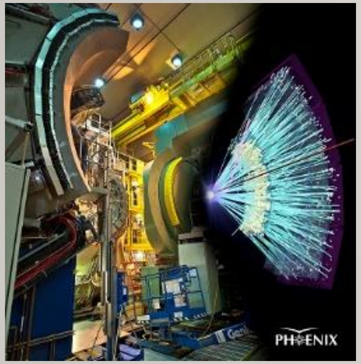
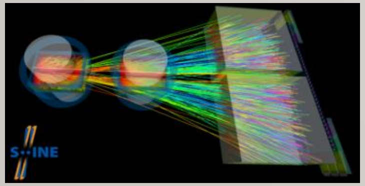








FEMTOSCOPY WITH LÉVY DISTRIBUTIONS FROM SPS TO LHC

MÁTÉ CSANÁD (EÖTVÖS UNIVERSITY)

52ND INTERNATIONAL SYMPOSIUM ON MULTIPARTICLE DYNAMICS, GYÖNGYÖS

ISMD 2023
Gyöngyös, Hungary

HUNGARIAN UNIVERSITY OF
AGRICULTURE AND LIFE SCIENCES

CONTENTS OF THIS TALK

- Basics of femtoscopy and Lévy sources
- First thorough Lévy HBT analysis in AA by PHENIX
- Recent phenomenological updates
- Recent experimental results
- Summary and outlook

CONTENTS OF THIS TALK

- Basics of femtoscopy and Lévy sources
- First thorough Lévy HBT analysis in AA by PHENIX
- Recent phenomenological updates
- Recent experimental results
- Summary and outlook

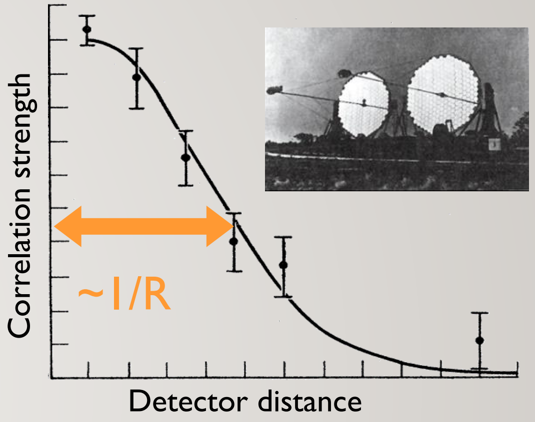
FEMTOSCOPY IN HIGH ENERGY PHYSICS

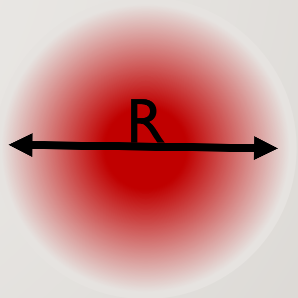
- R. Hanbury Brown, R. Q. Twiss - observing Sirius with radio telescopes
  - Intensity correlations vs detector distance ⇒ source size
  - Measure the sizes of apparently point-like sources!
- Goldhaber et al: applicable in high energy physics
- Understanding: Glauber, Fano, Baym, …
  Phys. Rev. Lett. 10, 84; Rev. Mod. Phys. 78 1267, …
  - Momentum correlation $C(q)$ related to source $S(r)$

    $C(q) \cong 1 + \left|\int S(r) e^{iqr} dr\right|^2$

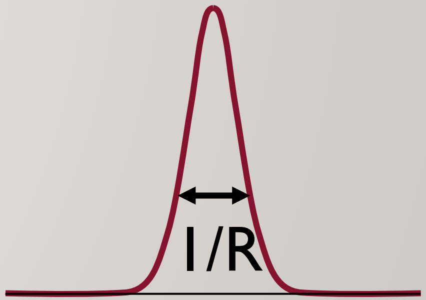
    (under some assumptions)
  - Also with distance distribution $D(r)$:

    $C(q) \cong 1 + \int D(r) e^{iqr} dr$
  - Neglected: pair reconstruction, final state interactions, multi-particle correlations, coherence, …
- What is the source shape? Can be explored via femtoscopy





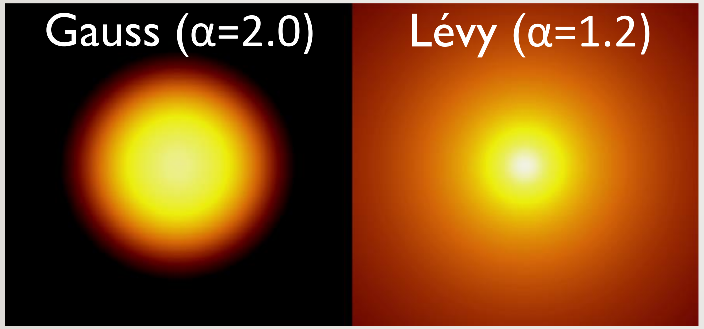
source function S(r)



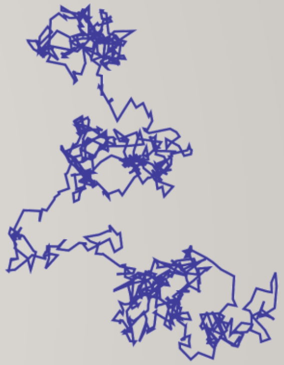
correlation funct. C(q)

LÉVY DISTRIBUTIONS IN HEAVY ION PHYSICS

- Central limit theorem (diffusion) and thermodynamics lead to Gaussians
- Measurements suggest phenomena beyond Gaussian distribution
- Lévy-stable distribution: $\mathcal{L}(\alpha,R;r) = (2\pi)^{-3}\int d^3q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$
  - From generalized central limit theorem, power-law tail ~ $r^{-(1+\alpha)}$
  - Special cases: α = 2 Gaussian, α = 1 Cauchy

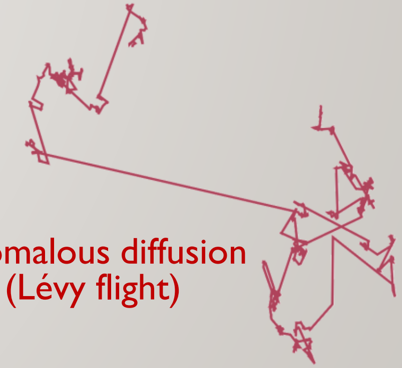


Normal diffusion



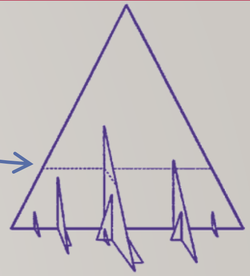
- Shape of the correlation functions with Lévy source:
  - $C_2(q) = 1 + \lambda \cdot e^{-|qR|^\alpha}$; $\alpha = 2$: Gaussian; $\alpha = 1$: exponential
    Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67-78
- A possible reason for Lévy source: anomalous diffusion, many others

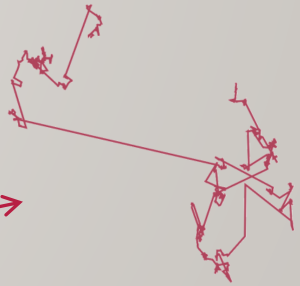
Anomalous diffusion
(Lévy flight)



WHY DO LÉVY SHAPES APPEAR, WHY IS IT IMPORTANT?

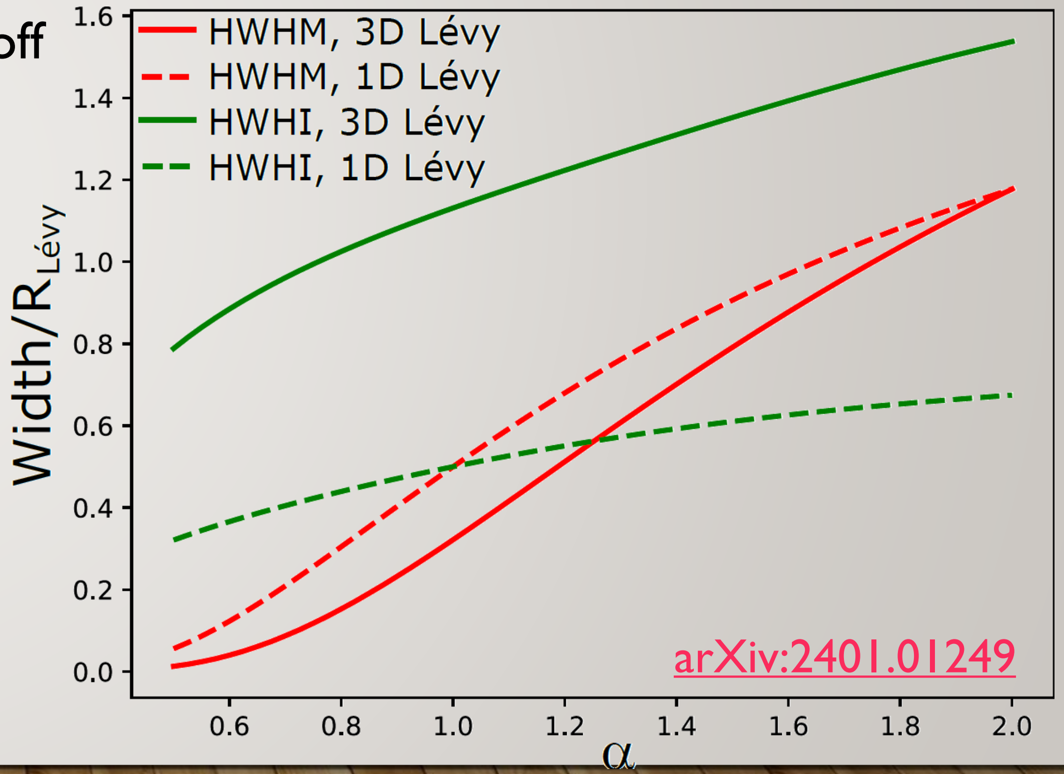
- A more comprehensive list of possible reasons:
  - Jet fragmentation (Csörgő, Hegyi, Novák, Zajc, Acta Phys.Polon. B36 (2005) 329-337)
    - See also talk by Yacine Mehtar-Tani at ExploreQGP workshop in Belgrade
  - Critical phenomena (Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828 (2006) no.1, 525-532)
  - Direction averaging and non-sphericality (Cimerman et al., Phys.Part.Nucl. 51 (2020) 282)
  - Event averaging (Cimerman et al., Phys.Part.Nucl. 51 (2020) 282)
  - Resonance decays (Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002; Kincses, Stefaniak, Csanád, Entropy 24 (2022) 308)
  - Hadronic rescattering, Lévy flight (Braz.J.Phys. 37 (2007) 1002; Entropy 24 (2022) 308)
- Importance of utilizing Lévy sources:
  - Measuring α and R
    - Order of quark-hadron transition, critical point search, understanding source dynamics
  - Measuring λ also requires correct shape assumption
    - In-medium mass modification, coherent pion production





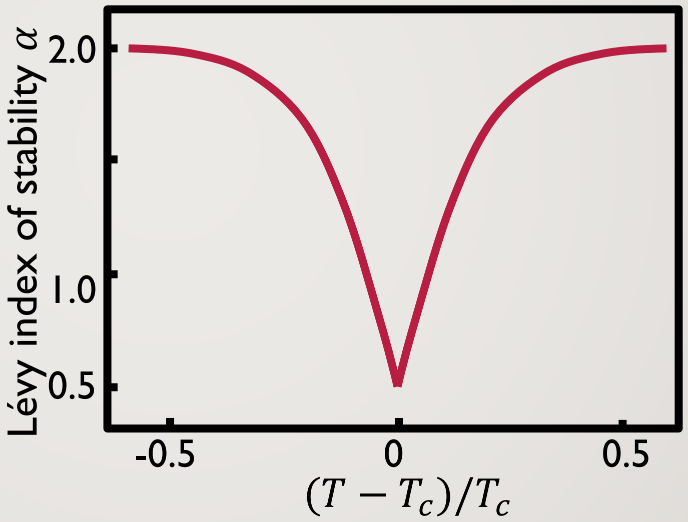
WHAT IS THE TRUE SIZE OF THE SOURCE?

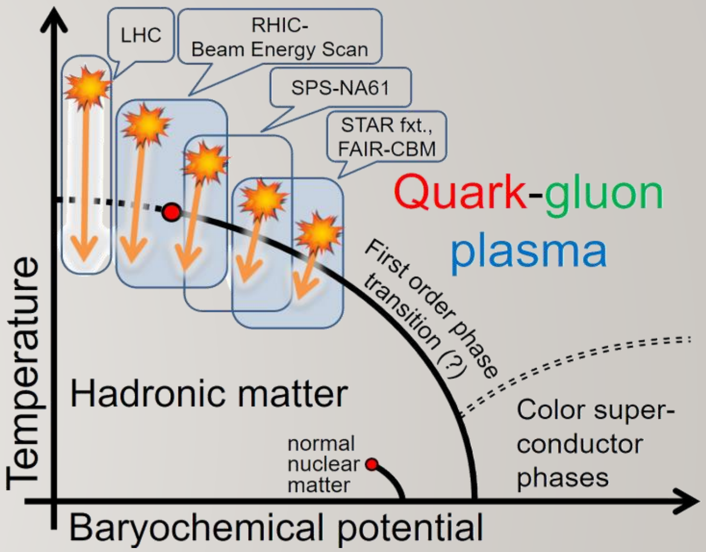
- No tail if $\alpha = 2$, power law if $\alpha < 2$; tail depends on α
- If $S(r)$ Lévy, $D(r)$ Lévy with same α and $R \to 2^{1/\alpha}R$
- In principle, RMS $= \infty$ if $\alpha < 2$, practice: depends on cutoff
- What do Gaussian HBT radii mean?
- Alternative measures:
  - HWHI: (half) width at half integral
  - HWHM: (half) width at half max
  - Large difference between 1D and 3D relative width
  - Width (normalized by R) nontrivially depends on α
  - If $\alpha = 2$ or $\alpha = 1$ assumed: deviation from true scale



LÉVY INDEX AS A CRITICAL EXPONENT?

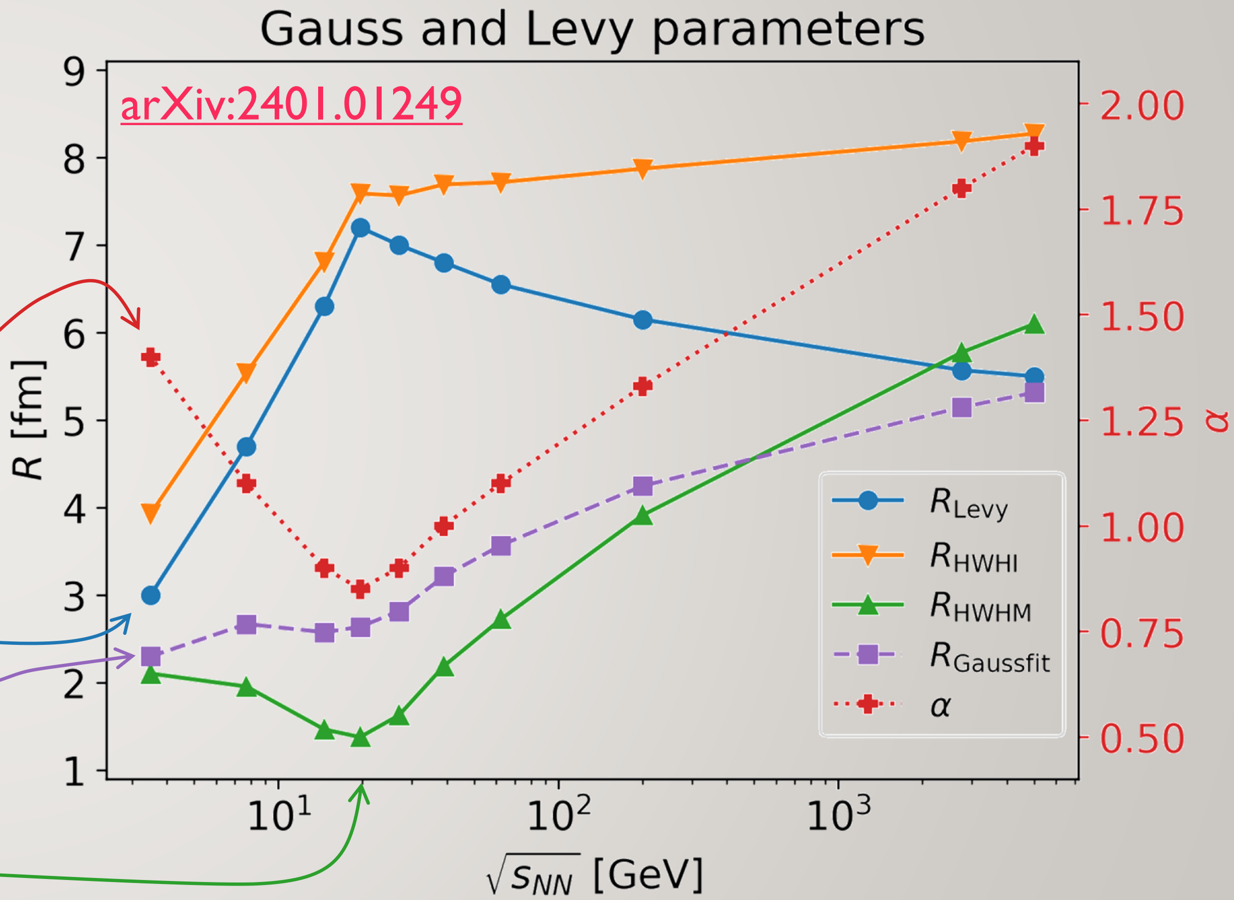
- Critical spatial correlation: $\sim r^{-(d-2+\eta)}$; Lévy source: $\sim r^{-(1+\alpha)}$; $\alpha \Leftrightarrow \eta$?
  Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67
- QCD universality class ↔ 3D Ising
  Halasz et al., Phys.Rev.D58 (1998) 096007
  Stephanov et al., Phys.Rev.Lett.81 (1998) 4816
- At the critical point:
  - Random field 3D Ising: η = 0.50±0.05
    Rieger, Phys.Rev.B52 (1995) 6659
  - 3D Ising: η = 0.03631(3)
    El-Showk et al., J.Stat.Phys.157 (4-5): 869
- Motivation for precise Lévy HBT!
- Change in α_{Levy} proximity of CEP?
- Finite size/time & non-equilibrium effects → what does power-law tail mean?





SOURCE SIZE MEASURES AROUND THE CRITICAL POINT?

- Lévy source parameters: R_{Levy}, α
- R_{Gaussfit}: $C(Q; R_{\text{Levy}}, \alpha)$ fitted with $\alpha = 2$ fixed
- R_{HWHM}: half width at half maximum
- R_{HWHI}: half width at half integral
- **Simulated scenario:**
  - minimum in α vs. s_{NN}
  - maximum in R_{Levy} vs. s_{NN}
- **Observation:**
  - R_{Gaussfit}: approximately monotonic increase
  - Minimum in R_{HWHM}!
  - Trend change in R_{HWHI}!



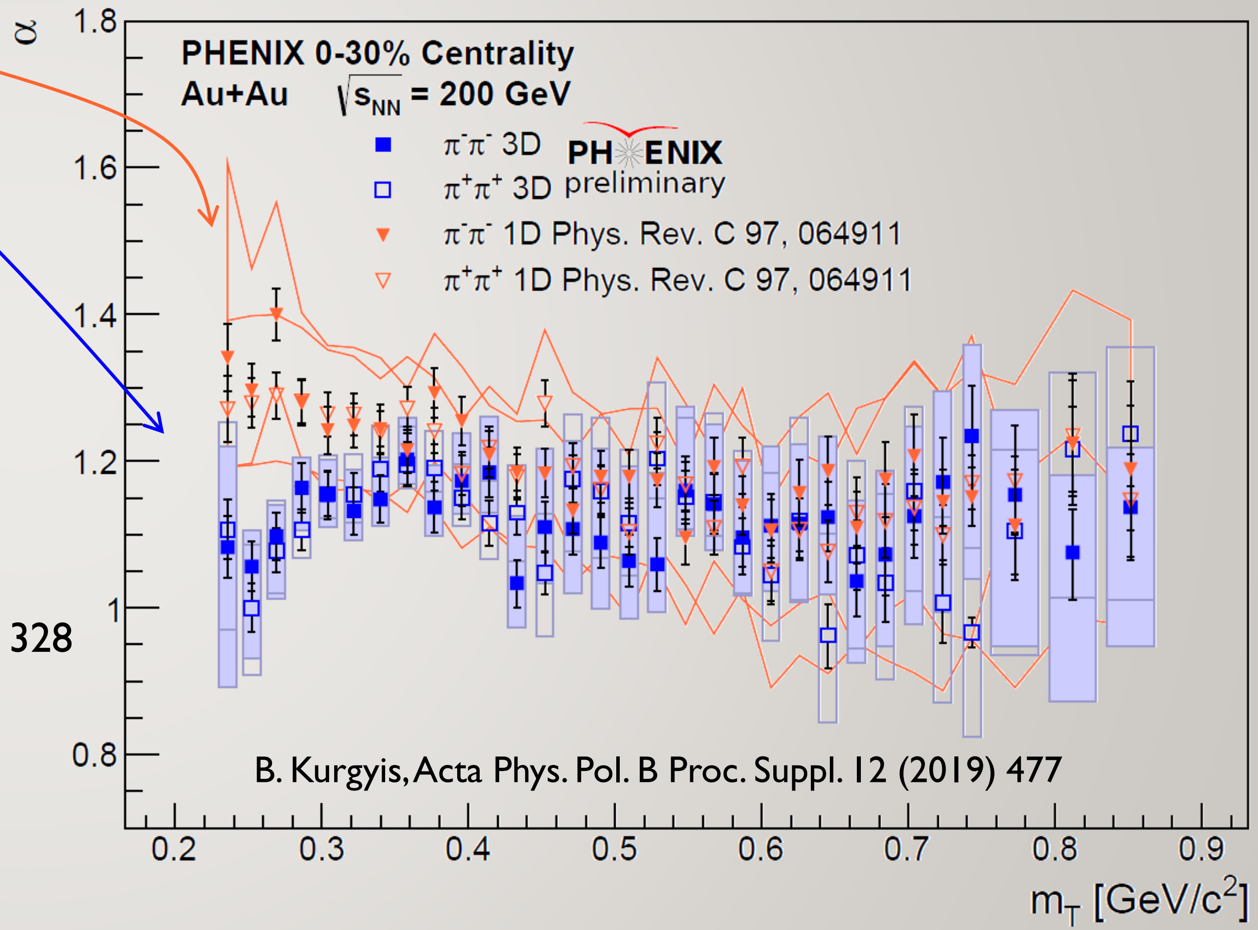
CONTENTS OF THIS TALK

- Basics of femtoscopy and Lévy sources
- First thorough Lévy HBT analysis in AA by PHENIX
- Recent phenomenological updates
- Recent experimental results
- Summary and outlook

LÉVY EXPONENT VERSUS TRANSVERSE MASS, 1D AND 3D

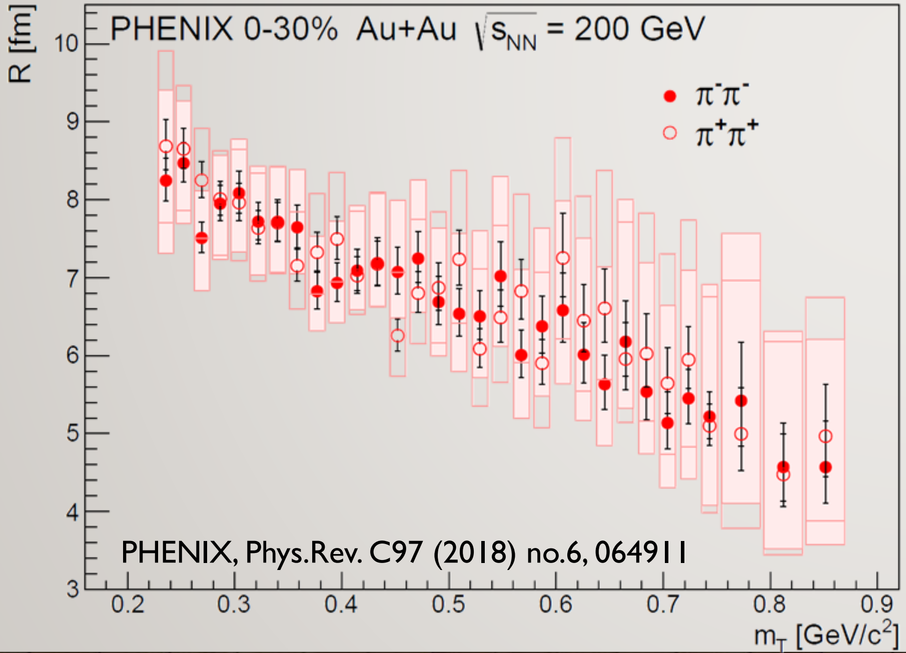
- Lévy exponent α in 3D close to 1D result
- On average still far from 2
- Observable differences at low m_T
- Maybe due to lack of spherical symmetry?
- Coulomb effect for non-spherical sources?
  - Approximation possible
    Kurgyis, Kincses, Csanád, Nagy, Universe 9 (2023) 7, 328
  - If spherical in LCMS, radius in PCMS:

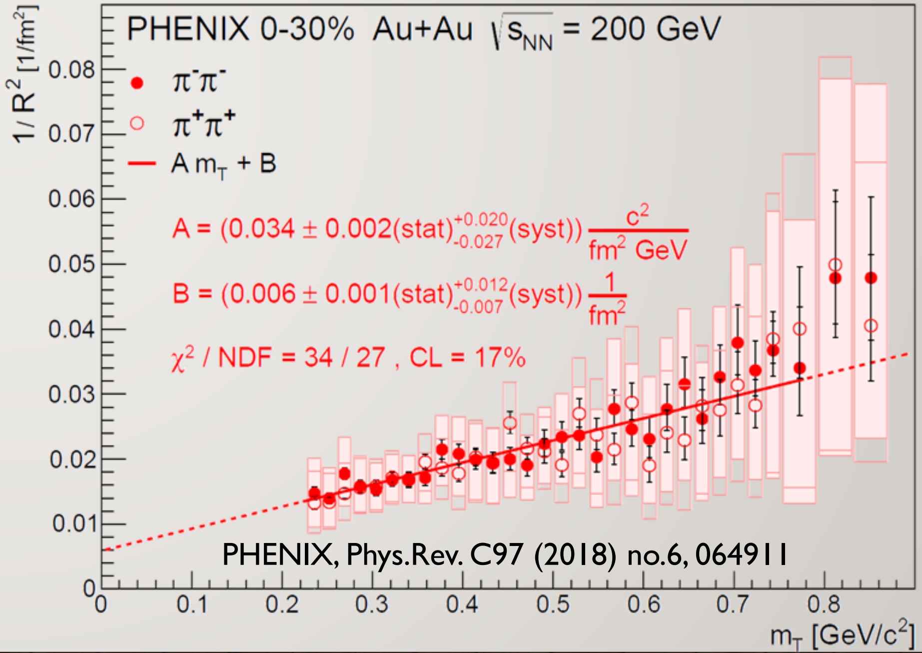
$$R_{PCMS} = \sqrt{\frac{1-2\beta_T^2/3}{1-\beta_T^2}} \cdot R_{LCMS}$$



LÉVY SCALE PARAMETER R AT RHIC

- Similar decreasing trend as Gaussian HBT radii, but it is not an RMS!
  - RMS of a Lévy source: in principle infinity, obtained value depends on cutoff
- What do model calculations, simulations say about this?
- Hydro behavior ($1/R^2 \sim m_T$) not invalid; but: predicted for Gaussian case only!

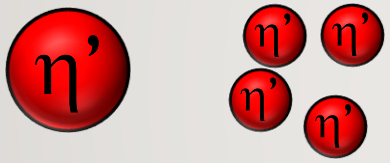




CORRELATION STRENGTH λ: IN-MEDIUM MASS?

- Connection to chiral restoration
  - Decreased η' mass → more η' produced → more decay pions → λ decreases
  - Kinematics: η' → ππππ with low m_T → decreased $\lambda(m_T)$ specifically at low m_T
  - Dependence on in-medium η' mass?

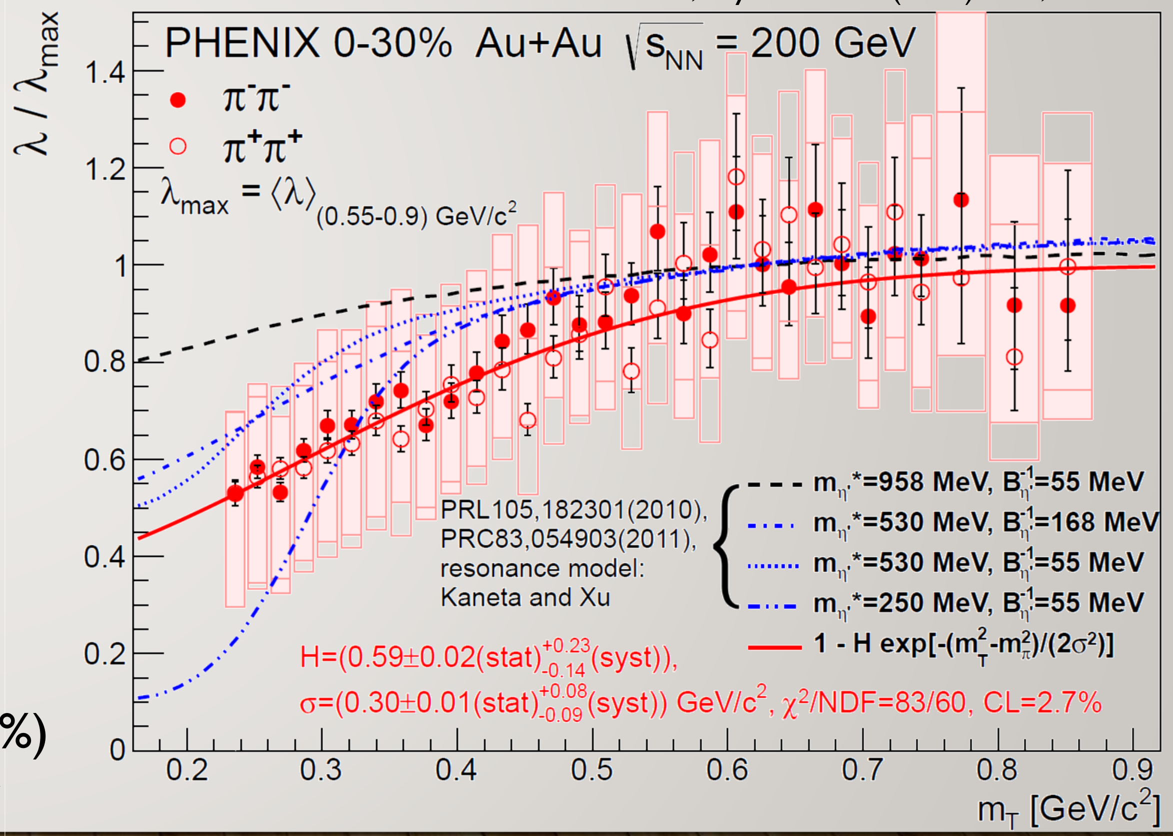
Kapusta, Kharzeev, McLerran, PRD53 (1996) 5028
Vance, Csörgő, Kharzeev, PRL 81 (1998) 2205
Csörgő, Vértesi, Sziklai, PRL105 (2010) 182301



$$T < T^*$$
$$m_{\eta'} > m^*_{\eta'}$$
$$N_{\eta'} < N^*_{\eta'}$$
$$\lambda > \lambda^*$$

- Results not incompatible with this
- Recall: 3D results similar to 1D
- Would need direct check with photons ($\eta' \to \gamma\gamma$, 2.3%)
- Centrality dependent analysis in collaboration review

PHENIX, Phys.Rev. C97 (2018) no.6, 064911

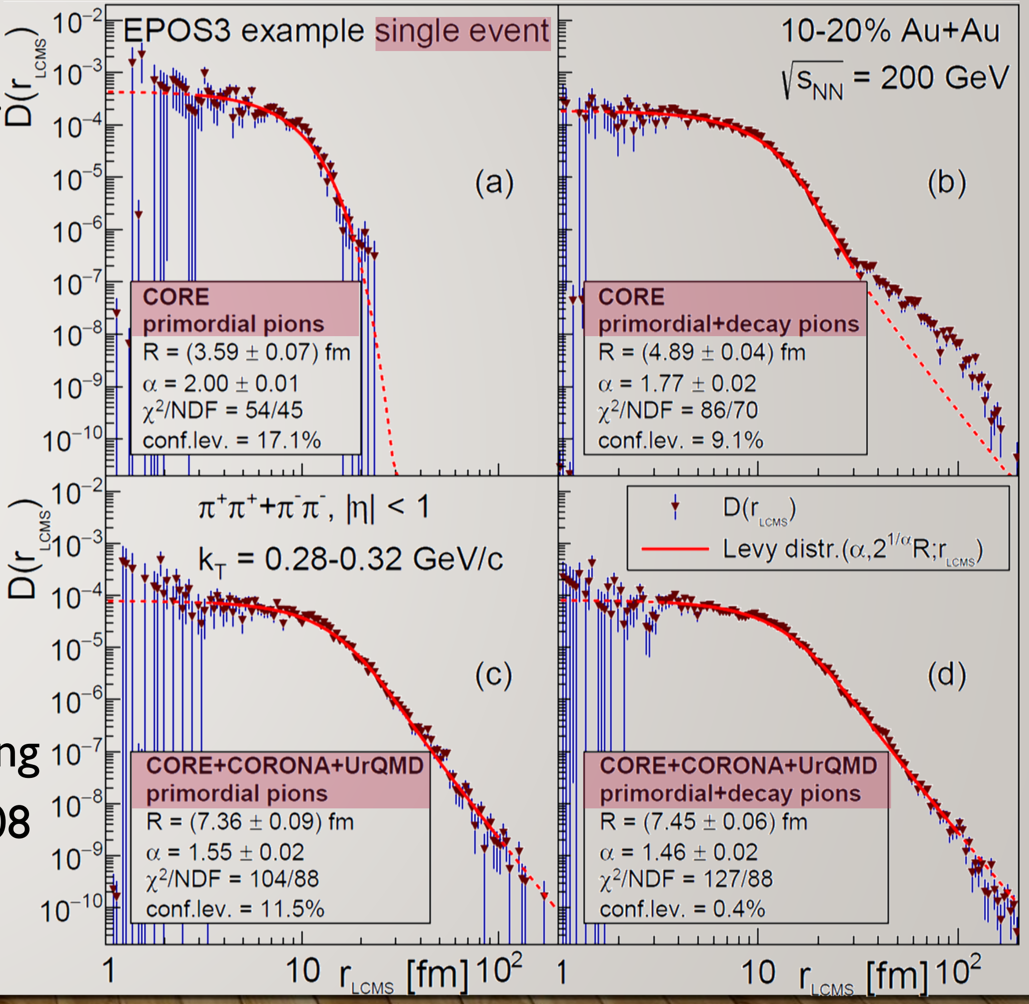


CONTENTS OF THIS TALK

- Basics of femtoscopy and Lévy sources
- First thorough Lévy HBT analysis in AA by PHENIX
- Recent phenomenological updates
- Recent experimental results
- Summary and outlook

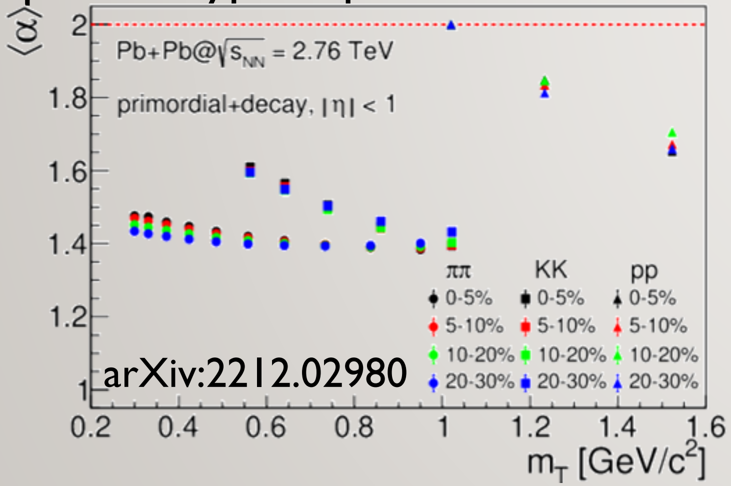
EVENT BY EVENT SHAPE ANALYSIS WITH EPOS

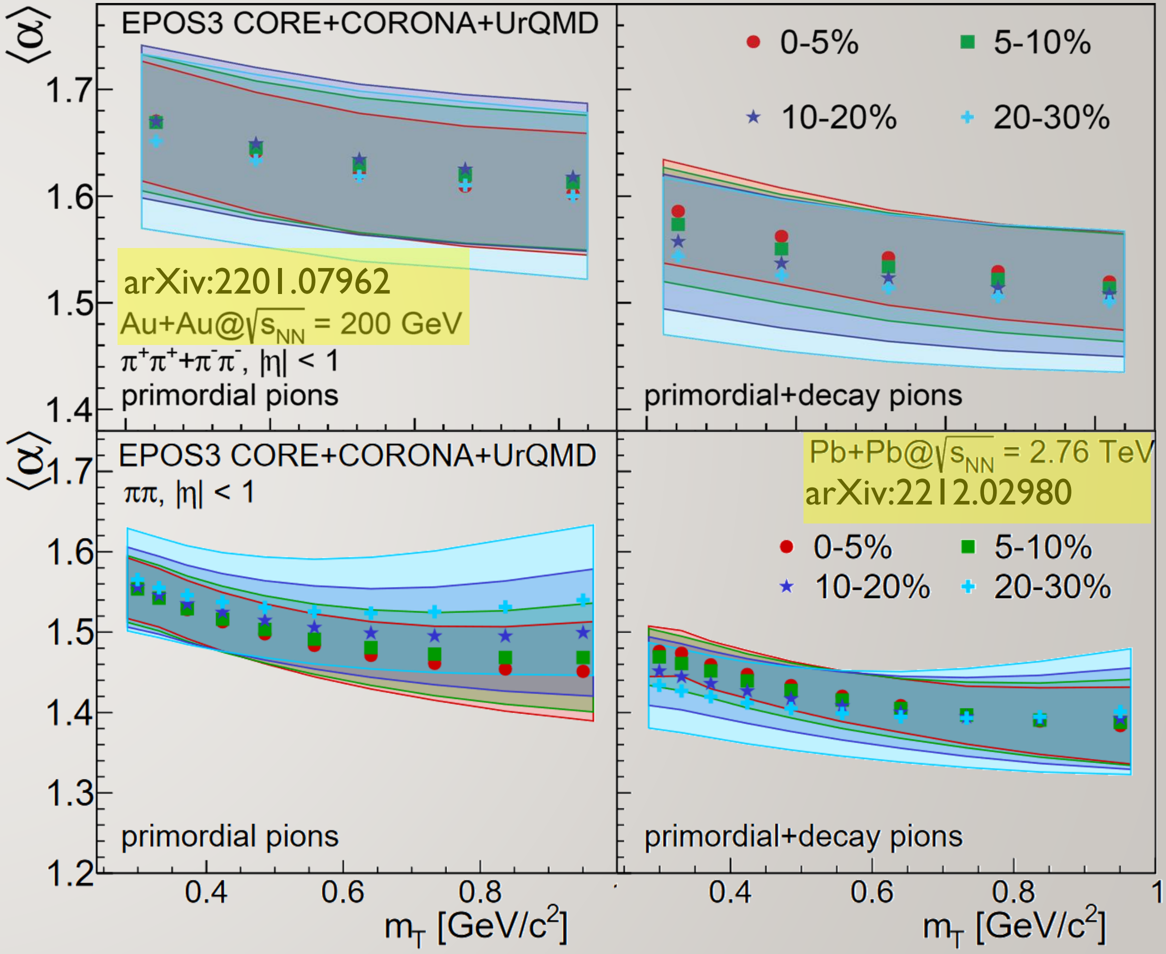
- EPOS model: parton-based Gribov-Regge theory (PBGRT)
  - Werner et al., PRC82 (2010) 044904, PRC89 (2014) 064903, …
  - Core-Corona, viscous hydro (vHLLE), cascades, UrQMD
- Pair distribution calculated: $D(r_{LCMS}) = \int d\Omega dt D(t, r_x, r_y, r_z)$
  - Angle-averaged radial source distribution of like-sign pion pairs
- Investigated cases:
  - a) CORE, primordial pions: close to Gaussian
  - b) CORE, with decay products: power-law structures
  - c) CORE+CORONA+UrQMD, primordial pions: Lévy shape
  - d) CORE+CORONA+UrQMD, with decay products: Lévy shape
- Lévy shape in single events; source size versus m_T: hydro scaling
  - 200 GeV AuAu: Kincses, Stefaniak, Cs., Entropy 24 (2022) 308
  - 2.76 TeV PbPb: Kórodi, Kincses, Cs., arXiv:2212.02980
  - See talk by B. Kórodi on Thursday



AVERAGE LÉVY EXPONENT VS TRANSVERSE MASS

- $\langle\alpha\rangle$ w.r.t. m_T and centrality: small dependendce
  - 200 GeV AuAu: Entropy 24 (2022) 308
  - 2.76 TeV PbPb: arXiv:2212.02980
- With or without decays at RHIC: $\alpha_{\rm EPOS} > \alpha_{\rm measured}$
  - Opposite at LHC energies, see talk by B. Kórodi on Thu
- Similary analysis at 2.76 TeV [arXiv:2212.02980]: particle type dependence as well





A NOVEL METHOD FOR LÉVY SHAPES WITH COULOMB FSI

- Calculating correlation functions with the Coulomb effect included: time consuming in the past
  1. Method used in early analyses: Coulomb correction calculated for fixed radius and shape (e.g. $R = 5$ fm, $\alpha = 2$)
  2. More consistent method: correlation function with Coulomb FSI precalculated in a tabular form
     - Iterative fitting, convergence in 2-3 iterations usually, see e.g. PHENIX Coll., Phys.Rev.C 97 (2018) 6, 064911
  3. Convenient, but somewhat restricted method: interpolating functional form, in a limited R, α range
     - See Csanád, Lökös, Nagy, Phys.Part.Nucl. 51 (2020) 3, 238, used e.g. in CMS Coll., arXiv:2306.11574 (HIN-21-011)
- New mathematical development: Coulomb integral $C_2(Q) = \int d^3r \left|\psi_Q(r)\right|^2 D(r)$ can be performed
  - $D(r)$ is expressible as a Fourier transform: $D(r) = \int d^3q e^{iqr} f(q)$, for example $D(r)$ Lévy: $f(q) = e^{-|qR|^\alpha}$
  - Integrals $\int d^3r$ and $\int d^3q$ unfortunately cannot be exchanged; calculation can still be performed
  - Via Lebesgue and Fubini theorems, see details in arXiv:2308.10745 (available on Aug 22, Tuesday)

$$C_2(Q) = |\mathcal{N}|^2 \left(1 + f(Q) + \frac{\eta}{\pi}\left[A_{1s}[f](Q) + A_{2s}[f](Q)\right]\right), \text{ where } |\mathcal{N}|^2 = \frac{2\pi\eta}{e^{2\pi\eta}-1} \text{ (Gamow)}, \eta = \frac{mc^2\alpha}{\hbar c Q}$$

- Details in Nagy, Purzsa, Csanád, Kincses arXiv:2308.10745, code at github.com/csanadm/CoulCorrLevyIntegral

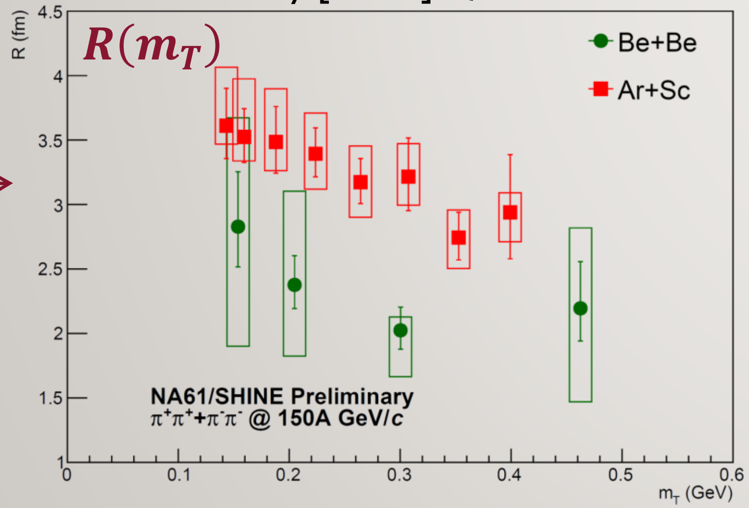
CONTENTS OF THIS TALK

- Basics of femtoscopy and Lévy sources
- First thorough Lévy HBT analysis in AA by PHENIX
- Recent phenomenology updates
- Recent experimental results
- Summary and outlook

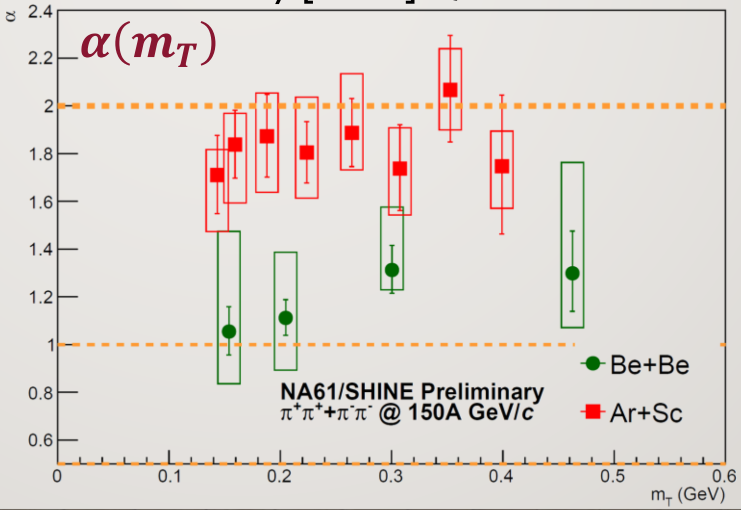
PION ANALYSIS AT SPS NA61/SHINE

- Lévy scale R of Ar+Sc [prelim.: Universe 9 (2023) 7, 298] and Be+Be [arXiv:2302.04593, EPJC accepted]:
  - Compatible with initial geometry factor 1.6 between Ar+Sc and Be+Be
  - Decrease with m_T due to transverse flow?
- No m_T dependence in λ, in contrast to RHIC result – can be turned off?
- Lévy index α: significant difference

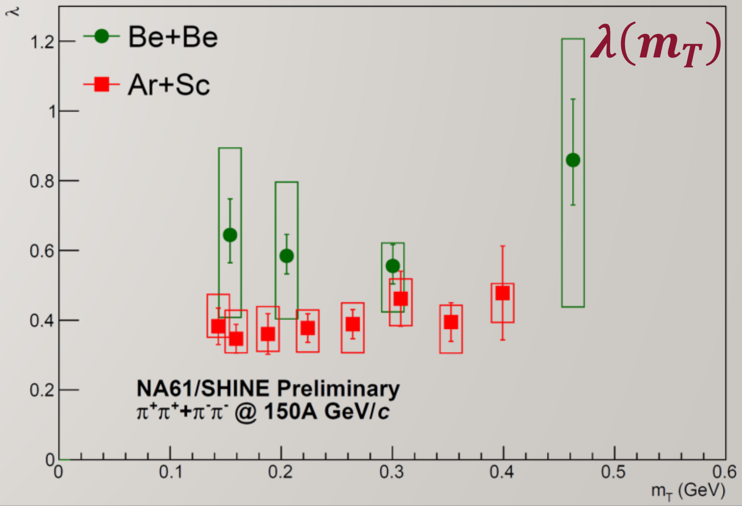
B. Pórfy [NA61], QM 22 & WPCF 22



B. Pórfy [NA61], QM 22 & WPCF 22

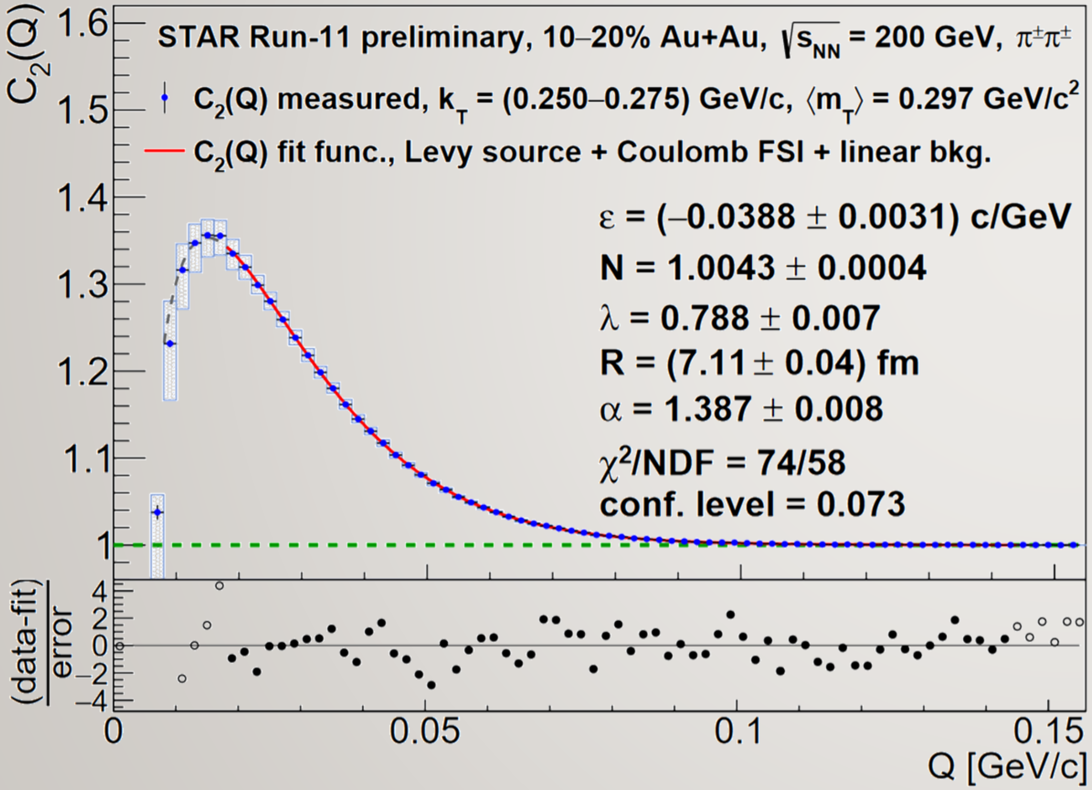


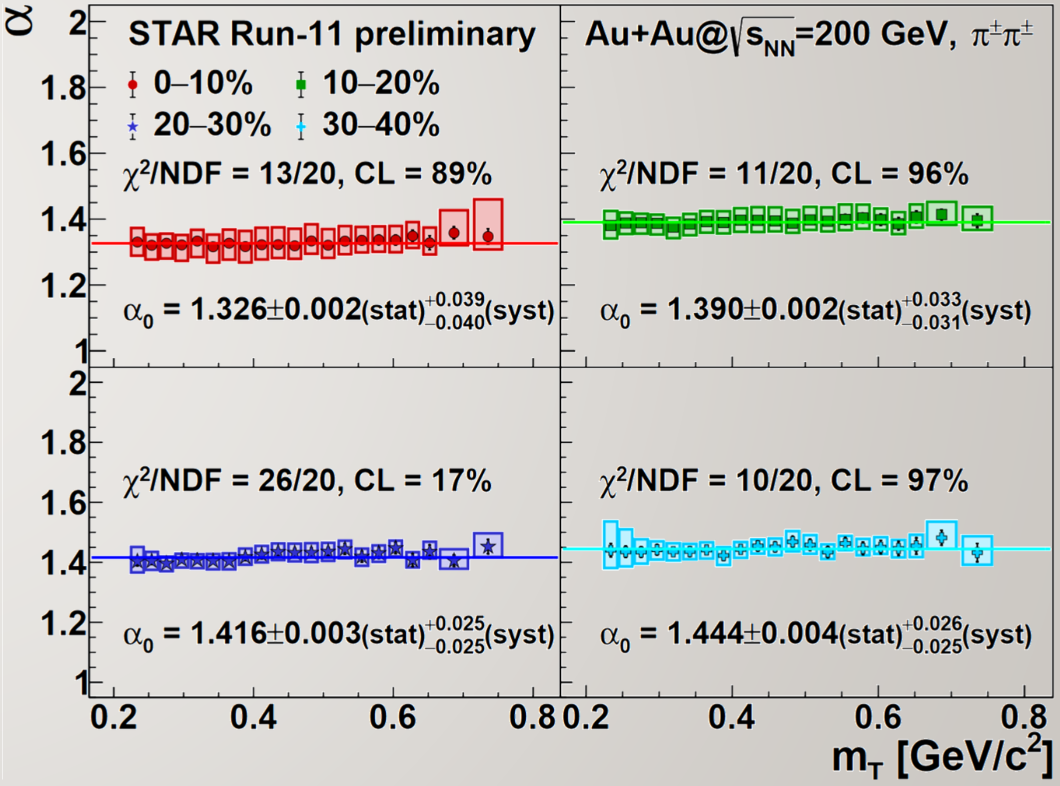
B. Pórfy [NA61], QM 22 & WPCF 22



PION ANALYSIS WITH LÉVY SOURCES AT STAR

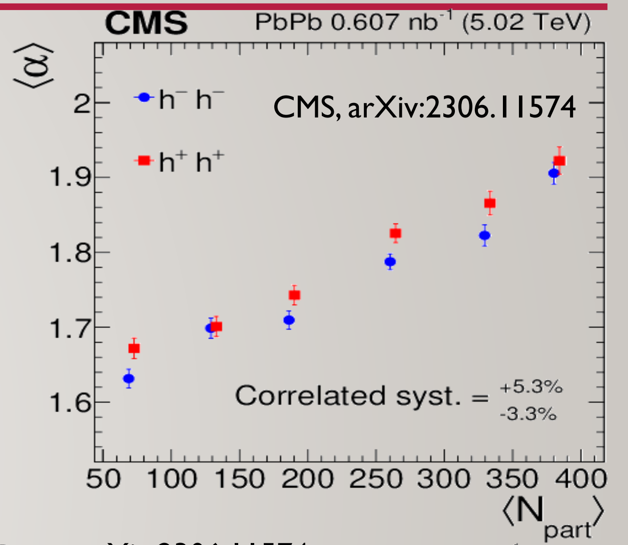
- Run-11 Au+Au at 200 GeV, ~ 550 M events, PID by TPC+TOF, 21 mT and 4 centrality bins
- Source far from Gaussian; more details on poster and flash talk by D. Kincses (and RHIC AUM poster)

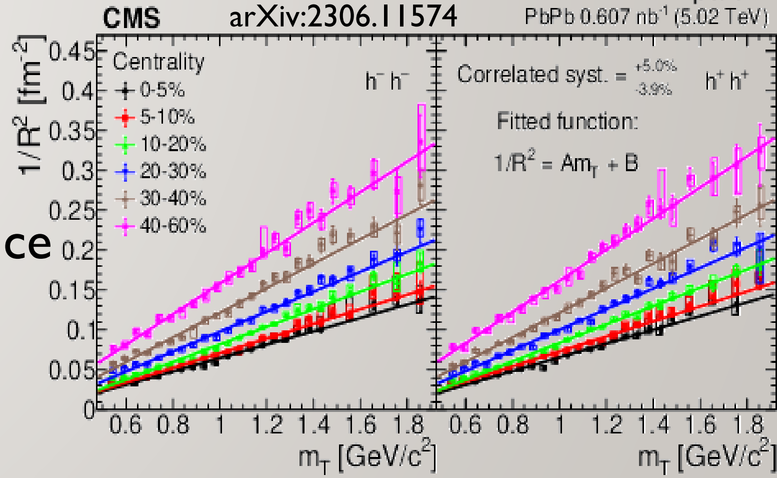




CHARGED HADRON ANALYSIS IN 5 TEV PB+PB

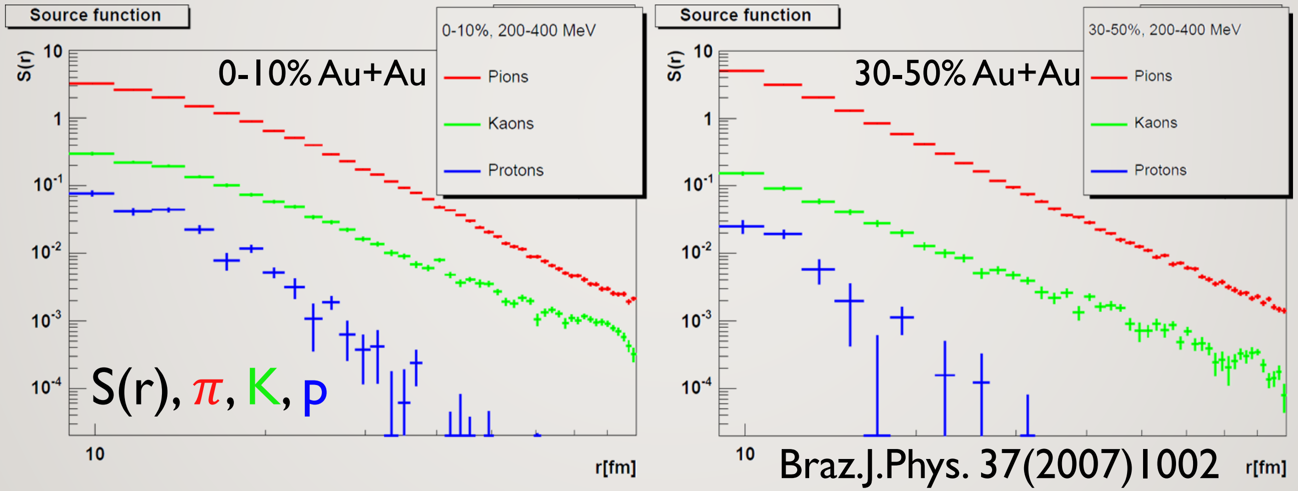
- Lévy index α measured:
  - Far from Cauchy
  - Not exactly Gaussian
  - Closer to Gaussian for large N_{part}, unlike RHIC
- Lévy scale R: hydro scaling confirmed
  - In every centrality class, despite non-Gaussianity
  - Hubble coefficient can be extracted: 0.12-0.18 c/fm, larger than at RHIC
- Correlation strength λ also analyzed
- Low-Q deviation cross-checked with Monte-Carlo: two-track acceptance
- Final CMS result: HIN-21-011, arXiv:2306.11574 [under review]
  - Preliminary results in proceedings: B. Kórodi, Universe 9 (2023) 7, 318





THE IMPORTANCE OF A KAON ANALYSIS

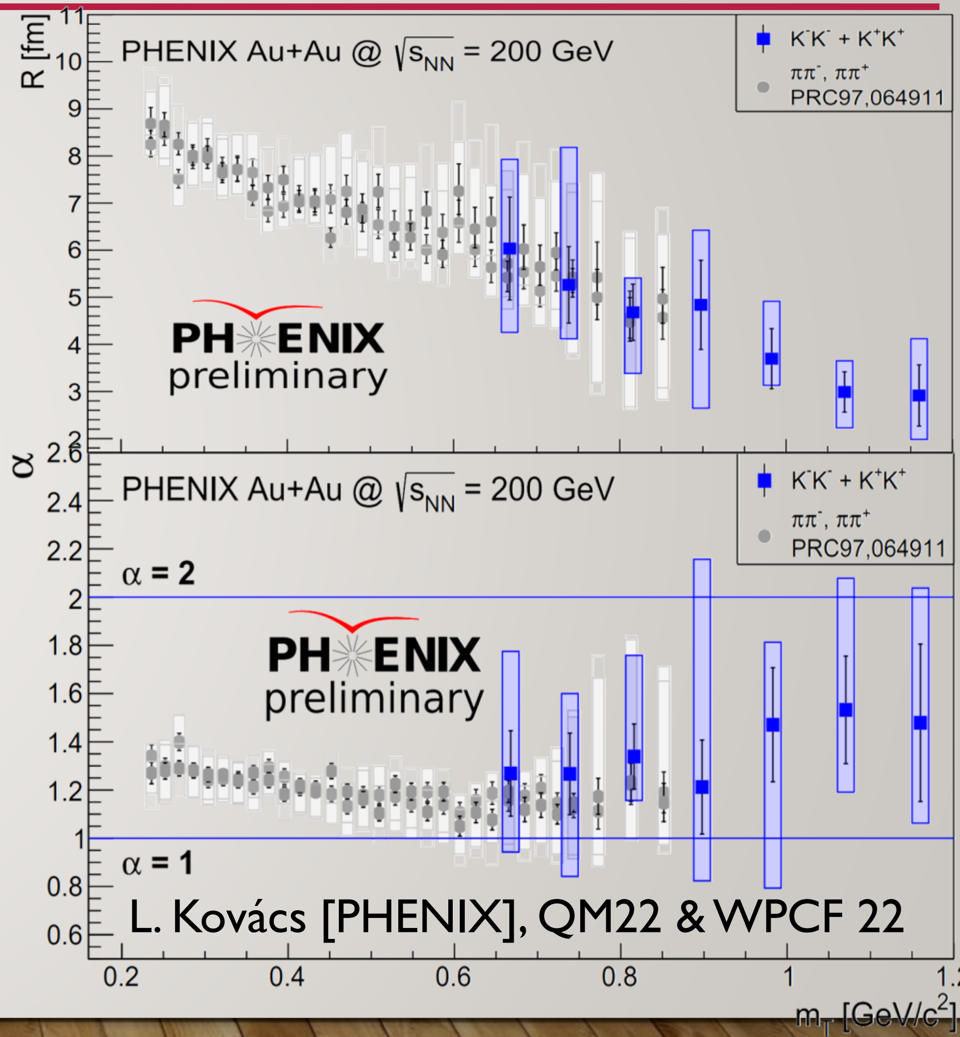
- Kaons: smaller cross-section, larger mean free path
- Mean free path increases more during a time-step → heavier power-law tail?
- Prediction for π, K, p based on Humanic's Resonance Model (HRM): anomalous diffusion due to rescattering
  Humanic, Int.J.Mod.Phys. E15 (2006) 197 [nucl-th/0510049]
  Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002 [hep-ph/0702032]



- Kaon HBT radii: m_T scaling or its violation for Lévy scale R?
- Prediction: $\alpha(p) > \alpha(\pi) > \alpha(K)$

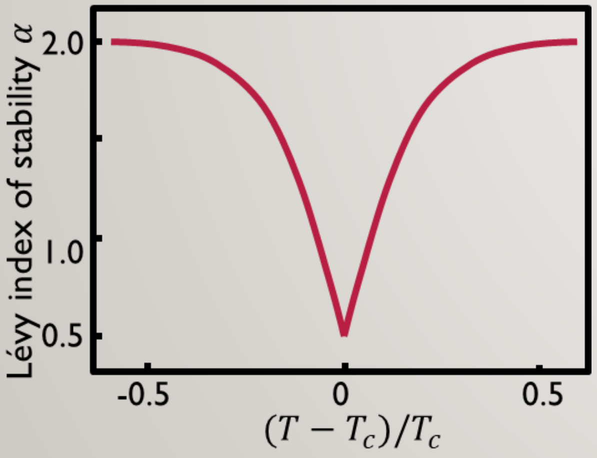
KAON ANALYSIS AT PHENIX AND STAR

- Preliminary analysis performed at PHENIX and STAR
- Kaon and pion data seem compatible at the same m_T
- Lévy scale R shows hydro type of scaling with m_T
  - R depending on m_T but not on particle type separately
- $\alpha(K) \geq \alpha(\pi)$, but anomalous diffusion suggests opposite
- Dominant mechanism creating Lévy source?
  - Not only rescattering?
  - Anomalous hydro at the sQGP stage?
- PHENIX prelim. results: L. Kovács, Universe 9 (2023) 7, 336
- STAR prelim. results: A. Mukherjee, Universe 9 (2023) 7, 300



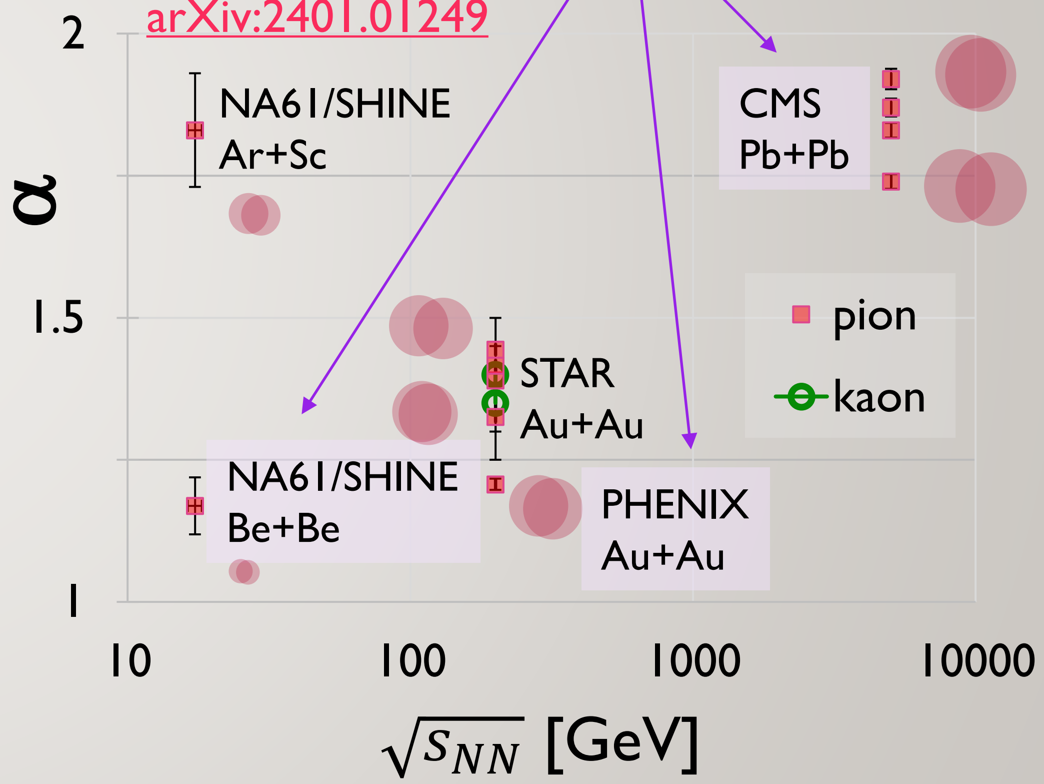
STABILITY PARAMETER α FROM SPS TO LHC

- Different values for small (Be+Be) & medium (Ar+Sc) systems at SPS
  - Also true for PbPb and pp at LHC? (pp: $\alpha = 1$ assumed so far)
- Medium and large systems: non-monotonic trend
- Compare to expectation cartoon based on Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67



Final data

arXiv:2401.01249

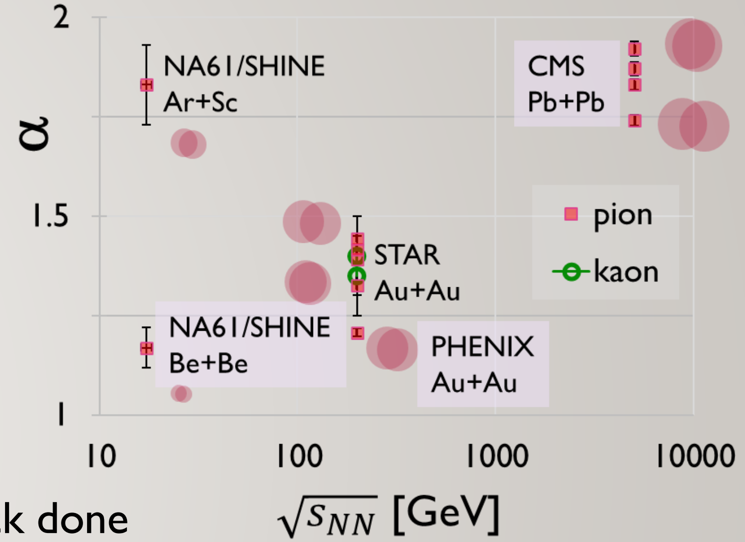


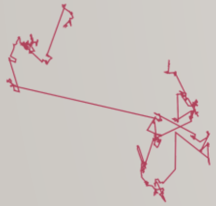
CONTENTS OF THIS TALK

- Basics of femtoscopy and Lévy sources
- First thorough Lévy HBT analysis in AA by PHENIX
- Recent phenomenology updates
- Recent experimental results
- Summary and outlook

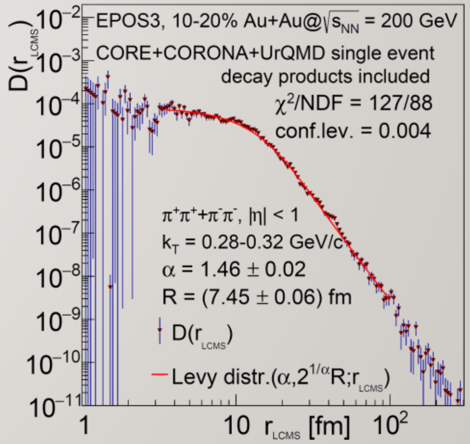
CONCLUSIONS AND OUTLOOK

- Lévy sources from SPS to RHIC and LHC
  - **Lévy α**: between 1 and 2, increases with $\sqrt{s_{NN}}$?
    - Contrary to expectations, $\alpha(K) \geq \alpha(\pi)$
  - **Lévy R**: hydro scaling, despite not Gaussian
  - **Lévy λ**: signs of η' in-medium mass modification

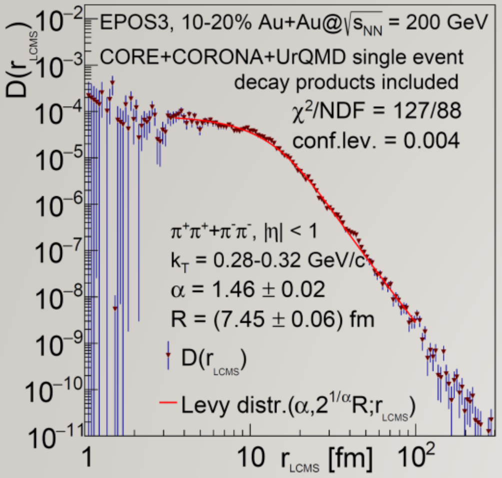


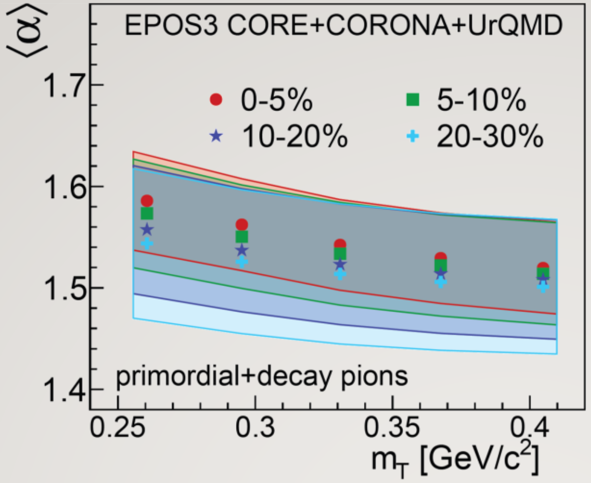


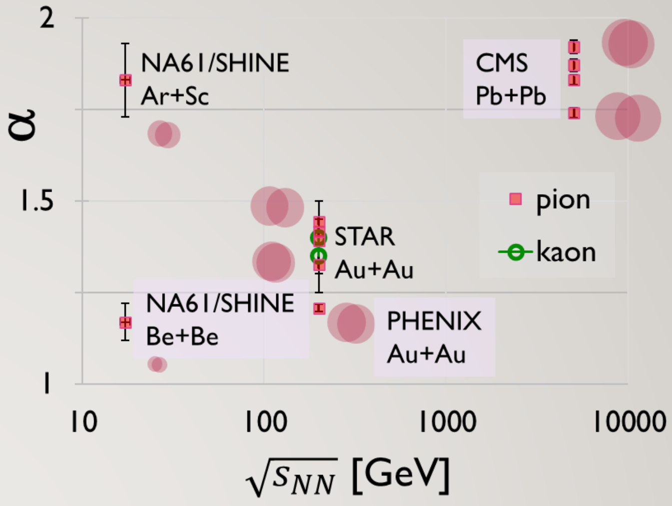
- Possible reasons:
  - Jet fragmentation → not dominant in AA collisions
  - **Critical phenomena** → maybe at lowest RHIC energies and SPS
  - Directional averaging → source is (approx.) spherical in LCMS, 3D cross-check done
  - Event averaging → event-by-event simulations show Lévy
  - **Resonance decays** → part of the reason, not enough alone
  - **Hadronic rescattering, Lévy flight** → $\alpha(K) \geq \alpha(\pi)$ puzzling



- Questions to be answered:
  - When measuring α, what effects need to be considered?
  - Can there be anomalous diffusion in the quark stage?
  - What is the role of finite size and finite time?







THANK YOU FOR YOUR ATTENTION

And if you are interested in these topics:

: https://agenda.infn.it/event/33324/ http://zimanyischool.kfki.hu/23/