Two
Pomeron Interaction

EX UMBRA IN SOLEM

C. Contreras

Departamento de Física, Universidad Técnica Federico Santa María
Chile

In collaboration with
J. Bartels and L. Cancino

Fondecyt 1231829

Outline

- Introduction
- N-Pomeron fields
- FRG approximation and interaction of Pomeron
- Numerical results for 2 Pomeron Interaction
- Summary and Outlook

JHEP 1603 (2016) 201
PRD 95 (2017) 014013
Two Pomeron: arXiv to appear

Introduction

- Scattering process: description in terms of QCD (parton distributions) Strong Interaction

- In the Regge Limit ($s > t$, t-channel exchange dominate)

Diffractive process, then becomes an effective 2+1 dimensional: transversal space and rapidity (Lipatov effective action)

- At very small transverse distances pQCD and BFKL Pomeron (1958)

- At large transverse distances and small x nonperturbative QCD BK/JIMWLK approach

see talk: C. Royon and M. Sadzikowsli

- There are another states with 3, 4 gluons…. BKP Equation

1980 J. Bartels; J. Kwiecinski and M. Praszalowicz

- Another option: RFT Reggeon Field Theory description Gribov

Soft Pomeron Reggeon Field Theory before QCD

P.D.B. Collins, *An introduction to Regge theory and high energy physics*, Cambridge University Press, Cambridge, 1977.

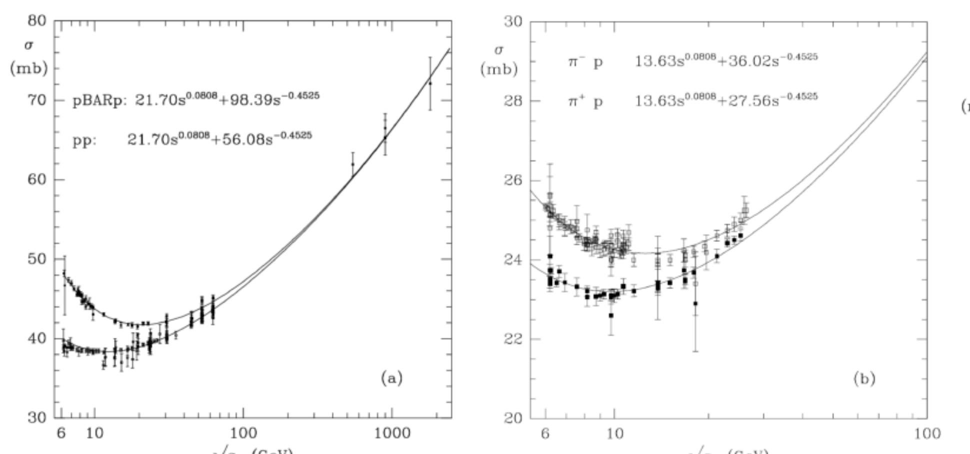
- V. N. Gribov introduce in the 60´s
- Scattering amplitude at high energies for hadrons is according Regge Theory
- The exchange are "quasi particles" characterized by its Regge trajectories : $\alpha_i\,(t)$
- Leading Pole: is Called Pomeron with vacuum quantum numbers

$$\alpha(t) = \alpha_0 + \alpha' t = 1 + (\alpha_0 - 1) + \alpha' t$$

- $\mu = \alpha_0 - 1$ is the Pomeron intercept and $\alpha´$ is the slope
- According to the Regge theory the contribution to the total Cross section, is given by:

$\sigma_T = A_i s^{\alpha_i(0)-1}$ A. Donnachie and Landshoff : 1992

- $\mu = \alpha_0 - 1 = 0.08$ and $\alpha' = 0.25$ GeV $\alpha_P(t) = 1.08 + 0.25\,(\mathrm{GeV}^{-2})t$



Soft Pomeron RFT

See Regge Pole analysis Talk Poblaguev

BFKL/Hard Pomeron pQCD

Balitsky Fadin, Kuraev Lipatov 1977

- For Hard processes short Transversal distances

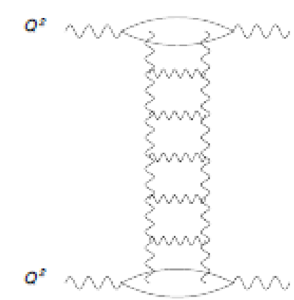
$$p + p \rightarrow p + p$$

we consider pQCD → Hard Pomeron dominate scattering

- The BFKL Pomeron which has been studied up to NLO in perturbation theory is a composite states of Reggeized gluons.

Resumation of the Ladder at leading log approximation (multi Regge Kinemmatics MRK) Lipatov, Bartels

- **The intercept of the Pomeron is related with the eigenvalues of the BFKL Kerne**



$$\boldsymbol{\omega_0(\gamma) = \alpha_s N_c / \pi \, [2\, \Psi(1) - \Psi(\gamma) - \Psi(1-\gamma)]}$$

$$\Psi(\gamma) = \frac{d}{d\gamma} Ln\, \Gamma(\gamma)$$ Digamma function

$$\alpha_P(0) = \omega_0(1/2) \approx 1 + 4\, \frac{\alpha_s N_c}{\pi} \ln 2 = 1 + 2.648\, \alpha_s \qquad Hard\ BFKL/QCD$$

$\alpha_P(0) \approx 1 + 0.5296$ for $\alpha_s = 0.2$

we can connect Hard –Soft Pomeron regions of different sizes and different sorts of Pomeron using Functional Renormalization Group

QCD Description and BFKL kernel

Balitsky Fadin, Kuraev Lipatov 1977

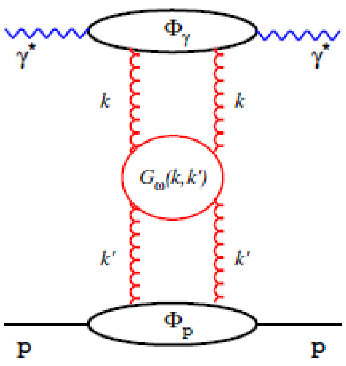
*A(s,***k***,***k´***)* is the amplitude for the scattering of a gluon with transverse momentum **k** off another gluon with transverse momentum **k´** at center of mass energy $\sqrt{s}$ and it is found to obey an evolution equation

$$\frac{\partial}{\partial Y} A(\mathbf{k},\mathbf{k}',Y) = \frac{\alpha_s N_c}{\pi}\int \frac{d^2q}{2\pi\,(k-q)^2}\left(A(q,k,Y') - \frac{k^2}{2\,q^2}A(k,k',Y)\right)$$

The kernel is obtained by summing all graphs which contribute an effective "gluon ladder".
Using a Mellin Transformation the BFKL evolution equation can be solved in terms of the $\phi_\omega(k)$ eigenfunctions of the Kernel

$$\omega\,\phi_\omega(k) = \bar{\alpha}\int \frac{d^2k'}{2\pi}\widetilde{K}(k,k')\phi_\omega(k')$$

$\widetilde{K}(k,k')$ **is the BFKL kernel**



Scattering process can be described using BFKL Green Function : G(t , t´, Y)

A(x, Q^2) = $\int dt\,dt'\,\Phi_\gamma(Q^2$, t) G(t , t´, Y) Φ_P(t´) where Y = ln(1/x) Rapidity

$\Phi_\gamma(Q^2$, t) describe the coupling of the gluon (perturbatively calculable) with transverse momentum k to a photon of virtuality Q^2 and Φ_P(t) describes the coupling of a gluon of transverse momentum k´ to the target proton

- In more general form we define the BFKL Greens functions G(q´, q - q´; q´´, q´´- q |w) as an infinite sum of ladder diagram and satisfies the integral equation: (Bethe Salpeter resummation)

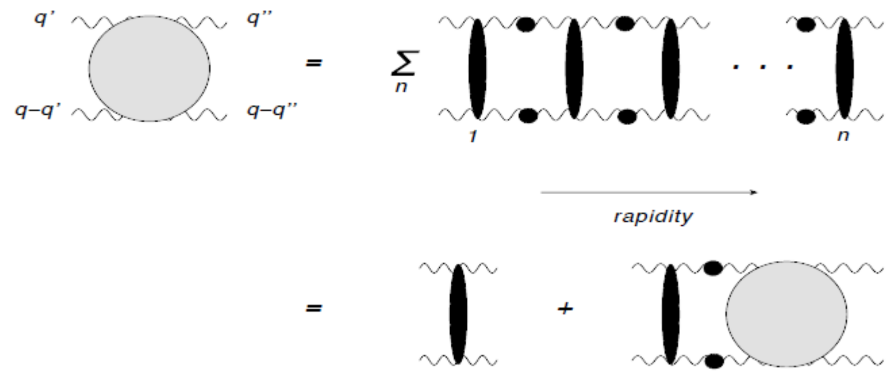


Figure 1: the BFKL ladder

$$G(q', q-q'; q'', q-q''|\omega) = K_{\mathrm{BFKL}}(q, q-q'; q'', q-q'') \qquad (2.1)$$
$$+ \int d^2k K_{\mathrm{BFKL}}(q, q-q'; k, q-k) \frac{1}{k^2(q-k)^2} \frac{1}{\omega - \omega_g(k) - \omega_g(q-k)} G(k, q-k; q'', q-q''|\omega)$$

- Where the wavy denote the Reggeized Gluons: $\frac{1}{k^2}\frac{1}{\omega - \omega_g(k^2)}$.
- And the real part

$$K_{\mathrm{BFKL}}(q, q-q'; q'', q-q'') = \frac{\bar{\alpha}_s}{2\pi}\left(-q^2 + \frac{q''^2(q-q')^2}{(q'-q'')^2} + \frac{q'^2(q-q'')^2}{(q'-q'')^2}\right),$$

- Virtual part (Gluon trajectory) $\omega_g(q^2) = -q^2 \frac{\bar{\alpha}_s}{4\pi} \int d^2k \frac{1}{k^2(q-k)^2}$

- One can reorganize this expression and the final BFKL kernel is given by

$$\tilde{K}(\mathbf{q}, \mathbf{q}-\mathbf{q}'', \mathbf{q}-\mathbf{q}'') = K(\mathbf{q}, \mathbf{q}-\mathbf{q}'; \mathbf{q}'', \mathbf{q}-\mathbf{q}'') + \delta^{(2)}(\mathbf{q}'-\mathbf{q}'')\left(\omega_g(q') + \omega_g(q-q')\right)$$

BFKL regularized equation and N-Pomeron

In the presence of an infrared cutoff and with running α_s the piece of the ω −cut between $\omega = \omega_0$ and zero is replaced by an infinite sequence of discrete poles, which accumulate at zero

L. N. Lipatov, Sov. Phys. JETP 63 (1986) 904

We need IR regulator in the propagator in order study the ladder diagram in the NLO.

BFKL with IR regulator it not new:

- **Lipatov 1986** — **IR Regulator**
- **Braun and Vacca 1999** — **bootstrap conditions**
- **Levin, Lipatov and Siddikov 2014** — **mass regulator**
- **Kowalski, Lipatov, Ross 2014** — **boundary conditions**
- **Bartels, Contreras and Vacca 2019** — **Wilsonian or ERG regulator**

- We introduce the following momentum regulator for the propagator for the Gluon

$$\frac{1}{q^2} \rightarrow \frac{1}{[q^2+R_k(q^2)]}$$

$$R_k(q^2) = (k^2 - q^2)\, \theta(k^2 - q^2)$$ Wilsonian Regulator

Trajectory Gluons

$$\omega_g(q^2) = -q^2 \frac{\bar{\alpha}_s}{4\pi} \int d^2k \frac{1}{k^2 (q-k)^2}$$

$$\omega_{gk}(q^2) = -q^2 \frac{\bar{\alpha}_s}{4\pi} \int d^2l \frac{1}{[l^2 + R_k(l^2)]\,[(q-l)^2 + R_k((q-l)^2]}$$

- BFKL Kernel

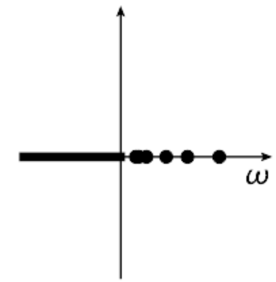
$$K_{\text{BFKL}}(q', q-q'; q'', q-q'') = \frac{\bar{\alpha}_s}{2\pi} \left(-q^2 \frac{(q'-q'')^2}{(q'-q'')^2 + R_k(q'-q'')^2} + \frac{q''^2 (q-q')^2 + q'^2 (q-q'')^2}{(q'-q'')^2 + R_k(q'-q'')^2)} \right)$$

What is important this result

BFKL kernel in the color singlet state of the t-channel

- ✓ **the spectral descomposition with the Wilsonian IR regulated BFKL kernel and with running coupling constant is discrete.**

$$\mathcal{G}_k = \frac{1}{\omega - \tilde{K}} = \sum_n \frac{\psi_{n,k}(q', q-q')\psi^*_{n,k}(q'', q-q'')}{\omega - \omega_{n.k}} + \text{continuous part},$$



$$\boldsymbol{\omega\, \phi_\omega(k) = \int \frac{d^2k'}{2\pi} \bar{\alpha}(k,k')\tilde{K}(k,k')\phi_\omega(k')}$$

- Numerical Solution: lattice formulation with N = 600
- Eigenvalues intercept
- Eigenfunctions: bound states n-Pomeron fields

$$\alpha_s(\mathrm{q}^2) = \frac{3.41}{\beta_0\left[\ln(\mathrm{q}^2 + R_0^2) + \ln\frac{m_h^2}{\Lambda^2_{QCD}}\right]}$$

Slope: we leave the forward direction and consider the q^2 dependence of the eigenvalues and the slope is

$$\bar{\alpha}_s(\mathrm{q}^2) = \alpha_s(\mathrm{q}^2)\frac{N_c}{\pi}$$

- ✓ $\alpha\,(q^2) = \omega(q^2) \sim \omega^0 + q^2\alpha'$ $\qquad \alpha' = \frac{d\omega}{dq^2}$ *when* $q^2 \to 0$

$$\omega_n(q^2) = \omega_n^{(0)} + q^2 \frac{\int d^2k \int d^2k' f_n(k'^2)\left[K^{(1)}(\mathrm{k},\mathrm{k}') + 2\delta^{(2)}(\mathrm{k}-\mathrm{k}')\omega_g^{(1)}(k^2)\right] f_n(k^2)}{\int d^2k |f_n(k^2)|^2}$$

Numerical Results NLO Pomeron

$$r_n = < \ln \mathbf{q}^2 > = \frac{\int d\mathbf{k}^2 |\psi_n(\mathbf{k})|^2 \ln \mathbf{k}^2}{\int d\mathbf{k}^2 |\psi_(\mathbf{k})|^2}$$

	energy			slope			radius		
n	k=0.54	k=1	k=5	k=0.54	k=1	k=5	k=0.54	k=1	k=5
1	-0.53	-0.43	-0.29	0.17	0.034	0.00065	2.2	4.7	3.4×10^{1}
2	-0.25	-0.22	-0.17	0.040	0.0091	0.00022	5.2×10^{1}	1.2×10^{2}	1.0×10^{3}
3	-0.16	-0.15	-0.12	0.017	0.0042	0.00012	1.4×10^{3}	3.3×10^{3}	2.8×10^{4}
4	-0.12	-0.11	-0.097	0.0097	0.0024	0.000070	3.7×10^{4}	8.9×10^{4}	7.8×10^{5}
5	-0.094	-0.089	-0.079	0.0061	0.0016	0.000048	1.0×10^{6}	2.5×10^{6}	2.2×10^{7}
6	-0.077	-0.074	-0.067	0.0042	0.0011	0.000035	2.8×10^{7}	6.7×10^{7}	6.0×10^{8}
7	-0.066	-0.064	-0.058	0.0040	0.00083	0.000026	7.8×10^{8}	1.9×10^{9}	1.7×10^{10}
8	-0.058	-0.056	-0.052	0.0023	0.00064	0.000021	2.2×10^{10}	5.2×10^{10}	4.6×10^{11}

Table 4: Numerical values for eigenvalues, slopes and radii, with running coupling and for different values of the IR cutoff k

Results independent of the regulator

see J. Bartels, C. Contreras G. P Vacca JHEP 1901(2019) 004

Wave Function Pomeron

- The support of the Wave function with $n > 2$, is defined in the UV region

- N=1 is concentrated in the soft region

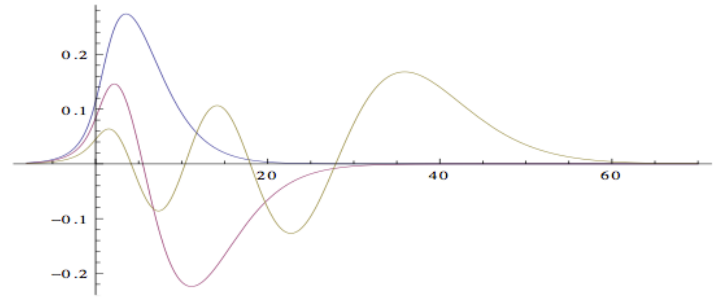


Figure 22: three leading wavefunctions (No 1,2,5) as a function of $\ln q^2$.

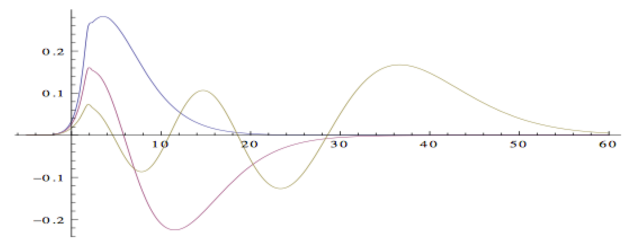


Figure 24: three leading wavefunctions (No 1,2,5) for the scale $k = 1\text{GeV}$, as a function of $\ln q^2$

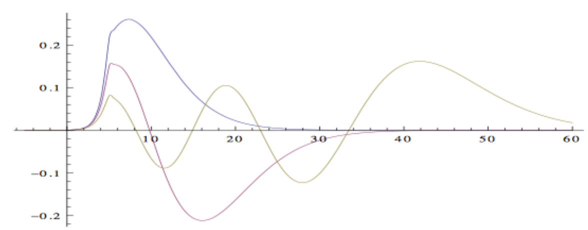


Figure 25: three leading wavefunctions (No 1,2,5) for the scale $k = 5\text{GeV}$, as a function of $\ln q^2$

How we can test this N - Pomeron

- Ellis, Kowalski and Ross: Using the N-BFKL Pomeron started to fit the small-x and low-Q^2 HERA data F2

J. Ellis, H. Kowalski and D. A. Ross, Phys. Lett. B 668 (2008) 51 Kowalski, Lipatov, Ross and Watt: arXiv: 1005.0355

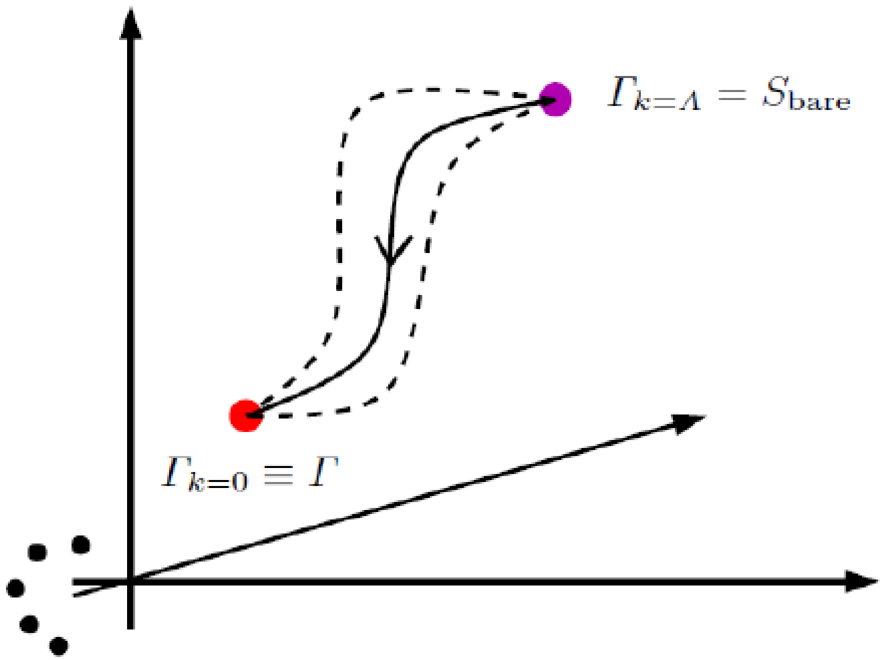
- All these approaches only use the contribution of the 3 discrete BFKL Pomeron
- No attempt has been made to introduce the triple Pomeron vertex and to study its QCD effect.
- In the field theory based upon reggeized gluons these vertex functions would lead to an extremely hard problem to solve **(see talk . Sadzikowsli)**
- To study the interactions of these Regge poles and in order to be able to move from larger to small distances it will be convenient to consider the local approximation and to make use of the well-known formalism of RFT
- BFKL Pomeron into a reggeon field theory which includes corrections to the BFKL Pomeron based upon interaction vertices, in particular the triple Pomeron vertex

$$\Gamma_k[\psi_i^\dagger,\psi_i] = \int d^D x d\tau [Z_i(\frac{1}{2}\psi_i^\dagger \hat{\partial}_\tau \psi_i - \alpha_i' \psi_i^\dagger \nabla^2 \psi_i) + m_{i,l}(\psi_i \psi_l^\dagger + \psi_i^\dagger \psi_l) - \mu_i \psi_i^\dagger \psi_i - V_k[\psi_i^\dagger, \psi_i; \lambda_j]\ ],$$

Functional Renormalization Group

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} Tr\left[\left(\frac{\delta^2 \Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} \partial_t R_k\right]$$

Wetterich Equation 93



Action $\Gamma_k(\psi_i, \psi_i^\dagger\,,\ \lambda_i)$

Regulator $R_k(q)$

Initial Condition $\Gamma_{k=\Lambda}(\phi, g_i)$

Interaction V = $\sum \lambda_{i;jk}\,(\psi_i^\dagger \psi_j \psi_k$ +cc)

$$R_1 = Z_1 \alpha_1'(k^2 - q^2)\Theta(k^2 - q^2)$$

$$R_2 = Z_2 \alpha_2'(k^2 - q^2)\Theta(k^2 - q^2) = r Z_2 \alpha_1'(k^2 - q^2)\Theta(k^2 - q^2),$$

$$r = \frac{\alpha_2{}'}{\alpha_1{}'}$$

Berges, Tetradis and Wetterich: hep-ph/0005122

Calculation

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} Tr\left[\left(\frac{\delta^2 \Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} \partial_t R_k\right]$$

$$\Gamma_k^{(2)} = \Gamma_{k,0}^{(2)} - V_k \quad ; \qquad G_{k,0} = \frac{1}{\Gamma_{k,0}^{(2)} + R_k},$$

$$(\Gamma_{k,0}^{(2)} + R_k - V_k)^{-1} = -G_{k,0}(1 + V_k G_{k,0} + V_k G_{k,0} V_k G_{k,0} + \ldots).$$

$$\Gamma_{k,0}^{(2)} = k^D \begin{pmatrix} 0 & (-i\omega + \alpha_1' q^2 - k^2 \alpha_1' \tilde{\mu}_1) & 0 & k^2 \alpha_1' \tilde{m} \\ (i\omega + \alpha_1' q^2 - k^2 \alpha_1' \tilde{\mu}_1) & 0 & k^2 \alpha_1' \tilde{m} & 0 \\ 0 & k^2 \alpha_1' \tilde{m} & 0 & (-i\omega + \alpha_2' q^2 - k^2 \alpha_1' \tilde{\mu}_2) \\ k^2 \alpha_1' \tilde{m} & 0 & (i\omega + \alpha_2' q^2 - k^2 \alpha_1' \tilde{\mu}_2) & 0 \end{pmatrix}.$$

$$Tr[\ \ldots\ ] \equiv \int\!\!\int \frac{dq^D}{(2\pi)^D} \frac{d\omega}{(2\pi)} Tr[\ \ldots\ ].$$

$\Gamma_k(\phi, \lambda_i, \mu_i) = \sum_i g_i(k) O_i(\phi)$ $\quad \partial_t \Gamma_k \rightarrow \sum_i \partial_t g_i(k) * O_i(\phi) \rightarrow$ $\beta_i(k)$ asociadas al $\{O_i\}$ operator basis.

Beta Funtions: $\beta_i(k) = \partial_t g_i(k)$

Fixed Points Conditions $\partial_t \Gamma_k^* \cong 0 \quad y \quad t = \ln(\frac{k}{\Lambda})$

Flow equation 10 parameter

Beta Funtions: $g_i(k) = (\mu_i, \lambda_{1:11}, \lambda_{2:22}, \lambda_{1:12}, \lambda_{1:22}, \lambda_{2:21}, \lambda_{2:11})$

Dimensionaless variable

$$\tilde{\psi} = \sqrt{Z_1} k^{-D/2} \psi \;,\; \tilde{\chi} = \sqrt{Z_2} k^{-D/2} \chi \;,\; \tilde{V} = \frac{V}{\alpha'_1 k^{D+2}}$$

$$\tilde{m} = \frac{m}{\sqrt{Z_1 Z_2 \alpha'_1 k^2}} \;,\; \tilde{\mu}_1 = \frac{\mu_1}{Z_1 \alpha'_1 k^2} \;,\; \tilde{\mu}_2 = \frac{\mu_2}{Z_2 \alpha'_1 k^2}$$

$$\tilde{\lambda}_1 = \frac{\lambda_1 k^{D/2}}{Z_1^{3/2} \alpha'_1 k^2} \;,\; \tilde{\lambda}_2 = \frac{\lambda_2 k^{D/2}}{Z_2^{3/2} \alpha'_1 k^2} \;,\; \tilde{\lambda}_{3,6} = \frac{\lambda_{3,6} k^{D/2}}{Z_1 \sqrt{Z_2} \alpha'_1 k^2} \;,\; \tilde{\lambda}_{4,5} = \frac{\lambda_{4,5} k^{D/2}}{Z_2 \sqrt{Z_1} \alpha'_1 k^2}$$

$$\eta_i = -\frac{1}{\alpha'_i} \partial_t \alpha'_i \quad ; \quad \xi_i = -\frac{1}{Z_i} \partial_t Z_i \quad ; \quad \dot{r} = r(\xi_2 - \xi_1)$$

Betas Functions and stability matrix

$$
\begin{aligned}
\dot{\mu}_1 &= (-2+\xi_1+\eta_1)\mu_1 + \textstyle\sum_{i,j}\lambda_i\lambda_j f_{1,ij}\\
\dot{\mu}_2 &= (-2+\xi_1+\eta_2)\mu_2 + \textstyle\sum_{i,j}\lambda_i\lambda_j f_{2,ij}\\
\dot{m} &= (-2+\xi_1+\tfrac{1}{2}(\eta_1+\eta_2))m + \textstyle\sum_{i,j}\lambda_i\lambda_j f_{m,ij}\\
\dot{\lambda}_1 &= (-2+\tfrac{D}{2}+\xi_1+\tfrac{3}{2}\eta_1)\lambda_1 + \textstyle\sum_{i,j,k}\lambda_i\lambda_j\lambda_k f_{1,ijk}\\
\dot{\lambda}_2 &= (-2+\tfrac{D}{2}+\xi_1+\tfrac{3}{2}\eta_2)\lambda_2 + \textstyle\sum_{i,j,k}\lambda_i\lambda_j\lambda_k f_{2,ijk}\\
\dot{\lambda}_3 &= (-2+\tfrac{D}{2}+\xi_1+\eta_1+\tfrac{1}{2}\eta_2)\lambda_3 + \textstyle\sum_{i,j,k}\lambda_i\lambda_j\lambda_k f_{3,ijk}\\
\dot{\lambda}_4 &= (-2+\tfrac{D}{2}+\xi_1+\eta_2+\tfrac{1}{2}\eta_1)\lambda_4 + \textstyle\sum_{i,j,k}\lambda_i\lambda_j\lambda_k f_{4,ijk}\\
\dot{\lambda}_5 &= (-2+\tfrac{D}{2}+\xi_1+\eta_2+\tfrac{1}{2}\eta_1)\lambda_5 + \textstyle\sum_{i,j,k}\lambda_i\lambda_j\lambda_k f_{5,ijk}\\
\dot{\lambda}_6 &= (-2+\tfrac{D}{2}+\xi_1+\eta_1+\tfrac{1}{2}\eta_2)\lambda_6 + \textstyle\sum_{i,j,k}\lambda_i\lambda_j\lambda_k f_{6,ijk}\\
\dot{r} &= r(\xi_2-\xi_1).
\end{aligned}
$$

$$M_{ij} \equiv \left|\frac{\partial\beta_i}{\partial\Lambda_j}\right|_{\Lambda=\beta*}$$

Numerical Solution

- Two decoupled Pomerons

μ_1	μ_2	m	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	r
0.1333	0.1333	0.	1.1676	1.1676	0.	0.	0.	0.	1.
0.1053	0.1053	0.	1.0001	1.0001	0.	0.	0.	0.	1.

- We reproduce the values of the critical exponents universality class of Percolation
- The convergence is under control with the increasing the local truncation

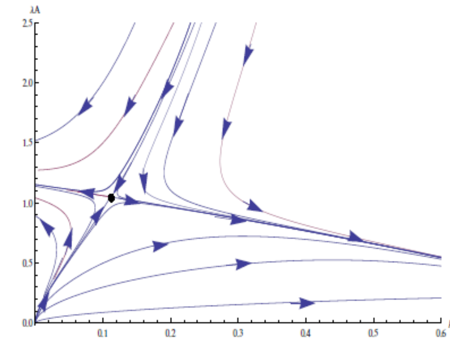
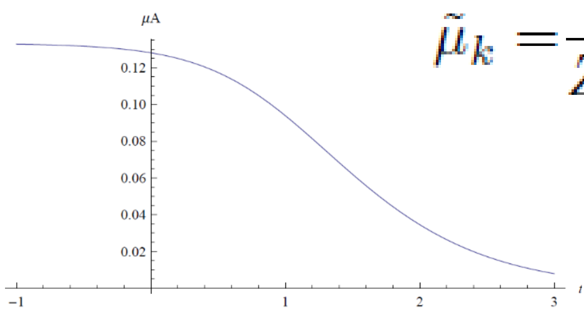
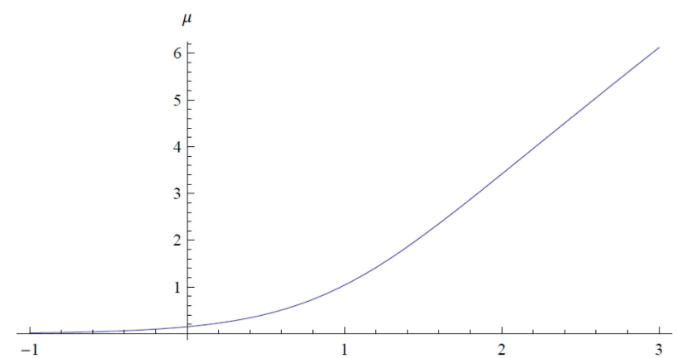


Fig.2: Some trajectories of the flow equation.

1980 Cardy y Sugar found that the RFT is in the same Universality class of "Percolation"



$$\tilde{\mu}_k = \frac{\mu_k}{Z_k \alpha'_k k^2}$$



Percolation and Monte Carlo Simulation:
The critical Exponent $\nu = 0.73$ with is related with our
$\nu = -1/($ most negative eigenvalue$)$ "
(L. Canet, B. Delamotte, N.Wschebor,…)

Numerical Solution

- Pomeron-Odderon Interaction

$\mu_1 \to \mu_1$	$\mu_2 \to \mu_2$	$\lambda \to \lambda_1$	$\lambda_2 \to \lambda_4$	$\lambda_3 \to \lambda_5$	$r \to r$
0.1111	0.1108	1.0503	1.4467	0.	0.9218

arXiv 1608.08836

- New Solution

Fixed Point IR

μ_1	μ_2	m	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	r
0.1384	0.1384	-0.0056	1.0118	1.0118	0.8150	0.8150	-0.1967	-0.1967	1.
0.1084	0.1084	-0.0042	0.8631	0.8631	0.6967	0.6967	-0.1664	-0.1664	1.

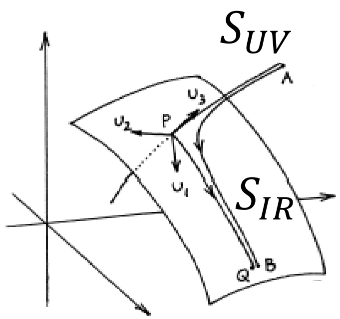
Critical Exponents θ_i of the stabilty matrix $\quad \boldsymbol{g_{k,i} = g^*{}_i + c_a e^{\theta_i t} v_i^a}$

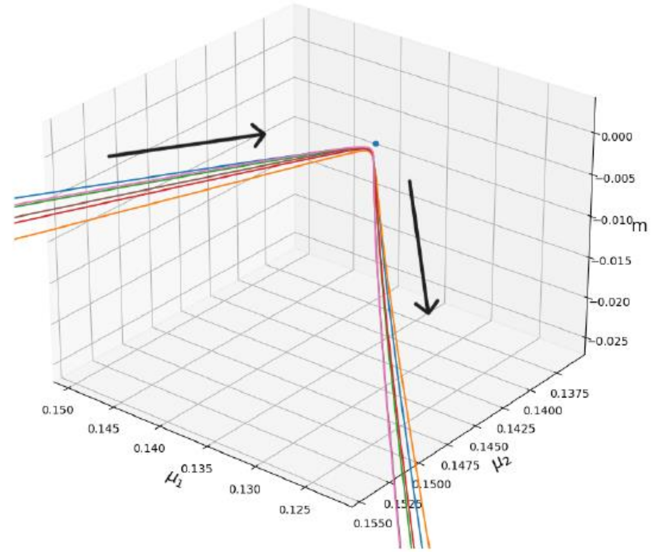
2.50	-2.23	-2.09	-2.03	1.94	1.41	0.36	-0.11	0.10	0.03

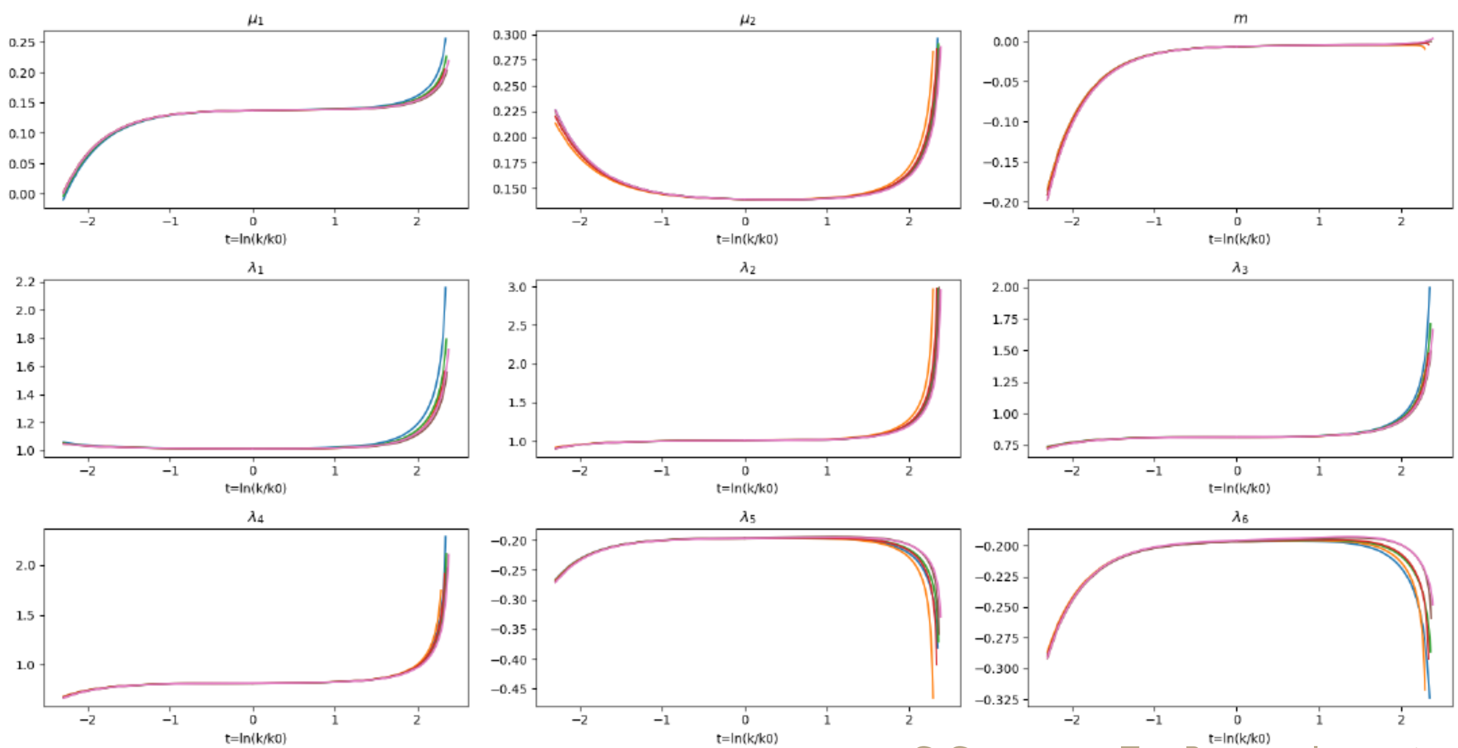
Linealizations of the Flow close to a FP:
If $\lambda_i > 0$ define a IR – Critical Surfase with relevant behaviour, where $\lambda_i < 0$ is UV

RG Trajectories at the IR Fixed point

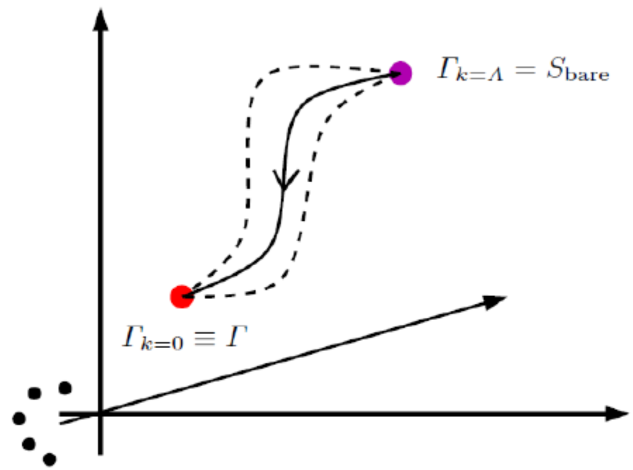
Numerical solution



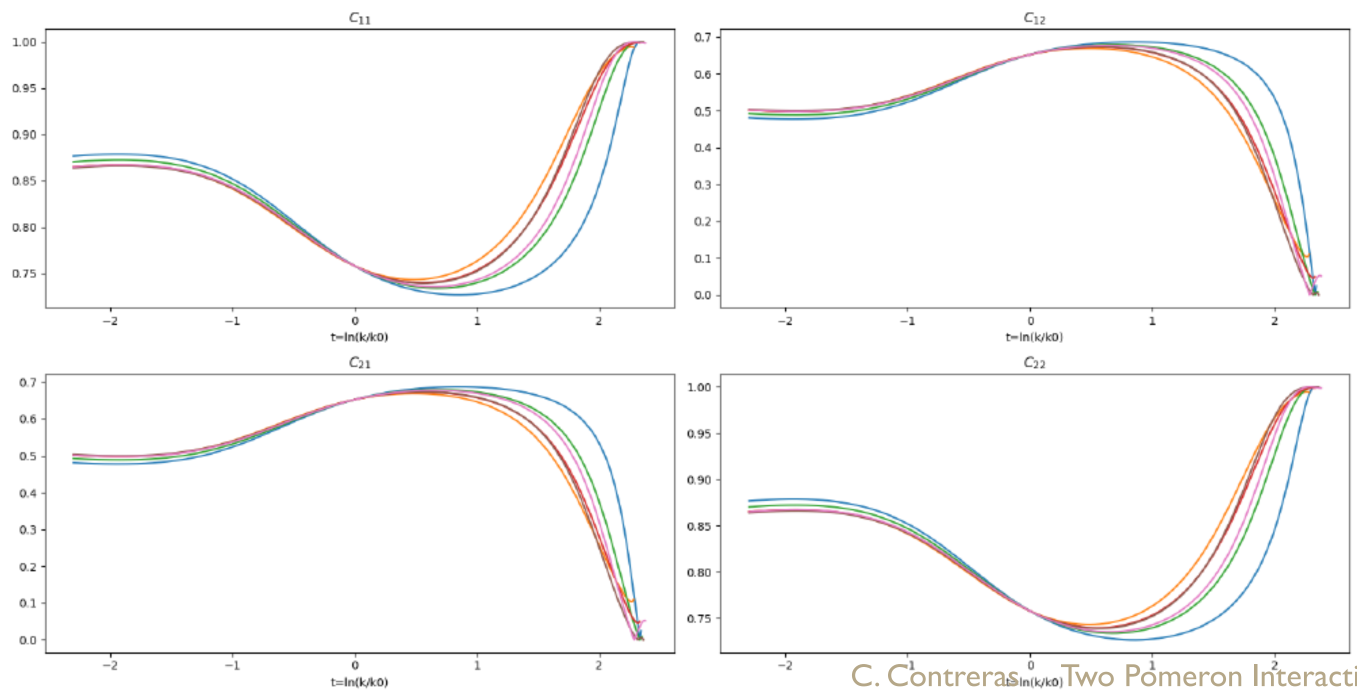




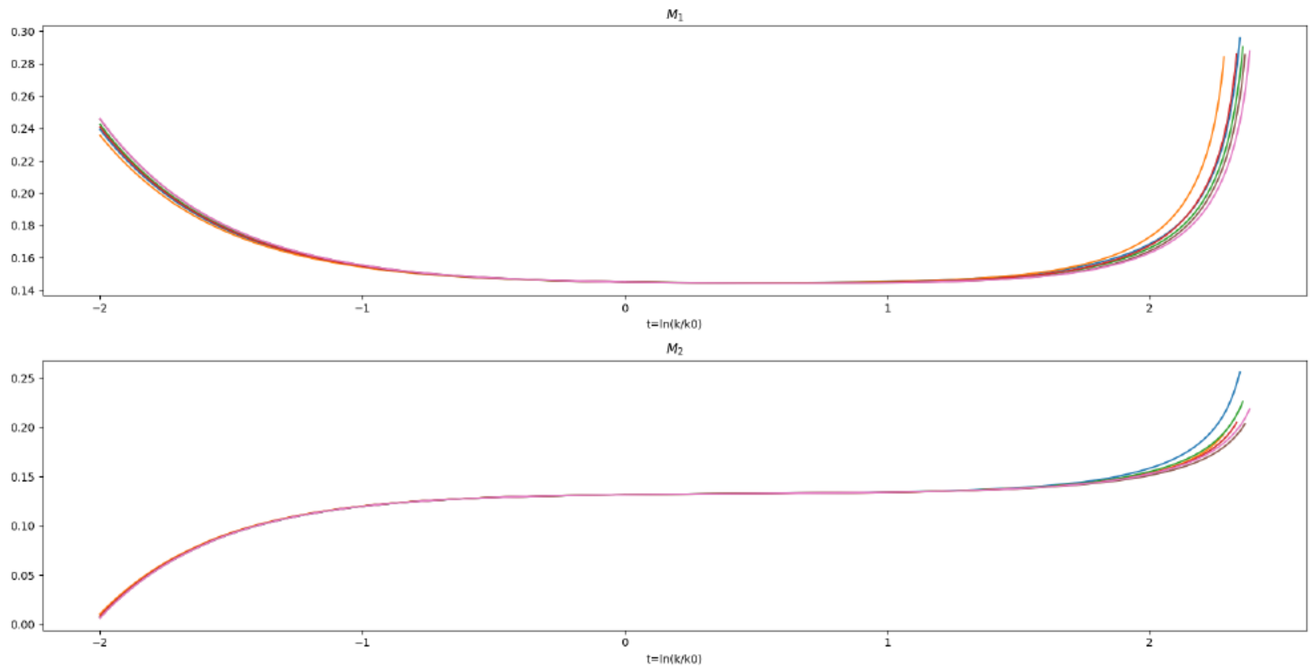
Results:



$$\psi' = C_{k,11}\psi + C_{k,12}\chi$$
$$\chi' = C_{k,21}\psi + C_{k,22}\chi,$$



Numerical evolution for the Pomeron Intercepts



Summary and outlook

- Using Numerical analysis of the RFT at the IR
- Wilsonian infrared regulator in the FRG

1. We studied the interaction of two Pomeron with triple Pomeron vertex

2. Can interpreted that the Pomeron really mixed between the two BFKL discrete Pomeron around the IR fixed point

3. The final states are intercept eigenstate of the 2 mixed discrete Pomeron

4. The intercept ω_n of the discrete n Pomeron which have a significant contribution to the gluon density at HERA now has a k-dependence $\omega_{n.k}$, and we need to study its effect.

In the future:

Extension to high order Pomeron vertex
N-Pomeron Interaction
N-Pomeron Interaction in diagonal basis

Thank you