Two Pomeron Interaction

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Fondecyt 1231829

ISDM 2023, 52nd International Symposium on Multiparticle Dynamics
21 - 26 August, Gyongyos, Hungary
Outline

- Introduction
- N-Pomeron fields
- FRG approximation and interaction of Pomeron
- Numerical results for 2 Pomeron Interaction
- Summary and Outlook

JHEP 1603 (2016) 201
PRD 95 (2017) 014013
Two Pomeron: arXiv to appear
Introduction

- Scattering process: description in terms of QCD (parton distributions)
  Strong Interaction

- In the Regge Limit \((s > t, t\)-channel exchange dominate) Diffractive process, then becomes an effective 2+1 dimensional: transversal space and rapidity (Lipatov effective action)

- At very small transverse distances pQCD and BFKL Pomeron (1958)

- At large transverse distances and small \(x\) nonperturbative QCD BK/JIMWLK approach

  see talk: C. Royon and M. Sadzikowsli

- There are another states with 3, 4 gluons…. BKP Equation

  1980 J. Bartels; J. Kwiecinski and M. Praszalowicz

- Another option: RFT Reggeon Field Theory description Gribov
Soft Pomeron Reggeon Field Theory before QCD

- V.N. Gribov introduce in the 60’s
- Scattering amplitude at high energies for hadrons is according Regge Theory
- The exchange are “quasi particles” characterized by its Regge trajectories: \( \alpha_i(t) \)
- Leading Pole: is Called Pomeron with vacuum quantum numbers
  \[ \alpha(t) = \alpha_0 + \alpha' t = 1 + (\alpha_0 - 1) + \alpha' t \]
- \( \mu = \alpha_0 - 1 \) is the Pomeron intercept and \( \alpha' \) is the slope
- According to the Regge theory the contribution to the total Cross section, is given by:
  \[ \sigma_T = A_i s^{\alpha_i(0) - 1} \]
  A. Donnachie and Landshoff: 1992

- \( \mu = \alpha_0 - 1 = 0.08 \) and \( \alpha' = 0.25 \text{ GeV} \)
- \( \alpha_P(t) = 1.08 + 0.25(\text{GeV}^{-2})t \)

See Regge Pole analysis
Talk Poblaguev
For Hard processes short Transversal distances

\[ p + p \rightarrow p + p \]

we consider \( \text{pQCD} \rightarrow \text{Hard Pomeron} \) dominate scattering

The BFKL Pomeron which has been studied up to NLO in perturbation theory is a composite states of Reggeized gluons.

Resumation of the Ladder at leading log approximation (multi Regge Kinemmatics MRK)

The intercept of the Pomeron is related with the eigenvalues of the BFKL Kernel

\[
\omega_0(\gamma) = \alpha_s N_c / \pi \left[ 2 \Psi(1) - \Psi(\gamma) - \Psi(1 - \gamma) \right]
\]

\[
\Psi(\gamma) = \frac{d}{d\gamma} \ln \Gamma(\gamma) \quad \text{Digamma function}
\]

\[
\alpha_P(0) = \omega_0(1/2) \approx 1 + 4 \frac{\alpha_s N_c}{\pi} \ln 2 = 1 + 2.648 \alpha_s \quad \text{Hard BFKL/QCD}
\]

\[
\alpha_P(0) \approx 1 + 0.5296 \text{ for } \alpha_s = 0.2
\]

we can connect Hard –Soft Pomeron regions of different sizes and different sorts of Pomeron using Functional Renormalization Group
A(s,k,k') is the amplitude for the scattering of a gluon with transverse momentum \(k\) off another gluon with transverse momentum \(k'\) at center of mass energy \(\sqrt{s}\) and it is found to obey an evolution equation

\[
\frac{\partial}{\partial Y} A(k, k', Y) = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 q}{2 \pi (k-q)^2} \left( A(q, k, Y') - \frac{k^2}{2 q^2} A(k, k', Y) \right)
\]

The kernel is obtained by summing all graphs which contribute an effective “gluon ladder”. Using a Mellin Transformation the BFKL evolution equation can be solved in terms of the \(\phi_\omega(k)\) eigenfunctions of the Kernel

\[
\omega \phi_\omega(k) = \bar{\alpha} \int \frac{d^2 k'}{2 \pi} \tilde{K}(k, k') \phi_\omega(k')
\]

\(\tilde{K}(k, k')\) is the BFKL kernel

Scattering process can be described using BFKL Green Function : \(G(t, t', Y)\)

\[
A(x, Q^2) = \int dt \, dt' \, \Phi_\gamma(Q^2, t) \, G(t, t', Y) \, \Phi_p(t') \quad \text{where} \quad Y = \ln(1/x) \quad \text{Rapidity}
\]

\(\Phi_\gamma(Q^2, t)\) describe the coupling of the gluon (perturbatively calculable) with transverse momentum \(k\) to a photon of virtuality \(Q^2\) and \(\Phi_p(t)\) describes the coupling of a gluon of transverse momentum \(k'\) to the target proton
In more general form we define the BFKL Greens functions $G(q', q - q'; q'', q'' - q|\omega)$ as an infinite sum of ladder diagram and satisfies the integral equation: (Bethe Salpeter resummation)

$$G(q', q - q'; q'', q'' - q|\omega) = K_{BFKL}(q,q - q'; q'', q'' - q) + \int d^2 k K_{BFKL}(q,q - q'; k,q - k) \frac{1}{k^2(q-k)^2} \frac{1}{\omega - \omega_g(k) - \omega_g(q-k)} G(k,q - k; q'', q'' - q|\omega)$$

Where the wavy denote the Reggeized Gluons:

$$\frac{1}{k^2} \frac{1}{\omega - \omega_g(k^2)} .$$

And the real part

$$K_{BFKL}(q,q - q'; q'', q'' - q) = \frac{\alpha_s}{2\pi} \left(-q^2 + \frac{q''(q-q')^2}{(q' - q'')^2} + \frac{q'^2(q'' - q'')^2}{(q' - q'')^2}\right),$$

Virtual part (Gluon trajectory)

$$\omega_g(q^2) = -\frac{q^2 \tilde{\alpha}_s}{4\pi} \int d^2 k \frac{1}{k^2(q-k)^2}$$

One can reorganize this expression and the final BFKL kernel is given by

$$\bar{K}(q, q - q'', q - q'') = K(q, q - q'; q'', q'' - q) + \delta^{(2)}(q' - q'') \left(\omega_g(q') + \omega_g(q - q')\right)$$
BFKL regularized equation and N-Pomeron

In the presence of an infrared cutoff and with running $\alpha_s$ the piece of the $\omega -$cut between $\omega = \omega_0$ and zero is replaced by an infinite sequence of discrete poles, which accumulate at zero.

We need IR regulator in the propagator in order to study the ladder diagram in the NLO.

**BFKL with IR regulator it not new:**

- Lipatov 1986
- Braun and Vacca 1999
- Levin, Lipatov and Siddikov 2014
- Kowalski, Lipatov, Ross 2014
- Bartels, Contreras and Vacca 2019

- **IR Regulator**
- **bootstrap conditions**
- **mass regulator**
- **boundary conditions**
- **Wilsonian or ERG regulator**

We introduce the following momentum regulator for the propagator for the Gluon

$$
\frac{1}{q^2} \to \frac{1}{[q^2+R_k(q^2)]}
$$

**Wilsonian Regulator**

$$
R_k(q^2) = (k^2 - q^2) \theta(k^2 - q^2)
$$

**Trajectory Gluons**

$$
\omega_{gk}(q^2) = -q^2 \frac{\bar{\alpha}_s}{4\pi} \int d^2 k \frac{1}{k^2(q-k)^2}
$$

**BFKL Kernel**

$$
K_{\text{BFKL}}(q', q-q'; q'', q-q'') = \frac{\bar{\alpha}_s}{2\pi} \left( -q^2 \frac{(q'-q'')^2}{(q'-q'')^2 + R_k(q'-q'')^2} + \frac{q''^2(q''-q')^2}{(q'-q'')^2 + R_k(q'-q'')^2} + \frac{q'^2(q'-q'')^2}{(q'-q'')^2 + R_k(q'-q'')^2} \right)
$$
What is important this result

BFKL kernel in the color singlet state of the t-channel

✓ the spectral decomposition with the Wilsonian IR regulated BFKL kernel and with running coupling constant is discrete.

\[ G_k = \frac{1}{\omega - \bar{K}} = \sum_n \frac{\psi_{n,k}(q', q - q')\psi_{n,k}(q'', q - q'')}{\omega - \omega_{n,k}} + \text{continuous part}, \]

\[ \omega \phi_\omega (k) = \int \frac{d^2k'}{2\pi} \overline{\alpha}(k, k') \bar{K}(k, k') \phi_\omega (k') \]

- Numerical Solution: lattice formulation with \( N = 600 \)
- Eigenvalues
- Eigenfunctions: bound states n-Pomeron fields

**Slope:** we leave the forward direction and consider the \( q^2 \) dependence of the eigenvalues and the slope is

\[ \alpha (q^2) = \omega (q^2) \sim \omega^0 + q^2 \alpha' \quad \alpha' = \frac{d\omega}{dq^2} \quad \text{when} \quad q^2 \to 0 \]

\[ \omega_n(q^2) = \omega_n^{(0)} + q^2 \frac{\int d^2k \int d^2k' f_n(k'^2) \left[ K^{(1)}(k, k') + 2\delta^{(2)}(k - k')\omega_n^{(1)}(k^2) \right] f_n(k^2)}{\int d^2k|f_n(k^2)|^2} \]
Numerical Results NLO Pomeron

\[ r_n = \langle \ln q^2 \rangle = \frac{\int d^2 k |\psi_n(k)|^2 \ln k^2}{\int d^2 k |\psi(k)|^2} \]

<table>
<thead>
<tr>
<th>n</th>
<th>k = 0.54</th>
<th>k = 1</th>
<th>k = 5</th>
<th>energy</th>
<th>slope</th>
<th>radius</th>
</tr>
</thead>
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<tr>
<td>1</td>
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<td>-0.43</td>
<td>-0.29</td>
<td>0.17</td>
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<td>-0.17</td>
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<td>0.00012</td>
</tr>
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</tr>
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</tr>
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<td>-0.067</td>
<td>0.0042</td>
<td>0.0011</td>
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<tr>
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<td>-0.064</td>
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<td>0.0040</td>
<td>0.00083</td>
<td>0.000026</td>
</tr>
<tr>
<td>8</td>
<td>-0.058</td>
<td>-0.056</td>
<td>-0.052</td>
<td>0.0023</td>
<td>0.00064</td>
<td>0.000021</td>
</tr>
</tbody>
</table>

Table 4: Numerical values for eigenvalues, slopes and radii, with running coupling and for different values of the IR cutoff \( k \)

Results independent of the regulator

see J. Bartels, C. Contreras G. P. Vacca  JHEP 1901(2019) 004
Wave Function Pomeron

The support of the Wave function with $n > 2$, is defined in the UV region

$N=1$ is concentrated in the soft region

Figure 22: three leading wavefunctions (No 1,2,5) as a function of $\ln q^2$.

Figure 24: three leading wavefunctions (No 1,2,5) for the scale $k = 1$GeV, as a function of $\ln q^2$.

Figure 25: three leading wavefunctions (No 1,2,5) for the scale $k = 5$GeV, as a function of $\ln q^2$. 
How we can test this N - Pomeron

- Ellis, Kowalski and Ross: Using the N-BFKL Pomeron started to fit the small-x and low-$Q^2$ HERA data $F_2$


- All these approaches only use the contribution of the 3 discrete BFKL Pomeron
- No attempt has been made to introduce the triple Pomeron vertex and to study its QCD effect.
- In the field theory based upon reggeized gluons these vertex functions would lead to an extremely hard problem to solve (see talk. Sadzikowsli)
- To study the interactions of these Regge poles and in order to be able to move from larger to small distances it will be convenient to consider the local approximation and to make use of the well-known formalism of RFT
- BFKL Pomeron into a reggeon field theory which includes corrections to the BFKL Pomeron based upon interaction vertices, in particular the triple Pomeron vertex

$$
\Gamma_k[\psi_1^\dagger, \psi_i] = \int d^Dx d\tau [Z_i(\frac{1}{2} \psi_i^\dagger \hat{\partial}\tau \psi_i - \alpha_i^\dagger \psi_i^\dagger \nabla^2 \psi_i) + m_i(\psi_i^\dagger \psi_i^\dagger + \psi_i^\dagger \psi_i) - \mu_i \psi_i^\dagger \psi_i - V_k[\psi_i^\dagger, \psi_i; \lambda_j]],
$$

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Wetterich Equation 93

\[ \partial_t \Gamma_k[\phi] = \frac{1}{2} Tr \left[ \left( \frac{\delta^2 \Gamma_k[\phi]}{\delta \phi \delta \phi} + R_k \right)^{-1} \partial_t R_k \right] \]

Action \( \Gamma_k(\psi_i, \psi_i^+, \lambda_i) \)
Regulator \( R_k(q) \)
Initial Condition \( \Gamma_{k=\Lambda}(\phi, g_i) \)
Interaction \( V = \sum \lambda_{i,j,k} (\psi_i^+ \psi_j \psi_k + \text{cc}) \)

\[ R_1 = Z_1 \alpha'_1 (k^2 - q^2) \Theta(k^2 - q^2) \]
\[ R_2 = Z_2 \alpha'_2 (k^2 - q^2) \Theta(k^2 - q^2) = r Z_2 \alpha'_1 (k^2 - q^2) \Theta(k^2 - q^2), \]

\[ r = \frac{\alpha'_2}{\alpha'_1} \]
Calculation

\[
\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr}\left[ \left( \frac{\partial^2 \Gamma_k[\phi]}{\partial \phi \partial \phi} + R_k \right)^{-1} \partial_t R_k \right]
\]

\[
\Gamma_k^{(2)} = \Gamma_{k,0}^{(2)} - V_k ; \quad G_{k,0} = \frac{1}{\Gamma_{k,0}^{(2)} \mid R_k} ,
\]

\[
(\Gamma_{k,0}^{(2)} + R_k - V_k)^{-1} = -G_{k,0}(1 + V_k G_{k,0} + V_k G_{k,0} V_k G_{k,0} + ...).
\]

\[
\Gamma_{k,0}^{(2)} = k^D \begin{pmatrix}
0 & (-i\omega + \alpha' q^2 - k^2 \alpha' \tilde{m}_1) & 0 & k^2 \alpha' \tilde{m} \\
(i\omega + \alpha' q^2 - k^2 \alpha' \tilde{m}_1) & 0 & k^2 \alpha' \tilde{m} & 0 \\
k^2 \alpha' \tilde{m} & 0 & k^2 \alpha' \tilde{m} & 0 \\
k^2 \alpha' \tilde{m} & 0 & (i\omega + \alpha' q^2 - k^2 \alpha' \tilde{m}_2) & 0
\end{pmatrix}.
\]

\[
\text{Tr}[... ] \equiv \int \int \frac{dq^D}{(2\pi)^D} \frac{d\omega}{(2\pi)} \text{Tr}[... ].
\]

\[
\Gamma_k(\phi, \lambda_i, \mu_i) = \sum_i g_i(k) O_i(\phi) \quad \partial_t \Gamma_k \rightarrow \sum_i \partial_t g_i(k) * O_i(\phi) \rightarrow \beta_i(k) \text{ asociadas al } \{ O_i \} \text{ operator basis.}
\]

**Beta Functions:**

\[
\beta_i(k) = \partial_t g_i(k)
\]

**Fixed Points Conditions**

\[
\partial_t \Gamma_k^* \cong 0 \quad y \quad t = \ln\left(\frac{k}{\Lambda}\right)
\]
Flow equation 10 parameter

Beta Functions:

\[ g_i(k) = (\mu_i, \lambda_{1:11}, \lambda_{2:22}, \lambda_{1:12}, \lambda_{1:22}, \lambda_{2:21}, \lambda_{2:11}) \]

Dimensionless variable

\[ \tilde{\psi} = \sqrt{Z_1} k^{-D/2} \psi, \quad \tilde{\chi} = \sqrt{Z_2} k^{-D/2} \chi, \quad \tilde{V} = \frac{V}{\alpha'_1 k^{D+2}} \]

\[ \tilde{m} = \frac{m}{\sqrt{Z_1 Z_2} \alpha'_1 k^2}, \quad \tilde{\mu}_1 = \frac{\mu_1}{Z_1 \alpha'_1 k^2}, \quad \tilde{\mu}_2 = \frac{\mu_2}{Z_2 \alpha'_1 k^2} \]

\[ \tilde{\lambda}_1 = \frac{\lambda_1 k^{D/2}}{Z_1^{3/2} \alpha'_1 k^2}, \quad \tilde{\lambda}_2 = \frac{\lambda_2 k^{D/2}}{Z_2^{3/2} \alpha'_1 k^2}, \quad \tilde{\lambda}_{3,6} = \frac{\lambda_{3,6} k^{D/2}}{Z_1 \sqrt{Z_2} \alpha'_1 k^2}, \quad \tilde{\lambda}_{4,5} = \frac{\lambda_{4,5} k^{D/2}}{Z_2 \sqrt{Z_1} \alpha'_1 k^2} \]

\[ \eta_i = -\frac{1}{\alpha'_i} \partial_t \alpha'_i \quad ; \quad \xi_i = -\frac{1}{Z_i} \partial_t Z_i \quad ; \quad \dot{r} = r(\xi_2 - \xi_1) \]
Betas Functions and stability matrix

\[ \dot{\mu}_1 = (-2 + \xi_1 + \eta_1) \mu_1 + \sum_{i,j} \lambda_i \lambda_j f_{1,ij} \]
\[ \dot{\mu}_2 = (-2 + \xi_1 + \eta_2) \mu_2 + \sum_{i,j} \lambda_i \lambda_j f_{2,ij} \]
\[ \dot{m} = (-2 + \xi_1 + \frac{1}{2}(\eta_1 + \eta_2)) m + \sum_{i,j} \lambda_i \lambda_j f_{m,ij} \]
\[ \dot{\lambda}_1 = (-2 + \frac{D}{2} + \xi_1 + \frac{3}{2} \eta_1) \lambda_1 + \sum_{i,j,k} \lambda_i \lambda_j \lambda_k f_{1,ijk} \]
\[ \dot{\lambda}_2 = (-2 + \frac{D}{2} + \xi_1 + \frac{3}{2} \eta_2) \lambda_2 + \sum_{i,j,k} \lambda_i \lambda_j \lambda_k f_{2,ijk} \]
\[ \dot{\lambda}_3 = (-2 + \frac{D}{2} + \xi_1 + \eta_1 + \frac{1}{2} \eta_2) \lambda_3 + \sum_{i,j,k} \lambda_i \lambda_j \lambda_k f_{3,ijk} \]
\[ \dot{\lambda}_4 = (-2 + \frac{D}{2} + \xi_1 + \eta_2 + \frac{1}{2} \eta_1) \lambda_4 + \sum_{i,j,k} \lambda_i \lambda_j \lambda_k f_{4,ijk} \]
\[ \dot{\lambda}_5 = (-2 + \frac{D}{2} + \xi_1 + \eta_2 + \frac{1}{2} \eta_1) \lambda_5 + \sum_{i,j,k} \lambda_i \lambda_j \lambda_k f_{5,ijk} \]
\[ \dot{\lambda}_6 = (-2 + \frac{D}{2} + \xi_1 + \eta_1 + \frac{1}{2} \eta_2) \lambda_6 + \sum_{i,j,k} \lambda_i \lambda_j \lambda_k f_{6,ijk} \]
\[ \dot{r} = r(\xi_2 - \xi_1) . \]

\[ M_{ij} \equiv \left| \frac{\partial \beta_i}{\partial \lambda_j} \right|_{\lambda=\beta^*} \]
Numerical Solution

- Two decoupled Pomerons

<table>
<thead>
<tr>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>$m$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$\lambda_3$</th>
<th>$\lambda_4$</th>
<th>$\lambda_5$</th>
<th>$\lambda_6$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
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<td>0.1333</td>
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<td>0</td>
<td>1.1676</td>
<td>1.1676</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.</td>
</tr>
<tr>
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<td>1.0001</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.</td>
</tr>
</tbody>
</table>

- We reproduce the values of the critical exponents universality class of Percolation
- The convergence is under control with the increasing the local truncation

1980 Cardy y Sugar found that the RFT is in the same Universality class of “Percolation”

$\tilde{\mu}_k = \frac{\mu_k}{Z_k \alpha'_k k^2}$

Percolation and Monte Carlo Simulation:
The critical Exponent $\nu = 0.73$ with is related with our
$\nu = -1/(\text{most negative eigenvalue})$”
(L. Canet, B. Delamotte, N. Wschebor, …)
Numerical Solution

- Pomeron-Odderon Interaction

\[
\begin{array}{cccccccc}
\mu_1 \rightarrow \mu_1 & \mu_2 \rightarrow \mu_2 & \lambda \rightarrow \lambda_1 & \lambda_2 \rightarrow \lambda_4 & \lambda_3 \rightarrow \lambda_5 & r \rightarrow r \\
0.1111 & 0.1108 & 1.0503 & 1.4467 & 0. & 0.9218 \\
\end{array}
\]

- New Solution

Fixed Point IR

\[
\begin{array}{cccccccccc}
\mu_1 & \mu_2 & m & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6 & r \\
0.1384 & 0.1384 & -0.0056 & 1.0118 & 1.0118 & 0.8150 & 0.8150 & -0.1967 & -0.1967 & 1. \\
0.1084 & 0.1084 & -0.0042 & 0.8631 & 0.8631 & 0.6967 & 0.6967 & -0.1664 & -0.1664 & 1. \\
\end{array}
\]

Critical Exponents $\theta_i$ of the stability matrix

\[
g_{k,i} = g^*_i + c_a e^{\theta_i} t v_i^a
\]

Linealizations of the Flow close to a FP:
If $\lambda_i > 0$ define a IR – Critical Surfase with relevant behaviour, where $\lambda_i < 0$ is UV
RG Trajectories at the IR Fixed point

Numerical solution

\[ S_{UV} \]

\[ S_{IR} \]

\[ S \]

\[ U \]

\[ \mu \]

\[ \lambda \]

\[ m \]

\[ \tilde{\lambda} \]

\[ \tilde{S} \]

\[ \tilde{U} \]

\[ \tilde{\mu} \]

\[ \tilde{\lambda} \]

\[ \tilde{m} \]

\[ \tilde{\tilde{\lambda}} \]

\[ \tilde{\tilde{S}} \]

\[ \tilde{\tilde{U}} \]

\[ \tilde{\tilde{\mu}} \]
Results:

\[ \psi' = C_{k,11} \psi + C_{k,12} \chi \]
\[ \chi' = C_{k,21} \psi + C_{k,22} \chi, \]
Numerical evolution for the Pomeron Intercepts
Summary and outlook

1. We studied the interaction of two Pomeron with triple Pomeron vertex

2. Can interpreted that the Pomeron really mixed between the two BFKL discrete Pomeron around the IR fixed point

3. The final states are intercept eigenstate of the 2 mixed discrete Pomeron

4. The intercept $\omega_n$ of the discrete n Pomeron which have a significant contribution to the gluon density at HERA now has a k-dependence $\omega_{n,k}$, and we need to study its effect.

In the future:
Extension to high order Pomeron vertex
N-Pomeron Interaction
N-Pomeron Interaction in diagonal basis
Thank you