Structure of the real amplitude in forward pp scattering at the LHC

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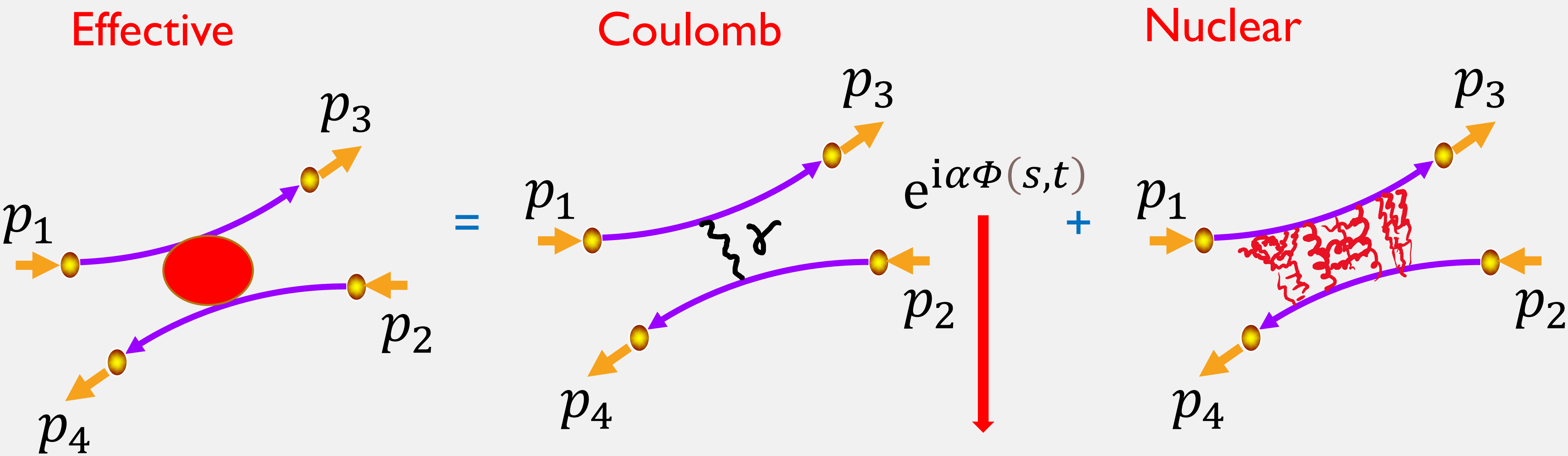
Some challenges:

- Seemingly simple kinematics but complicated dynamics
- Non-perturbative phenomenon
- Long range force interplays with short ranged
- Experimental gap among energies
- Differential cross sections with fluctuations (and/or) large uncertanties
- Lack of cross-symmetric experiments at the same energies

Relativistic Elastic Scattering



Coulomb phase

H. Bethe, Ann. Phys. (N.Y.) 3, 190 (1958) 27;
L.D. Solov'ev, JETP 22, 205 (1966) 26;
G.B. West, D.R. Yennie, Phys. Rev. 172 (5), 1413 (1968);
V. Kundrat and M. Lokajcek, Phys. Lett. B 611 (2005) 102 ;
R. Cahn, Z. Phys. C 15 (1982) 253.

Mandelstam variables

$$s = (p_1 + p_2)^2$$

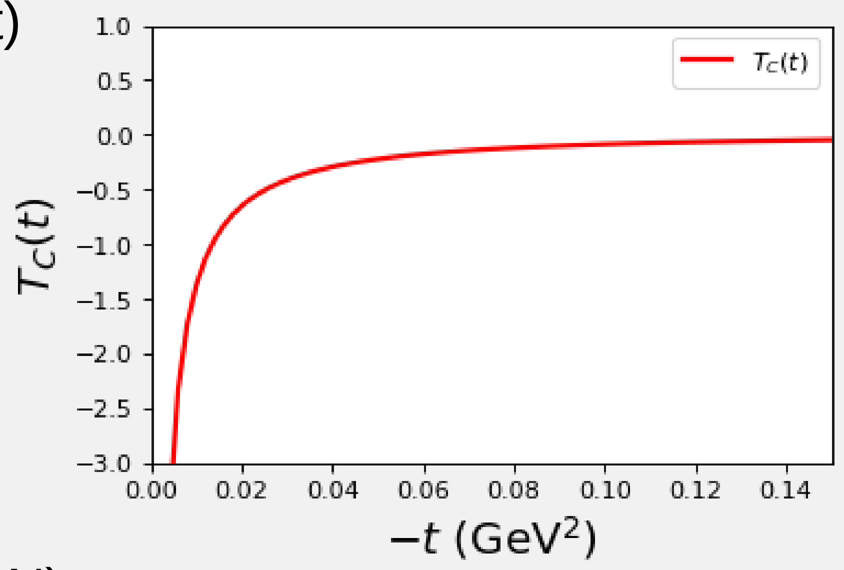
$$t = (p_1 - p_3)^2$$

$$u = (p_1 - p_4)^2$$

Scattering amplitudes

Coulomb part (purelly real – energy independent)

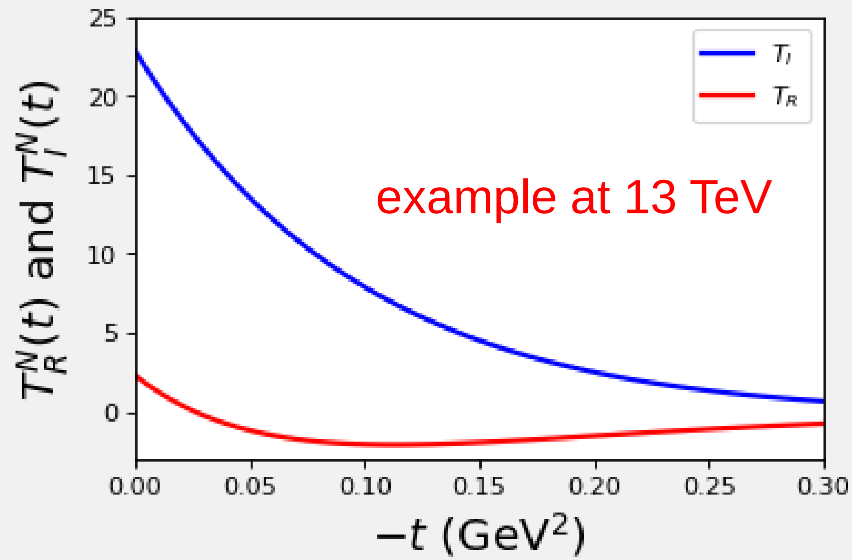
$$T_c(t) = \mp \frac{2\alpha}{|t|}\left(\frac{\Delta^2}{\Delta^2 + |t|}\right)^2$$



Complex nuclear part (FORWARD DESCRIPTION)
(different t dependences for real and imaginary parts)

$$T_R^N(t) = \frac{\sigma(\rho - \mu_R t)e^{B_R t/2}}{4\sqrt{\pi}(\hbar c)^2}$$

$$T_I^N(t) = \frac{\sigma(1 - \mu_I t)e^{B_I t/2}}{4\sqrt{\pi}(\hbar c)^2}$$



AKK, E. Ferreira, T. Kodama and M. Rangel, Eur. Phys. J. C 77 (2017) 12, 877

Basic physical quantities we are interested in:

Forward quantities (t=0)

Optical theorem $\sigma_{tot} = 4\pi(\hbar c)^2 T_I^N(s,0)$

Ratio of real and imaginary amplitudes $$\rho = \frac{T_R^N(s,0)}{T_I^N(s,0)}$$

effective slopes

$$B_I^{eff} = \frac{2}{T_I^N(s,t)}\frac{\mathrm{d}}{\mathrm{d}t}T_I^N(s,t)\Big|_{t=0} = (B_I - 2\mu_I)$$

$$B_R^{eff} = \frac{2}{T_R^N(s,t)}\frac{\mathrm{d}}{\mathrm{d}t}T_R^N(s,t)\Big|_{t=0} = \left(B_R - \frac{2\mu_R}{\rho}\right)$$

$$B_I^{eff} = \frac{\rho^2\left(B_R - \frac{2\mu_R}{\rho}\right) + (B_I - 2\mu_I)}{\rho^2 + 1}$$

Differential cross section (s,t) dependent

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = |T(s,t)|^2$$

Why the parameters μ_R and μ_I ?

μ_I

The dip of the $\frac{d\sigma}{dt}$ is due to the passage of the imaginary amplitude through zero

$(1-\mu_I t_{dip})=0$ Once μ_I is determined we know the dip position

...But this is not our concern in the present work!!!

μ_R

Theorem 1: ""*"If, for sufficiently large values of ξ, σ(ξ) is non decreasing and approaches infinity as ξ→∞. Then Re F (ξ) is positive for all sufficiently large values of ξ""* $\xi=(s-u)/2$

A. Martin and T. T. Wu, Phys. Rev. D 97 (2018) 1, 014011

Theorem 2: ""*"We show that if for fixed negative (physical) square of the momentum transfer, the differential cross-section dσ/dt tends to zero and if the total cross-section tends to infinity, When the energy goes to infinity, the real part of the even signature amplitude cannot have a constant sign near t=0.""*

A. Martin, Phys. Lett. B 404, 137 (1997).

To sumarize, the real amplitude in the forward range at high energies is positive, and at some point within the diffractive cone it crosses zero.

$$(\rho-\mu_R t_{zero})=0$$

A forward description in terms of analytic functions

Following A. Martin (1977)

$$T^N(E,t) \sim i\, C\, E(log(E) - \pi i/2)^2 f(\tau)$$

with $\tau = t\log^2(E)$ the scaling variable (real)

$E = (s-u)/4\mathrm{m}$

$f(\tau) = T^N(E,t)/\, T^N(E,0)$

geometric scaling function

A. Martin, Lett. Nuovo Cim. 7 (1973) 811.

We extended the idea to a complex scaling variable $\tau' \sim t(log(E) - \pi i/2)^2$

and as a consequence in the limit of large *s* we obtain $f(\tau') = e^{\tau'} = \mathrm{e}^{\Omega_R(s,t)+\Omega_I(s,t)}$

$$T_R^N(s,0) = s[\beta(P_1 + 2H\log s) - R_1 s^{-\eta_1}\sin(\eta_1\beta) \mp R_2 s^{-\eta_2}\cos(\eta_2\beta)]$$

$$T_I^N(s,0) = s[P + P_1\log s + H(\log^2 s - \beta^2) + R_1 s^{-\eta_1}\cos(\eta_1\beta) \pm R_2 s^{-\eta_2}\sin(\eta_2\beta)]$$

$$\Omega_R(s,t) = [b_0 + b_1\log s + b_2(\log^2 s - \beta^2) + b_3 s^{-\eta_3}\cos(\eta_3\beta)]t$$

$$\Omega_I(s,t) = -[b_1\beta + 2b_2\beta\log s - b_3 s^{-\eta_3}\sin(\eta_3\beta)]t$$

The complex amplitude is

AKK., J. Phys. G 46 (2019) 12, 125001

$$\begin{pmatrix} T_R^N(s,0) \\ T_I^N(s,0) \end{pmatrix} = s\sigma_\mp(s) \begin{pmatrix} \cos\Omega_I(s,t) & -\sin\Omega_I(s,t) \\ \sin\Omega_I(s,t) & \cos\Omega_I(s,t) \end{pmatrix} \begin{pmatrix} \rho_\mp(s) \\ 1 \end{pmatrix} \mathrm{e}^{\Omega_R(s,t)}$$

To describe the data we add to the real part a shape $G^R_{\mp}(s,t) = \sigma_{\mp} \frac{s\, t}{\Delta^2 - t} e^{\Omega_R(s,t)}$

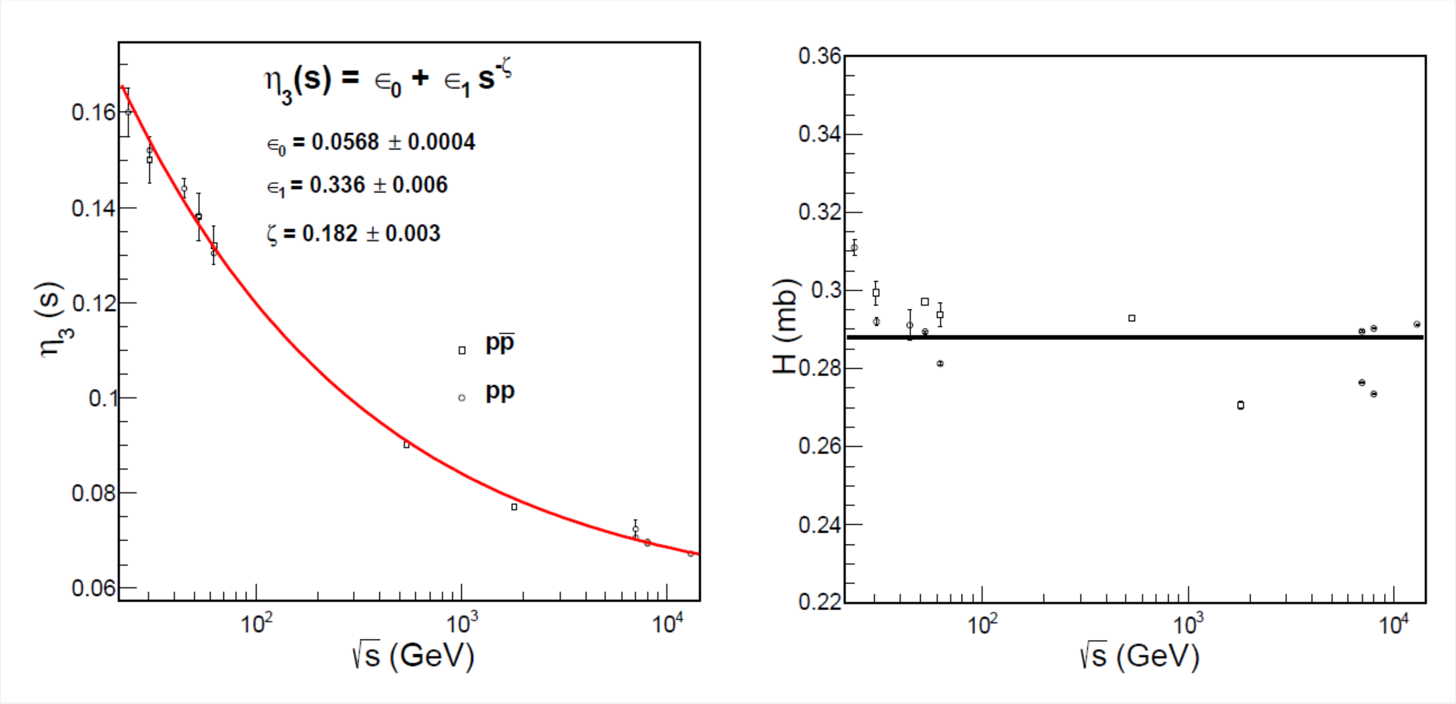
$$T^N_R(s,t) \rightarrow T^N_R(s,t) + G^R_{\mp}(s,t)$$

And the differential cross section is

$$\frac{d\sigma}{dt} = s^2\, \sigma_{\mp}{}^2(\rho^2_{\mp} + 1)e^{2\Omega_R(s,t)} + 2\, s\, \sigma_{\mp}\, G^R_{\mp}(s,t)[\rho_{\mp} \cos\Omega_I\ (s,t) - \sin\Omega_I\ (s,t)]\, e^{\Omega_R(s,t)} + \left|G^R_{\mp}(s,t)\right|^2$$

	parameters		derived quantities				
$\sqrt{s}$ (GeV)	H (mb)	η_3	σ (mb)	ρ	B (GeV^{-2})	$\sigma_{\text{elas.}}$ (mb)	χ^2/ndf
pp							
23.882	0.311±0.002	0.16 (fix)	39.57	0.034	11.77	7.27	90.7/62
30.6	0.292±0.001	0.1522±0.001	39.79	0.049	12.23	7.03	94.1/68
44.7	0.291±0.0004	0.144 (fix)	41.51	0.077	13.12	7.08	87.3/67
52.8	0.2894±0.0003	0.1383±0.0003	42.41	0.088	13.26	7.30	245/88
62.5	0.2812±0.0005	0.1304±0.0004	42.76	0.092	13.15	7.49	111.1/62
200*	0.2887 (fix)	0.106 (fix)	52.05	0.133	14.61	9.94	-
900*	0.2887 (fix)	0.085 (fix)	68.38	0.145	16.43	15.15	-
2760*	0.2887 (fix)	0.076 (fix)	84.04	0.143	18.23	20.51	-
7000	0.2895±0.0003	0.0735±0.0002	99.51	0.138	20.39	25.57	74.4/59
7000	0.2764±0.0002	0.0707±0.0001	95.43	0.136	19.90	24.11	42.9/33
8000	0.2903±0.0001	0.0694±0.0001	102.12	0.137	19.65	27.99	72.5/58
8000	0.2735±0.0001	0.0698±0.0001	96.74	0.135	20.11	24.51	28.8/25
13000	0.2913±0.0001	0.0673±0.0001	111.43	0.134	20.99	31.11	149.4/126
14000*	0.2887 (fix)	0.067 (fix)	112.65	0.132	21.13	31.15	-
57000*	0.2887 (fix)	0.063 (fix)	141.59	0.123	24.19	42.93	-
p$\bar{\text{p}}$							
30.4	0.2994±0.003	0.15 (fix)	41.32	0.076	12.05	7.73	22.4/25
52.6	0.2971±0.001	0.138 (fix)	43.44	0.102	13.24	7.69	29.9/27
62.3	0.2938±0.003	0.132 ±0.004	44.13	0.107	13.32	7.89	19.9/15
540	0.2930±0.0004	0.0901±0.0003	62.95	0.145	15.65	13.52	164.9/97
1800	0.2706±0.001	0.0771±0.0004	73.71	0.141	17.19	16.78	43.8/53

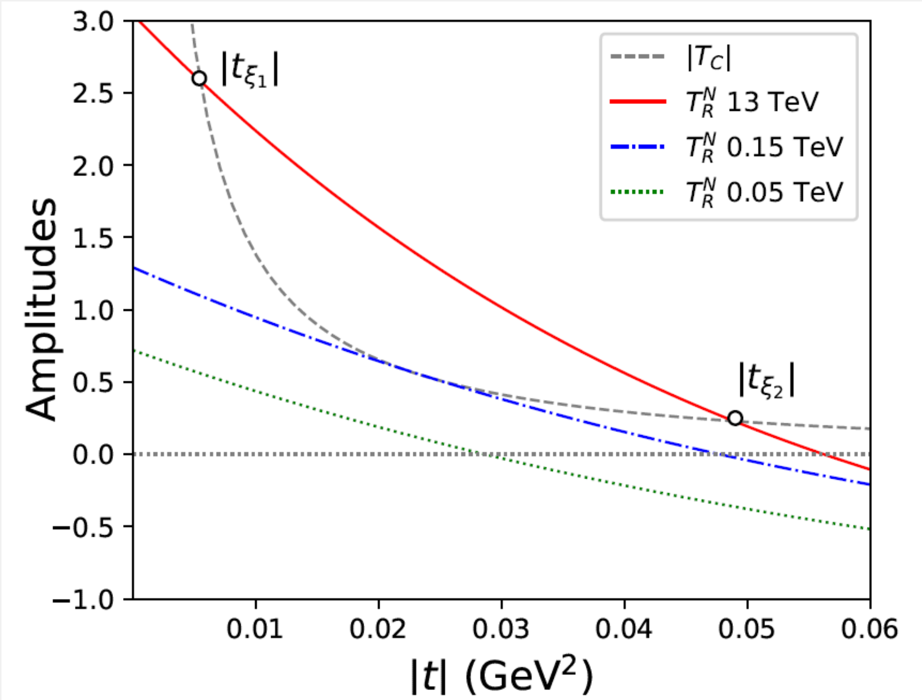
We leave two free parameters H and η_3



$\eta_3(s) = \epsilon_0 + \epsilon_1 s^{-\zeta}$
ϵ_0 = 0.0568 ± 0.0004
ϵ_1 = 0.336 ± 0.006
ζ = 0.182 ± 0.003
$p\bar{p}$
pp
η_3 (s)
$\sqrt{s}$ (GeV)
H (mb)
$\sqrt{s}$ (GeV)

The real-Coulomb interference
(very forward scattering)

As the energy increases the real nuclear amplitude also increases



Note: pp Coulomb amplitude is negative

Let $T_R(s,t)$ be the real part of the sum of the nuclear and Coulomb pp amplitudes,

$$T_R(s,t) \equiv T_R^N(s,t) + T_C(s) \; , \tag{8}$$

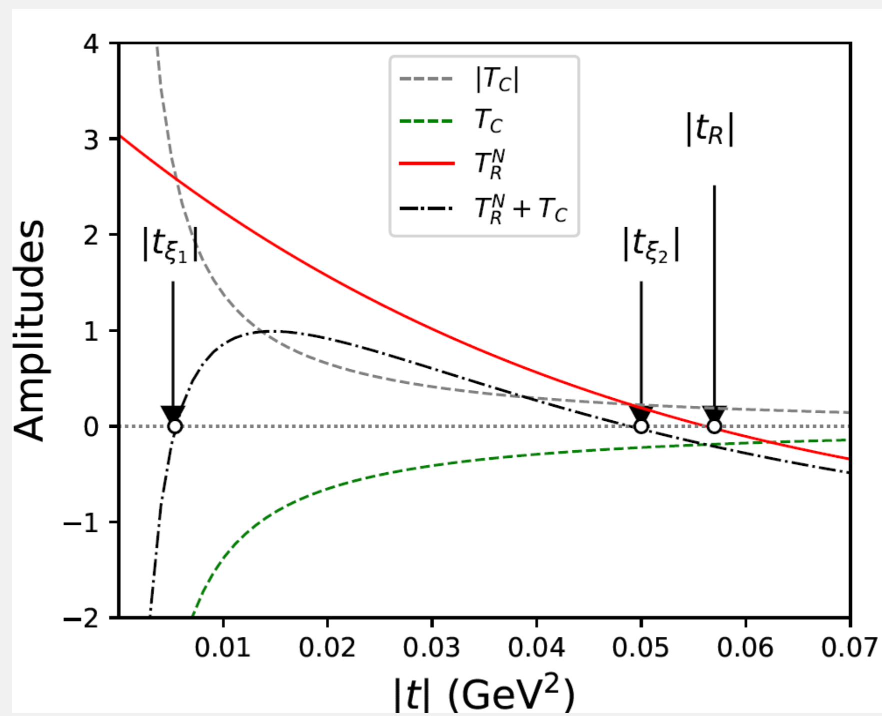
then, for s large, if $T_R^N(s,t) > |T_C(t)|$ in a region $0 < |t| < |t_R|$ then $T_R(s,t)$ has two zeros,

$$T_R(s,t_{\xi_1}) = T_R(s,t_{\xi_2}) = 0, \quad 0 < |t_{\xi_1}| < |t_{\xi_2}| < |t_R| \tag{9}$$

AKK, Eur. Phys. J. C 83 (2023) 2, 126

The real nuclear amplitude is positive in the forward range (Martin's theorem)

A. Martin, Phys. Lett. B 404, 137 (1997).



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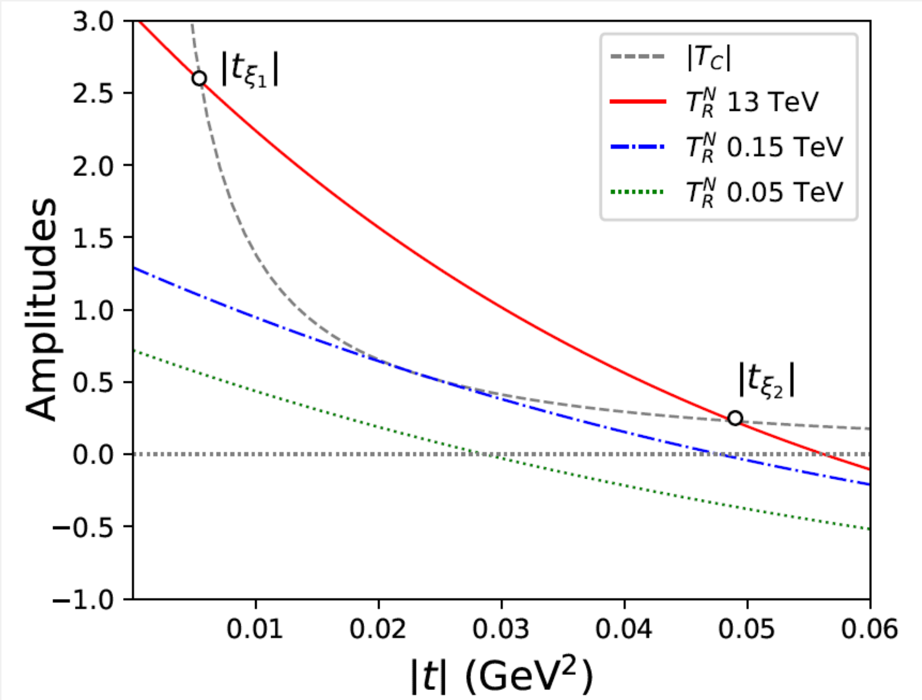

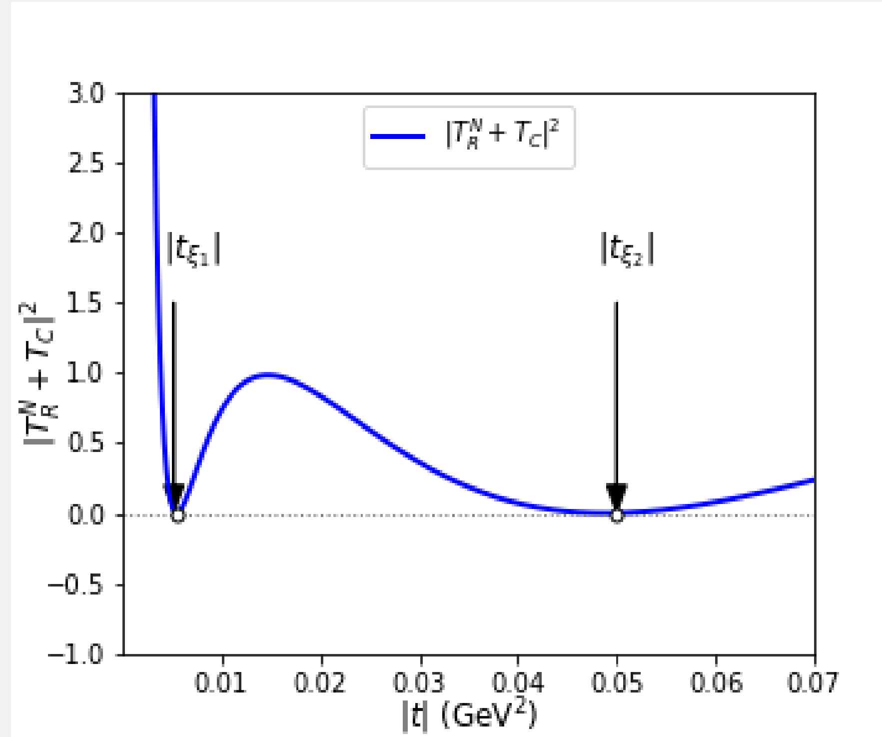

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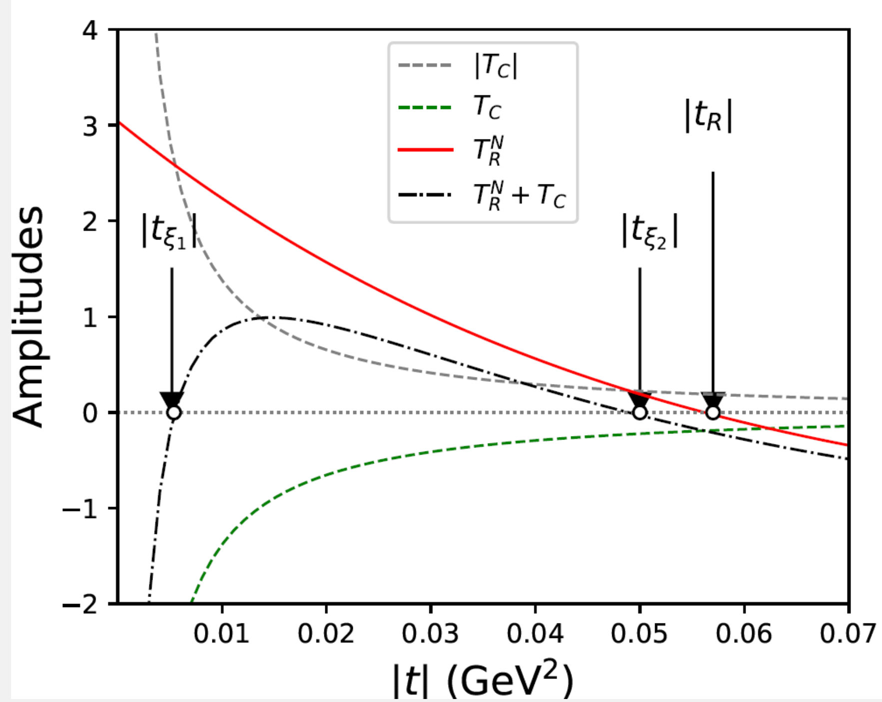
AKK, Eur. Phys. J. C 83 (2023) 2, 126

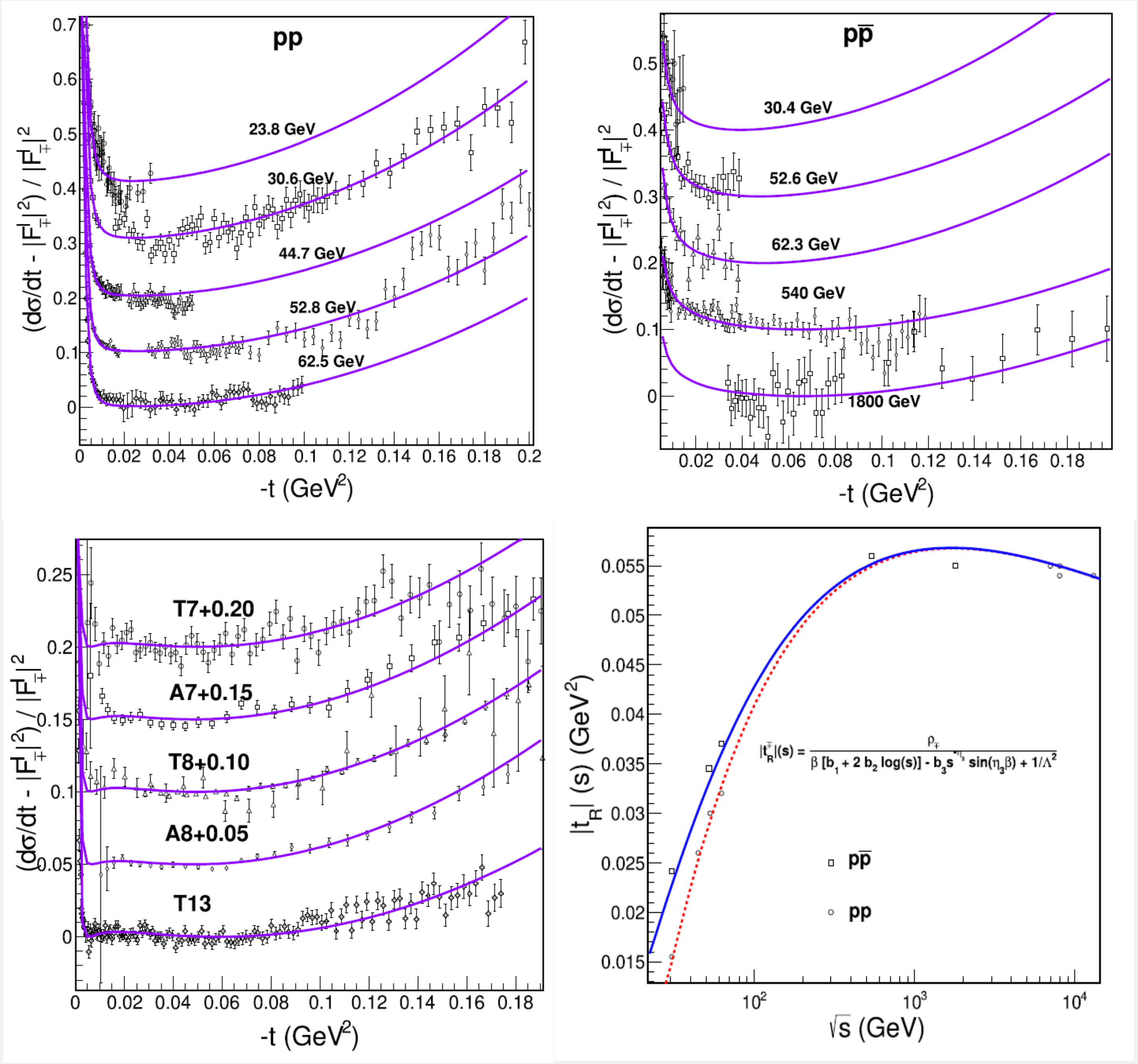


How to observe the effects of the real part?

Subtracting the square of the imaginary part we have

$$\frac{\frac{\mathrm{d}\sigma}{\mathrm{d}t} - \pi(\hbar c)^2 \left|T_I^N\right|^2}{\pi(\hbar c)^2 \left|T_I^N\right|^2} = \frac{\left|T_R^N + T_C\right|^2}{\left|T_I^N\right|^2}$$

The so-called non-exponential behaviour can be a manifestation of the real amplitude



Zero of Martin
(first zero of the real part)

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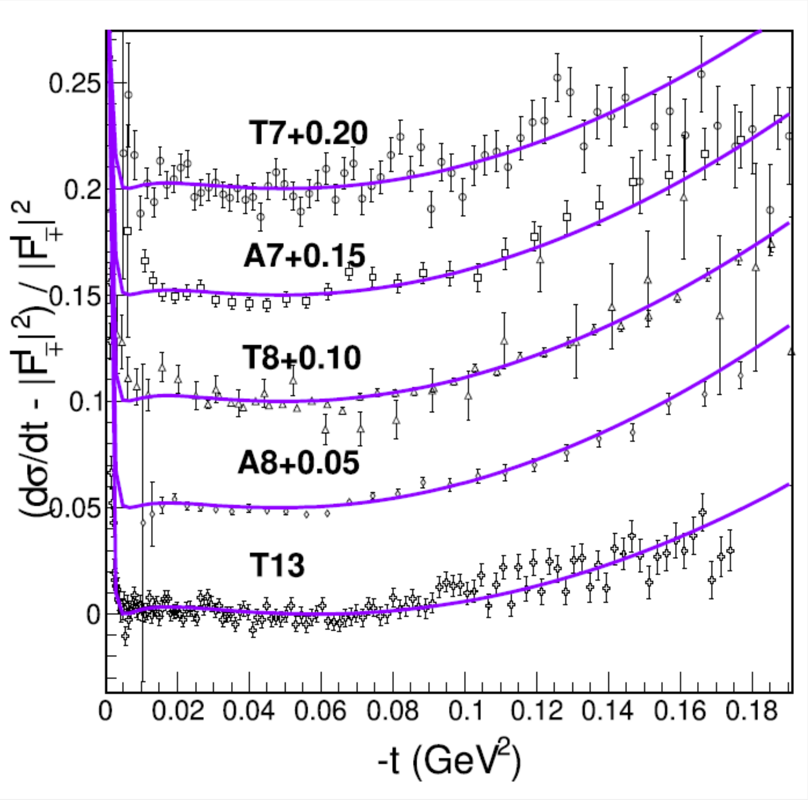
Is it possible to observe any dip due to the interplay between real and Coulomb amplitude?

$$T_R(s,t) + T_C(t) = 0$$



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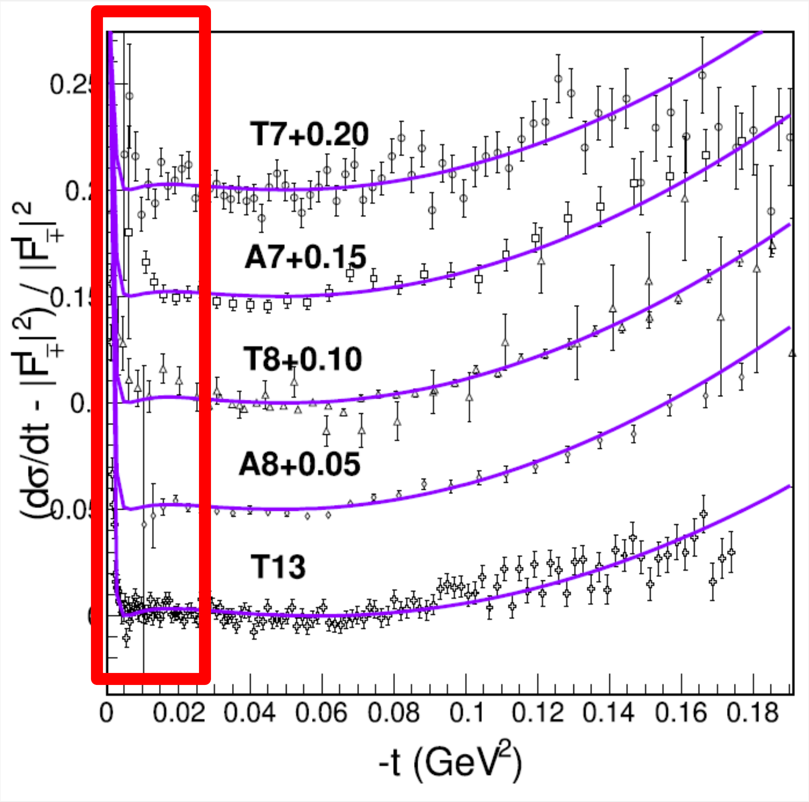
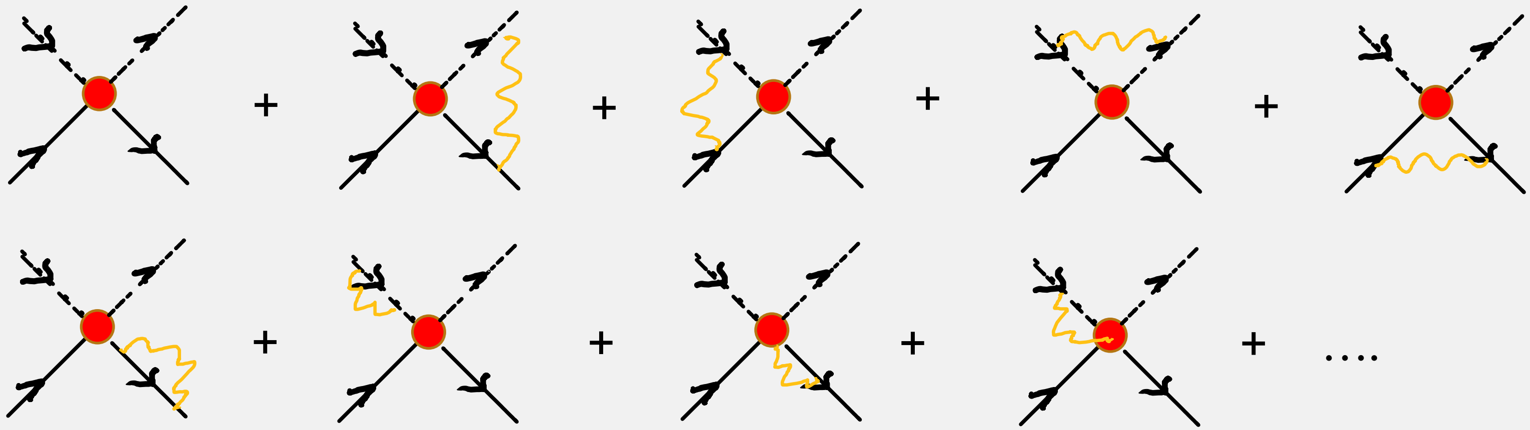
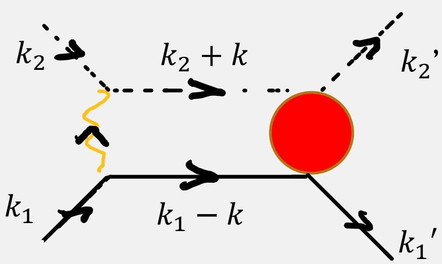


What about the relative Coulomb phase?

$$T_{total}(s,t) = T_R(s,t) + iT_I(s,t) + T_C(t)\mathrm{e}^{\mathrm{i}\alpha\Phi(s,t)}$$

G.B. West, D.R. Yennie, Phys. Rev. ***172****(5), 1413 (1968)*





$$I_N = -4\pi \mathrm{i}\alpha(4k_1 \cdot k_2) \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{f_N(2k_2 \cdot k + k^2, -2k_1 \cdot k + k^2, s, t')}{(k^2 + i_\epsilon)[(k_2 + k)^2 - m^2 + \mathrm{i}\epsilon][(k_1 - k)^2 - M^2 + \mathrm{i}\epsilon]}$$

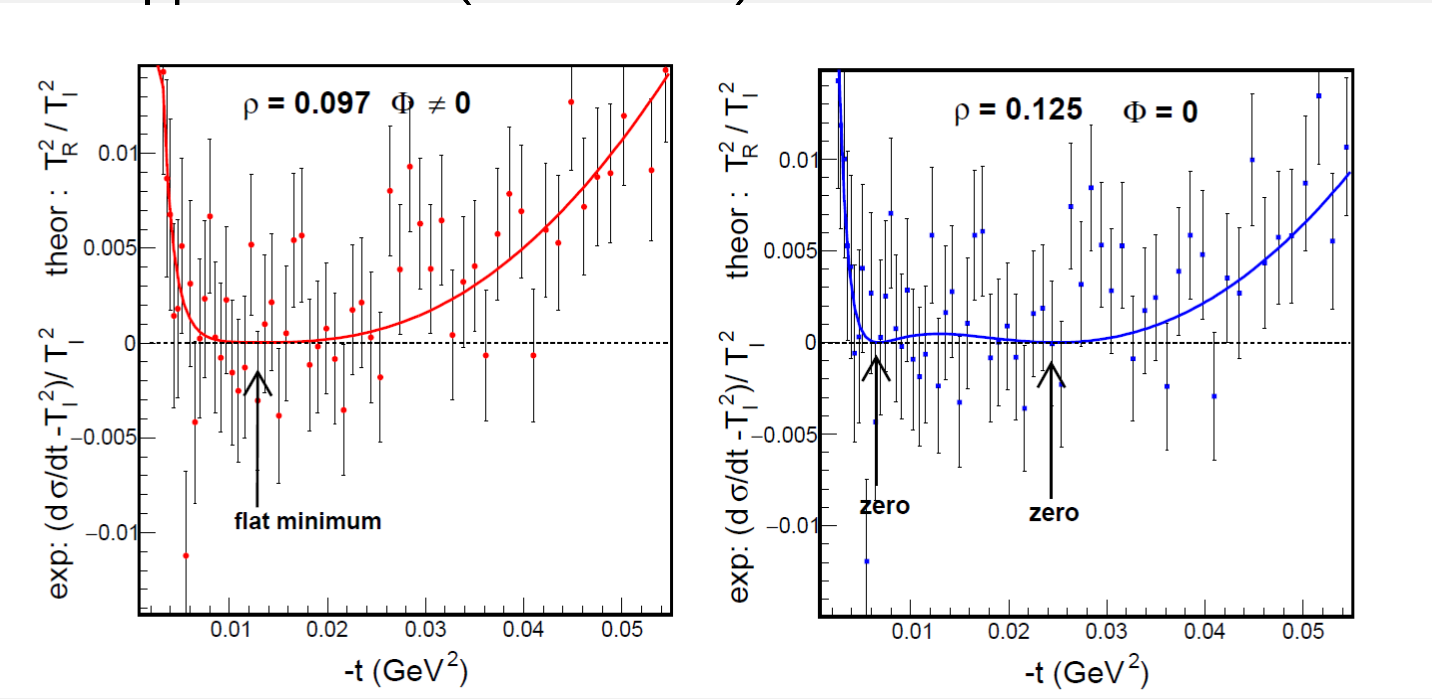
Similar results from adding Coulomb and Strong force eikonals *R. Cahn, Z. Phys. C* ***15*** *(1982) 253.*

$$\left.\begin{aligned} T_{C+N}(s,t) &= \frac{s}{4\pi \mathrm{i}} \int \mathrm{d}^2 b\, \mathrm{e}^{iq\cdot b}\left[\mathrm{e}^{2\mathrm{i}(\chi_C + \chi_N)} - 1\right] \\ T_C(t) &= \frac{s}{4\pi \mathrm{i}} \int \mathrm{d}^2 b\, \mathrm{e}^{iq\cdot b}\left[\mathrm{e}^{2\mathrm{i}\chi_C} - 1\right] \\ T_N(s,t) &= \frac{s}{4\pi \mathrm{i}} \int \mathrm{d}^2 b\, \mathrm{e}^{iq\cdot b}\left[\mathrm{e}^{2\mathrm{i}\chi_N} - 1\right] \end{aligned}\right\} \quad T_{C+N}(s,t) = T_C(t) + T_N(s,t) + \frac{s}{4\pi \mathrm{i}} \int \mathrm{d}^2 b\, \mathrm{e}^{iq\cdot b}\left[\mathrm{e}^{2\mathrm{i}\chi_C} - 1\right]\left[\mathrm{e}^{2\mathrm{i}\chi_N} - 1\right]$$

Relative Coulomb phase $$\phi(s,t) = -\ln\left(\frac{-t}{s}\right) + \int_0^s \frac{\mathrm{d}t'}{|t' - t|}\left[1 - \frac{F^{\mathrm{N}}(s,t')}{F^{\mathrm{N}}(s,t)}\right]$$

Explicity $\phi(s,t) \sim -\left[\gamma + \ln\left(-\frac{Bt}{2}\right) + O(-Bt)\right]$

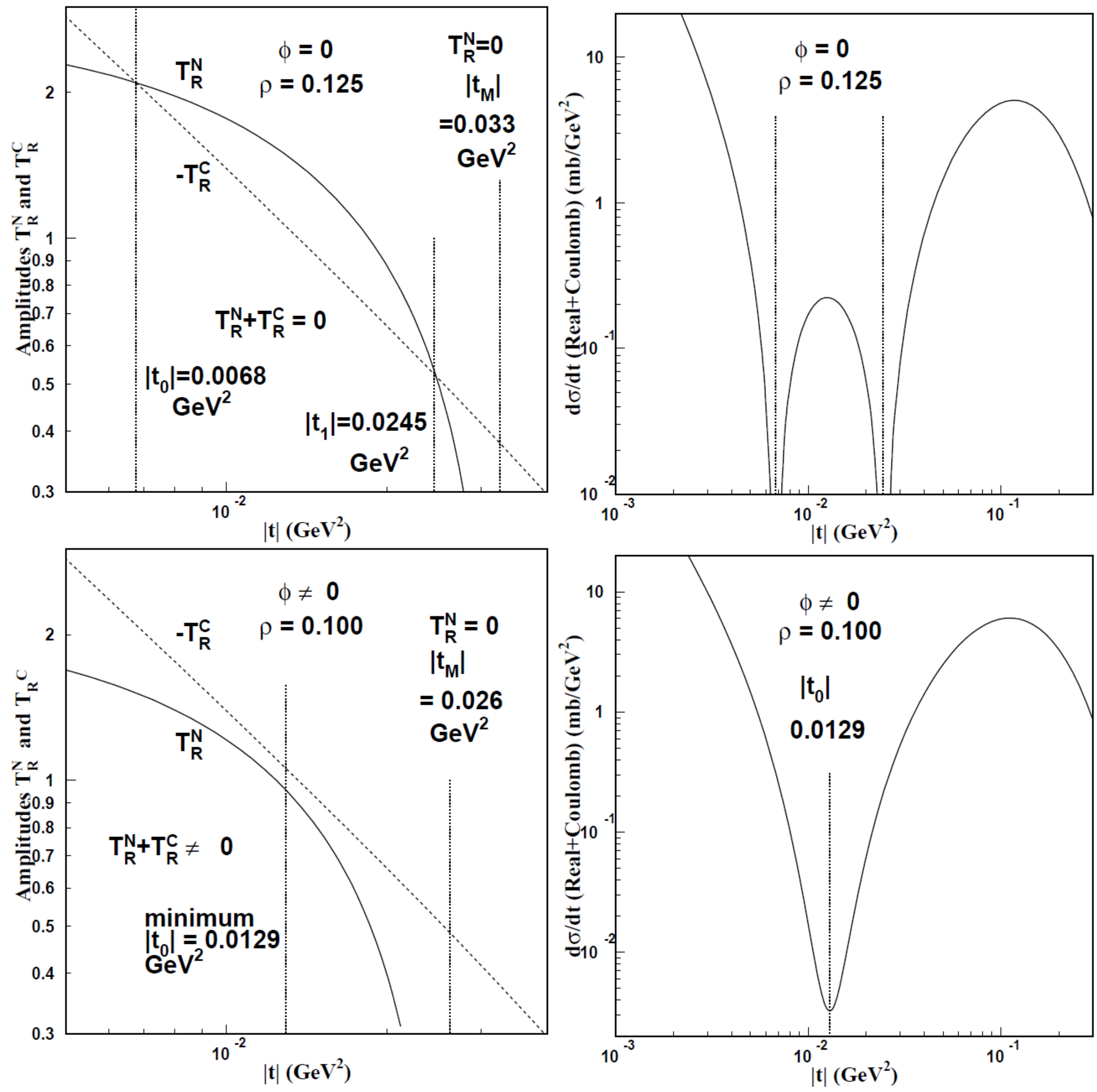
pp at 13 TeV (Totem data)



AKK., E. Ferreira and M. Rangel, Phys.Lett.B 789 (2019) 1-6

No matter which prescription is used to represent the relative phase,

the presence of the Coulomb phase reduces the magnitude of ρ


T_R^N
$\phi = 0$
$\rho = 0.125$
$T_R^N=0$
$|t_M|$
$=0.033$
GeV^2
$-T_R^C$
$T_R^N+T_R^C = 0$
$|t_0|=0.0068$
GeV^2
$|t_1|=0.0245$
GeV^2
Amplitudes T_R^N and T_R^C
$|t|$ (GeV^2)
$\phi = 0$
$\rho = 0.125$
$d\sigma/dt$ (Real+Coulomb) (mb/GeV^2)
$|t|$ (GeV^2)
$\phi \neq 0$
$\rho = 0.100$
$-T_R^C$
$T_R^N = 0$
$|t_M|$
$= 0.026$
GeV^2
T_R^N
$T_R^N+T_R^C \neq 0$
minimum
$|t_0| = 0.0129$
GeV^2
Amplitudes T_R^N and T_R^C
$|t|$ (GeV^2)
$\phi \neq 0$
$\rho = 0.100$
$|t_0|$
0.0129
$d\sigma/dt$ (Real+Coulomb) (mb/GeV^2)
$|t|$ (GeV^2)

"**Entities must not be multiplied beyond necessity**"

Illustration of William of Ockham (from Wikipedia)

Thank you!

Hadronic Collider Experiments

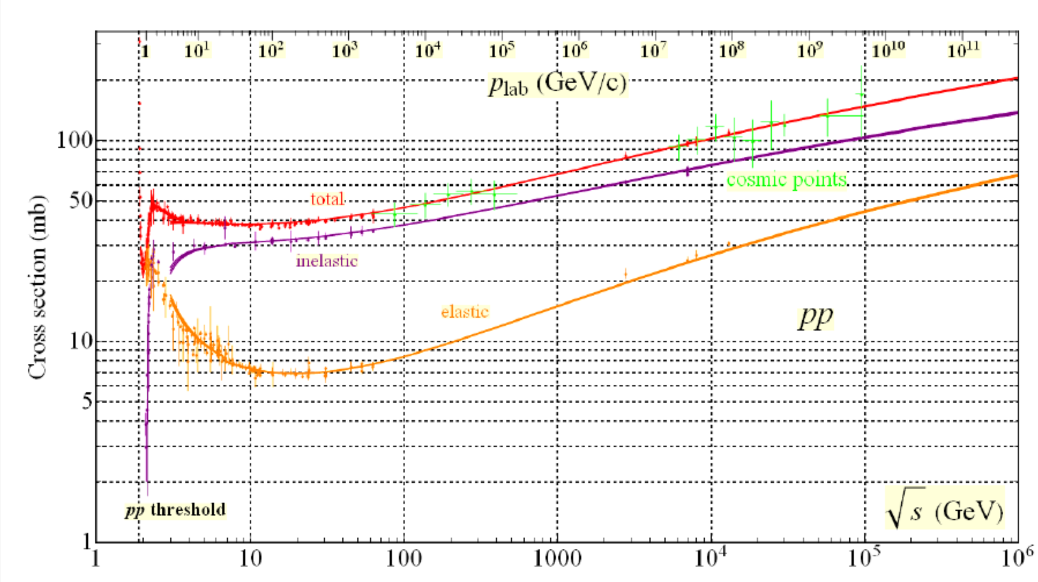
Intersecting Storage Rings-CERN, 1971–1984

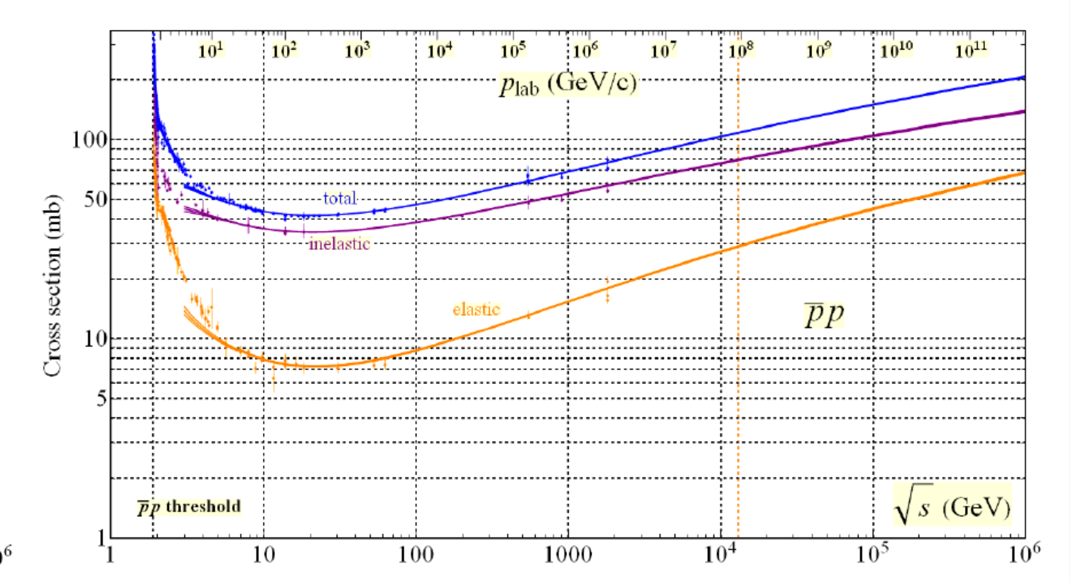
Proton-Antiproton Collider(SPS)-CERN, 1981–1991

Tevatron-Fermilab, 1987–2011

Relativistic Heavy Ion Collider-BNL, 2000–...

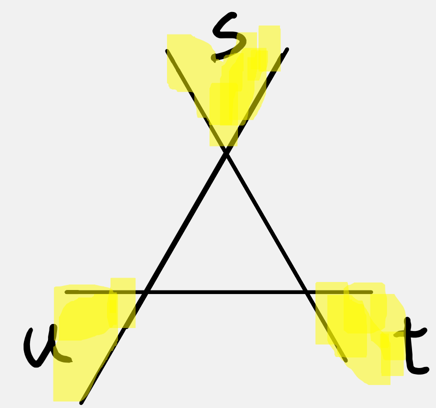
Large Hadron Collider-CERN, 2009–...





P.A. Zyla et al. (Particle Data Group), *Prog. Theor. Exp. Phys.* 2020, 083C01 (2020)

Assumptions



Mandelstam plane

Analytic nuclear amplitude $A(s,t,u)$

Singularities have a physical meaning

Crossing symmetric amplitudes $A_{pp}(s,t,u) = A_{p\bar{p}}(u,t,s)$

Unitarity of S matrix $ss^{\dagger} = 1$

Theorems

Optical theorem $$\sigma_T = \frac{1}{2|p|\sqrt{s}} \operatorname{Im} A(s,t)$$

Froissart theorem/bound $$\sigma_T(s) \le C \log^2\left(\frac{s}{s_0}\right) \qquad s \to \infty$$

Pomeranchuck theorem $$\frac{\sigma_T^{pp}(s)}{\sigma_T^{p\bar{p}}(s)} \to 1 \qquad s \to \infty$$