Structure of the real amplitude in forward pp scattering at the LHC

Anderson Kendi Kohara
Faculty of Physics, AGH-University of Science and Technology
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Some challenges:

- Seemingly simple kinematics but complicated dynamics
- Non-perturbative phenomenon
- Long range force interplays with short ranged
- Experimental gap among energies
- Differential cross sections with fluctuations (and/or) large uncertainties
- Lack of cross-symmetric experiments at the same energies
Relativistic Elastic Scattering

Mandelstam variables

\[ s = (p_1 + p_2)^2 \]
\[ t = (p_1 - p_3)^2 \]
\[ u = (p_1 - p_4)^2 \]

Coulomb phase

L.D. Solov’ev, JETP 22, 205 (1966) 26;
G.B. West, D.R. Yennie, Phys. Rev. 172 (5), 1413 (1968);
Scattering amplitudes

Coulomb part (purely real – energy independent)

\[ T_c(t) = \mp \frac{2\alpha}{|t|} \left( \frac{\Delta^2}{\Delta^2 + |t|} \right)^2 \]

Complex nuclear part (FORWARD DESCRIPTION)
(different \( t \) dependences for real and imaginary parts)

\[ T_R^N(t) = \frac{\sigma(\rho - \mu_R t)e^{B_R t/2}}{4\sqrt{\pi}(\hbar c)^2} \]

\[ T_I^N(t) = \frac{\sigma(1 - \mu_I t)e^{B_I t/2}}{4\sqrt{\pi}(\hbar c)^2} \]


example at 13 TeV
Basic physical quantities we are interested in:

**Forward quantities** \((t=0)\)

Optical theorem \[ \sigma_{tot} = 4\pi(\hbar c)^2 T_I^N(s, 0) \]

Ratio of real and imaginary amplitudes \[ \rho = \frac{T_R^N(s, 0)}{T_I^N(s, 0)} \]

effective slopes

\[
B_{I}^{\text{eff}} = \frac{2}{T_I^N(s, t)} \frac{d}{dt} T_I^N(s, t) \bigg|_{t=0} = (B_I - 2\mu_I)
\]

\[
B_{R}^{\text{eff}} = \frac{2}{T_R^N(s, t)} \frac{d}{dt} T_R^N(s, t) \bigg|_{t=0} = \left( B_R - \frac{2\mu_R}{\rho} \right)
\]

Differential cross section \((s,t)\) dependent

\[
\frac{d\sigma}{dt} = |T(s, t)|^2
\]
Why the parameters $\mu_R$ and $\mu_I$ ?

$\mu_I$

The dip of the $\frac{d\sigma}{dt}$ is due to the passage of the imaginary amplitude through zero

$(1 - \mu_I t_{dip}) = 0$ Once $\mu_I$ is determined we know the dip position

...But this is not our concern in the present work!!!

$\mu_R$

Theorem 1: ““If, for sufficiently large values of $\xi$, $\sigma(\xi)$ is non decreasing and approaches infinity as $\xi \to \infty$. Then $\text{Re} F(\xi)$ is positive for all sufficiently large values of $\xi”$” $\xi=(s-u)/2$


Theorem 2: ““We show that if for fixed negative (physical) square of the momentum transfer, the differential cross-section $d\sigma/dt$ tends to zero and if the total cross-section tends to infinity, When the energy goes to infinity, the real part of the even signature amplitude cannot have a constant sign near $t=0.””


To sumarize, the real amplitude in the forward range at high energies is positive, and at some point within the diffractive cone it crosses zero.

$(\rho - \mu_R t_{zero}) = 0$
A forward description in terms of analytic functions

Following A. Martin (1977)

\[ T^N(E, t) \sim i C E (\log(E) - \pi i / 2)^2 f(\tau) \]

with \( \tau = t \log^2(E) \) the scaling variable (real)

\[ E = (s - u)/4m \]

\[ f(\tau) = T^N(E, t) / T^N(E, 0) \]

generic scaling function


We extended the idea to a complex scaling variable

\[ \tau' \sim t (\log(E) - \pi i / 2)^2 \]

and as a consequence in the limit of large \( s \) we obtain

\[ f(\tau') = e^{\tau'} = e^{\Omega_R(s,t) + \Omega_I(s,t)} \]

\[ T^N_R(s, 0) = s[\beta (P_1 + 2H \log s) - R_1 s^{-\eta_1} \sin(\eta_1 \beta) \mp R_2 s^{-\eta_2} \cos(\eta_2 \beta)] \]

\[ T^N_I(s, 0) = s[P + P_1 \log s + H (\log^2 s - \beta^2) + R_1 s^{-\eta_1} \cos(\eta_1 \beta) \pm R_2 s^{-\eta_2} \sin(\eta_2 \beta)] \]

\[ \Omega_R(s, t) = [b_0 + b_1 \log s + b_2 (\log^2 s - \beta^2) + b_3 s^{-\eta_3} \cos(\eta_3 \beta)] t \]

\[ \Omega_I(s, t) = -[b_1 \beta + 2 b_2 \beta \log s - b_3 s^{-\eta_3} \sin(\eta_3 \beta)] t \]

The complex amplitude is

\[ \begin{pmatrix} T^N_R(s, 0) \\ T^N_I(s, 0) \end{pmatrix} = s \sigma_+(s) \begin{pmatrix} \cos \Omega_I(s, t) & -\sin \Omega_I(s, t) \\ \sin \Omega_I(s, t) & \cos \Omega_I(s, t) \end{pmatrix} \begin{pmatrix} \rho_+(s) \\ 1 \end{pmatrix} e^{\Omega_R(s,t)} \]

To describe the data we add to the real part a shape

\[ G^R_{\pi^+}(s, t) = \sigma^+ \frac{s \, t}{\Delta^2 - t} e^{\Omega_R(s,t)} \]

\[ T_R^N(s, t) \to T_R^N(s, t) + G^R_{\pi^+}(s, t) \]

And the differential cross section is

\[ \frac{d\sigma}{dt} = s^2 \sigma^2 \left( \rho^2 + 1 \right) e^{2\Omega_R(s,t)} + 2 \, s \, \sigma^+ \, G^R_{\pi^+}(s, t) \left[ \rho^+ \cos \Omega_I(s, t) - \sin \Omega_I(s, t) \right] e^{\Omega_R(s,t)} + |G^R_{\pi^+}(s, t)|^2 \]

We leave two free parameters \( H \) and \( \eta_3 \).
\[ \eta_3(s) = \varepsilon_0 + \varepsilon_1 s^{\zeta} \]

\[ \varepsilon_0 = 0.0568 \pm 0.0004 \]
\[ \varepsilon_1 = 0.336 \pm 0.006 \]
\[ \zeta = 0.182 \pm 0.003 \]
The real-Coulomb interference
(very forward scattering)
As the energy increases the real nuclear amplitude also increases.

\[
|t_{\xi_1}|, |t_{\xi_2}|
\]

Note: pp Coulomb amplitude is negative

The real nuclear amplitude is positive in the forward range (Martin’s theorem)


Let \( T_R(s,t) \) be the real part of the sum of the nuclear and Coulomb pp amplitudes,

\[
T_R(s,t) = T_R^N(s,t) + T_C(s), \tag{8}
\]

then, for \( s \) large, if \( T_R^N(s,t) > |T_C(t)| \) in a region \( 0 < |t| < |t_R| \) then \( T_R(s,t) \) has two zeros,

\[
T_R(s, t_{\xi_1}) = T_R(s, t_{\xi_2}) = 0, \quad 0 < |t_{\xi_1}| < |t_{\xi_2}| < |t_R| \tag{9}
\]

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$$T_R(s,t) = T^N_R(s,t) + T_C(s),$$  \hspace{1cm} (8)

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How to observe the effects of the real part?

Subtracting the square of the imaginary part we have

\[
\frac{d\sigma}{dt} - \pi (\hbar c)^2 |T_I|^2 = \left| T_R + T_C \right|^2 - \pi (\hbar c)^2 |T_I^N|^2 \]
The so-called non-exponential behaviour can be a manifestation of the real amplitude

Zero of Martin
(first zero of the real part)
\[ T_R(s, t) + T_C(t) = 0 \]

Is it possible to observe any dip due to the interplay between real and Coulomb amplitude?
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\[ T_R(s, t) + T_C(t) = 0 \]
What about the relative Coulomb phase?

\[ T_{\text{total}}(s, t) = T_R(s, t) + iT_I(s, t) + T_C(t)e^{i\alpha \Phi(s, t)} \]
$I_N = -4\pi i\alpha(4k_1 \cdot k_2) \int \frac{d^4k}{(2\pi)^4} \frac{f_N(2k_2 \cdot k + k^2, -2k_1 \cdot k + k^2, s, t')}{(k^2 + i\epsilon)[(k_2 + k)^2 - m^2 + i\epsilon][(k_1 - k)^2 - M^2 + i\epsilon]}$

**Similar results from adding Coulomb and Strong force eikonals**


$$T_{C+N}(s, t) = \frac{S}{4\pi i} \int d^2b \ e^{i\vec{q} \cdot \vec{b}} \ [e^{2i(\chi_c + \chi_N)} - 1]$$

$$T_C(t) = \frac{S}{4\pi i} \int d^2b \ e^{i\vec{q} \cdot \vec{b}} \ [e^{2i\chi_c} - 1]$$

$$T_N(s, t) = \frac{S}{4\pi i} \int d^2b \ e^{i\vec{q} \cdot \vec{b}} \ [e^{2i\chi_N} - 1]$$

$$T_{C+N}(s, t) = T_C(t) + T_N(s, t) + \frac{S}{4\pi i} \int d^2b \ e^{i\vec{q} \cdot \vec{b}} \ [e^{2i\chi_c} - 1][e^{2i\chi_N} - 1]$$

**Relative Coulomb phase**

$$\phi(s, t) = -\ln \left(\frac{-t}{s}\right) + \int_0^s \frac{dt'}{|t' - t|} \left[1 - \frac{F_N(s, t')}{F_N(s, t)}\right]$$


No matter which prescription is used to represent the relative phase, the presence of the Coulomb phase reduces the magnitude of $\rho$
"Entities must not be multiplied beyond necessity"

Illustration of William of Ockham (from Wikipedia)
Thank you!
Hadronic Collider Experiments


Tevatron-Fermilab, 1987–2011

Relativistic Heavy Ion Collider-BNL, 2000–...

Large Hadron Collider-CERN, 2009–...
Assumptions

Analytic nuclear amplitude \( A(s, t, u) \)

Singularities have a physical meaning

Crossing symmetric amplitudes \( A_{pp}(s, t, u) = A_{p\bar{p}}(u, t, s) \)

Unitarity of S matrix \( ss^\dagger = 1 \)

Theorems

Optical theorem \( \sigma_T = \frac{1}{2|p|\sqrt{s}} \text{Im} A(s, t) \)

Froissart theorem/bound \( \sigma_T(s) \leq C \log^2 \left( \frac{s}{s_0} \right) \quad s \to \infty \)

Pomeranchuck theorem \( \frac{\sigma_{pp}^T(s)}{\sigma_T^{-1}(s)} \to 1 \quad s \to \infty \)