Twist decomposition of non-linear effects in Balitsky-Kovchegov evolution of proton structure functions

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Introduction

- The standard description of proton hard interactions based on OPE and hard factorization theorem at twist-2 is very successful.
- Higher twist (HT) effects are expected to be enhanced at small x: twist 4/twist 2$\sim (Q_0^2/Q^2)(x_0/x)^\lambda$.
- Data are not conclusive - this may affect the accuracy of extracted PDFs at small x and moderate Q^2.
- The perturbative evolution of HT operators are known, but the non-perturbative input is not.
- Use the small x models and small x evolution equations to describe the HT contributions:
  - Saturation models: Golec-Biernat - Wüsthoff (GBW), (J. Bartels, K. Peters and K. Golec-Biernat, 2000; J. Bartels, K. Golec-Biernat and L. Motyka, 2009),
  - BFKL equation: L. Motyka and MS, 2014.
  - New results: BK equation, L. Motyka and MS, arXiv:2306.02118.

- Precise set of the combined HERA data show problems of DGLAP fits of DIS and DDIS when $Q^2 < 5$ GeV^2 data are included in fits; problems at small x. Example: $F_{L/T} = F_{L/T}^{(\tau=2)} + \Delta F_{L/T}^{(\tau=4)}$,

$$\Delta F_{L/T}^{(\tau=4)} = \frac{Q_0^2}{Q^2} x^{-2\lambda} \left[c_{L/T} \log\left(\frac{Q_0^2}{Q^2} + \lambda \log \frac{1}{x} \right) + c_{L/T}^{(0)} \right].$$

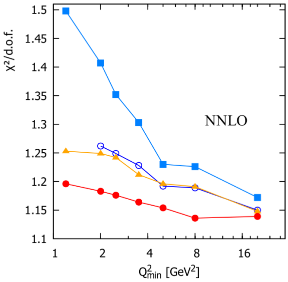
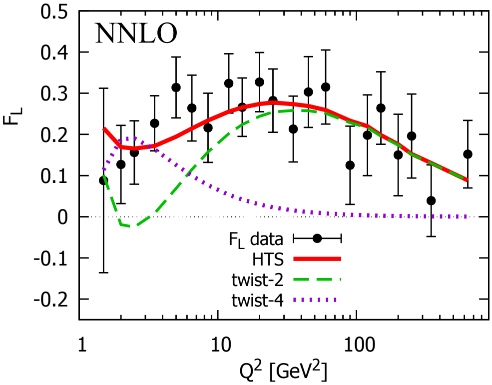


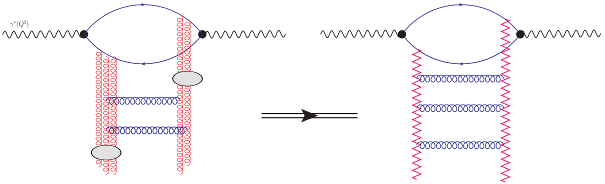
Figure: DIS, L. Motyka, MS, W. Słomiński and K. Wichmann, 2017.

Findings from DGLAP+higher twist fits:

- small F_2 corrections,
- significant and positive corrections to F_L.



- BFKL picture: gluon reggeizetion binds the t-channel gluons that couple to the color dipole produced by γ^* into two Reggeized gluons, that span a single gluon ladder, dominated by twist 2 contribution.



BFKL bootstrap — gluon Reggeization

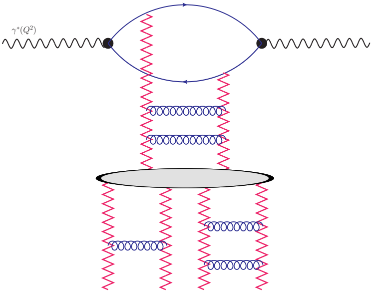
BFKL equation — two interacting Reggeized gluons

- HT from BFKL equation (L. Motyka and MS, Acta Phys. Polon., 2014) at LL($1/x$) found to decrease with decreasing x in the asymptotic regime.
- HT effects: small in F_2, small negative corrections in F_L.

Round-up

1. Problems of DGLAP fits at small Q^2, small x region.
2. Explanation proposed: HT corrections
   - DGLAP+HT fits to HERA data:
     - small HT effects in F_2,
     - large positive HT corrections in F_L.
   - BFKL:
     - small HT effects in F_2,
     - small negative HT corrections in F_L.
3. There are other possible explanations: small x resummation beyond DGLAP (R. Ball et. al, 2017).

HT from BK equation: the Triple Pomeron Vertex allows for a transition from single BFKL ladder to two and more ladders, that carry the HT contributions with the strongest enhancement due to evolution. This is the mechanism that should provide reliable estimate of HT effects in the proton at $LL(1/x)$ approximation in QCD.



Triple Pomeron (BK) vertex — transition to 4 Reggeized gluons

The basic object: the Fourier transform of the dipole scattering amplitude $N(y, r)$:

$$\phi(y, k_\perp^2) = \int \frac{d^2 r}{2\pi} e^{-i\vec{k}_\perp \cdot \vec{r}} \, \frac{N(y, r)}{r^2} \, ,$$

which is related to the unintegrated and collinear gluon density by linear integral transformations. $\phi(y, k_\perp^2)$ satisfies the BK equation:

$$\frac{\partial \phi(y, k_\perp^2)}{\partial y} = \bar{\alpha}_s \int d^2 q \, K_{BFKL}(q_\perp, k_\perp) \phi(y, q_\perp^2) - \bar{\alpha}_s \phi^2(y, k_\perp^2).$$

The solution may be presented as a power series in a number of Triple Pomeron Vertices $\phi = \sum_{n=0}^{\infty} \phi_n$ which is convergent for momenta above the saturation scale [Kovchegov, 1999]. For $n = 0$ the BFKL equation is reproduced, for $n = 1$ the $1 \to 2$ BFKL ladders transition is described.

We solve the equation in the Mellin space

$$\frac{\partial \tilde{\phi}_0(y,\gamma)}{\partial y} = \bar{\alpha}_s \chi(\gamma) \tilde{\phi}_0(y,\gamma)$$

$$\frac{\partial \tilde{\phi}_1(y,\gamma)}{\partial y} = \bar{\alpha}_s \chi(\gamma) \tilde{\phi}_1(y,\gamma)$$

$$-2\pi i\, \bar{\alpha}_s \int_{c_1 - i\infty}^{c_1 + i\infty} \frac{d\gamma_1}{2\pi i} \int_{c_2 - i\infty}^{c_2 + i\infty} \frac{d\gamma_2}{2\pi i} \delta(\gamma - \gamma_1 - \gamma_2) \tilde{\phi}_0(y,\gamma_1) \tilde{\phi}_0(y,\gamma_2)$$

The solution is obtained iteratively. $\tilde{\phi}_0$ is well known BFKL solution in the Mellin space and

$$\tilde{\phi}_1(y,\gamma) = \int \frac{d\gamma_1}{2\pi i} \frac{d\gamma_2}{2\pi i} 2\pi i \delta(\gamma - \gamma_1 - \gamma_2) C_0(\gamma_1) C_0(\gamma_2) \frac{\exp\left(\bar{\alpha}_s y \chi(\gamma)\right) - \exp\left(\bar{\alpha}_s y \chi(\gamma_1) + \bar{\alpha}_s y \chi(\gamma_2)\right)}{\chi(\gamma_1) + \chi(\gamma_2) - \chi(\gamma)},$$

The $\gamma^*_{T,L}$ cross sections may be also expanded: $\sigma^{\gamma^* A}_{T,L} = \sum_{i=0}^{\infty} \sigma^{(i)\gamma^* A}_{T,L}$

$$\sigma^{(i)\,\gamma^* A}_{T,L}(x, Q^2) = \sigma_0 \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{Q_0^2}{Q^2}\right)^{-\gamma} \tilde{H}_{T,L}(-\gamma) \frac{\Gamma(1+\gamma)}{2^{-2\gamma-1}\Gamma(-\gamma)} \tilde{\phi}_i(y, -\gamma)$$

Functions $\tilde{H}_{T,L}$ are Mellin transforms of the photon wave functions.
The lowest BFKL component

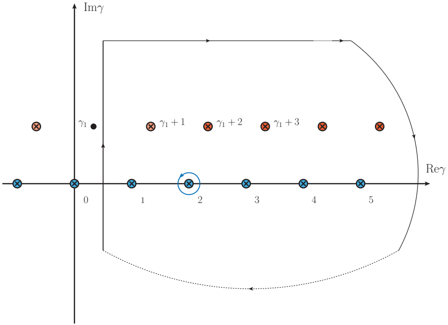
$$\sigma^{(0)\,\gamma^* A}_{T,L}(x, Q^2) = -\sigma_0 \int_{-1/2-i\infty}^{-1/2+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{Q_0^2}{Q^2}\right)^{-\gamma} \tilde{H}_{T,L}(-\gamma)\Gamma(\gamma) e^{\bar{\alpha}_s y \chi(\gamma)}$$

where the eigenvalue of the BFKL kernel $\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$ has simple poles for all integer values of γ.

- The first nonlinear correction:

$$\sigma^{(1)\,\gamma^* A}_{T,L} = \sigma_0 \int_c \frac{d\gamma}{2\pi i}\left(\frac{Q_0^2}{Q^2}\right)^{\gamma} \int_{c_1} \frac{d\gamma_1}{2\pi i} \tilde{H}_{T,L}(\gamma) \frac{\Gamma(1-\gamma)B(\gamma_1,\gamma-\gamma_1)}{2\gamma_1(\gamma-\gamma_1)}$$
$$\times \frac{e^{(\bar{\alpha}_s y \chi(\gamma))} - e^{(\bar{\alpha}_s y \chi(\gamma_1)+\bar{\alpha}_s y \chi(\gamma-\gamma_1))}}{\chi(\gamma_1)+\chi(\gamma-\gamma_1)-\chi(\gamma)}$$

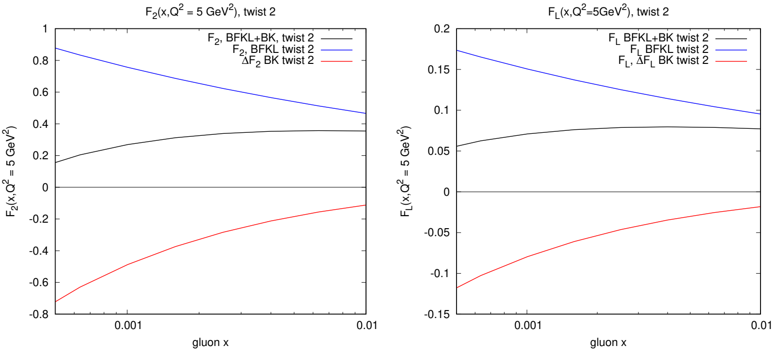
- The cross section have isolated singularities in the double Mellin plane (γ,γ_1) in the right half-planes. The first line contributes poles, however essential singularities appear in the second line.
- Singularities are found for integer γ and γ_1 but also for integer $\gamma-\gamma_1$. In our convention the twist poles correspond to singularities in positive integer values.
- Decomposition strategy: represent the cross sections as sums of contributions coming from singularities in γ at fixed γ_1



- The integration around γ-singularities performed numerically. The resulting expression has terms with γ_1 dependence factored out from Q^2 dependence and terms of the form of $\int d\gamma_1 (Q_0^2/Q^2)^{\gamma_1} G(\gamma_1)$ where $G(\gamma_1)$ has singularities for integer γ_1.
- First two twists give 95% (F_L) and 99% (F_T) of the full result.

$Q^2 = 5\ \text{GeV}^2$, twist 2 only

The curves: BFKL, BK correction, **BFKL+BK**

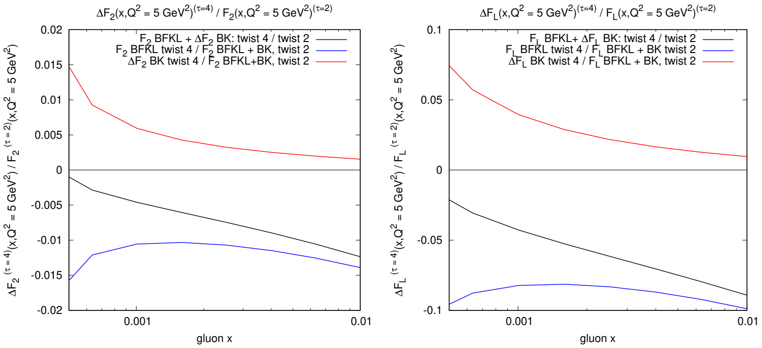


The calculations are performed for the BFKL term adjusted to data. The plots show the strength of the BK effects at twist 2 assuming a single iteration of the Triple Pomeron Vertex.

HT effects in proton structure functions at small x

$Q^2 = 5$ GeV^2, the ratio of higher twist contributions to the BFKL+BK twist-2 contribution.

The curves: BFKL, BK correction, **BFKL+BK**



The BFKL/BK higher twist corrections are found to be a small (F_2) or moderate (F_L) fraction of the leading twist contributions.

- Unintegrated gluon distribution: $f(x,k^2) = \frac{Q_s^2(x)}{Q_0^2} h(k^2/Q_s^2(x))$, where $Q_s^2(x) = Q_0^2(x_{in}/x)^\lambda$ is the saturation scale.
- The profile $h(\xi) \approx \xi^2$ for $\xi \to 0$, $h(\xi) \approx \xi^{\gamma_c}$, $\gamma_c > 0$ for $\xi \gg 1$.
- Using the simplest model: $h(\xi) = A[\xi^2\theta(1-\xi) + \xi^{\gamma_c}\theta(\xi-1)]$ one gets: $xg(x,Q^2) \approx \int^{Q^2} dk^2/k^2 f(x,k^2) \sim (x_{in}/x)^\lambda \log(Q^2/Q_s^2(x))$.
- Therefore, due to a strong suppression of unintegrated gluon distribution $f(x,k^2)$ in the region $k^2 < Q_s^2(x)$, the saturation scales acts as a lower cut-off on the logarithmic integration in $xg(x,Q^2)$. Dominance of the $k^2 > Q_s^2(x)$ momenta.
- Hence the relative correction due to non-linearity reads

$$\frac{xg(x,Q^2) - xg(x,Q^2)|_{linear}}{xg(x,Q^2)|_{linear}} = -\frac{\log(Q_s^2(x)/\mu_0^2)}{\log(Q^2/\mu_0^2)}$$

The correction enters without a suppressing factor of $1/Q^2$ - at the leading twist.

Summary and outlook

- We have performed the twist decomposition of the proton structure functions at small x from the $LL(1/x)$ BK equation, assuming a single iteration of the Triple Pomeron Vertex.
- The nonlinear BK corrections enter very strongly at twist 2 (leading twist shadowing). This is true both for F_2 and F_L. The unitarization of the full BK scattering amplitudes occurs mostly at twist 2.
- At reference scale $Q^2 = 5$ GeV^2, the relative higher twist to the twist 2 contributions to F_2 are of the order of 1% from both BFKL and BK (with different signs). For F_L the relative HT effects are smaller than 10% also with opposite signs.
- For a reliable estimate of the BK amplitude in the region where the first BK correction becomes comparable to the BFKL linear term, more iterations of the TPV vertex are necessary, then the full BK solution is recovered.

- Precise set of the combined HERA data show problems of DGLAP fits of DIS and DDIS when $Q^2 < 5$ GeV^2 data are included in fits; problems at small x.

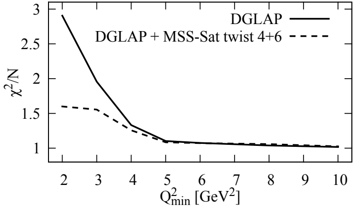
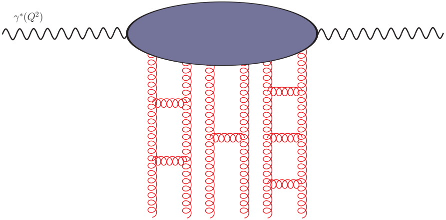


Figure: Example: DDIS, L. Motyka, MS and W. Słomiński, 2012.

- Explanation proposed: HT corrections (L. Motyka, MS and W. Słomiński, 2012; L. Harland-Lang et al., 2016; I. Abt et al., 2016; L. Motyka, MS, W. Słomiński and K. Wichmann, 2017).
- There are other possible explanations: small x resummation beyond DGLAP (R. Ball et. al, 2017).

- Golec-Biernat – Wüsthoff or Bartels – Golec-Biernat – Kowalski saturation models assume multiple independent (eikonal) scattering: $\sigma(x, Q^2) = \sigma_0[1 - \exp(-\sigma_1(x, Q^2)/\sigma_0)]$.



Independent multiple scattering
— eikonal approximation —
GBW saturation model

- HT contributions increase with decreasing x: $\sigma^{(\tau=2n)} \sim \sigma_1^n$.
- HT effects: small in F_2, large negative corrections in F_L.

Comment: this picture does not fully agree with QCD-based analysis of HT components.