Small-$x$ Quark and Gluon Helicity Contributions to the Proton Spin Puzzle

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Based on: 2204.11898, 2306.01651, 2308.07461, and earlier publications
Proton Spin

- In the past, proton spin was thought to be the sum of constituent quarks spins.
- Now, we believe it to be the sum of spins of valence quarks, sea quarks and gluons, together with their orbital angular momenta (OAM).
Helicity PDF

- Helicity-dependent generalization of PDFs
- For each parton $f$,
  \[ \Delta f(x, Q^2) = f(x, Q^2, +) - f(x, Q^2, -) \]
- For quarks, we often consider the “flavor singlet” quark hPDF:
  \[ \Delta \Sigma(x, Q^2) = \sum_{q=u,d,s} [\Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2)] \]
  and the “flavor non-singlet” quark hPDF: \[ \Delta q^-(x, Q^2) = \Delta q(x, Q^2) - \Delta \bar{q}(x, Q^2) \]
- Gluon hPDF: \[ \Delta G(x, Q^2) \]
Proton Helicity Sum Rule

- Jaffe-Manohar sum rule: \[ \frac{1}{2} = S_q + S_G + L_q + L_G \]

where the helicity of quarks \((S_q)\) and gluons \((S_G)\) are

\[ S_q(Q^2) = \frac{1}{2} \int_0^1 dx \Delta \Sigma(x, Q^2) \] and \[ S_G(Q^2) = \int_0^1 dx \Delta G(x, Q^2) \]

- In the late 1980’s, EMC measurement implied that \(S_q \approx 0.05\), much lower than what would have been \((1/2)\) had all the proton spin been carried by the constituent quarks.
Current Knowledge of Proton Helicity

- More recently, the proton spin carried by quarks and gluon are estimated to be

\[ S_q(Q^2 = 10 \ \text{GeV}^2) \approx \frac{1}{2} \int_{0.001}^{1} dx \Delta \Sigma(x, 10 \ \text{GeV}^2) \in [0.15, 0.20] \]

\[ S_G(Q^2 = 10 \ \text{GeV}^2) \approx \int_{0.01}^{1} dx \Delta G(x, 10 \ \text{GeV}^2) \in [0.13, 0.26] \]

- They do not add to 1/2. The missing spin can come from:
  - Orbital angular momenta, \( L_q \) and \( L_G \).
  - Small-\( x \) region of \( \Delta \Sigma \) and \( \Delta G \). Scattering experiments can only access finitely small \( x \). The limit will improve with EIC.

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Scattering experiments can only access $\Delta \Sigma$ and $\Delta G$ down to finitely small $x$.

Fill the gap by finding small-$x$ asymptotics for $\Delta \Sigma$ and $\Delta G$ via evolution in $x$, resumming $\alpha_s \ln^2(1/x)$. (Unpolarized BK/JIMWLK resums $\alpha_s \ln(1/x)$.)
Small-x Evolution for Helicity: KPS-CTT Evolution

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- Helicity evolution must keep track of both quark and gluon helicity, in contrast to unpolarized small-$x$ evolution that is dominated by gluons.

$\Delta \Sigma$ and $\Delta G$ are the differences between quark and gluon helicities, respectively. The polarized incoming proton is shown on the left, transitioning to a polarized quark in the middle, and a polarized gluon on the right. The diagrams illustrate the evolution process, with arrows indicating the flow of particles and helicities.
Small-x Evolution for Helicity

- The KPS-CTT evolution in $x$ is complementary to the existing polarized DGLAP evolution.
Small-x Asymptotics with Quark Exchanges

- At small $x$, gluons dominate $\rightarrow N_c \gg 1$
- Still important to include quark exchanges ($\sim N_f/N_c$) for helicity evolution
- Flavor non-singlet hPDF:

$$\Delta q^-(x, Q^2) = \Delta q(x, Q^2) - \Delta \bar{q}(x, Q^2) \sim \left(\frac{1}{x}\right)^{\sqrt{\frac{\alpha_s N_c}{\pi}}}$$  [1610.06197]

- Flavor singlet hPDF:

$$\Delta \Sigma(x, Q^2) = \sum_{q=u,d,s} \left[ \Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2) \right]$$

$$\sim \Delta G(x, Q^2) \sim g_1(x, Q^2) \sim \left(\frac{1}{x}\right)^{3.43 \sqrt{\frac{\alpha_s N_c}{2\pi}}}$$  [2306.01651]
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  \]
  \[2306.01651\]
  Exceed 1 for $\alpha_s \gtrsim 0.18$
  Infinite spin from small $x$???
Corrections to the DLA Evolution

- So far, KPS-CTT evolution resums $\alpha_s \ln^2(1/x)$.
- Potentially significant **single-log corrections**, resumming $\alpha_s \ln(1/x)$.
  - Convoluting with unpolarized dipoles, which obey BK evolution
  - Likely to include saturation mechanism
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- Recently, a **running coupling correction** (daughter dipole prescription) is employed to the DLA evolution in a global fit with polarized DIS & SIDIS data.
- KPS-CTT evolution (with $rc$) starts at $x_0 = 0.1$.
- At larger $x$, employ generalized Born-level initial condition: $\text{Dipole} \sim a \ln(\text{rapidity}) + b \ln(\text{dipole size}) + c$
Global Fit

- Polarized DIS and SIDIS data \((A_1, A_\parallel, A_1^h)\) from SLAC, EMC, SMC, COMPASS and HERMES at \(0.005 \leq x \leq 0.1\) and \(1.69 \text{ GeV}^2 \leq Q^2 \leq 10.4 \text{ GeV}^2\).
  - Include proton, deuteron and helium-3 targets
  - For SIDIS, include \(\pi^\pm\), \(K^\pm\) and unidentified charged hadron productions
- In total, \(N_{\text{pts}} = 226\) data points
- Overall, \(\chi^2 / N_{\text{pts}} = 1.03\)
Global Fit

\[ \int_{10^{-5}}^{0.1} dx \left( \frac{1}{2} \Delta \Sigma + \Delta G \right)(x) = -0.64 \pm 0.60 \]

Significant spin from small \( x \)

[10^{-5}, x_{\text{max}}]

[\text{[} x_{\text{min}}, 0.1\text{]}]

[2308.07461]
Future EIC Impact

- Significant reduction of uncertainty at small $x$ with future EIC data.
Conclusion

- Already at DLA, KPS-CTT evolution provides a promising small-$x$ description of parton helicity, with potential improvement from future EIC results.

- Future work (stay tuned):
  - Improved global fit that includes $pp$ particle production data
  - More deterministic initial condition using a valence-quark wave function
  - Complete single-logarithmic corrections, which will incorporate saturation

- The framework can be modified to calculate OAM’s [1901.07453, 2307.09544] and other TMD’s, e.g. Sivers Function [2108.03667, 2209.03538].

- Collaboration is very welcome 😊
Global Fit: hPDF Results