

# Small- $x$ Quark and Gluon Helicity Contributions to the Proton Spin Puzzle

Yossathorn (Josh) Tawabutr

University of Jyväskylä,  
Department of Physics, Centre of Excellence in Quark Matter



CoE  QM



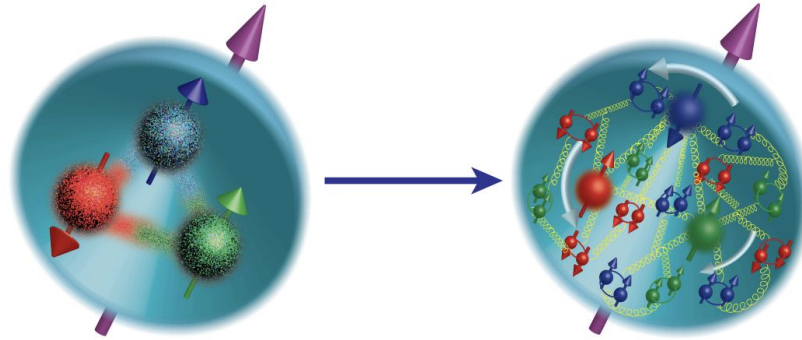
In collaboration with:

Y. Kovchegov, F. Cougoulic, A. Tarasov  
D. Pitonyak, M. Sievert, N. Baldonado  
D. Adamiak, W. Melnitchouk, N. Sato

Based on: 2204.11898, 2306.01651,  
2308.07461, and earlier publications

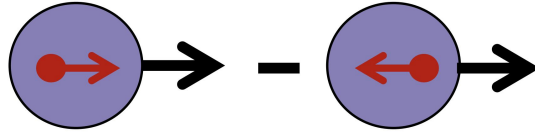


# Proton Spin



- In the past, proton spin was thought to be the sum of constituent quarks spins.
- Now, we believe it to be the sum of spins of valence quarks, sea quarks and gluons, together with their orbital angular momenta (OAM).

# Helicity PDF



- Helicity-dependent generalization of PDFs
- For each parton  $f$ ,

$$\Delta f(x, Q^2) = f(x, Q^2, +) - f(x, Q^2, -)$$

- For quarks, we often consider the “flavor singlet” quark hPDF:

$$\Delta\Sigma(x, Q^2) = \sum_{q=u,d,s} [\Delta q(x, Q^2) + \Delta\bar{q}(x, Q^2)]$$

and the “flavor non-singlet” quark hPDF:  $\Delta q^-(x, Q^2) = \Delta q(x, Q^2) - \Delta\bar{q}(x, Q^2)$

- Gluon hPDF:  $\Delta G(x, Q^2)$

# Proton Helicity Sum Rule

- Jaffe-Manohar sum rule:  $\frac{1}{2} = S_q + S_G + L_q + L_G$

where the helicity of quarks ( $S_q$ ) and gluons ( $S_G$ ) are

$$S_q(Q^2) = \frac{1}{2} \int_0^1 dx \Delta\Sigma(x, Q^2) \quad \text{and} \quad S_G(Q^2) = \int_0^1 dx \Delta G(x, Q^2)$$

- In the late 1980's, EMC measurement implied that  $S_q \approx 0.05$ , much lower than what would have been (1/2) had all the proton spin been carried by the constituent quarks.

# Current Knowledge of Proton Helicity

- More recently, the proton spin carried by quarks and gluon are estimated to be

$$S_q(Q^2 = 10 \text{ GeV}^2) \approx \frac{1}{2} \int_{0.001}^1 dx \Delta\Sigma(x, 10 \text{ GeV}^2) \in [0.15, 0.20]$$

$$S_G(Q^2 = 10 \text{ GeV}^2) \approx \int_{0.01}^1 dx \Delta G(x, 10 \text{ GeV}^2) \in [0.13, 0.26]$$

- They do not add to 1/2. The missing spin can come from:
  - Orbital angular momenta,  $L_q$  and  $L_G$ .
  - Small- $x$  region of  $\Delta\Sigma$  and  $\Delta G$ . Scattering experiments can only access finitely small  $x$ . The limit will improve with EIC.

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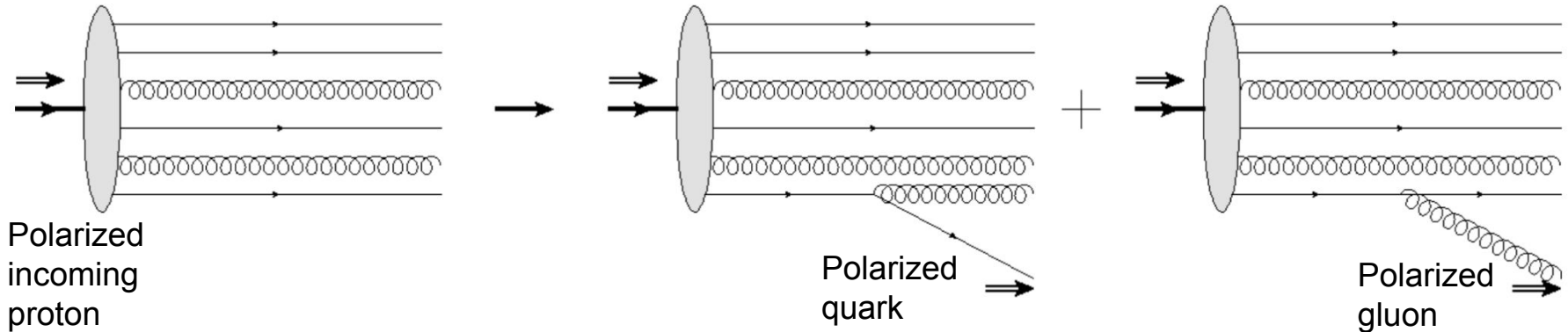
# Small-x Evolution for Helicity: KPS-CTT Evolution

- Scattering experiments can only access  $\Delta\Sigma$  and  $\Delta G$  down to finitely small  $x$ .
- Fill the gap by finding small-x asymptotics for  $\Delta\Sigma$  and  $\Delta G$  via evolution in  $x$ , resumming  $\alpha_s \ln^2(1/x)$ . (Unpolarized BK/JIMWLK resums  $\alpha_s \ln(1/x)$ .)

# Small-x Evolution for Helicity: KPS-CTT Evolution

[2204.11898]

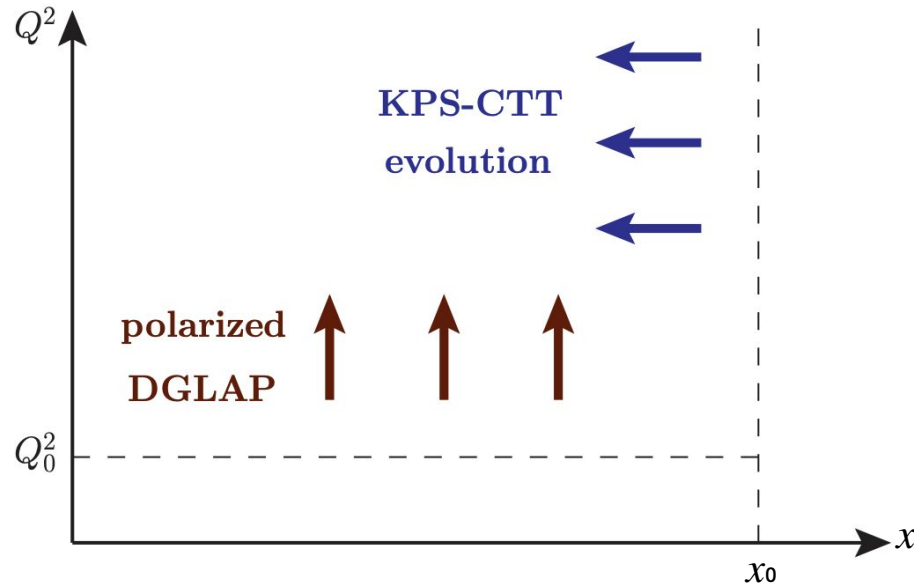
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- Helicity evolution must keep track of both quark and gluon helicity, in contrast to unpolarized small- $x$  evolution that is dominated by gluons.

# Small-x Evolution for Helicity

- The KPS-CTT evolution in  $x$  is complementary to the existing polarized DGLAP evolution.





# Small-x Asymptotics with Quark Exchanges

- At small  $x$ , gluons dominate  $\rightarrow N_c \gg 1$
- Still important to include quark exchanges ( $\sim N_f/N_c$ ) for helicity evolution
- Flavor non-singlet hPDF:

$$\Delta q^-(x, Q^2) = \Delta q(x, Q^2) - \Delta \bar{q}(x, Q^2) \sim \left(\frac{1}{x}\right)^{\sqrt{\alpha_s N_c / \pi}} \quad [1610.06197]$$

- Flavor singlet hPDF:

$$\begin{aligned} \Delta \Sigma(x, Q^2) &= \sum_{q=u,d,s} [\Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2)] \\ &\sim \Delta G(x, Q^2) \sim g_1(x, Q^2) \sim \left(\frac{1}{x}\right)^{3.43\sqrt{\alpha_s N_c / 2\pi}} \quad [2306.01651] \end{aligned}$$

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Exceed 1 for  $\alpha_s \gtrsim 0.18$

Infinite spin from small  $x$ ???

[2306.01651]

# Corrections to the DLA Evolution

- So far, KPS-CTT evolution resums  $\alpha_s \ln^2(1/x)$ .
- Potentially significant **single-log corrections**, resumming  $\alpha_s \ln(1/x)$ .
  - Convoluting with unpolarized dipoles, which obey BK evolution
  - Likely to include saturation mechanism
  - Stay tuned

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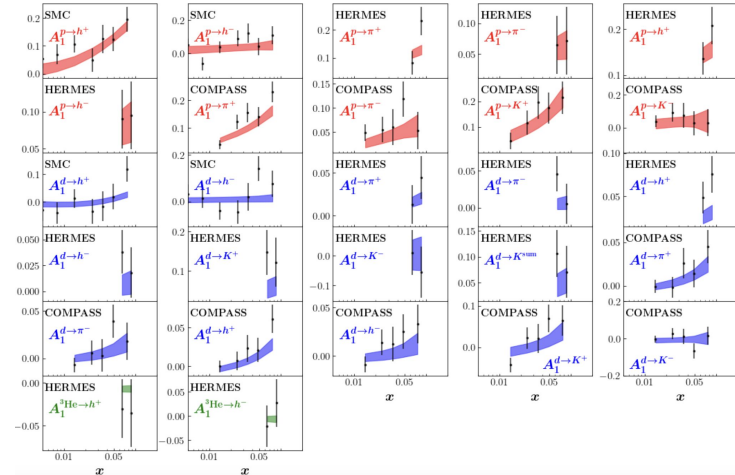
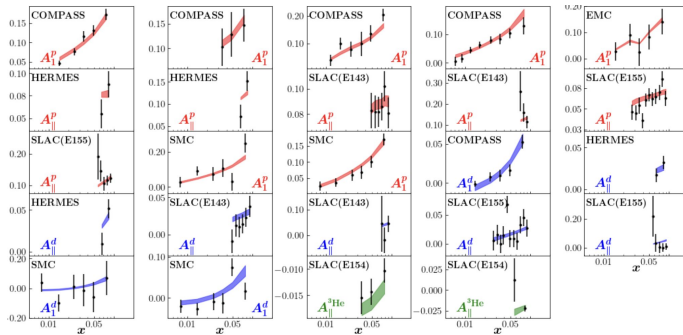
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- Recently, a **running coupling correction** (daughter dipole prescription) is employed to the DLA evolution in a global fit with polarized DIS & SIDIS data.
- KPS-CTT evolution (with rc) starts at  $x_0 = 0.1$ .
- At larger  $x$ , employ generalized Born-level initial condition: [2308.07461]

$$\text{Dipole} \sim a \ln(\text{rapidity}) + b \ln(\text{dipole size}) + c$$

# Global Fit

[2308.07461]

- Polarized DIS and SIDIS data ( $A_1$ ,  $A_{\parallel}$ ,  $A_1^h$ ) from SLAC, EMC, SMC, COMPASS and HERMES at  $0.005 \leq x \leq 0.1$  and  $1.69 \text{ GeV}^2 \leq Q^2 \leq 10.4 \text{ GeV}^2$ .
  - Include proton, deuteron and helium-3 targets
  - For SIDIS, include  $\pi^{\pm}$ ,  $K^{\pm}$  and unidentified charged hadron productions
- In total,  $N_{\text{pts}} = 226$  data points
- Overall,  $\chi^2 / N_{\text{pts}} = 1.03$



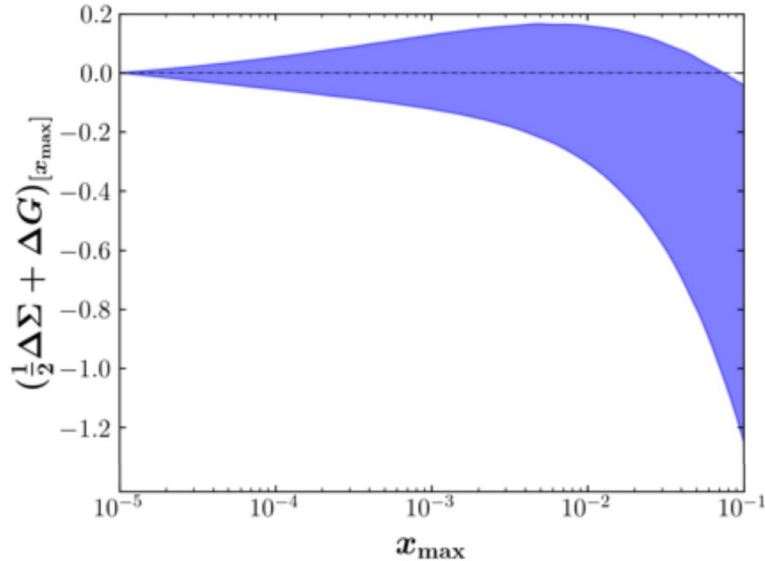
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[2308.07461]

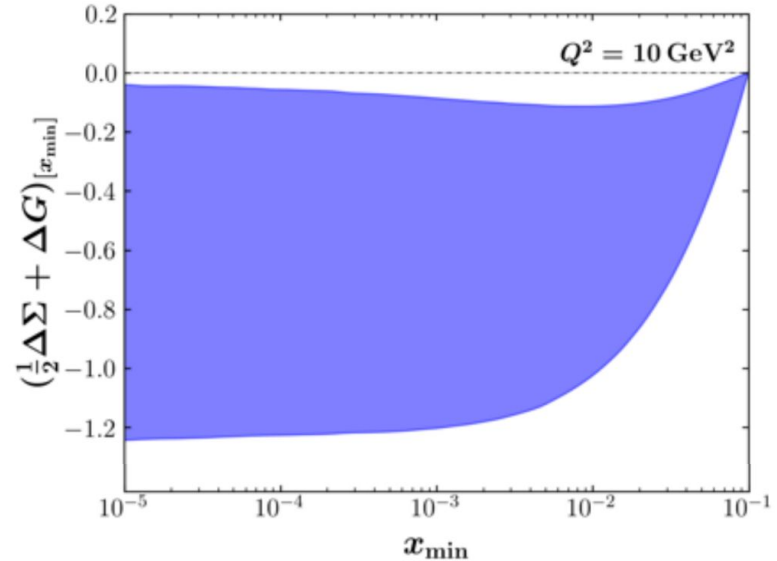
$$\int_{10^{-5}}^{0.1} dx \left( \frac{1}{2} \Delta\Sigma + \Delta G \right) (x) = -0.64 \pm 0.60$$

Significant spin from small  $x$

$[10^{-5}, x_{\max}]$

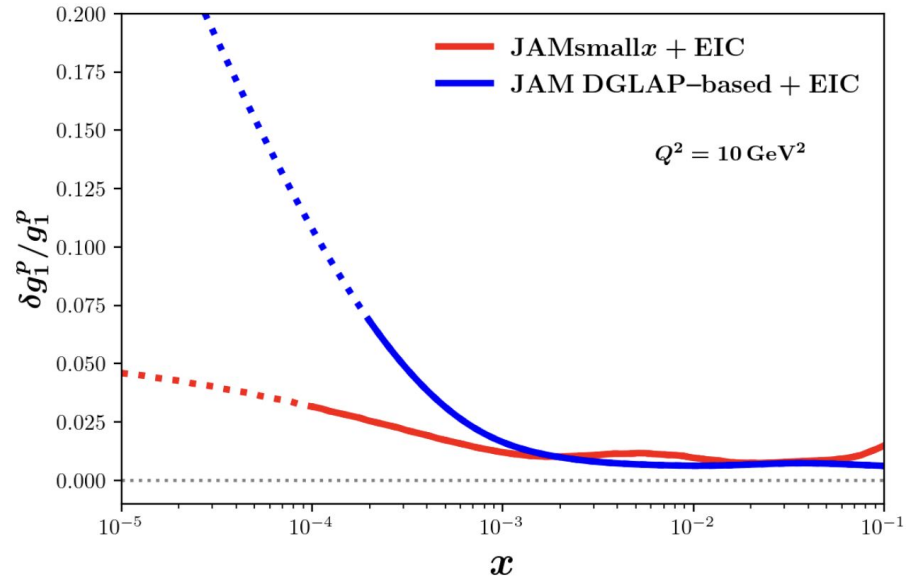
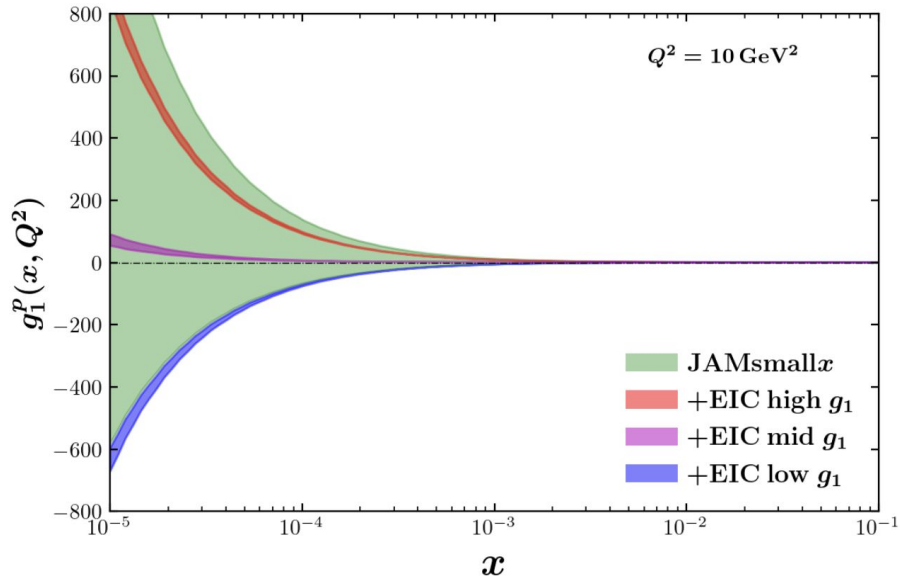


$[x_{\min}, 0.1]$



# Future EIC Impact

- Significant reduction of uncertainty at small  $x$  with future EIC data.





# Conclusion

- Already at DLA, KPS-CTT evolution provides a promising small- $x$  description of parton helicity, with potential improvement from future EIC results.
- Future work (stay tuned):
  - Improved global fit that includes  $pp$  particle production data
  - More deterministic initial condition using a valence-quark wave function
  - Complete single-logarithmic corrections, which will incorporate saturation
- The framework can be modified to calculate OAM's [1901.07453, 2307.09544] and other TMD's, e.g. Sivers Function [2108.03667, 2209.03538].
- Collaboration is very welcome 😊

# Global Fit: hPDF Results

