Diquark and heavy hadron production

Su Houng Lee

- Motivations
- Diquarks
- Ξ_c/D production in pp and heavy ion collisions

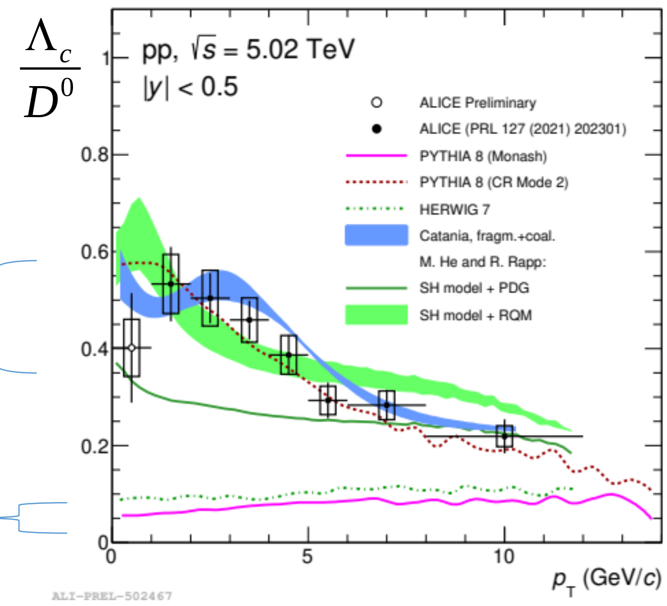
Acknowledgments:

Hyeongock Yun, Sungsik No, Sanghoon Lim, Taesoo Song, Juhee Hong, Aaron Park, Su Houng Lee, Benjamin Doenigus: arXiv:2308.06760

Some recent data

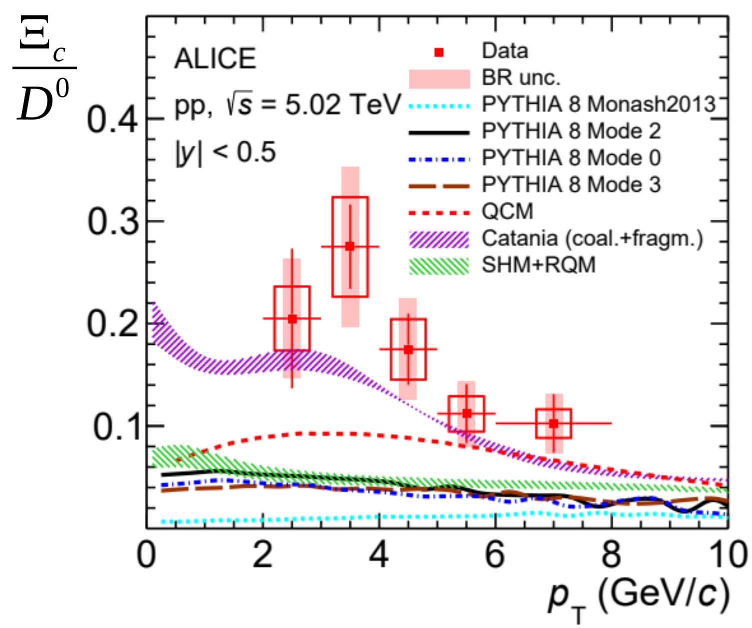
Additional color source (medium)

Tuned to e+e- collision



PLL 127,272001 (21)

Above models can not explain data



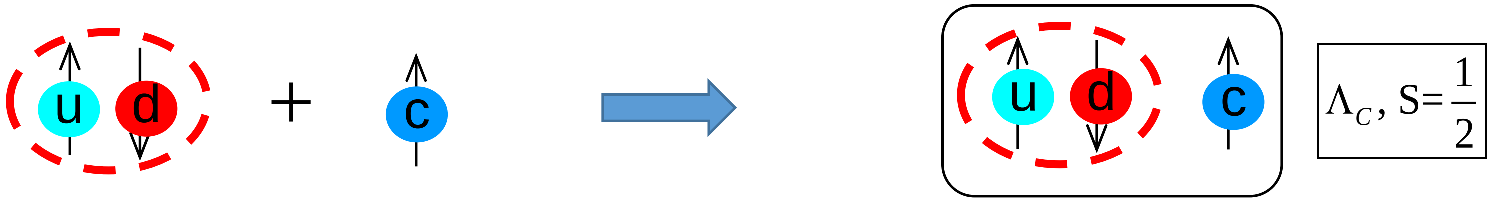
Charmed baryon

SU(3) Flavor $\boxed{1}\otimes\boxed{2} = \boxed{1\,|\,2} \oplus \begin{array}{|c|}\hline 1\\ \hline 2\\ \hline\end{array}$

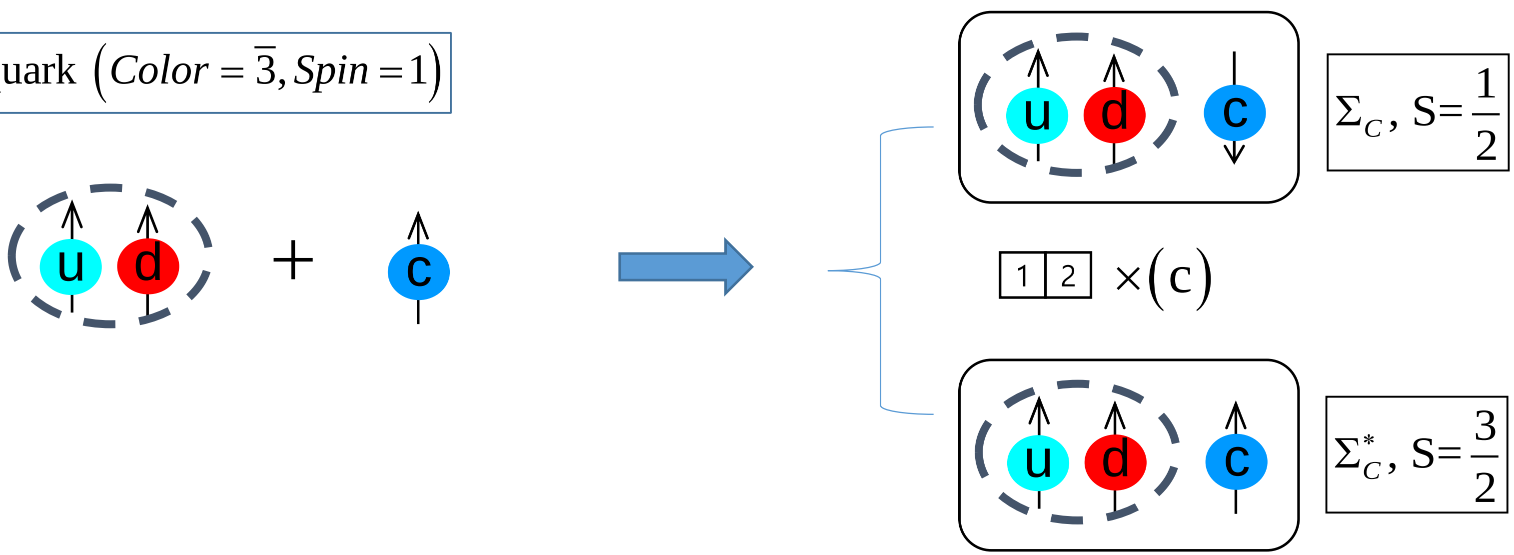
symmetric product   anti-symmetric product

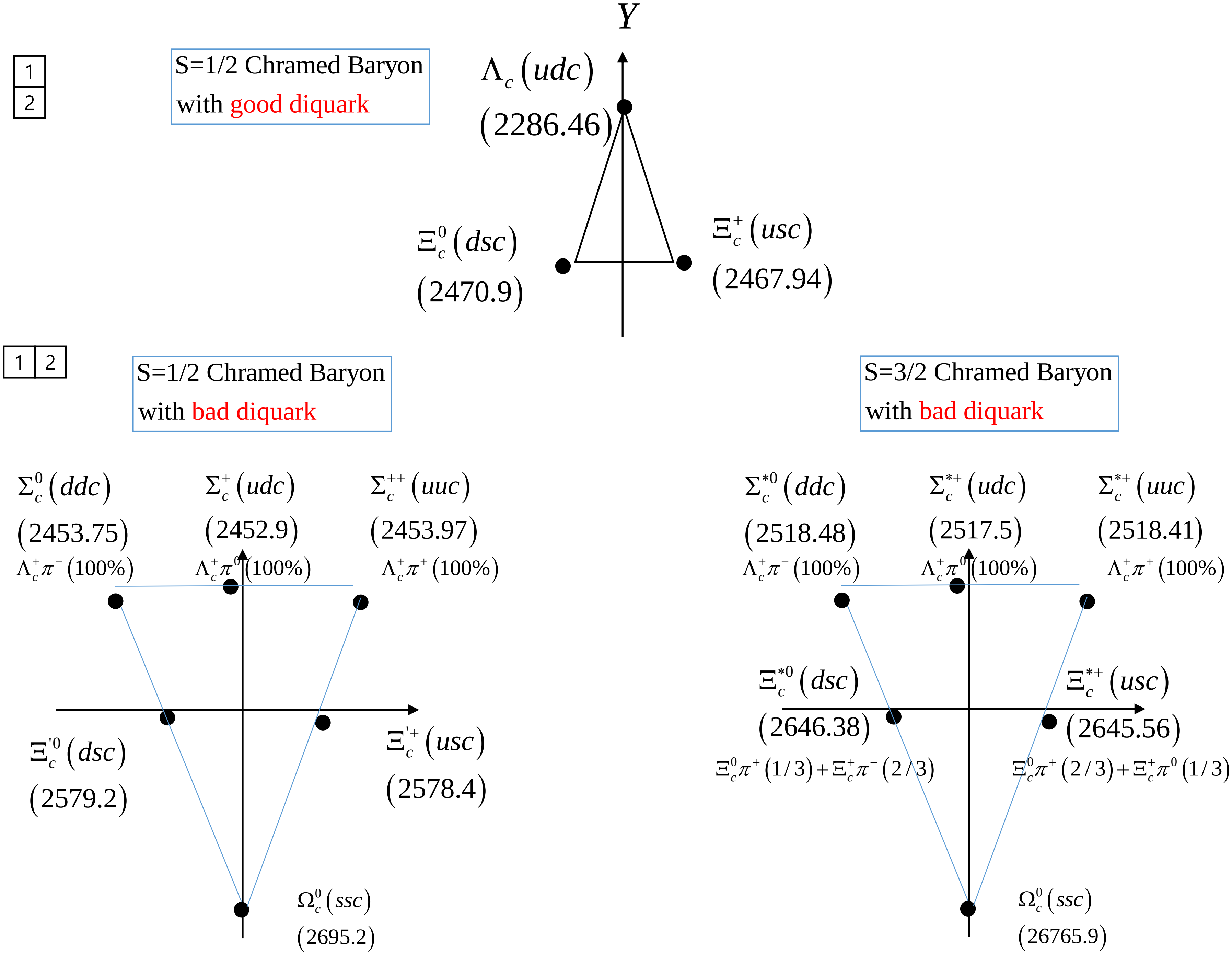
$\begin{array}{|c|}\hline 1\\ \hline 2\\ \hline\end{array}$ Good diquark $\left(Color=\bar{3}, Spin=0\right)$

$\begin{array}{|c|}\hline 1\\ \hline 2\\ \hline\end{array} \times (\mathrm{c})$



$\boxed{1\,|\,2}$ Bad diquark $\left(Color=\bar{3}, Spin=1\right)$





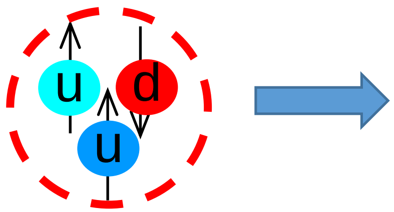
Y
1
2
S=1/2 Chramed Baryon
with good diquark
$\Lambda_c(udc)$
(2286.46)
$\Xi_c^0(dsc)$
(2470.9)
$\Xi_c^+(usc)$
(2467.94)
1 2
S=1/2 Chramed Baryon
with bad diquark
$\Sigma_c^0(ddc)$
(2453.75)
$\Lambda_c^+\pi^-$ (100%)
$\Sigma_c^+(udc)$
(2452.9)
$\Lambda_c^+\pi^0$ (100%)
$\Sigma_c^{++}(uuc)$
(2453.97)
$\Lambda_c^+\pi^+$ (100%)
$\Xi_c^{\prime 0}(dsc)$
(2579.2)
$\Xi_c^{\prime +}(usc)$
(2578.4)
$\Omega_c^0(ssc)$
(2695.2)
S=3/2 Chramed Baryon
with bad diquark
$\Sigma_c^{*0}(ddc)$
(2518.48)
$\Lambda_c^+\pi^-$ (100%)
$\Sigma_c^{*+}(udc)$
(2517.5)
$\Lambda_c^+\pi^0$ (100%)
$\Sigma_c^{*+}(uuc)$
(2518.41)
$\Lambda_c^+\pi^+$ (100%)
$\Xi_c^{*0}(dsc)$
(2646.38)
$\Xi_c^0\pi^+(1/3)+\Xi_c^+\pi^-(2/3)$
$\Xi_c^{*+}(usc)$
(2645.56)
$\Xi_c^0\pi^+(2/3)+\Xi_c^+\pi^0(1/3)$
$\Omega_c^0(ssc)$
(26765.9)

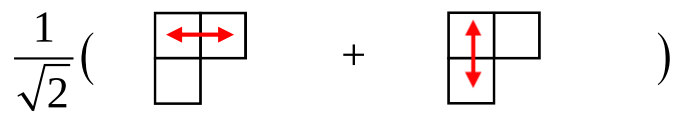
Baryon Octet

SU(3) Flavor 

Contains both good and bad diquarks:→ Smaller contribution from good diquark

p, S= $\frac{1}{2}$

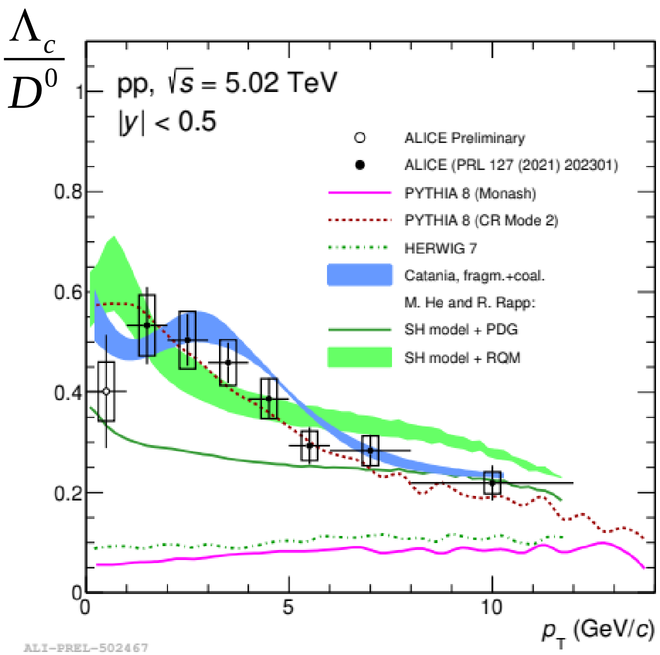


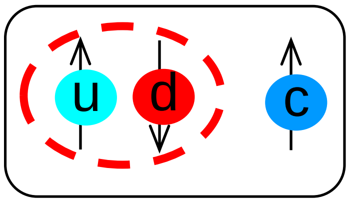


Baryon	Good diquark			Bad diquark		
q=u,d	(qq)	(qs)	(ss)	(qq)	(qs)	(ss)
p, n	1/2	0	0	1/2	0	0
Λ	1/3	1/6	0	0	1/2	0
Σ	0	1/2	0	1/3	1/6	0
Ξ	0	1/2	0	0	1/6	1/3

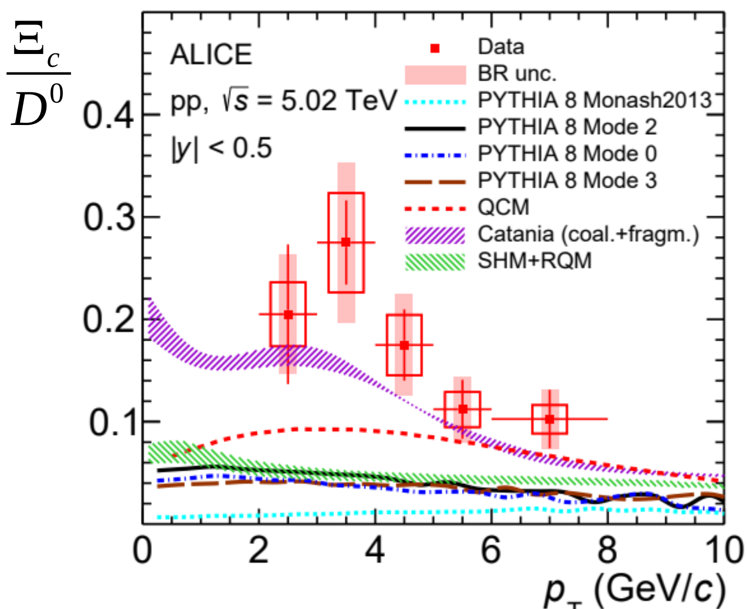
Back to recent data

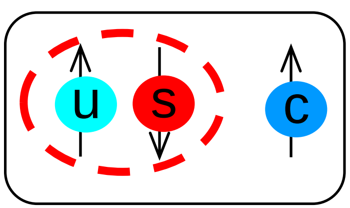
Both contain only good diquarks but different flavors





$(ud)_{S=0}$ - diquark in Λ_c





$(us)_{S=0}$ - diquark in Ξ_c

What does quark model tell us about diquarks?

Naively, it seems strange that (us) diquark is more stable than (ud) diquark

A Quark Model perspectives

$$H = \sum_{i=1}^{n}\left(m_i + \frac{p_i^2}{2m_i}\right) - \sum_{i<j}^{n}\left(\lambda_i^c \lambda_j^c\right) V_{ij}^C\left(r_{ij}\right) - \sum_{i<j}^{n}\frac{\left(\lambda_i^c \lambda_j^c\right)\left(\sigma_i \sigma_j\right)}{m_i m_j} V_{ij}^{CS}\left(r_{ij}\right)$$

☞ Can fit baryon masses

$$V_{ij}^C = -\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0^2} - D,$$

$$V_{ij}^{CS} = \frac{\hbar^2 c^2 \kappa'}{m_i m_j c^4} \frac{e^{-(r_{ij})^2/(r_{0ij})^2}}{(r_{0ij}) r_{ij}} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\,,$$

$$\kappa = 130.0\,\mathrm{MeV\ fm}, \quad a_0 = 0.0318119\,(\mathrm{MeV^{-1} fm})^{1/2}$$
$$D = 975\,\mathrm{MeV}, \quad m_u = m_d = 335\,\mathrm{MeV},$$
$$m_s = 652\,\mathrm{MeV}, \quad m_c = 1940\,\mathrm{MeV},$$
$$\alpha = 1.2499\,\mathrm{fm}^{-1}, \ \beta = 0.0008314\,(\mathrm{MeV\ fm})^{-1},$$
$$\gamma = 0.00168\,\mathrm{MeV}^{-1}, \ \kappa_0 = 185.144\,\mathrm{MeV}.$$

Particle	Experimental Value (MeV)	Mass (MeV)	Variational Parameters (fm^{-2})
Λ	1115.7	1116.3	$a_1 = 3.0,\ a_2 = 2.9$
Λ_c	2286.5	2272.2	$a_1 = 3.1,\ a_2 = 3.9$
Σ_c	2452.9	2446.2	$a_1 = 2.2,\ a_2 = 4.0$
Σ_c^*	2517.5	2531.4	$a_1 = 2.0,\ a_2 = 3.5$
Σ	1192.6	1202.2	$a_1 = 2.2,\ a_2 = 3.3$
Σ^*	1383.7	1401.1	$a_1 = 1.9,\ a_2 = 2.5$
Ξ	1314.9	1331.8	$a_1 = 3.6,\ a_2 = 3.1$
Ξ^*	1531.8	1544.1	$a_1 = 3.1,\ a_2 = 2.3$
Ξ_c	2467.8	2474.2	$a_1 = 3.5,\ a_2 = 5.0$
Ξ_c^*	2645.9	2654.9	$a_1 = 2.6,\ a_2 = 4.6$
Ξ_c'	2579.2	2570.2	$a_1 = 2.8,\ a_2 = 5.2$
Ω_c	2695.2	2684.7	$a_1 = 3.9,\ a_2 = 6.1$
Ω_c^*	2765.9	2768.8	$a_1 = 3.6,\ a_2 = 5.3$
p	938.27	951.42	$a_1 = 2.5,\ a_2 = 2.5$
Δ	1232	1246.9	$a_1 = 1.8,\ a_2 = 1.8$

Color spin interaction

$$H=\sum_{i=1}^{n}\left(m_i+\frac{p_i^2}{2m_i}\right)-\sum_{i<j}^{n}\left(\lambda_i^c\lambda_j^c\right)V_{ij}^C\left(r_{ij}\right)-\sum_{i<j}^{n}\frac{\left(\lambda_i^c\lambda_j^c\right)\left(\sigma_i\sigma_j\right)}{m_i m_j}V_{ij}^{SS}\left(r_{ij}\right)$$

☞ Color-spin interaction for 2 body:

$$K=-\sum_{i<j}^{N}\left(\lambda_i^c\lambda_j^c\right)\left(\sigma_i^s\sigma_j^s\right)$$

$K<0$ attraction;

$K>0$ repulsion

	$Q-Q$				$Q-\bar{Q}$			
Color	A	S	A	S	1	8	1	8
Flavor	A	A	S	S				
Spin	A(0)	S(1)	S(1)	A(0)	0	0	1	1
K	-8	-4/3	8/3	4	-16	2	16/3	-2/3

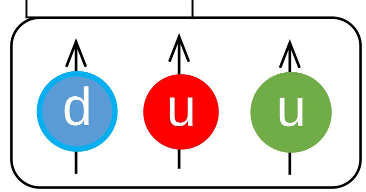
good diquark (A, A, A(0): $K=-8$)

bad diquark (A, S, S(1): $K=8/3$)

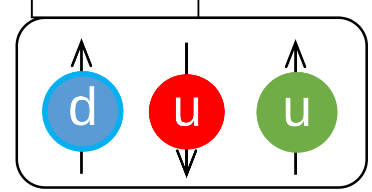
☞ $M_\Delta - M_P \approx 290$ MeV $\rightarrow$ K factors $3\times\left(\frac{8}{3}\right) - \left(-8\right)=16$

K factor of $1 \rightarrow$ 18 MeV

Delta $S=3/2$



Proton $S=1/2$

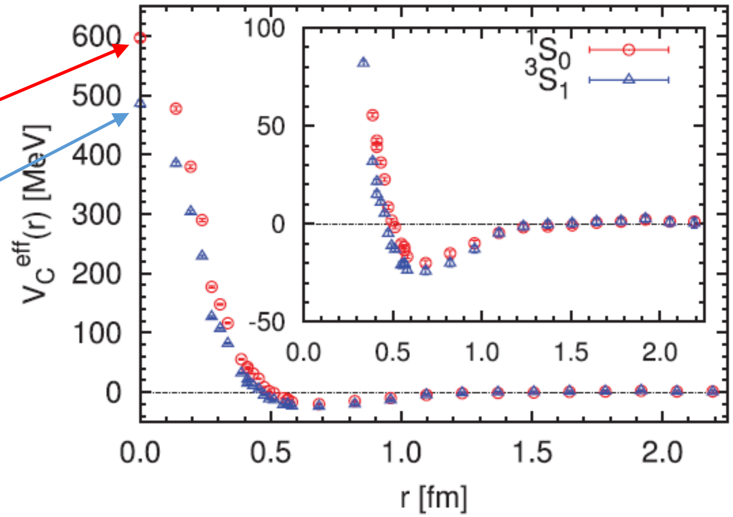


☞ NN force in SU(2) spin 1 vs spin 0 channel: comparison to lattice

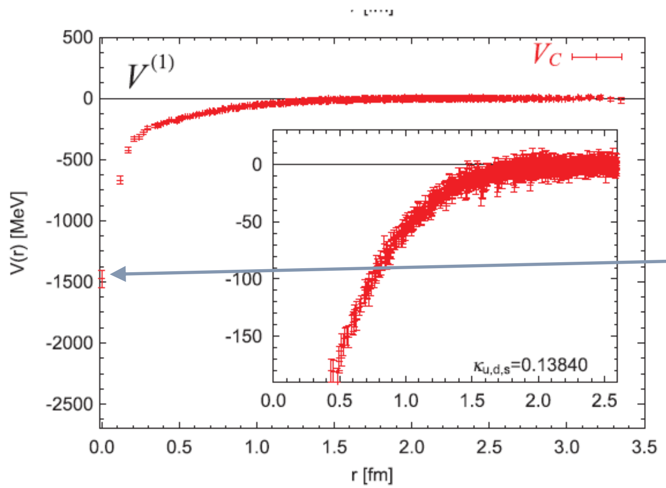
$$K_{2-N} = K_{6-quark} - \left(K_{1N} + K_{1N}\right)$$

$$\frac{K_{2-N}^{S=0}}{K_{2-N}^{S=1}} = 1.29$$ → comparison

QCD HAL collaboration

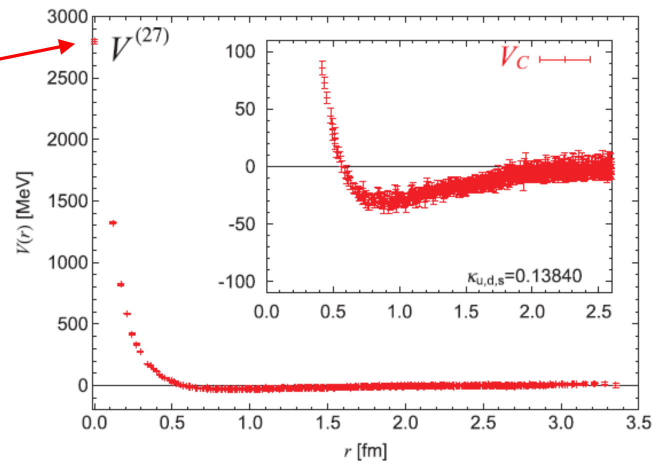


☞ H dibaryon channel: Flavor 1 vs Flavor 27



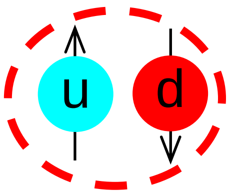
$$\frac{K_{2-N}^{F=27}}{K_{2-N}^{F=1}} = -3$$

(HAL QCD Collaboration)



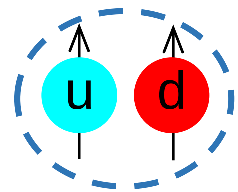
$$K_m = -\frac{1}{m_i m_j}\left(\lambda_i^c \lambda_j^c\right)\left(\sigma_i^s \sigma_j^s\right)$$

Isolated diquarks



good diquark

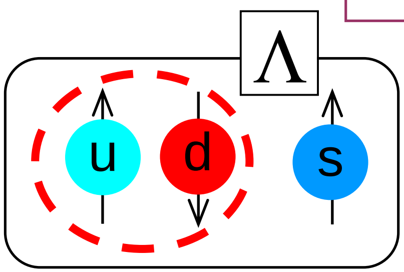
$$K_m = -8\frac{1}{m_u m_d}$$



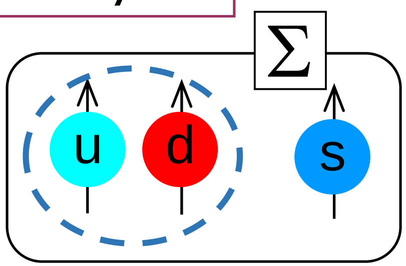
bad diquark

$$K_m = \frac{8}{3}\frac{1}{m_u m_d}$$

Diquarks inside a Baryon



$$K_m^{\Lambda_c} = -8\frac{1}{m_u m_d}$$



$$K_m^{\Sigma_c} = +\frac{8}{3}\frac{1}{m_u m_d} - \frac{32}{3}\frac{1}{m_u m_s}$$

$$m_u = m_d = 300 \text{ MeV}, \quad m_s = 500 \text{ MeV}, \quad m_c = 1500 \text{ MeV}, \quad m_b = 4700 \text{ MeV}$$

Mass diff	$M_\Delta - M_N$	$M_\Sigma - M_\Lambda$	$M_{\Sigma c} - M_{\Lambda c}$	$M_{\Sigma b} - M_{\Lambda b}$
Formula	290 MeV	77 MeV	154 MeV	180 MeV
Experiment	290 MeV	75 MeV	170 MeV	192 MeV

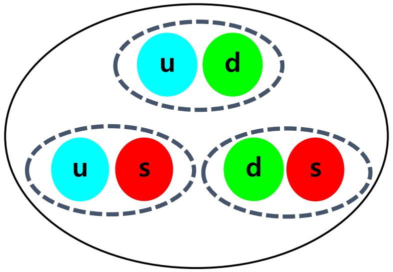
Color spin: Exotics

$$K_m = -\sum_{i<j}^{n} \frac{1}{m_i m_j} \left(\lambda_i^c \lambda_j^c\right)\left(\sigma_i^s \sigma_j^s\right)$$

☞ H-dibaryon

$$K^{Exact}_{Flavor=1} = -\frac{5}{m_u^2} - \frac{22}{m_u m_s} + \frac{3}{m_s^2}$$

In SU(3) symmetric limit $K \to -\frac{24}{m_u^2}$



☞ Pentaquark

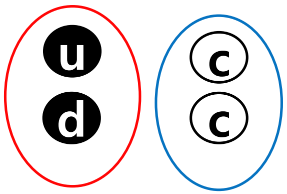
([3], 1/2)	$\begin{pmatrix} -\frac{14}{m_1{}^2} - \frac{22}{m_1 m_5} & -\frac{2}{\sqrt{3}m_1{}^2} - \frac{2}{\sqrt{3}m_1 m_5} \\ -\frac{2}{\sqrt{3}m_1{}^2} - \frac{2}{\sqrt{3}m_1 m_5} & -\frac{46}{3m_1{}^2} + \frac{26}{3m_1 m_5} \end{pmatrix}$
$SU(6)_{CS}$	[21] $[421^3]$
Eigenvalue	$-\frac{8}{3}(\sqrt{31}+8),\ \frac{8}{3}(\sqrt{31}-8)$

Park, Cho, SHL, PRD99(2019) 010087

LHCb discovery → $\left(uds c\bar{c}\right)_{I=0,\ S=1/2}$ $\Lambda + J/\psi$

Pc (Lipkin 87) → $\left(uuds\bar{c}\right)_{I=1/2,\ S=1/2}$

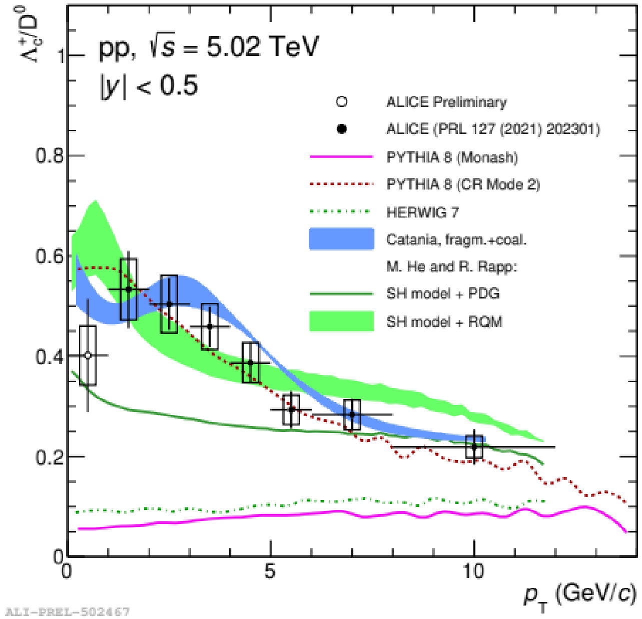
☞ Tetraquark Tcc

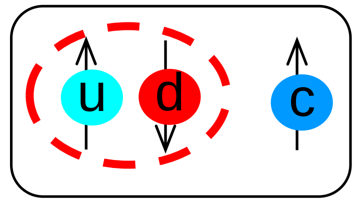


$$K_{T_{cc}(3875)} - K_D - K_{D*} = \begin{pmatrix} -8\frac{1}{m_q^2} + \frac{8}{3}\frac{1}{m_c^2} + \frac{32}{3}\frac{1}{m_c m_q} & -8\sqrt{2}\frac{1}{m_c m_q} \\ -8\sqrt{2}\frac{1}{m_c m_q} & -\frac{4}{3}\frac{1}{m_q^2} + 4\frac{1}{m_c^2} + \frac{32}{3}\frac{1}{m_c m_q} \end{pmatrix}$$

Back to recent data and color-spin interaction

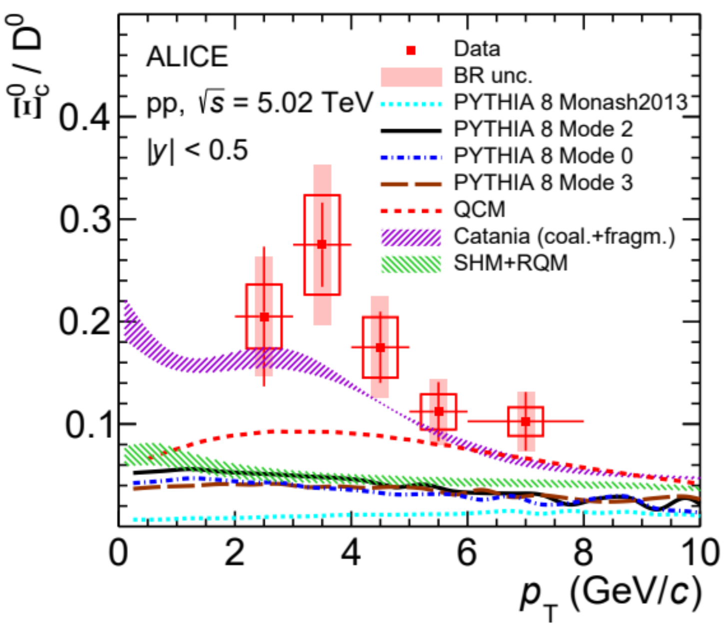
$$K_m = -\sum_{i<j}^{n} \frac{1}{m_i m_j} \left(\lambda_i^c \lambda_j^c\right)\left(\sigma_i^s \sigma_j^s\right)$$

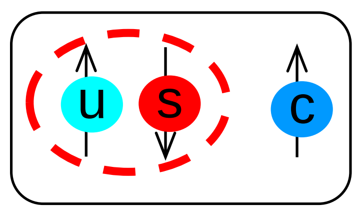




$(ud)_{S=0}$ - diquark in Λ_c $\quad K_m = -\frac{8}{m_u m_d}$

Stronger color-spin attraction





$(us)_{S=0}$ - diquark in Ξ_c $\quad K_m = -\frac{8}{m_u m_s}$

However Color-color interaction

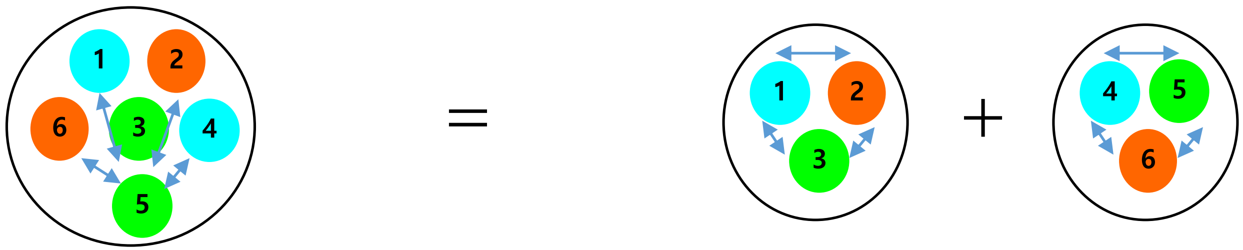
$$H=\sum_{i=1}^{n}\left(m_i+\frac{p_i^2}{2m_i}\right)-\sum_{i<j}^{n}\left(\lambda_i^c\lambda_j^c\right)V_{ij}^C\left(r_{ij}\right)-\sum_{i<j}^{n}\frac{\left(\lambda_i^c\lambda_j^c\right)\left(\sigma_i\sigma_j\right)}{m_i m_j}V_{ij}^{SS}\left(r_{ij}\right)$$

☞ Color-Color interaction is not important for short range N-N interaction

$$\sum_{i<j}^{N}\left(\lambda_i^c\lambda_j^c\right)=\frac{1}{2}\left[\left(\lambda_1^c+\ldots+\lambda_N^c\right)^2-\lambda_1^2-\ldots\lambda_N^2\right]$$

$$N=N_{B_1}+N_{B_2}$$

$$=0-\frac{8}{3}\left(N_{B_1}+N_{B_2}\right)=\sum_{i<j}^{N_{B_1}}\left(\lambda_i^c\lambda_j^c\right)+\sum_{i<j}^{N_{B_2}}\left(\lambda_i^c\lambda_j^c\right)$$



Mass effect in Color-Color interaction: Example Karliner and Rosner

☞ Coulomb interaction: heavier particles will have larger Coulomb attraction

$$H_{cc} = \ldots + \lambda_i^c \lambda_j^c \left(\frac{g}{r_{ij}} \right) + \ldots . \qquad r \approx \frac{1}{mg^2}, \qquad E_C \approx -mg^4$$

☞ Solving for diquark states in the quark model with vacuum parameters.

$$\text{Binding energy} = \text{M}_{diquark} - m_{q1} - m_{q2}$$

Diquark	Mass(MeV)	Binding energy(MeV)	Size(fm)
ud	680.3	10.22	0.761
us	970.7	-16.30	0.714
uc	2244.8	-30.23	0.700

Contribution	ud	us	uc
$m_u + m_q - \frac{1}{2}D$	182.5	499.5	1787.5
$\sum_{i=1}^{2} \frac{\mathbf{p}_i^2}{2m_i}$	383.6	329.9	265.8
V^C	267.1	236.6	227.5
V^{CS}	-152.9	-95.34	-36.0
Total	680.3	970.7	2244.8

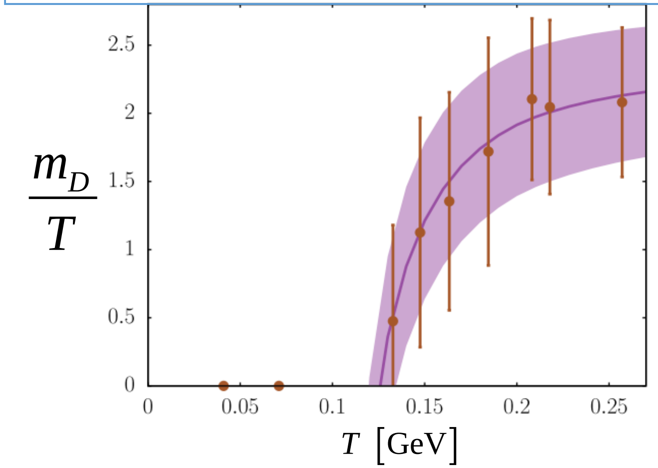
Including color-color interaction (us) diquark is more attractive than (ud) diquark

At Finite temperature

☞ Finite temperature potential

$$V_{ij}^C = -\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0^2} - D, \quad \Rightarrow \quad V_{ij}^C = -\kappa\left[m_D + \frac{e^{-m_D r_{ij}}}{r_{ij}}\right] + \frac{1}{a_0^2}\left[\frac{2}{m_D} - \frac{e^{-m_D r_{ij}}(2+m_D r_{ij})}{m_D}\right] - D.$$

Lafferty, Rothkopf, PRD101(2020)056010



☞ Use thermal quark masses used

$$m_{u,d} = 300\text{ MeV}, \quad m_s = 400\text{ MeV}$$

☞ (us) diquark is more attractive than (ud)

$$B_{(us)} = M_{(us)\,diquark} - m_u - m_s$$

At $T=0$ → Scheme 1

At $T = 181$ MeV → Scheme 4

Type	$B_{(us)} - B_{(ud)}$	m_{us}(MeV)	Size(fm)
Scheme 1	-26.52	970.7	0.714
Scheme 2	-11.14	710.5	0.753
Scheme 3	-9.67	673.2	0.800
Scheme 4	-7.81	613.8	0.889

Effects of Diquarks in Ξ_c/D production

Strong (us) diquark correlation

→ Enhanced Ξ_c production.

→ Smaller effects on Hyperon production

Model it through the presence of (us) diquarks in the hadronization

→ Use coalescence model

Model diquark P_T distribution

☞ Blast-wave fit to ALICE data

$$\frac{1}{m_T}\frac{dN}{dm_T} \propto m_T \int_0^R I_0\left(\frac{p_T \sinh\rho}{T_{kin}}\right) K_1\left(\frac{m_T \cosh\rho}{T_{kin}}\right) r\,dr$$

$$\rho = \tanh^{-1}\beta_r \qquad \beta_r = \left(\frac{r}{R}\right)^n \beta_s,$$

Collision system	$\langle dN_{ch}/d\eta \rangle$	T_{kin} (GeV)	$\langle \beta \rangle$	n
pp 13 TeV MB	6.88	0.184	0.270	3.878
pp 5.02 TeV MB	4.30	0.181	0.198	6.248
p–Pb 5.02 TeV NSD	17.81	0.177	0.423	1.846
Pb–Pb 5.02 TeV 0-10%	1781	0.113	0.659	0.650

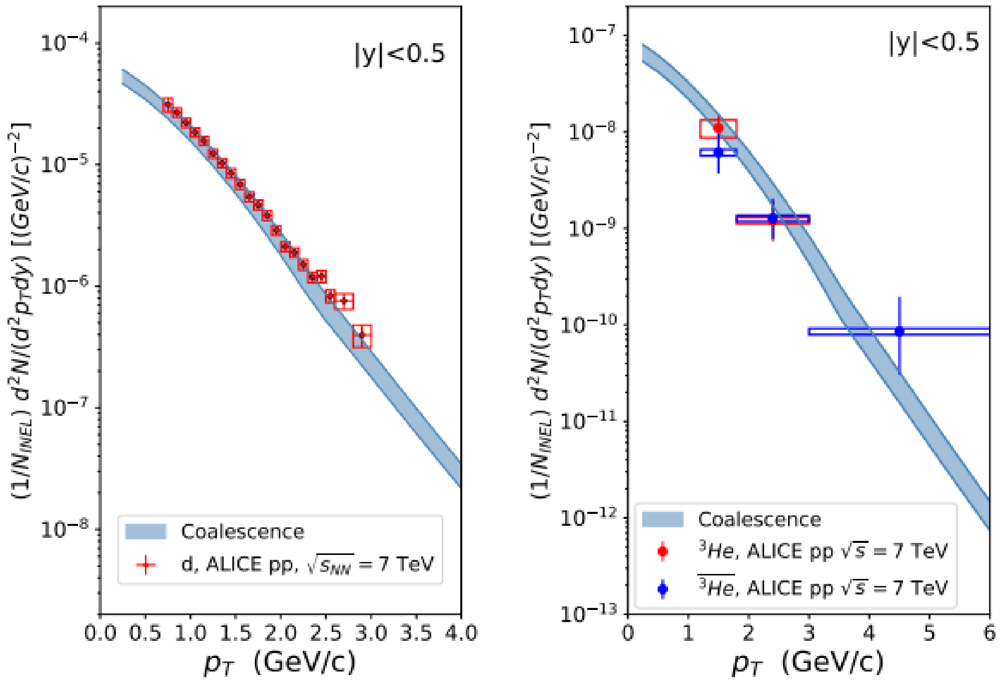
Use coalescence model to calculate extra Ξ_c from (us)+c for pp collision

☞ 2-Dim Coalescence model

$$\frac{d^2 N_{\Xi_c^0}}{d^2 P_T} = \frac{g_{\Xi_c^0}}{g_{[us]} g_c} \left(2\sqrt{\pi}\sigma\right)^2 \frac{1}{A} \int d^2 p_{1T} d^2 p_{2T} \frac{d^2 N_{[us]}}{d^2 p_{[us]T}} \frac{d^2 N_c}{d^2 p_{cT}} \times \exp\left[-\sigma^2 p'^2\right] \delta^{(2)}(\vec{P}_T - \vec{p}_{[us]T} - \vec{p}_{cT}),$$

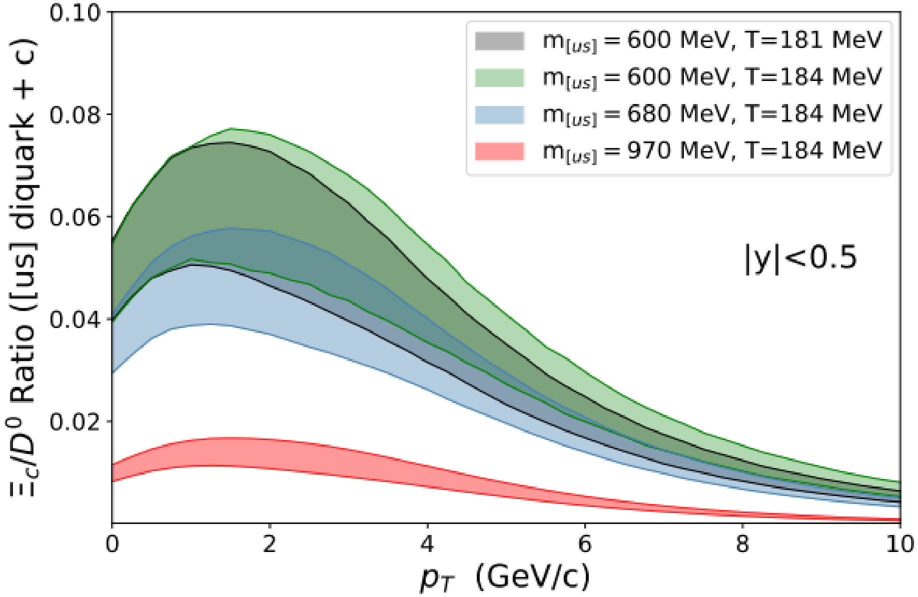
$\sigma = \sqrt{8/3} r_{\Xi_c^0}$ and we used $r_{\Xi_c^0} = 0.222$ fm quark model calculation.

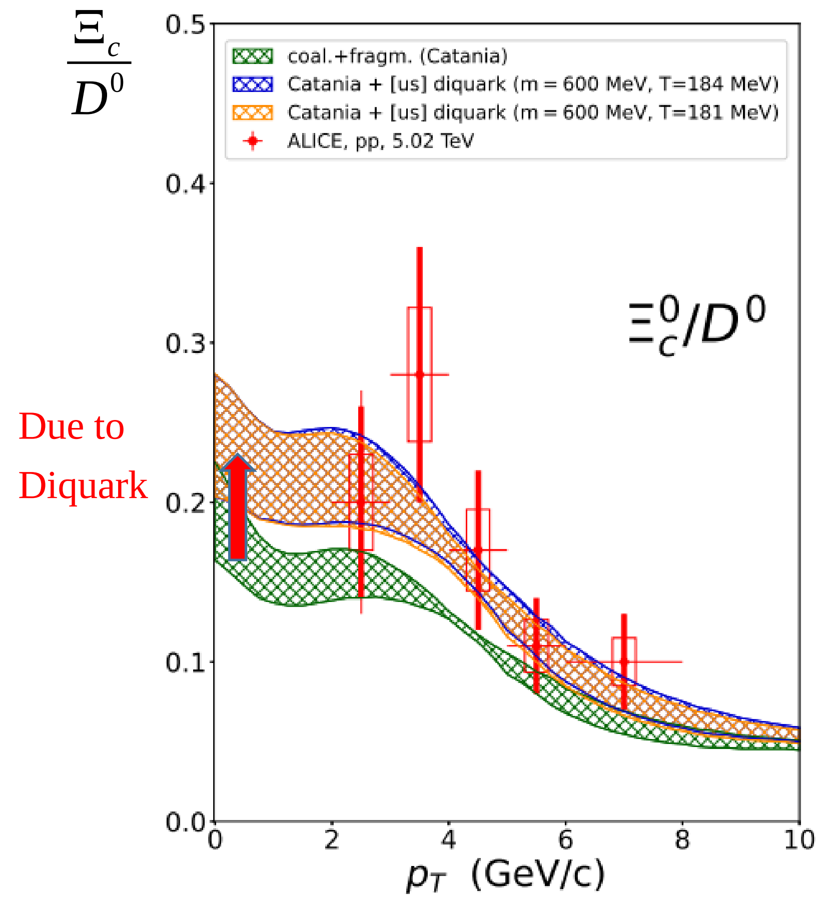
☞ A from fit to deuteron and He3



Result

☞ Ξ_c from (us)+c coalescence





Summary

- Quark model shows that (us) or (ds) diquarks are more stable than an (ud) diquark

- Such a strong correlation in (us) or (ds) will enhance the production of baryons composed dominated by such configurations.

- The anomalous enhancement of $\frac{\Xi_c}{D^0}$ in pp can be explained by the presence of (us) diquarks in the hadronization process.

Similar enhancement is expected in heavy ion collision