Exploring Strongly-Interacting Matter in Heavy-ion Collisions

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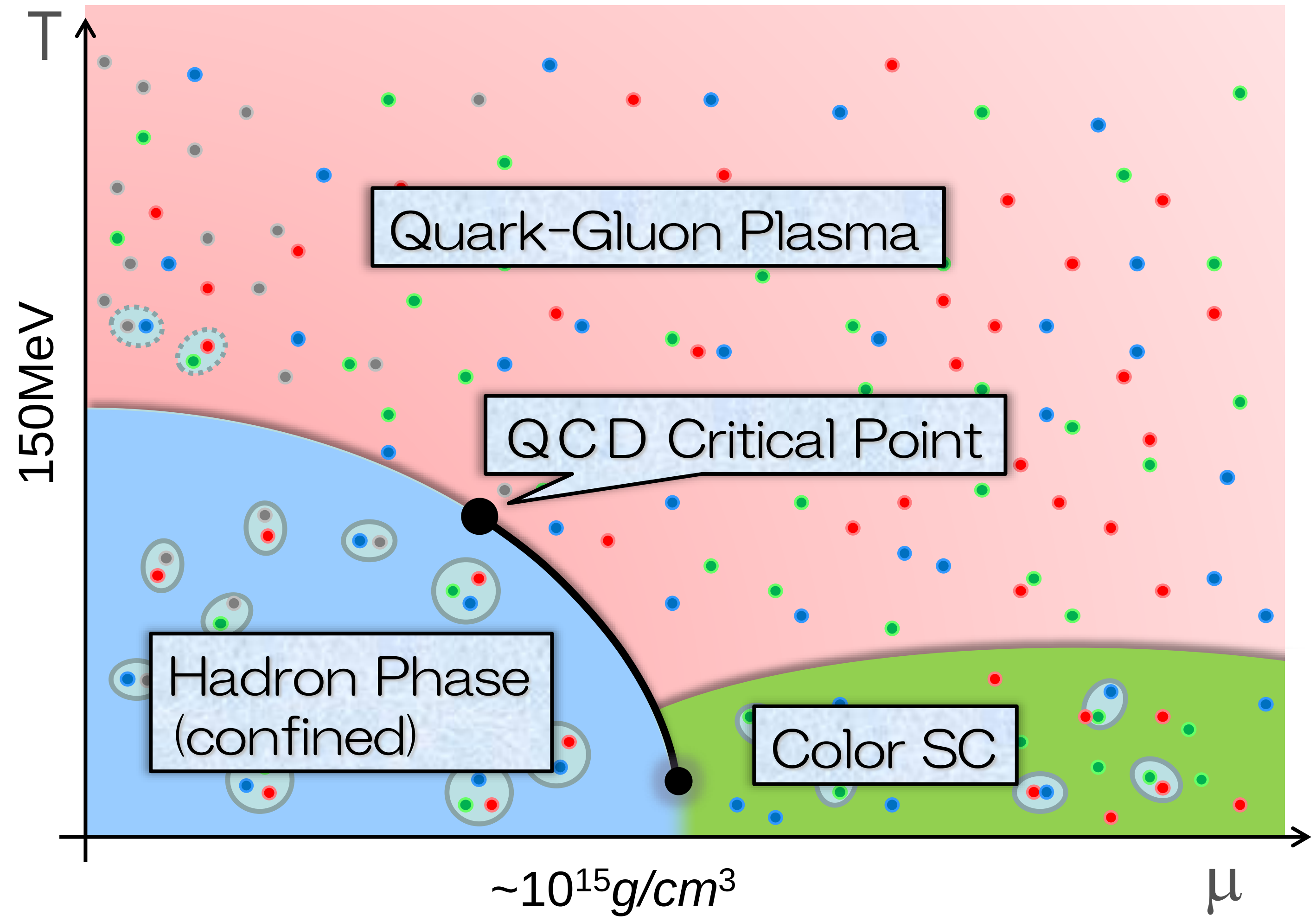
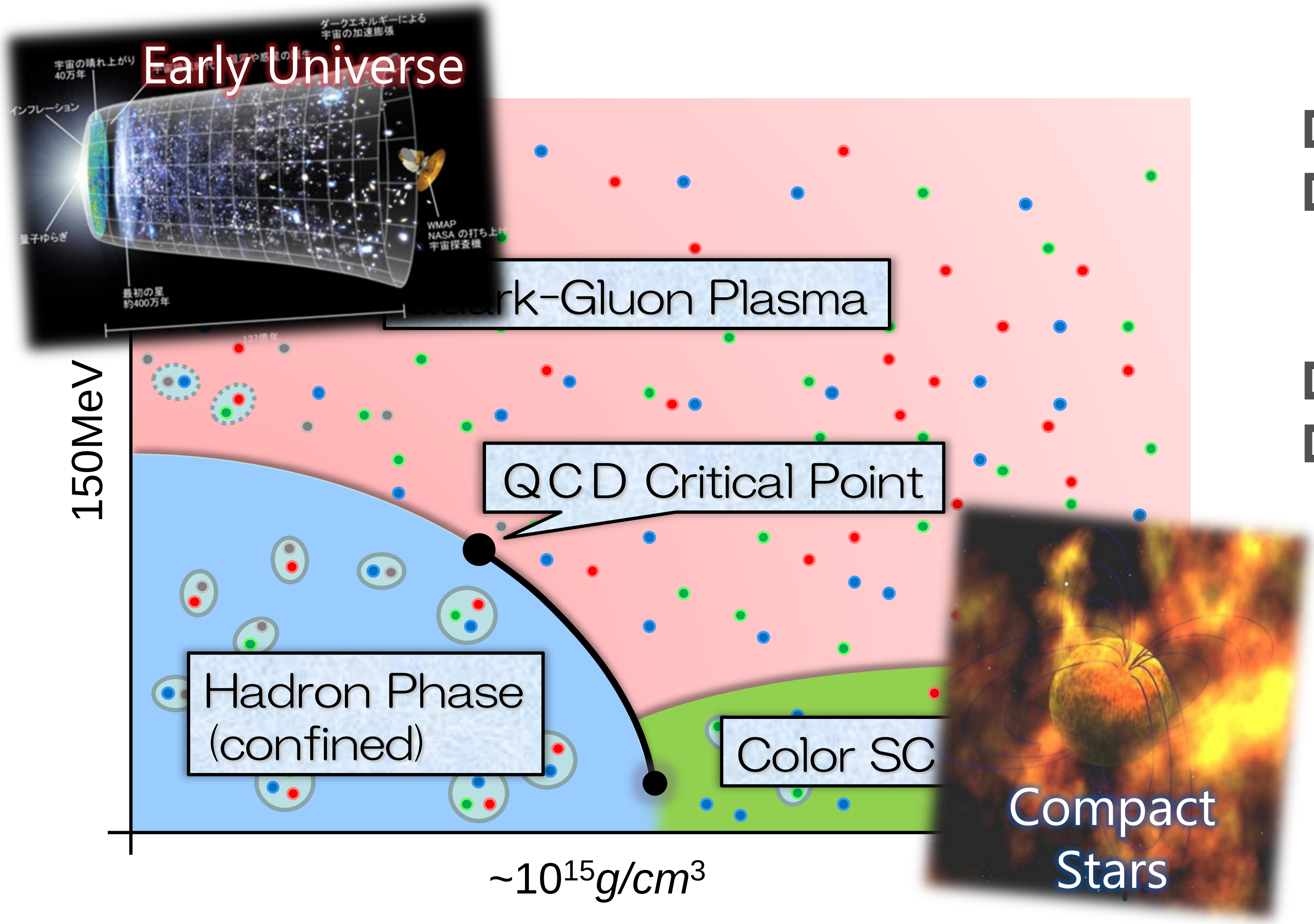
ISMD 2023, Gyöngyös, Hungary, August. 22, 2023

QCD Phase Diagram



- Crossover at $\mu = 0$
- Possible first-order transition and QCD critical point in dense region
- Multiple QCD-CP? MK+ ('02)
- Color superconducting phases in dense and cold quark matter

QCD Phase Diagram



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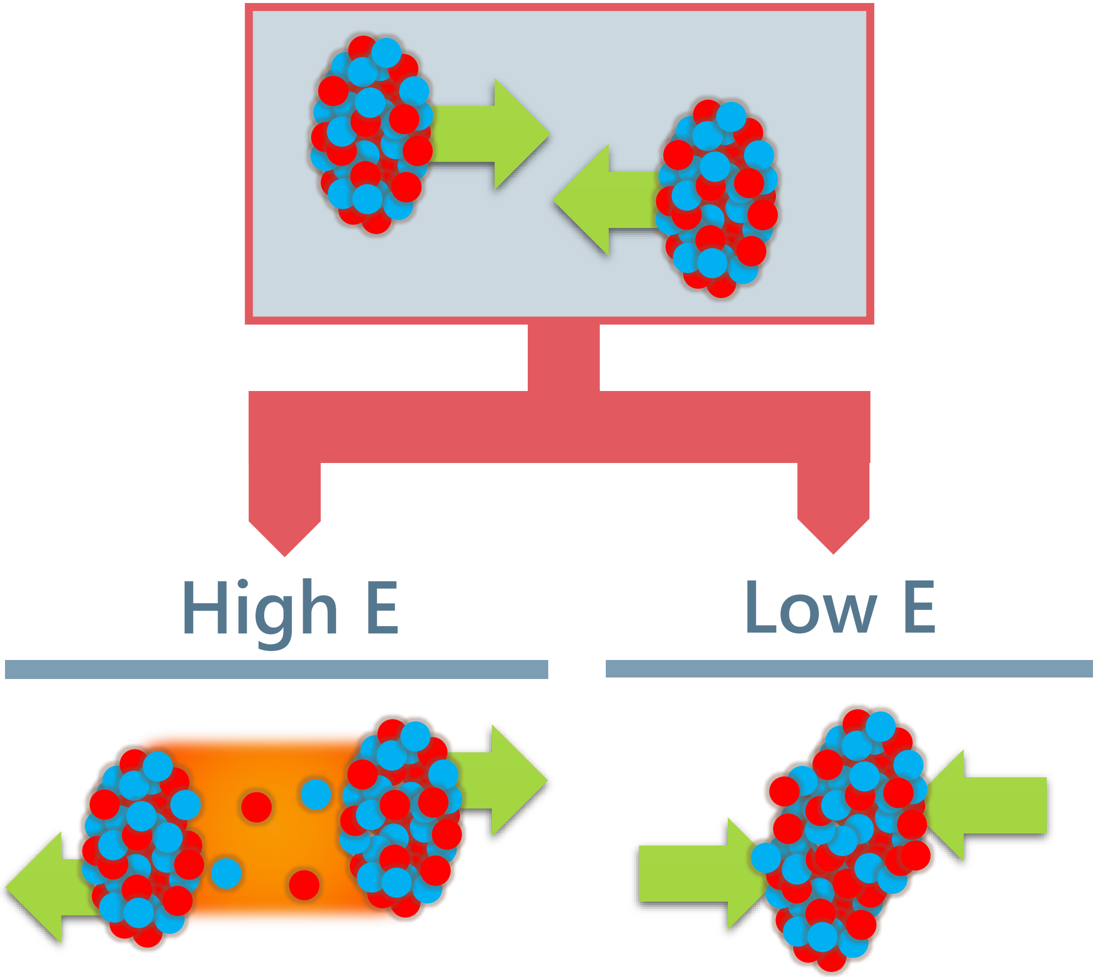
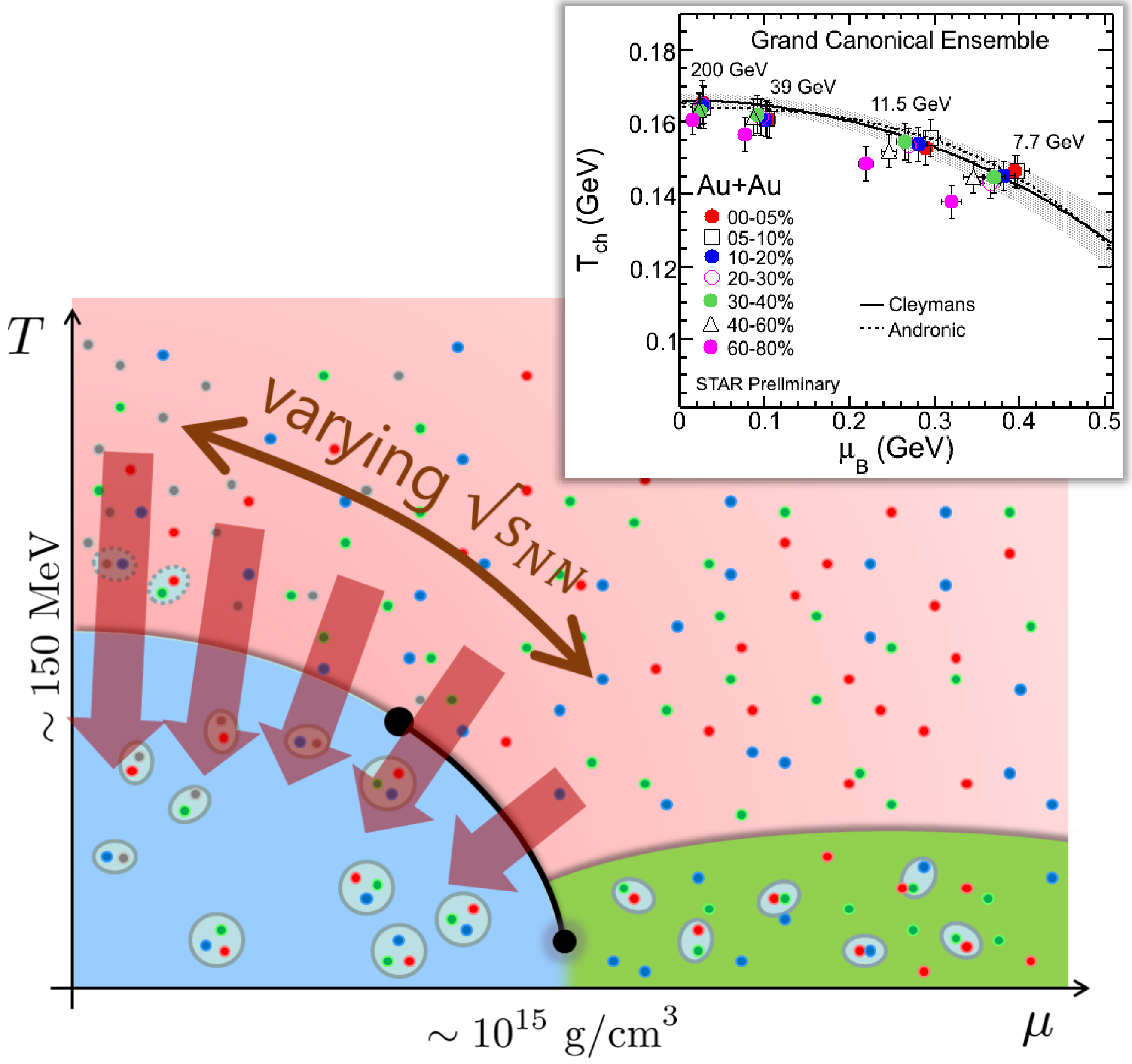
- Color superconducting phases in dense and cold quark matter

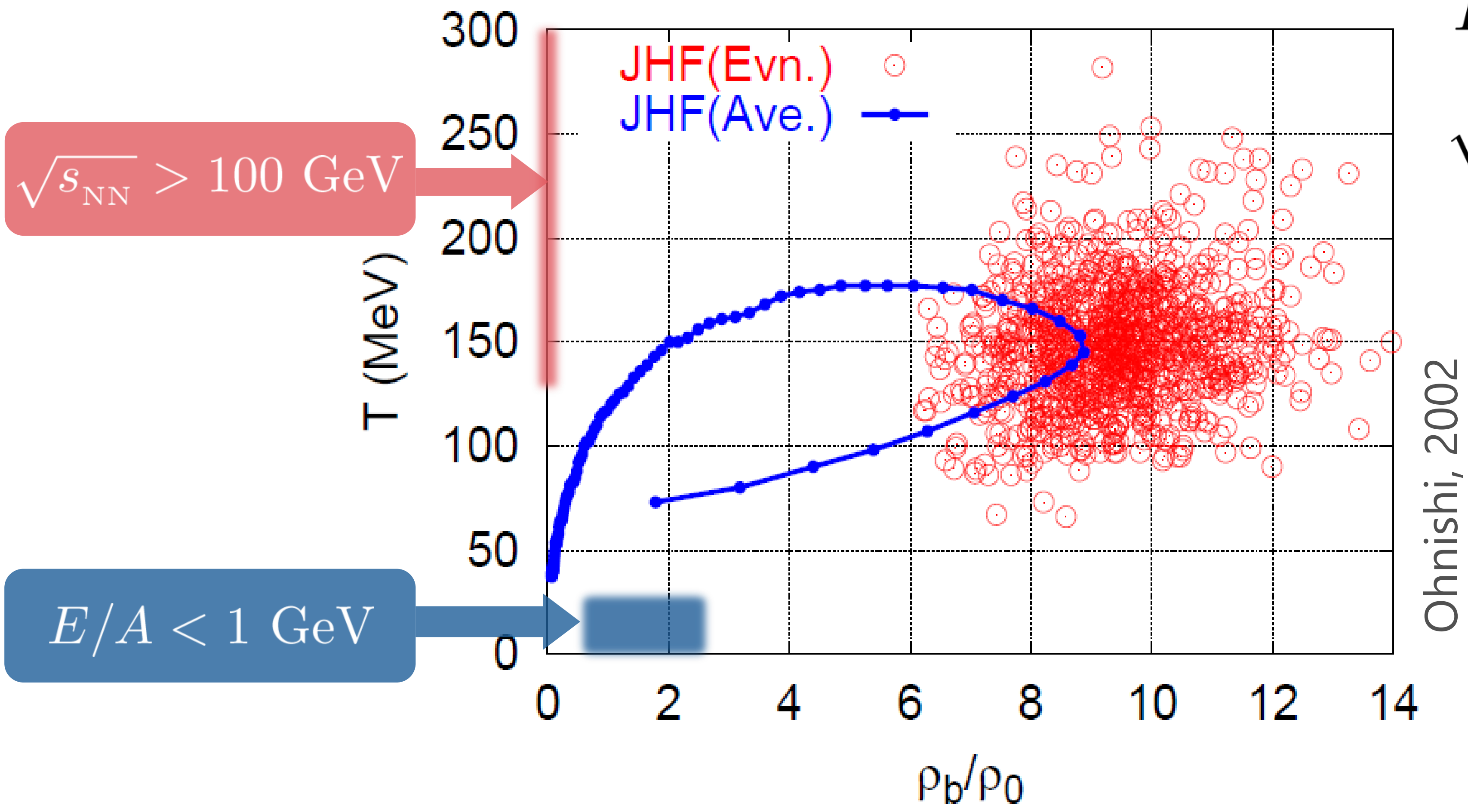
Beam-Energy Scan

STAR, 2012





Highest Baryon Density



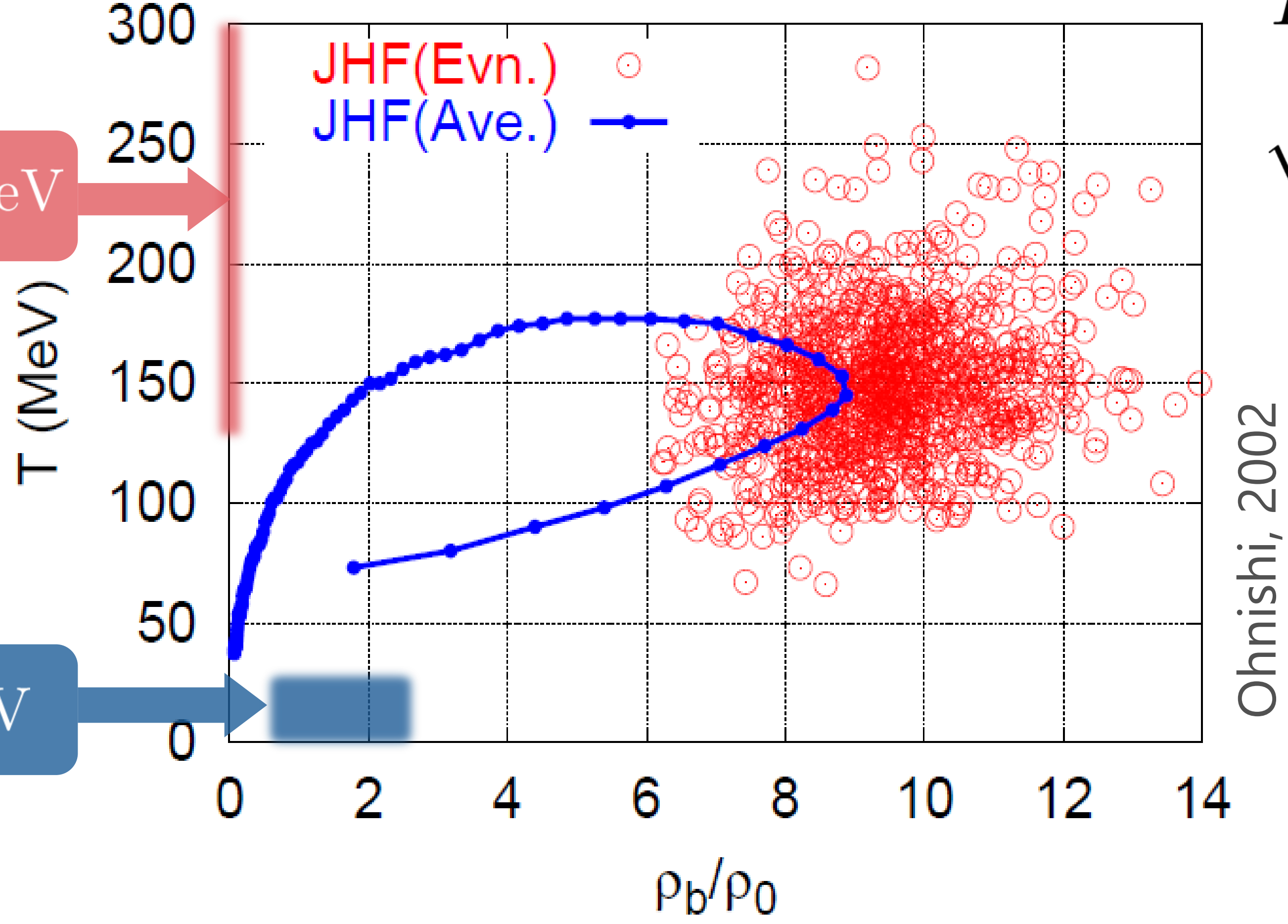
$E/A = 20\mathrm{GeV}$

$\sqrt{s_{NN}} \simeq 6\mathrm{GeV}$

Highest Baryon Density

$\sqrt{s_{\rm NN}} > 100$ GeV

$E/A < 1$ GeV

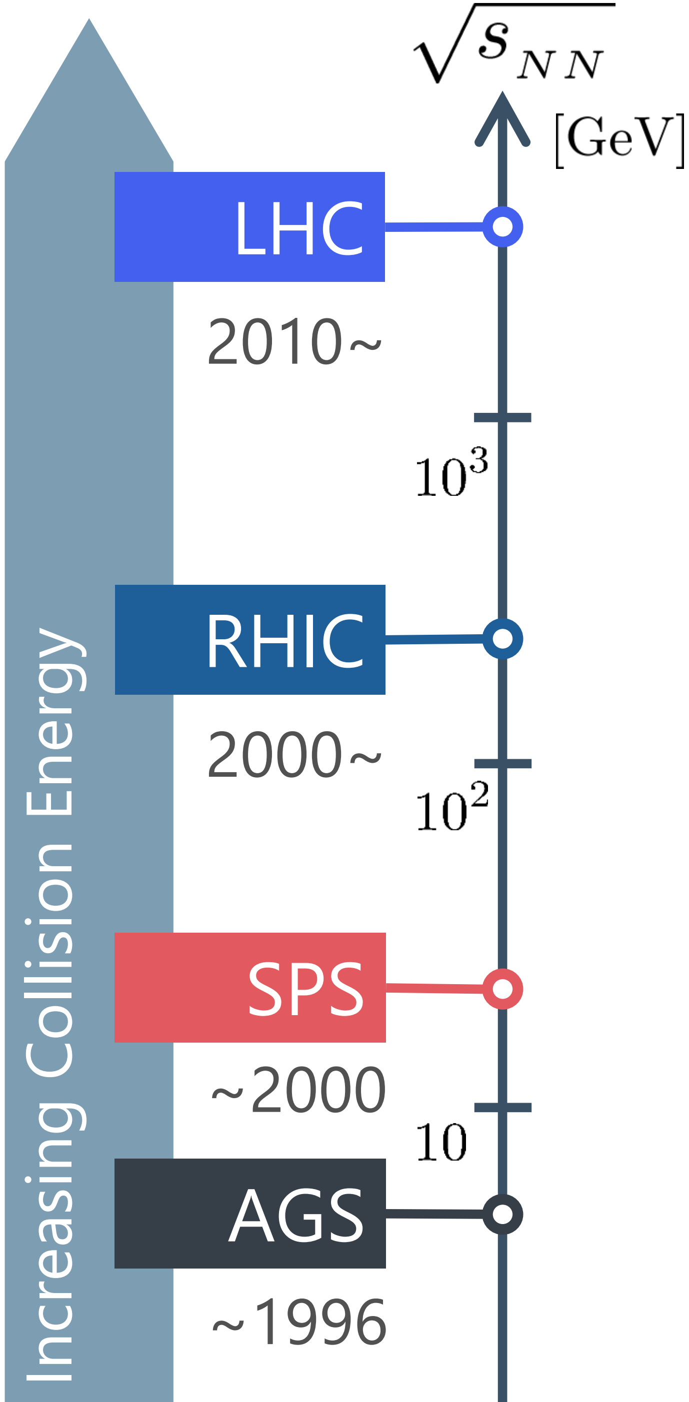


$$E/A = 20\text{GeV}$$

$$\sqrt{s_{NN}} \simeq 6\text{GeV}$$

Akira Ohnishi
1964-2023
passed away silently
on May. 16, 2023

History / Current Status of HIC



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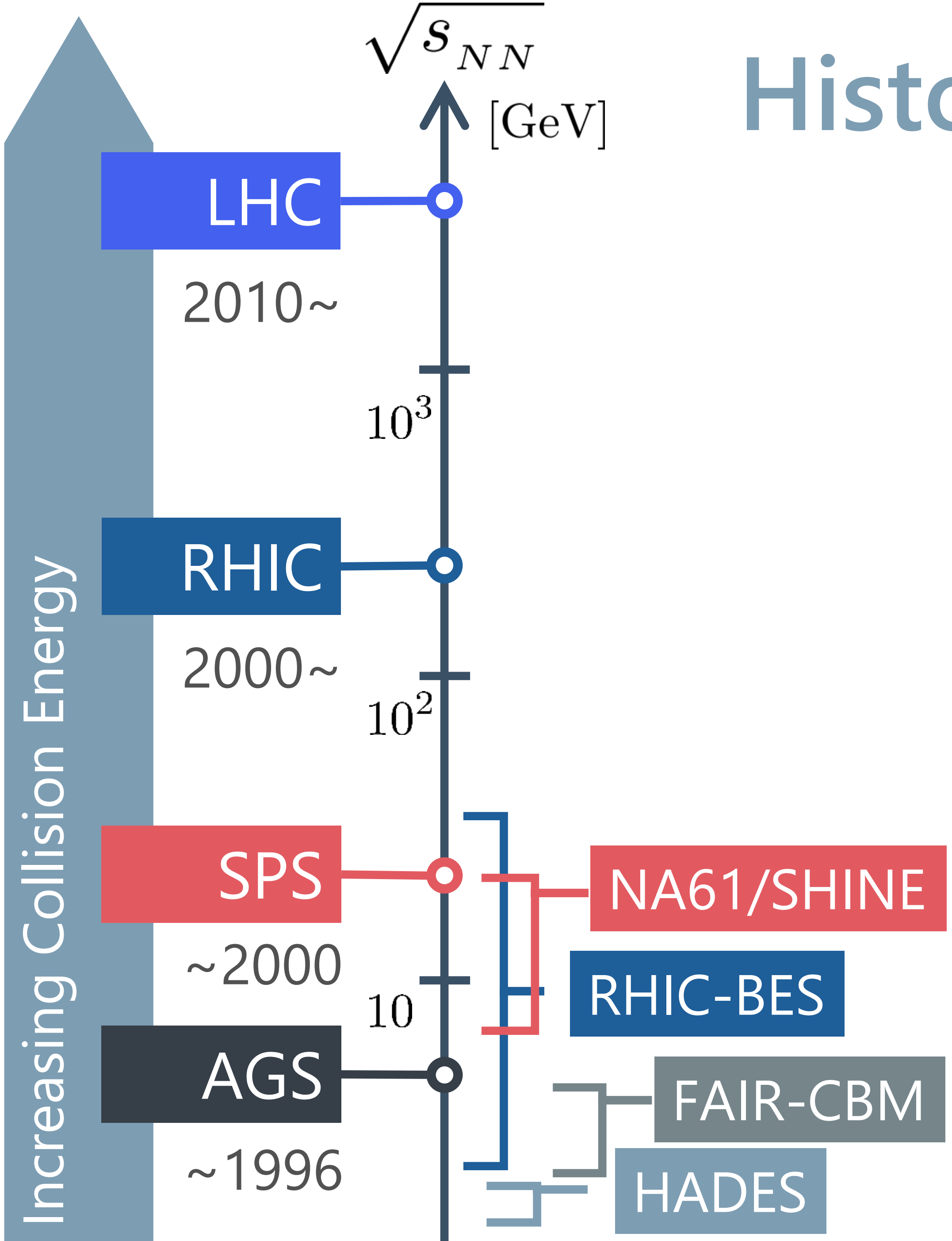

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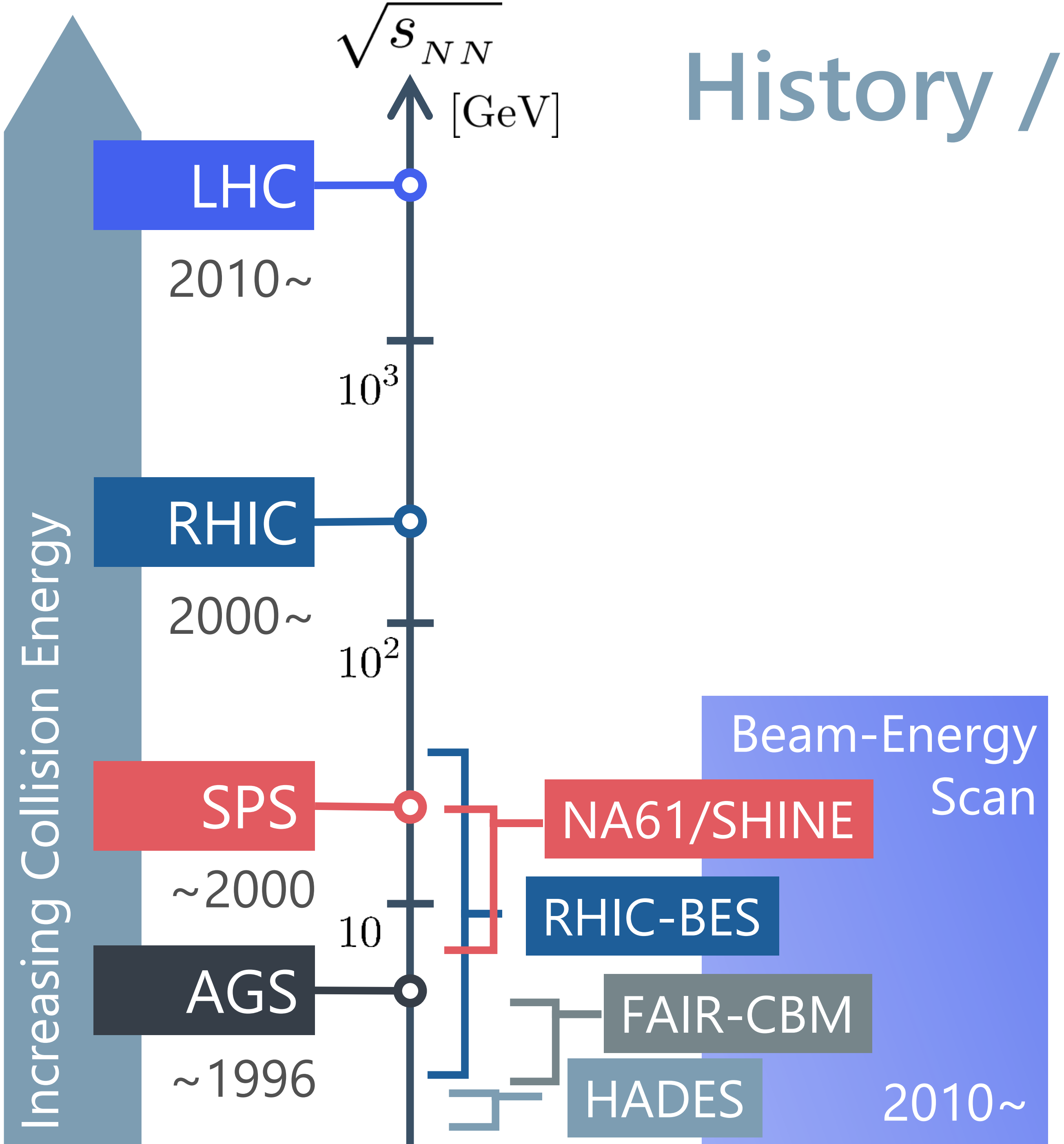

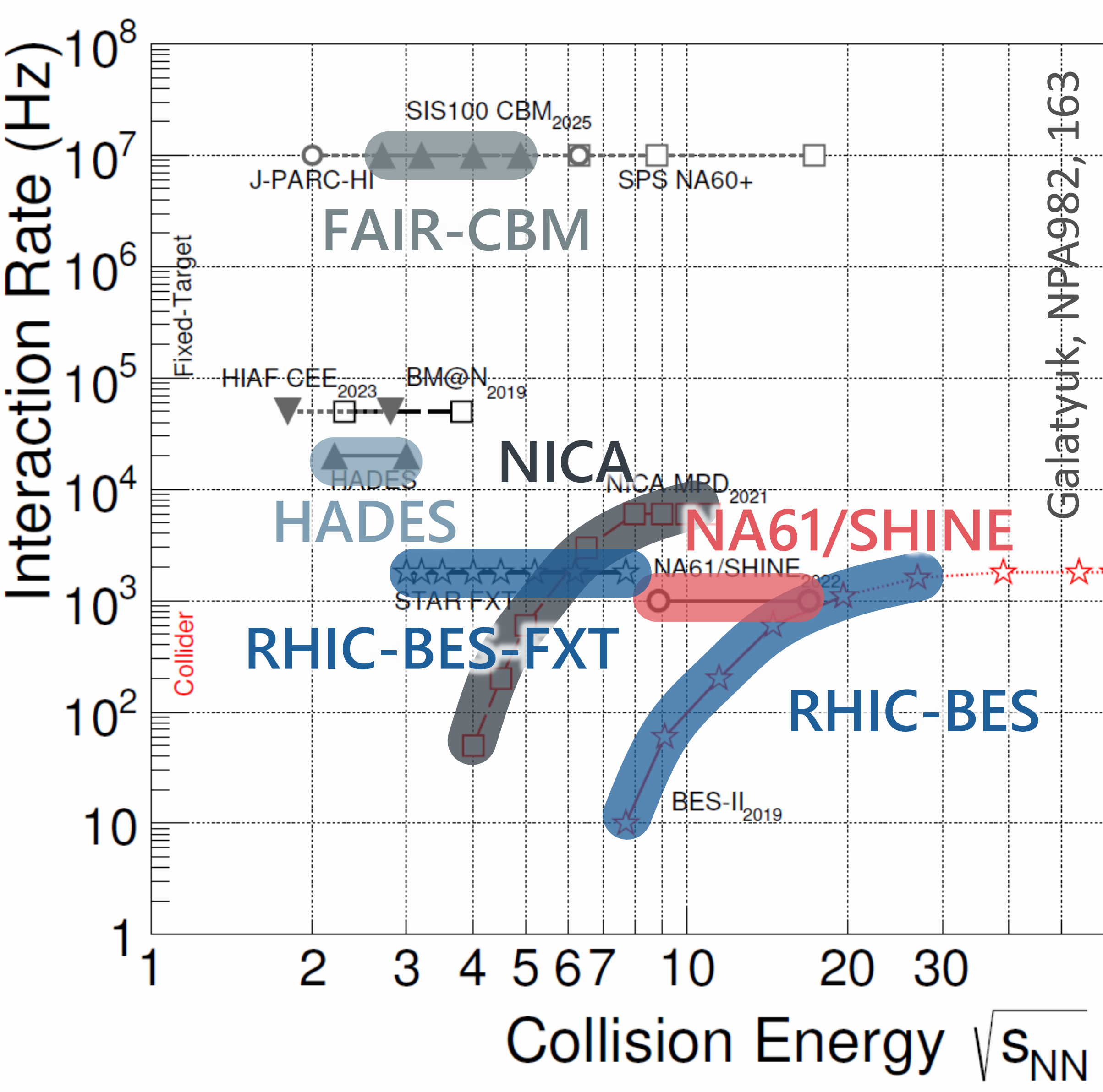

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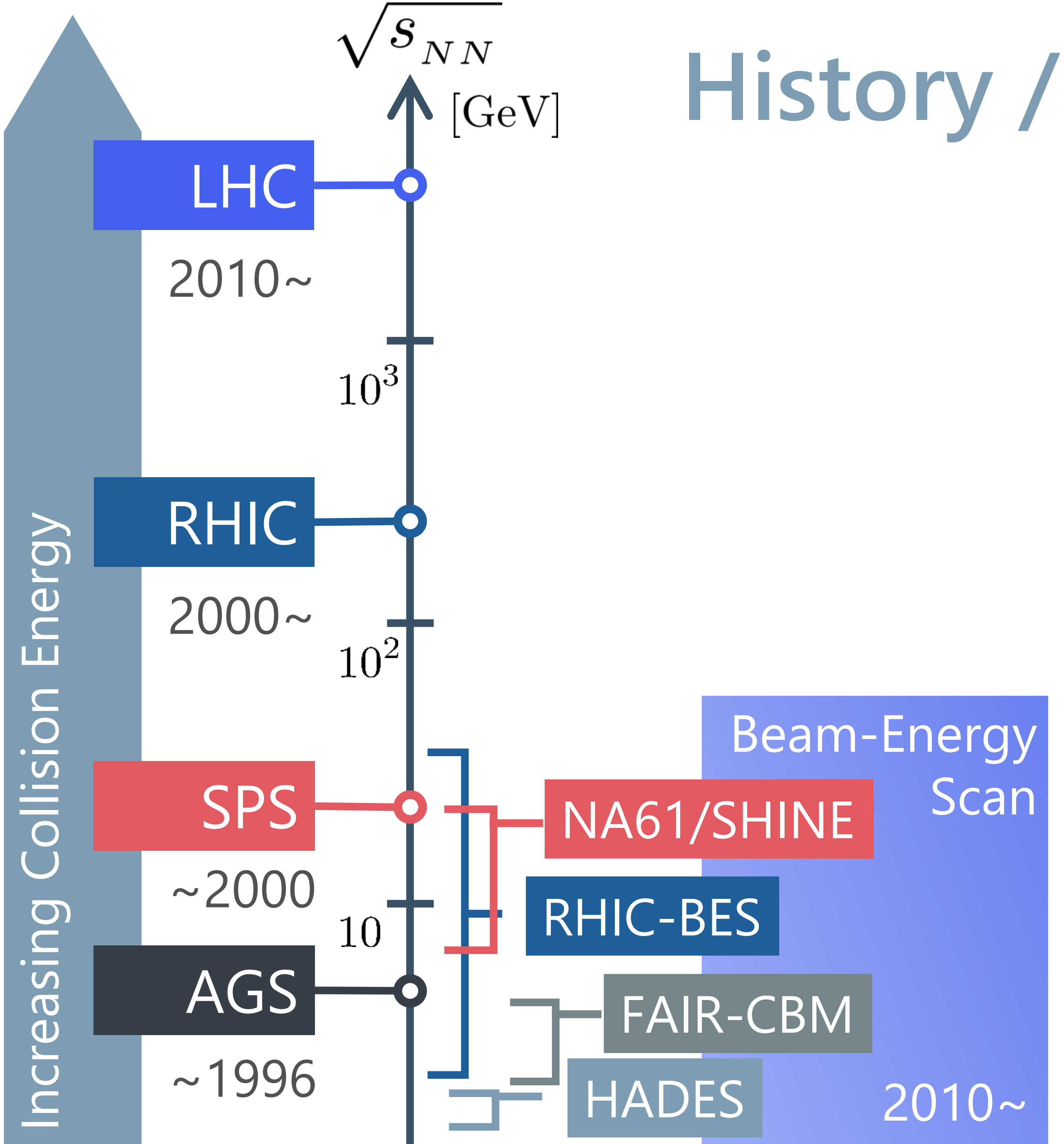
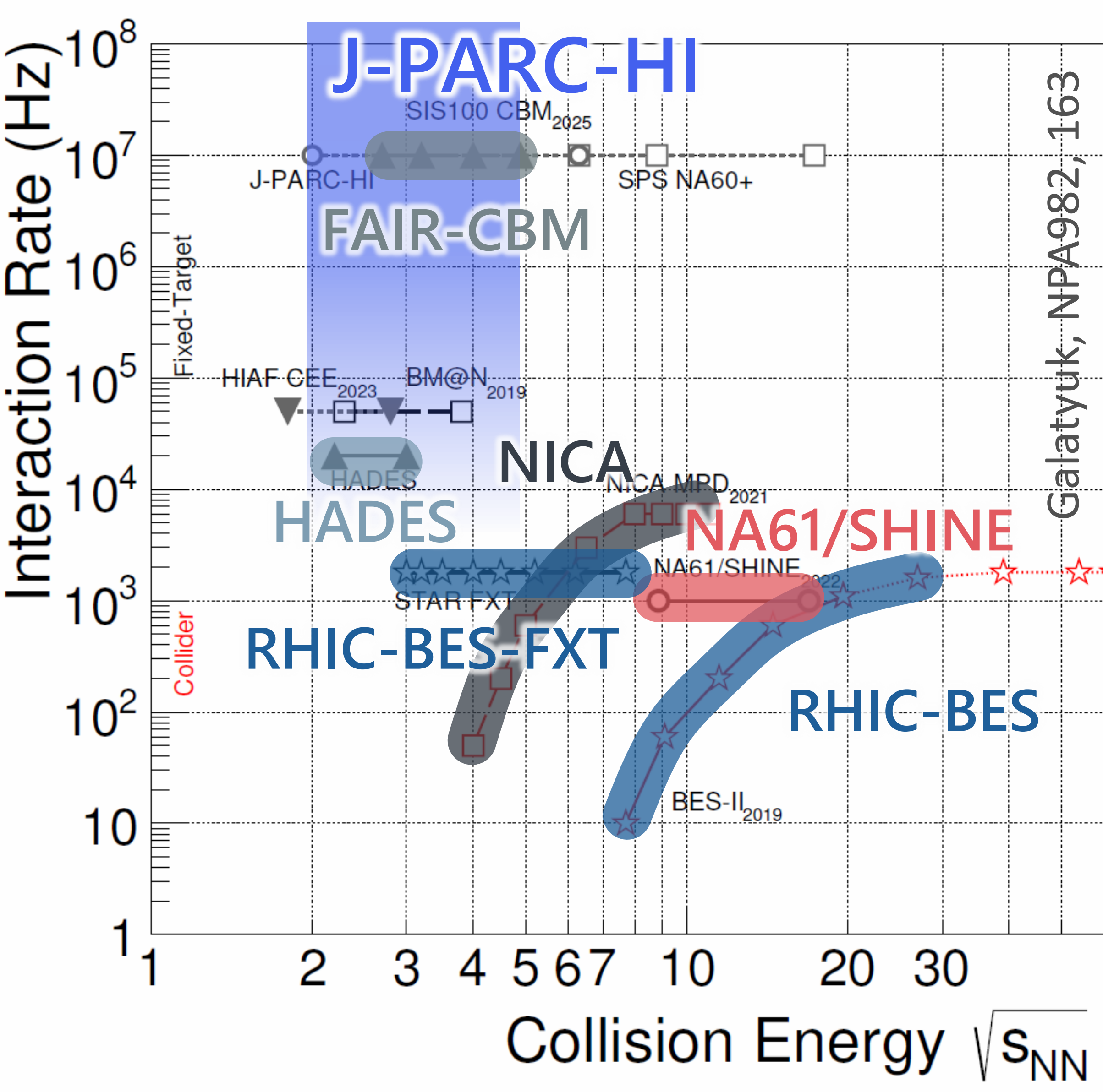
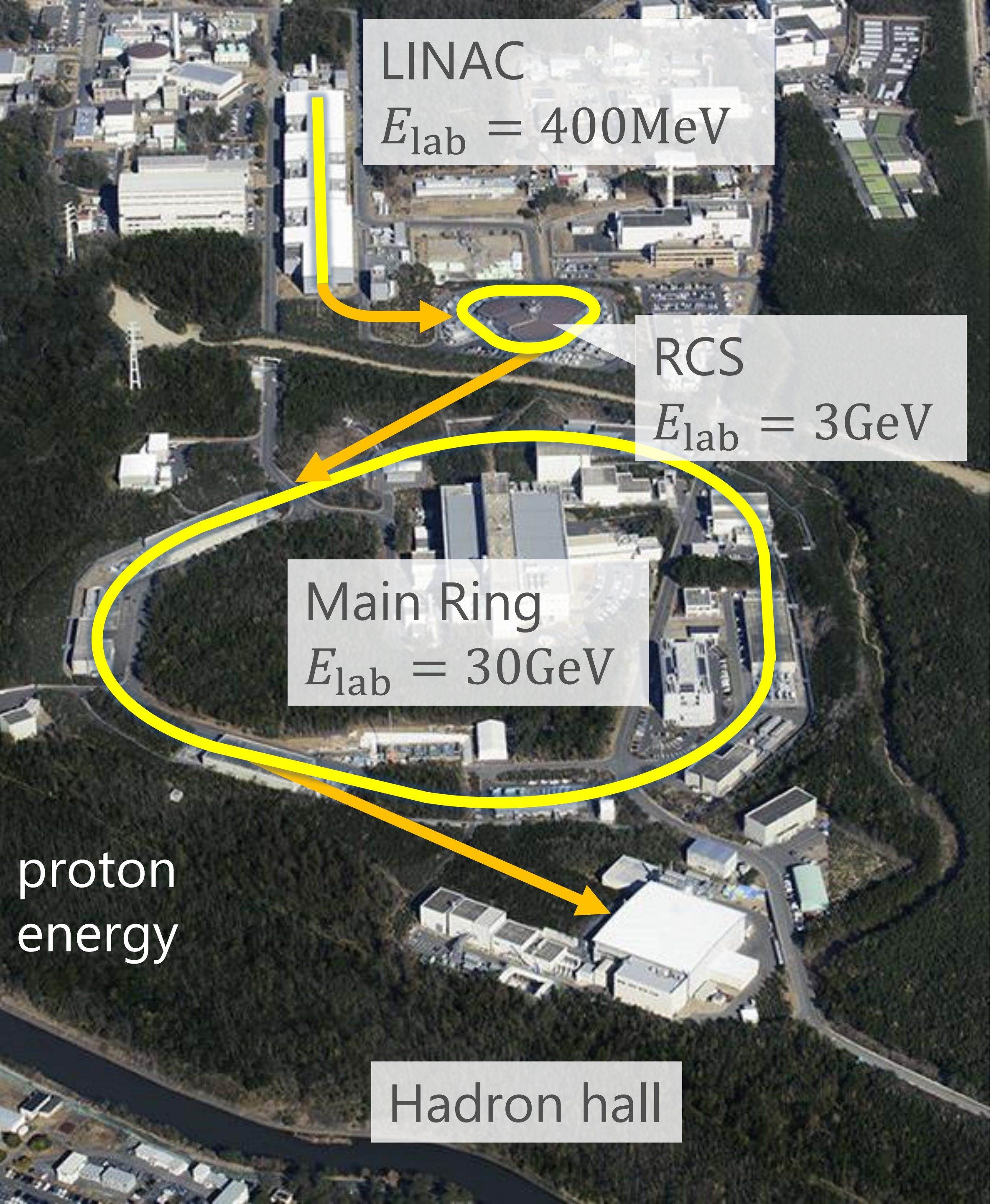






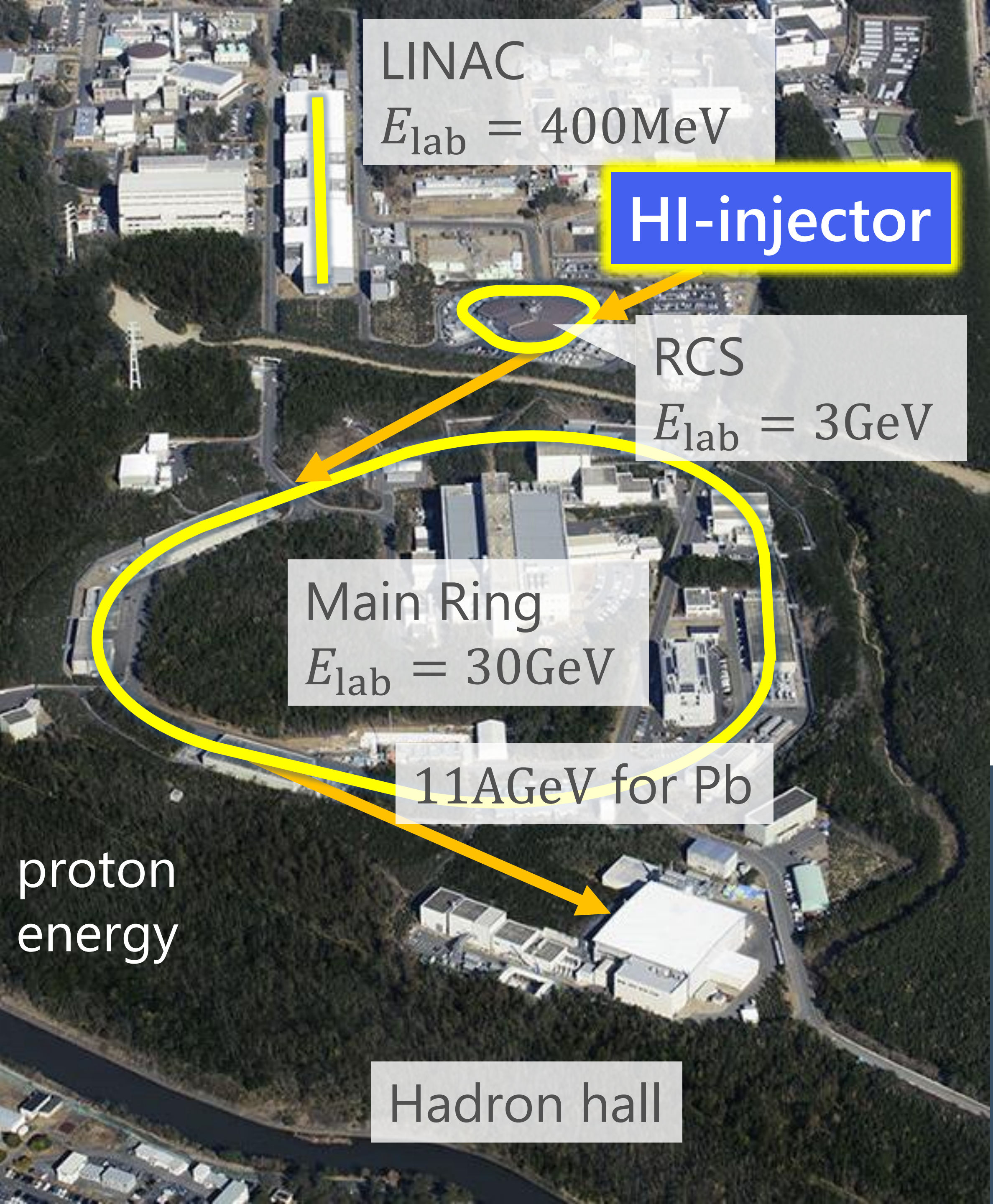
J-PARC

Accelerators

- LINAC
- RCS
- Main Ring(MR)
- High intensity $I = 1{\rm MW}$

Purposes

- Hadron/Nuclear physics
- Neutrino physics
- Material/Life science



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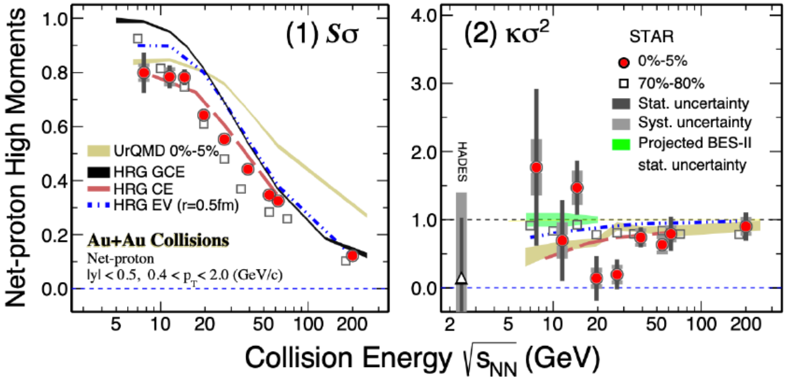
J-PARC-HI

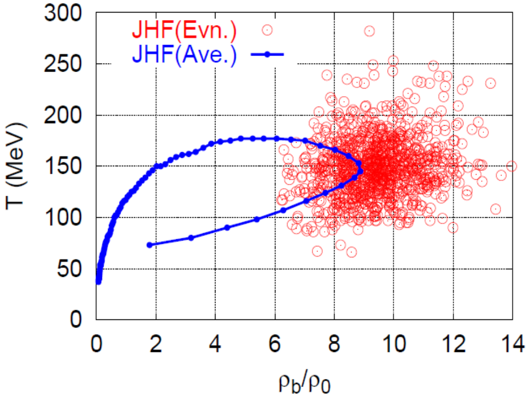
J-PARC Heavy Ion Program

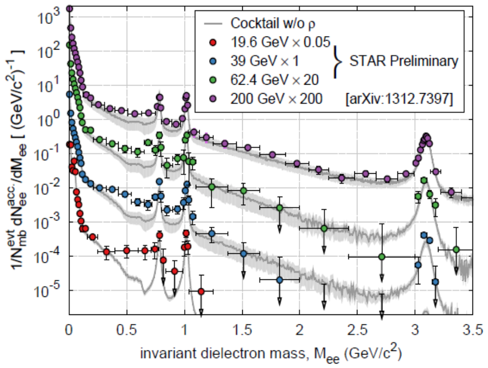
High intensity × Intermediate energy

Exploring Dense Medium

- Equation of state
- QCD critical point / 1st order transition / Color superconductivity
- Dilepton production rate
- Event selection / Higher correlations







Dilepton Production

as experimental observables of

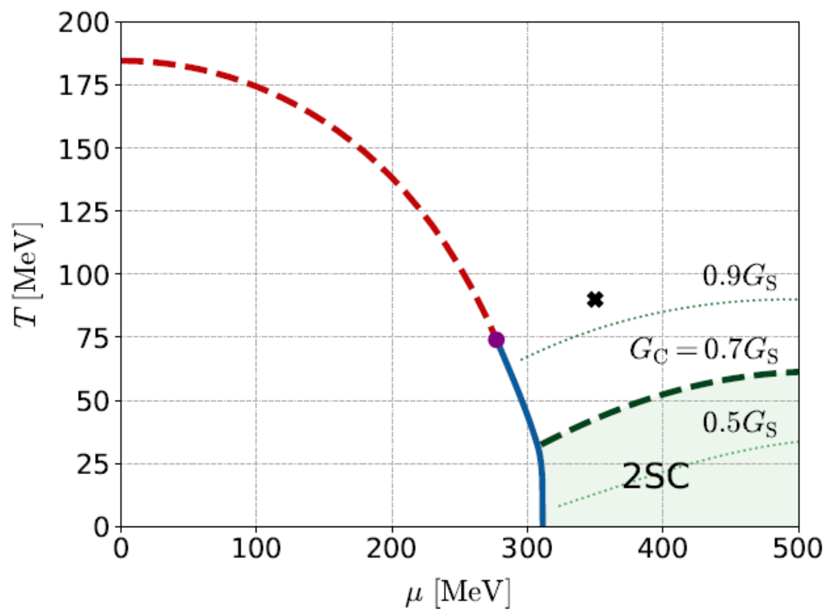
Color Superconductivity & QCD-CP

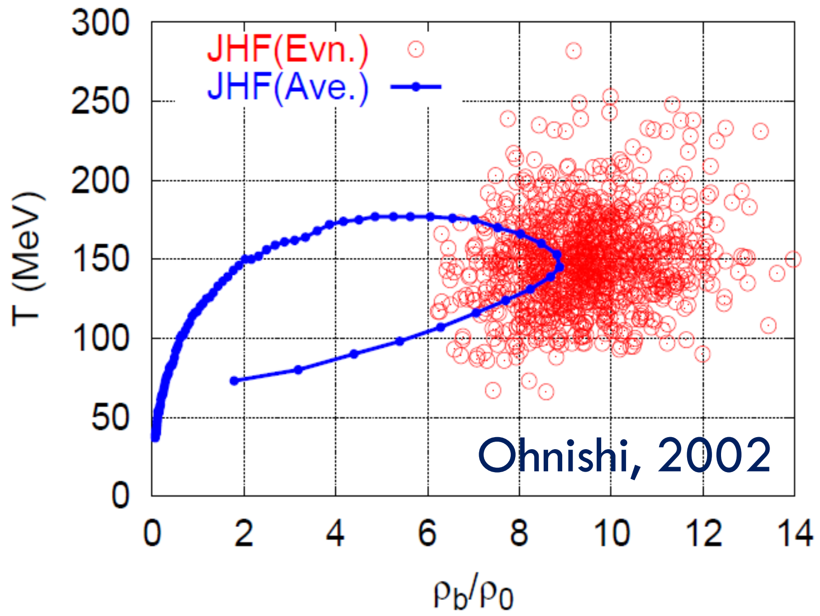
Nishimura, MK, Kunihiro, PTEP2022, 093D02
Nishimura, MK, Kunihiro, PTEP2023, 053D01

Observing CSC in HIC

- **Difficulties**
  - CSC would not be created if Tc is not high enough.
  - Even if created, its lifetime would be short.
  - Since CSC is created in the early stage, its signal would be blurred during the evolution in later stage.

- **Strategy** in the present study:
  - Use dilepton production as an observable
  - Focus on precursory phenomena of CSC



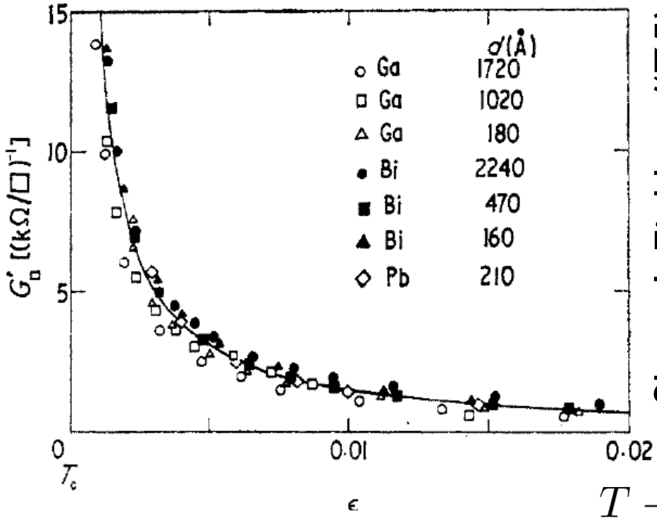


Precursor of CSC

- **Anomalous behavior of observables near but above Tc of SC**
  - electric conductivity
  - magnetic susceptibility
  - pseudogap

- Enhanced pair fluctuations is one of the origins of precursory phenomena.
- More significant phenomena in strongly-coupled systems.

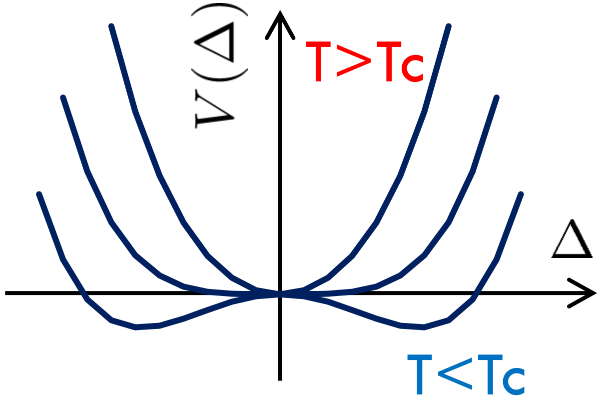
Electric conductivity



Skopal, Tinkham('75)

$$\varepsilon = \frac{T - T_c}{T_c}$$

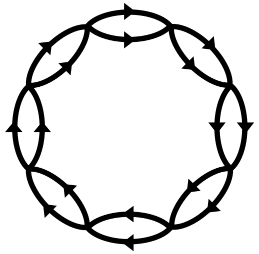
Landau's free energy



Precursor of Color Superconductivity

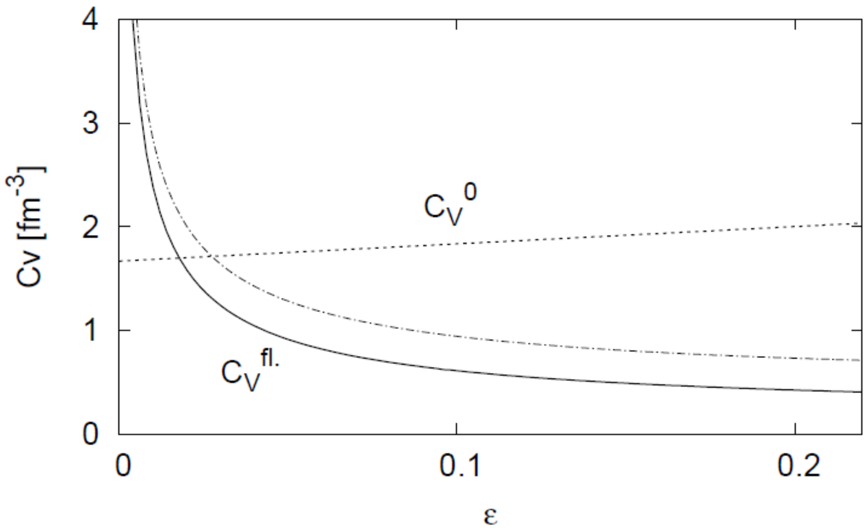
MK, Koide, Kunihiro, Nemoto, ’03, ‘05

□Thermodynamic Potential

$\Omega =$ 

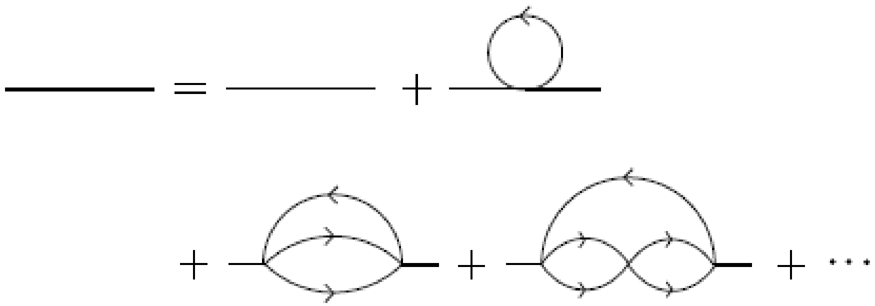
Specific heat

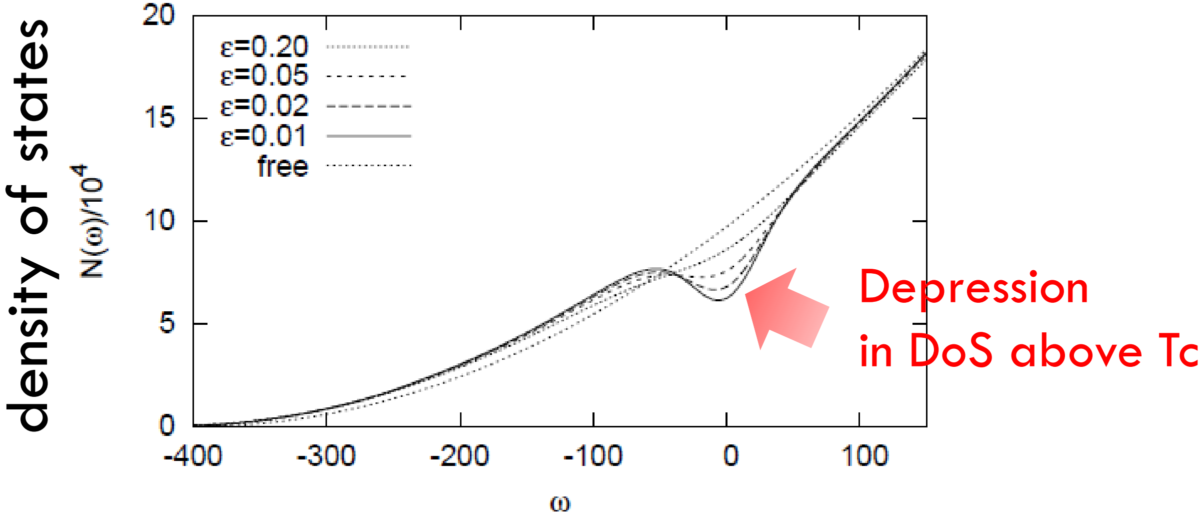
$$c = -T\frac{\partial^2 \Omega}{\partial T^2}$$



$$\varepsilon = \frac{T - T_c}{T_c}$$

□Pseudogap





Model

NJL model (2-flavor)

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \mathcal{L}_S + \mathcal{L}_C$$

$$\mathcal{L}_S = G_S \left((\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \tau \psi)^2 \right)$$

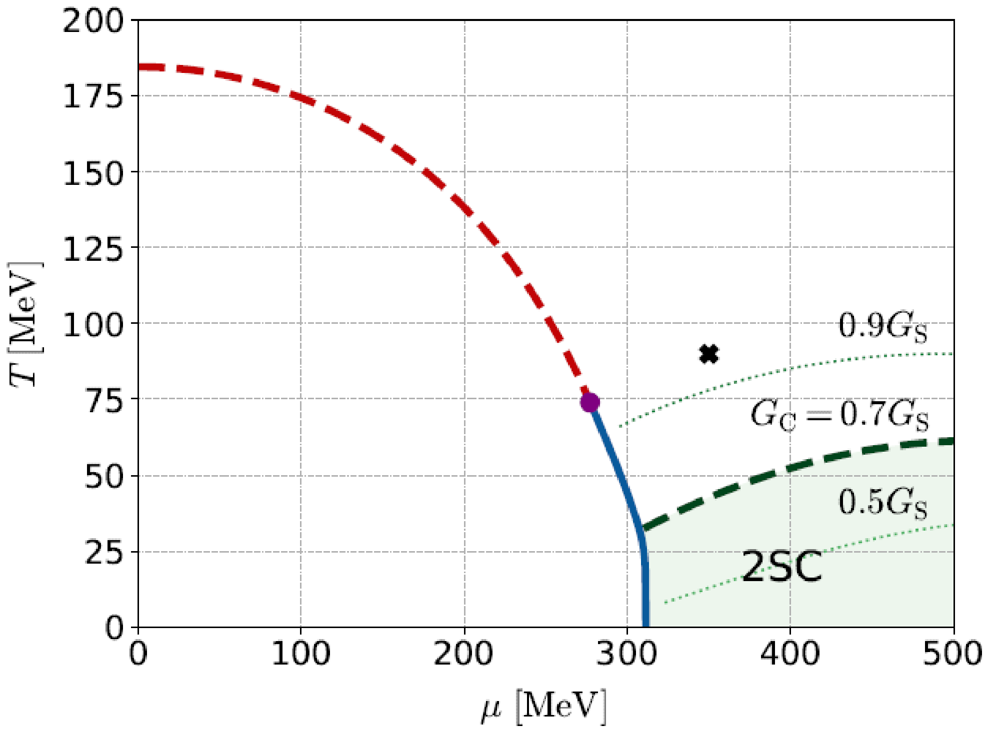
$$\mathcal{L}_C = G_C \left((\bar{\psi} i\gamma_5 \tau_A \lambda_A \psi^C)(\text{h.c.}) \right)$$

diquark interaction

Parameters

$$G_S = 5.01\ \text{GeV}^{-2}, \quad \Lambda = 650\text{MeV}, \quad m_q = 0$$

Phase Diagram in MFA



- Order of phase transition
  - 2nd in the MFA
  - can be 1st due to gauge fluctuation

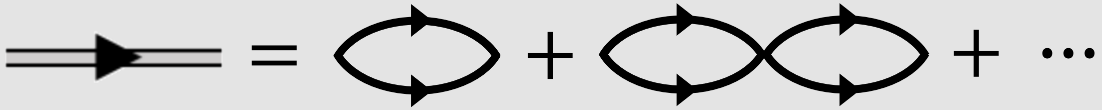
Matsuura+('04), Giannakis+('04)
Noronha+('06), Fejos, Yamamoto('19)

Di-quark Fluctuations

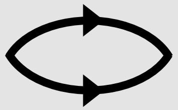
Diquark Propagator

$$D^R(x) = \langle[\Delta^\dagger(x), \Delta(0)]\rangle\theta(t) =$$ 

- Random Phase Approximaion



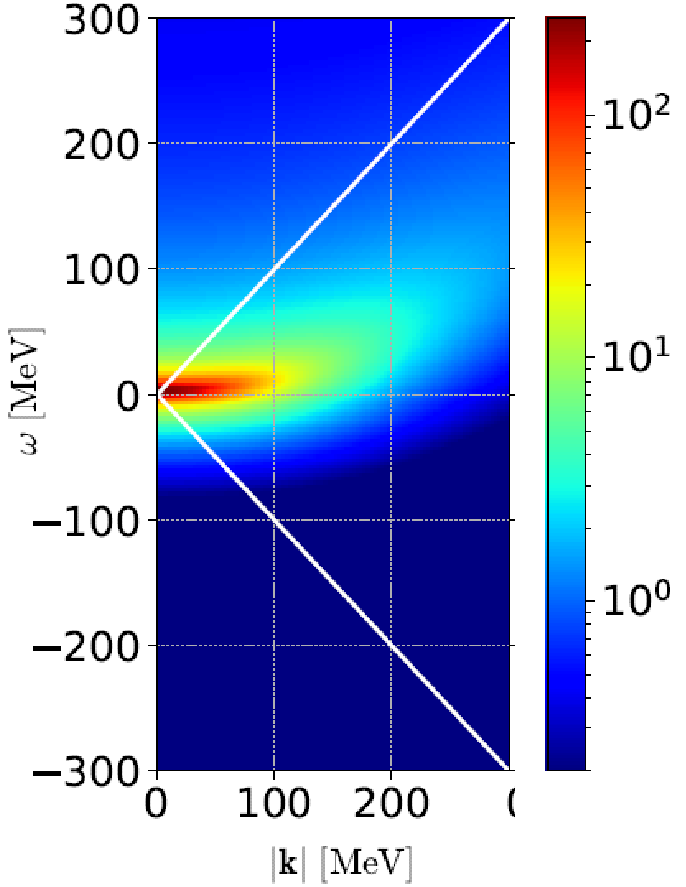
$$= \frac{Q^R(\boldsymbol{k},\omega)}{1 + G_C Q^R(\boldsymbol{k},\omega)}$$

$$Q^R(\boldsymbol{k},\omega) =$$ 

- Diquark field becomes massless at T=Tc
- Soft mode of CSC transition
- Strength in the space-like region

Dynamical Structure Factor

$$S(\boldsymbol{k},\omega) = -\frac{1}{\pi}\frac{1}{1-e^{-\beta\omega}}\mathrm{Im}D^{\mathrm{R}}(\mathrm{k},\omega)$$

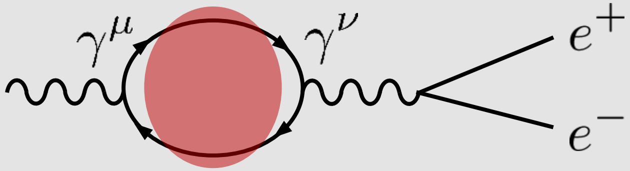


MK, Koide, Kunihiro, Nemoto, '01,'05

Photon Self-Energy: Precursor of CSC

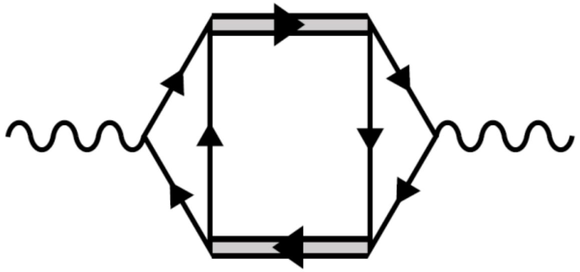
☐ Dilepton Production Rate

$$\frac{d^4\Gamma}{dk^4} = \frac{\alpha}{12\pi^4}\frac{1}{k^2}\frac{1}{e^{\beta\omega-1}}\mathrm{Im}\Pi^{R\mu}_{\mu}(k)$$

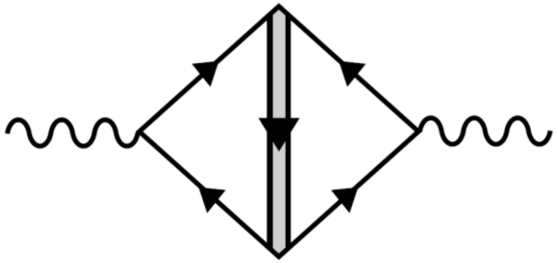


☐ Effect of Di-quarks on $\Pi^{\mu\nu}(k)$

Aslamasov-Larkin term



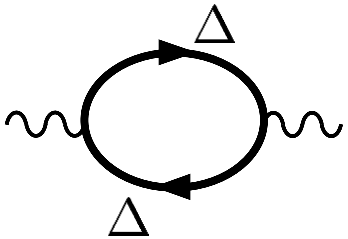
Maki-Thompson term

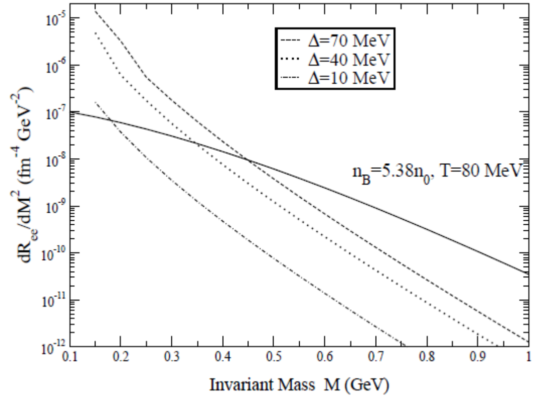


Well-known diagrams in metallic SC
for describing paraconductivity

☐ DPR from CFL phase

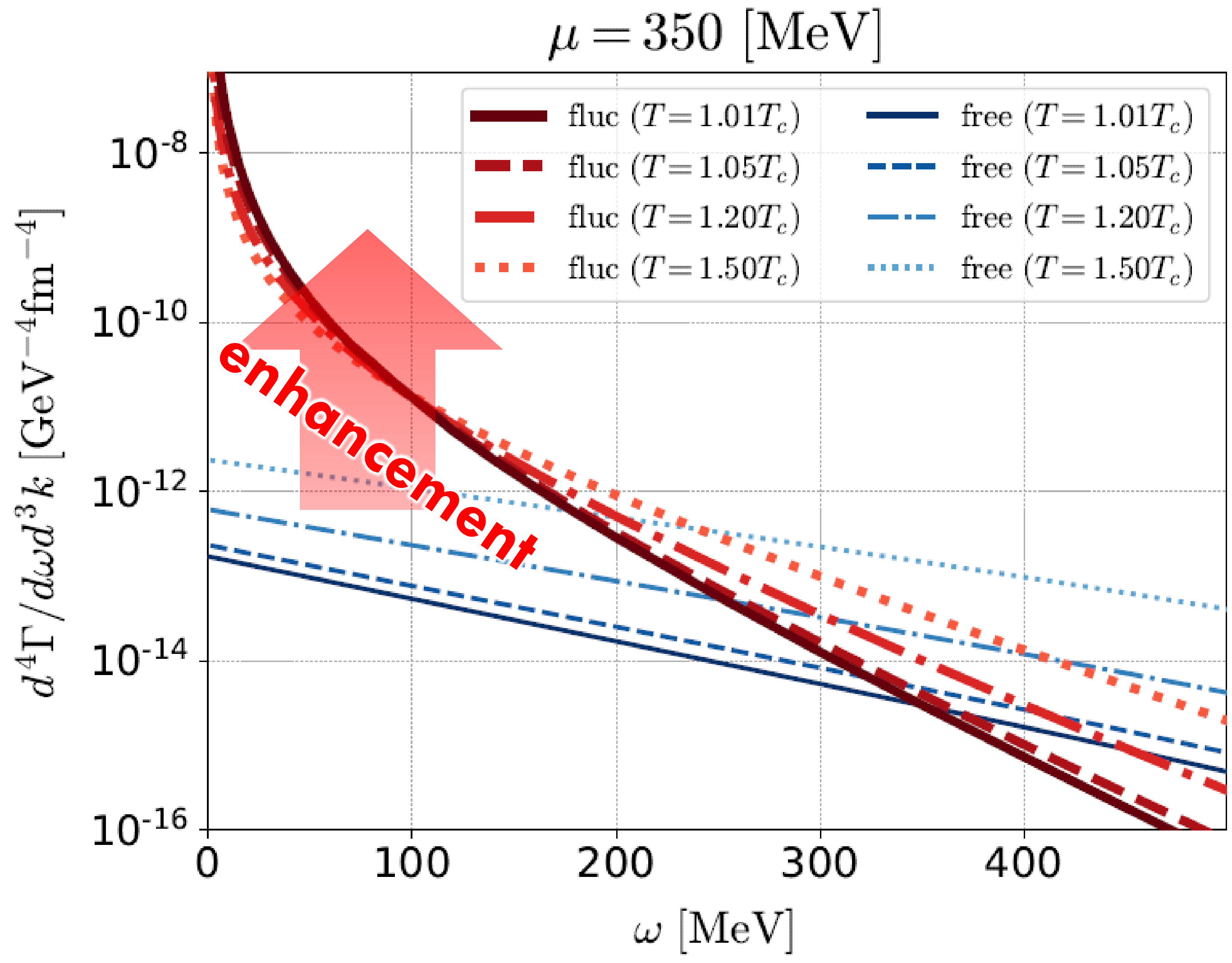
Jaikumar, Rapp, Zahed ('02)





Production Rate at $\boldsymbol{k} = 0$

Nishimura, MK, Kunihiro ('22)



Red: fluctuation contribution

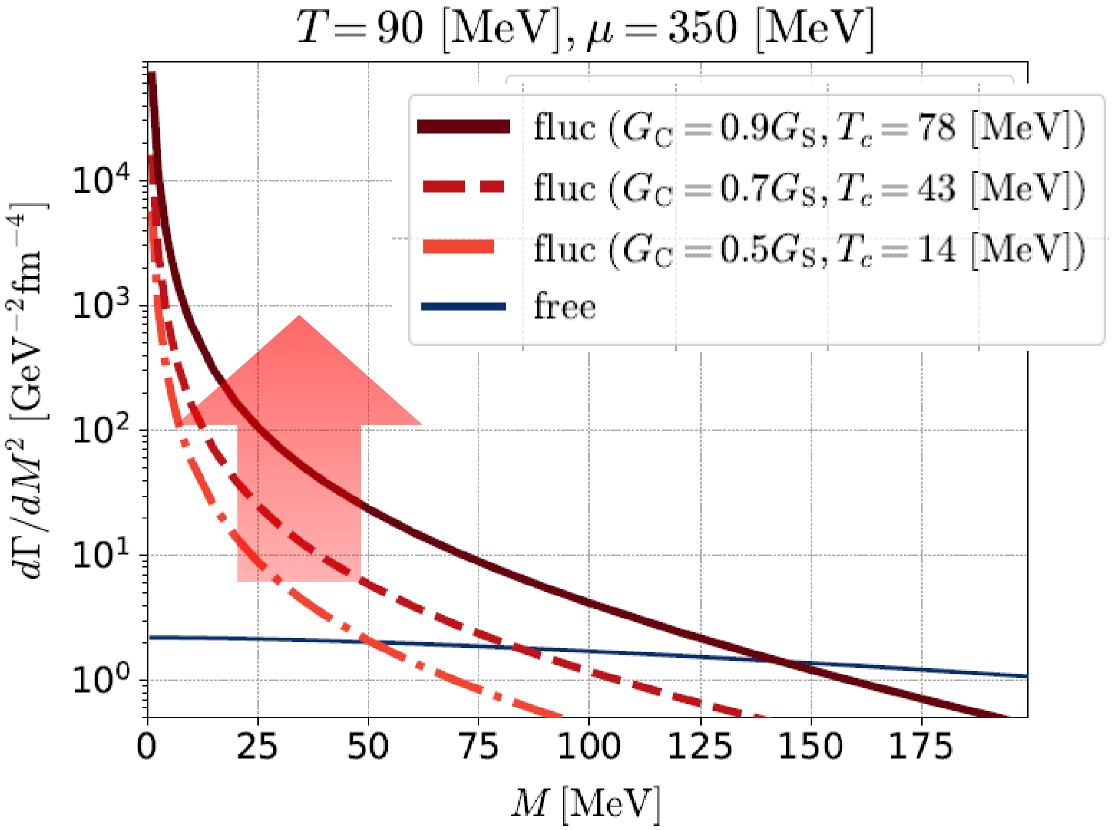
Blue: free quarks

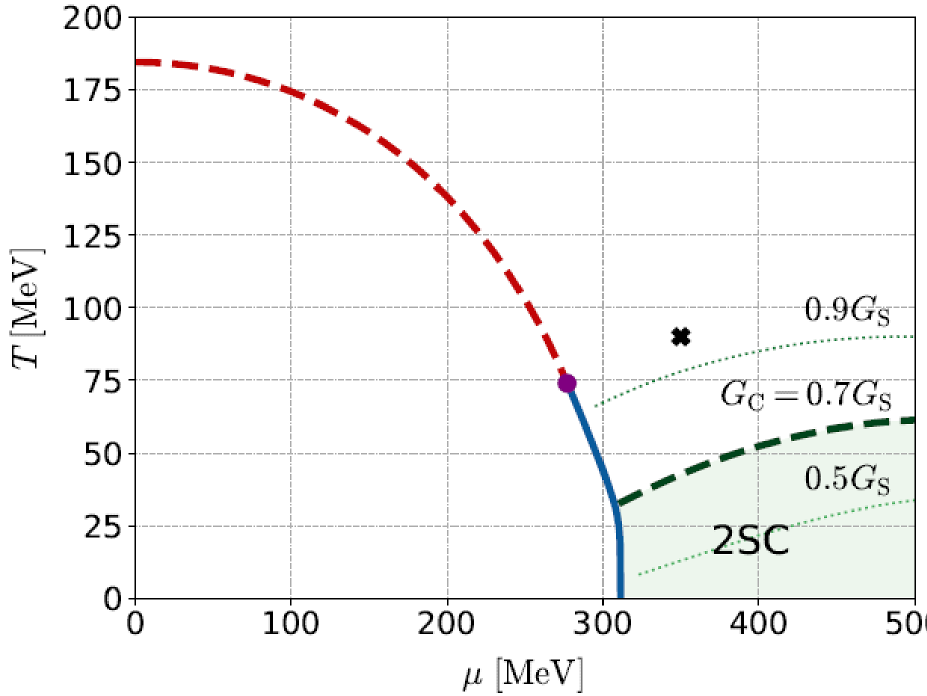
$G_C = 0.7G_s, T_c \simeq 45$ MeV

- Di-quark fluctuations give rise to large enhancement in the low energy region $\omega < 200$ MeV and $T < 1.5T_c$.
- Anomalous enhancement is not sensitive to T.

Invariant-Mass Spectrum

Nishimura, MK, Kunihiro ('22)





- ☐ Strong enhancement at low invariant mass.
- ☐ **Observable in the HIC?**

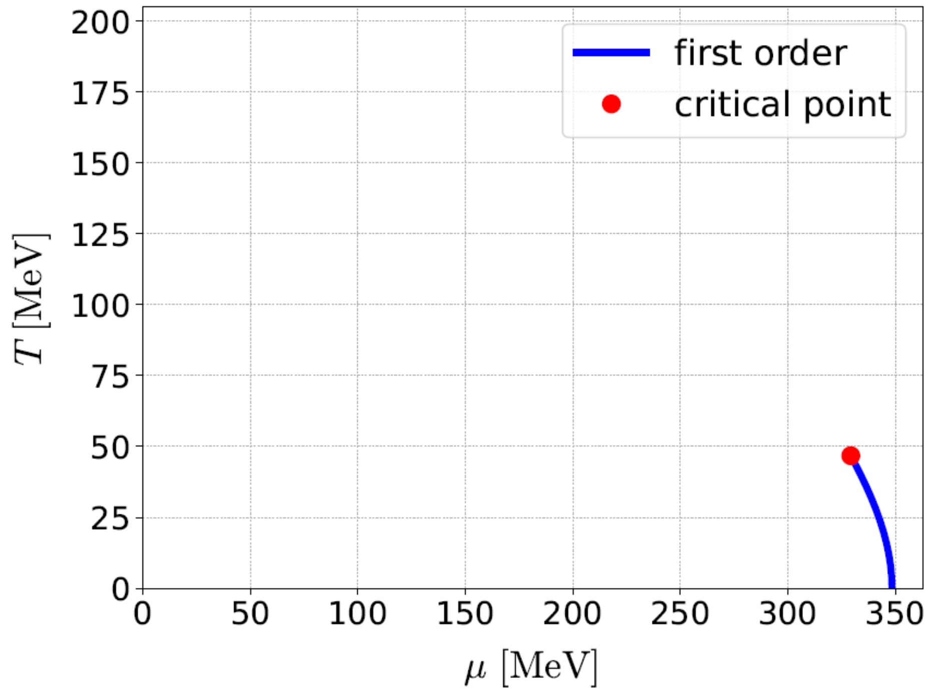
Dileptons from QCD Critical Point

NJL model (2-flavor)

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G_S\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2\right)$$

Parameters

$$G_S = 5.5\ \mathrm{GeV}^{-2},\ \Lambda = 631\mathrm{MeV},\ m_q = 5.5\ \mathrm{MeV}$$

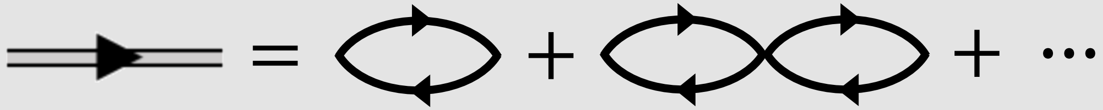


Soft Mode of QCD-CP

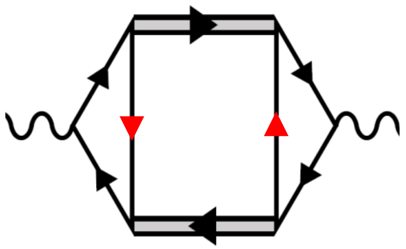
= fluctuation of scalar ($\bar{q}q$) channel

$$D^R(x) = \langle[\bar{\psi}\psi(x), \bar{\psi}\psi(0)]\rangle\theta(t) =$$

- Random Phase Approximaion



Modification of dilepton production through

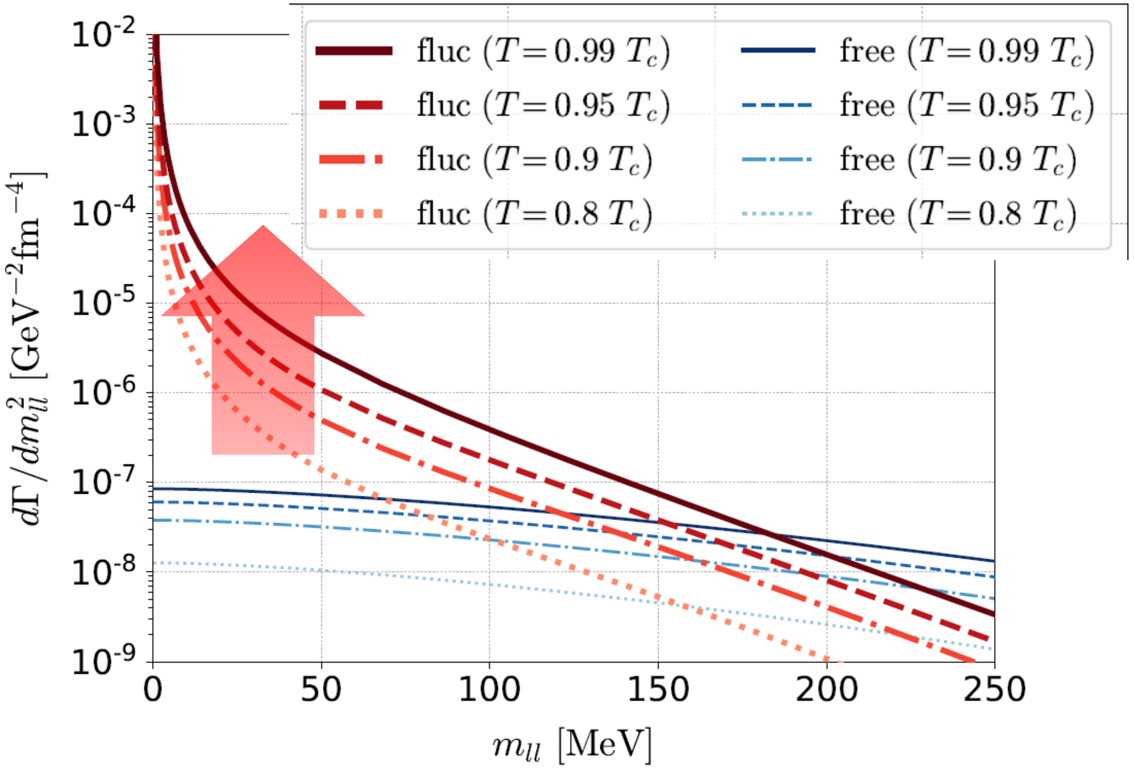


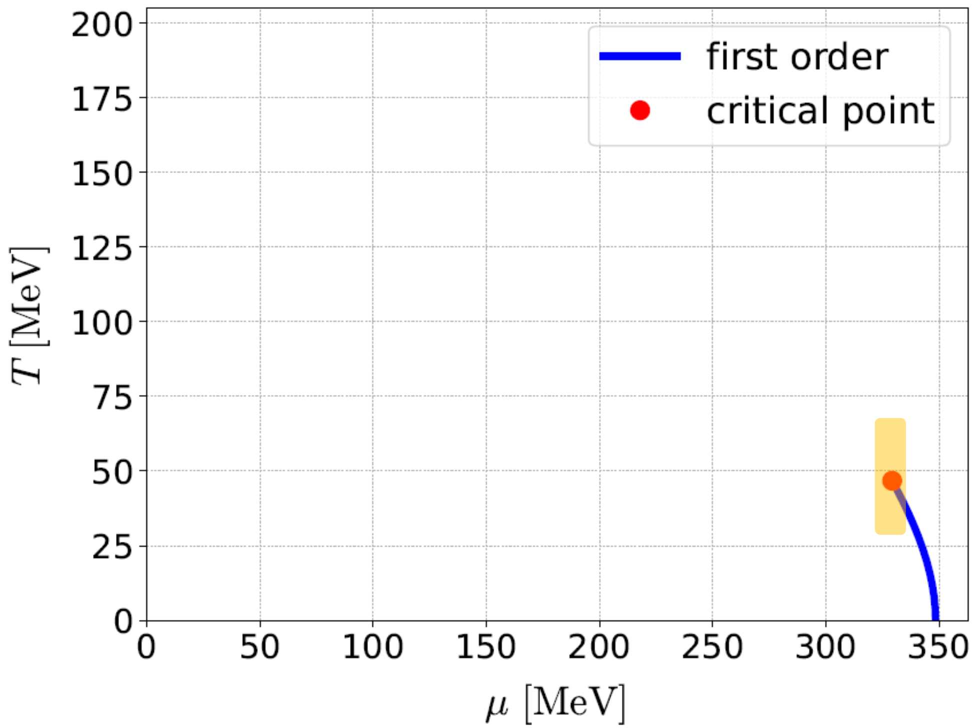
$+ \cdots$ MT, DoS

Dilepton production rate near QCD-CP

Nishimura, MK, Kunihiro ('23)

Invariant mass spectrum





for fixed chem. pot.: $\mu = \mu_c$

- ☐ Enhancement at low M_{ll} region near QCD-CP
- ☐ Distinguishment from diquark soft mode may be difficult.

Summary

Exploring dense quark matter

— is an interesting subject in heavy-ion collisions that are investigated actively all over the world.

- J-PARC-HI will accelerate this research field.

Dilepton production at ultra-low mass region

- can be used for experimental signals to detect
  — onset of color-superconducting phase transition
  — existence of QCD critical point

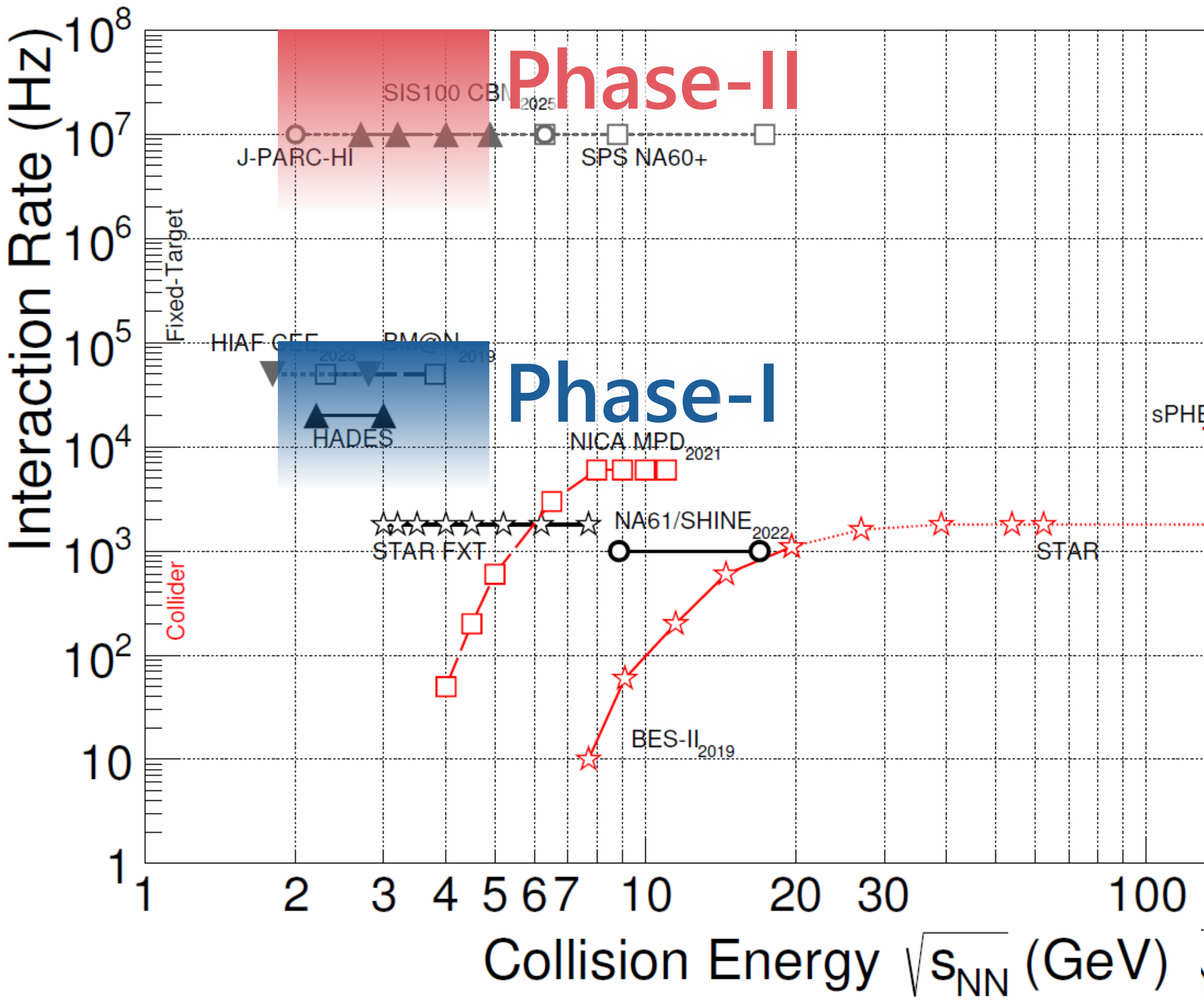
J-PARC-HI Staging Plan

Phase-I

- KEK-BS booster
- E16+α spectrometer

Phase-II

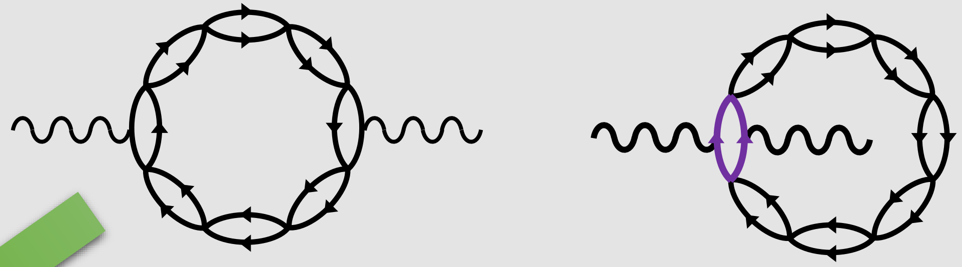
- NEW HI booster
- NEW spectrometer



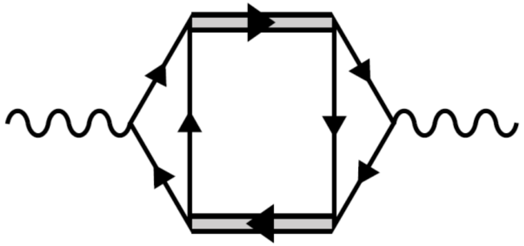
Gauge-Invariant Construction of $\Pi_{\mu\nu}(k)$

Insert two photon vertices in thermodynamic potential

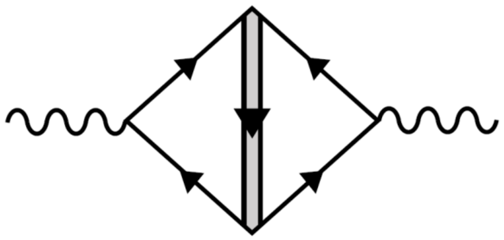
$$\Pi^{\mu\nu}(k) =$$



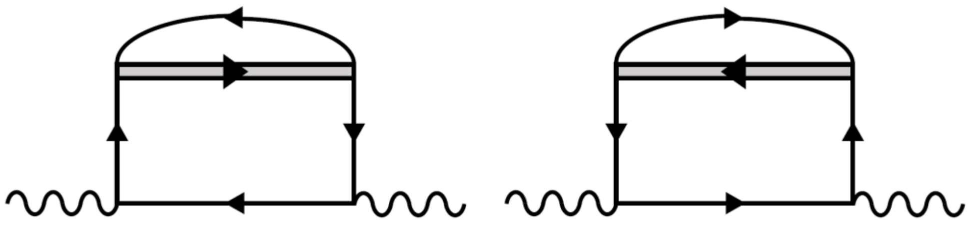
Aslamasov-Larkin (AL)



Maki-Thompson (MT)



Density of States (DoS)



- ☐ WT identity $k_\mu \Pi^{\mu\nu}(k) = 0$ is satisfied with AL, MT and DoS terms.

(Modified) Time-Dependent Ginzburg-Landau Approximation

TDGL approximation for T-matrix

$$\Xi^R(\boldsymbol{k},\omega) = \frac{G_C}{1+G_C Q^R(\boldsymbol{k},\omega)} \simeq \frac{1}{c\omega + \Xi^R(\boldsymbol{k},0)^{-1}} \qquad c = \left.\frac{\partial(\Xi^R)^{-1}}{\partial\omega}\right|_{\omega=0}$$

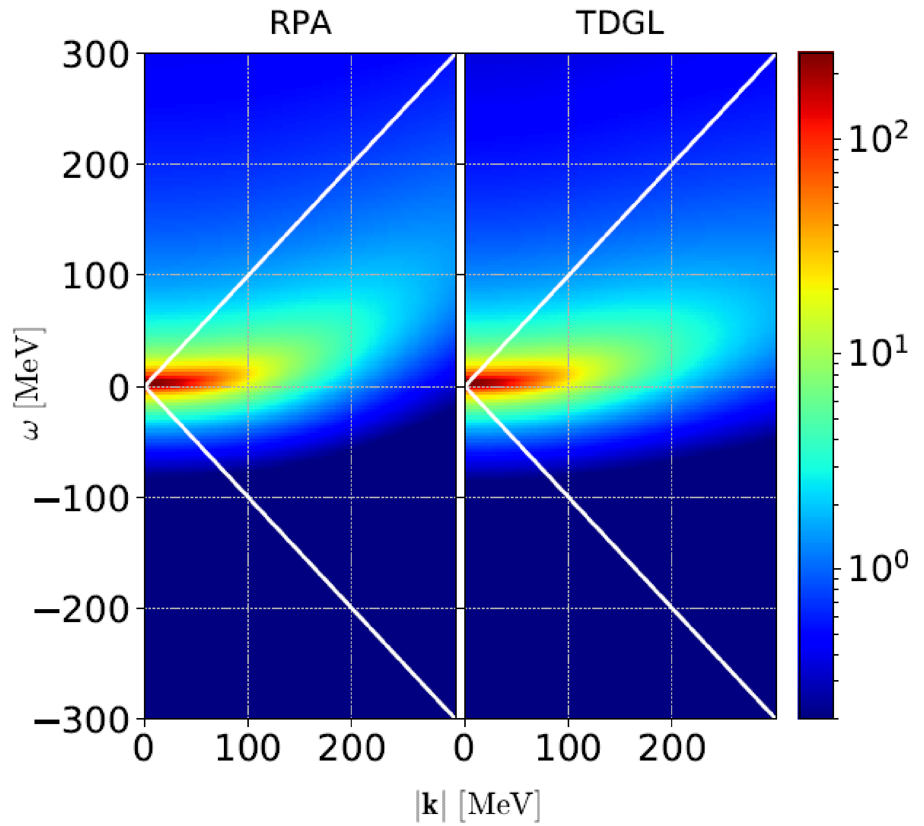
$$\Xi^R(\boldsymbol{k},\omega) = \frac{G_C}{Q^R(\boldsymbol{k},\omega)} D^R(\boldsymbol{k},\omega)$$

Note:

- ☐ Valid in low energy region
- ☐ $[\Xi^R(0,0)]^{-1} = 0$ at $T = T_c$
- ☐ We do not expand w.r.t. k

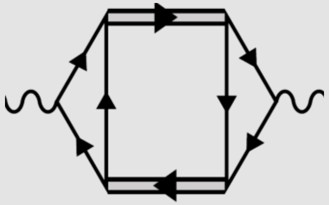
$$\Xi^R(\boldsymbol{k},\omega) \simeq \frac{1}{c\omega + \Xi^R(\boldsymbol{k},0)^{-1}} \simeq \frac{1}{c\omega + a + b\boldsymbol{k}^2}$$

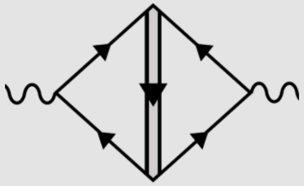
⇔ TDGL equation: $ic\frac{\partial}{\partial t}\Delta + a\Delta - b\nabla^2\Delta = 0$



Vertices

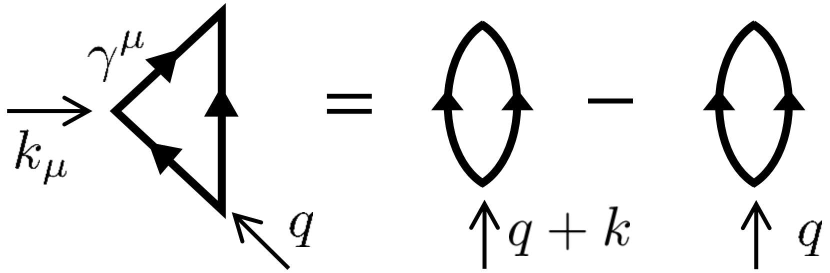
Vertices must be determined to be consistent with the TDGL approx.

$\Pi^{\mu\nu}_{\mathrm{AL}}(k) =$ 

$\Pi^{\mu\nu}_{\mathrm{MT}}(k) =$ 

☐ WT identity for AL vertex

$$k_\mu \Gamma^\mu(q, q+k) = \Xi^{-1}(q+k) - \Xi^{-1}(k)$$

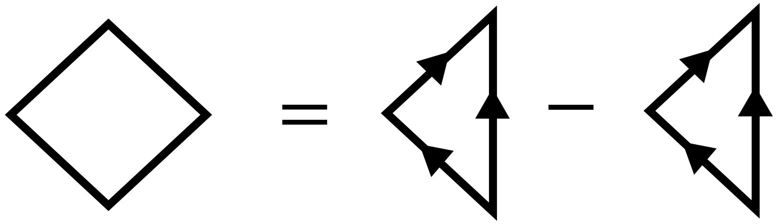


At the lowest order in k

$$\begin{cases} \Gamma^0 = e_\Delta c \\ \Gamma^i = e_\Delta \dfrac{\partial^2 \Xi(q)^{-1}}{\partial q^2}(2q^i + k^i) \end{cases}$$

e_Δ: electric charge of diquarks

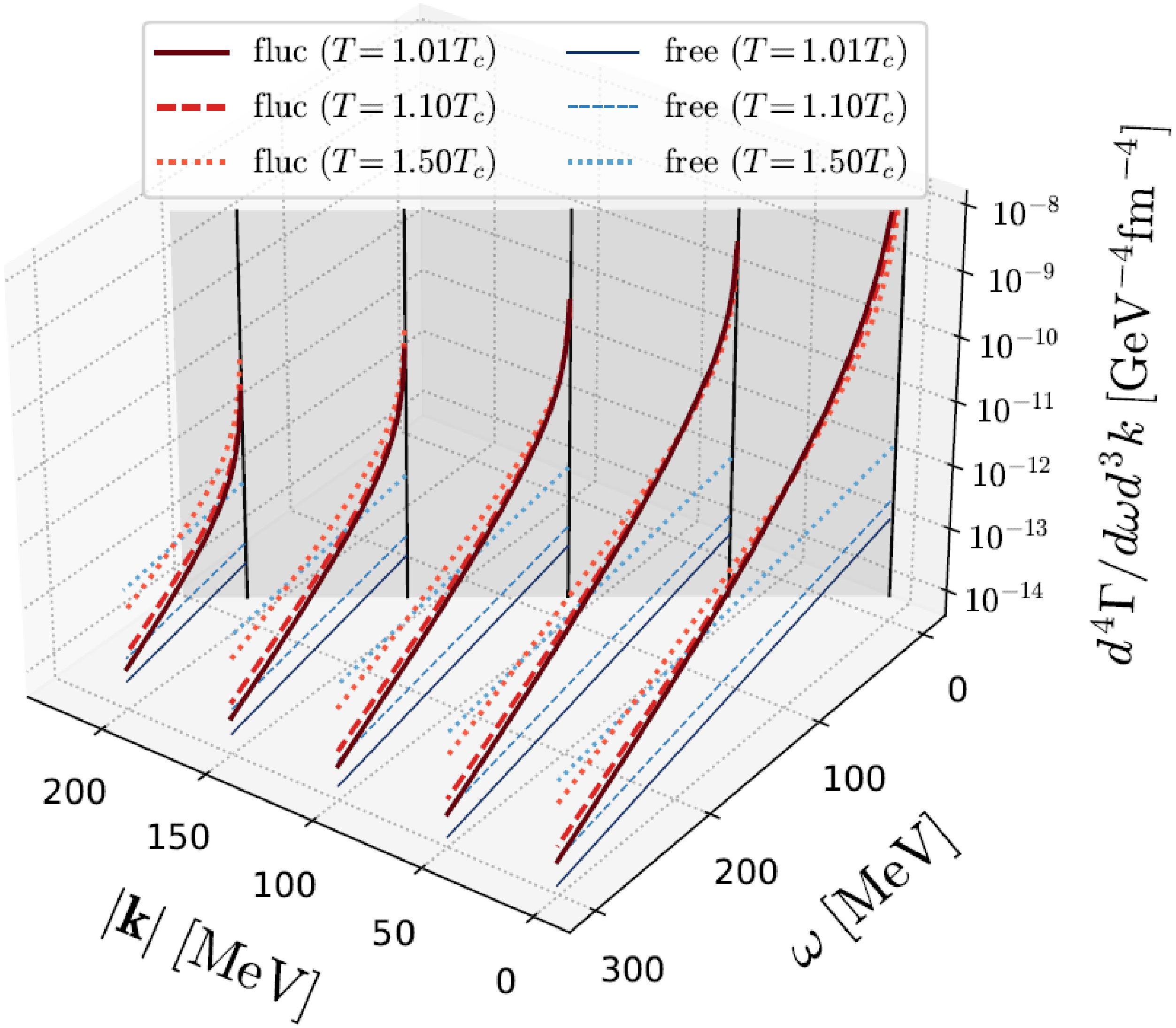
☐ MT+DoS



Similar formula for MT+DoS vertex

Energy-Momentum Dependence

Nishimura, MK, Kunihiro ('22)



Red: fluctuation contribution

Blue: free quarks

$G_C = 0.7G_S, T_c \simeq 45$ MeV

- Enhancement due to diquark fluctuations is more suppressed for larger k.

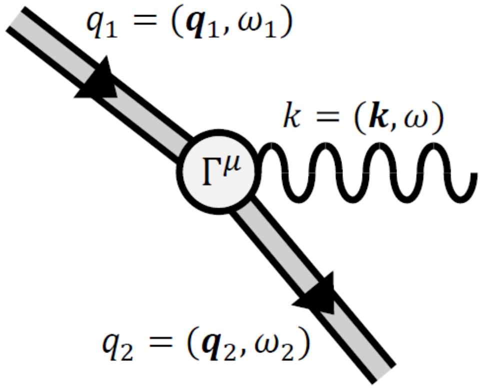
Production Mechanism of Virtual Photons

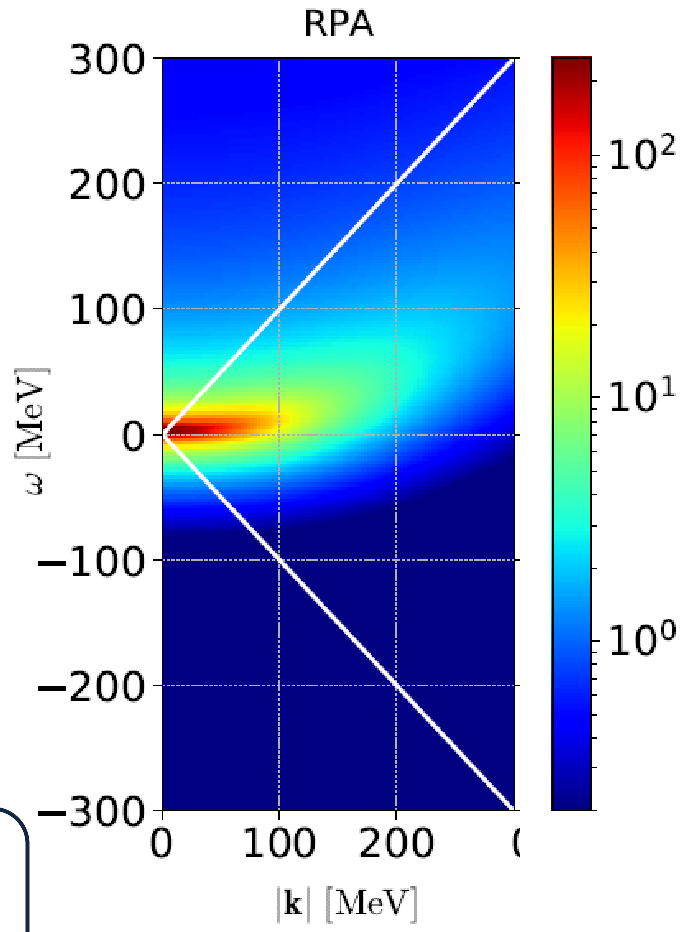
☐ Production mechanism

- ☐ scattering of diquarks
- ☐ diquarks: space-like region

$$\omega = \omega_1 - \omega_2$$

$$\boldsymbol{k} = \boldsymbol{q}_1 - \boldsymbol{q}_2$$



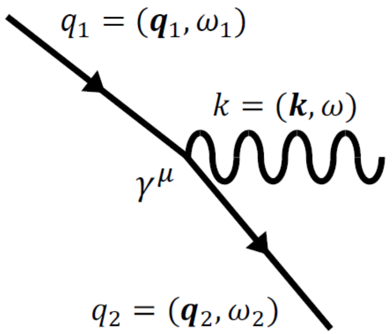


➡ Production in the time-like region is possible.

⬄

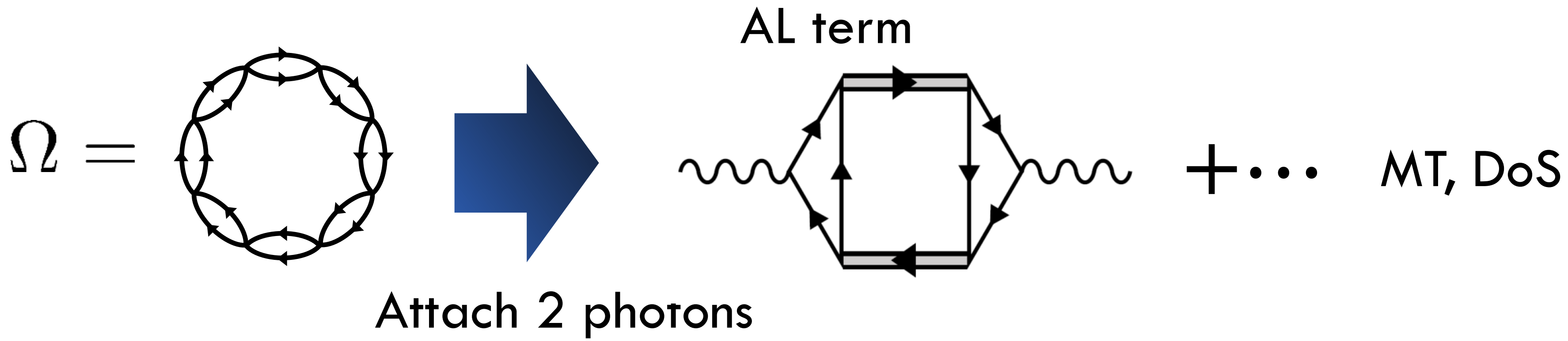
☐ C.f.) Scattering of free quarks produces virtual photons only in the space-like region.

$$|\boldsymbol{q}_1 - \boldsymbol{q}_2| \geq \omega_1 - \omega_2$$

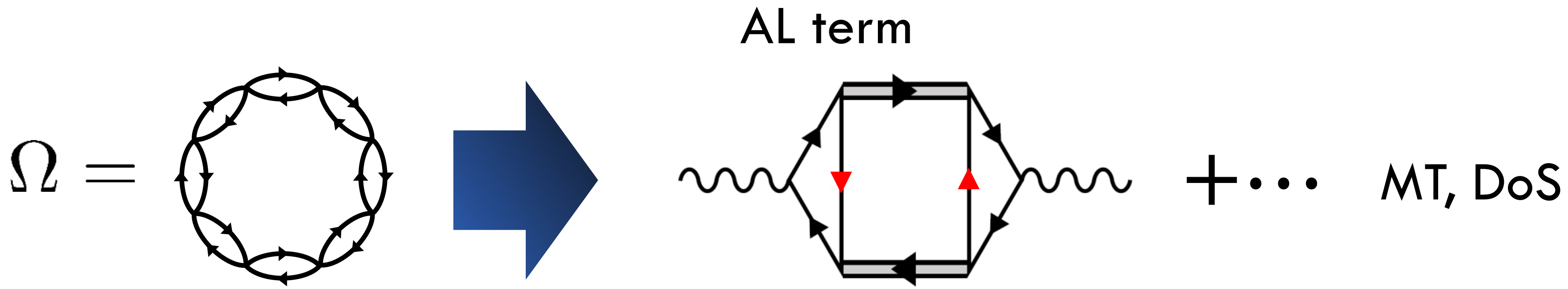


Formulation

☐ Diquark Fluctuations



☐ Scalar Fluctuations



→ Photon self-energy including the soft mode of QCD-CP can be constructed in a similar manner as before.

Dilepton production rate near QCD-CP

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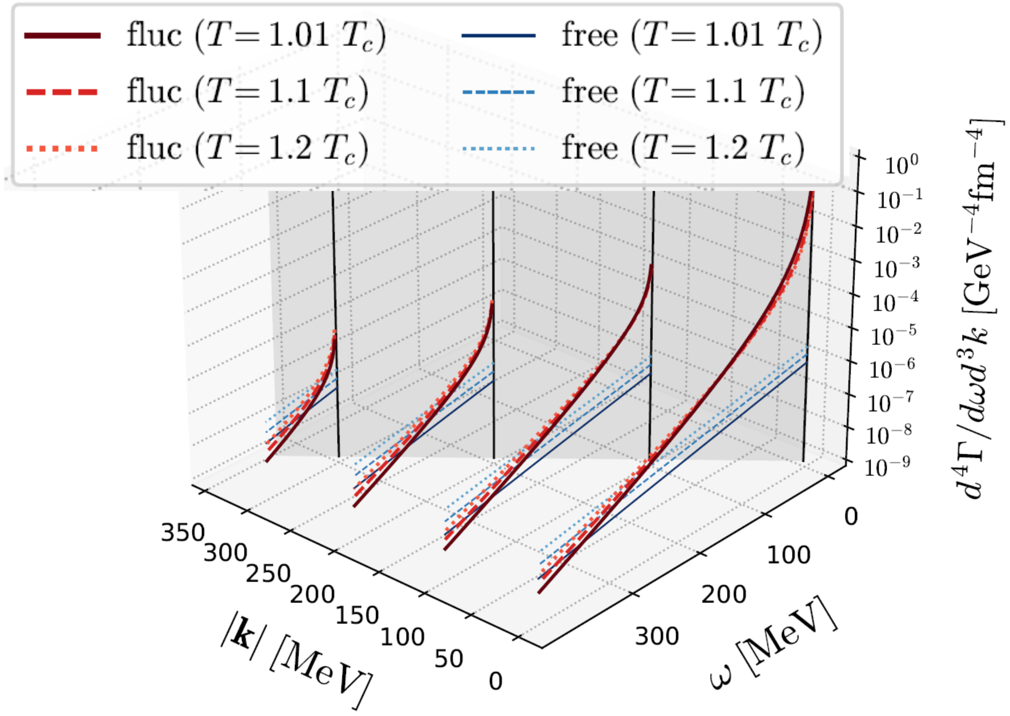
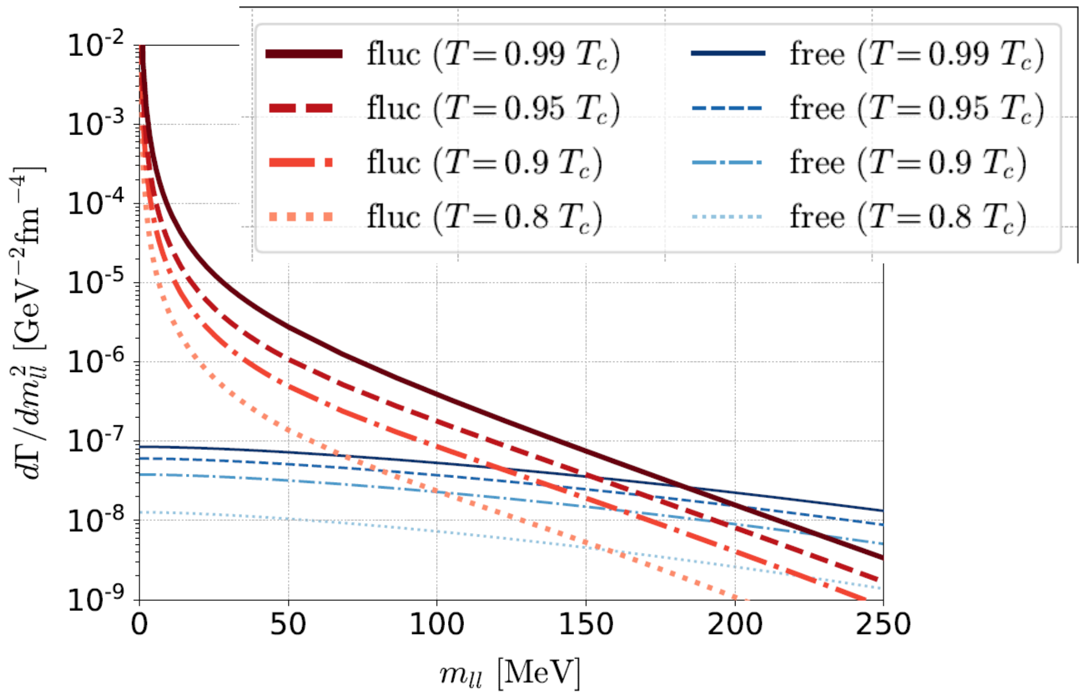
□ $\omega - k$ plane



□ Invariant mass spectrum

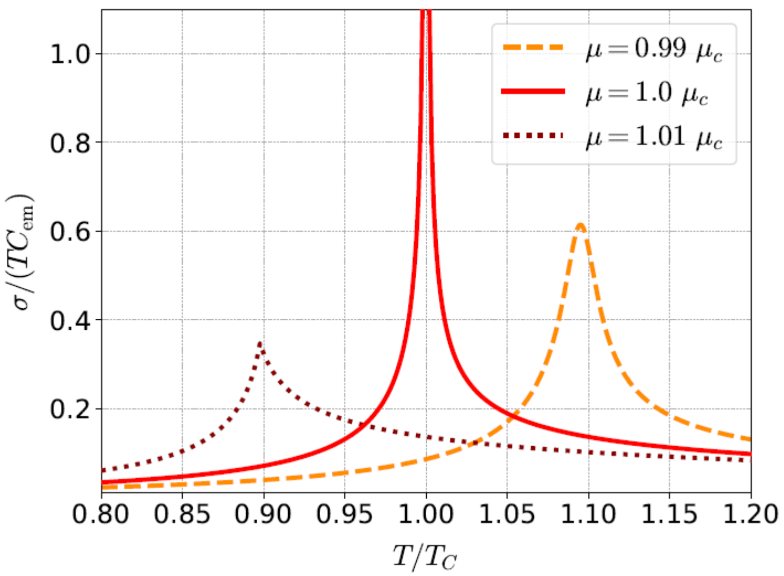


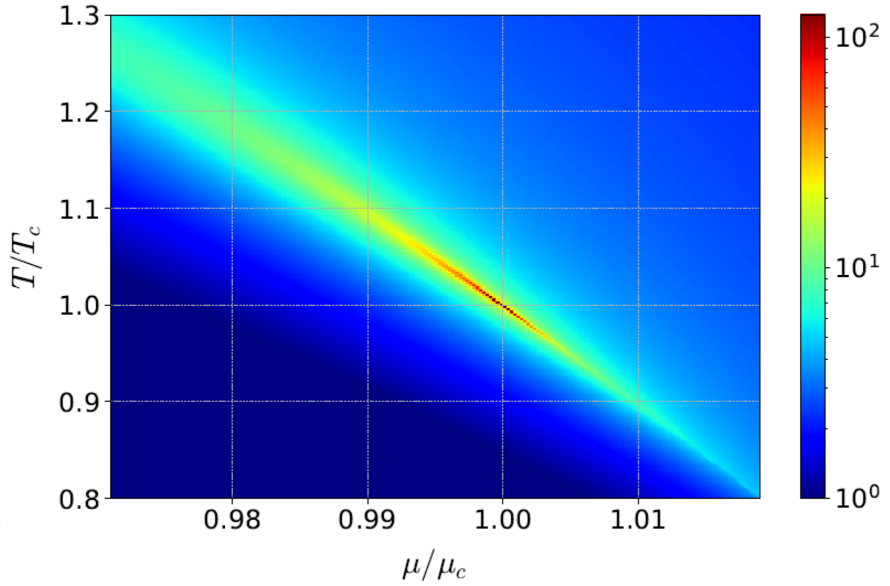
chemical potential: $\mu = \mu_c$

- □ Enhancement at low ω, k, m_{ll} regions near QCD-CP
- □ Distinguishment from diquark soft mode may be difficult.

Electric Conductivity

- Soft mode leads to enhancement of conductivity σ.





Note:

Both DPR and σ are given from photon self-energy.

$$\frac{d^4\Gamma}{d^4k} = -\frac{\alpha}{12\pi^4}\frac{1}{k^2}\frac{1}{e^{\omega/T}-1} g_{\mu\nu} \mathrm{Im}\Pi^{R\mu\nu}(\boldsymbol{k},\omega),$$

$$\sigma = \frac{1}{3}\lim_{\omega\to 0}\frac{1}{\omega}\sum_{i=1,2,3}\mathrm{Im}\Pi^{Rii}(\boldsymbol{0},\omega).$$

Critical Exponents

	QCD-CP	CSC
σ	$\lvert T-T_c\rvert^{-2/3}$	$\lvert T-T_c\rvert^{-1/2}$
τ	$\lvert T-T_c\rvert^{-1}$	$\lvert T-T_c\rvert^{-1}$

- Conductivity diverges with different critical exponents in QCD-CP & CSC.
- Can they distinguishable in HIC??

Nishimura, MK, Kunihiro, in prep.