QCD mesonic screening masses: Beyond perturbative study

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Introduction

- At finite temperature, Lorentz symmetry is broken $\Rightarrow$ temporal and spatial directions are, in general, unrelated.
- Correlation function in the time direction is used to define spectral functions, which gives information about the plasma's real-time properties, such as particle production rate.
- Correlation functions in spatial direction give info about
  (1) The length scale at which thermal fluctuation are correlated
  (2) The length scale at which the external charges are screened.
- These "static" observables are physical and eminently suited to measurements in lattice experiments.
- Different operator gives different correlation lengths depending on their discrete and continuous global symmetry properties.

Screening mass

- Screening mass can show us how perturbative the medium is.
- Vector-like excitations can reach the perturbative estimate more quickly than pseudo-scalar excitation (HotQCD 19).
- Perturbative estimation: $M/T = 2\pi + \frac{g^2 C_F}{2\pi}(\frac{1}{2} + E_0)$

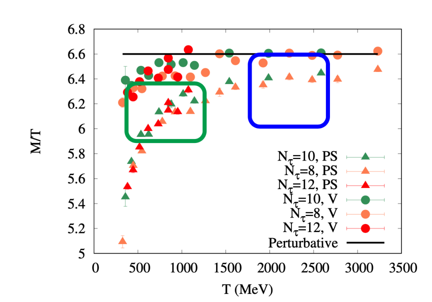

Figure: Credit: Sayantan Sharma's talk in WHEPP 2019, India.

Detailed setup

- Defination:

$$C_z\left[O^a, O^b\right] = \int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}^2\mathrm{x}_\perp \left\langle O^a\left(\tau, \mathrm{x}_\perp, z\right) O^b(0, \mathbf{0}, 0)\right\rangle$$

- In the limit of $z \to \infty, C_z\left[O^a, O^b\right] \sim \mathrm{e}^{-2\omega_0 z} = \mathrm{e}^{-mz}$,

$$\omega_n = 2\pi T\left(n + \frac{1}{2}\right), \qquad \zeta^{-1} = 2\pi T = m \to \text{Screening mass}$$

- For the correlation lengths ζ of mesonic observables, $\mathcal{O} = \bar{\psi}\Gamma F^a \psi$, where

$$\Gamma = \left\{1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5\right\},$$

- Physical significance of some of these operators:
  $\bar{\psi}\gamma_5 F^s \psi \propto \eta'$-meson ,
  $\bar{\psi}\gamma_5 F^n \psi \propto$ pion,
  $\bar{\psi}\gamma_0 F^s \psi \propto$ baryon number density,
  $\bar{\psi}\gamma_0 F^n \psi \propto$ electric charge density (for $N_\mathrm{f} = 3$).

Next-to-leading order for flavour non-singlet correlators

- Infinitely many higher order graphs that need to be considered.

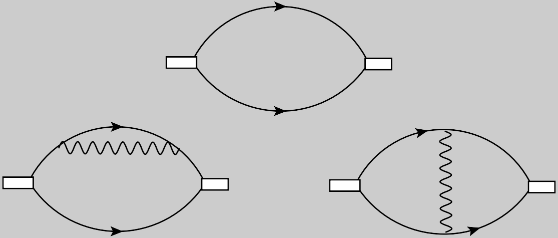

Figure: The diagrams which contribute to meson correlation function: (a) free theory correlator (b) quark self-energy graph (c) interaction of quark and antiquark through gluon exchange.

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- A convenient way of resummation of all the diagrams is offered by the effective field theory, namely NRQCD.
- Correlation lengths can be seen as (2+1)-dimensional bound states of heavy particles of mass "p_0", which is much larger than infrared scale gT, g^2T.
- Correlation function in leading order dominates only at zero Matsubara mode.

$$\mathcal{L}_E^{\psi} = \bar{\psi}\left[i\gamma_0 p_0 - ig\gamma_0 A_0 + \gamma_k D_k + \gamma_3 D_3\right]\psi$$

- The "diagonalized" on-shell effective lagrangian for two independent light modes with a non-relativistic structure:

$$\begin{aligned}\mathcal{L}_E^{\psi} \approx & i\chi^{\dagger}\left[p_0 - gA_0 + D_3 - \frac{1}{2p_0}\left(D_k^2 + \frac{g}{4i}\left[\sigma_k, \sigma_l\right]F_{kl}\right)\right]\chi + \\ & + i\phi^{\dagger}\left[p_0 - gA_0 - D_3 - \frac{1}{2p_0}\left(D_k^2 + \frac{g}{4i}\left[\sigma_k, \sigma_l\right]F_{kl}\right)\right]\phi + \mathcal{O}\left(\frac{1}{p_0^2}\right)\end{aligned}$$

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- Since we are interested in results up to $\mathcal{O}(g^2)T$, we can ignore the spatial gauge fields.

$$\mathcal{L}_E^{\psi} = i\chi^\dagger \left(M - g_{\mathrm{E}} A_0 + D_t - \frac{\nabla_\perp^2}{2p_0} \right) \chi + i\phi^\dagger \left(M - g_{\mathrm{E}} A_0 - D_t - \frac{\nabla_\perp^2}{2p_0} \right) \phi$$

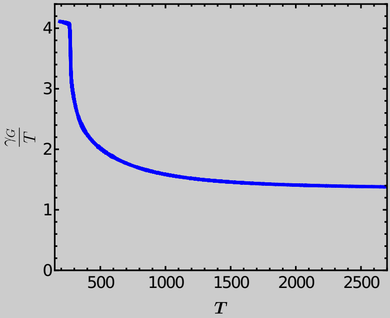
- To be consistent at $\mathcal{O}(g^2 T)$, we should replace p_0 of the tree-level effective Lagrangian by a matching coefficient $M = p_0 + \mathcal{O}(g^2 T)$ that needs to be determined.

Matching conditions from QCD to NRQCD, with Gribov

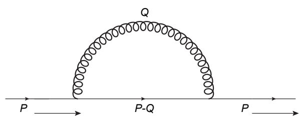
- Gluon Propagator (Gribov modified):

$$D^{\mu\nu}(P) = \left[\delta^{\mu\nu} - (1-\xi)\frac{P^\mu P^\nu}{P^2}\right]\frac{P^2}{P^4+\gamma_G^4}$$

- The Gribov parameter γ_G can be fixed from lattice thermodynamics.



- With Gribov propagator, quark self-energy becomes



$$\begin{aligned}\Sigma(P) &= -ig^2 C_F \sum\!\!\!\!\!\!\int_Q \frac{\gamma_\mu(\not{P}-\not{Q})\gamma_\mu}{(P-Q)_f^2}\left(\frac{Q^2}{Q^4+\gamma_G^4}\right) + ig^2 \\ &\times \quad C_F \sum\!\!\!\!\!\!\int_Q \frac{\not{Q}(\not{P}-\not{Q})\not{Q}}{Q^2(P-Q)_f^2}\left(\frac{Q^2}{Q^4+\gamma_G^4} - \frac{\xi\, Q^2}{Q^4+\gamma_G^4}\right)\end{aligned}$$

- With the above quark self-energy, the Euclidean dispersion relation on the QCD becomes

$$p_3 \approx i\Big[p_0 - g^2 C_F (I_1 + I_2)\Big]$$

Continued...

- $$I_1 = \frac{-1}{p_0}\int_0^\infty \frac{q^2 dq}{(2\pi)^2}\left[\frac{n^+}{E_+} + \frac{n^-}{E_-}\right], \quad I_2 = \frac{1}{p_0}\left[\frac{-T^2}{24} + X\right]$$

  with $n^\pm \rightarrow$ B.E distribution function, $\tilde{n} \rightarrow$ F.D distribution function and $E_\pm = \sqrt{q^2 \pm \gamma_G^2}$ and

  $$X = \frac{\gamma_G^4}{T^2}\int \frac{d^3q}{(2\pi)^3}\frac{1}{8EE_+E_-}\left[\left\{\frac{\tilde{n}+n^-}{i\pi - E + E_-} - \frac{\tilde{n}+n^-}{i\pi + E - E_-}\right\} - (n^- \rightarrow n^+)\right]\frac{1}{E_+ - E_-}$$

- On NRQCD$_3$ side, the pole location is simply $p_3 = iM$. Now, after doing the matching, we will get

  $$M = p_0 - g^2 C_F (I_1 + I_2)$$

Solution for the screening states

- Correlators we have considered are

$$C_z\left[O^a, O^b\right] \sim \int \mathrm{d}^2 x_\perp \left\langle O^a\left(x_\perp, z\right) O^b\left(\mathbf{0}_\perp, 0\right)\right\rangle,$$

- The E.O.M obeyed by the Green's function (at large z), is of the form

$$(\partial_z - H)G(z) = C\,\delta(z)$$

- The tree-level contribution reads as

$$\left[\partial_z + 2M - \frac{1}{p_0}\nabla_{\boldsymbol{r}}^2\right] C^{(0)}(r,z) \propto \delta(z)\delta^{(2)}(r).$$

- Similarly, the 1-loop contribution can be written as

$$\left[\partial_z + 2M - \frac{1}{p_0}\nabla_{\boldsymbol{r}}^2\right] C^{(1)}(\boldsymbol{r},z) = -g_{\mathrm{E}}^2 C_F \mathcal{K}\left(\frac{1}{zp_0}, \frac{\nabla_{\boldsymbol{r}}}{p_0}, \frac{\gamma_G^4}{p_0^4}, rp_0\right) C^{(0)}(r,z).$$

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- The kernel $\mathcal{K}$ gives the one-loop static potential as

$$V(r) = g_{\mathrm{E}}^2 \frac{C_F}{2\pi} \left[\ln \frac{\gamma_{\mathrm{G}} r}{2} + \gamma_E - K_0 \left(\gamma_{\mathrm{G}} r \right) \right].$$

- This potential determines the coefficient of the exponential fall-off, $\xi^{-1} \equiv m$, through

$$\left[2M - \frac{\nabla_r^2}{p_0} + V(r) \right] \Psi_0 = m \Psi_0$$

- To find the solution numerically, we rescale $m - 2M \equiv g_{\mathrm{E}}^2 \frac{C_F}{2\pi} E_0$
- EOM becomes $\left[-\frac{\nabla_r^2}{p_0} + V(r) \right] \Psi_0 = g_{\mathrm{E}}^2 \frac{C_F}{2\pi} E_0 \Psi_0$
- After solving for E_0, the Screening mass can be obtained as $m = 2\pi T + g^2 T \frac{C_F}{2\pi} \left[E_0 - \frac{4\pi}{T} \left(I_1 + I_2 \right) \right].$

Results and Discussion

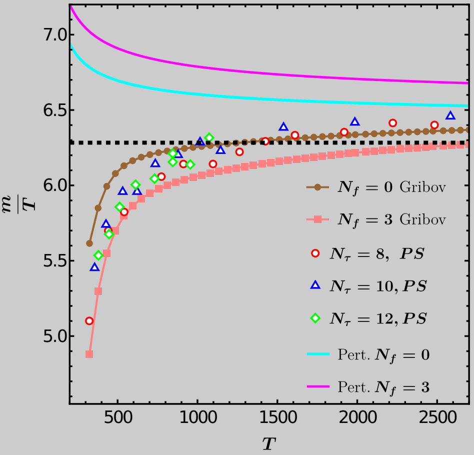

Figure: The temperature dependence of the scaled screening mass. The dashed line represents the free theory result from $m/T = 2\pi$.

Thank you for your attention.