$\eta'$ in medium

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- Chiral symmetry restoration
- $\eta'$ in medium
- Outlook
Why $\eta'$, Chiral symmetries

Massless fermions: helicity, left- and right handed particles

$m_u \approx 3$ MeV, $m_d \approx 6$ MeV, $m_s \approx 95$ MeV

if the masses are 0, then the symmetry group:

$U_{QCD} = SU_V(3) \otimes SU_A(3) \otimes U_A(1) \otimes U_V(1)$

$m_q \ll m_p$, QCD nearly chiral symmetric

$U_A(1)$ symmetry is broken by anomaly

the small quark masses break $SU_A(3) \otimes SU_V(3)$ and

remains $U_V(1) \otimes SU_V(3)$ if all 3 quark masses are equal

remains $U_V(1) \otimes SU_V(2)$ if 2 masses are equal

remains $U_V(1)$ if all 3 masses are different.
The order parameter of the chiral symmetry restauration: $\langle \bar{q}q \rangle$
Anomaly $U_A(1)$ problem, $\eta'$

$U_A(1)$ is broken by the ABJ anomaly

$$\partial^\mu J^5_\mu = 2N_f \frac{g^2}{16\pi^2} \text{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu}) + \sum_f 2im_f \bar{\psi}_f \gamma_5 \psi_f.$$

The breaking of the $U_A(1)$ symmetry is an operator relation that remains valid even when the spontaneously broken chiral symmetry is restored. $U_A(1)$ symmetry is restored at very high temperature, where $g^2/4\pi \to 0$ as suggested by Kapusta, Kharzeev, McLerran.

However, it was shown that in the chiral limit, with $N_f$ flavors, the symmetry will effectively be restored in correlation functions composed of up to $N_f - 1$ points.


What is its effect on the $m_{\eta'}$
Extended linear sigma model (ELSM)

\[ \Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \left( \begin{array}{ccc} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{array} \right) (\sim \bar{q}_i \gamma_5 q_j) \]

\[ \Phi_S = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \left( \begin{array}{ccc} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{array} \right) (\sim \bar{q}_i q_j) \]

Particle content:

Pseudoscalars: \( \pi(138), K(495), \eta(548), \eta'(958) \)

Scalars: \( a_0(980 \text{ or } 1450), K_0^*(800 \text{ or } 1430), (\sigma_N, \sigma_S) : 2 \text{ of } f_0(500, 980, 1370, 1500, 1710) \)
\[
V^\mu = \sum_{i=0}^{8} \rho^\mu_i T_i = \frac{1}{\sqrt{2}} \left( \begin{array}{ccc}
\frac{\omega_N + \rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\
\rho^- & \frac{\omega_N - \rho^0}{\sqrt{2}} & K^{*0} \\
K^{*-} & \bar{K}^{*0} & \omega_S
\end{array} \right)^\mu
\]

\[
A^\mu = \sum_{i=0}^{8} b^\mu_i T_i = \frac{1}{\sqrt{2}} \left( \begin{array}{ccc}
\frac{f_{1N} + a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\
a_1^- & \frac{f_{1N} - a_1^0}{\sqrt{2}} & K_1^0 \\
K_1^- & \bar{K}_1^0 & f_{1S}
\end{array} \right)^\mu
\]

Particle content:
Vector mesons: \( \rho(770), K^*(894), \omega_N = \omega(782), \omega_S = \phi(1020) \)
Axial vectors: \( a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426) \)
\[ \mathcal{L}_{\text{Tot}} = \text{Tr}[(D_\mu \Phi)\dagger(D_\mu \Phi)] - m_0^2 \text{Tr}(\Phi\dagger \Phi) - \lambda_1 \text{Tr}(\Phi\dagger \Phi)^2 - \lambda_2 \text{Tr}(\Phi\dagger \Phi)^2 \\
- \frac{1}{4} \text{Tr}(L^2_{\mu\nu} + R^2_{\mu\nu}) + \text{Tr} \left[ \left( \frac{m_1^2}{2} + \Delta \right)(L^2_\mu + R^2_\mu) \right] + \text{Tr}[H(\Phi + \Phi\dagger)] \\
+ c_1 (\text{det} \Phi + \text{det} \Phi\dagger) + \frac{i g_2}{2} (\text{Tr}\{L_{\mu\nu} [L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu} [R^\mu, R^\nu]\}) \\
+ \frac{h_1}{2} \text{Tr}(\Phi\dagger \Phi)\text{Tr}(L^2_\mu + R^2_\mu) + h_2 \text{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \text{Tr}(L_\mu \Phi R^\mu \Phi\dagger) \\
+ \bar{\Psi} i \partial_\mu \Psi - g_F \bar{\Psi} (\Phi_S + i \gamma_5 \Phi_{PS}) \Psi + g_V \bar{\Psi} \gamma^\mu \left( V_\mu + \frac{g_A}{g_V} \gamma_5 A_\mu \right) \Psi \\
+ \text{Polyakov loops} \]

$D^\mu \Phi = \partial^\mu \Phi - ig_1 (L^\mu \Phi - \Phi R^\mu) - ieA_e^\mu [T_3, \Phi]$

$\Phi = \sum_{i=0}^{8} (\sigma_i + i\pi_i)T_i, \quad H = \sum_{i=0}^{8} h_i T_i \quad T_i : U(3) \text{ generators}$

$R^\mu = \sum_{i=0}^{8} (\rho_i^\mu - b_i^\mu) T_i, \quad L^\mu = \sum_{i=0}^{8} (\rho_i^\mu + b_i^\mu) T_i$

$L^{\mu \nu} = \partial^\mu L^\nu - ieA_e^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu [T_3, L^\mu]\}$

$R^{\mu \nu} = \partial^\mu R^\nu - ieA_e^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu [T_3, R^\mu]\}$

$\bar{\Psi} = (\bar{u}, \bar{d}, \bar{s})$

non strange – strange base:

$\varphi_N = \sqrt{2/3}\varphi_0 + \sqrt{1/3}\varphi_8,$

$\varphi_S = \sqrt{1/3}\varphi_0 - \sqrt{2/3}\varphi_8, \quad \varphi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i)$

broken symmetry: non-zero condensates $\langle \sigma_N \rangle, \langle \sigma_S \rangle \longleftrightarrow \phi_N, \phi_S$
16 unknown parameters \((m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F, g_V, g_A)\) → Determined by the min. of \(\chi^2\):

\[
\chi^2(x_1, \ldots, x_N) = \sum_{i=1}^{M} \left[ \frac{Q_i(x_1, \ldots, x_N) - Q_i^{\text{exp}}}{\delta Q_i} \right]^2,
\]

where \((x_1, \ldots, x_N) = (m_0, \lambda_1, \lambda_2, \ldots)\), \(Q_i(x_1, \ldots, x_N)\) calculated from the model, while \(Q_i^{\text{exp}}\) taken from the PDG multiparametric minimalization → MINUIT

PCAC → 2 physical quantities: \(f_\pi, f_K\)

Tree-level masses → 15 physical quantities:

\(m_u/d, m_s, m_\pi, m_\eta, m_\eta', m_K, m_\rho, m_\Phi, m_{K^*}, m_{a_1}, m_{f_1^H}, m_{a_0}, m_{K_S}, m_{f_0L}, m_{f_0H}\)

Decay widths → 12 physical quantities:

\(\Gamma_{\rho \to \pi\pi}, \Gamma_{\Phi \to KK}, \Gamma_{K^* \to K\pi}, \Gamma_{a_1 \to \pi\gamma}, \Gamma_{a_1 \to \rho\pi}, \Gamma_{f_1 \to KK^*}, \Gamma_{a_0}, \Gamma_{K_S \to K\pi}, \Gamma_{f_0L \to \pi\pi}, \Gamma_{f_0L \to KK}, \Gamma_{f_0H \to \pi\pi}, \Gamma_{f_0H \to KK}\)

\(T_c = 155\) MeV from lattice
Extremum equations for $\phi_{N/S}$ and $\Phi, \bar{\Phi}$

$\Omega$: grand canonical potential

\[
\frac{\partial \Omega}{\partial \Phi} = \left. \frac{\partial \Omega}{\partial \Phi} \right|_{\varphi_{N}=\phi_{N}, \varphi_{S}=\phi_{S}} = 0
\]

\[
\frac{\partial \Omega}{\partial \phi_{N}} = \left. \frac{\partial \Omega}{\partial \phi_{S}} \right|_{\Phi, \bar{\Phi}} = 0, \quad \text{(after the SSB)}
\]
Order parameters and masses in medium $m_\sigma = 500$ MeV

Condensates and Polyakov loop variables with $m_\sigma = 500$ MeV

Masses

Order parameters and masses in medium $m_\sigma = 290$ MeV
Consider the gluonic correlation function:

\[ U(k) = i \int d^4x \, e^{ik \cdot x} \langle TG\tilde{G}(x) G\tilde{G}(0) \rangle. \]

\[ U(0) = 0 \] if there is a massless fermion

\[ U(k) = -\sum_n \frac{|\langle 0|G\tilde{G}|^{n\text{th} \text{glueball}} \rangle|^2}{k^2 - M_n^2} - \sum_n \frac{|\langle 0|G\tilde{G}|^{n\text{th} \text{meson}} \rangle|^2}{k^2 - m_n^2} \]

\[ \equiv U_0(k) + U_1(k). \]

\[ U_0(k) \propto N_c^2 \]
\[ U_1(k) \propto N_c. \]
$U(0) = 0$ is possible only if in $U_1$ there is a meson, $\eta'$, for $m_{\eta'}^2 \sim 1/N_c$

$$U_0(0) = -\frac{|\langle 0|G\tilde{G}|\eta'\rangle|^2}{m_{\eta'}^2}.$$

$$\langle 0|G\tilde{G}|\eta'\rangle = \frac{4\pi}{\alpha_s} \frac{1}{N_f} \langle 0|\partial_\mu J_5^\mu|\eta'\rangle = \frac{4\pi}{\alpha_s} \frac{1}{N_f} \sqrt{N_f} m_{\eta'} f_\pi,$$

Witten-Veneziano equation:

$$U_0(0) = -\frac{1}{N_f} m_{\eta'} f_\pi^2 \left(\frac{4\pi}{\alpha_s}\right)^2,$$

$$m_{\eta'} = \sqrt{\frac{8}{33}} \frac{1}{f_\pi} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle^{1/2} \approx 464\text{ MeV},$$

This is the mass which is originated from the anomaly.
Witten-Veneziano theorem in matter

\[ U_0(0) = -\frac{|\langle 0 | G \tilde{G} | \eta' \rangle|^2}{m_{\eta'}^2}. \]

The equation is valid in matter, the derivation is the same. Y. Kwon, S.H. Lee, K. Morita, Gy. Wolf, Phys. Rev. D86 (2012) 034014

Low energy theorem:

\[ U(k = 0) = -\frac{2}{b} \left( \frac{4\pi}{3\alpha_s} \right)^2 \left( d - T \frac{\partial}{\partial T} - \mu \frac{\partial}{\partial \mu} \right) \langle \alpha_s \pi G^2 \rangle_{T,\mu}. \]


From the 2 equations we obtain

\[ m_{\eta'}^2 = \left( \frac{3\alpha_s}{4\pi} \right)^2 \frac{2}{b} \left( d - T \frac{\partial}{\partial T} \right) \langle \alpha_s \pi G^2 \rangle_{T,\text{puregauge}}. \]
The denominator depends weakly on the temperature

\[(4 - T(\partial/\partial T))\langle (\alpha/\pi)G^2 \rangle\]

\[4\langle (\alpha/\pi)G^2 \rangle\]

\[T(\partial/\partial T)\langle (\alpha/\pi)G^2 \rangle\]
If the chiral symmetry is restored

\[
U(k) = i \int d^4 x \, e^{i k \cdot x} \langle TG \tilde{G}(x) G \tilde{G}(0) \rangle
\]

\[
= k^{\mu} k^{\nu} i \int d^4 x \, e^{i k \cdot x} \left( \frac{4\pi}{\alpha_s N_f} \right)^2 \left[ \langle T \bar{q} i \gamma_\mu \gamma_5 q(x) \bar{q} i \gamma_\nu \gamma_5 q(0) \rangle - \langle T \bar{q} \gamma_\mu q(x) \bar{q} \gamma_\nu q(0) \rangle \right],
\]

\[
\propto \text{Tr}[S_A(x, x)]^2
\]  

(2)

The connected diagrams do not contribute, since they appear with opposite signs.

Using the Banks-Cashier equation:

\[
\langle 0 | G \tilde{G} | \eta' \rangle \propto \text{Tr}[S_A(x, x)] \propto \langle \bar{q} q \rangle^*
\]

So:

\[
m_{\eta'}^* \propto \langle \bar{q} q \rangle^*
\]

Important to note that this is only for the mass arising from the anomaly, which is in vacuum 464 MeV.
Experimental results

\[ \lambda / \lambda_{\text{max}} \]

**PHENIX 0-30% Au+Au \( \sqrt{s_{\text{NN}}} = 200 \text{ GeV} \)**

- \( \pi^\pm \pi^\mp \)
- \( \pi^+ \pi^+ \)

\[ \lambda_{\text{max}} = \langle \lambda \rangle_{(0.55-0.9) \text{ GeV}/c^2} \]

**Legend**:
- \( m_r^* = 958 \text{ MeV}, B_{\eta} = 55 \text{ MeV} \)
- \( m_r^* = 530 \text{ MeV}, B_{\eta} = 168 \text{ MeV} \)
- \( m_r^* = 530 \text{ MeV}, B_{\eta} = 55 \text{ MeV} \)
- \( m_r^* = 250 \text{ MeV}, B_{\eta} = 55 \text{ MeV} \)

**Fits**:
- \( H = (0.59 \pm 0.02_{\text{stat}}^{0.022_{\text{syst}}}) \)
- \( \sigma = (0.30 \pm 0.01_{\text{stat}}^{0.08_{\text{syst}}}) \) \text{ GeV}/c^2, \( \chi^2/\text{NDF} = 83/60 \), \( \text{CL} = 2.7\% \)
Summary

- We studied the chiral transition in a linear sigma model. We determined the $m_{\eta'}$ as a function of the temperature.  

- We generalized the Witten-Veneziano theorem for nonzero temperature and found a limit (actually a rough estimate at the chiral restoration temperature) for $m_{\eta'}$.  

  Note that we do not state the $SU_A(1)$ symmetry is restored at temperatures relevant for experimental signals.

- The two values differ we wait for the experimental result (according to rumours the second estimate is not so bad).