η' in medium

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- Chiral symmetry restoration
- $\eta\prime$ in medium
- Outlook

Why η', Chiral symmetries

Massless fermions:helicity, left- and right handed particles

$m_u \approx 3$ MeV, $m_d \approx 6$ MeV, $m_s \approx 95$ MeV

if the masses are 0, then the symmetry group:

$U_{QCD} = SU_V(3) \otimes SU_A(3) \otimes U_A(1) \otimes U_V(1)$

$m_q \ll m_p$, QCD nearly chiral symmetric

$U_A(1)$ symmetry is broken by anomaly

the small quark masses break $SU_A(3) \otimes SU_V(3)$ and

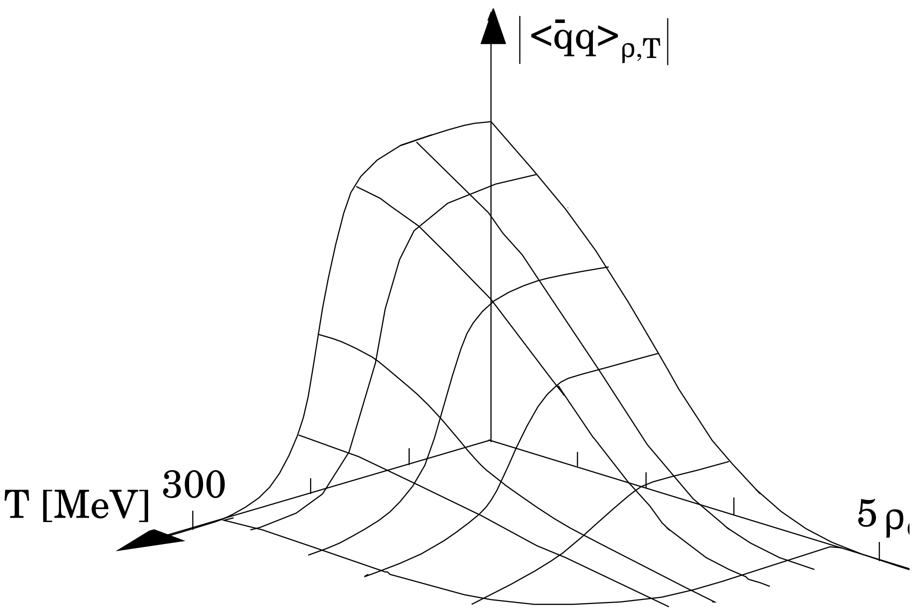
remains $U_V(1) \otimes SU_V(3)$ if all 3 quark masses are equal

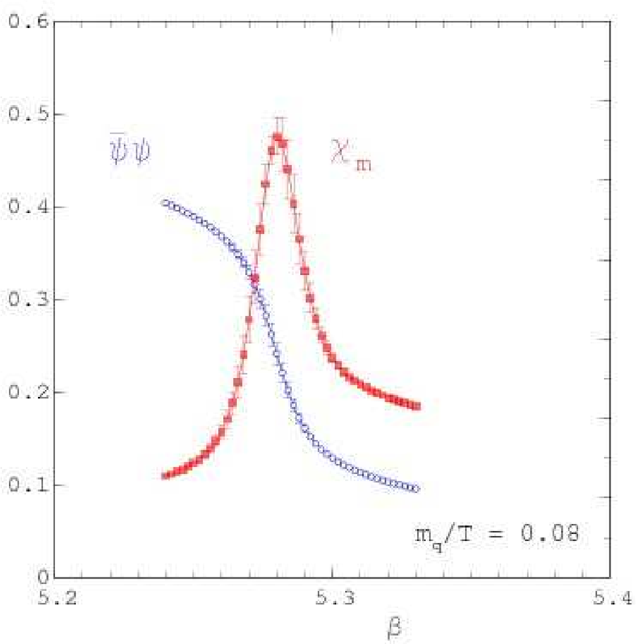
remains $U_V(1) \otimes SU_V(2)$ if 2 masses are equal

remains $U_V(1)$ if all 3 masses are different.

Quark Condensate

The order parameter of the chiral symmetry restauration: $< \bar{q}q >$





Anomaly $U_A(1)$ problem, η'

$U_A(1)$ is broken by the ABJ anomaly

$$\partial^\mu J_\mu^5 = 2N_f \frac{g^2}{16\pi^2} \mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu}) + \sum_f 2im_f \overline{\psi_f}\gamma_5\psi_f.$$

The breaking of the $U_A(1)$ symmetry is an operator relation that remains valid even when the spontaneously broken chiral symmetry is restored. $U_A(1)$ symmetry is restored at very high temperature, where $g^2/4\pi \to 0$ as suggested by Kapusta, Kharzeev, McLerran

However, it was shown that in the chiral limit, with N_f flavors, the symmetry will effectively be restored in correlation functions composed of up to $N_f - 1$ points

T. D. Cohen, Phys. Rev. D 54, 1867 (1996)

S. H. Lee and T. Hatsuda, Phys. Rev. D 54, 1871 (1996)

What is its effect on the $m_{\eta'}$

Extended linear sigma model (ELSM)

$$\Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} \quad (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_{S} = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} \quad (\sim \bar{q}_i q_j)$$

Particle content:

Pseudoscalars: $\pi(138), K(495), \eta(548), \eta'(958)$

Scalars: $a_0(980 \text{ or } 1450), K_0^\star(800 \text{ or } 1430)$,

(σ_N, σ_S) : 2 of $f_0(500, 980, 1370, 1500, 1710)$

$$V^\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star +} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star -} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

Particle content:

Vector mesons: $\rho(770), K^\star(894), \omega_N = \omega(782), \omega_S = \phi(1020)$

Axial vectors: $a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426)$

Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Tot}} = \; & \mathrm{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \mathrm{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\mathrm{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \mathrm{Tr}(\Phi^\dagger \Phi)^2 \\
- \; & \frac{1}{4} \mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \mathrm{Tr}[H(\Phi + \Phi^\dagger)] \\
+ \; & c_1 (\det \Phi + \det \Phi^\dagger) + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
+ \; & \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger \Phi)\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2 \mathrm{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \mathrm{Tr}(L_\mu \Phi R^\mu \Phi^\dagger) \\
+ \; & \bar{\Psi} i \partial\!\!\!/ \Psi - g_F \bar{\Psi}\left(\Phi_S + i\gamma_5 \Phi_{PS}\right)\Psi + g_V \bar{\Psi}\gamma^\mu \left(V_\mu + \frac{g_A}{g_V}\gamma_5 A_\mu\right)\Psi \\
+ \; & \text{Polyakov loops}
\end{aligned}
$$

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, D.H. Rischke, Phys. Rev. D87 (2013) 014011

Lagrangian 2

$$
\begin{aligned}
D^\mu \Phi &= \partial^\mu \Phi - i g_1 (L^\mu \Phi - \Phi R^\mu) - ieA_e^\mu [T_3, \Phi] \\
\Phi &= \sum_{i=0}^{8} (\sigma_i + i\pi_i) T_i, \quad H = \sum_{i=0}^{8} h_i T_i \qquad T_i : U(3) \text{ generators} \\
R^\mu &= \sum_{i=0}^{8} (\rho_i^\mu - b_i^\mu) T_i, \quad L^\mu = \sum_{i=0}^{8} (\rho_i^\mu + b_i^\mu) T_i \\
L^{\mu\nu} &= \partial^\mu L^\nu - ieA_e^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu [T_3, L^\mu]\} \\
R^{\mu\nu} &= \partial^\mu R^\nu - ieA_e^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu [T_3, R^\mu]\} \\
\bar{\Psi} &= (\bar{u}, \bar{d}, \bar{s})
\end{aligned}
$$

non strange – strange base:

$$
\begin{aligned}
\varphi_N &= \sqrt{2/3}\varphi_0 + \sqrt{1/3}\varphi_8, \\
\varphi_S &= \sqrt{1/3}\varphi_0 - \sqrt{2/3}\varphi_8, \qquad \varphi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i)
\end{aligned}
$$

broken symmetry: non-zero condensates $\langle \sigma_N \rangle, \langle \sigma_S \rangle \longleftrightarrow \phi_N, \phi_S$

Parametrization at $T = 0$

16 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F, g_V, g_A) \longrightarrow$ Determined by the min. of χ^2:

$$\chi^2(x_1, \ldots, x_N) = \sum_{i=1}^{M} \left[\frac{Q_i(x_1, \ldots, x_N) - Q_i^{\exp}}{\delta Q_i} \right]^2 , \tag{1}$$

where $(x_1, \ldots, x_N) = (m_0, \lambda_1, \lambda_2, \ldots)$, $Q_i(x_1, \ldots, x_N)$ calculated from the model, while $Q_i^{\exp}$ taken from the PDG

multiparametric minimalization $\longrightarrow$ MINUIT

PCAC $\rightarrow$ 2 physical quantities: f_π, f_K

Tree-level masses $\rightarrow$ 15 physical quantities:

$m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$

Decay widths $\rightarrow$ 12 physical quantities:

$\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi},$

$\Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$

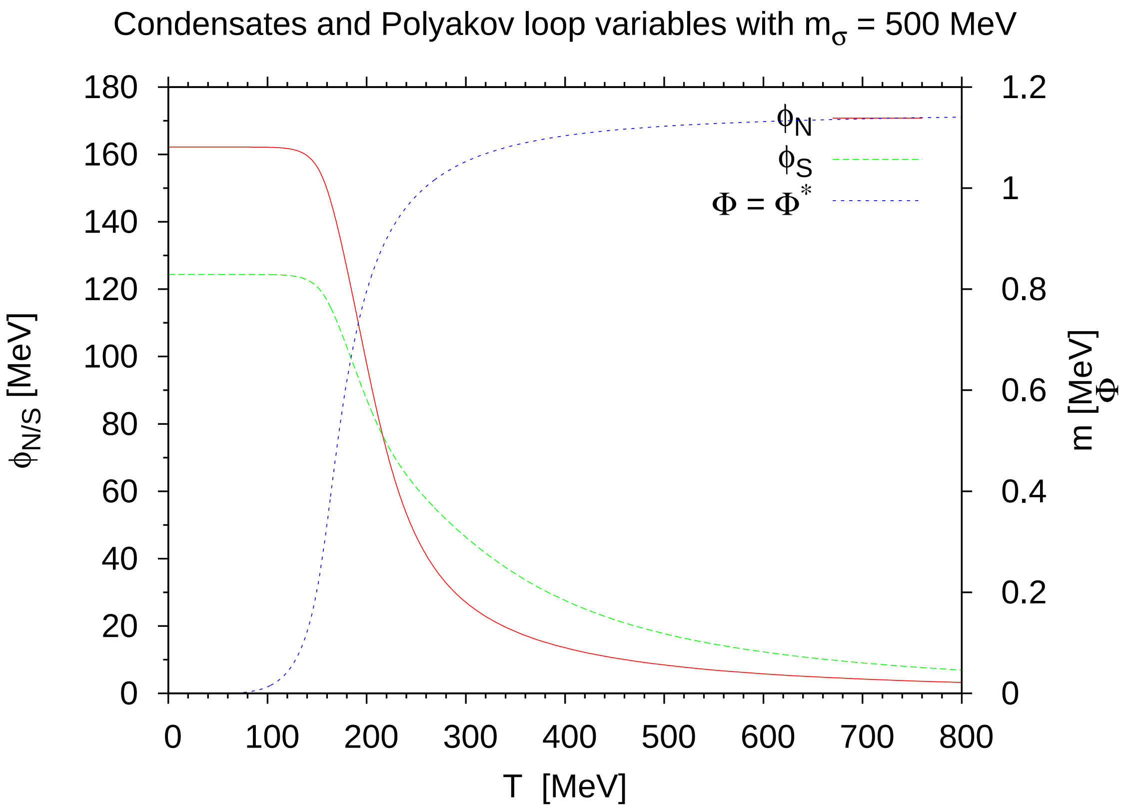
$T_c = 155$ MeV from lattice

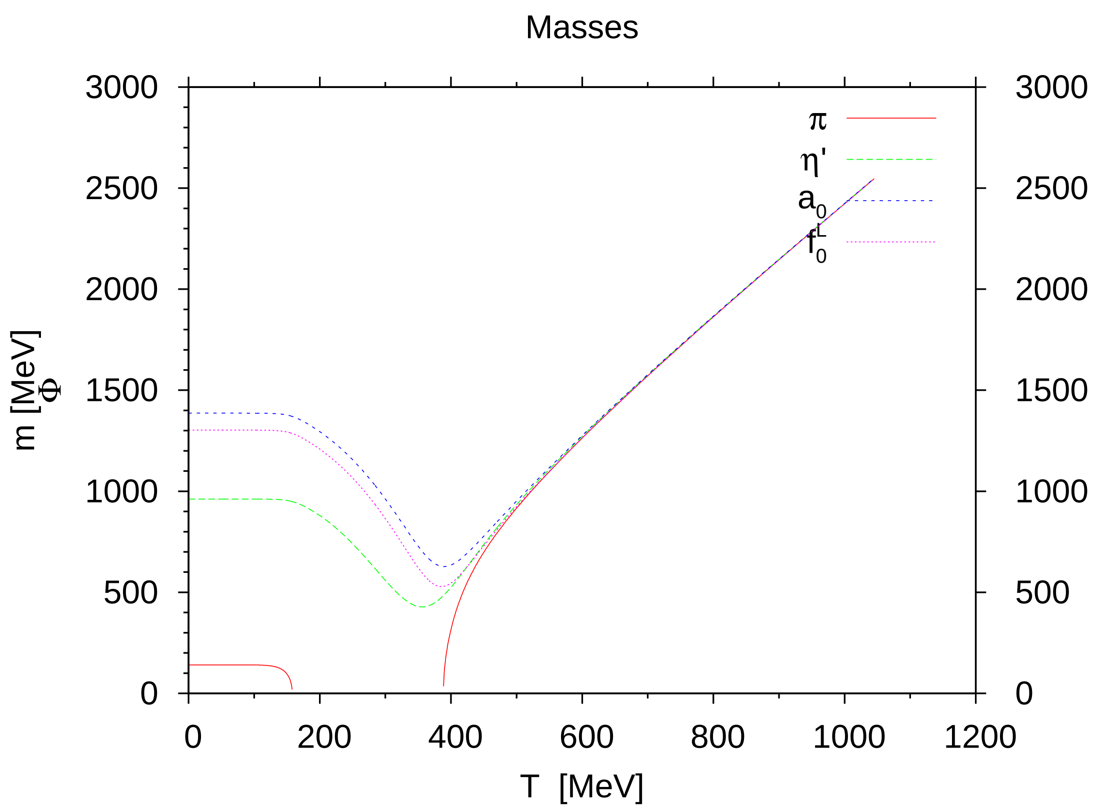
Extremum equations for $\phi_{N/S}$ and $\Phi, \bar{\Phi}$

Ω: grand canonical potential

$$
\begin{aligned}
\frac{\partial \Omega}{\partial \Phi} &= \left.\frac{\partial \Omega}{\partial \bar{\Phi}}\right|_{\varphi_N=\phi_N,\varphi_S=\phi_S} = 0 \\
\frac{\partial \Omega}{\partial \phi_N} &= \left.\frac{\partial \Omega}{\partial \phi_S}\right|_{\Phi,\bar{\Phi}} = 0, \qquad \text{(after the SSB)}
\end{aligned}
$$

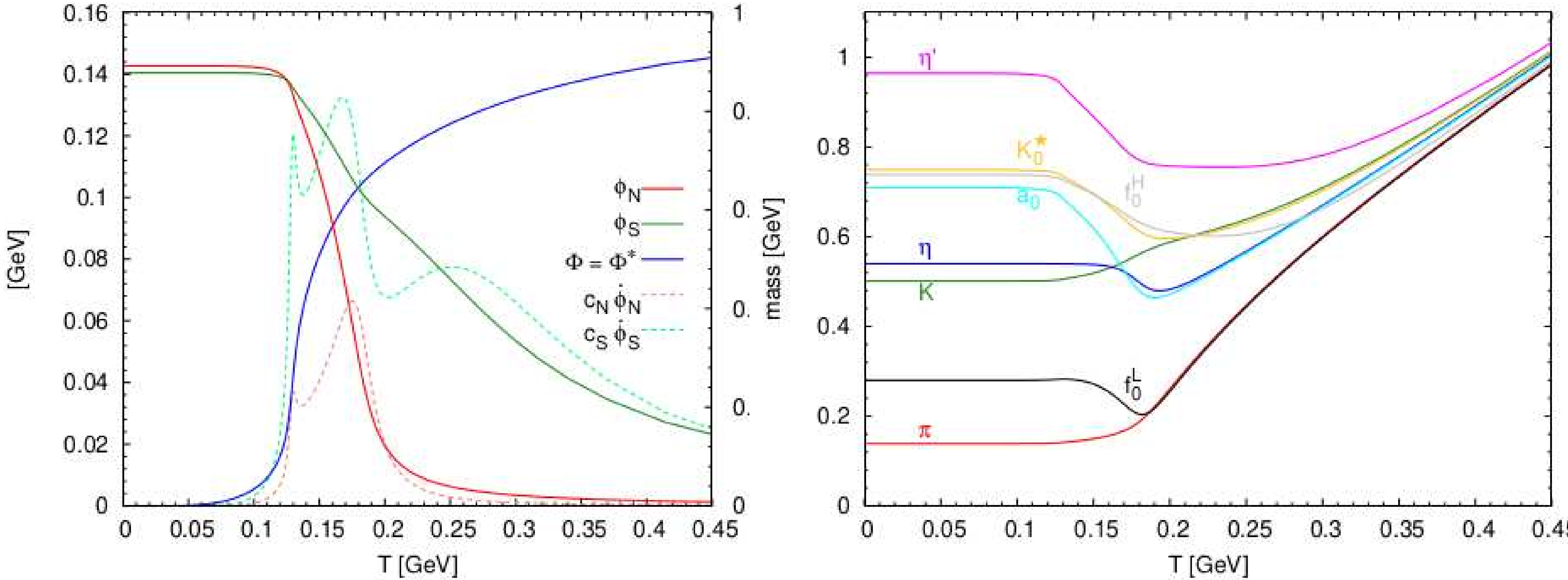
Order parameters and masses in medium $m_\sigma = 500$ MeV





P. Kovács, Zs. Szép, Gy. Wolf, Phys. Rev. D93, (2016) 114014

Order parameters and masses in medium $m_\sigma = 290$ MeV



Witten-Veneziano theorem

Consider the gluonic correlation function:

$$U(k) = i\int d^4x\, e^{ik\cdot x}\langle T G\tilde{G}(x)\, G\tilde{G}(0)\rangle.$$

$U(0) = 0$ if there is a massles fermion

$$\begin{aligned} U(k) &= -\sum_n \frac{|\langle 0|G\tilde{G}|\, n^{th} glueball\rangle|^2}{k^2 - M_n^2} - \sum_n \frac{|\langle 0|G\tilde{G}|\, n^{th} meson\rangle|^2}{k^2 - m_n^2} \\ &\equiv U_0(k) + U_1(k). \end{aligned}$$

$U_0(k) \propto N_c^2$

$U_1(k) \propto N_c.$

$U(0) = 0$ is possible only if in U_1 there is a meson, η', for $m^2_{\eta'} \sim 1/N_c$

$$U_0(0) = -\frac{|\langle 0|G\tilde{G}|\eta'\rangle|^2}{m^2_{\eta'}}.$$

$$\langle 0|G\tilde{G}|\eta'\rangle = \frac{4\pi}{\alpha_s}\frac{1}{N_f}\langle 0|\partial_\mu J_5^\mu|\eta'\rangle = \frac{4\pi}{\alpha_s}\frac{1}{N_f}\sqrt{N_f}\, m^2_{\eta'} f_\pi,$$

Witten-Veneziano equation:

$$U_0(0) = -\frac{1}{N_f} m^2_{\eta'} f^2_\pi \left(\frac{4\pi}{\alpha_s}\right)^2,$$

$$m_{\eta'} = \sqrt{\frac{8}{33}\frac{1}{f_\pi}\langle\frac{\alpha_s}{\pi}G^2\rangle^{1/2}} \approx 464 \ \ \text{MeV},$$

This is the mass which is originated from the anomaly.

Witten-Veneziano theorem in matter

$$U_0(0) = -\frac{|\langle 0|G\tilde{G}|\eta'\rangle|^2}{m^2_{\eta' *}}.$$

equation is valid in matter, the derivation is the same

Y. Kwon, S.H. Lee, K. Morita, Gy. Wolf, Phys.Rev. D86 (2012) 034014

Low energy theorem:

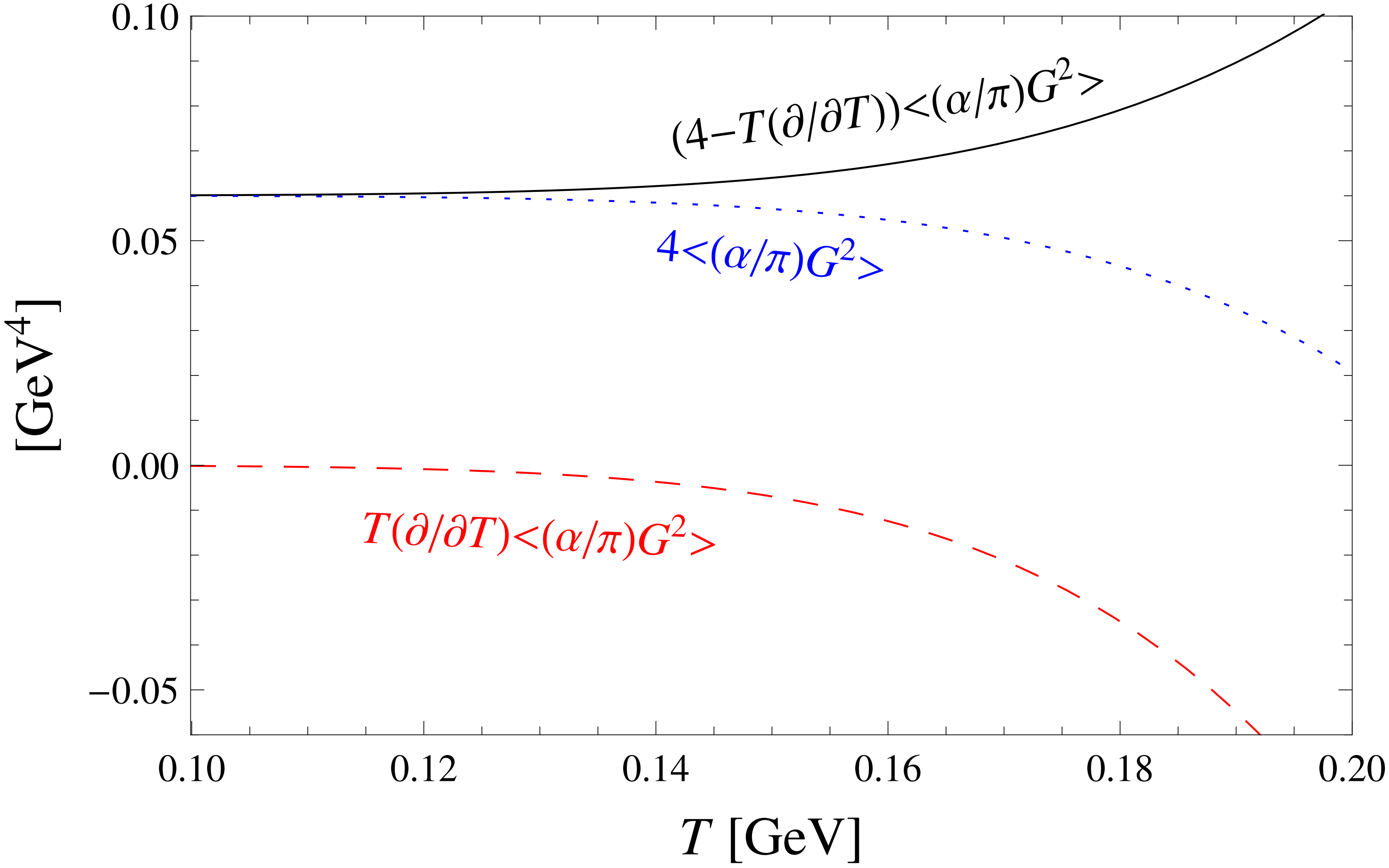
$$U(k=0) = -\frac{2}{b}\left(\frac{4\pi}{3\alpha_s}\right)^2\left(d - T\frac{\partial}{\partial T} - \mu\frac{\partial}{\partial \mu}\right)\left\langle \frac{\alpha_s}{\pi}G^2\right\rangle_{T,\mu}.$$

S. H. Lee and I. Zahed, Phys. Rev. C 63, 045204 (2001)

From the 2 equations we obtain

$$m^2_{\eta'} = \left(\frac{3\alpha_s}{4\pi}\right)^2 \frac{|\langle 0|G\tilde{G}|\eta'\rangle|^2}{\frac{2}{b}\left(d - T\frac{\partial}{\partial T}\right)\left\langle \frac{\alpha_s}{\pi}G^2\right\rangle_{T,puregauge}}.$$

The denominator depends weakly on the temperature



If the chiral symmetry is restored

$$
\begin{aligned}
U(k) &= i\int d^4x\, e^{ik\cdot x}\langle T G\tilde{G}(x)\, G\tilde{G}(0)\rangle \\
&= k^\mu k^\nu i\int d^4x\, e^{ik\cdot x}\left(\frac{4\pi}{\alpha_s N_f}\right)^2\Big[\langle T\bar{q}i\gamma_\mu\gamma_5 q(x)\,\bar{q}i\gamma_\nu\gamma_5 q(0)\rangle \\
&\quad -\langle T\bar{q}\gamma_\mu q(x)\,\bar{q}\gamma_\nu q(0)\rangle\Big], \\
&\quad \propto \mathrm{Tr}[S_A(x,x)]^2
\end{aligned}
\tag{2}
$$

The connected diagrams do not contribute, since they appear with opposite signs.
Using the Banks-Cashier equation:

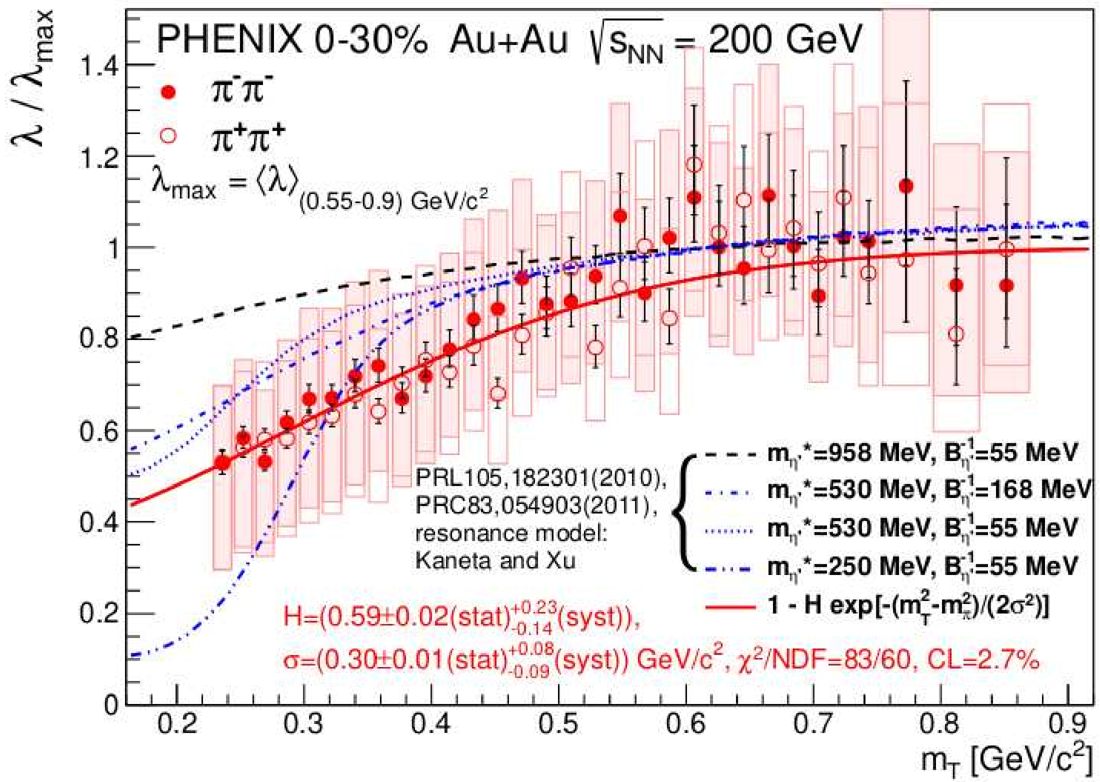
$$
\langle 0|G\tilde{G}|\eta'\rangle \propto \mathrm{Tr}[S_A(x,x)] \propto \langle\bar{q}q\rangle^*
$$

So:

$$
m^*_{\eta'} \propto \langle\bar{q}q\rangle^*
$$

Important to note that this is only for the mass arising from the anomaly, which is in vacuum 464 MeV.

Experimental results



Summary

- We studied the chiral transition in a linear sigma model. We determined the $m_{\eta'}$ as a function of the temperature.
  P. Kovács, Zs. Szép, Gy. Wolf, Phys. Rev. D93, (2016) 114014

- We generalized the Witten-Veneziano theorem for nonzero temperature and found a limit (actually a rough estimate at the chiral restaration temperature) for $m_{\eta'}$
  Y. Kwon, S.H. Lee, K. Morita, Gy. Wolf, Phys.Rev. D86 (2012) 034014
  Note that we do not state the $SU_A(1)$ symmetry is restored at temperatures relevant for experimental signals

- The two value difer we wait for the experimental result (according to rumours the second estmate is not so bad).