Proposal for an electromagnetic mass formula for the X17 particle.

Sándor Varró,
[Wigner RCP] Budapest. ELI-Attosecond, Szeged, Hungary.

Varró S, Proposal for an electromagnetic mass formula for the X17 particle.
[Talk presented at ISMD 2023 - 52nd International Symposium on Multiparticle Dynamics – August 21-25 2023 - Gyöngyös, Hungary.] File:"Varro_S_Talk_B_ISMD_2023_X17_v_1.ppt"

Outline.

- Introduction. Motivation. Wentzel's consideration on the photon self energy.

- Exact solution of the Dirac equation in a quantized electromagnetic plane wave; 'Quantized Volkov states' of charged particles.

- Derivation of the electromagnetic mass formula for the X17 from the quantized Volkov state. A possible plasma interpretation.

- Summary.

New experiments are also pointing to the existence of the X17 vector boson. Considerations on the fifth force explaining the ATOMKI anomaly...

PHYSICAL REVIEW C **106**, L061601 (2022)

Letter

New anomaly observed in ^{12}C supports the existence and the vector character of the hypothetical X17 boson

A. J. Krasznahorkay,* A. Krasznahorkay,† M. Begala, M. Csatlós, L. Csige, J. Gulyás, A. Krakó, J. Timár, I. Rajta, and I. Vajda

Institute for Nuclear Research (ATOMKI), P.O. Box 51, H-4001 Debrecen, Hungary

N. J. Sas

University of Debrecen, Faculty of Science and Technology, Physics Institute, 4010 Debrecen, PO Box 105, Hungary

PHYSICAL REVIEW D **102,** 036016 (2020)

Dynamical evidence for a fifth force explanation of the ATOMKI nuclear anomalies

Jonathan L. Feng,* Tim M. P. Tait,† and Christopher B. Verhaaren‡

Department of Physics and Astronomy, University of California, Irvine, California 92697-4575, USA

QED meson description of the X17 particle and dark matter.

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Open string QED meson description of the X17 particle and dark matter

Cheuk-Yin Wong

Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, U.S.A.

E-mail: wongc@ornl.gov

ABSTRACT: As a quark and an antiquark cannot be isolated, the intrinsic motion of a composite $q\bar{q}$ system in its lowest-energy states lies predominantly in 1+1 dimensions, as in an open string with the quark and the antiquark at its two ends. Accordingly, we study the lowest-energy states of an open string $q\bar{q}$ system in QCD and QED in 1+1 dimensions. We show that π^0, η, and η' can be adequately described as open string $q\bar{q}$ QCD mesons. By extrapolating into the $q\bar{q}$ QED sector in which a quark and an antiquark interact with the QED interaction, we find an open string isoscalar $I(J^\pi) = 0(0^-)$ QED meson state at 17.9±1.5 MeV and an isovector $(I(J^\pi) = 1(0^-), I_3 = 0)$ QED meson state at 36.4±3.8 MeV. The predicted masses of the isoscalar and isovector QED mesons are close to the masses of the hypothetical X17 and E38 particles observed recently, making them good candidates for these particles. The decay products of QED mesons may show up as excess e^+e^- and $\gamma\gamma$ pairs in the anomalous soft photon phenomenon associated with hadron productions in high-energy hadron-proton collisions and e^+-e^- annihilations. Measurements of the invariant masses of excess e^+e^- and $\gamma\gamma$ pairs will provide tests for the existence of the open string $q\bar{q}$ QED mesons. An assembly of gravitating QED mesons are expected to emit electron-positron pairs and/or gamma rays and their decay energies and lifetimes will be modified by their gravitational binding energies. Consequently, a self-gravitating isoscalar QED meson assembly whose mass M and radius R satisfy $(M/M_\odot)/(R/R_\odot) \gtrsim 4.71 \times 10^5$ will not produce electron-positron pairs nor gamma rays and may be a good candidate for the primordial dark matter.

KEYWORDS: Phenomenological Models, QCD Phenomenology

ARXIV EPRINT: 2001.04864

Wentzel's deduction of the 'non-vanishing self-energy of the photon' (1948).

PHYSICAL REVIEW VOLUME 74, NUMBER 9 NOVEMBER 1, 1948

New Aspects of the Photon Self-Energy Problem

GREGOR WENTZEL
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received June 29, 1948)

A finite but non-vanishing value for the self-energy of the photon, corresponding to a finite rest-mass, can be deduced from the new invariant formulation of quantum electrodynamics developed by Tomonaga and Schwinger, in the e^2 order approximation. The implications of this result are discussed.

state considered, divided by κ_0^2. Therefore, a photon of momentum $\hbar\boldsymbol{\kappa}$, having the total energy $\hbar c\kappa_0 = \hbar c|\boldsymbol{\kappa}|$ in the zero-order approximation, appears in the second-order approximation as having the energy

$$\hbar c\kappa_0\left(1+\frac{1}{8\pi^2}\frac{e^2}{\hbar c}\frac{\mu^2}{\kappa_0^2}+\cdots\right)$$
$$=c\left[(\hbar|\boldsymbol{\kappa}|)^2+\frac{1}{4\pi^2}\frac{e^2}{\hbar c}(mc)^2\right]^{\frac{1}{2}}. \quad (25)$$

This corresponds to a "photon rest-mass" amounting to one electron mass divided by

Perturbation theory plus an extrapolation:

$$m_\gamma = \sqrt{\frac{e^2}{\pi\hbar c}}\,m_e = \frac{m_e}{\sqrt{\pi 137}}$$

"CONCLUSION. The results of the last section are hardly encouraging in view of higher approximations. We have tried to take the quantum theory of fields seriously, without admitting any ad hoc subtractions inconsistent with the principles of quantum mechanics. The outcome shows that the empirical fact, that the photon has no rest-mass, does not fit naturally into the framework of quantum electrodynamics. It seems questionable to what extent the predictions of such a theory in higher order effects are trustworthy.
Finally, it should be remembered that the pair creation of other charged particles (mesons, protons) is likely to contribute to the photon selfenergy. Therefore, the phenomena involving electrons, positrons, and photons only, can hardly be expected to be quite independent, in the higher order effects, of the existence of other particles and their nature."

Wentzel G, New aspects of the photon self-energy problem. Physical Rewiev 74 (1), 1070-1075 (1948).

Volkov states [1935]. Exact solutions of the Dirac equation of a charged particle interacting with a classical plane electromagnetic field.

$$[\gamma_\mu (i\partial - \varepsilon A_{rad})^\mu - \kappa] \left| \Psi \right\rangle = 0 \quad (\varepsilon \gamma_0 V \left| \Psi \right\rangle)$$

$$\varepsilon \equiv \mathrm{e} / \hbar c$$

$$\kappa \equiv mc / \hbar$$

$$A_{rad}(\xi) = e_x A_0 f(\xi)$$

$$\xi = k_\mu x^\mu = \omega(t - z/c)$$

$$\Psi_{ps}^{(\pm)}(x) = \left[1 \pm \frac{\varepsilon(\gamma \cdot k)[\gamma \cdot A(\xi)]}{2k \cdot p} \right] u_{ps}^{(\pm)}$$

$$\times \exp\left[\mp i[p \cdot x + \int I_p^{(\pm)}(\xi) d\xi] \right]$$

$$I_p^{(\pm)}(\xi) = (1/2k \cdot p)[\pm 2\varepsilon p \cdot A(\xi) - \varepsilon^2 A^2(\xi)$$

Wolkow D M, Über eine Klasse von Lösungen der Diracschen Gleichung. *Zeitschrift für Physik* 94, 250-260 (1935). [Application to strong-field and multiphoton processes: from ~1960...]

Exact solutions of the Dirac equation in a quantized plane wave. Nonlinear Compton scattering (HHG) beyond the semiclassical description (1981). The generalization of the Klein–Nishina formula. [The effect of depletion of the laser field; e.g. altered kinematics (spectrum) .]

The calculation of the nonlinear Compton process was based on the Exact solutions for the 'Dirac electron + quantized e.m. radiation mode' system [1-3]:

$$\psi_{E,\boldsymbol{P}} = \left[1 + g\frac{\not k \not \varepsilon}{2Qk}(\hat{a}\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}} + \hat{a}^{+}\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}) \right] u_Q e^{i[\boldsymbol{Q}+\boldsymbol{k}g(n)]\cdot\boldsymbol{r}} \hat{S}_\theta \hat{D}_\tau |n\rangle$$

$$\omega_n' = \frac{n\omega_0 + \omega_C \mu_0^2 \Delta}{1 + \left(2\frac{n\omega_0}{\omega_C} + \frac{\mu_0^2}{2} \right) \sin^2 \frac{\theta}{2}}$$

$$\Delta = \frac{n_0 - n}{n_0}$$

,depletion factor' [1-3]

The generalization of the Klein–Nishina formula (complete depletion of the photon mode):

$$| t_{fi}^{(n)} |_{av}^2 = \frac{1}{4}\left[\frac{n\omega_0}{\omega'} + \frac{\omega'}{n\omega_0} - 2 + 4(\boldsymbol{\varepsilon}\cdot\boldsymbol{\varepsilon}')^2 \right] \frac{(nb)^n}{n!} e^{-nb}$$

$$b = \tfrac{1}{2}\mu_0^2 \, | \hat{\boldsymbol{k}}' \cdot \boldsymbol{\varepsilon} |^2$$

[1] S. Varró, Theoretical study of the interaction of free electrons with intense light. (Ph.D. dissertation, 1981). (in Hungarian) Hungarian Physical Journal XXXI, 399-454 (1983). [2] J. Bergou, S. Varró:J. Phys. A: Math. Gen. 14, l469-l482 (1981) . [3] J. B. S. V., Nonlinear scattering processes in the presence of a quantised radiation field: II. Relativistic treatment. *Ibid.* 14, 2281 (1981).

The matrix elements of the squeezing operator between (unperturbed) photon number eigenstates. Expression in terms of classical Gegenbauer polynomials.

$$S = \exp(\tfrac{1}{2}\xi a^{+2} - \tfrac{1}{2}\xi^* a^2) \qquad H_{\text{int}} = \gamma(a^{+2}\mathrm{e}^{i\varphi} + a^2\mathrm{e}^{-i\varphi})$$

$$\langle m|S(\xi)|n\rangle = \mathrm{e}^{-\eta/4}(-2\mathrm{e}^{-i\varphi}\tanh|\xi|)^{\lambda}\frac{\Gamma(\lambda+\frac{1}{2})}{\Gamma(\frac{1}{2})}\sqrt{\frac{m!}{n!}}C_m^{(\lambda+\frac{1}{2})}\left(\frac{1}{\cosh|\xi|}\right) \qquad m \le n, \quad \lambda = \tfrac{1}{2}(n-m)$$

$$\langle m|S(\xi)|n\rangle = \mathrm{e}^{-\eta/4}(2\mathrm{e}^{i\varphi}\tanh|\xi|)^{\alpha}\frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\frac{1}{2})}\sqrt{\frac{n!}{m!}}C_n^{(\alpha+\frac{1}{2})}\left(\frac{1}{\cosh|\xi|}\right) \qquad m \ge n, \quad \alpha = \tfrac{1}{2}(m-n)$$

Remark: SU(1,1); SL(2,R) group. Lorentz group in 2 + 1 dimensions (osc. repr.).

$$K_+ = \tfrac{1}{2}(a^+)^2, \quad K_- = \tfrac{1}{2}(a)^2, \quad K_0 = \tfrac{1}{4}(aa^+ + a^+a), \quad [K_0, K_\pm] = \pm K_\pm \quad [K_-, K_+] = 2K_0$$

Two-photon emission | **Two-photon absorption** | Phiton energy

General transformation: $S(\xi) = \exp(\xi K_+ - \xi^* K_-)$

$$U(g) = \exp(-itK_0)S(\xi), \qquad U(g_1)U(g_2) = U(g), \qquad g = g_1 \circ g_2$$

Varró S, Coherent and incoherent superposition of transition matrix elements of the squeezing operator. *New Journal of Physics* 24, 053035 (2021). *E-print*: arXiv: 2112.08430 [quant-ph] . 29th Int. Las. Phys. (LPHYS'21). J. Phys. Conf. Ser. 2249 012013 (2022).

Derivation of the electromagnetic mass formula on the basis of quantized Volkov states. Contributions of the sub-systems to the energy and momentum.

Stationary states of the system:

$$\psi_{E,\boldsymbol{P}} = \exp[-i\boldsymbol{k}\cdot\boldsymbol{r}(a_1^+ a_1 + a_3^+ a_3 + 1)]\left\{1 + g\frac{\not k}{2(k\cdot p)}[\not\epsilon_1(a_1 + a_1^+) + \not\epsilon_3(a_3 + a_3^+)]\right\}u_p$$

$$\times S_1(\theta)S_3(\theta)D_1(\tau_1)D_3(\tau_3)\left|n_1, n_3\right\rangle \exp\{i[\boldsymbol{p} + g_{\boldsymbol{p}}(n)\boldsymbol{k}]\cdot\boldsymbol{r}\}$$

$$\boxed{g = \frac{e}{\hbar}\sqrt{\frac{2\pi\hbar}{\omega V}} = \frac{e}{\sqrt{\hbar c}}\sqrt{\frac{\lambda}{V}}}$$

Parameters:

$\boxed{\tau_{1,2} = p_{x,z}g/(k\cdot p)}$ $\boxed{e^{2\theta} = \sqrt{1+2b}}$ $\boxed{b = g^2/(k\cdot p)}$ $\boxed{g_p(n) = \sqrt{1+2b}(n+1-\tau^2)}$

Total energy and momentum:

$$K_0 = p_0 - k_0\frac{g^2}{(k\cdot p)}\frac{p_x^2 + p_z^2}{(k\cdot p)} + \sqrt{1+2\beta}k_0(n_1 + n_3 + 1) \qquad \boldsymbol{K} = \boldsymbol{p} - \boldsymbol{k}\frac{g^2}{(k\cdot p)}\left[\frac{p_x^2 + p_z^2}{(k\cdot p)} - (n_1 + n_3 + 1)\right] + \boldsymbol{k}(n_1 + n_3 + 1)$$

$\boxed{K_0 = \overline{p}_0 + \sqrt{1+2b}k_0(n_1 + n_2 + 1)}$ $\boxed{\boldsymbol{K} = \overline{\boldsymbol{p}} + \boldsymbol{k}(n_1 + n_2 + 1)}$ $\boxed{\tilde{k}_0^2 - |\boldsymbol{k}|^2 = 2bk_0^2}$

~ (mass)^2 term of the radiation subsystem

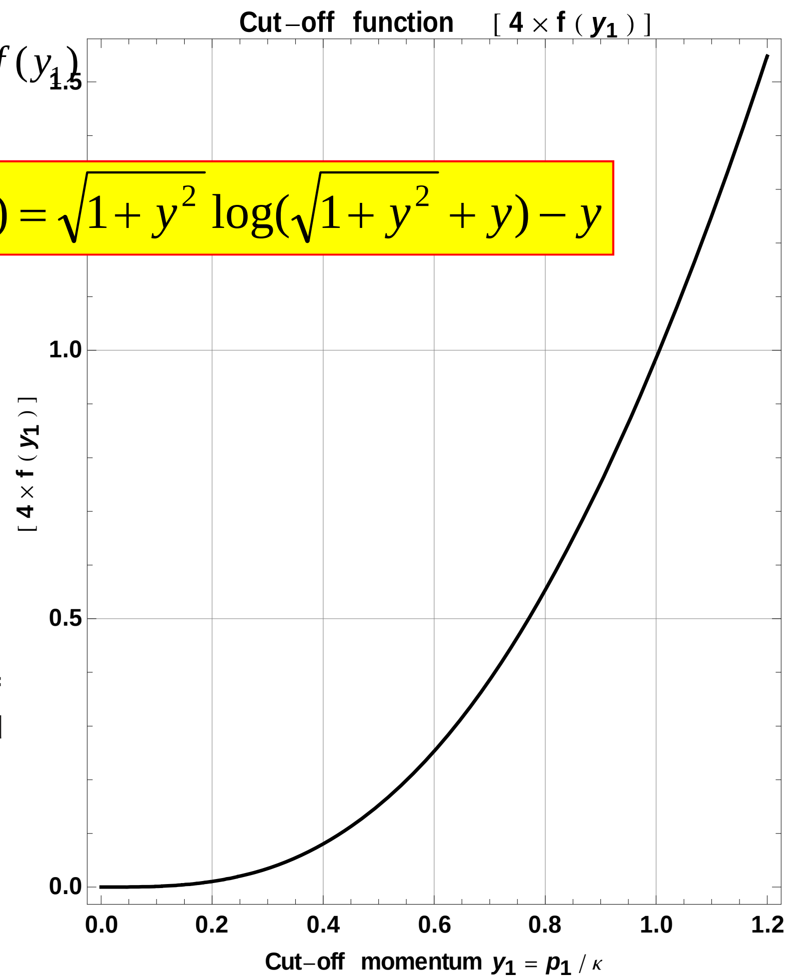
Varró S, Proposal for an electromagnetic mass formula for the X17 particle. [Talk presented at ISMD 2023 - 52nd International Symposium on Multiparticle Dynamics – August 21-25 2023 - Gyöngyös, Hungary.]
File:"Varro_S_Talk_B_ISMD_2023_X17_v_1.ppt"

Derivation of the electromagnetic mass formula for X17 on the basis of quantized Volkov states. Sum over the charged particle's momentum; cut-off function.

$$m'_\gamma c^2 \leftarrow \hbar^2 c^2 \times 2 \sum \Delta n_x \Delta n_y \Delta n_z \frac{\kappa}{q_0} k_0^2 2\beta = \frac{1}{\pi}\frac{e^2}{\hbar c}(mc^2)^2 \times 4 f(y_1)$$

$$\sqrt{\frac{1}{\pi}\frac{e^2}{\hbar c}}(m_p c^2) = 45.2206 \;\; MeV$$

$$f(y) = \sqrt{1+y^2}\log(\sqrt{1+y^2}+y) - y$$



Here we have taken m_p as the proton mass. This is 'Wentzel-type' mass term, though we did not consider vacuum polarization. Rather, we have taken a real positive energy charge. We now proceed with a Fermi gas picture, and define the Fermi radius

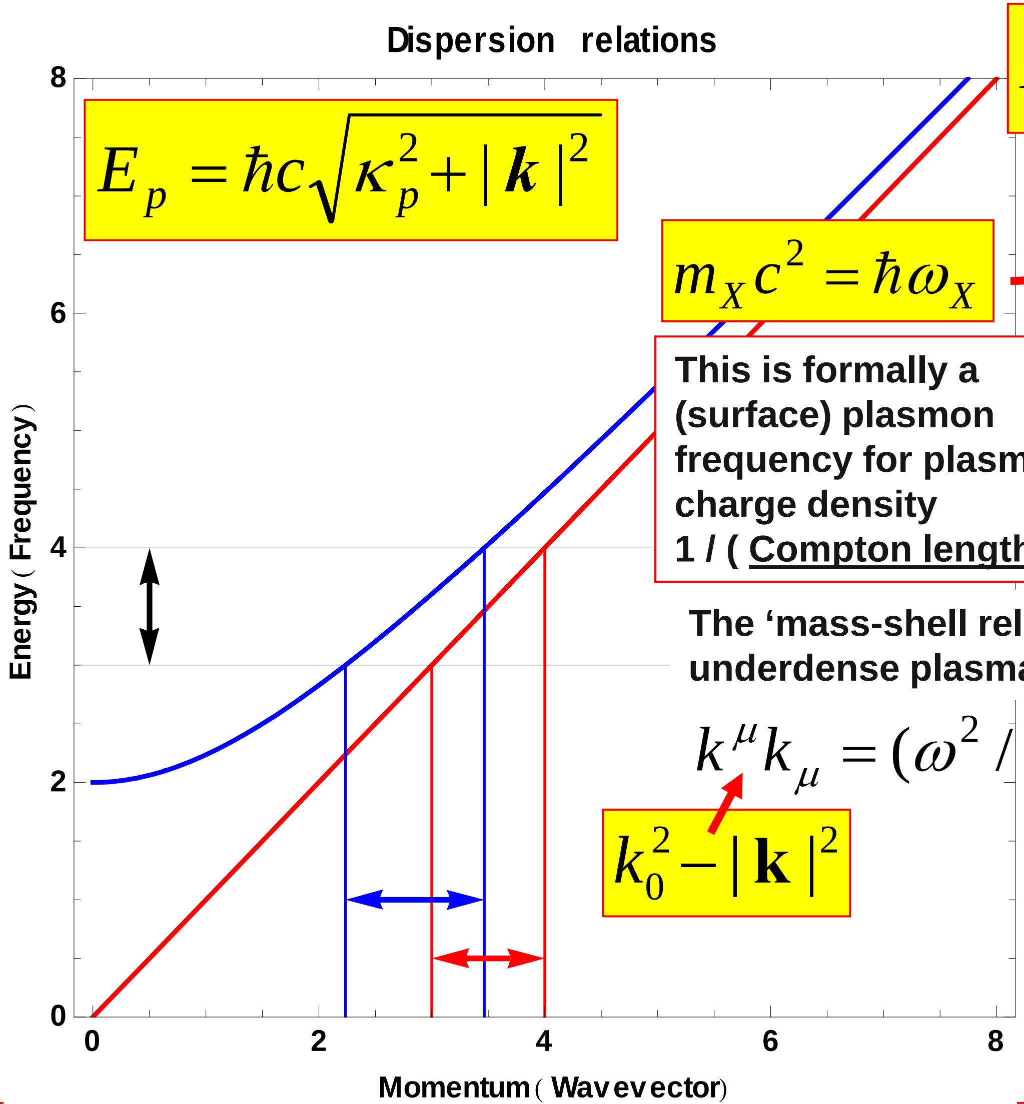
$$n = \frac{N}{V} = \frac{1}{3\pi^2} k_F^3, \; V = \frac{4\pi}{3} r_F^3. \;\; N = \frac{1}{4\pi}\left(\frac{4}{3}\right)^2 (k_F r_F)^3$$

Taking $k_F r_F=1$, and multiplying with the average number of particles N within the Fermi sphere, we have the proposed mass formula for the X17:

$$m_X c^2 = \frac{1}{2\pi}\frac{4}{3}\sqrt{\frac{e^2}{\hbar c}}(m_p c^2) = 17.0087 \;\; MeV$$

Varró S, Proposal for an electromagnetic mass formula for the X17 particle. [Talk presented at ISMD 2023 - 52nd International Symposium on Multiparticle Dynamics – August 21-25 2023 - Gyöngyös, Hungary.]
File:"Varro_S_Talk_B_ISMD_2023_X17_v_1.ppt"

Dispersion relations, rest mass. Plasma consideration.



$$E_p = \hbar c \sqrt{\kappa_p^2 + |\boldsymbol{k}|^2}$$

$$E_{ph} = \hbar c \sqrt{0^2 + |\boldsymbol{k}_{ph}|^2}$$

$$m_X c^2 = \hbar \omega_X$$

$$\omega_X = \frac{1}{\sqrt{2}} \sqrt{\frac{4\pi \cdot e^2}{\bar{m}_p \bar{\lambda}_p^3}}$$

This is formally a (surface) plasmon frequency for plasma charge density 1 / (<u>Compton length</u>)³.

$$\bar{\lambda}_p = \frac{h}{\bar{m}_p c} \qquad \bar{m}_p = \frac{4}{3} m_p$$

The ‘mass-shell relation’ of e.m. radiation in an underdense plasma *does not depend on the frequency* !

$$k^\mu k_\mu = (\omega^2 / c^2)(1 - n_m^2(\omega)) = (\omega_p^2 / c^2)$$

$$k_0^2 - |\mathbf{k}|^2$$

$$\omega(\boldsymbol{k}) = \sqrt{\omega_p^2 + c^2 |\boldsymbol{k}|^2}$$

‘rest mass’: $m_{ph} = \hbar \omega_p / c^2$

Varró S, Proposal for an electromagnetic mass formula for the X17 particle. [Talk presented at ISMD 2023 - 52nd International Symposium on Multiparticle Dynamics – August 21-25 2023 - Gyöngyös, Hungary.] File:”Varro_S_Talk_B_ISMD_2023_X17_v_1.ppt”

Summary.

• **We have briefly shown Wentzel's consideration on the non-vanishing photon self energy.**

• **We have given the exact solution of the Dirac equation of a proton in a quantized electromagnetic plane wave;. This leads to a 'Wentzel-type' mass term for the dressed radiation. We did not consider vacuum polarization. Rather, we have taken a real positive energy charge, which dresses the radiation.**

• **According to the present analysis, the dressed radiation is a massive vector boson field with (rest mass)c^2 = 17.0087 MeV, which may perhaps be identified with the X17 resonace, found by the ATOMKI group in several experiments.**

Varró S, Proposal for an electromagnetic mass formula for the X17 particle.
[Talk presented at ISMD 2023 - 52nd International Symposium on Multiparticle Dynamics – August 21-25 2023 - Gyöngyös, Hungary.] File:"Varro_S_Talk_B_ISMD_2023_X17_v_1.ppt"

Appendix.

Varró S, Proposal for an electromagnetic mass formula for the X17 particle.
[Talk presented at ISMD 2023 - 52nd International Symposium on Multiparticle Dynamics – August 21-25 2023 - Gyöngyös, Hungary.] File:”Varro_S_Talk_B_ISMD_2023_X17_v_1.ppt”

New experiments are also pointing to the existence of the X17 vector boson.

PHYSICAL REVIEW C **106**, L061601 (2022)

Letter

New anomaly observed in ^{12}C supports the existence and the vector character of the hypothetical X17 boson

A. J. Krasznahorkay,[*] A. Krasznahorkay,[†] M. Begala, M. Csatlós, L. Csige, J. Gulyás, A. Krakó, J. Timár, I. Rajta, and I. Vajda

Institute for Nuclear Research (ATOMKI), P.O. Box 51, H-4001 Debrecen, Hungary

N. J. Sas

University of Debrecen, Faculty of Science and Technology, Physics Institute, 4010 Debrecen, PO Box 105, Hungary

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Employing the $^{11}\mathrm{B}(p,\gamma)^{12}\mathrm{C}$ nuclear reaction, the angular correlation of e^+e^- pairs was investigated in the angular range of $40^\circ \leqslant \Theta \leqslant 175^\circ$ for five different proton energies between $E_p = 1.50$–2.5 MeV. At small angles ($\Theta \leqslant 120^\circ$), the results can be well interpreted by the internal pair creation process of electromagnetic radiations with $E1$ and $M1$ multipolarities and by the external pair creation in the target backing. However, at angles greater than 120°, additional count excesses and anomalies were observed, which could be well accounted for by the existence of the previously suggested hypothetical X17 particle. Our results suggest that the X17 particle was generated mainly in $E1$ radiation. The derived mass of the particle is $m_X c^2 = 17.03 \pm 0.11(\mathrm{stat}) \pm 0.20(\mathrm{syst})$ MeV. According to the mass, and to the derived branching ratio [$B_x = 3.6(3) \times 10^{-6}$], this is likely the same X17 particle that we recently suggested for describing the anomaly observed in the decay of ^{8}Be and ^{4}He.

DOI: 10.1103/PhysRevC.106.L061601

Considerations on the fifth force explaining the ATOMKI anomaly.

PHYSICAL REVIEW D **102,** 036016 (2020)

Dynamical evidence for a fifth force explanation of the ATOMKI nuclear anomalies

Jonathan L. Feng,* Tim M. P. Tait,† and Christopher B. Verhaaren‡

Department of Physics and Astronomy, University of California, Irvine, California 92697-4575, USA

(Received 18 June 2020; accepted 30 July 2020; published 17 August 2020)

Recent anomalies in ^{8}Be and ^{4}He nuclear decays can be explained by postulating a fifth force mediated by a new boson X. The distributions of both transitions are consistent with the same X mass, 17 MeV, providing kinematic evidence for a single new particle explanation. In this work, we examine whether the new results also provide dynamical evidence for a new particle explanation, that is, whether the observed decay rates of both anomalies can be described by a single hypothesis for the X boson's interactions. We consider the observed ^{8}Be and ^{4}He excited nuclei, as well as a ^{12}C excited nucleus; together these span the possible J^P quantum numbers up to spin 1 for excited nuclei. For each transition, we determine whether scalar, pseudoscalar, vector, or axial vector X particles can mediate the decay, and we construct the leading operators in a nuclear physics effective field theory that describes them. Assuming parity conservation, the scalar case is excluded and the pseudoscalar case is highly disfavored. Remarkably, however, the protophobic vector gauge boson, first proposed to explain only the ^{8}Be anomaly, also explains the ^{4}He anomaly within experimental uncertainties. We predict signal rates for other closely related nuclear measurements, which, if confirmed by the ATOMKI group and others, would provide overwhelming evidence that a fifth force has been discovered.

DOI: 10.1103/PhysRevD.102.036016

The Bloch-Nordsieck model:The very first „non-perturbative true multiphoton description”. [Elimination of the ‘infrared divergence’ from QED.]

JULY 15, 1937 PHYSICAL REVIEW VOLUME 52

Note on the Radiation Field of the Electron

F. BLOCH AND A. NORDSIECK*
Stanford University, California

(Received May 14, 1937)

Previous methods of treating radiative corrections in non-stationary processes such as the scattering of an electron in an atomic field or the emission of a β-ray, by an expansion in powers of $e^2/\hbar c$, are defective in that they predict infinite low frequency corrections to the transition probabilities. This difficulty can be avoided by a method developed here which is based on the alternative assumption that $e^2\omega/mc^3$, $\hbar\omega/mc^2$ and $\hbar\omega/c\Delta p$ (ω = angular frequency of radiation, Δp = change in momentum of electron) are small compared to unity. In contrast to the expansion in powers of $e^2/\hbar c$, this permits the transition to the classical limit $\hbar = 0$. External perturbations on the electron are treated in the Born approximation. It is shown that for frequencies such that the above three parameters are negligible the quantum mechanical calculation yields just the directly reinterpreted results of the classical formulae, namely that the total probability of a given change in the motion of the electron is unaffected by the interaction with radiation, and that the mean number of emitted quanta is infinite in such a way that the mean radiated energy is equal to the energy radiated classically in the corresponding trajectory.

I. INTRODUCTION

THE quantum theory of radiation has been successfully applied to radiative emission and absorption processes. If the methods which lead to these results are used to obtain more general radiative corrections, a characteristic difficulty arises. This difficulty is clearly visible in the formulae given by Mott, Sommerfeld[1] and Bethe ... ical theory, the former has no counterpart there. There is, however, a feature in the classical theory which indicates the cause of the difficulty: If for simplicity one considers only frequencies which are small compared to the reciprocal of the collision time, the mechanism of emission may be described as follows. The amplitude of each Fourier component of the proper field of the electron before the impact retains its value after

Displacement operator

$$D_\sigma = \mathrm{e}^{\sigma a^+ - \sigma^* a}$$

$$D_\sigma^+ a D_\sigma = a + \sigma$$

F. Bloch and A. Nordsieck, Note on the radiation field of the electron. *Phys. Rev.* 15, 54-59 (1937).

Matrix elements of D Poisson; Laguerre polynomial

$$\langle n + n_0 | \mathrm{e}^{\sigma a^+ - \sigma^* a} | n_0 \rangle \; [\to J_n(z)]$$

Non-relativistic illustration. Diagonalization: elimination of the p.A and A^2 terms. The appearance of the „quantized space-translated potential”.

$$|\Psi(t)\rangle = \hat{S}\hat{D}|\Phi_{SD}(t)\rangle$$

$$\hat{S} = \Pi_k \hat{S}_k(\theta_k)$$

$$\hat{D} = \Pi_k \hat{D}_k(\sigma_k)$$

$$\hat{S}_k(\theta_k) = \exp[\tfrac{1}{2}\theta_k(\hat{a}_k^2 - \hat{a}_k^{+2})]$$

$$\theta_k = \tfrac{1}{4}\log(1+2\beta_k)$$

(<u>squeezing transformation</u>)

$$\hat{D}_k(\sigma_k) = \exp[\sigma_k(\hat{a}_k^+ - \hat{a}_k)]$$

$$\sigma_k = -\sqrt{\beta_k / m\hbar\omega_k}\, e^{-3\theta_k}(\hat{\boldsymbol{p}}\cdot\boldsymbol{\varepsilon}_k)$$

(<u>displacement</u> op.)

„quantized space-translated potential”

$$\left[\frac{\hat{\boldsymbol{p}}^2}{2m} + V(\boldsymbol{r}+\hat{\boldsymbol{\alpha}}) + \tilde{H}_{rad} + \hat{D}^+\hat{S}^+\hat{M}\hat{S}\hat{D}\right]|\Phi_{SD}(t)\rangle = i\hbar\partial_t|\Phi_{SD}(t)\rangle$$

New equation:

where

$$\hat{\boldsymbol{\alpha}} = \sum_k \hat{\boldsymbol{\alpha}}_k$$

$$\hat{\boldsymbol{\alpha}}_k = -\sqrt{\tilde{\beta}_k / m\hbar\tilde{\omega}_k}\,\boldsymbol{\varepsilon}_k(\hat{a}_k^+ - \hat{a}_k) = \frac{e}{mc^2}\hat{\boldsymbol{Z}}_k$$

Hertz vector

$$\tilde{\omega}_k = \sqrt{c^2|\boldsymbol{k}|^2 + \omega_p^2}$$

plasmon dispersion, „blue-shift”, „attochirp”...

$$\tilde{H}_{rad} = \sum_{\boldsymbol{k},s}\hbar\tilde{\omega}_k(\hat{a}_{\boldsymbol{k},s}^+\hat{a}_{\boldsymbol{k},s} + \tfrac{1}{2})$$

Varró S, Quantum optical aspects of high-harmonic generation. *Photonics* 2021, *8* (7), 269 (2021). The algebraic elimination technique is the same as in V S (1983), B J and V S, J. Phys. A 14, 1469 (1981), ibid. 14, 2281 (1981) used for free Schrödinger and Dirac electrons, resp.

Transformation properties of the ‘disentengling operators’ have the same structure for any SU(1,1) generators. Thus, the elimination of the interaction terms for exactly solving the Dirac equation in a quantized field, relies on the SU(1,1) algebra.

Exact solutions for the ‘Dirac electron + quantized e.m. radiation mode’ system in case of two co-propagating circularly polarized quantized modes. E.g. subharmonic EPR pair generation.

$$\psi_{E,\boldsymbol{P}} = \left[1 + \sigma \frac{\not{k}}{2Qk}(\not{A}_{+} + \not{A}_{-})\right] u_Q e^{i[\boldsymbol{Q}+\boldsymbol{k}g(n)]\cdot\boldsymbol{r}} U_g \hat{D}_1 \hat{D}_2 \left|n\right\rangle_1 \left|n+n_0\right\rangle_2$$

$$K_{+} = a^{+}b^{+} \qquad K_{-} = ab \qquad K_0 = \tfrac{1}{2}(a^{+}a + b^{+}b + 1) \qquad \hat{C} = \tfrac{1}{4}(a^{+}a - b^{+}b)^2 - \tfrac{1}{4}$$

$$U(g) = \exp(-itK_0)S(\xi), \qquad UK_iU^{+} = M_{ij}K_j, \qquad n_1 - n_2 = const.$$

Remark: SU(1,1); SL(2,R) group. Lorentz group in 2 + 1 dimensions (osc. repr.).

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & \sin\gamma \\ 0 & -\sin\gamma & \cos\gamma \end{pmatrix} \begin{pmatrix} \cosh\beta & -\sinh\beta & 0 \\ -\sinh\beta & \cosh\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix}.$$

The expansion coefficients of the Dirac stationary states, i.e. The photon statistics is, at the same time, are the transition probabilities in the parametric down-conversion; generation of entangled photon pairs.

Gordon-Volkov states (1927, 1935): Exact solutions of the Klein–Gordon and Dirac equations of an electron in an arbitrary intense 'laser field' propagating in vacuum. After ~ 80 years; the only new exact, closed form solutions for the 'monochromatic problem' in a medium [S. V. (2013, 2014)].

Der Comptoneffekt nach der Schrödingerschen Theorie.

Von **W. Gordon** in Berlin.

(Eingegangen am 29. September 1926.)

Über eine Klasse von Lösungen der Diracschen Gleichung.

Von **D. M. Wolkow** in Leningrad.

(Eingegangen am 12. Februar 1935.)

$$e^{-iz\sin\omega_0 t} = \sum_n J_n(z) e^{-in\omega_0 t}$$

$$e^{\frac{i}{\hbar}(E_f - E_i)t - in\omega_0 t} = e^{\frac{i}{\hbar}(E_f - E_i - n\hbar\omega_0)t}$$

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LETTER

New exact solutions of the Dirac equation of a charged particle interacting with an electromagnetic plane wave in a medium

Sándor Varró

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ELSEVIER

New exact solutions of the Dirac and Klein–Gordon equation of a charged particle propagating in a strong laser field in an underdense plasma

Sándor Varró

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Letter

A new class of exact solutions of the Klein–Gordon equation of a charged particle interacting with an electromagnetic plane wave in a medium

Sándor Varró

Origin of the dielectric function and index of refraction. Elementary (textbook) considerations. Maxwell equations in a plasma; field equations of a massive vector boson. [‘Massive photon’.]

Displacement vector:

$$\boldsymbol{D} = \boldsymbol{E} + 4\pi\boldsymbol{P} = (1 + 4\pi\chi_e)\boldsymbol{E} = \varepsilon\boldsymbol{E}$$

Induced polarization density:

$$m\ddot{\boldsymbol{r}} = e\boldsymbol{E}_0 e^{-i\omega t} \qquad \boldsymbol{r} = -\frac{e}{m\omega^2}\boldsymbol{E}_0 e^{-i\omega t} \qquad \boldsymbol{P} = e\boldsymbol{r}n_e = -\frac{e^2 n_e}{m\omega^2}\boldsymbol{E}$$

Electric susceptibility, plasma frequency, dielectric function:

$$4\pi\chi_e = -\frac{\omega_p^2}{\omega^2} \qquad \omega_p = \sqrt{\frac{4\pi n_e e^2}{m}} \qquad \varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

Index of refraction, propagation of e.m. radiation in an underdense plasma:

$$n_m^2(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \qquad e^{-i\zeta}, \qquad \zeta = k_0(x_0 - n_m z) = \omega_0(t - n_m z / c)$$

‘Equivalence’ of the Maxwell equations in a plasma medium with the field equations of a massive vector boson in vacuum; ‘mass’: $\kappa \equiv \omega_p / c$

Gauss — **Maxwell fields**

$$\nabla \cdot \boldsymbol{D}_\omega = 4\pi\rho_\omega,$$

No magnetic charge:

$$\nabla \cdot \boldsymbol{B}_\omega = 0,$$

Faraday:

$$\nabla \times \boldsymbol{E}_\omega = -\frac{1}{c}\frac{\partial \boldsymbol{B}_\omega}{\partial t},$$

Ampére:

$$\nabla \times \boldsymbol{B}_\omega = \frac{1}{c}\frac{\partial \boldsymbol{D}_\omega}{\partial t} + \frac{4\pi}{c}\boldsymbol{j}_\omega.$$

Massive vector fields

$$\nabla \cdot \boldsymbol{E} = -\kappa^2 V + 4\pi\rho,$$

$$\nabla \cdot \boldsymbol{B} = 0,$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t},$$

$$\nabla \times \boldsymbol{B} = \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} - \kappa^2 \boldsymbol{A} + \frac{4\pi}{c}\boldsymbol{j}.$$

Potentials; **Lorentz gauge;** **Klein-Gordon equation:**

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}, \quad \boldsymbol{E} = -\partial_0 \boldsymbol{A} - \nabla V, \quad \partial_0 V + \nabla \cdot \boldsymbol{A} = 0, \quad (\partial^2 + \kappa^2) A_\mu = 0$$

Covariant formalism for the massive vector field

$\kappa \equiv \omega_p / c$

Lagrange density:

$$\mathscr{L} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} + \frac{\kappa^2}{8\pi} A_\mu A^\mu - \frac{1}{c} j_\mu A^\mu ,$$

Field strength tensor:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Field equations (Euler-Lagrange equations):

$$\partial^\mu F_{\mu\nu} + \kappa^2 A_\nu = \frac{4\pi}{c} j_\nu , \qquad \partial^\alpha F^{\beta\gamma} + \partial^\gamma F^{\alpha\beta} + \partial^\beta F^{\gamma\alpha} = 0$$

No gauge freedom (only Lorentz gauge) if the current is conserved (continuity).

$$\partial_\mu j^\mu = \frac{c\kappa^2}{4\pi} \partial_\alpha A^\alpha$$

That is, if $\dfrac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$ **, then** $\partial_0 V + \nabla \cdot \mathbf{A} = 0.$

Inhomogeneous Klein-Gordon equation:

$$(\partial^2 + \kappa^2) A_\mu = \frac{4\pi}{c} j_\mu , \qquad \partial^2 = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 .$$

Lánczos C, Die tensoranalytischen Beziehungen der Diracschen Gleichung. *Z. für Physik* 57, 447 (1929). Proca A, Sur la théorie ondulatoire des électrons positifs et négatifs. *J. Phys. Radium* 7, 347-353 (1936). See recent review Goldhaber A S and Nieto M N, Photon and graviton mass limits. RMP 82, 939 (2010).

The »Lánczos–Proca vector field«

[A] „Neither the titles nor the detailed texts of Proca's papers indicate explicitly that this is an equation for the electromagnetic field. Indeed, from the context it is clear that he was thinking of a charged massive spin-1 field. The idea that this could be identified with a massive photon came later." **[A] Goldhaber and Nieto (2010).**

[B] „Since 1928, Dirac's relativistic wave-equation has been written in various forms in order to facilitate its interpretation. For example, in 1930 Sauter [1] and Proca [2] rewrote it using Clifford numbers. However, the most direct road was taken by Lanczos in 1929 [3].... Hence, (1) [Lánczos's coupled quaternionic equations, SV] can be seen as Maxwell's equations with feed-back. This was a very important idea, and precisely the one that lead Proca in 1936 to discover the correct equation for the massive spin one particle [5]. The concept was easily generalized, and Kemmer finally wrote down the wave-equation of the (peudo-) scalar and (pseudo-) vector particles in the form we still use today [6]. Later, Gürsey [7] showed that Proca's and Kemmer's equations are just degenated cases of Lanczos's." **[B] Gsponer and Hurni (1993).**

[A] Goldhaber A S and Nieto M M, Photon and graviton mass limits. *Rev. Mod. Phys*. 82, 939-978 (2010). Quote; de Broglie L, *La Méchanique Ondulatoire du Photon, Une Novelle Théorie de la Lumiere* (Hermann, Paris, 1940).

[B] Gsponer A and Hurni J-P, Lanczos' equation to replace Dirac's equation? In J. D. Brown et al., eds., Proceedings of the Cornelius Lanczos International Centennary Conference, Raleigh, NC, 12-17 December 1993 (SIAM, Philadelpia, 1994) pp. 509-512.