Phenomenological Equation of State of Strongly Interacting Matter with Modified Excluded-Volume Mechanism

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The main goal of heavy-ion collisions experiments is to gain a better understanding of the theory of strong interactions – QCD, by detecting critical phenomena.

Exploring of the QCD phase diagram:
- Detect signals of deconfinement PT
- Detect signals of (partial) chiral symmetry restoration
- Locate (tri)critical endpoint(s) if such exists

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\[ p_i = g_i^{\text{eff}} \int \frac{d^3 p}{(2\pi)^3} \frac{p^2}{3E_i} f(T, \mu_i^{\text{eff}}, m_i^{\text{eff}}) \]

\[ n_i = g_i^{\text{eff}} \int \frac{d^3 p}{(2\pi)^3} \frac{m_i^{\text{eff}}}{E_i} f(T, \mu_i^{\text{eff}}, m_i^{\text{eff}}) \]

\[ n_i^s = g_i^{\text{eff}} \int \frac{d^3 p}{(2\pi)^3} \frac{m_i^{\text{eff}}}{E_i} f(T, \mu_i^{\text{eff}}, m_i^{\text{eff}}) \]

For details see S. Typel, EPJ A (2016) 52: 16 and S. Typel, D. Blaschke, Universe 2018, 4, 32
Modified Excluded Volume

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\[ g_i V_i \rightarrow g_i^{\text{eff}} V \quad g_i^{\text{eff}} = g_i \Phi_i \quad n_i^{s} = g_i^{\text{eff}} \int \frac{d^3p}{(2\pi)^3} \frac{m_i^{\text{eff}}}{E_i} f(T, \mu_i^{\text{eff}}, m_i^{\text{eff}}) \]

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Where effective mass \( m_i^{\text{eff}} = m_i - S_i^\Phi \) and chemical potential \( \mu_i^{\text{eff}} = \mu_i - V_i^\Phi \)

\[ S_i^\Phi = \sum_j p_j \frac{\partial \ln \Phi_j}{\partial n_i^{s}}, \quad V_i^\Phi = - \sum_j p_j \frac{\partial \ln \Phi_j}{\partial n_i} \]

\[ P = P_0 + P_\Phi = \sum_i p_i + \sum_i (n_i V_i^\Phi - n_i^{s} S_i^\Phi) \]

Our goal is to construct the model that effectively describes the transition from hadronic matter to quark-gluon plasma. To do this, first we divide hadrons into four sets:

\[ S_1^0 = \{ p, n, \Lambda \} + \text{a.p.} \]
\[ S_2^0 = \{ \pi^+, \pi^0, \pi^- \} \]
\[ S_3^0 = \{ \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0, \Delta^-, \Delta^0, \Delta^+, \Delta^{++}, \Sigma^{*-}, \Sigma^{*0}, \Sigma^{*+}, \Xi^{*-}, \Xi^{*0}, \Omega \} + \text{a.p.} \]
\[ S_4^0 = \{ \omega, \eta, K^0, K^+, \bar{K}^0, K^-, \rho^-, \rho^0, \rho^+, \phi, \eta', a_0^-, a_0^+, a_1^0, f_0, K^{*0}, K^{*+}, \bar{K}^{*0}, K^{*-} \} \]
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S_1^\infty = \{u, d, s\} + \text{a.p.} \\
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Now, using the modified excluded volume mechanism one can change the effective number of degrees of freedom to transform hadron-like quasiparticles into quark- and gluon-like quasiparticles:

\[
\begin{align*}
  g_i^0 = g(S_i^0) &= \begin{cases} 
    6, & i \in S_1 \\
    3, & i \in S_2 \\
    50, & i \in S_3 \\
    37, & i \in S_4 
  \end{cases} \\
  g_i^\infty = g(S_i^\infty) &= \begin{cases} 
    18, & i \in S_1 \\
    16, & i \in S_2 \\
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S_4^0 &= \{\omega, \eta, K^0, K^+, \bar{K}^0, K^-, \rho^-, \rho^0, \rho^+, \phi, \eta', \alpha_0, \alpha_0^0, \alpha_0^+, \alpha_0^0, f_0, K^{*0}, K^{*+}, \bar{K}^{*0}, K^{*-}\}
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Here, we assume that form of \(\Phi_i\) is identical for all particles and depends only on the total scalar density:

\[
\Phi_i(x_i) = \left(\frac{(g_i^\infty)^{1/\alpha_i}x_i^2 + 1}{(g_i^0)^{1/\alpha_i}x_i^2 + 1}\right)^{\alpha_i}
\]

if \(x_i > 0\) and 1 otherwise, \(x_i = \frac{n_{i,\text{tot}} - n_{i,c}}{n_i^0}\) and \(n_{i,\text{tot}} = \sum_i n_i^s\).
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S^0_1 &= \{p, n, \Lambda\} + \text{a.p.} & S^\infty_1 &= \{u, d, s\} + \text{a.p.} \\
S^0_2 &= \{\pi^+, \pi^0, \pi^-\} & S^\infty_2 &= \{g\} \\
S^0_3 &= \{\Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0, \Delta^0, \Delta^+, \Delta^{-}, \Sigma^{*-}, \Sigma^{*0}, \Sigma^{*+}, \Xi^{-}, \Xi^{*0}, \Omega\} + \text{a.p.} & S^\infty_3 &= \emptyset \\
S^0_4 &= \{\omega, \eta, K^0, K^+, \bar{K}^0, K^-, \rho^-, \rho^0, \rho^+, \phi, \eta', a_0^-, a_0^+, a_0^0, f_0, K^{*0}, K^{*+}, \bar{K}^{*0}, K^{*-}\} & S^\infty_4 &= \emptyset
\end{align*}

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\[ g^0_i = g(S^0_i) = \begin{cases} 6, & i \in S_1 \\ 3, & i \in S_2 \\ 50, & i \in S_3 \\ 37, & i \in S_4 \end{cases} \quad g^\infty_i = g(S^\infty_i) = \begin{cases} 18, & i \in S_1 \\ 16, & i \in S_2 \\ 0, & i \in S_3 \\ 0, & i \in S_4 \end{cases} \]

Here, we assume that form of \( \Phi_i \) is identical for all particles and depends only on the total scalar density:

\[ \Phi_i(x_i) = \left(\frac{(g_i^{\infty})^{1/\alpha_i} x_i^2 + 1}{(g_i^0)^{1/\alpha_i} x_i^2 + 1}\right)^{\alpha_i} \text{ if } x_i > 0 \text{ and } 1 \text{ otherwise, } x_i = \frac{n^s_{i_{tot}} - n^c_{i_0}}{n^0_i} \text{ and } n^s_{i_{tot}} = \sum_i n^s_i \]

\[ \lim_{x \to 0} \Phi_i(x) = 1, \quad \lim_{x \to \infty} \Phi_i(x) = \frac{g_i^{\infty}}{g_i^0} \]
Results: $\mu = 0$

\[
\begin{align*}
  n^c_i &= \begin{cases} 
    4.02 \text{ fm}^{-3}, & i \in S_1 \\
    0.03 \text{ fm}^{-3}, & i \in S_2 \\
    0.30 \text{ fm}^{-3}, & i \in S_3 \\
    0.28 \text{ fm}^{-3}, & i \in S_4
  \end{cases} \\
  n^0_i &= \begin{cases} 
    176.08 \text{ fm}^{-3}, & i \in S_1 \\
    36.07 \text{ fm}^{-3}, & i \in S_2 \\
    7.79 \text{ fm}^{-3}, & i \in S_3 \\
    93.20 \text{ fm}^{-3}, & i \in S_4
  \end{cases} \\
  \alpha_i &= \begin{cases} 
    0.356, & i \in S_1 \\
    0.231, & i \in S_2 \\
    0.939, & i \in S_3 \\
    0.884, & i \in S_4
  \end{cases}
\end{align*}
\]

[Data: S. Borsanyi et al., PLB 730 (2014) 99; A. Bazavov et al., PRD 90, 094503 (2014)]
Results: $\mu_B \neq 0$

$\mu_s = \mu_Q = 0$ MeV

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<thead>
<tr>
<th>$\mu_B / T$</th>
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<td>1</td>
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$\frac{n_Q}{n_B} = 0.4$, $n_s = 0$ fm$^3$

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[Data: A. Bazavov et al., PRD 95, 054504 (2017)]
Conclusions & Outlook

Conclusions:
• We presented an effective equation of state of strongly interacting matter based on modified excluded volume mechanism
• The transition from hadron- to quark/gluon-like quasiparticles realized with the help of effective medium dependent degeneracy factors
• The obtained equation of state shows good agreement with lattice QCD data and can be used for hydrodynamical simulations of relativistic heavy ion collisions

Outlook:
• Revise particle sets, develop another ansatz with 1\textsuperscript{st} order phase transition as in [S. Typel, D. Blaschke, Universe 2018, 4, 32]
• Add mean-field interactions to constrain model parameters at high density region
• Goal: Construct a tool to investigate the correlation between the existence and position of the critical endpoint and observables in heavy ion collisions
\[ \Phi_i \text{ T-dependence} \]
$T \to \infty$