Phenomenological Equation of State of Strongly Interacting Matter with Modified Excluded-Volume Mechanism

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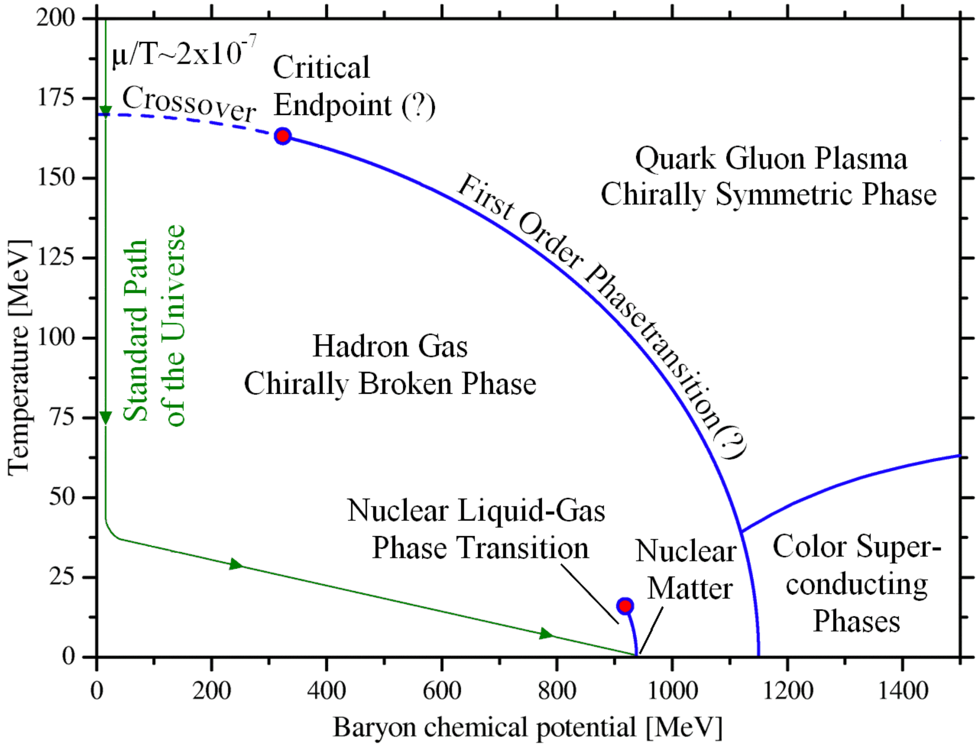
52nd International Symposium on Multiparticle Dynamics
August 25th, 2023

Uniwersytet Wrocławski

NATIONAL SCIENCE CENTRE POLAND

Grant № 2022/45/N/ST2/02391

Motivation



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Exploring of the QCD phase diagram:

- Detect signals of deconfinement PT
- Detect signals of (partial) chiral symmetry restoration
- Locate (tri)critical endpoint(s) if such exists

[Figures: Universe 4 (2018) 52 & PoS (KMI 2013) 025]

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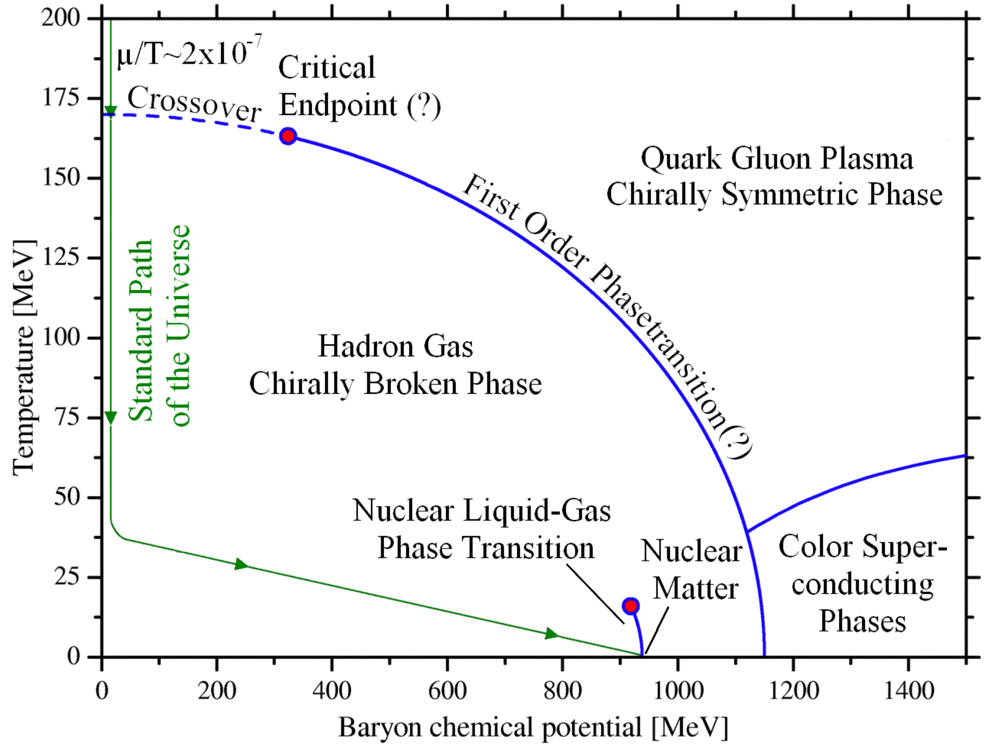

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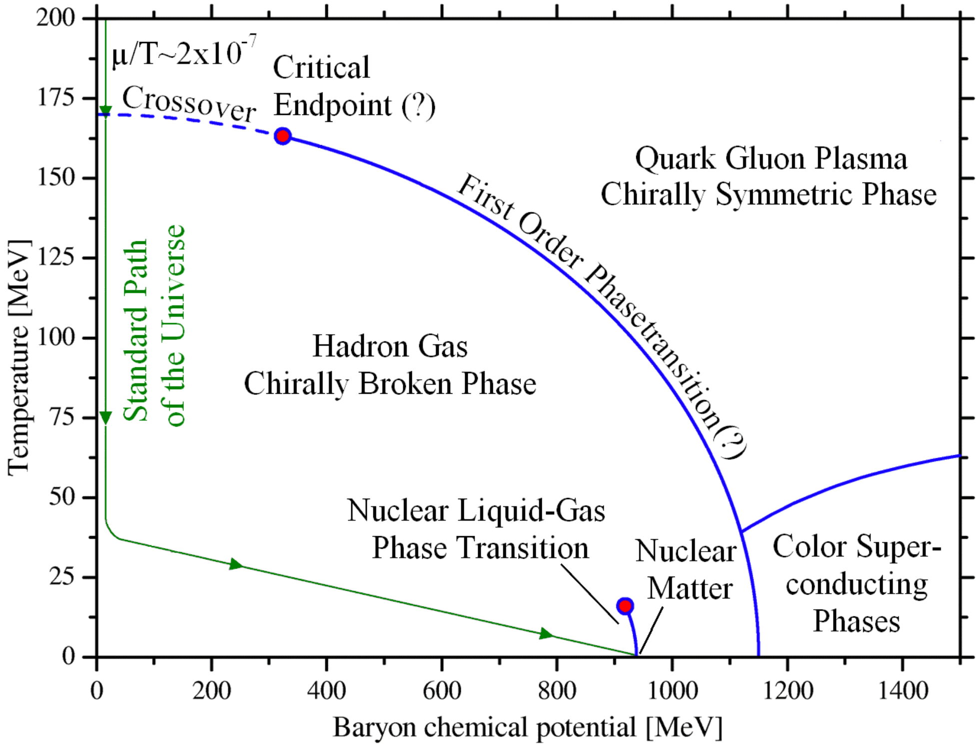

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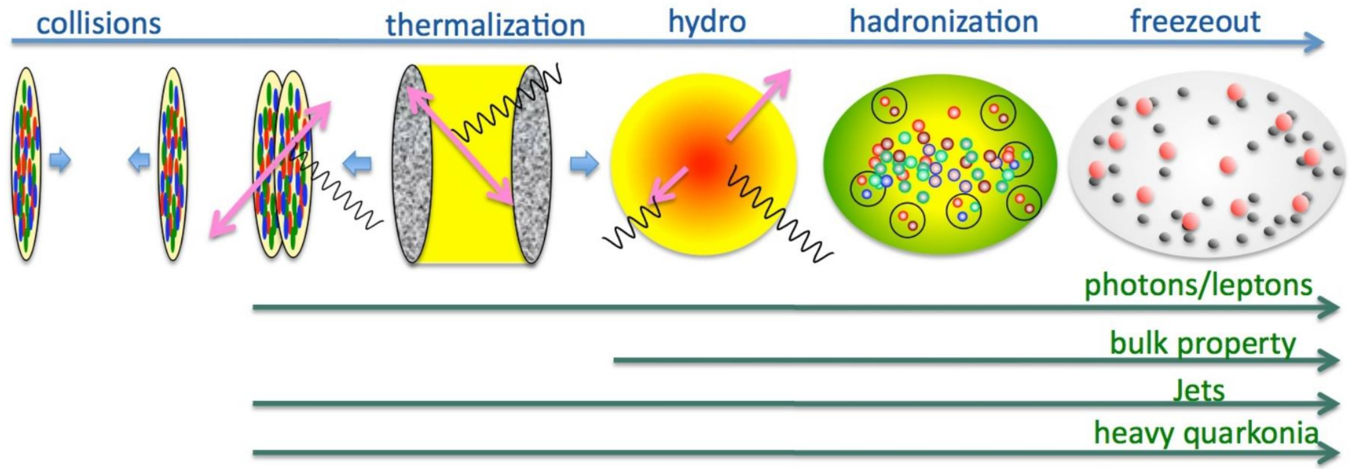

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Modified Excluded Volume

The effect of finite particle volumes can be seen as a modification of the degeneracy factors.
E. g. one can introduce medium dependent degeneracy factors instead of available volume fraction:

$$g_i V_i \rightarrow g_i^{eff} V$$

[For detailes see S. Typel, EPJ A (2016) 52: 16 and S. Typel, D. Blaschke, Universe 2018, 4, 32]

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$$n_i^s = g_i^{eff} \int \frac{d^3p}{(2\pi)^3} \frac{m_i^{eff}}{E_i} f\left(T, \mu_i^{eff}, m_i^{eff}\right)$$

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Where effective mass $m_i^{eff} = m_i - S_i^\Phi$ and chemical potential $\mu_i^{eff} = \mu_i - V_i^\Phi$

$$S_i^\Phi = \sum_j p_j \frac{\partial \ln \Phi_j}{\partial n_i^s}, \qquad V_i^\Phi = -\sum_j p_j \frac{\partial \ln \Phi_j}{\partial n_i}$$

$$P = P_0 + P_\Phi = \sum_i p_i + \sum_i \left(n_i V_i^\Phi - n_i^s S_i^\Phi\right)$$

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Model Description

Our goal is to construct the model that effectively describes the transition from hadronic matter to quark-gluon plasma. To do this, first we divide hadrons into four sets:

$$\mathcal{S}_1^0 = \{p, n, \Lambda\} + \mathrm{a.p.}$$
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$\Rightarrow$

$$\mathcal{S}_1^\infty = \{u, d, s\} + \text{a. p.}$$
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Now, using the modified excluded volume mechanism one can change the effective number of degrees of freedom to transform hadron-like quasiparticles into quark- and gluon-like quasiparticles:

$$g_i^0 = g(\mathcal{S}_i^0) = \begin{cases} 6, & i \in \mathcal{S}_1 \\ 3, & i \in \mathcal{S}_2 \\ 50, & i \in \mathcal{S}_3 \\ 37, & i \in \mathcal{S}_4 \end{cases} \quad \Rightarrow \quad g_i^\infty = g(\mathcal{S}_i^\infty) = \begin{cases} 18, & i \in \mathcal{S}_1 \\ 16, & i \in \mathcal{S}_2 \\ 0, & i \in \mathcal{S}_3 \\ 0, & i \in \mathcal{S}_4 \end{cases}$$

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$$\Phi_i(x_i) = \left(\frac{(g_i^\infty)^{1/\alpha_i} x_i^2 + 1}{(g_i^0)^{1/\alpha_i} x_i^2 + 1} \right)^{\alpha_i}$$ if $x_i > 0$ and 1 otherwise, $x_i = \dfrac{n_{tot}^s - n_i^c}{n_i^0}$ and $n_{tot}^s = \sum_i n_i^s$

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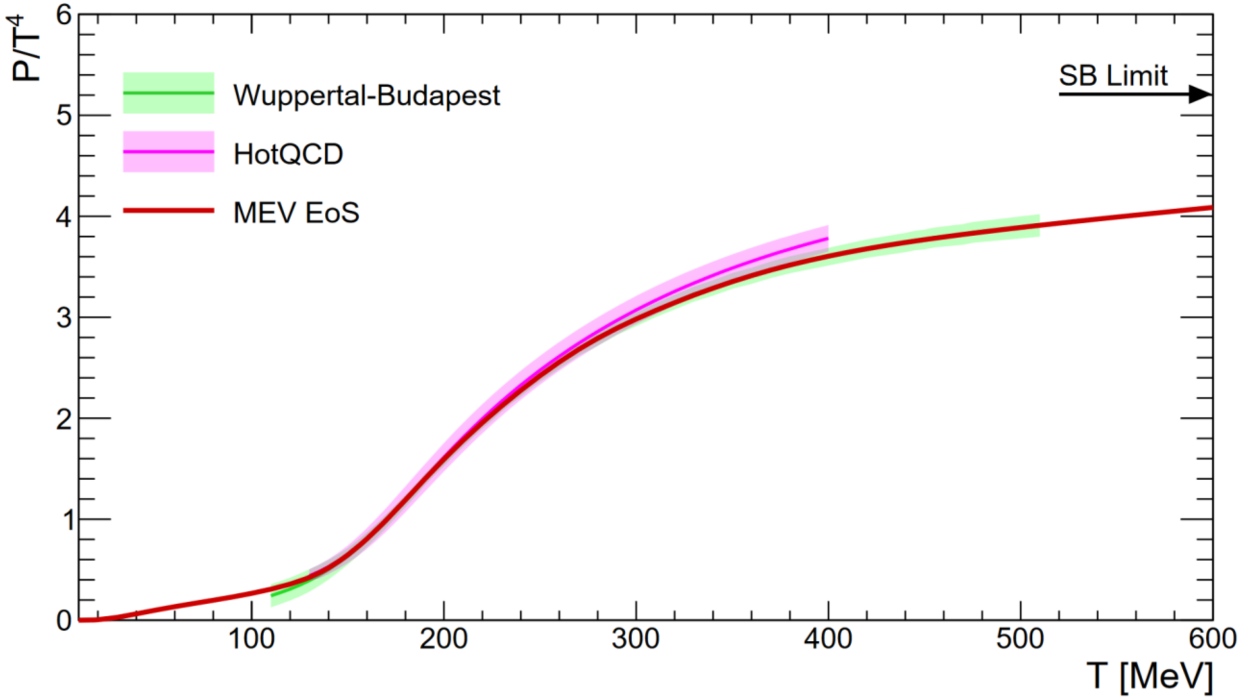
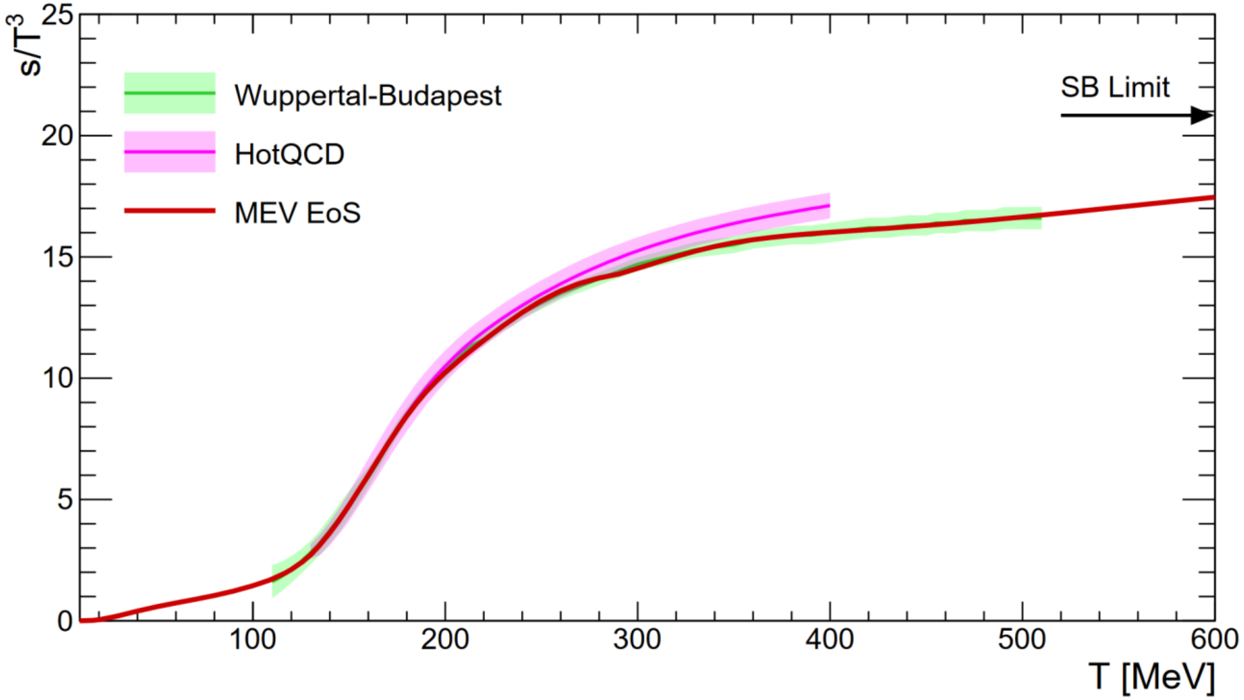
$$\lim_{x \to 0} \Phi_i(x) = 1, \qquad \lim_{x \to \infty} \Phi_i(x) = \frac{g_i^\infty}{g_i^0}$$

Results: $\mu = 0$





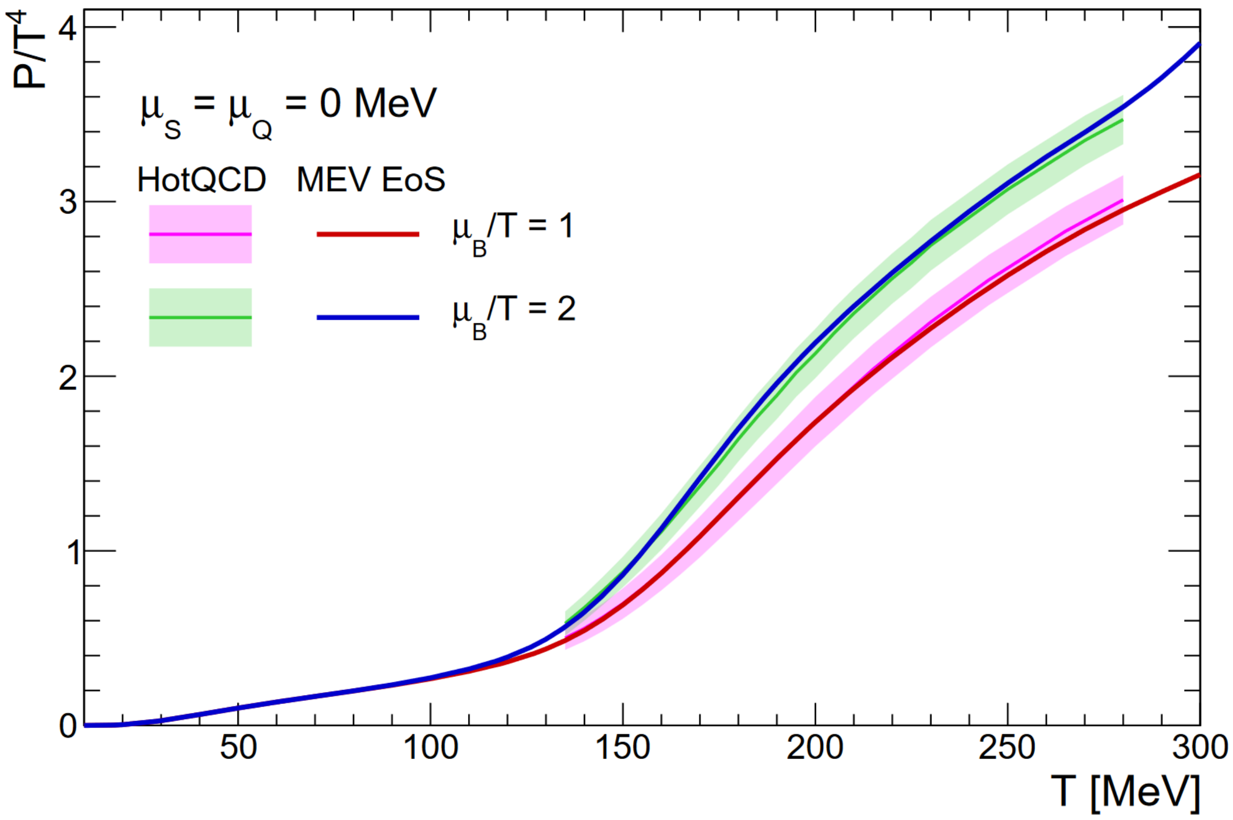
$$n_i^c = \begin{cases} 4.02\ \mathrm{fm}^{-3}, i \in \mathcal{S}_1 \\ 0.03\ \mathrm{fm}^{-3}, i \in \mathcal{S}_2 \\ 0.30\ \mathrm{fm}^{-3}, i \in \mathcal{S}_3 \\ 0.28\ \mathrm{fm}^{-3}, i \in \mathcal{S}_4 \end{cases}$$

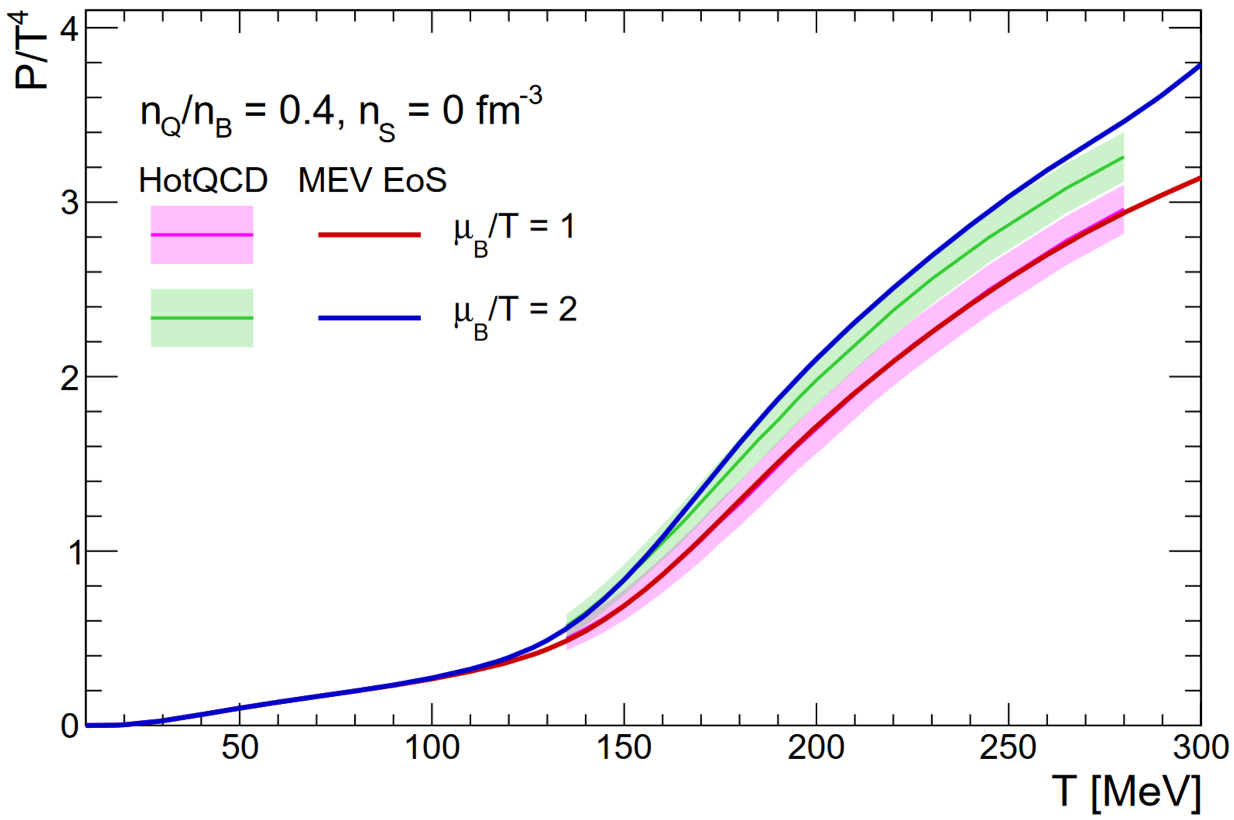
$$n_i^0 = \begin{cases} 176.08\ \mathrm{fm}^{-3}, i \in \mathcal{S}_1 \\ 36.07\ \ \mathrm{fm}^{-3}, i \in \mathcal{S}_2 \\ 7.79\ \ \ \mathrm{fm}^{-3}, i \in \mathcal{S}_3 \\ 93.20\ \ \mathrm{fm}^{-3}, i \in \mathcal{S}_4 \end{cases}$$

$$\alpha_i = \begin{cases} 0.356\ , i \in \mathcal{S}_1 \\ 0.231\ , i \in \mathcal{S}_2 \\ 0.939\ , i \in \mathcal{S}_3 \\ 0.884, i \in \mathcal{S}_4 \end{cases}$$

[Data: S. Borsanyi et al., PLB 730 (2014) 99; A. Bazavov et al., PRD 90, 094503 (2014)]

Results: $\mu_B \neq 0$





[Data: A. Bazavov et al., PRD 95, 054504 (2017)]

Conclusions & Outlook

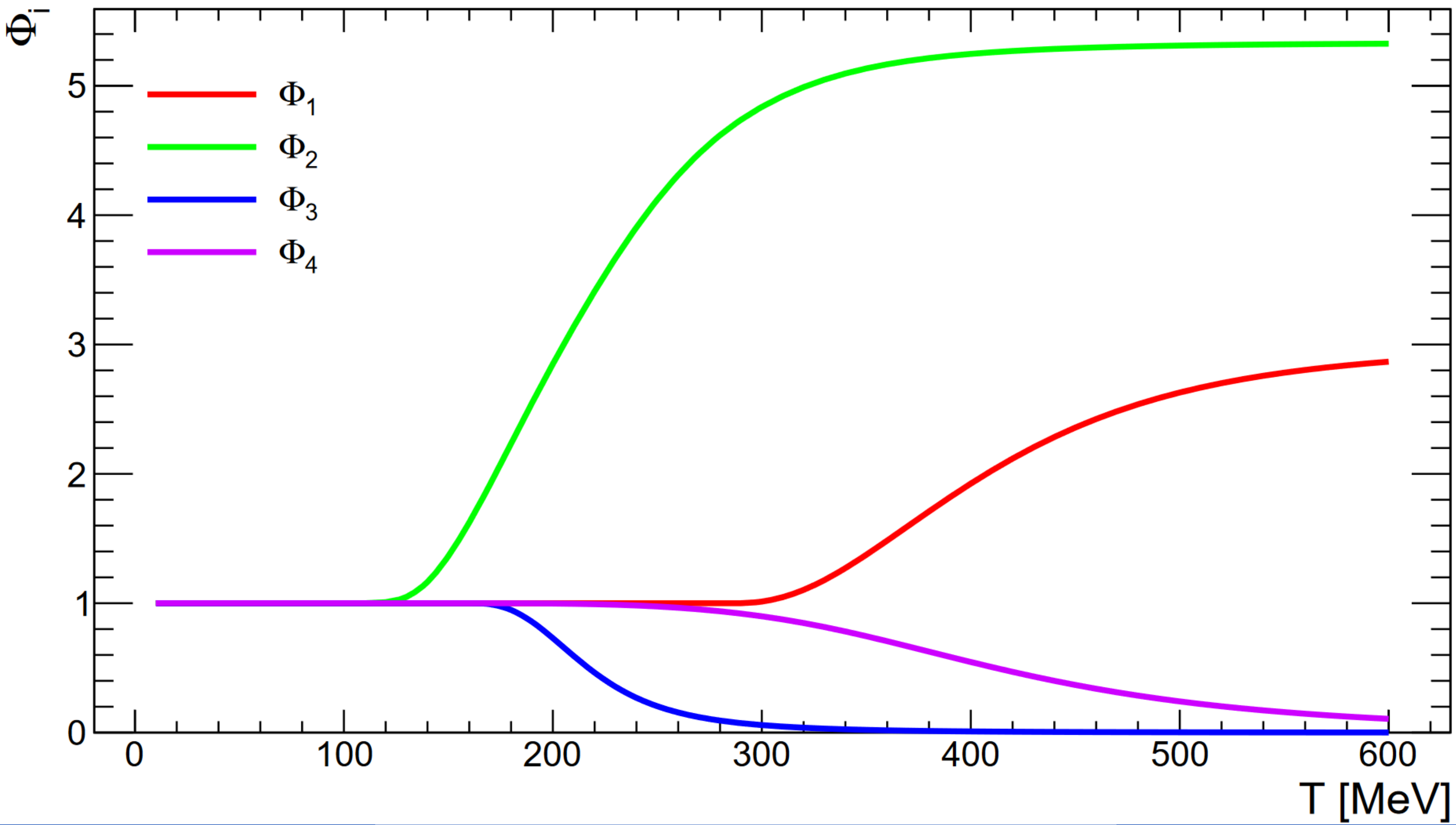
Conclusions:

- We presented an effective equation of state of strongly interacting matter based on modified excluded volume mechanism
- The transition from hadron- to quark/gluon-like quasiparticles realized with the help of effective medium dependent degeneracy factors
- The obtained equation of state shows good agreement with lattice QCD data and can be used for hydrodynamical simulations of relativistic heavy ion collisions

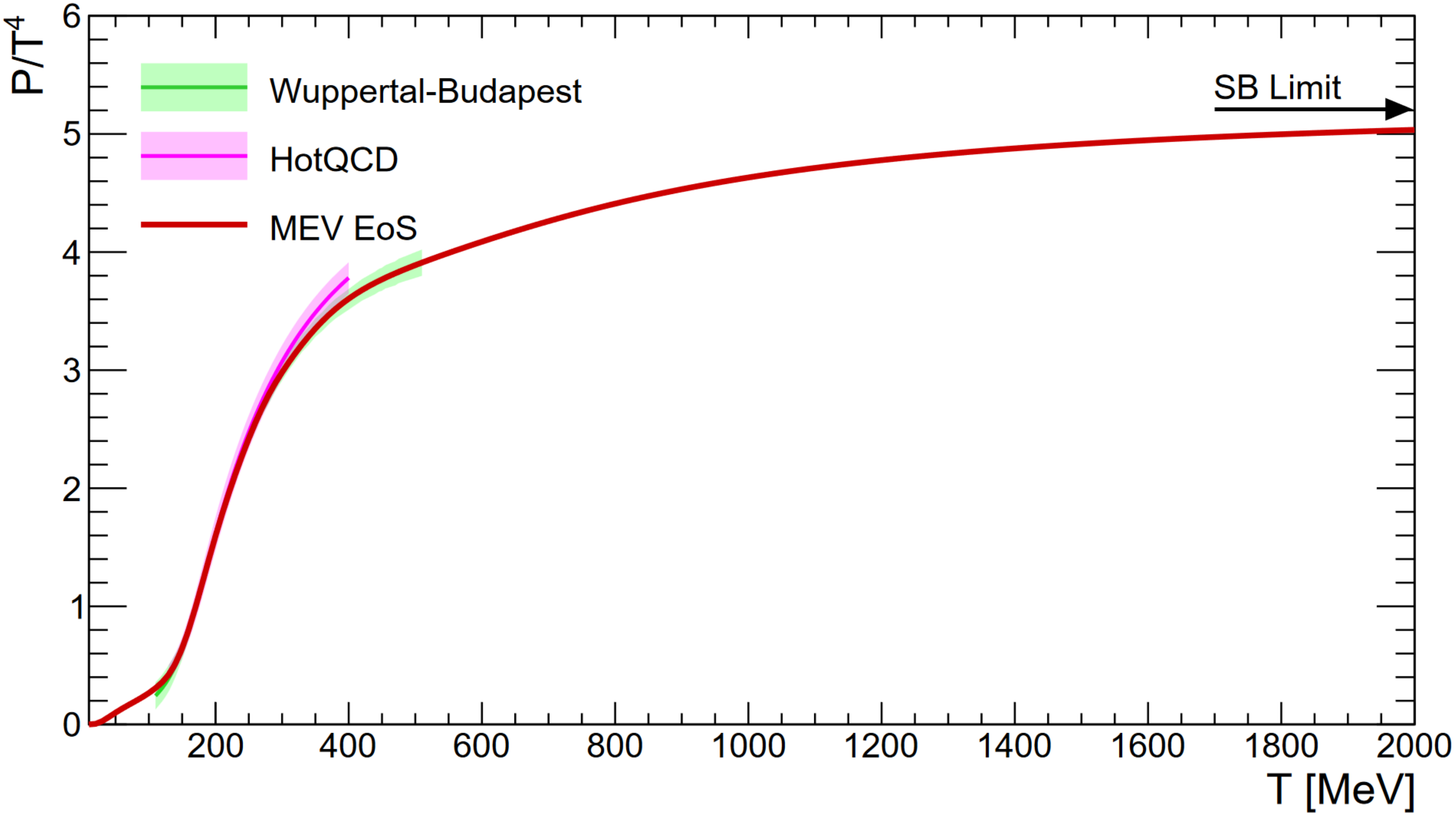
Outlook:

- Revise particle sets, develop another ansatz with 1^{st} order phase transition as in [S. Typel, D. Blaschke, Universe 2018, 4, 32]
- Add mean-field interactions to constrain model parameters at high density region
- Goal: Construct a tool to investigate the correlation between the existence and position of the critical endpoint and observables in heavy ion collisions

Φ_i T-dependence



$T \to \infty$



P/T^4
6
5
4
3
2
1
0
Wuppertal-Budapest
HotQCD
MEV EoS
SB Limit
200
400
600
800
1000
1200
1400
1600
1800
2000
T [MeV]