Abundant radiation of soft photons: a puzzle lasting four decades

Boris Kopeliovich
UTFSM Valparaiso

In collaboration with
Irina Potashnikova,
Ivan Schmidt

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FEDERICO SANTA MARIA

Soft photon puzzle

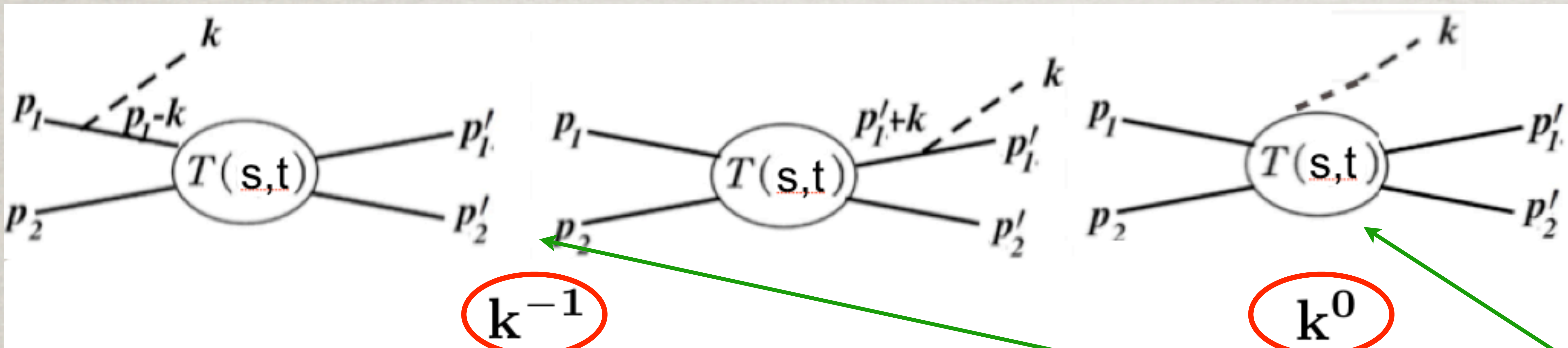
Experiment	Year	Collision energy	Photon p_T	Photon / Brems Ratio	Detection method	Reference
π^+p	1979	10.5 GeV	p_T < 30 MeV/c	~ 1	bubble chamber	Goshaw et al., Phys. Rev. Lett. 43, 1065 (1979)
K^+p WA27, CERN	1984	70 GeV	p_T < 60 MeV/c	4.0 ± 0.8	bubble chamber (BEBC)	Chliapnikov et al., Phys. Lett. B 141, 276 (1984)
π^+p CERN, EHS, NA22	1991	250 GeV	p_T < 40 MeV/c	6.4 ± 1.6	bubble chamber	Botterweck et al., Z. Phys. C 51, 541 (1991)
K^+p CERN, EHS, NA22	1991	250 GeV	p_T < 40 MeV/c	6.9 ± 1.3	bubble chamber	Botterweck et al., Z. Phys. C 51, 541 (1991)
π^-p, CERN, WA83, OMEGA	1993	280 GeV	p_T < 10 MeV/c (0.2 < E_γ < 1 GeV)	7.9 ± 1.4	calorimeter	Banerjee et al., Phys. Lett. B 305, 182 (1993)
p-Be	1993	450 GeV	p_T < 20 MeV/c	< 2	pair conversion, calorimeter	Antos et al., Z. Phys. C 59, 547 (1993)
p-Be, p-W	1996	18 GeV	p_T < 50 MeV/c	< 2.65	calorimeter	Lissauer et al., Phys.Rev. C54 (1996) 1918
π^-p, CERN, WA91, OMEGA	1997	280 GeV	p_T < 20 MeV/c (0.2 < E_γ < 1 GeV)	7.8 ± 1.5	pair conversion	Belogianni et al., Phys. Lett. B 408, 487 (1997)
π^-p, CERN, WA91, OMEGA	2002	280 GeV	p_T < 20 MeV/c (0.2 < E_γ < 1 GeV)	5.3 ± 1.0	pair conversion	Belogianni et al., Phys. Lett. B 548, 122 (2002)
pp, CERN, WA102, OMEGA	2002	450 GeV	p_T < 20 MeV/c (0.2 < E_γ < 1 GeV)	4.1 ± 0.8	pair conversion	Belogianni et al., Phys. Lett. B 548, 129 (2002)
$e^+e^- \to$ 2 jets CERN, DELPHI	2006	91 GeV (CM)	p_T < 80 MeV/c	4.0 ± 0.3 ± 1.0	pair conversion	DELPHI, Eur. Phys. J. C 47, 273 (2006)
$e^+e^- \to \mu^+\mu^-$ CERN, DELPHI	2008	91 GeV (CM)	p_T < 80 MeV/c	~ 1	pair conversion	DELPHI, Eur. Phys. J. C57, 499 (2008)

Soft photons | K. Reygers

The Low theorem, revisited

$$\mathbf{h_1 + h_2 \to h_1' + \gamma + h_2'}$$

F. Low, Phys. Rev. 110, 974 (1958)



The amplitude $\mathbf{M} = \mathbf{e}_\mu \mathbf{M}_\mu$ gets contributions from external and internal radiation.

$$\mathbf{M}_\mu = \mathbf{M}_\mu^{\mathbf{ext}} + \mathbf{M}_\mu^{\mathbf{int}}$$

$$\mathbf{M}_\mu^{\mathbf{ext}} = \left(\frac{\mathbf{p}'_{1\mu}}{\mathbf{p}'_1 \mathbf{k}} - \frac{\mathbf{p}_{1\mu}}{\mathbf{p}_1 \mathbf{k}} \right) \mathbf{T}(\mathbf{s}, \mathbf{t}),$$

is infrared divergent

The two terms are related by gauge invariance $\mathbf{k}_\mu \mathbf{M}_\mu = \mathbf{0}$

Therefore $\mathbf{M}^{\mathbf{int}}$ is not divergent at $\mathbf{k} \to \mathbf{0}$, i.e. is suppressed in comparison with $\mathbf{M}_\mu^{\mathbf{ext}}$

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Landau-Pomeranchuk principle

L.Landau, I.Pomeranchuk,
Dokl. Akad. Nauk 92, 535(1953)

The Low theorem can also be treated as a formal proof of the Landau-Pomeranchuk principle, which states that any variation of the electric current within a short distance do not affect the spectrum of radiation with much longer coherence length,

$$\mathbf{l_c^\gamma = \frac{2E_{h1}x_1(1-x_1)}{k_T^2 + x_1 m_h^2}} \qquad \mathbf{x_1 = \frac{k_+^\gamma}{p_+^{h_1}}}$$

Low: "the distance a particle can move with energy imbalance ΔE=k"

Important is to keep the incoming ($l < -l_c$) and outgoing ($l > l_c^\gamma$) currents unaffected by the current variations on shorter length scales. This means that only extrinsic radiation from initial and final hadrons h1, h'1 matters.

$\mathbf{l_c^\gamma}$ and $\mathbf{E_{h1}}$ are not Lorentz invariant, must be taken within the same reference frame

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Fock state representation

The Low process $\mathbf{h_1 + h_2 \to h_1' + \gamma + h_2'}$ is diffractive excitation $\mathbf{h_1 \to h_1' + \gamma}$

The Good-Walker picture of diffraction:

M.L.Good & W.D.Walker, Phys.Rev. 120, 1857(1960)

$$|\mathbf{h}\rangle = \mathbf{C_0}|\mathbf{h}\rangle_\mathbf{0} + \mathbf{C_1}|\mathbf{h}\gamma\rangle_\mathbf{0} + ...$$

The Fock components $|\mathbf{i}\rangle$ are eigenstates of interaction, i.e. $\hat{\mathbf{f}}|\mathbf{i}\rangle = \mathbf{f_i}|\mathbf{i}\rangle$

Diffractive excitation occurs only due to diversity of the elastic eigenamplitudes, otherwise the incoming wave packet remains unchanged.

$$\langle \mathbf{h}\gamma|\hat{\mathbf{f}}|\mathbf{h}\rangle = \mathbf{C_1^* C_0}(\mathbf{f_{h\gamma}} - \mathbf{f_h})$$

Differently from the Feynman diagrams, one cannot say whether the photon is radiated before or after the interaction. The radiation amplitude is a linear combination of elastic eigenamplitudes.

Fock state expansion

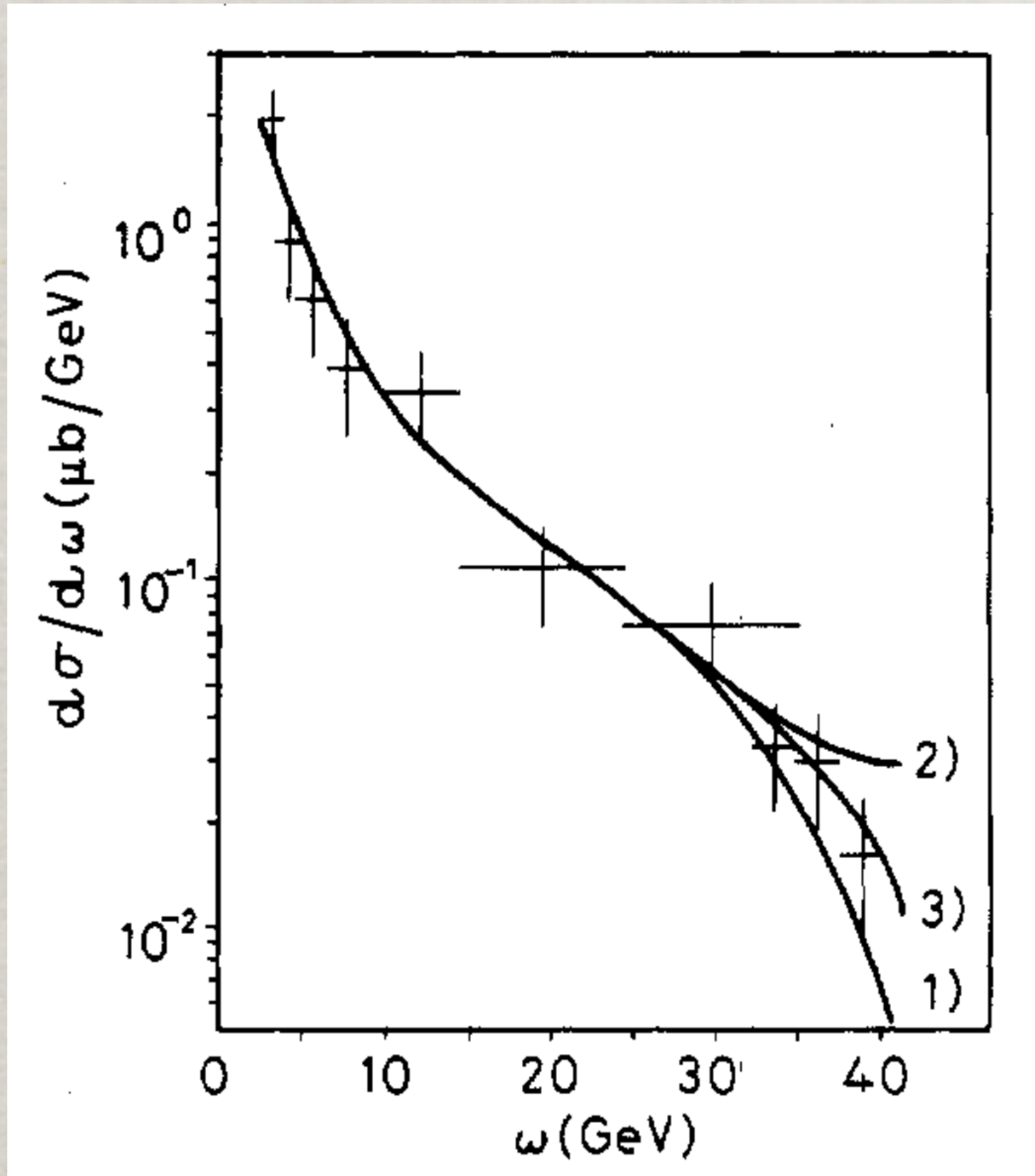
The Fock component lifetime is the coherence time/length $\mathbf{l_c^\gamma}$

In both Fock components $|\mathbf{h}\rangle$ and $|\mathbf{h}\gamma\rangle$ only hadron **h** interacts, so at first glance $\mathbf{f_{h\gamma} = f_h}$ and the diffractive amplitude vanishes. However, the impact parameters **b** are shifted, so the eigen-amplitudes $\mathbf{f_{h\gamma}(b) \neq f_h(b')}$ cancel only in **b**-integrated amplitude, i.e. at t=0.

The Low amplitude $\mathbf{M_\mu^{ext}} = \left(\frac{\mathbf{p'_{1\mu}}}{\mathbf{p'_1 k}} - \frac{\mathbf{p_{1\mu}}}{\mathbf{p_1 k}}\right)\mathbf{T(s,t)}$, indeed vanishes at t->0.

The Low theorem was successfully tested experimentally for the radiative process $\pi^- p \to \pi^- \gamma p$ at 43 GeV

Yu. M. Antipov et al. Europhys. Lett., 11 (8),725(1990)



The Low theorem: small-ω expansion

$$\frac{\mathbf{d}\sigma}{\mathbf{d}\omega} = \frac{\sigma_\mathbf{0}}{\omega} + \sigma_\mathbf{1} + \ldots$$

$$\sigma_\mathbf{0} = \lim_{\omega\to\mathbf{0}} \omega\sigma$$

$$\sigma_\mathbf{1} = \lim_{\omega\to\mathbf{0}} \frac{\mathbf{d}\omega\sigma}{\mathbf{d}\omega}$$

σ_0 is elastic pion-proton cross section, well measured at this energy.

$$\sigma_\mathbf{1} = \mathbf{12.0 \pm 1.2_{stat} \pm 1.3_{syst}}\ \mu\mathbf{b}$$

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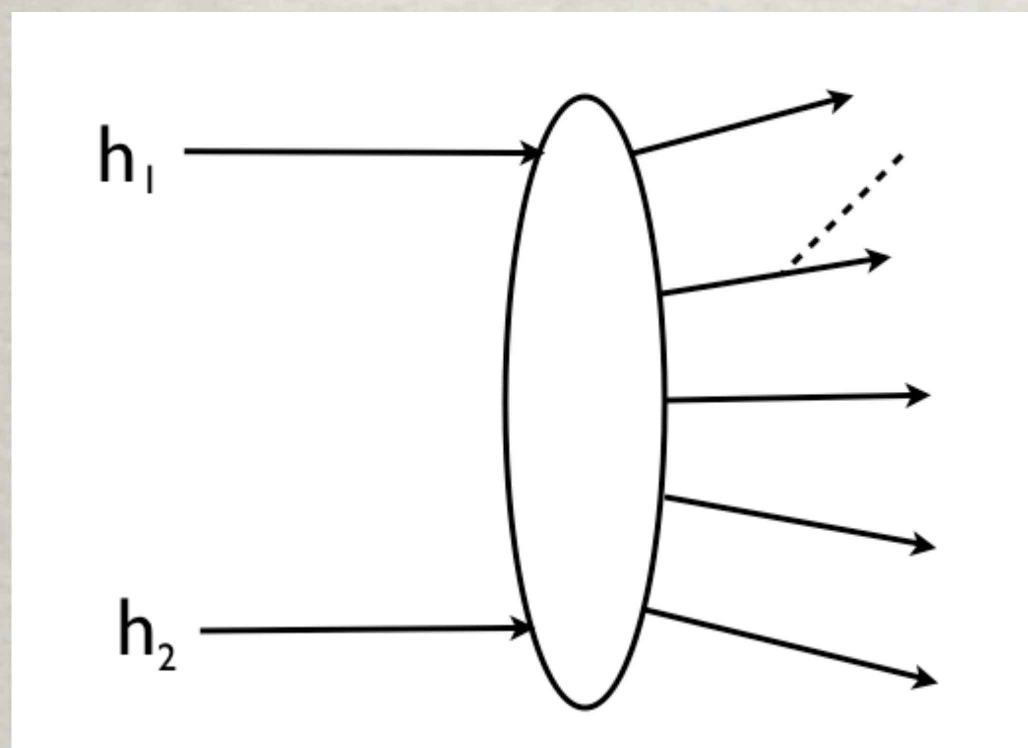
Photon radiation in inelastic collisions

The so-called **bremsstrahlung model (BM)** pretends to extend the Low theorem from the radiation in elastic scattering to inelastic collisions with multi-particle production



Photons are assumed to be radiated by participating charge particles, either the incoming, or outgoing.

$$\mathbf{M(p_1, p_2; p_3 ... p_N k) = M_0(p_1, p_2; p_3 ... p_N)} \left(\sum_{\mathbf{i}}^{\mathbf{charged}} \frac{\eta_\mathbf{i} \mathbf{e_i p_i \cdot} \epsilon}{\mathbf{2p_i \cdot k}} \right)$$

where $\eta_i = \pm 1$ for outgoing and incoming particles respectively, and M_0 is the amplitude of $2 \to N$ inelastic collision without radiation.

A.T. Goshaw et al., PRL 43(1979)1065

T. H. Burnettt & N. M. Kroll, PRL 20(1968)86

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The model turns out to contradict data significantly

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BM "extension" of the Low theorem is not only unjustified, but in fact contradicts the Low theorem

First of all, the notion of incoming and outgoing particles is poorly defined.
The process is characterized by two length scales:

(i) One scale is related to the coherence length of photon radiation (defined by Low as **"the distance a particle can move with energy imbalance"**)

$$\mathbf{l_c^\gamma = \frac{2E_1 x_1(1-x_1)}{k_T^2 + x_1 m_h^2}}$$

E1 is the energy of h1 in the rest frame of h2.
x1 is the fractional light-cone momentum of the photon

$$\mathbf{x_1 = k_+^\gamma / p_+^{h_1}}$$

(ii) The range of strong interactions, is assumed to be short. In the target rest frame it is given by the inverse pion mass $l_h \sim 1$ fm. The main condition of the Low theorem is $\mathbf{l_c^\gamma \gg l_h}$

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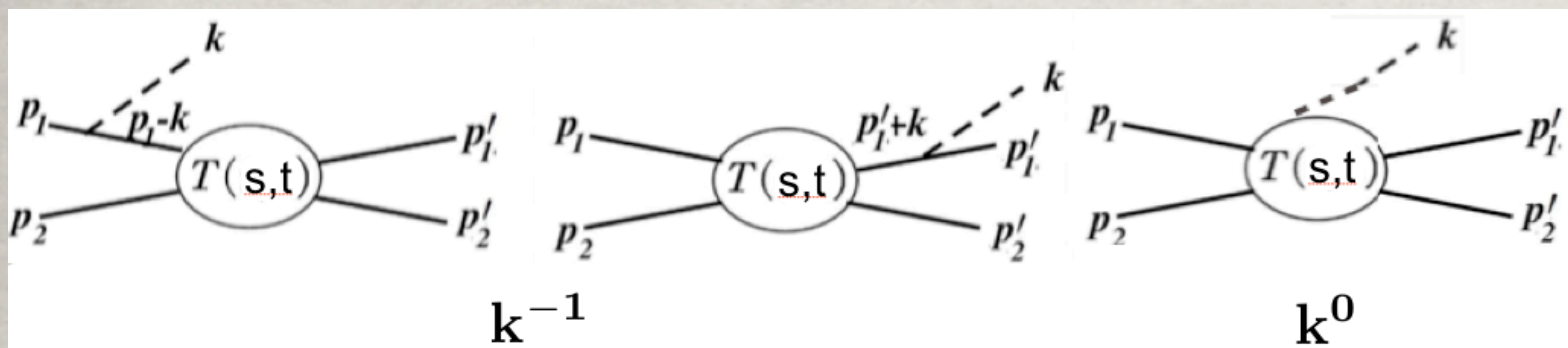
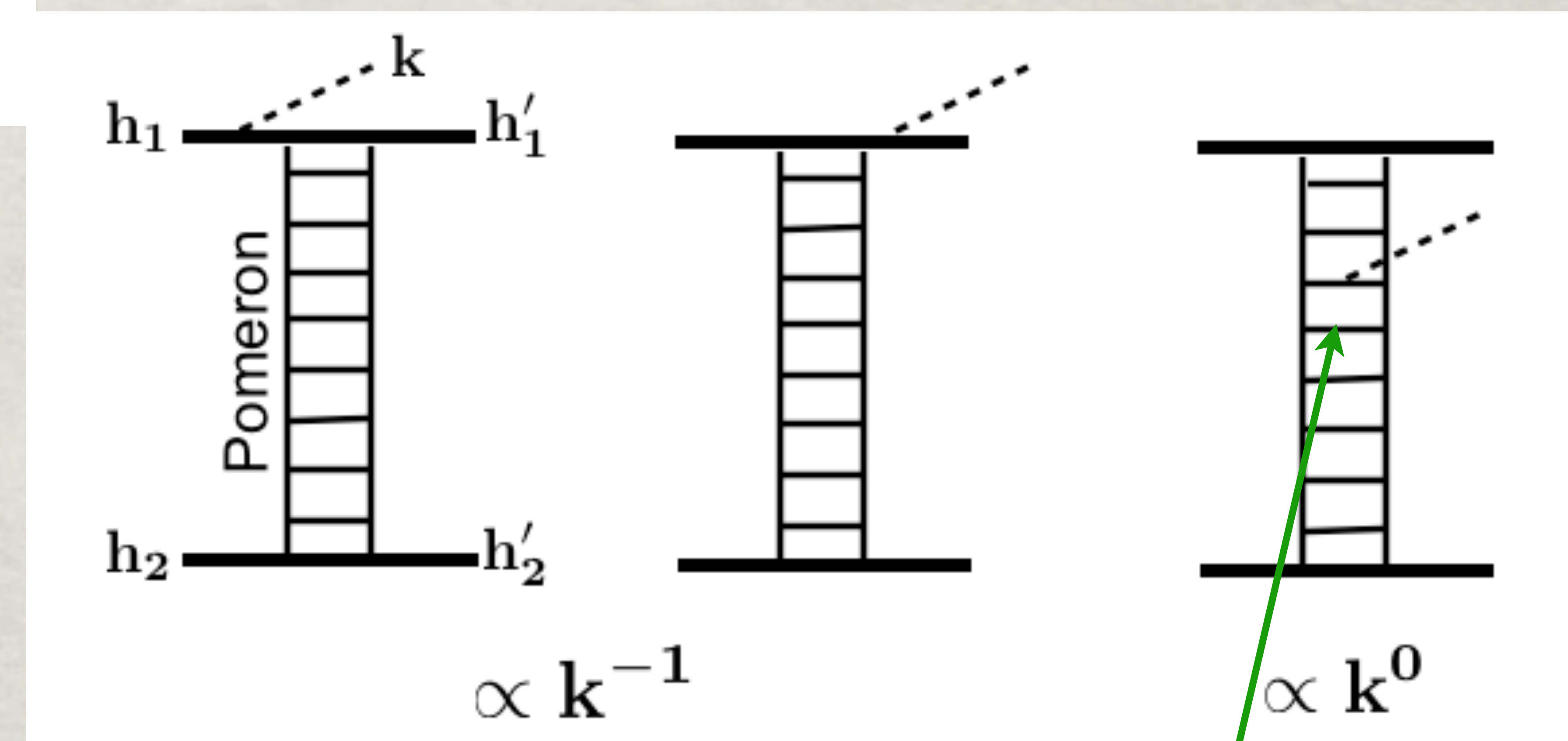
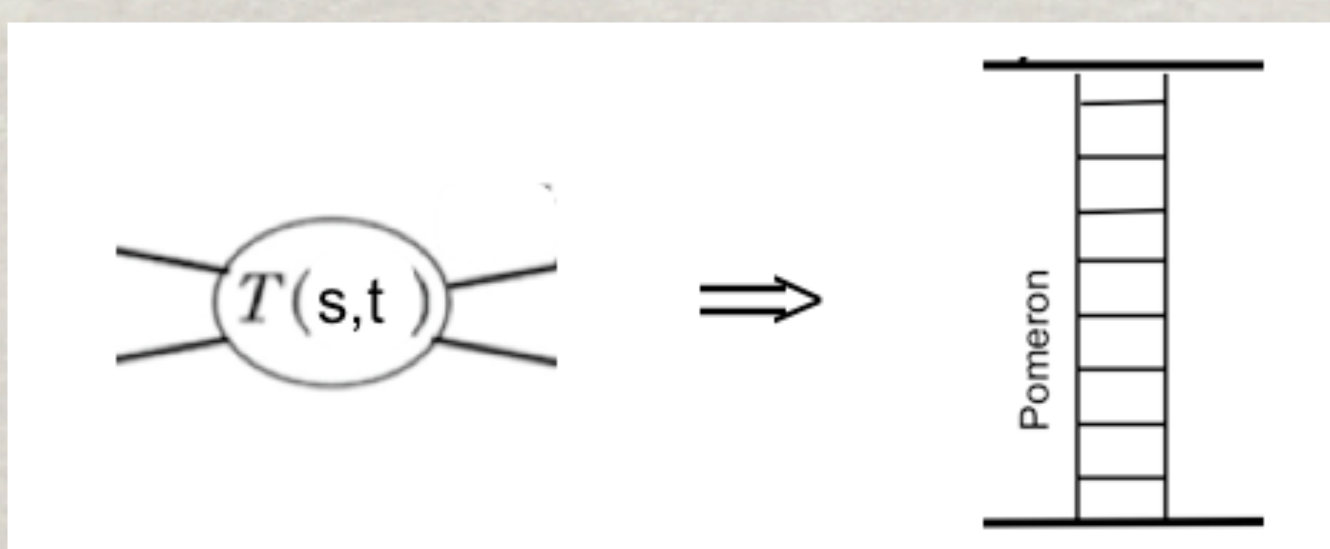
At high **energies** the Low process essentially simplifies. Elastic scattering dominates, so $\mathbf{h_1 = h_1'}$ and T(s,t) is the elastic amplitude, dominated by Pomeron exchange.

Besides, the energy dependence of T(s,t) can be neglected, $\frac{\partial \mathbf{T(s,t)}}{\partial \mathbf{s}} \approx \frac{\mathbf{1}}{\mathbf{s}}$





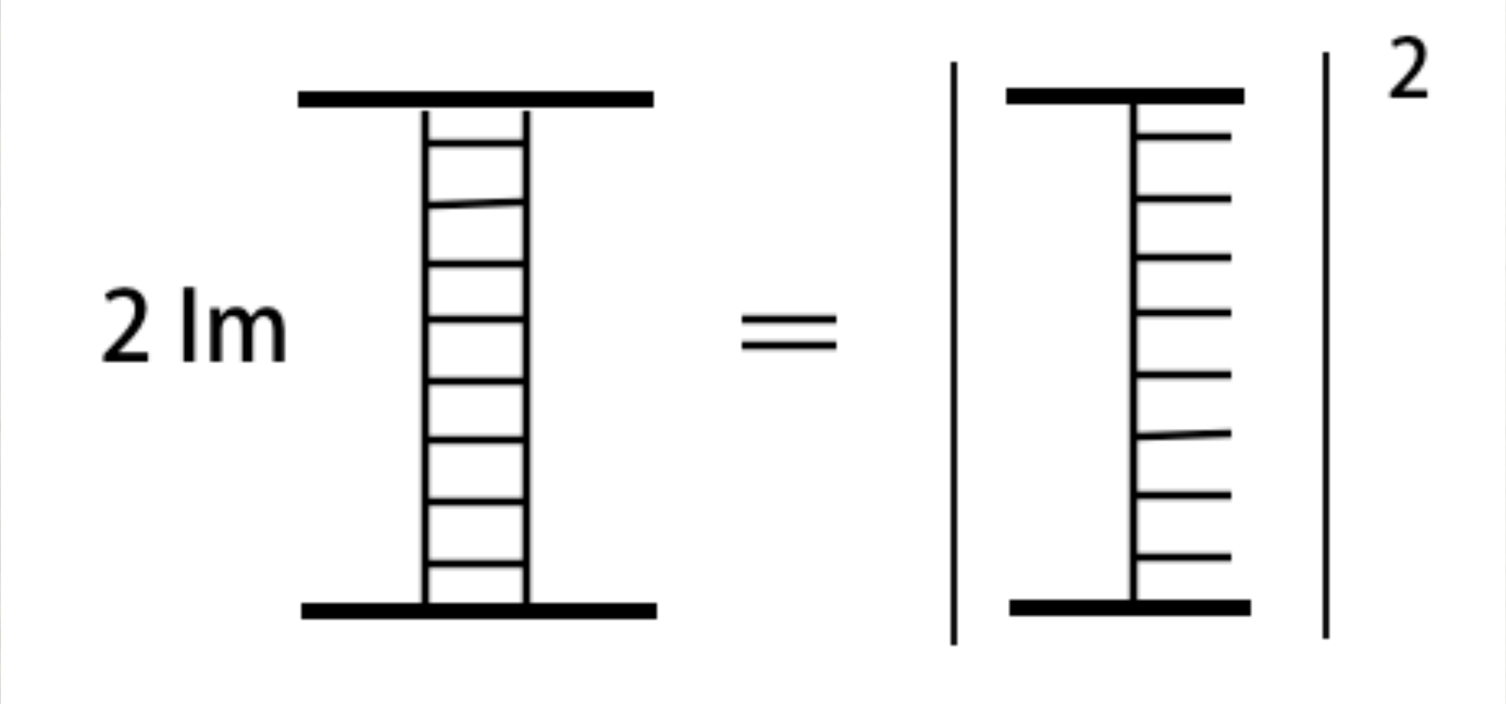


$\mathbf{M^{ext}}$ $\mathbf{M^{int}}$

The key result of the Low theorem is suppression of internal radiation.

Unitarity relation

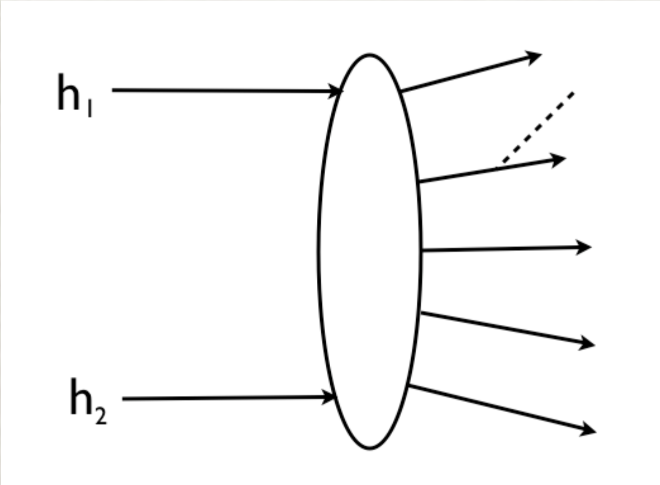
Optical theorem relates multi-particle production with the imaginary part of the forward elastic amplitude.



Thus, radiation by produced charged particles, within the "bremsstrahlung model", corresponds to the strongly suppressed internal radiation term $\mathbf{M}^{\mathbf{int}}$ of the Low precess.

On the contrary to the claim that the "bremsstrahlung model" is an extended version of the Low theorem, in fact it strictly contradicts it.

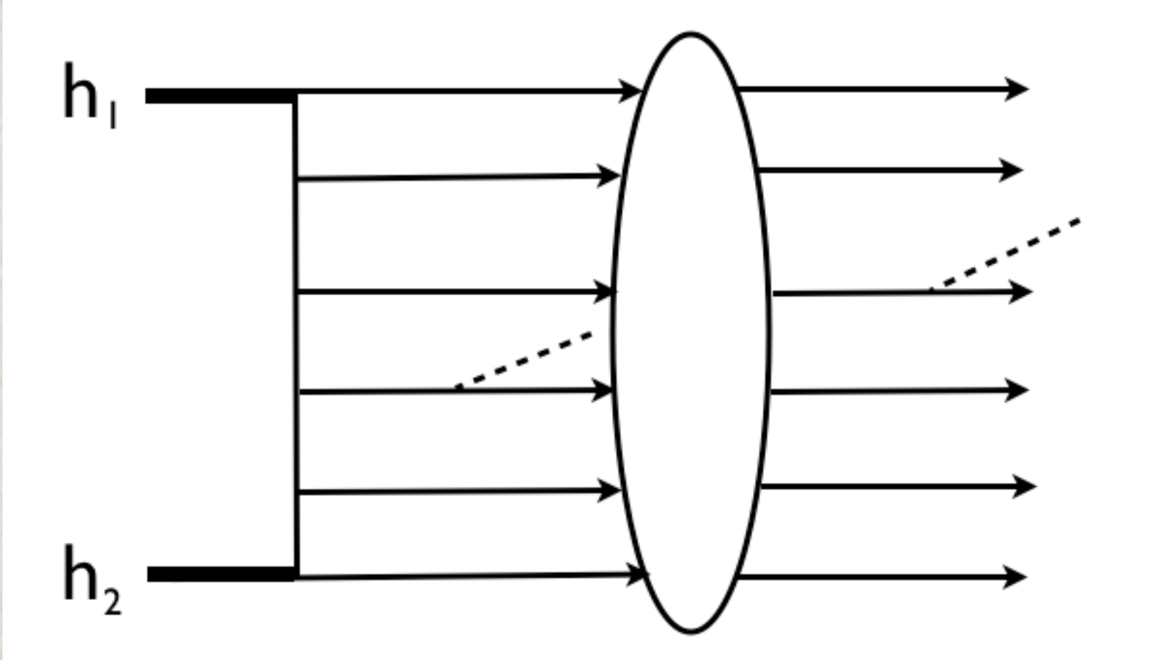
One might wonder what is wrong in the BM, why Feynman rules cannot be applied ?



?

The propagator in coordinate representation acquires an infinitely long radiation length.

$$\sum_{\mathbf{i}}^{\mathbf{charged}} \frac{\eta_\mathbf{i} \mathbf{e_i} \mathbf{p_i} \cdot \epsilon}{\mathbf{2 p_i} \cdot \mathbf{k}} \longrightarrow \frac{\mathbf{1}}{\mathbf{p^2 - m_h^2}} = \frac{\mathbf{x_1 (1 - x_1)}}{\mathbf{k_T^2 + x_1 m_h^2}} = \frac{\mathbf{l_c^\gamma}}{\mathbf{2E_1}}$$



Thus, the process cannot be treated as radiation either from the two incoming, or from N outgoing charges. All the produced partons pre-exist the interaction.

This explains the observed contradiction between the BM and Low/optical theorems.

As far as the bremsstrahlung model turns out to be incorrect, an alternative description of soft photon radiation is required.

BK, I.Potashnikova, M.Krelina, K.Reygers, 2212.03429

The usual factorization relation for photon radiation

$$\frac{d^4\sigma}{dM^2\,dx_F\,d^2k_T} = \frac{\alpha_{em}}{3\pi M^2}\frac{x_1}{x_1+x_2}\int_{x_1}^{1}\frac{d\alpha}{\alpha^2}\sum_f Z_f^2\left\{q_f\left(\frac{x_1}{\alpha}\right)+q_{\bar{f}}\left(\frac{x_1}{\alpha}\right)\right\}\frac{d\sigma(qN\to\gamma^* X)}{d\ln\alpha\,d^2k_T}$$

$$\alpha = \mathbf{p}_+^{\gamma}/\mathbf{p}_+^{\mathbf{q}}$$

$$\mathbf{x_1x_2 = M^2/s;\quad 2x_2 = \sqrt{x_F^2+4M^2/s}-x_F}$$

cannot be used in the soft domain of massless photons and small kT

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For the quark distribution function we rely on the Quark-Gluon String model (QGSM)

A. Kaidalov, Phys.Lett. B116(1982)459

A. Kaidalov and M. Poghosyan, *Eur.Phys.J.C* 67 (2010) 397

or Dual Parton Model

A. Capella, U. Sukhatme, C-I Tan, J. Tran Thanh Van, Phys.Rept. 236(1994)225

A.Capella, A.Kaidalov, C.Merino, J.Tran Thanh Van, Phys.Lett.B 337 (1994) 358

which are nearly the same, both based on Regge phenomenology

The radiation cross section $\frac{\mathbf{d}\sigma(\mathbf{qp}\to\gamma\mathbf{p})}{\mathbf{d}\ln\alpha\mathbf{d^2k_T}}$

is calculated within the color dipole approach

B.K., Hirschegg 1995 hep-ph/9609385

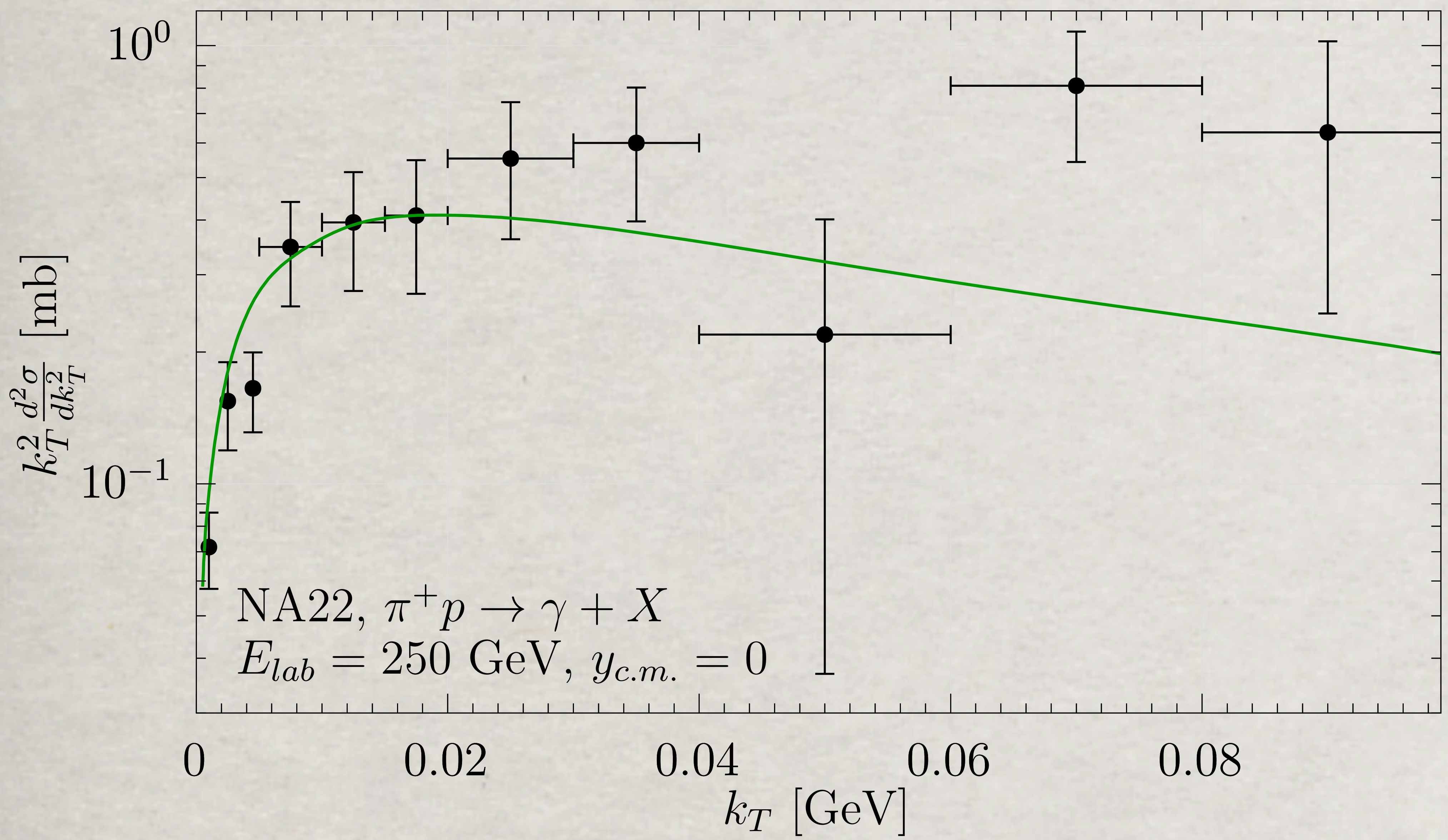
B.K., A.Schäfer, A.Tarasov,
Phys.Rev.C 59(1999)1609;
Phys.Rev.D 62 (2000) 054022

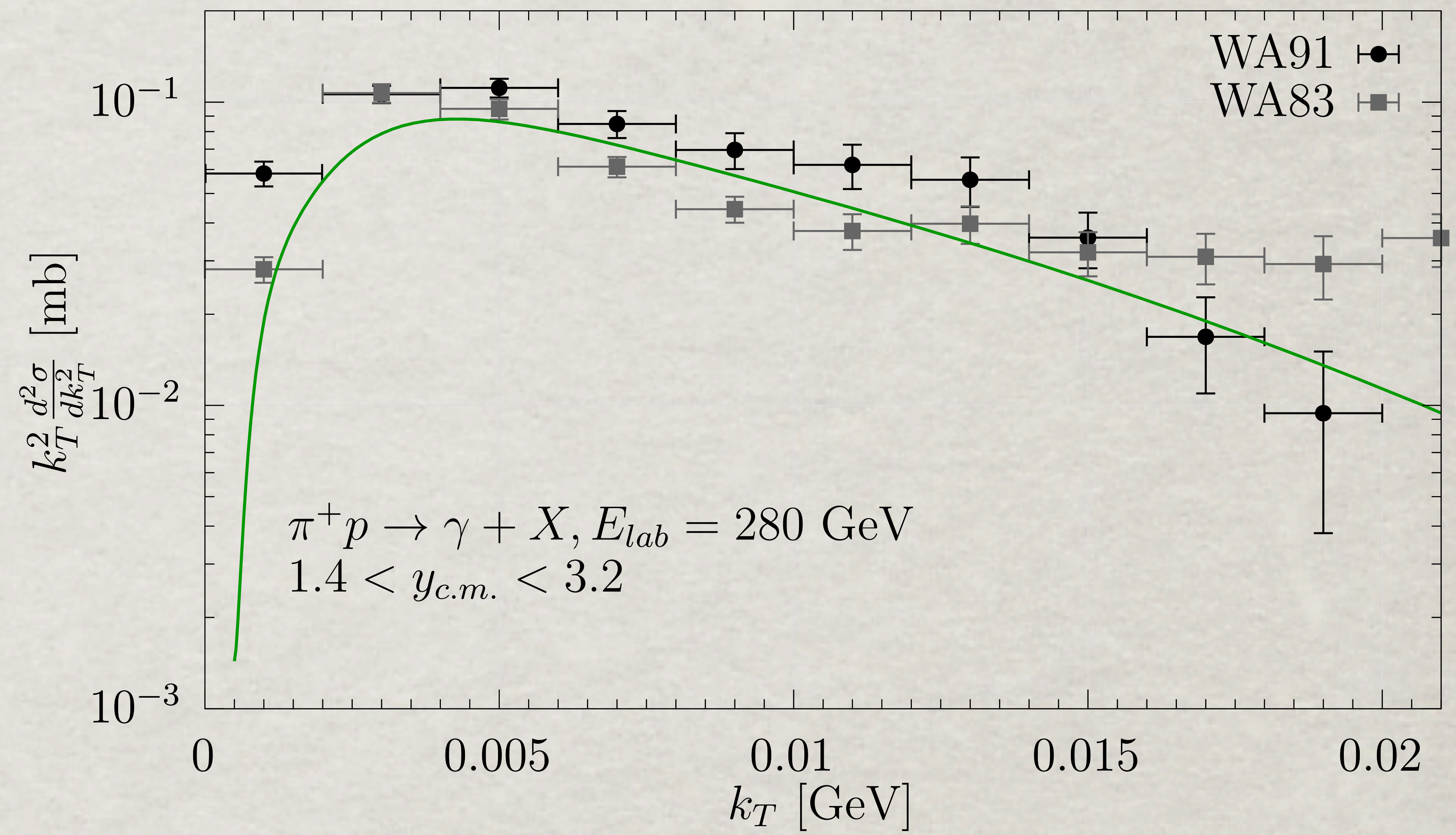
B.K., J.Raufeisen and A.Tarasov, Phys.Lett. B503(2001)91

B.K., A.Rezaeian, H-J.Pirner, I.Schmidt, Phys.Lett. B653(2007)210

B.K., E.Levin, A.Rezaeian, I.Schmidt, Phys.Lett. B675(2009)190

Data

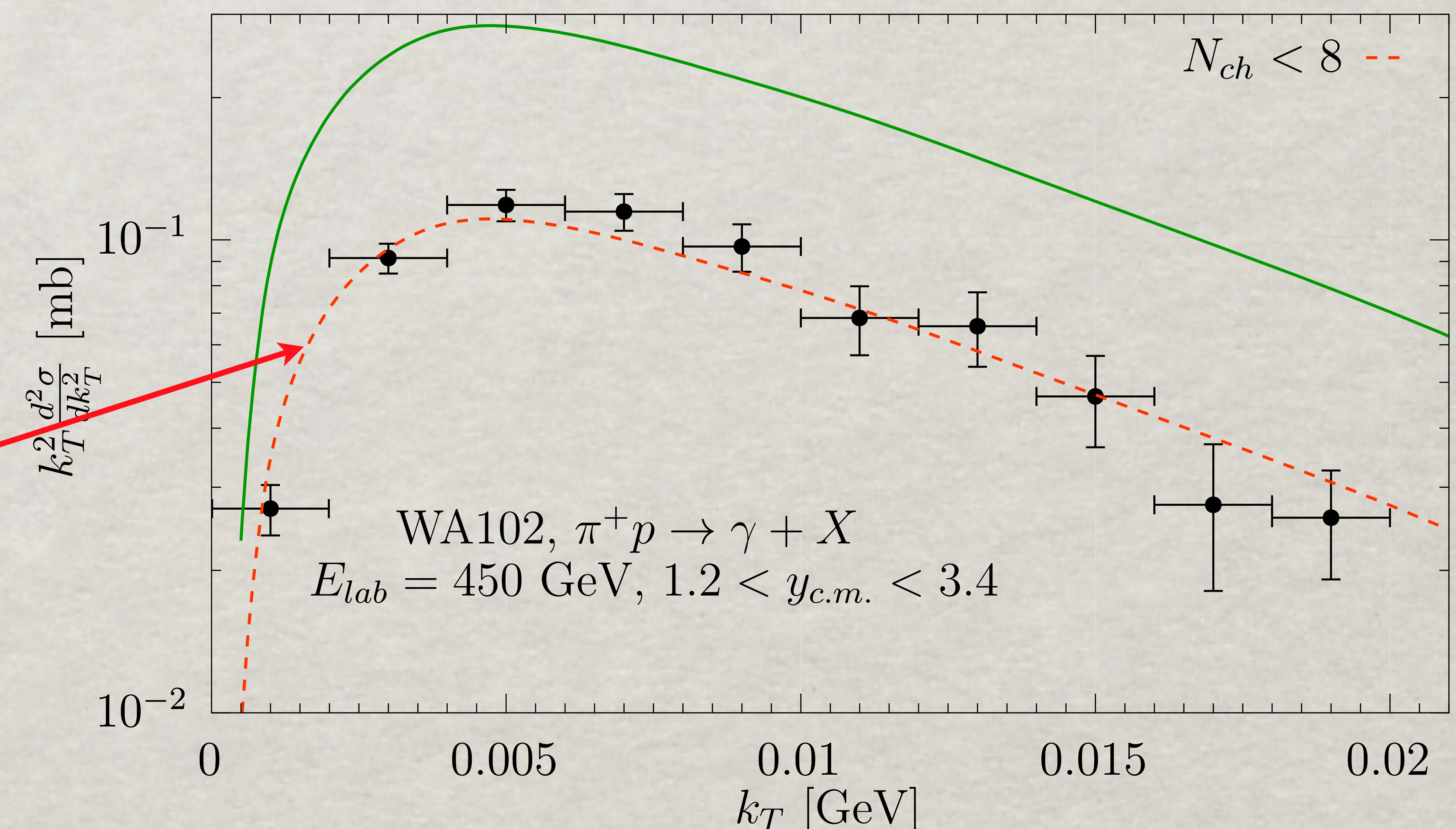




The primordial qT distribution of the quarks was taken into account: $\mathbf{\tilde{k}'_T} = \mathbf{\tilde{k}_T} - \alpha \mathbf{\tilde{q}_T}$

WA102 had a cut for number of charged tracks $\mathbf{N_{ch} < 8}$

That leads to the suppression factor 0.39 (QGSM)



Conclusions

The observed enhancement of low-kT photons in comparison with incorrect calculations, should not be treated as a puzzle.

Low's paper considered a large rapidity gap process of diffractive photon radiation $\mathbf{h} \to \mathbf{h} + \gamma$, which has little to do with multiple production of hadrons spanning all over the rapidity interval between colliding hadrons.

According to the unitarity relation what final state external radiation corresponds to internal radiation in the elastic amplitude, which was proven by Low to be suppressed

Radiation of small-kT photons takes long time $\mathbf{l}^{\gamma} \propto \mathbf{1}/\mathbf{k_T}$, so they cannot appear momentarily from the interaction blob, as is assumed in the Bremsstrahlung Model.

Calculations based on QGSM and color dipole description of photon radiation well agree with data.