# Neutral pion screening mass in a magnetized medium

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# Overview

- Introduction (pole and screening masses)
- LQCD and NJL results for the screening mass of the neutral pion
- Linear sigma model with quarks (LSMq)
- Calculation of the neutral pion self-energy in the LSMq.
- Results
- Summary and perspectives

# Pole and screening masses ($B \neq 0$, $T = 0$)

Interactions $\rightarrow$ mass modifications (parametrized in the self energy)

- Pole mass

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_3^2 = p_\perp^2 = 0} = 0$$

- Screening mass $B$ breaks Lorentz invariance and defines $\parallel$ and $\perp$
  - Longitudinal

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2 = p_\perp^2 = 0} = 0$$

  - Transverse

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2 = p_3^2 = 0} = 0$$

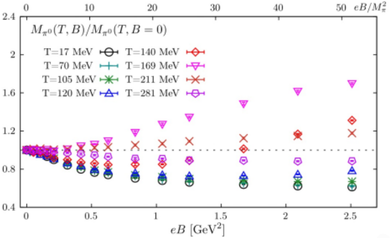
Relations:

$$m_{\pi^0, scr.\perp} = \frac{m_{\pi^0, pole}}{u_\perp}$$

$$m_{\pi^0, scr.\parallel} = \frac{m_{\pi^0, pole}}{u_\parallel}$$

$$m_{\pi^0, pole} = m_{\pi^0, scr.\parallel} < m_{\pi^0, scr.\perp}$$

# Comparison with the NJL model



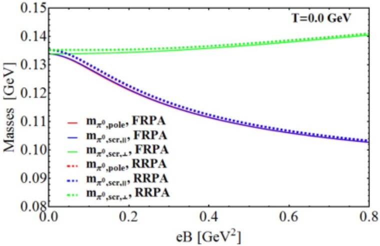




Figure: H. T. Ding, S. T. Li, J. H. Liu, and X. D. Wang, Phys. Rev D105, 034514 (2022), and B. Sheng, Y. Wang, X. Wang, and L. Yu, Phys. Rev. D103 (2021) 9, 094001.

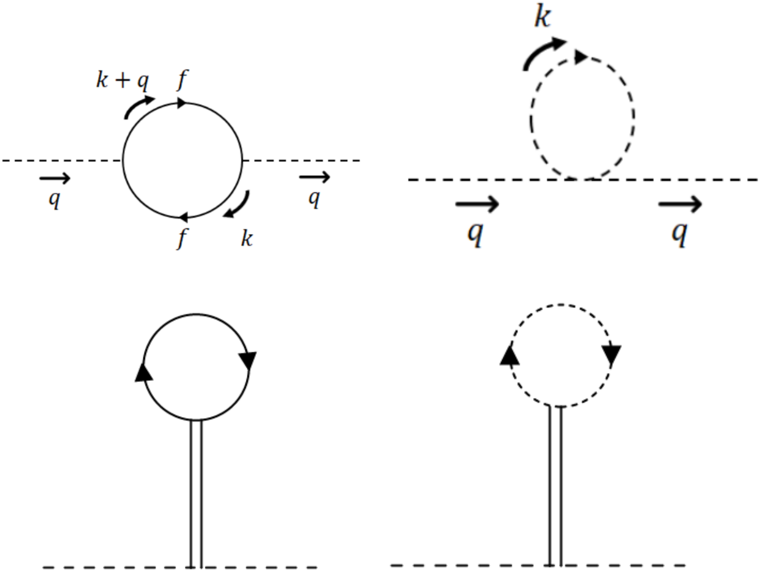
# LSMq Lagrangian and features

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 + i\bar{\psi}\gamma^\mu \partial_\mu \psi$$
$$- ig\bar{\psi}\gamma^5\bar{\psi}\vec{\tau} \cdot \vec{\pi}\psi - g\bar{\psi}\psi\sigma\,.$$

- it implements the SSB of: $SU(2)_L \times SU(2)_R \to SU(2)_V$.
- $m_\pi(v) = \sqrt{\lambda v^2 - a^2} = 0$ at VEV.
- $m_f(v) = gv$.
- $m_\sigma(v) = \sqrt{3\lambda v^2 - a^2}$

$\mathcal{L} \to \mathcal{L} + h(\sigma + v)$ in order to give the correct vacuum pion mass.

# Relevant Feynman diagrams



# Feynman rules for the fermionic contribution to $\pi_{f\bar{f}}$ (vertices and propagator)

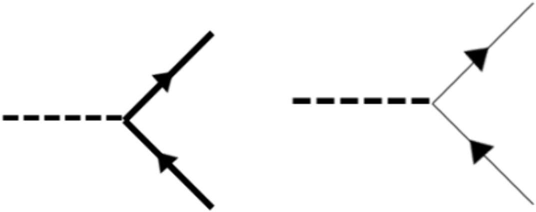


Figure: $\pi_0$-$u$ quark vertex=g$\gamma^5$; $\pi_0$-$d$ quark vertex=-g$\gamma^5$

$$S(p) = \int_0^\infty \frac{ds}{\cos(qBs)} \exp\left[ is\left( p_\parallel^2 - p_\perp^2 \frac{\tan(qBs)}{qBs} - m_f^2 + i\epsilon \right)\right]$$

$$\times \left\{ (m_f + \not{p}_\parallel)\Big( \cos(qBs) + \gamma^1\gamma^2 \sin(qBs) \Big) - \frac{\not{p}_\perp}{\cos(qBs)} \right\}$$

$$S(p) \to i\frac{(m_f + \not{p})}{p^2 - m_f^2 + i\epsilon}$$

# Neutral pion self-energy (fermion contribution)

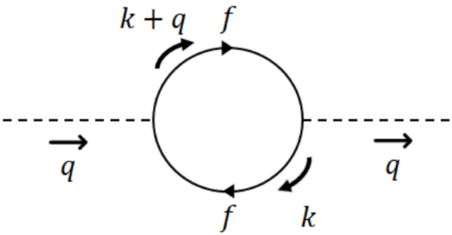



Figure: Neutral pion self-energy

$$-i\pi_{f\bar{f}} = -g^2 \int \frac{d^4k}{(2\pi)^4} Tr\left[\gamma^5 S(k)\gamma^5 S(k+q)\right] + c.c..$$

# Neutral pion self-energy (fermion contribution)

$$
-i\pi_{f\bar{f}} = -4g^2 \int_0^\infty \int_0^\infty \frac{ds ds'}{\cos(qBs)\cos(qBs')}
$$

$$
\times \int \frac{d^4k}{(2\pi)^4} e^{is\left(k_\parallel^2 - k_\perp^2 \frac{\tan(qBs)}{qBs} - m_f^2 + i\epsilon\right)} e^{is'\left((k+p)_\parallel^2 - (k+p)_\perp^2 \frac{\tan(qBs')}{qBs'} - m_f^2 + i\epsilon\right)}
$$

$$
\left\{ \cos[qB(s+s')][m_f^2 - k_\parallel \cdot (k+p)_\parallel] + \frac{k_\perp \cdot (k_\perp + p_\perp)}{\cos(qBs)\cos qBs'} \right\}
$$

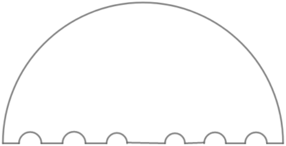
# Analysis

$$f(p_0, p_\perp, p_\parallel, B) = \pi_{f\bar{f}} - \lim_{B\to 0} \pi_{f\bar{f}}$$

We start with the simplest case ($p_\perp^2 = p_0^2 = 0$). This is the 'longitudinal' screening mass which is found by solving the equation:

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2 = p_\perp^2 = 0} = 0$$

# Final expression



$$\Re f = lim_{\epsilon\to 0}\Bigg( - \frac{4g^2}{(4\pi)^2}\int_0^1 dv \Bigg\{ \Big( - 2v(1-v)p_3^2 \Big)$$

$$\times \left[ \frac{\pi}{2}\frac{\sin(\frac{a\pi}{qB})}{\cosh(\frac{\epsilon\pi}{qB}) - \cos(\frac{a\pi}{qB})} - \tan^{-1}\left( \frac{\sin(\frac{a\pi}{qB})e^{\left(\frac{-\epsilon\pi}{qB}\right)}}{1 - e^{\left(\frac{-\epsilon\pi}{qB}\right)}\cos\left(\frac{a\pi}{qB}\right)} \right) \right]$$

$$+ qB\left[ \frac{\epsilon\pi}{2qB} - \ln\sqrt{2\cosh\left(\frac{\epsilon\pi}{qB}\right) - 2\cos\left(\frac{a\pi}{qB}\right)} \right]$$

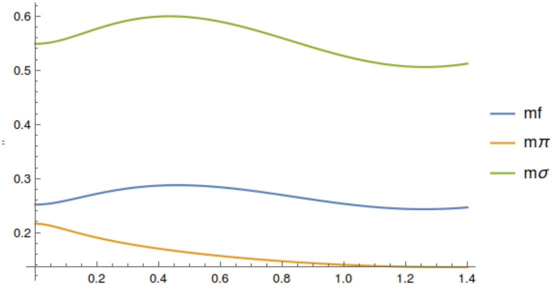
$$- \frac{qB}{\pi}\left[ \Re\Big( Li_2\Big[ e^{-(ia+\epsilon)\frac{\pi}{qB}} \Big] \Big) \right] \Bigg\} \Bigg)$$

# Parameters

Parameters at zero magnetic field

$$g = \frac{m_f}{v} \approx 1.5; \quad \lambda = \frac{m_\sigma^2 - m_\pi^2}{2f_\pi^2} \approx 10$$

Magnetic field dependence of masses

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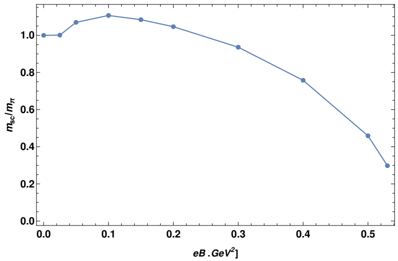
[1] S. S. Avancini, R. Farias, M. B. Pinto, W. R. Tavares, Phys. Letters B 767 (2017) 247-252.

# Fermionic contribution with fixed g



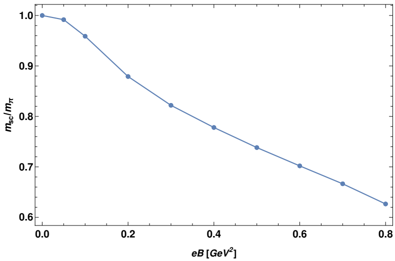
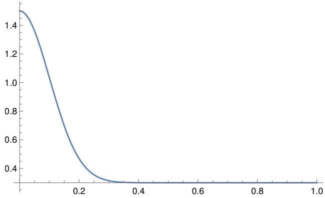




Figure: 'Longitudinal' SC mass as function of $B$, $g = 1.5$ (left). 'Longitudinal' SC mass as function of $B$, $g_{eff}(B) = 0.3 + 1.2\,\exp[-(7B)^2]$(right).



# Fermionic contribution + Tadpoles

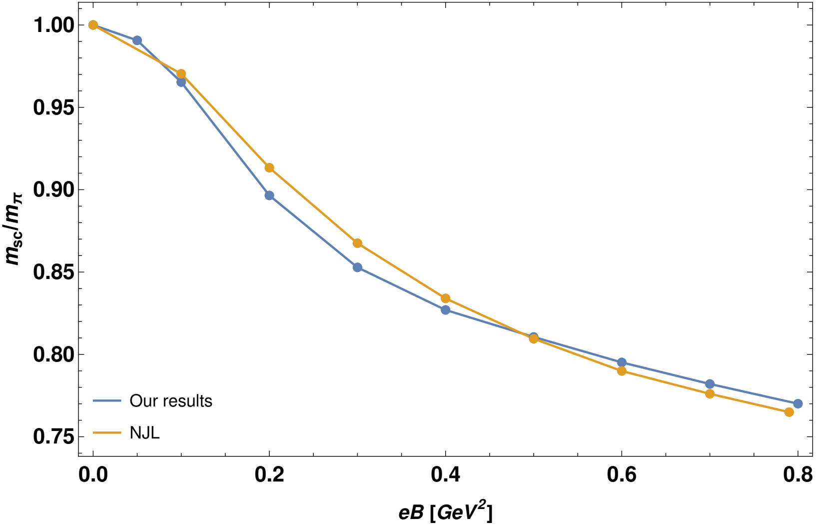



Figure: 'Longitudinal' Screening mass as a function of $B$, $g_{eff}(B) = 0.3 + 1.2\,\exp[-(7B)^2], \lambda = 1.5$.

# Summary and perspectives

Summary:

- We have calculated the neutral pion self-energy in the LSMq.
- We have obtained the 'longitudinal' screening mass as a function of $B$.
- We have compared our results with LQCD and NJL, and we have found a nice agreement only when we have a magnetic field dependence on the couplings and masses.

Perspectives:

- We will study the 'transverse' screening mass as a function of $B$.
- We will study the case where $T \neq 0$.

Thank You

# Important integrals and convenient change of variables

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$$

$$u = s + s'$$

$$s = u(1 - v)$$

$$s' = uv$$

$$\frac{\partial(s, s')}{\partial(u, v)} = u$$

# Neutral pion self-energy (fermion contribution)

$$\pi_{\bar{f}f} = \frac{-4g^2 qB}{(4\pi)^2} \int_0^1 dv \int_0^\infty du \exp(-ix) \exp(-u\epsilon) \left[ \frac{m_f^2}{\tan(qBu)} \right.$$
$$- \frac{qB}{\sin^2(qBu)} \left( \frac{p_\perp^2}{|qB|} \frac{\sin(qBu(1-v)) \sin(qBuv)}{\sin(qBu)} \right) - \frac{v(1-v)(p_3^2 - p_0^2)}{\tan(qBu)}$$
$$\left. - \frac{iqB}{\sin(qBu)} - \frac{i}{u \tan(qBu)} \right]$$

where $x$ is given by:

$$x = \frac{p_\perp^2}{qB} \frac{\sin(qBu(1-v)) \sin(qBuv)}{\sin(qBu)} + p_3^2 uv(1-v) - p_0^2 uv(1-v) + m_f^2 u$$

# $B \to 0$ limit of $\pi_{\bar{f}f}$

$$\lim_{B\to 0} \pi_{\bar{f}f} = -\frac{4g^2}{(4\pi)^2}\int_0^1 dv \int_0^\infty \frac{du}{u} e^{-ix_0} e^{-u\epsilon}$$
$$\times \left\{ m_f^2 - v(1-v)(p_\perp^2 - p_3^2) + v(1-v)p_0^2 - \frac{2i}{u} \right\}$$

where

$$x_0 = uv(1-v)(p_\perp^2 + p_3^2) - p_0^2 uv(1-v) + m_f^2$$

# Fermionic contribution with $g_{eff}$ as a function of $B$

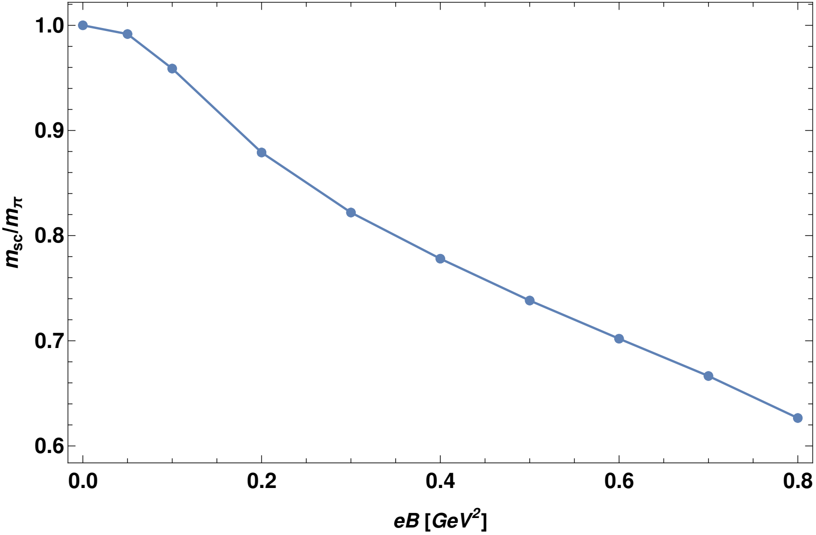



Figure: 'Longitudinal' Screening mass as a function of $B$, $g_{eff}(B) = 0.3 + 1.2\exp[-(7B)^2]$.

# Fermionic contribution + Tadpoles

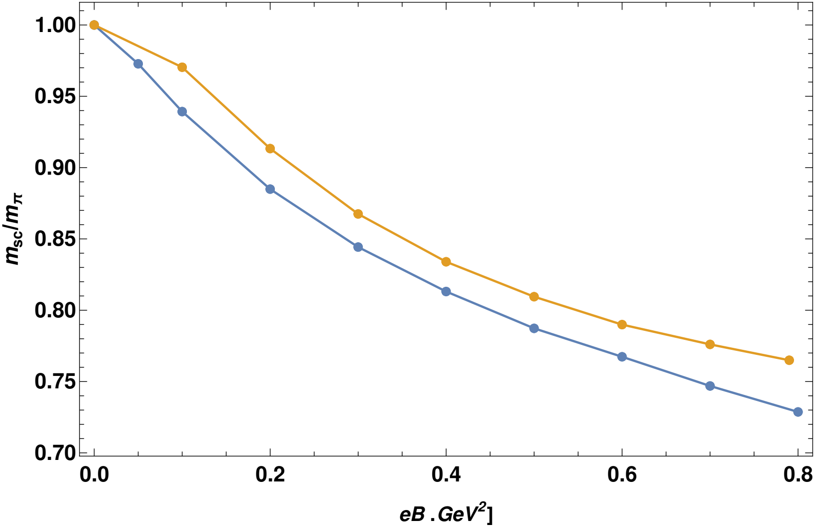



Figure: 'Longitudinal' Screening mass as a function of $B$ vs NJL $g_{eff}(B) = 0.3, \lambda = 1.$

# Preliminary results

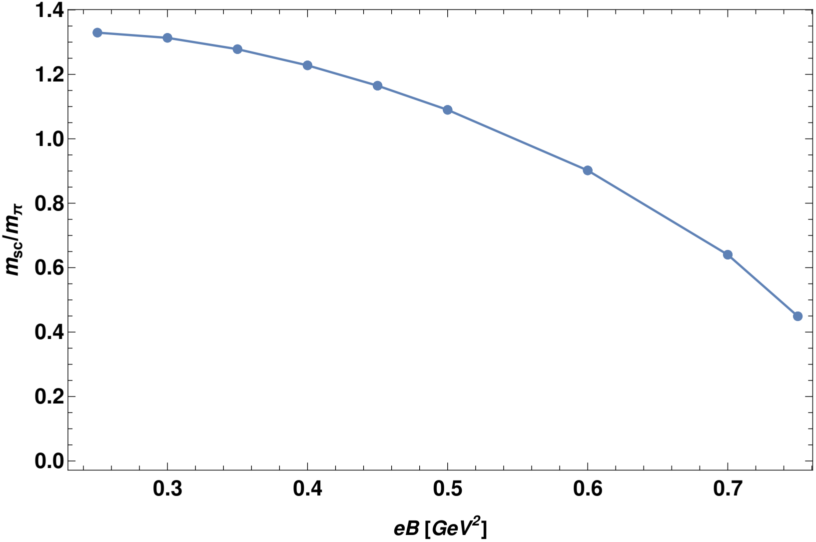



Figure: 'Longitudinal' Screening mass as a function of $B$.