Status of MUonE theory

F. Piccinini

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Sixth Plenary Workshop of the Muon $g-2$ Theory Initiative

Bern, 4-8 September 2023

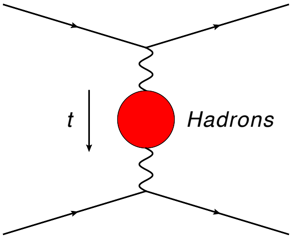
- ⋆ G. Abbiendi, C.M. Carloni Calame, U. Marconi, C. Matteuzzi, G. Montagna, O. Nicrosini, M. Passera, F. Piccinini, R. Tenchini, L. Trentadue, G. Venanzoni,
  Measuring the leading hadronic contribution to the muon g-2 via μe scattering
  Eur. Phys. J. C **77** (2017) no.3, 139 - arXiv:1609.08987 [hep-ph]
- ⋆ C. M. Carloni Calame, M. Passera, L. Trentadue and G. Venanzoni,
  A new approach to evaluate the leading hadronic corrections to the muon g-2
  Phys. Lett. B **746** (2015) 325 - arXiv:1504.02228 [hep-ph]

Master formula

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

e.g. Lautrup, Peterman, De Rafael, Phys. Rept. 3 (1972) 193



⇝ The hadronic VP correction to the running of α enters

- ⋆ **$\Delta\alpha_{\rm had}(t)$ can be directly measured in a (single) experiment involving a space-like scattering process and $a_\mu^{\rm HLO}$ obtained through numerical integration**

  Carloni Calame, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

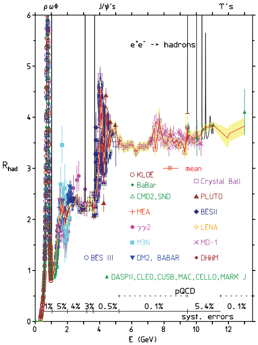
- ⋆ **A data-driven evaluation of $a_\mu^{\rm HLO}$, but with space-like data**

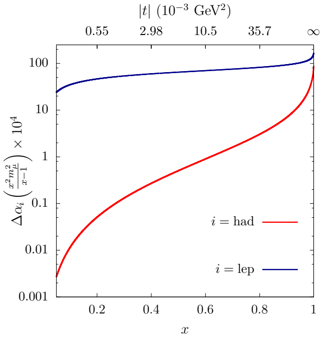
  - By modifying the kernel function $\frac{\alpha}{\pi}(1-x)$, also $a_\mu^{\rm HNLO}$ and $a_\mu^{\rm HNNLO}$ can be provided

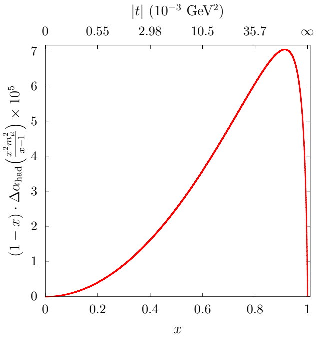
    Balzani, Laporta, Passera, Phys.Lett.B834 (2022) 137462, Nesterenko, J.Phys.G 49 (2022) 055001

From time-like to space-like evaluation of a_μ^{HLO}

Time-like $\mapsto$ **Space-like**







Smooth function

- $\mapsto$ **Time-like:** combination of many experimental data sets, control of RCs better than $\mathcal{O}(1\%)$ on hadronic channels required
- $\mapsto$ **Space-like:** in principle, one single experiment, *it's a one-loop effect, very high accuracy needed*

MUonE: $\mu e \to \mu e$ @CERN

Abbiendi et al., EPJC 77 (2017) 3, 139

Abbiendi et al., *Letter of Intent: the MUonE project, CERN-SPSC-2019-026, SPSC-I-252 (2019)*

- Scattering μ's on e's in a low Z target looks like an ideal process (fixed target experiment)
- **It is a pure t-channel process at tree level**
- **The M2 muon beam ($E_\mu \simeq 160$ GeV) is available at CERN**
- $\sqrt{s} \simeq 0.42$ GeV and $-0.153 < t < 0$ GeV^2
- With ~ 3 years of data taking, a statistical accuracy of $\sim 0.3\%$ on $a_\mu^{\rm HLO}$ $(\sim 20 \cdot 10^{-11})$ can be achieved

Requirement: systematics on ratio of cross sections in the signal and normalization regions at 10 ppm level

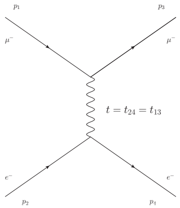
Main sources of systematics on the theory side

- **Radiative corrections**

- **Background processes**

High precision Monte Carlo simulation tools required

First step towards precision: QED NLO and MC (2018)

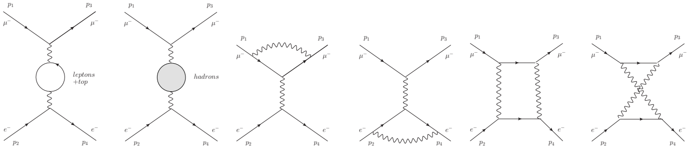


- analytical expression for tree level

$$\frac{d\sigma}{dt} = \frac{4\pi\alpha^2}{\lambda(s, m_\mu^2, m_e^2)} \left[\frac{(s - m_\mu^2 - m_e^2)^2}{t^2} + \frac{s}{t} + \frac{1}{2} \right]$$

- VP gauge invariant subset of NLO rad. corr.
- factorized over tree-level: $\alpha \to \alpha(t)$

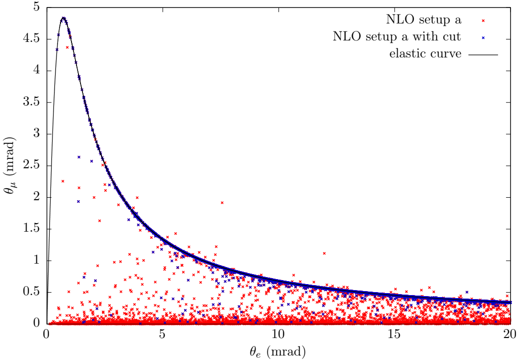
- QED NLO virtual diagrams and real emission diagrams with exact finite m_e and m_μ effects

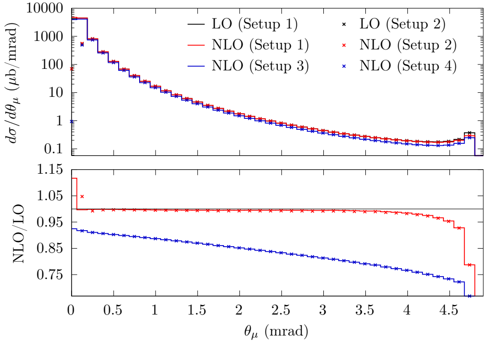


- **tree-level Z-exchange important** at the 10^{-5} level ($\sim tG_\mu/4\pi\alpha\sqrt{2}$ in the Fermi theory)
- SM weak RCs at most at a few 10^{-6} level, negligible

Alacevich et al. JHEP 02 (2019) 155

First realistic description of scattering events





- many points fall out of the $2 \to 2$ correlation curve $\theta_\mu - \theta_e$ because of the radiative events
- NLO QED radiative corrections at the % level, enhanced by exclusive event selections

Second step, towards *photonic* radiative corrections at **NNLO** (2020)

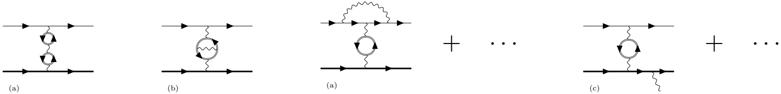
- **exact calculation of corrections along one lepton line with all finite mass effects**
  - two independent calculations, with different subtraction procedures — Carloni Calame et al., JHEP 11 (2020) 028, P. Banerjee, T. Engel, A. Signer, Y. Ulrich, SciPost Phys. 9 (2020) 027
  - implemented in **`Mesmer`** and **`McMule`**, perfect numerical agreement
- **NNLO with finite mass effects and approximate up-down interference in `Mesmer`**
  - interference of LO $\mu e \to \mu e$ amplitude with

⇝ NNLO double-virtual amplitudes where at least 2 photons connect the e and μ lines are approximated according to the Yennie-Frautschi-Suura ('61) formalism to catch the infra-red divergent structure

- **complete calculation of the amplitude $f^+f^- \to F^+F^-$ with $m_f = 0$, $m_F \neq 0$** — R. Bonciani *et al.*, PRL 128 (2022)
- **"massification" to recover the leading m_e terms** — T. Engel, C. Gnendiger, A. Signer and Y. Ulrich, JHEP 02 (2019) 118
- **NNLO approximate calculation which includes leading log $\propto \ln(m_e^2/Q^2)$ and m_e^0 terms in `McMule`**

A. Broggio et al., JHEP 01 (2023) 112

- any lepton (and hadron) in the VP blobs
- interfered with $\mu e \to \mu e$ or $\mu e \to \mu e \gamma$ amplitudes



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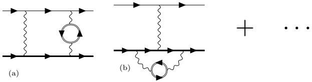


- 2-loop integral evaluated with dispersion relation techniques in **Mesmer**

used e.g. in the past for Bhabha: Actis et al., Phys. Rev. Lett. 100 (2008) 131602; Carloni Calame et al., JHEP 07 (2011) 126

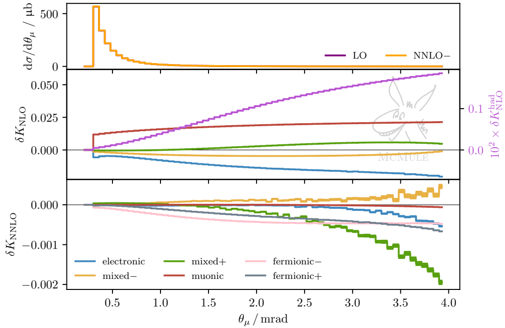
$$\frac{g_{\mu\nu}}{q^2+i\epsilon} \to g_{\mu\nu}\frac{\alpha}{3\pi}\int_{4m_\ell^2}^{\infty}\frac{dz}{z}\frac{R_\ell(z)}{q^2-z+i\epsilon} = g_{\mu\nu}\frac{\alpha}{3\pi}\int_{4m_\ell^2}^{\infty}\frac{dz}{z}\frac{1}{q^2-z+i\epsilon}\left(1+\frac{4m_\ell^2}{2z}\right)\sqrt{1-\frac{4m_\ell^2}{z}}$$

- 2-loop integral evaluated with hyperspherical method in **McMule**

M. Fael, JHEP02 (2019) 027

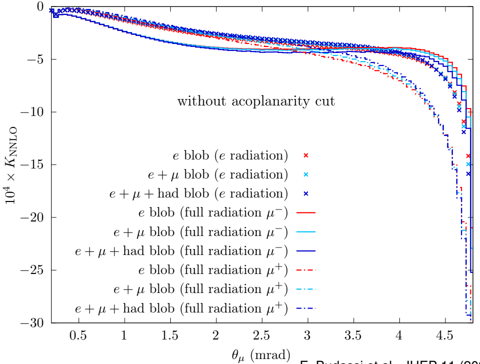
NNLO order of magnitude

McMule



M. Rocco, IV MUonE General Meeting, 16-17/05/2023 A. Broggio et al., JHEP 01 (2023) 112

Mesmer



E. Budassi et al., JHEP 11 (2021) 098

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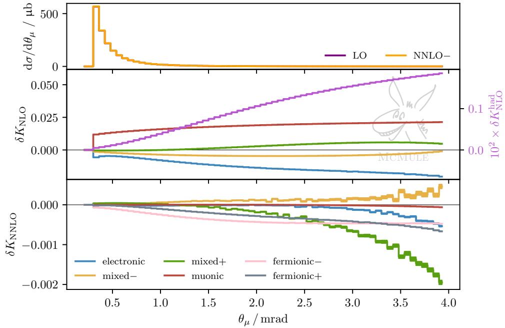

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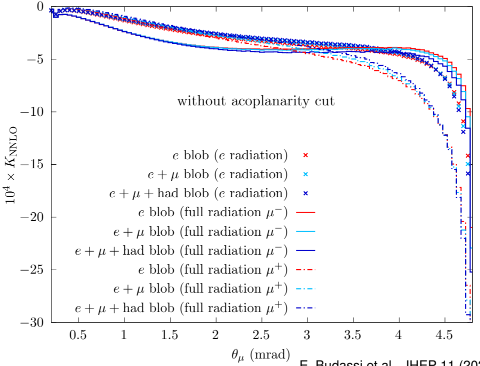


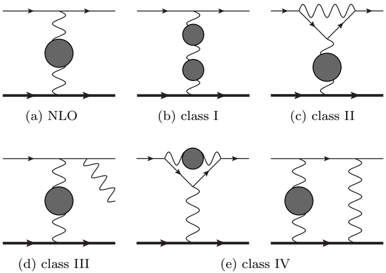
M. Rocco, IV MUonE General Meeting, 16-17/05/2023 A. Broggio et al., JHEP 01 (2023) 112

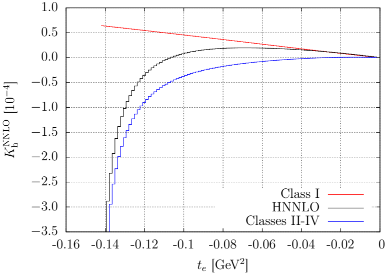
E. Budassi et al., JHEP 11 (2021) 098

- **NNLO corrections at the $10^{-4} - 10^{-3}$ level**
- **fixed order calculations need to be matched to resummation of higher order corrections, through Parton Shower techniques (e.g. BaBayaga) or YFS techniques (e.g. KKMC/SHERPA)**

NNLO hadronic contributions (2019)

- using the dispersion relation approach





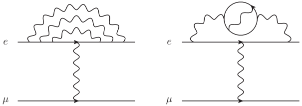
Fael, Passera, Phys. Rev. Lett. 122 (2019) 192001

- corrections of the order of 10^{-4}
- hyperspherical integration method to calculate hadronic NNLO corrections, where the hadronic vacuum polarization is employed in the space-like region (used in **McMule**)

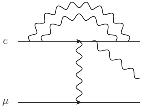
M. Fael, JHEP02 (2019) 027

Towards N^{3}LO on the electron line

- All virtual (three loops)



- Single real emission (two loops)



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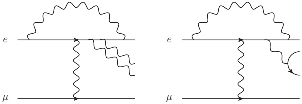
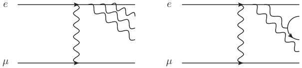


- Triple real



M. Fael, MUonE Collaboration Meeting, 16/05/2023, CERN

- **this contribution will allow improved perturbative predictions and more reliable theoretical uncertainty estimates**

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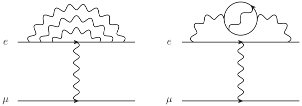

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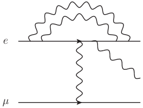

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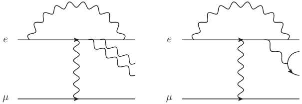

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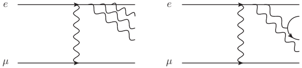

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- **this contribution will allow improved perturbative predictions and more reliable theoretical uncertainty estimates**
- **the three-loop form factor with finite fermion mass is now available**

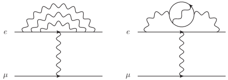
M. Fael, F. Lange, K. Schönwald, M. Steinhauser, Phys.Rev.D 107 (2023) 094017

N^{3}LO kick-off workstop/thinkstart

Y. Ulrich et al., 3-5 August 2022, IPPP Durham

- KFS^3 subtraction of IR divergences
  Engel, Signer, Ulrich, JHEP 01 (2020) 085

- All virtual (three loops) ✓



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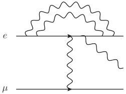

  - $\gamma^\star \to 3j$
    Gehrmann, Remiddi, Nucl.Phys.B 601 (2001) 248
  - Difficult with $m_e \neq 0$

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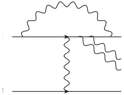
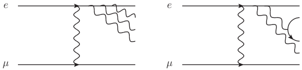


  - OpenLoops
  - NTS Stabilisation
    Engel, Signer, Ulrich, JHEP 04 (2022) 097

- Triple real



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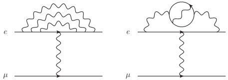

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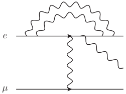

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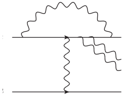

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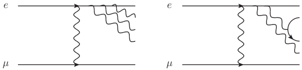

M. Fael, MUonE Collaboration Meeting, 16/05/2023, CERN

- very recent calculation of NNLO QED heicity amplitudes for $0 \to \ell\bar{\ell}\gamma\gamma^*$ with $m_\ell = 0$

S. Badger, J. Krys, R. Moodie, S. Zoia, arXiv:2307.03098

Backgrounds

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- **pion pair production forbidden** kinematically with the available $\sqrt{s}$

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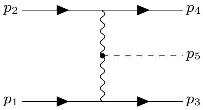
- **single π^0 production possible**



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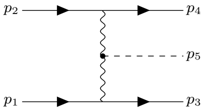

- π^0 **production** calculated and shown to be **well below 10^{-5}** w.r.t. $\mu e \to \mu e$

E. Budassi et al., PLB 829 (2022) 137138

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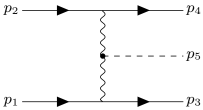

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E. Budassi et al., PLB 829 (2022) 137138

- **lepton pair production**

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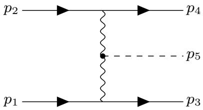

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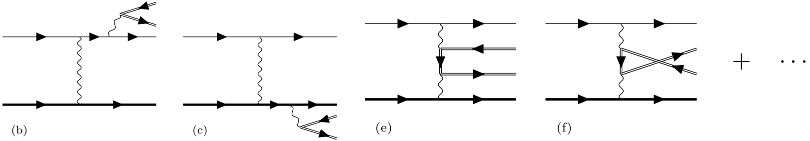
E. Budassi et al., PLB 829 (2022) 137138

- **lepton pair production**
  - $\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$
  - $\mu^\pm N \to \mu^\pm N \ell^+ \ell^-$

$\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$ (same pert. order as virtual pairs)

E. Budassi et al., JHEP 11 (2021) 098

- it also contributes at NNLO accuracy



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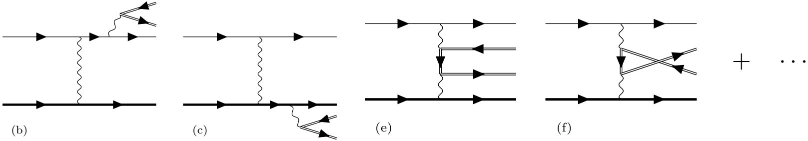

- the emission of an extra electron pair $\mu e \to \mu e\, e^+ e^-$ is potentially a dramatically large background, because of the presence of "peripheral" diagrams which develop powers of collinear logarithms upon integration

G. Racah, Il Nuovo Cimento 14 (1937) 83-113; L.D. Landau, E.M. Lifschitz, Phys. Z. Sowjetunion 6 (1934) 244; H.J. Bhabha, Proc. Roy. Soc. Lond. A152 (1935) 559; R.N. Lee, A.A. Lyubyakin, V.A. Smirnov, arXiv:2309.02904[hep-ph]

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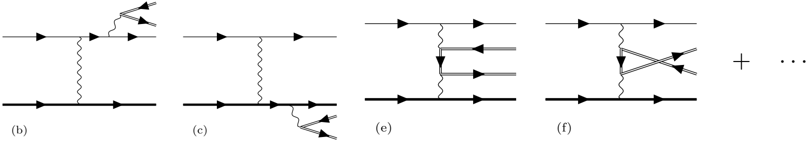

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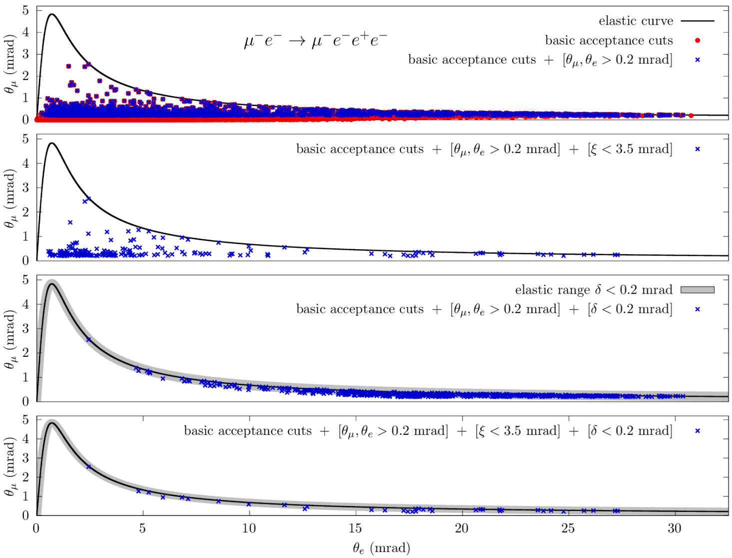
- **$\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$ calculated with finite mass effects and implemented in `Mesmer`**

simulation of $5 \cdot 10^5$ points of $\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$



Real pair emission from scattering on nucleus: $\mu^\pm N \to \mu^\pm N \ell^+ \ell^-$

Pavia, work in progress

- **it can mimic the signal if one particle is not reconstructed or two tracks overlap within resolution**
- **cross section scaling $\sim Z^2$**

Real pair emission from scattering on nucleus: $\mu^\pm N \to \mu^\pm N \ell^+ \ell^-$

Pavia, work in progress

- **it can mimic the signal if one particle is not reconstructed or two tracks overlap within resolution**
- **cross section scaling $\sim Z^2$**

- **GEANT4: "for the process of e^+e^- pair production the muon deflection is neglected"**

A.G. Bogdanov et al., IEEE transactions on nuclear science, 53, n. 2, April 2006

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$\Longrightarrow$ **we need a dedicated calculation and Monte Carlo generator**

- **approximation: scattering on the external nucleus field**

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$$F_Z(q) = \frac{1}{Ze} \int_0^\infty dr\, r^2 \rho_Z(r) \frac{\sin(qr)}{qr}$$

  - q : momentum transferred to the nucleus
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- **different models for charge density**

J. Heeck, R. Szafron, Y. Uesaka, PRD 105 (2022) 053006

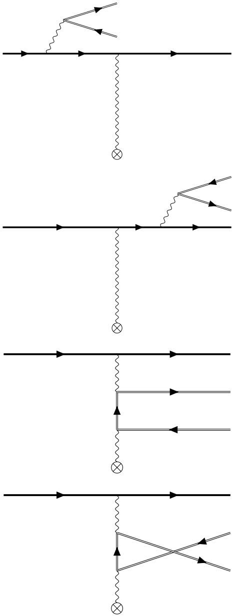
  - $F_Z(q) = 1$ (conservative)
  - 1 parameter Fermi model (1pF)

$$\rho_Z(r) = \frac{\rho_0}{1 + \exp\frac{r-c}{z}}$$

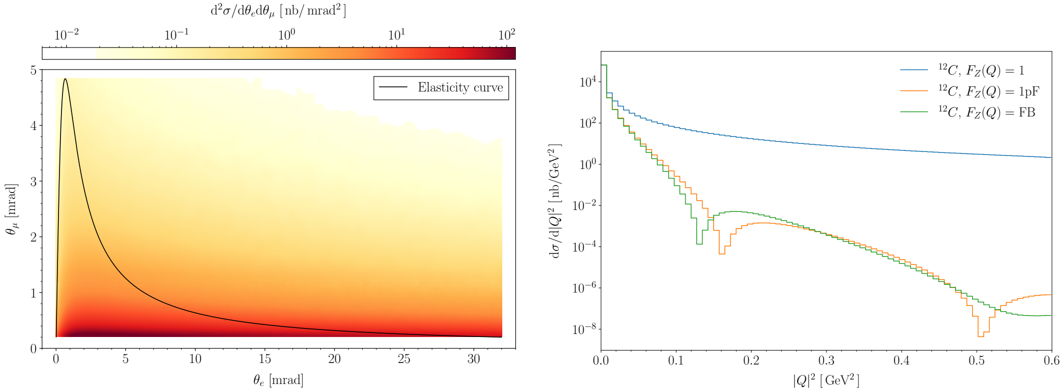
  - Fourier Bessel expansion (FB)

$$\begin{aligned} \rho_Z(r) &= \sum_k^n a_k\, j_0\left(\frac{k\pi r}{R}\right), \quad r \geq R \\ &= 0 \qquad\qquad\qquad\qquad > R \end{aligned}$$

  - modified-harmonic oscillator model



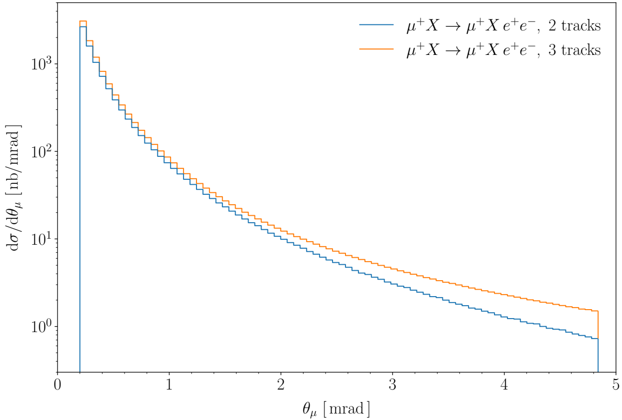
Preliminary results with `Mesmer`



- For $|Q| \gtrsim 300$ MeV the form factor effectively cuts away the cross section
- For $|Q| \leq 300$ MeV
  - two different form factors give results wich differ by less than 1%
  - including the form factor w.r.t FF=1 gives a 10% difference

Preliminary results with `Mesmer`

- 0.2 mrad $< \vartheta_\mu < 4.84$ mrad, $E_\mu \gtrsim 10.23$ GeV; $\vartheta_e < 32$ mrad, $E_e > 0.2$ GeV;
- $|Q|^2 < 0.6$ GeV^2



- events with three tracks could be an handle to check the independence of two tracks background from the nuclear form factor

Possible New Physics contamination in the $\Delta\alpha(t)$ measurement?

A. Masiero, P. Paradisi and M. Passera, Phys. Rev. D102 (2020) 075013

P.S.B. Dev, W. Rodejohann, X.-J. Xu and Y. Zhang, JHEP 05 (2020) 053

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HVP determination with MUonE data will be robust against New Physics

Possible New Physics studies with MUonE (in complementary regions to $\Delta\alpha_h$)

- **interesting proposals for NP searches at MUonE (new light mediators) in $2 \to 3$ processes**
  - invisibly decaying light Z' in $\mu e \to \mu e Z'$

    Asai et al., Phys. Rev. D106 (2022) 5
    - a relevant background can be $\mu e \to \mu e \pi^0$, in addition to $\mu e \to \mu e \gamma$
  - long-lived mediators with displaced vertex signatures $\mu e \to \mu e A' \to \mu e e^+ e^-$

    Galon et al., Phys.Rev.D 107 (2023) 095003
  - through scattering off the target nuclei $\mu N \to \mu N X \to \mu N e^+ e^-$

    Grilli di Cortona and E. Nardi, Phys. Rev. D105 (2022) L111701

Summary

- **Given its precision requirements, MUonE represents a challenge for**
  - **QED corrections**
  - **background calculation**

- **at present we have two independent Monte Carlo tools, `Mesmer` and `McMule` featuring**
  - NLO QED corrections
  - NNLO QED corections from single lepton legs
  - YFS inspired approximation to the full NNLO QED in **`Mesmer`**
  - full NNLO QED with electron “massification” in **`McMule`**
  - pair production in **`Mesmer`**
    - $\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$
    - $\mu^\pm N \to \mu^\pm N \ell^+ \ell^-$

- **efforts for N^{3}LO started**

- **work in progress for matching with higher order QED corrections**

Theoretical progress, thanks also to past

- MUonE theory workshops
  - Theory Kickoff Workshop, Padova, 4-5 September 2017
  - MITP Workshop, Mainz 19-23 February 2018
  - 2^{nd} Workstop/ThinkStart, Zürich, 4-7 February 2019
  - N^3LO kick-off workstop/thinkstart IPPP Durham, 3-5 August 2022
  - MITP Workshop, Mainz 14-18 November 2022
- Four General MUonE Collaboration Meetings

A collection of references on calculation developments

- Carloni Calame et al., PLB 746 (2015), 325
- Abbiendi et al., Eur. Phys. J. C77 (2017), 139
- Mastrolia et al., JHEP 11 (2017) 198
- Di Vita et al., JHEP 09 (2018) 016
- Alacevich et al., JHEP 02 (2019) 155
- Fael and Passera, PRL 122 (2019) 19, 192001
- Fael, JHEP 02 (2019) 027
- Carloni Calame et al., JHEP 11 (2020) 028
- Banerjee et al., SciPost Phys. 9 (2020), 027
- Banerjee et al., EPJC 80 (2020) 6, 591
- Budassi et al., JHEP 11 (2021) 098
- Balzani et al., Phys.Lett.B834 (2022) 137462
- Bonciani et al., PRL 128 (2022) 2, 022002
- Budassi et al., PLB 829 (2022) 137138
- Broggio et al., JHEP 01 (2023) 112
- Fael et al., PRD 107 (2023) 094017
- Badger et al., arXiv:2307.0398
- Ahmed et al., arXiv:2308.05028

↦ Independent numerical codes (Monte Carlo generators and/or integrators) are developed and cross-checked to validate high-precision calculations. Chiefly

- ✓ **Mesmer** in Pavia
  `github.com/cm-cc/mesmer`
- ✓ **McMule** at PSI/IPPP
  `gitlab.com/mule-tools/mcmule`

↦ An international MUonE collaboration is growing

THANK YOU!