Nature of neutrino mass, refraction and Cosmology

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The Nobel prize in physics 2015

Takaaki Kajita

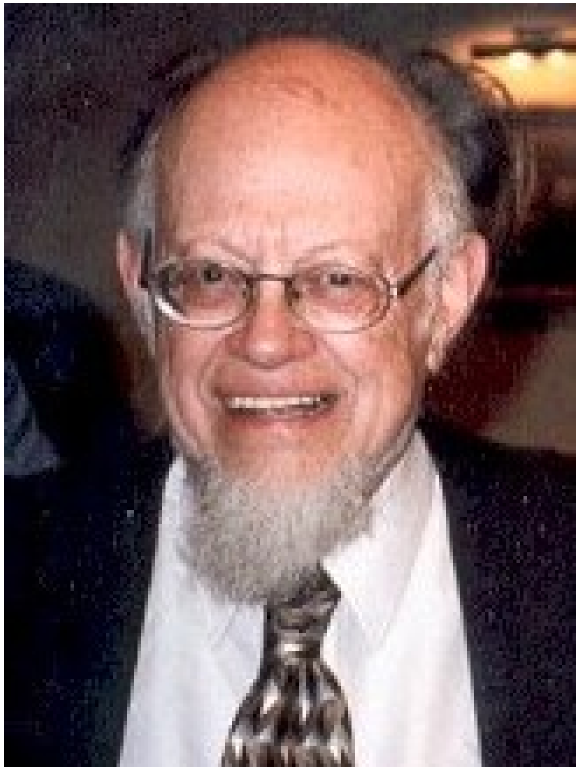
Arthur B. McDonald

" for the discovery of neutrino oscillations, which shows that neutrinos have mass"

How do we know that the mass behind oscillation?

Oscillations without mass

Lincoln Wolfenstein



Oscillations of massless neutrinos

General conditions for oscillations:

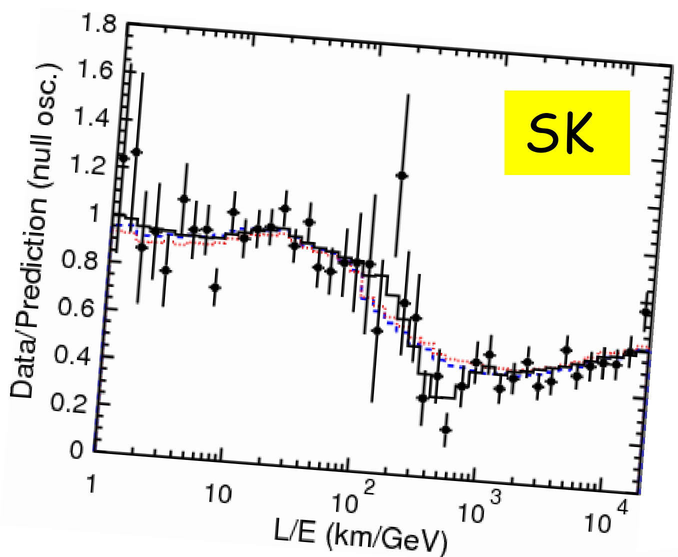
- neutrinos with different dispersion relations
- production of mixed states of these neutrinos

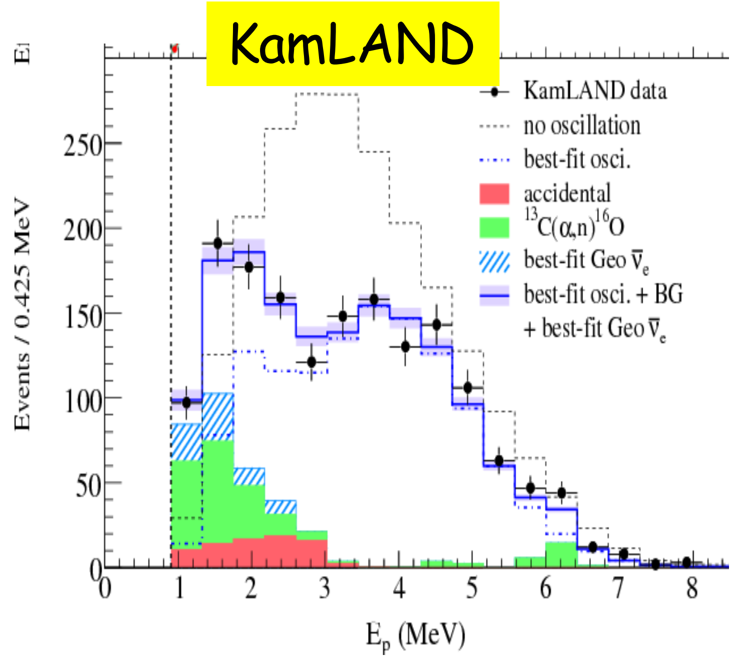
Non-standard interactions of neutrinos –
Non-diagonal in the flavor basis → potentials

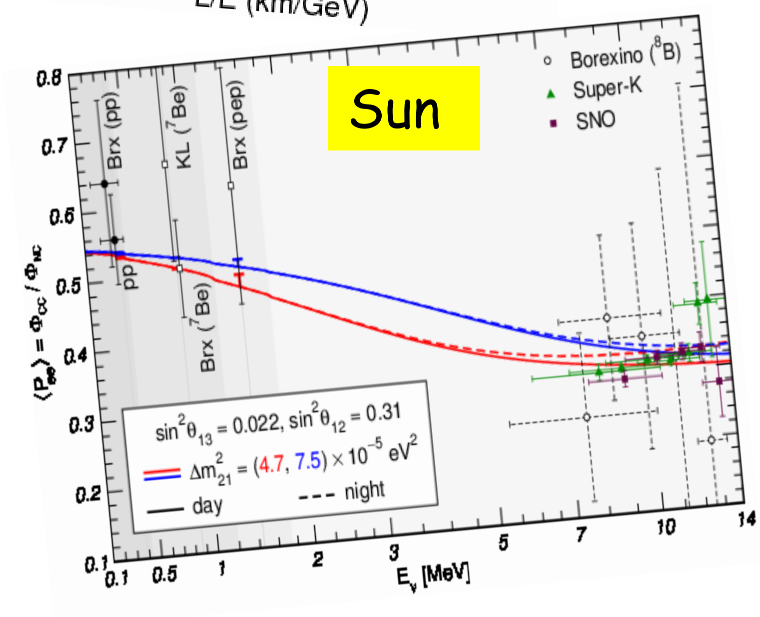
$$E_i = p + V_i$$

Introduced 4 –fermionic (local) interactions
→ imply heavy mediators
→ no energy dependence of the oscillation effects

The energy dependence found







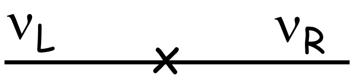
also MINOS, Daya Bay, RENO ...

In agreement with the presence of the mass term in the Hamiltonian of evolution:

$$H = E = \sqrt{p^2 + m^2} = p + m^2/2E$$

What is the mass?

Mass term changes of chirality of fermion:



In the SM mass generated due to coupling with classical scalar field,
Vacuum expectation value VEV of the Higgs field:

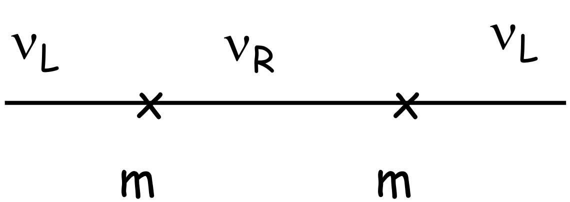
$$m = h \langle H_0 \rangle$$

= value of the field in the minimum of potential

Variations: $m = h \langle \Delta_0 \rangle$ triplet for Majorana

$m = h^2 \langle H_0 \rangle^2 / M_R$ Seesaw

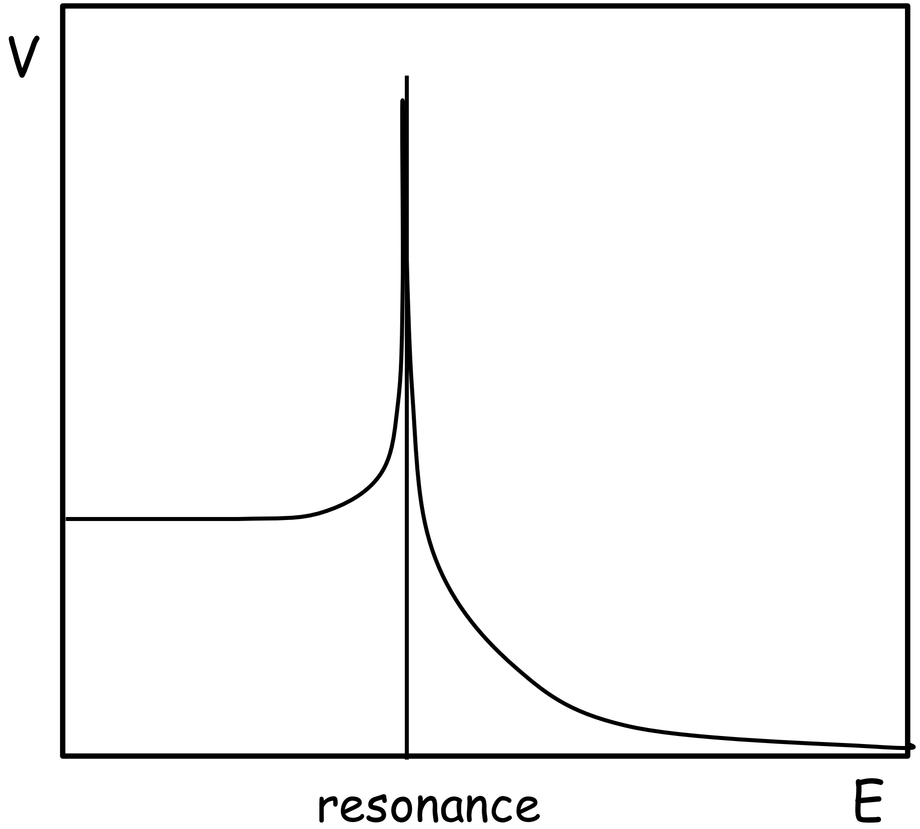
In oscillations: no change of chirality: (spin state does not change)



effects ~ m^2

Matter potential

Any contribution to the Hamiltonian of evolution which has A/E form with constant A can reproduce the oscillation data



Even in the SM:

$$V \sim \begin{cases} 1/m_W^2, & s \ll m_W^2 \\ 1/\,2m_W E, & s \gg m_W^2 \end{cases}$$

C. Lunardini, A.S.

Above resonance V ~ 1/E → potential can substitute the mass term

If mediator is light as well as target particle is light, the 1/E dependence shows up at low explored energies.

Ki-Yong Choi, Eung Jin Chun, Jongkuk Kim, 1909.10478 2012.09474 [hep-ph],

Effective mass squared

m_{ref}^2 = singlet of SM symmetry group in contrast to mass

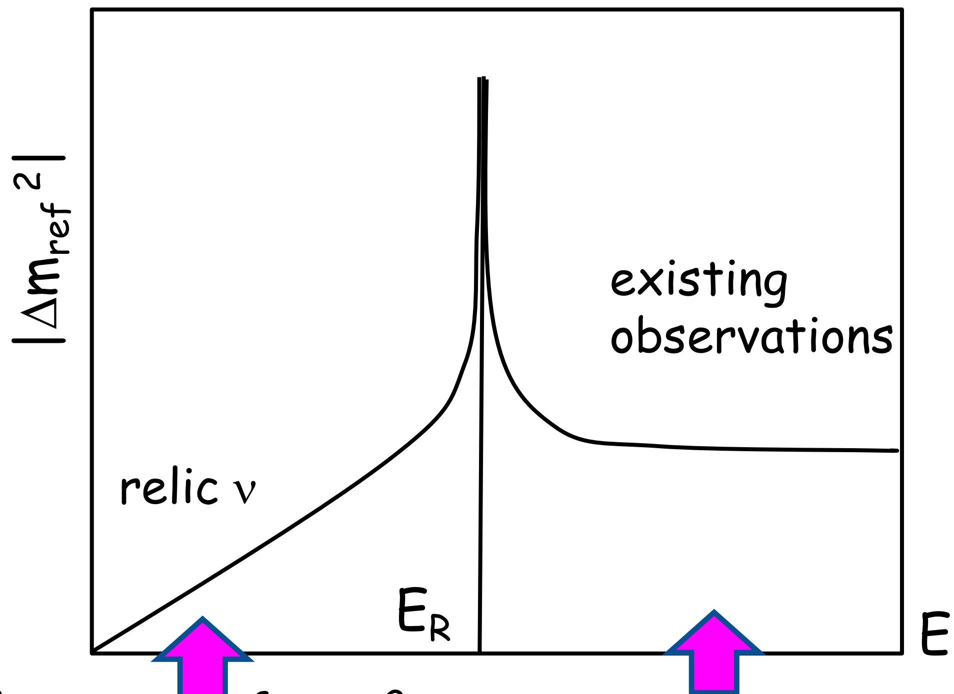
Can introduce the effective or refractive mass squared as

$$V = m_{ref}^2/2E$$

$$m_{ref}^2 = 2EV$$

m_{ref}^2 = constant – checked down to 0.1 MeV

→ take E_R << 0.1 MeV



The decrease of m_{ref}^2 with E allows to avoid the cosmological bound on sum of neutrino masses

~ constant m_{ref}^2 explains oscillation data

Large number density of target particles is required → form substantial part of whole DM

Is this scenario excluded?

Manibrata Sen, A. Y. S.
to appear

Outline:

Refraction in cold gas

Cosmological consequences

Refraction in classical scalar field

Refraction in a cold gas

A realization

Target (DM): complex scalar field ϕ with mass m_ϕ

Medator: χ_k – light Majorana fermions with masses $m_{\chi k}$

At least two χ are needed to explain data

$$L = g_{\alpha k} \bar{\nu}_{\alpha L} \chi_{kR} \phi + \frac{1}{2} m_{\chi k} \chi_{kR}{}^T \chi_{kR} + h.c.$$

$k = 1,2$

$\alpha = e, \mu, \tau$

$g_{\alpha k} < 10^{-7}$

We assume zero VEV $\langle \phi \rangle = 0$

The interaction can be generated via mixing of ϕ with SM Higgs boson

In general (depending on production) the field has classical and quantum components:

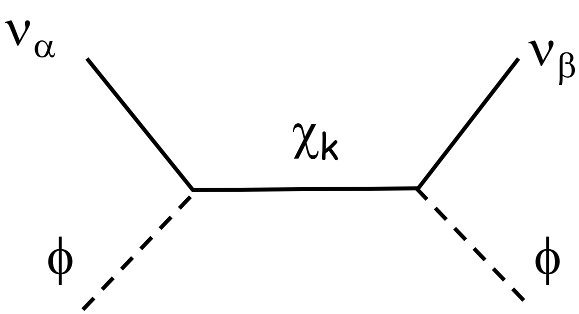
$$\phi = \phi_c + \phi_q$$

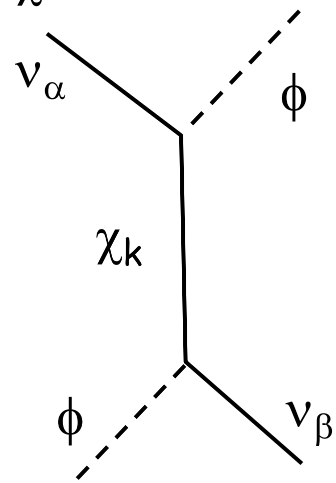
First consider the quantum one

Refraction on scalar DM

S. F Ge and H Murayama, 1904.02518 [hep-ph]
Ki-Yong Choi, Eung Jin Chun, Jongkuk Kim, 1909.10478 [hep-ph]
2012.09474 [hep-ph]

Elastic forward scattering of ν on background scalars ϕ with fermionic χ mediator





Resonance: $s = m_\chi^2$

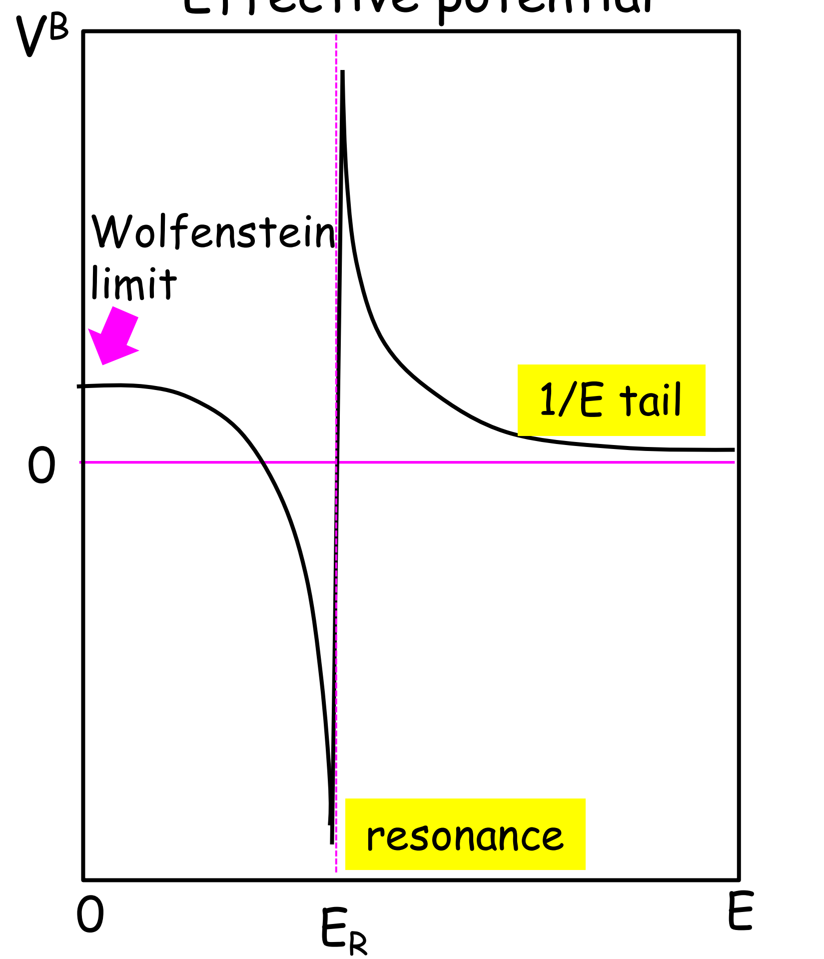
for ϕ at rest the resonance ν energy:

$$E_R = \frac{m_\chi^2}{2m_\phi}$$

For small m_ϕ resonance at low, observable energies

A.Y.S. , V. Valera, 2106.13829 [hep-ph]

Effective potential



Potential: standard computations

$$V_{\alpha\beta} = \Sigma_k V^0_{\alpha\beta k} \left(\frac{(1 - \varepsilon)(y - 1)}{(y - 1)^2 + \xi_k^2} + \frac{1 + \varepsilon}{y + 1} \right)$$

$$V^0_{\alpha\beta k} = \frac{g_{\alpha k}\, g_{\beta k}^*}{2m_\chi^2} (\bar{n}_\phi + n_\phi)$$

n_ϕ and $\bar{n}_\phi$ – the number densities of ϕ and ϕ^*

For simplicity $m_{\chi 1} = m_{\chi 2} = m_\chi$

$y = E/E_R$ $\quad E_R = m_\chi^2/2m_\phi$

$\varepsilon = (\bar{n}_\phi - n_\phi)/(\bar{n}_\phi + n_\phi)$ $\quad$ C-asymmetry of the ϕ gas

$\xi = \Gamma/E_R$ $\quad \Gamma = \frac{g^2}{4\pi} m_\chi$ $\quad$ width of resonance

we neglect ξ

Refraction mass squared

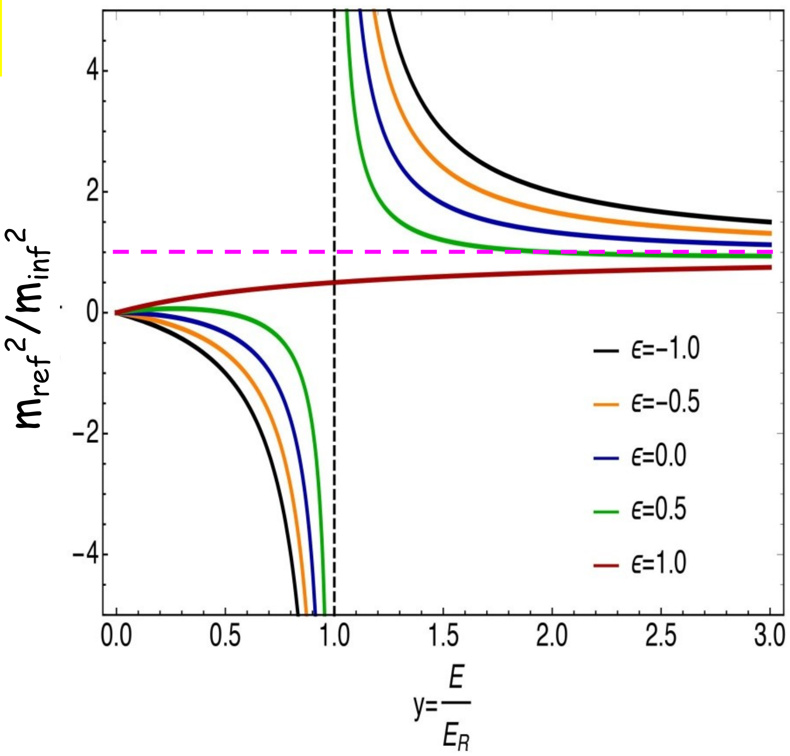
Effective mass squared $m_{ref}^2 = 2EV$

$$m_{ref}^2 = m_{inf}^2 \frac{y(y - \varepsilon)}{y^2 - 1}$$

where

$$m_{inf}^2 = \Sigma_k\, g_{\alpha k}\, g_{\beta k}^* \frac{(\overline{n_\phi} + n_\phi)}{m_\phi}$$

is the refraction mass squared at $y \rightarrow$ infty



$$m_{inf}^2 = \Sigma_k\, g_{\alpha k}\, g_{\beta k}^* \frac{\rho_\phi}{m_\phi^2}$$

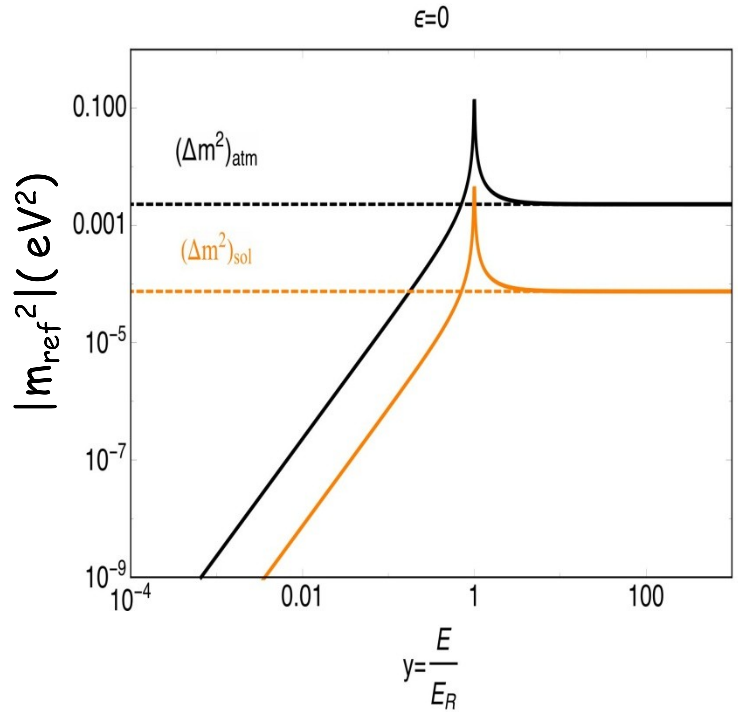
$\rho_\phi = m_\phi (\overline{n_\phi} + n_\phi)$ is the energy density in ϕ

Properties of the refraction mass squared

$y \ll 1$ $\quad m_{ref}^2/m_{inf}^2 = y(y - \varepsilon) = -\varepsilon y$

reproducing the Wolfenstein result

For C-symmetric background
$m_{ref}^2/m_{inf}^2 = y^2$ - decreases faster



$y \gg 1$

$$m_{ref}^2/m_{inf}^2 = \begin{cases} 1 - \varepsilon/y, & \varepsilon \neq 0 \\ 1 + y^{-2} & \varepsilon = 0 \end{cases}$$

converges to constant faster

For antineutrinos $\varepsilon \rightarrow -\varepsilon$

$m_{inf}^2\,(\text{nu}) = m_{inf}^2\,(\text{antinu})$

m_{inf}^2 has all the properties of usual mass

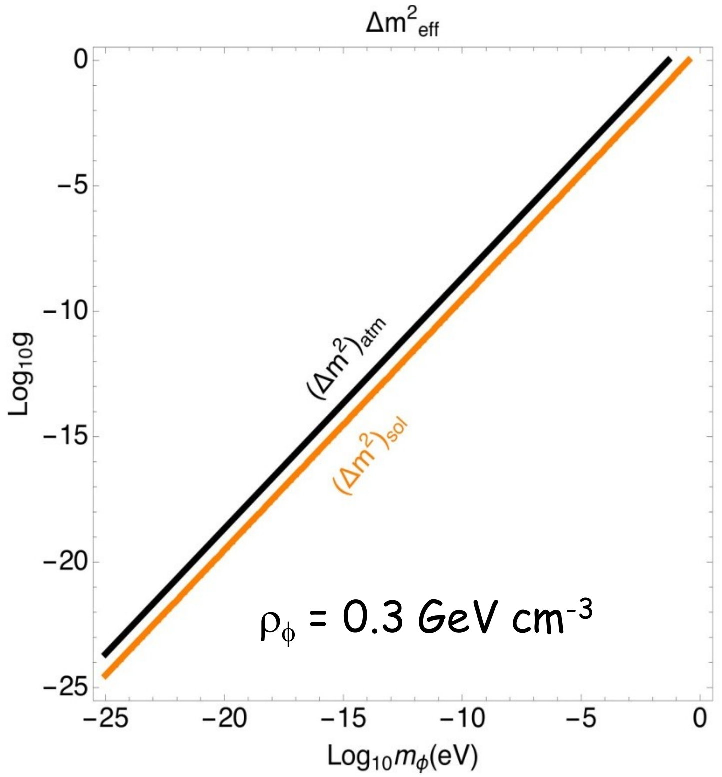
Fitting the oscillation data

Nearly TBM mixing can be obtained for

$g_{e1} = g_{\mu 1} = g_{\tau 1} = g_1 \quad g_{e2} = 0 \quad g_{\mu 2} = - g_{\tau 2} = g_2$

Masses (normal hierarchy) $m_1 = 0$

$$g_1 = m_\phi \sqrt{\frac{\Delta m_{sol}^2}{3\rho_\phi}} \qquad g_2 = m_\phi \sqrt{\frac{\Delta m_{atm}^2}{2\rho_\phi}}$$



These results do not depend on m_χ

m_χ is determined by m_ϕ and the resonance energy:

$$m_\chi = \sqrt{2 m_\phi E_R}$$

Astrophysical bounds

Dissipation of the astrophysical neutrino fluxes due to inelastic scattering on background (energy loss, scattering angle)

$\nu\, \phi \rightarrow \nu\, \phi$

Upper bound on

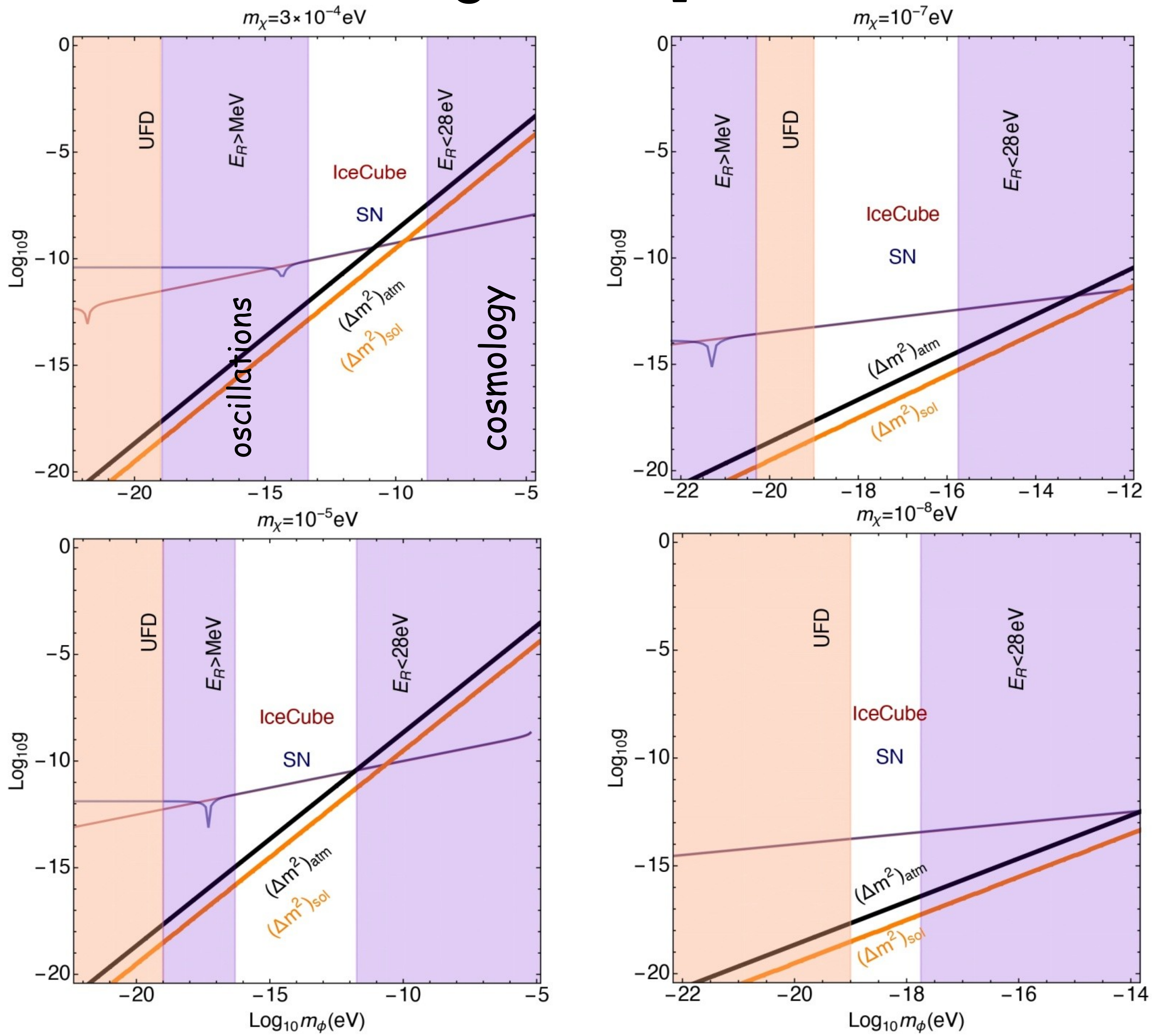
σ_ν / m_ϕ → bounds on *g* as functions of m_ϕ

SN1987A, 50 kpc

K.-Y. Choi, J. Kim, C Rott
PRD99 (2019) 8, 083018

Ice Cube observation of neutrino event IC-170922A with E =290 TeV in association with blazar TXS0506+56 (z = 0.3365, 1421 Mpc)

Viable ranges of parameters



Bounds and regions required for explanation of oscillation data by refraction in $g - m_\phi$ plane for different values of m_χ

UFD – bound from heating of ultra-faint galaxies

Refraction mass vs. VEV mass

Refraction mass is different in different space time points and also depend on energy:

$$m_{ref}^2\,(x, t, E) = n_\phi(x, t)\, f(E)$$

m_{ref}^2 is different in solar system, center of Galaxy, intergalactic space

The average $m_{ref}^2\,(z)$ in the Universe increased in the past.

VEV mass is determined by minimum of the potential, can depend on t and x in the presence of topological defects and due to thermal corrections to the potential in the Early Universe

Cosmological

Evolution, bounds

Evolution of the refractive mass

In the present epoch, $z = 0$, the average refraction mass in the Universe

$$m_{ref}^2(0) \sim \xi\, m_{ref}^2(loc)$$

$\xi \sim 10^{-5}$ - inverse of local (near the Earth) over-density of background

With explicit energy dependence at small y

$$m_{ref}^2(0) \sim \xi\, m_{inf}^2(loc)\, y(y - \varepsilon) \qquad y = E/E_R$$

With redshift $n_\phi(z) = (1 + z)^3 n_\phi(0)$, $E(z) = (1 + z)E(0)$

$$m_{ref}^2(z) \sim \xi\, m_{inf}^2(loc)\,(1 + z)^4\, E(0)/E_R\, [(E(0)/E_R\,(1 + z) - \varepsilon]$$

$E(0) \sim 10^{-3}$ eV is the present average energy of relic neutrinos

For large enough E_R the mass $m_{ref}^2(z)$ can satisfy cosmological bound on sum of neutrino masses from structure formation

Bound on resonance energy

In the epoch of matter-radiation equality, $z \sim 1000$,
DM should already exist and structures start to form

We require that

$$m_{ref}^2\,(1000) \sim (\Sigma m_\nu)^2 < 10^{-2}\ \mathrm{eV}^2$$

$$m_{inf}^2\,(loc) = \Delta m_{atm}^2$$

For $y \ll \varepsilon$

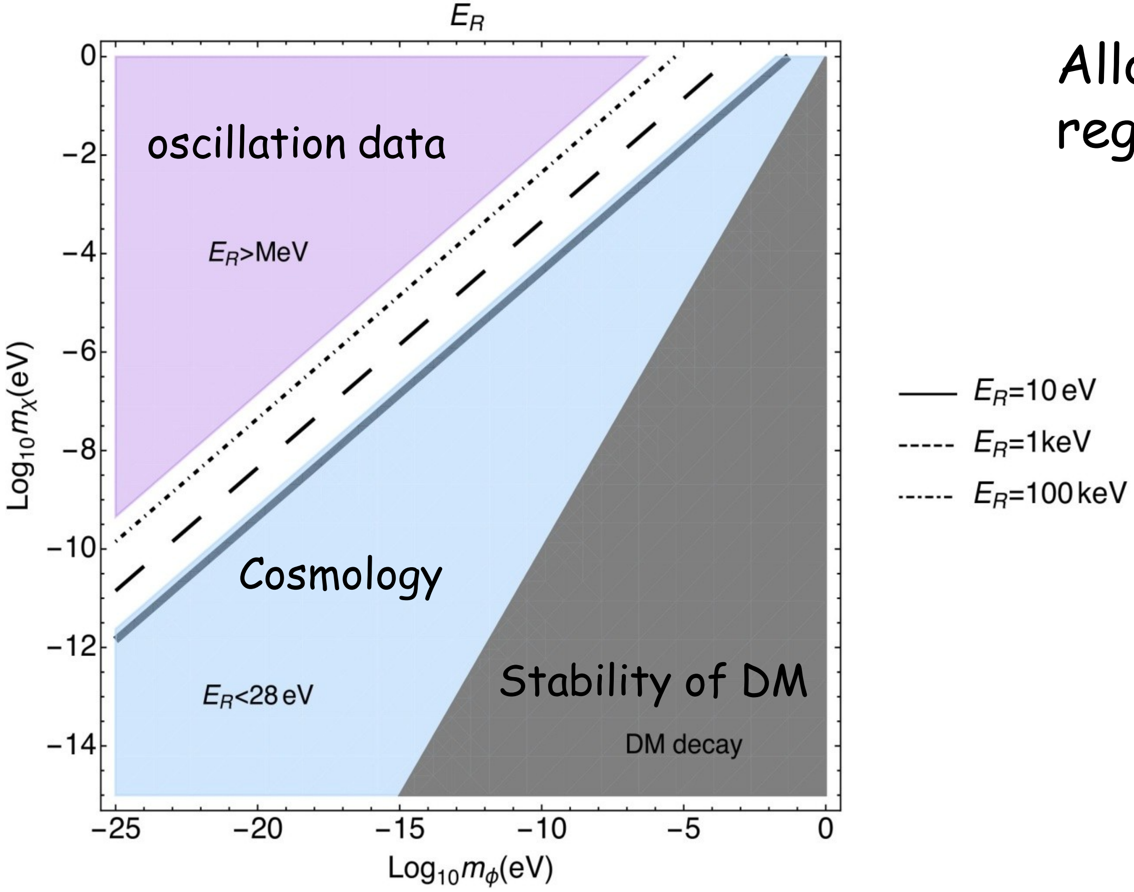
$$E_R > E(0)\,\xi\,\varepsilon\,(1+z)^4\,\frac{\Delta m_{atm}^2}{(\Sigma m_\nu)^2} \quad \Rightarrow \quad E_R > 1.2\,\varepsilon\ \mathrm{keV}$$

For $\varepsilon = 0$

$$E_R > E(0)\,[\,\xi\,(1+z)^5\,]^{1/2}\,\frac{\sqrt{\Delta m_{atm}^2}}{\Sigma m_\nu} \quad \Rightarrow \quad E_R > 30\ \mathrm{eV}$$

The bound for refractive (dynamical) masses should be reconsidered
(group velocities, mass in density perturbations etc…)

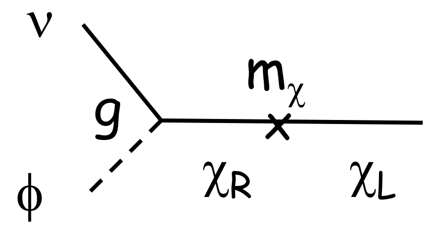
Viable ranges of parameters



Allowed and excluded regions in m_χ - m_ϕ plane

$\nu - \chi$ mixing and transitions

$\nu\, \phi \rightarrow \chi_R \rightarrow \chi_L$



Coherence:
States of medium with ϕ being absorbed from different space-time points are coherent once $\Delta x < \lambda_{DB} = 2\pi/v\, m_\phi$

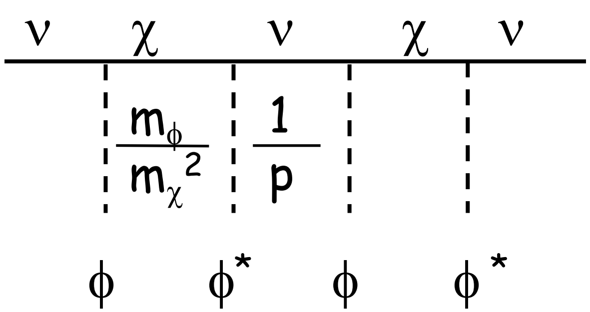
Energy - momentum conservation OK within $\Delta p < 1/L$ (baseline)

Perturbativity and resummation

Radius of interactions below resonance : $1/m_\chi$

Large number of scatterers ϕ within interaction volume.
Processes with many ϕ should be taken into account

$\nu\,\phi\,\phi \rightarrow \nu\,\phi\,\phi \qquad \nu\,\phi\,\phi\,\phi \rightarrow \nu\,\phi\,\phi\,\phi$ with Bose enhancement?



$$V = \frac{\varepsilon g^2 n_\phi}{2 m_\chi^2}\left(1 - \frac{\varepsilon g^2 n_\phi}{2E m_\chi^2} + \ldots\right)$$

$$= \frac{\varepsilon g^2 n_\phi}{2 m_\chi^2}\left(1 + \frac{\varepsilon g^2 n_\phi}{2E m_\chi^2}\right)^{-1}$$

< 1

At low energies and high densities the perturbativity can be broken

$$E_{pert} > \frac{\varepsilon g^2 n_\phi}{2 m_\chi^2} = \frac{m_{inf}^2}{2E_R} = \frac{\Delta m_{atm}^2}{2E_R}$$

For $E_R = 30$ eV, $E_{pert} > 10^{-4}$ eV

important for relic neutrinos?

Refraction in classical field

Coherent classical field

System of ϕ with large occupation number can be treated as classical scalar field

Condition $\lambda_\phi^3 n_\phi >> 1$

$\lambda_\phi = 2\pi/k_\phi = 2\pi/v m_\phi$ - de Broglie wave of ϕ

$v \sim 10^{-3}$ - virial velocity in Galaxy

$$m_\phi << 2\pi\left(\frac{\rho_\phi}{2\pi v^3}\right)^{1/4}$$

$m_\phi << 30$ eV is well satisfied

In terms of QFT such a scalar field ϕ_c can be introduced as expectation value of the field operator in the coherent state:

$$\phi_c = \langle\phi_{coh}|\ \phi\ |\phi_{coh}\rangle$$

$$|\phi_{coh}\rangle = \exp\left(\int\frac{d\mathbf{k}}{(2\pi)^3}[f_a(\mathbf{k})a_k^+ + f_b(\mathbf{k})b_k]\right)|0\rangle \qquad \mathbf{k} = m_\phi\,\mathbf{v}$$

It can be parameterized as

$$\phi_c(x) = F(x\ t)\ e^{-i\Phi}$$

$$F^2 \sim \rho_\phi / m_\phi^2$$

Neutrino mass in classical field

In the Lagrangian: $\phi \rightarrow \phi_c$

$$L = g_{\alpha k}\, \bar{\chi}_{kR}\, \nu_{\alpha L}\, \phi_c^* + h.c.$$

➡ mass terms $m_{\alpha k} = g_{\alpha k}\, \phi_c^*$

Mass matrix in the basis $(\nu_{f,} \chi^c{}_L) = (\nu_e, \nu_\mu, \nu_\tau, \chi_1, \chi_2)$

$$M = \begin{pmatrix} 0 & g_{\alpha k}\, \phi_c^* \\ g_{k\alpha}\, \phi_c^* & \mathrm{diag}\,(m_{\chi 1}, m_{\chi 2}) \end{pmatrix}$$

The Hamiltonian

$$H = \frac{1}{2E} M M^+ = \frac{1}{2E} \begin{pmatrix} |F|^2 \Sigma_k\, g_{\alpha k}\, g_{\beta k}^* & g_{\alpha k}\, F\, m_{\chi k}\, e^{i\Phi} \\ g_{k\alpha}^*\, F^*\, m_{\chi k}\, e^{-i\Phi} & M_\chi^2 \end{pmatrix}$$

$$M_\chi^2 = f(\,|F|^2, |\, g_{\alpha k}|^2, m_{\chi k}^2\,)$$

Properties of the Hamiltonian

3x3 flavor block has the same form as refraction matrix m_{inf}^2

Additional time dependence can appear in F:

For real field

$$|F|^2 \sim \rho_\phi / m_\phi^2 \cos^2 m_\phi t$$

For C-asymmetric background the amplitude of oscillations can be suppressed

Averaging?

No resonance dependence of mass on energy
No decrease of the mass with energy at low energies

$\nu - \chi$ mixing

ν - χ mixing and oscillations

After TBM rotation of active neutrinos

- one (masless) state decouples
- rest 4 states split into two pairs which evolve independently

$$M_k = \begin{pmatrix} 0 & m_{ak}\, e^{i\Phi} \\ m_{ak}\, e^{i\Phi} & m_{\chi k} \end{pmatrix} \qquad k = 1, 2$$

Oscillation parameters of active-sterile systems:

$$\Delta m_{ak}^2 = 2\sqrt{(m_{\chi k}^2 - 2E\, d\Phi/dt)^2 + m_{ak}^2\, m_{\chi k}^2}$$

$$\tan 2\theta_{ak} = \frac{2 m_{ak}\, m_{\chi k}}{m_{\chi k}^2 - 2E\, d\Phi/dt} \qquad m_{\chi k}^2 \ll m_{a1}^2 = \Delta m_{sol}^2$$

Two viable cases to avoid bounds from active-sterile oscillations

$d\Phi/dt = 0$ - pseudo Dirac neutrinos with $\Delta m_{ak}^2 < 10^{-12}\ eV^2$

$E\, d\Phi/dt \sim E m_\phi \gg m_{\chi k}^2$ - small mixing

Summary

We can not exclude that neutrino oscillations are explained the refractive mass squared

Refraction in cold gas: energy dependent mass at low energies: avoid the cosmological bound on sum of neutrino masses from structure formation

Refraction in classical coherent field: energy independent mass: Problem with cosmology? Late formation of the field?

Nature of neutrino mass can be related the nature of Dark matter and the Cosmological evolution

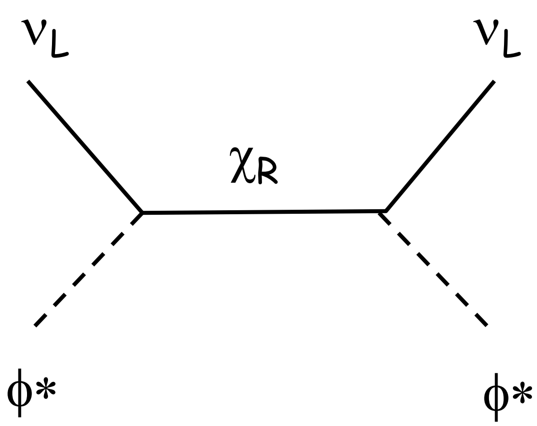
Backup slides

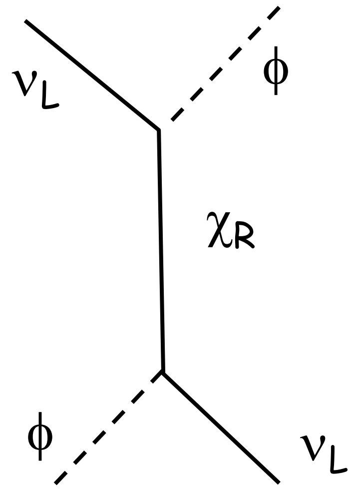
Neutrino refraction on scalar DM

S. F Ge and H Murayama, 1904.02518 [hep-ph]

Ki-Yong Choi, Eung Jin Chun, Jongkuk Kim, 1909.10478 [hep-ph]

2012.09474 [hep-ph]





Neutrino scattering on DM particles ϕ (target) with χ_R - mediator

$$V_s \sim \frac{(s - m_\chi^2)\,\bar{n}}{(s - m_\chi^2)^2 + s\,\Gamma^2}$$

$$V_u \sim \frac{n}{u - m_\chi}$$

m_f^2

n and $\bar{n}$ – the number densities of ϕ and ϕ^*

$$\Gamma = \frac{g^2}{32\pi}\, m_\chi$$

Resonance: $s = m_f^2 \rightarrow E_R = m_\chi^2/2m_\phi$

Effect of classical component

A. Berlin, 1608.01307 [hep-ph]

Ultra-light scalar DM, large number density – as a classical field, solution

$$\phi(t, x) \sim \frac{\sqrt{2\rho(x)}}{m_\phi} \cos(\omega t - k x)$$

$\omega \sim m_\phi$ $k = m_\phi v$ $v \sim 10^{-3}$ – virialized velocity in the Galaxy

generates the mass term $m' = g\,\phi_c$ $m' \nu_L f_R + m_f f_L f_R + h.c.$

Oscillating mass with period $T_{osc} = \frac{2\pi}{m_\phi} = 4\;10^{-15}$ sec $(1\text{ eV}/m_\phi)$

Lost of coherence due to velocity dispersion $\Delta v \sim v$ ⇨ $\Delta\omega = m_\phi v \Delta v \sim m_\phi v^2$

Coherence time: $\tau_{coh} = \frac{2\pi}{\Delta\omega} = 4\;10^{-9}$ sec $(1\text{ eV}/m_\phi)$

Coherence length: $L_{coh} = \frac{2\pi}{\Delta v\, m_\phi} = 1.2\;\;10^{-3}$ m $(1\text{ eV}/m_\phi)$

System transforms in the cold gas of individual scatterers. Still in some aspects can be considered as classical field without t variations

Phenomenology. Bounds on parameters

ν - DM inelastic scattering

$$\sigma = \frac{g^4}{16\pi}\begin{cases} \dfrac{s}{m_f^4} & s \ll m_f^2 \\ \dfrac{1}{s} & s \gg m_f^2 \end{cases}$$

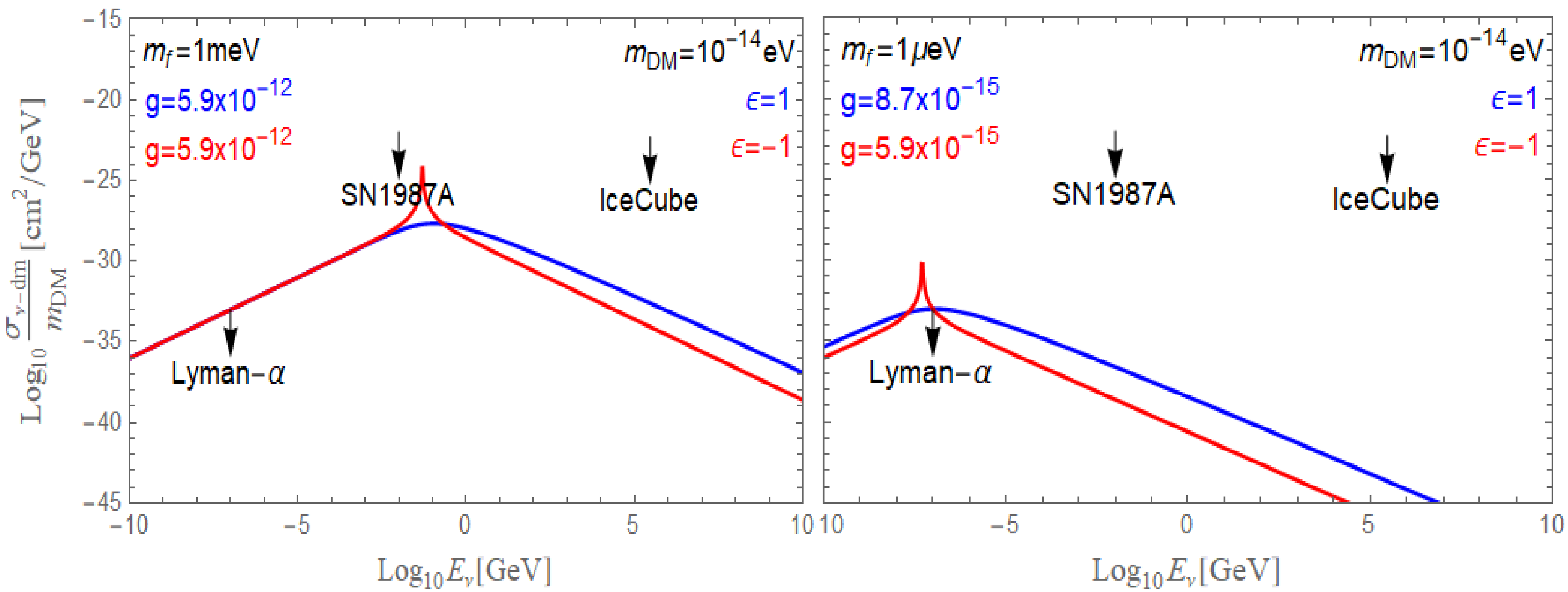
Upper phenomenological bounds on σ / m_ϕ

$\sigma / m_\phi < \xi$ for certain neutrino energies E_ξ

$$m_f > g\left(\frac{E_\xi}{8\pi\,\xi}\right)^{1/4} \qquad m_\phi < m_f^2/2E_\xi$$

$$m_\phi > g^2\left(\frac{1}{32\pi\,E_\xi\,\xi}\right)^{1/2} \qquad m_\phi > m_f^2/2E_\xi$$

Bounds from neutrino DM interactions



The most stringent bound
from Lyα
relic neutrinos

ξ < 10^{-33} cm^2 / GeV

R.J. Wilkerson, C. Boehm, L. Lesgourges
JCAP 1405 (2014) 011

SN87A, E = 10 MeV

Ice Cube

Relic SN neutrinos

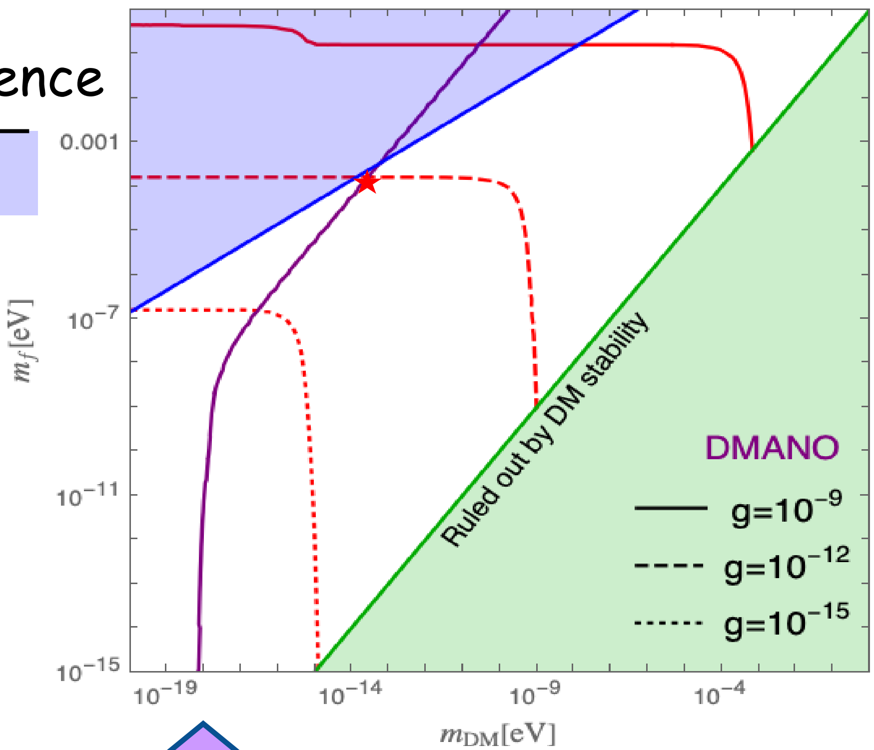
Stability of DM

Bounds on parameters

Ki-Yong Choi, Eung Jin Chun, Jongkuk Kim, A.S.

Bound from 1/E dependence

$m_f < \sqrt{2 m_\phi E_B}$



$m_\phi > m_f$

Red lines – lower bounds on masses of DM and mediator from Lyα - data for different values of g

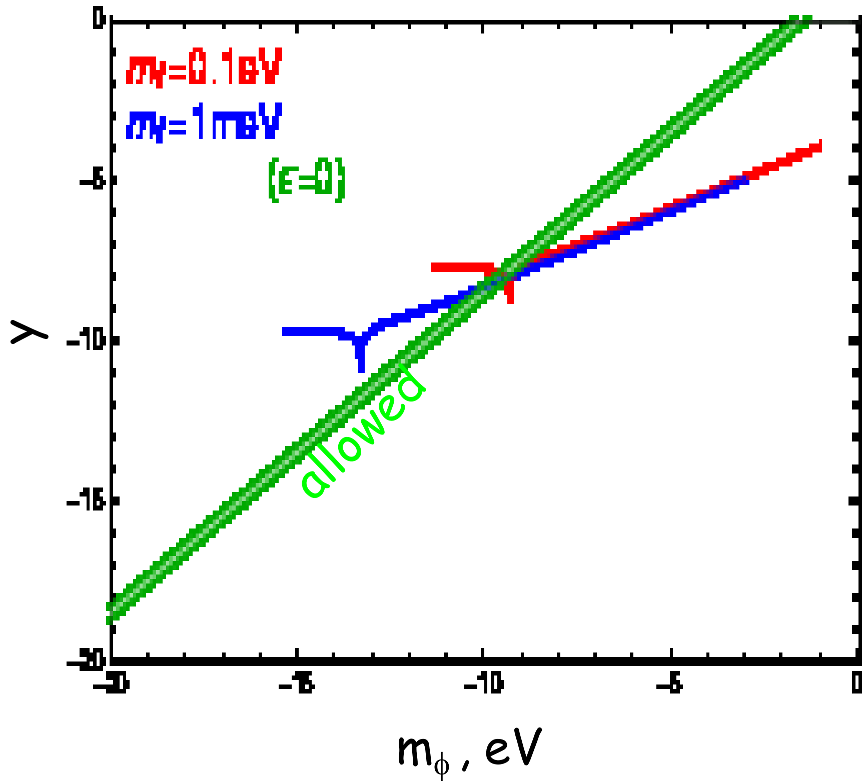
Lower bound for parameters which can reproduce observed oscillation effects

Bounds

$m_f < 10^{-4}$ eV $m_\phi < 10^{-13}$ eV $g < 10^{-12}$

Bounds on parameters

Ki-Young Choi, Eung Jin Chun, Jongkuk Kim, 2012.09474 [hep-ph]



Green band: $\Delta m_{eff}^2 = \Delta m_{atm}^2$

Upper bounds on y from scattering of neutrinos from SN1987A on DM ϕ with zero C- asymmetry and two different masses of mediator f

Similar bound from Lyα (relic neutrinos) .

Allowed values:

$m_f < 10^{-3}$ eV
$m_\phi < 10^{-10}$ eV
$y < 10^{-9}$

the corresponding resonance energy E_R = 0.01 MeV

Cosmological bound is satisfied

Dependence of the effective mass on density and energy

$$m_{eff}(z) \sim [\xi (1 + z)^3]^{1/2}\, m_{eff}(loc)$$

where $1/\xi \sim 10^5$ - local (near the Earth) over-density of the background

In the epoch of matter-radiation equality, $z = 1000$, DM should already be formed and structures start to form.

For $m_{eff}(loc) = 0.05$ eV and $1/\xi \sim 10^5$ $\Rightarrow$ $m_{eff}(1000) \sim 5$ eV
- violates cosmological bound on the sum of neutrino masses

For not very small E_R one should take into account dependence (decrease) of $m_{eff}(loc)$ with neutrino energy

$$\Delta m_{eff}^2(E) \sim \frac{y(y - \varepsilon)}{y^2 - 1} \Delta m^2 \qquad y = E/E_R$$

and for relic neutrinos $m_{eff}(loc)$ can be very small