COMPARISON OF CHAPLYGIN GAS AND BULK VISCOUS COSMOLOGICAL MODELS

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Introduction

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- Cosmology is a science based on the study of the origin, and evolution of the universe.
- This is both theoretical and observational development that began over 100 years ago.
- A key to the development of cosmology was Einstein's static universe model derived in 1917.
- The origin was scientifically proposed as a "singularity" over 13.8 billion years ago, leading to the prediction of the Big Bang model(BB).
- BB explains the universe's origin as a big explosion that eventually started to cool forming stars, planets, and galaxies(Smeenk and Ellis 2017).

Introduction.........continues

- Observational relations between distance and recession velocity of galaxies led to Hubble's discovery in 1929 of the expanding universe.
- It has been discovered in 1998 by observing distant supernovae that the expanding universe is undergoing acceleration (Tonry et al. 2003).
- This acceleration is supported by Cosmic Microwave Background radiation (CMBR), Large Scale Structure acoustic oscillations, and weak lensing.
- The basic ΛCDM model has a fluid with constant density, and negative pressure.

Introduction.........continues

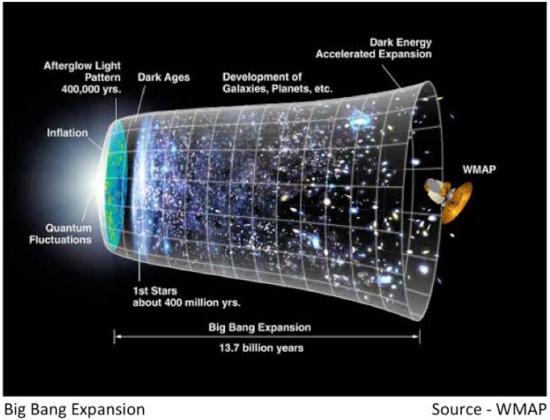

Figure: (1)The phases of the universe

Chaplygin Gas (CG)

Chaplygin gas

- For Chapylgin gas [Kamenshchik et al PLB 511(01)265], the EoS is:

$$p = -\frac{A}{\rho} \tag{1}$$

where A is a positive constant

- Einstein's field equations are:

$$R_{ab} - \frac{1}{2}Rg_{ab} = T_{ab} \tag{2}$$

- The Friedmann–Lemaitre–Robertson–Walker (FLRW) metric is:

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta^2 d\phi^2)\right] \tag{3}$$

where the symbols have their usual meanings.

- From (1)-(2) we get the Friedmann-type equation:

$$3\frac{\dot{a}^2}{a^2} = \rho \tag{4}$$

where ρ is the energy density, and the energy conservation equation:

$$d(\rho a^3) = -pd(a^3) \tag{5}$$

- From the above eqns, we get

$$\rho = \sqrt{A + \frac{B}{a^6}} \tag{6}$$

where B is an integration constant.

- From this equation (6), useful conclusions can be drawn. For positive B and small a ($a^6 << B/A$), we get:

$$\rho = \sqrt{\frac{B}{a^3}} \tag{7}$$

which is similar to the eq for the matter-dominated era in general relativity ($p = 0$).

- For large values of a, eqs (5) & (6) yield:

$$\rho \sim \sqrt{A}, \quad p \sim \; -\sqrt{A} \tag{8}$$

resulting in the deSitter-type (accelerated) universe.

- Let us consider now expanding ρ from Eq (6) in a Taylor series, and retaining terms up to the first order only:

$$\rho = \sqrt{A} + \frac{B}{\sqrt{4A}} a^{-6} \tag{9}$$

- Also by expanding p from (5) and (6) we get

$$p = -\sqrt{A} + \frac{B}{\sqrt{4A}} a^{-6} \tag{10}$$

Eqns (9) and (10) describe a mixture of stiff matter at "intermediate" times ($p = \rho$) with cosms const at late times $\sqrt{A}$

Thus we have 3 stages with just one EoS (5): dust ($p = 0$), stiff matter ($p = \rho$), and finally deSitter ($p = -\rho$)

Other forms of Chaplygin gas

Other forms of Chaplygin gas

These are some of the EoS for other types of CG models:

- **Generalized Chaplygin gas (GCG) model** (Bento 2002)

$$p = -\frac{A}{\rho^{\alpha}} \tag{11}$$

  with A is a positive constant, and α is a constant, $\alpha \geq -1$.

- **Modified Chaplygin gas (MCG) model** (Debnath et al 2004)

$$p = A\rho - \frac{B}{\rho^{\alpha}} \tag{12}$$

  with A, B are positive constants, and α is a constant, $\alpha \geq -1$.

- **Variable Modified Chaplygin (VMG) gas model** (Debnath 2007)

$$p = A\rho - \frac{B(a)}{\rho^{\alpha}} \tag{13}$$

  with A is positive constant and B now is a function of a scale factor, $\alpha \geq -1$.

Other forms of Chaplygin gas.................

- **Generalized Chaplygin gas (GCG) model** (Bento 2002)

$$p = -\frac{A}{\rho^{\alpha}} \tag{14}$$

with A is a positive constant, and α is a constant, $\alpha \geq -1$.

- solution:

$$\rho = \sqrt{A + \frac{B}{a^6}} \tag{15}$$

where B is an integration constant.

Other forms of Chaplygin gas..................

- **Modified Chaplygin gas (MCG) model** (Bento 2002)

$$p = A\rho - \frac{B}{\rho^{\alpha}} \tag{16}$$

with A is a positive constant, and α is a constant, $\alpha \geq -1$.

- solution:

$$\rho = \left(\frac{B}{A+1}\right)^{\frac{1}{\alpha+1}} \left(1 + \frac{1}{AW_r^{\alpha+1}}\right)^{\frac{1}{\alpha+1}} \tag{17}$$

where B is an integration constant and where $W_r = W/W_0$

- **Variable Modified Chaplygin gas (MCG) model** (Bento 2002)

$$p = A\rho - \frac{B(a)}{\rho^{\alpha}} \tag{18}$$

with A is positive constant and B now is a function of a scale factor, $\alpha \geq -1$.

- solution:

$$\rho = \left[\frac{3(1+\alpha)B_0}{\{3(1+\alpha)(1+B)-n\}} \frac{1}{a^n} + \frac{C}{a^{3(1+A)(1+\alpha)}} \right]^{\frac{1}{1+\alpha}} \tag{19}$$

where B, is an integration constant and $C, B_0 > 0$ and n are constants

Bulk viscous models

- To incorporate bulk viscosity, the pressure p has to be modified as follwos:

$$\overline{p} = p - 3\eta H \qquad (20)$$

where $\overline{p}$ is the total pressure, p is the perfect fluid pressure and η is the coefficient of bulk viscosity.

- We usually take a barotropic equation of state for the perfect fluid, $\omega\rho$, $\omega = const$, and get:

$$\overline{p} = \omega\rho - 3\eta H \qquad (21)$$

- Since $H = \dot{a}/a$, the bulk viscosity eq above becomes:

$$\overline{p} = \omega\rho - 3\eta\frac{\dot{a}}{a} \qquad (22)$$

Comparing Bulk viscosity and Chaplygin gas

Comparing Bulk viscosity and Chaplygin gas

- The coefficient of bulk viscosity η is usually (but not always) taken as

$$\eta = \eta_0 \rho^m \tag{23}$$

where m is a constant and $\eta_0 \geq 0$

- Substituting equation (23) and $\frac{\dot{a}}{a} = \sqrt{\rho/3}$ into equation (22), we get the following equation:

$$\overline{p} = \omega\rho - \sqrt{3}\eta_0 \rho^{\mathrm{m}+1/2} \tag{24}$$

We can now compare the above equation for bulk viscosity with the eqns for Chaplygin gas, viz., (5), (11)-(13)

- Putting for $m = -3/2$, $\omega = 0$ and $\sqrt{3}\eta_0 = A$ in Eq (24), we get Eq (1), which is the basic Chaplygin Gas equation.
- Putting $m = -\alpha - 1/2$, $\omega = 0$ and $\sqrt{3}\eta_0 = A$ in Eq (24) for , results in Eq (11) which is the Generalized Chaplygin gas equation.
- Putting $m = -\alpha - 1/2$, $\omega = A$ and $\sqrt{3}\eta_0 = B$ in Eq (24), results in Eq (12) which is the Modified Chaplygin gas equation.
- For the last form, VMG, we go back to the basic eq (16), and comapare with (13). Remembering that we have freedom of choice for η, we put

$$\eta = \frac{B(a)}{\rho^{\alpha+1/2}3^{3/2}} \tag{25}$$

and then find equivalence between VMG and bulk viscosity again

Conclusion

Conclusion

- In this talk, we have shown that for all the different forms of Chaplygin gas, they can be written as bulk viscous models for different forms of viscosity.
- The viscous models have been studied in great detail [e.g., Barrow...], and by means of simple transformations, the Chaplygin gas solutions may be obtained.
- This is an example showing the richness of bulk viscous models.

The End. Thank you!

References

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