https://www.overleaf.com/project/642c1f00ca7dfff6b1de6032

Workshop: Applications of Quantum Information in Astrophysics and Cosmology

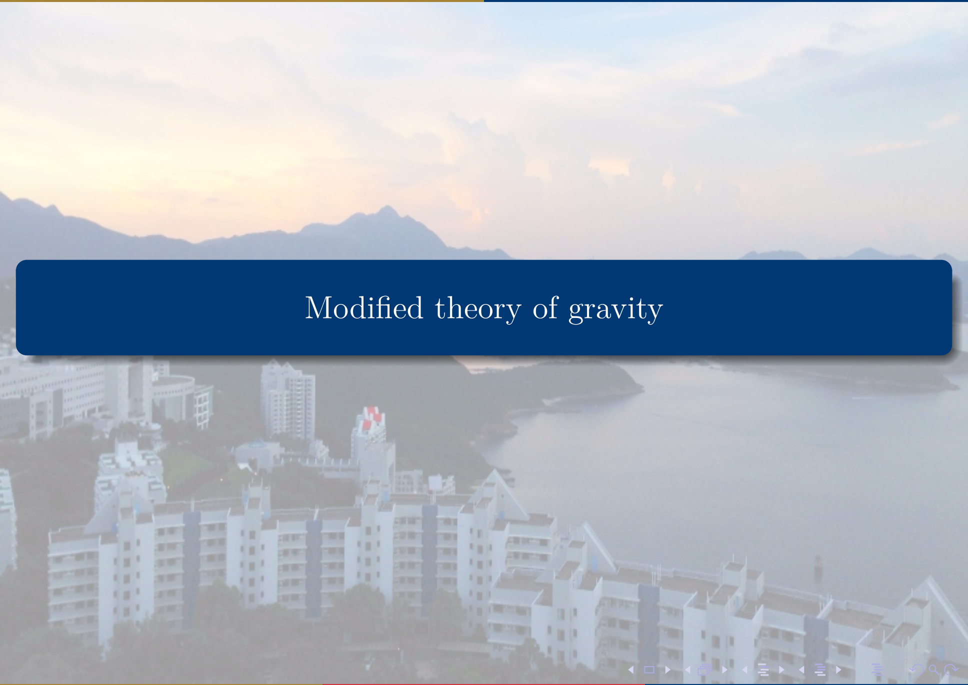
Captown, South Africa, 2023

Postdoc: Shambel Sahlu, Host: Prof. Amare Abebe

Cosmology of Modified Theory of Gravity

April 26, 2023

Table of Contents



Modified theory of gravity

Modified theory of gravity

- The standard model of cosmology so-called ΛCDM model is going to be a big challenge due to the presence of the most extremely dramatic behavior of the universe.
- For instance, the discovery of dark energy and dark matter still becomes a puzzle in physics.
- Dark energy responsible for current accelerating expansion universe.
- To solve such kind of probes in cosmology, the modification of the general theory of gravity count as an alternative approach to explain the acceleration expansion universe without dark energy scenarios.
- There are many modified theories of gravity, such as:
  - Gauss Bonnet $f(G)$, G-being the universal gravitational constant,
  - $f(R)$, R-being scalar curvature ,
  - $f(T)$, T-being torsion scalar and $f(T, B)$, B-being the boundary term .
- Most modified gravity studies start with a standard gravitational formulation based on curvature and change the Einstein-Hilbert action, while teleparallel gravity is torsion based.

$f(T)$-gravity

- In this talk, the second-order gravity so called $f(T)$-gravity only considered.
- In GR, torsion T is assumed to vanish and in **teleparallel gravity theory**, curvature R is assumed to vanish.
- Fortunately, the two basic theories of gravity describe the gravitational interaction equivalently TEGR.
- So, torsion is an alternative direction of describing the gravitational field interaction the energy-momentum tensor is the source in both theories the source of the curvature in GR and the source of torsion in teleparallel gravity theory
- One of the basic differences from the usual internal gauge models in many ways, the most significant being the presence of a tetrad field.

- In teleparallel gravity theory, we use the new connection which is Weitzenböck connection instead of the usual connection affine or Levi-Civita connection, and we use the torsion scalar T to describe the Lagrangian density rather than curvature scalar R.
- Ricci scalar yields as

$$R = T + 2\nabla^a T_{ab}{}^b = T + 2\nabla^a T_a \ , \tag{1}$$
$$= T + B \ , \tag{2}$$

where $T_a = T_{ab}{}^b$ and $B = 2\nabla^a T_a$ is the boundary term.
- Then Eq. (2) gives us a clear hint on how we can explore the relationship between R and T.

- One of the significant advantages of $f(T)$ gravity over $f(R)$ gravity is that its field equation is a second-order instead of a fourth-order.
- This has been considered as an indication that $f(T)$ gravity has been obtaining great attention relative to GR to study different aspects of cosmological interests.

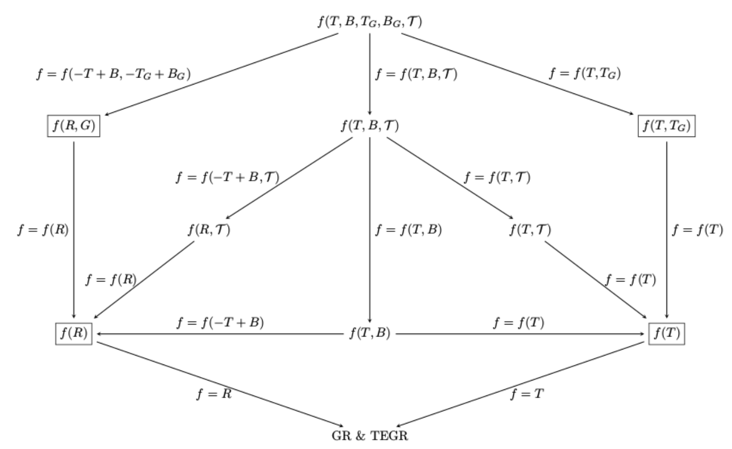


Figure: Different teleparalel gravity theories.

Accelerating universe in modified teleparallel gravity theory

Accelerating universe in modified teleparallel gravity theory

- https://doi.org/10.1017/S1743921320003567
- The thermodynamic quantities in $f(T,B)$ gravity is given as the energy density ρ_{TB} and pressure p_{TB} of the torsion-like fluid as presented in Bahamonde and Capozziello [2017]

$$\kappa^2\rho_m = -3H^2(3f_B + 2f_B) + 3H\dot{f}_B - 3\dot{H}f_B + \frac{f(T,B)}{2}\,, \tag{3}$$

$$\kappa^2 p_m = 3H^2 + \dot{H})(3f_B + 2f_T)2H\dot{f}_T - \ddot{f}_B - \frac{f(T,B)}{2}\,. \tag{4}$$

Therefore, the new general form of w_{TB} can be constructed from the above equations Eqs. (3) and (4) for the effective torsion fluid defined as $w_{TB} = p_{TB}/\rho_{TB}$ and Sahlu et al. [2019] is given by

$$w_{TB} = -1 + \frac{\ddot{f}_B - 3H\dot{f}_B - 2\dot{H}f_T - 2H\dot{f}_T}{3H^2(3f_B + 2f_T) - 3H\dot{f}_B + 3\dot{H}f_B - f(T,B)/2}\,. \tag{5}$$

Generally, in all models that are treated above, we numerically computed the effective equation of the state parameter form of $w(z)$ through the use of Eq. (5) and the value of $w(z)$ is favored with the observed value of the effective equation of state parameter of cosmological constant $w \approx -1$ in the present Universe Sahlu et al. [2020]

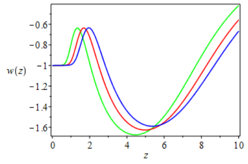


Figure: $w(z)$ versus red-shift z for exponential model

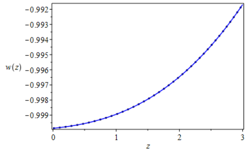


Figure: $w(z)$ versus red-shift z for linder model

- So, we clearly show that all three $f(T, B)$ gravity models can be regarded as an alternative way of the cosmological constant model to describe the late-time accelerating expansion of the Universe.
- Surprisingly, in all models the value of $w(z)$ in the present and near past Universe asymptotically approaches to the equation of state parameter of the cosmological constant $w = -1$.

The Chaplygin Gas as a Model for Modified Teleparallel Gravity?

The Chaplygin Gas as a Model for Modified Teleparallel Gravity?

https://doi.org/10.1140/epjc/s10052-019-7226-1

- In this paper, we start our main works by considering torsion as a CG with cosmological implications to try to understand the nature of dark matter and dark-energy as presented in Sahlu et al. [2019]. The Universe comprises different components, amongst them matter, radiation, and dark matter. For instance, the equation state is

$$P_T = -\frac{A}{\rho_T^{\alpha}} \tag{6}$$

- we reconstructed $f(T)$ gravity models from the different variants of the CG model, namely the original CG (OCG), the Generalized CG (GCG), and the modified generalized CG (MGCG) models.
- The idea is that, in that way, we will be able to mimic the same expansion behavior (as the CG) with a theory of gravity, in this case, $f(T)$, but without having to worry about the exoticism of the fluid with the above-mentioned problems.

- The background evolution of the Universe is investigated.
- The equation of the state parameter and the density parameter of the fluid is presented in various CG models

Linear cosmological perturbations in the modified theory of gravity

Cosmological perturbations

1. Paper 1 https://doi.org/10.1140/epjc/s10052-020-7961-3
2. Paper 2 https://link.springer.com/article/10.1140/epjc/s10052-021-09615-6
3. Paper 3 https://doi.org/10.1142/S0218271820501205

Paper 1: Scalar perturbations in $f(T)$ gravity using the $1+3$ covariant approach

- The cosmological scalar perturbations of standard matter are investigated in the context of extended teleparallel $f(T)$ gravity theories using the $1+3$ covariant formalism.
- After a review of the background gravitational field equations of $f(T)$ gravity and the introduction of the covariant perturbation variables, the usual scalar and harmonic decomposition have been performed.
- And the analysis of the growth of the density contrast in the quasi-static approximation for two non-interacting fluids scenarios, namely torsion-dust and torsion-radiation mixtures is presented for the generic $f(T)$ gravity theory.
- Special applications to two classes of $f(T)$ gravity toy models, namely $f(T)=\mu T_0\left(\frac{T}{T_0}\right)^n$ and $f(T)=T+\mu T_0\left(-\frac{T}{T_0}\right)^n$, have then been made within the observationally viable regions of their respective parameter spaces, and the growth of the matter density contrast for both torsion-dust and torsion-radiation epochs of the Universe has been examined.

- The exact solutions of the dust perturbations, with growing amplitudes in cosmic time, are obtained for some limiting cases of n.
- Similarly, the long- and short-wavelength modes in the torsion-radiation case are treated, with the amplitudes either oscillating or monotonically growing with time.
- Overall, it is noted that $f(T)$ models contain a richer set of observationally viable structure growth scenarios that can be tested against up-and-coming observational data and can accommodate currently known features of the large-scale structure power spectrum in the general relativistic and ΛCDM limits.

The Evolution equations for matter-energy density perturbations yield

$$\ddot{\Delta}^k_m = \Bigg\{ \frac{1+3w}{2f'}(1-w)\rho_m + \frac{w}{f'}(f - Tf') - \frac{2f''w}{3f'}\theta\dot{T} - w\frac{k^2}{a^2} \\ +(1+w)\left[\frac{3\rho_m f''}{2f'^2} + \frac{2f''^2}{3f'^2}\theta\dot{T} - \frac{2f'''}{3f'}\theta\dot{T}\right]\frac{2w\dot{T}\ddot{T}}{\dddot{T}}\Bigg\}\Delta^k_m \\ +\Bigg[\frac{f''}{3f'}\dot{T} + \theta\left(w - \frac{2}{3}\right) + \left(\frac{3\rho_m f''}{2f'^2} + \frac{3w\rho_m f''}{2f'^2} + \frac{2f''^2}{3f'^2}\theta\dot{T} - \\ \frac{2f'''}{3f'}\theta\dot{T}\right)\frac{w\dot{T}^2}{\dddot{T}}\Bigg]\dot{\Delta}^k_m \,. \tag{7}$$

For the case of $f(T) = T + 2\Lambda$, Eq. (7) is reduced to the well-known evolution equation of ΛCDM:

$$\ddot{\Delta}^k_m = \left[\frac{3}{2}\Omega_m(1+3w)(1-w)H^2 + 6wH^2\Omega_\Lambda - w\frac{k^2}{a^2}\right]\Delta^k_m \\ +3H\left(w - \frac{2}{3}\right)\dot{\Delta}^k_m \,, \tag{8}$$

Results

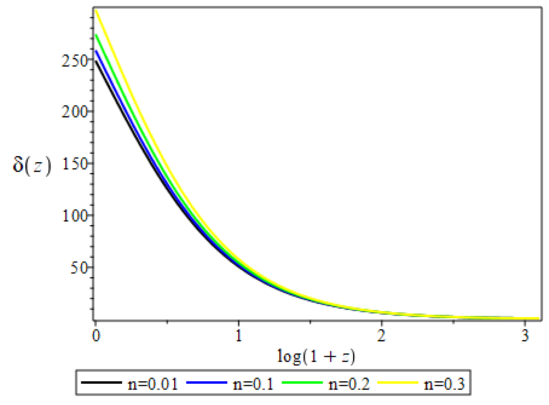


Figure: $\delta(z)$ versus z for $f(T) = T + \mu T_0 (T/T_0)^n$ in the torsion-dust system for for different values of n. We used $\Omega_d = 0.32$ for illustrative purposes.

Paper 2: Covariant density and velocity perturbations of the quasi-Newtonian cosmological model in $f(T)$ gravity

- We investigate classes of shear-free cosmological dust models with the irrotational fluid flow within the framework of $f(T)$ gravity. In particular, we use the $1+3$ covariant formalism and present the covariant linearised evolution and constraint equations describing such models.
- We then derive the integrability conditions describing a consistent evolution of the linearised field equations of these quasi-Newtonian universes in the $f(T)$ gravitational theory. Finally, we derive the evolution equations for the density and velocity perturbations of the quasi-Newtonian universe.
- Our numerical solutions show that these f (T) theories can be suitable alternatives to study the background dynamics, whereas the growth of energy density fluctuations changes dramatically from the expected LCDM behavior even for small deviations from the general relativistic limits of the underlying f (T) theory.

Paper 3: On multifluid perturbations in scalar-tensor cosmology

- In this paper, the scalar-tensor theory is applied to the study of perturbations in a multi-fluid universe.
- It is observed that the fluctuations of the energy density decrease with increasing redshift, for different values of n of a power-law R^n model.

Inflationary Universe in the modified theory of gravity

Inflationary Universe in the modified theory of gravity

1. Paper 1 https://doi.org/10.1142/S0219887821500274
2. Paper 2 https://doi.org/10.1142/S0219887818502092

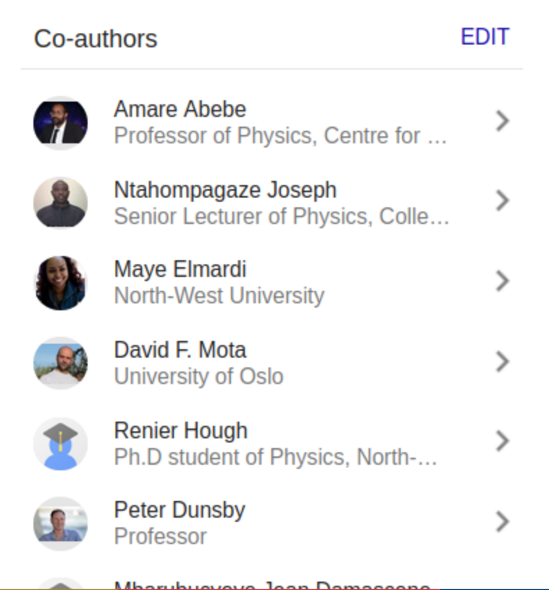
Paper 1: Inflationary constraints in teleparlallel gravity

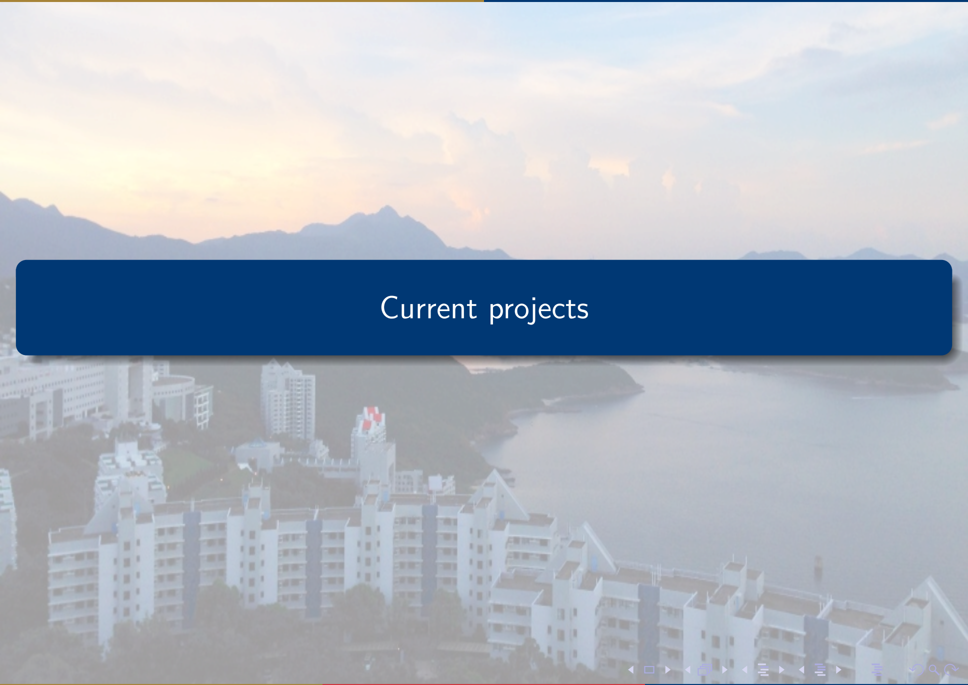
- In this work, the cosmological inflationary parameters in the correspondence of teleparallel gravity for the scalar-tensor theory is investigated.
- After the review of $f(T)$ and $f(T,B)$ gravity cosmology, we use the slow-roll approximations to study the behavior of the inflationary parameters namely: the spectral index n_s and tensor-to-scalar ratio r, and a comparison with observational data for different paradigmatic $f(T)$ gravity models such as exponential, Linder and power-law models are considered.

Paper 2: Inflation constraints for classes of $f(R)$ models

- In this paper, we explore the equivalence between two theories, namely $f(R)$ and scalar–tensor theories of gravity. We use this equivalence to explore several f(R) toy models focusing on the inflation epoch of the early universe.

Authors and Co-Authors





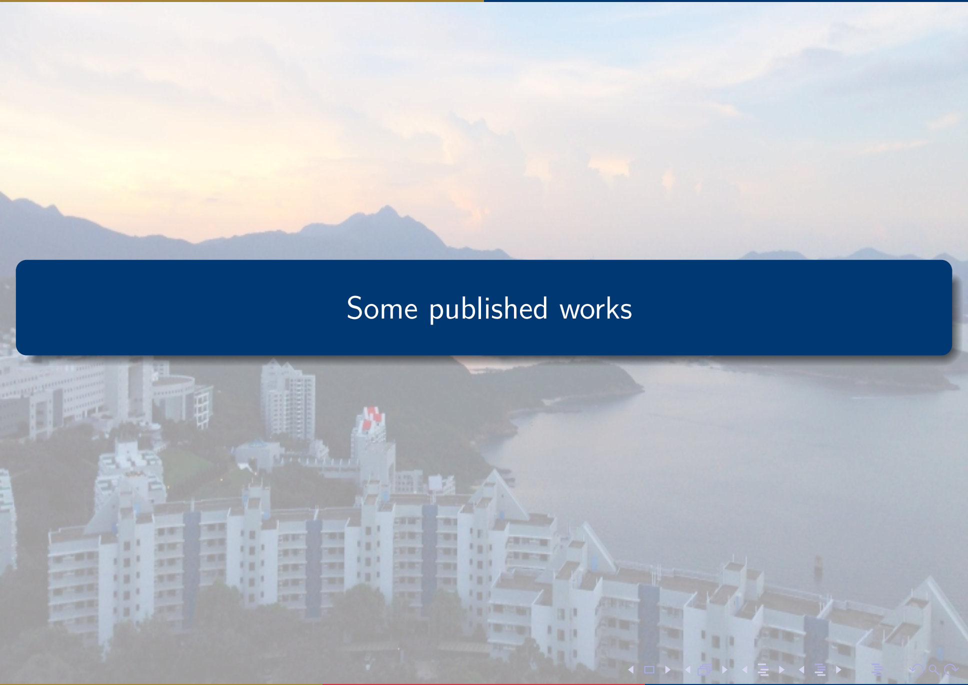
Current projects

- Confronting the Chaplygin gas with data: background and perturbed cosmic dynamics **Under Review**
- Scalar perturbations in cosmological $f(Q)$ gravity model **Submited, Part my my student's Work**
- Modified Telleparallel Gravity with SNIa, GRB and OHD data. **Manuscript Development, My student's Work**

Current project: **Chameleon gravity**

Current project: **Gravitational waves in Various Cosmological Models**

Current project: Constraint alternative cosmological models



Some published works

Some published works

- https://link.springer.com/article/10.1140/epjc/s10052-019-7226-1
- https://link.springer.com/article/10.1140/epjc/s10052-020-7961-3
- https://link.springer.com/article/10.1140/epjc/s10052-021-09615-6
- https://www.cambridge.org/core/services/aop-cambridge-core/content/view/7D402E5089A2723B4A7B3E5F573997A1/S1743921320003567a.pdf/accelerating_universe_in_modified_teleparallel_gravity_theory.pdf
- https://www.worldscientific.com/doi/abs/10.1142/S0219887821500274
- https://www.worldscientific.com/doi/abs/10.1142/S0218271820501205
- https://events.saip.org.za/event/144/papers/1359/files/776-CG_proceedings2019v3.pdf

THANK YOU!

Sebastian Bahamonde and Salvatore Capozziello. Noether symmetry approach in $f(T, B)$ teleparallel cosmology. *The European Physical Journal C*, 77(2):107, 2017.

Sebastian Bahamonde et al. Modified teleparallel theories of gravity. *Physical Review D*, 92(10):104042, 2015.

Shambel Sahlu et al. The Chaplygin gas as a model for modified teleparallel gravity? *The European Physical Journal C*, 79(9):749, 2019.

Shambel Sahlu et al. Accelerating universe in modified teleparallel gravity theory. *arXiv preprint arXiv:2003.10908*, 2020.