Quantum Information Theoretic Aspects in Neutrino Oscillations

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Plan of this talk

Motivation

- Foundations of quantum mechanics are usually studied in optical or electronic systems. Quantum correlation is a central topic of investigations in the quest for an understanding as well as for harvesting the power of quantum mechanics in a plethora of systems like quantum optics, spin systems etc.
- Recently, some measures of quantum correlations have been investigated for the systems of unstable mesons viz. B and K-mesons and for neutrino oscillations.
- Neutrinos can be potential candidates for transmitting quantum information and was demonstrated by Stancil et al. (2012).
- Open problem in neutrino sector:
  Neutrino mass hierarchy (Unknown sign of Δ_{31}, $+$ or $-$),
  Is there CP-violation?
- We study some measures of quantum correlation such as Bell-type inequalities viz. Mermin inequality, Svetlichny inequality and some other measures like flavor entropy and geometric entanglement. These quantities are found to be sensitive to the neutrino mass ordering as well as to the effects of nonstandard interaction (NSI).

Quantum mechanics in neutrino oscillations

- The three flavor states (eigenstates of weak interaction, which are detectable in lab) of neutrinos, ν_e, ν_μ and ν_τ mix via a 3×3 unitary matrix to form the three mass eigenstates (which are the propagation eigenstates) ν_1, ν_2 and ν_3. Neutrino oscillations occur only if the three corresponding masses, m_1, m_2 and m_3, are non-degenerate.
- In three flavor neutrino oscillation
  Propagation states $\rightarrow \{|\nu_1\rangle\,, \;\; |\nu_2\rangle\,, \;\; |\nu_3\rangle\}$;
  Flavor states $\rightarrow \{|\nu_e\rangle\,, \;\; |\nu_\mu\rangle\,, \;\; |\nu_\tau\rangle\}$
- The general state of a neutrino can be expressed in flavor basis as:

$$|\Psi(t)\rangle = \nu_e(t)\,|\nu_e\rangle + \nu_\mu(t)\,|\nu_\mu\rangle + \nu_\tau(t)\,|\nu_\tau\rangle$$

- Same state in propagation basis looks like:

$$|\Psi(t)\rangle = \nu_1(t)\,|\nu_1\rangle + \nu_2(t)\,|\nu_2\rangle + \nu_3(t)\,|\nu_3\rangle$$

- The coefficients in two representations are connected by a *unitary* matrix

$$\begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \\ \nu_\tau(t) \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1(t) \\ \nu_2(t) \\ \nu_3(t) \end{pmatrix}.$$

  or,

$$\nu_\alpha(t) = \mathbf{U}\nu_i(t). \tag{1}$$

Quantum mechanics in neutrino oscillations

- A convenient parametrization for **U** or $U(\theta_{12},\theta_{23},\theta_{13},\delta)$ is given by the PMNS matrix

$$U(\theta_{12},\theta_{23},\theta_{13},\delta)=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{23}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{13}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- where $c_{ij}=\cos\theta_{ij}$, $s_{ij}=\sin\theta_{ij}$, θ_{ij} being the mixing angles and δ the CP (Charge-Parity) violating phase.
- The mass eigenstates evolve as

$$\begin{pmatrix}\nu_1(t)\\ \nu_2(t)\\ \nu_3(t)\end{pmatrix}=\begin{pmatrix} e^{-iE_1t} & 0 & 0\\ 0 & e^{-iE_2t} & 0\\ 0 & 0 & e^{-iE_3t}\end{pmatrix}\begin{pmatrix}\nu_1(0)\\ \nu_2(0)\\ \nu_3(0)\end{pmatrix},$$

or,

$$\nu_{\mathbf{m}}(t)=\mathbf{E}\nu_{\mathbf{m}}(0) \tag{2}$$

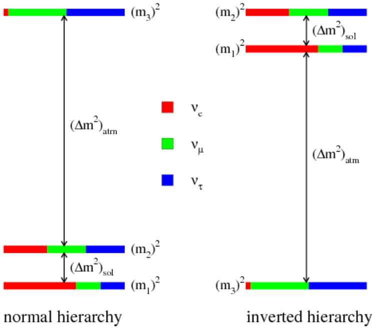
- From 1 and 2, $\nu_{\mathbf{f}}(t)=\mathbf{U}\ \mathbf{E}\mathbf{U}^{-1}\ \nu_{\mathbf{f}}(0)=\mathbf{U}_f\ \nu_{\mathbf{f}}(0)$.

$$\begin{aligned} P_{\alpha\beta} &= \delta_{\alpha\beta}-4\sum_{i>j} Re(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\sin^2\left(1.27\frac{\Delta_{ij}L}{E}\right)\\ &\quad +2\sum_{i>j} Im(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\sin\left(2.54\frac{\Delta_{ij}L}{E}\right) \end{aligned} \tag{3}$$

where $\Delta_{ij}=m_j^2-m_i^2\equiv E_j-E_i$.

Problems not resolved yet . . .

- Neutrino mass hierarchy problem i,e., whether $m_1 \leq m_2 \leq m_3$ or $m_3 \leq m_1 \leq m_2$).
- CP violation ($\delta \neq 0$).
  $P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \neq P(\nu_\alpha \to \nu_\beta)$
- Absolute mass



Neutrino experimental facilities

We included accelerator ν_μ- neutrino experiments experimental conditions in our study such as

DUNE (L = 1300 Km, E = 1 - 10 GeV, $A = 1.7\times10^{-13}$ eV)

NOνA (L = 810 Km, E = 1 - 4 GeV, $A = 1.7\times10^{-13}$ eV)

T2K (L = 295 Km, E = 0.1 - 1 GeV, $A = 1.01\times10^{-13}$ eV)

($L \rightarrow$ baseline, $E \rightarrow$ neutrino-energy, $A \rightarrow$ matter density potential)

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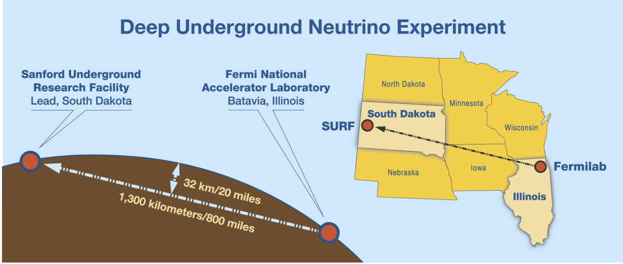


Foundational issues in quantum mechanics

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

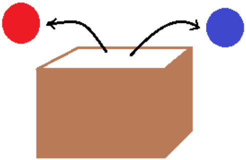
Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

Spatial Quantum Correlations



Classical Correlation

Quantum Correlation

Measurement process discerns the two types of correlations.

Observables don't have preassigned values. The measurement process forces an observable to take a particular value.

Entanglement $\equiv$ nonseparability

$\underline{\textit{Separable states}}$: $|1\rangle_A \otimes |0\rangle_B \equiv |10\rangle_{AB}$;

$$\frac{1}{2}\left(|00\rangle_{AB} + |01\rangle_{AB} + |10\rangle_{AB} + |11\rangle_{AB}\right) = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)_A \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)_B$$

$\underline{\textit{Entangled states}}$: $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right)$

Spatial Quantum Correlations

A system is separable if

$$\rho_{\alpha\beta} = \rho_\alpha \otimes \rho_\beta$$

where ρ_α and $\rho_\beta \rightarrow$ pure states
System is entangled if

$$\rho_{\alpha\beta} \neq \rho_\alpha \otimes \rho_\beta$$

- **Von-Neumann Entanglement Entropy:**

$$S(\rho_j) = -Tr(\rho_j \log_2(\rho_j))$$

and

$$S(\rho_j) = \begin{cases} 0 & \textit{for pure state} \\ \log_2 d & \textit{for mixed state}, \end{cases} \qquad d \rightarrow \textit{dimension}$$

For a system defined by $\rho_{\alpha\beta}$, entanglement can be measured in the form of the von-Neumann entropy of the reduced density ρ_α matrix representing one of the sub-systems.

Spatial Quantum Correlations

LOCALITY: A measurement made on a system cannot influence other systems instantaneously.
REALISM: A system has well defined values of an observable whether someone measures it or not. Measurement process simply reveals these values to us.

Bell's inequality

Probability of a coincidence between separated measurements of particles with correlated (e.g. identical or opposite) orientation properties

$$P(a,b) = \int d\lambda \rho(\lambda) p_A(a,\lambda) p_B(b,\lambda)$$

where, $p_A(a,\lambda)$ is the probability of detection of particle A with hidden variable λ by detector A, set in direction a, and similarly $p_B(b,\lambda)$ is the probability at detector B, set in direction b, for particle B, sharing the same value of λ. The source is assumed to produce particles in the state λ with probability $\rho(\lambda)$

The inequality $P(a,b) - P(a,c) \leq 1 + P(b,c)$
or
$|\langle M_a M_b\rangle - \langle M_a M_c\rangle| \leq 1 + \langle M_b M_c\rangle$

Temporal quantum correlations

Leggett-Garg inequality (LGI) (PRL **54**, 857 (1985)) follows two concepts:

- *macrorealism (MR):* the system which has available to it two or more macroscopically distinct states, pertaining to an observable $\hat{Q}$, always exists in one of these states irrespective of any measurement performed on it.
- *noninvasive measurability (NIM):* we can perform the measurement without disturbing the future dynamics of the system.

The simplest form of LGI is the one involving three measurements performed at time t_0, t_1 and t_2 ($t_0 \leq t_1 \leq t_2$) (three-time measurement)

$$K_3 = C_{01} + C_{12} - C_{02}$$

where, $C_{ij} = \langle Q(\hat{t}_i) Q(\hat{t}_j) \rangle$ (the two-time correlation function) and bounds on K_3 are obtained as $-3 \leq K_3 \leq 1$. $\hat{Q} \rightarrow$ dichotomic observable (with possible outcomes ± 1), $\hat{Q}^\dagger = \hat{Q}$.

Single particle (mode) entanglement in 2-flavor neutrino oscillations

- One can establish the following correspondence of a two flavor state with two-qubit state (Blasone et al., PRD **77**, 096002 (2008))

$$\left.\begin{aligned} |\nu_e\rangle &\equiv |1\rangle_e |0\rangle_\mu \\ |\nu_\mu\rangle &\equiv |0\rangle_e |1\rangle_\mu \end{aligned}\right\} \text{occupation no. represention} \tag{4}$$

  where $|0\rangle_\alpha \rightarrow$ absence of neutrino in mode α
  $|1\rangle_\alpha \rightarrow$ presence of neutrino in mode α.

- State of a neutrino in two flavor neutrino oscillation scheme

$$\begin{aligned} |\psi(t)\rangle &= U_{ee} |\nu_e\rangle + U_{e\mu} |\nu_\mu\rangle \\ &= U_{ee} |1\rangle_e |0\rangle_\mu + U_{e\mu} |0\rangle_e |1\rangle_\mu \end{aligned}$$

- Entanglement (non-separability) is established among flavor modes, in a single-particle setting.

Quantum correlations in terms of neutrino oscillation probabilities (Alok et al., NPB 909 (2016))

If a state is given by ρ (density matrix operator) and matrix $T = Tr(\rho(\sigma_m \otimes \sigma_n))$ is defined, then

- Bell-CHSH inequality : $M(\rho) = max(u_i + u_j) \leq 1$, where u_i and $u_j \rightarrow$ eigenvalues of T (violation shows nonlocality). for neutrinos
$$M(\rho) = 1 + 4P_{sur}P_{osc}$$
- Concurrence (entanglement measure) : $C = max(\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0)$, $\lambda_i \rightarrow$ square roots of eigenvalues of $\rho\tilde{\rho}$, $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ for neutrino system (nonzero C $\equiv$ entanglement)
$$C = 2\sqrt{P_{sur}P_{osc}}$$

Leggett-Garg inequality (LGI) in neutrino oscillations
(Formaggio et al., PRL **117**, 050402 (2016))

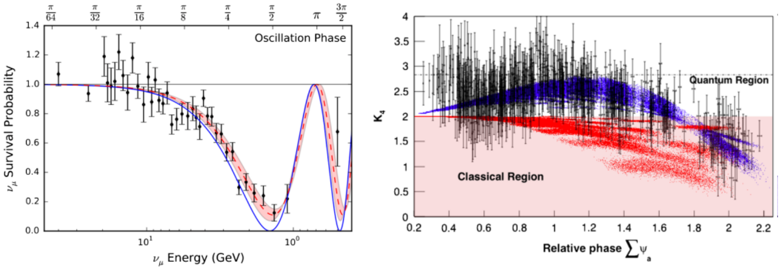


Figure: Experimental verification of LGI-violation in neutrino-system

- MINOS experiment's data shows a greater than 6σ violation.

I. Correlation Measures in 3-flavor neutrino oscillations

State of a neutrino in three flavor neutrino oscillation framework mapped over three qubit system with ν_μ as initial state

$$|\psi(t)\rangle = U_{\mu e}\,|1\rangle_e\,|0\rangle_\mu\,|0\rangle_\tau + U_{\mu\mu}\,|0\rangle_e\,|1\rangle_\mu\,|0\rangle_\tau + U_{\mu\tau}\,|0\rangle_e\,|0\rangle_\mu\,|1\rangle_\tau$$

- **Flavor Entropy** : The von Neumann entropy for a state ρ in a d-dimensional space is defined as

$$\mathcal{S}(|\psi(t)\rangle) = -\sum_{j=e,\mu,\tau} Tr(\rho_j \log \rho_j) \qquad \left(\rho_j = Tr_{\text{all but not subsystem j}}\,|\psi(t)\rangle\,\langle\psi(t)|\,\right)$$

$$= -\sum_{\beta} \mid U_{\mu\beta}\mid^2 \ \log_2 \mid U_{\mu\beta}\mid^2 - \sum_{\beta}(1-\mid U_{\mu\beta}\mid^2)\ \log_2(1-\mid U_{\mu\beta}\mid^2)$$

$$S(\rho) = \begin{cases} 0 & \textit{for separable state} \\ d\ \ log_2(d)\ \ -(d-1)log_2(d-1) & \textit{for totally entangled state} \end{cases}$$

- **Tripartite entanglement** : For a tripartite system, the geometric entanglement is defined as the cube of the geometric mean of Shanon entropy over every bipartite section.

$$G = H(U^2_{\mu e})H(U^2_{\mu\mu})H(U^2_{\mu\tau})$$

where $H(U^2_{\mu\beta}) = -U^2_{\mu\beta}log_2(U^2_{\mu\beta}) - (1-U^2_{\mu\beta})\ log_2(1-U^2_{\mu\beta}),\ \beta \equiv e, \mu, \tau$

I. Correlation Measures in 3-flavor neutrino oscillations

Tripertite nonlocality :

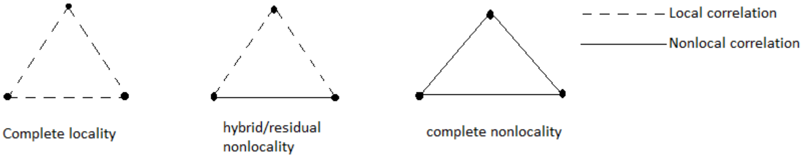
- A three qubit system may be nonlocal if nonclassical correlations exist between two of the three qubits. Such a state will be absolute nonlocal and will violate the *Mermin inequality* for a detector setting A, B and C. Mermin inequalities are:

$$M_1 \equiv \langle ABC' \rangle + \langle AB'C \rangle + \langle A'BC \rangle - \langle A'B'C' \rangle \leq 2$$

$$M_2 \equiv \langle ABC \rangle - \langle A'B'C \rangle - \langle A'BC' \rangle - \langle AB'C' \rangle \leq 2$$

- A state violating a Mermin inequality may fail to violate a *Svetlichny inequality* which provides a sufficient condition for genuine tripartite nonlocality. Svetlichny inequality is:

$$\sigma \equiv M_1 + M_2 \leq 4$$



K. Dixit, J. Naikoo, S. Banerjee, A. K. Alok, Euro. Phys. J. C, 78 914 (2018)

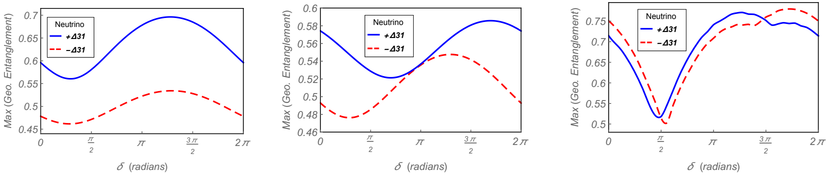


Figure: The maximum of geometric entanglement (GE) is plotted against CP-phase for DUNE (left), NOνA (middle), T2K (right) experiments with neutrino (ν_μ) beam. Solid(blue) and dashed(red) curves correspond to the positive and negative signs of Δ_{31}, respectively. The mixing angles and the squared mass differences used are $\theta_{12} = 33.48^o$, $\theta_{23} = 42.3^o$, $\theta_{13} = 8.5^o$, $\Delta_{21} = 7.5 \times 10^{-5} eV^2$, $\Delta_{32} \approx \Delta_{31} = 2.457 \times 10^{-3} eV^2$. The neutrinos pass through a matter density of 2.8 gm/cc

- Entanglement exists both in terms of absolute and genuine manner, Mermin and Svetlichny inequalities are violated.
- Various correlation measures show sensitivity to the neutrino mass ordering.
- DUNE is the most prominent experimental setup to discriminate the effects of normal and inverted mass orderings. It can be attributed to its long baseline and higher neutrino-energy range.

K. Dixit, J. Naikoo, S. Banerjee, A. K. Alok, Euro. Phys. J. C, 78 914 (2018)

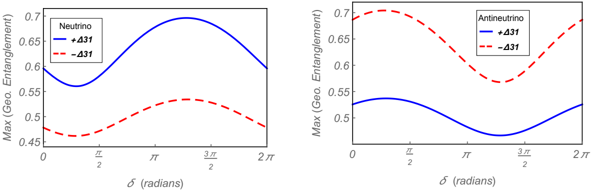


Figure: The maximum of geometric entanglement (GE) is plotted against CP-phase for DUNE with neutrino (ν_μ) beam (left) and for antineutrino ($\bar{\nu}_\mu$) beam source (right). Solid(blue) and dashed(red) curves correspond to the positive and negative signs of Δ_{31}, respectively.

- From QIP point of view, to test the nonclassicality embedded in the neutrino system, one should employ (anti)neutrino beam as source in case of (inverted)normal mass ordering.

II. NSI effect on quantum correlations

Motivation: Neutrino stands the test of entanglement and nonlocality. It becomes pertinent to characterize the quantum nature of neutrinos under different circumstances. The simplest parameter defining quantumness of a system can be *quantum coherence*.

- Coherence: A measure of quantumness embedded in a system; a key concept in quantum mechanics & information theory.
- Quantum coherence is closely related to various measures of quantum correlations, such as entanglement.
- Recently, quantum coherence has been quantified in terms of experimentally observed neutrino survival and transition probabilities (Song et al., PRA **98**, 050302(R) (2018)).
- In this work we study the effects of nonstandard neutrino-matter interaction on coherence in the oscillating neutrino system in a model-independent approach in the context of DUNE experimental setup.

Definition

Coherence For a d-dimensional state

$$\rho = |\psi\rangle \langle\psi| ,$$

the l_1-norm of coherence parameter is formulated as

$$C = \sum_{i \neq j} |\rho_{ij}| \leq d - 1$$

NSI effect: Model independent analysis

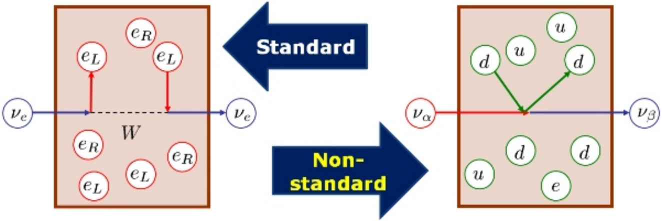
The Lagrangian for neutral-current nonstandard neutrino-matter interactions (*NSI*)

$$\mathcal{L}_{NSI} = -2\sqrt{2}G_F \sum_{f,P,\alpha,\beta} \epsilon^{f,P}_{\alpha,\beta} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta)(\bar{f}\gamma_\mu P f)$$

with $\epsilon^{f,P}_{\alpha,\beta} \equiv \epsilon^{\eta}_{\alpha\beta}\, \xi^{f,P} \sim \mathcal{O}(G_x/G_F)$. The matter part V_f of Hamiltonian $\mathcal{H}_m = H_m + U^{-1} V_f U$ ($H_m = \mathrm{diag}(\mathrm{E}_1, \mathrm{E}_2, \mathrm{E}_3)$), defined for the evolution of neutrino-state, becomes

$$V_f = \sqrt{2}G_F N_e(x) \begin{pmatrix} 1+\epsilon_{ee}(x) & \epsilon_{e\mu}(x) & \epsilon_{e\tau}(x) \\ \epsilon^*_{e\mu}(x) & \epsilon_{\mu\mu}(x) & \epsilon_{\mu\tau}(x) \\ \epsilon^*_{e\tau}(x) & \epsilon^*_{\mu\tau}(x) & \epsilon_{\tau\tau}(x) \end{pmatrix} \text{ with } \epsilon_{\alpha\beta} = \sum_{f=e,u,d} \frac{N_f(x)}{N_e(x)} \epsilon^f_{\alpha\beta}.$$

U is the 3×3 unitary (PMNS) matrix.



K. Dixit and A. K. Alok, Eur. Phys. J. Plus (2021) 136:334

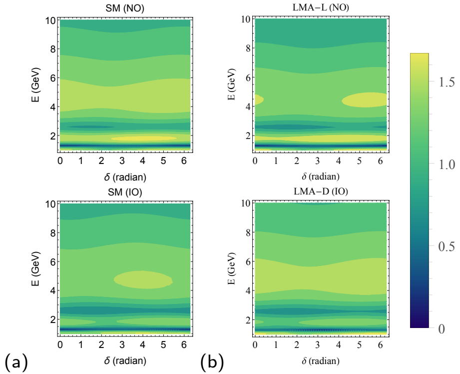


Figure: C plotted in (E - δ) plane in the context of DUNE ($L = 1300$ km & $E = 1 - 10$ GeV) experiment: (a) Upper panel (SM+NO) and lower panel (SM+IO). (b) Upper panel (LMA-Light + NO) and lower panels (LMA-Dark + IO). Minimum value (zero) of χ represents the complete loss of coherence whereas for a maximally coherent state $\chi = 2$.

- LMA-Light + NO solution decreases the coherence in comparison to the SM + NO.
- For LMA-Dark + IO, coherence is enhanced in comparison to the case of SM + IO for E $\approx$ 4 GeV, the energy corresponding to maximum neutrino flux at DUNE, for almost all values of δ.
- The maximum value $C = 2$ cannot be achieved by three flavour neutrino oscillation in the context of DUNE experiment.

K. Dixit and A. K. Alok, Eur. Phys. J. Plus (2021) 136:334

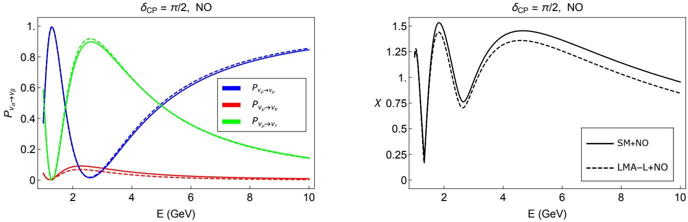


Figure: In the left panel probabilities $P_{\nu_\mu \to \nu_\mu}$ (blue), $P_{\nu_\mu \to \nu_e}$ (red) and $P_{\nu_\mu \to \nu_\tau}$ (green) are plotted with respect to E in the context of DUNE ($L = 1300$ km) experiment for $\delta = \pi/2$ and normal ordering, where solid and dashed lines correspond to the SM and NSI interaction, respectively. The right panel shows the variations of C parameter with E for $\delta = \pi/2$ and NO.

- A small change in probabilities due to NSI effects can trigger relatively large alteration in the coherence inherent in the neutrino-system.

Recent work

with Prof. S. Razzaque, U. of Johannesburg & Dr. S. Haque., U. of Cape Town

Complexity of spread of neutrino flavor states

Quantum complexity of spread of states

Motivation

- Quantum computational complexity estimates the difficulty of constructing quantum states from elementary operations, a problem of prime importance for quantum computation.
- Neutrinos have shown features such as entanglement and nonlocal correlations that proves their efficiency to perform QIP tasks.
- It gives us motivation to see how complex is a evolution of neutrino system and if complexity can also probe any open issue in the neutrino sector.

Complexity of spread of states

Balasubramanian et al., PRD 106, 046007 (2022)

- The complexity of the state can be defined by minimizing the spread of the wavefunction over all possible bases.
- This minimum is uniquely attained by an orthonormal basis produced by applying the Gram-Schmidt procedure.

Schrodinger equation for a system represented by $|\psi(t)\rangle$

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle$$

Then, the time evolution of the state $|\psi(t)\rangle$ is obtained as

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle .$$

One can also write

$$|\psi(t)\rangle = \sum_{n=0}^{\infty}\frac{(-it)^n}{n!}H^n|\psi(0)\rangle = \sum_{n=0}^{\infty}\frac{(-it)^n}{n!}|\psi_n\rangle ,$$

where, $|\psi_n\rangle = H^n|\psi(0)\rangle$. Hence, we can see that the time evolved system-state $|\psi(t)\rangle$ is represented as superposition of infinite $|\psi_n\rangle$ states.

Complexity of spread of states

We have $|\psi_n\rangle = H^n |\psi(0)\rangle$. These states $\{|\psi_0\rangle, |\psi_1\rangle, |\psi_2\rangle, \ldots\}$ are not orthonomalized. Gram-Schmidt procedure to obtain an ordered orthonomalized basis

$$|K_0\rangle = |\psi_0\rangle \,,$$
$$|K_1\rangle = |\psi_1\rangle - \frac{\langle K_0|\psi_1\rangle}{\langle K_0|K_0\rangle} |K_0\rangle \,,$$
$$|K_2\rangle = |\psi_2\rangle - \frac{\langle K_0|\psi_2\rangle}{\langle K_0|K_0\rangle} |K_0\rangle - \frac{\langle K_1|\psi_2\rangle}{\langle K_1|K_1\rangle} |K_1\rangle \,, \quad \text{and so on.}$$

$\mathcal{K} = \{|K_n\rangle\,, n = 0, 1, 2 \ldots\} \Rightarrow$ Krylov basis

Cost function to quantify the complexity (Balasubramanian et al., PRD 106, 046007 (2022))
For a time evolved state $|\psi(t)\rangle$ and the Krylov basis defined as $\{|K_n\rangle\}$, the cost function is

$$\chi = \sum_{n=0}^{\infty} n|\langle K_n|\psi(t)\rangle|^2,$$

where $n = 0, 1, 2 \ldots$ For such Krylov basis the above defined cost function becomes minimum.

Spread complexity in two flavor neutrino oscillations

The evolution of flavor states can be represented by Schrodinger equation as

$$i\frac{\partial}{\partial t}\begin{pmatrix}|\nu_e(t)\rangle\\|\nu_\mu(t)\rangle\end{pmatrix} = H_f\begin{pmatrix}|\nu_e(t)\rangle\\|\nu_\mu(t)\rangle\end{pmatrix} \tag{5}$$

where $H_f = UH_mU^{-1}$, U being the mixing matrix and H_m is the Hamiltonian (diagonal) that governs the time evolution of neutrino mass eigenstate

$$H_m = \begin{pmatrix}E_1 & 0\\0 & E_2\end{pmatrix}, \qquad U = \begin{pmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{pmatrix}.$$

$$|\nu_e(0)\rangle = \begin{pmatrix}1\\0\end{pmatrix}, \qquad |\nu_\mu(0)\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

We have

$$\{|\psi_n\rangle\} = \begin{cases}\{|\nu_e(0)\rangle, H_f|\nu_e(0)\rangle, H_f^2|\nu_e(0)\rangle\dots\} & \textit{for initial } \nu_e \textit{ flavor}\\ \{|\nu_\mu(0)\rangle, H_f|\nu_\mu(0)\rangle, H_f^2|\nu_\mu(0)\rangle\dots\} & \textit{for initial } \nu_\mu \textit{ flavor}\end{cases}$$

After applying Gram-Schmidt procedure we get $\{|K_n\rangle\} = \{|K_0\rangle, |K_1\rangle\}$, *i.e.*,

$$\{|K_n\rangle\} = \begin{cases}\{|K_0\rangle = \begin{pmatrix}1\\0\end{pmatrix}, |K_1\rangle = \begin{pmatrix}0\\1\end{pmatrix}\} = \{|\nu_e\rangle, |\nu_\mu\rangle\} & \textit{for initial } \nu_e\\ \{|K_0\rangle = \begin{pmatrix}0\\1\end{pmatrix}, |K_1\rangle = \begin{pmatrix}1\\0\end{pmatrix}\} = \{|\nu_\mu\rangle, |\nu_e\rangle\} & \textit{for initial } \nu_\mu\end{cases}$$

Spread complexity in two flavor neutrino oscillations

For a time evolved state $|\nu_e(t)\rangle = \begin{pmatrix} A_{ee}(t) \\ A_{e\mu}(t) \end{pmatrix} = \begin{pmatrix} \cos^2\theta e^{-iE_1 t} + \sin^2\theta e^{-iE_2 t} \\ \sin\theta\cos\theta(e^{-iE_2 t} - e^{-iE_1 t}) \end{pmatrix}$
(with $\{|K_n\rangle\} = \{|\nu_e(0)\rangle, |\nu_\mu(0)\rangle\}$)

$$\chi_e = \sum_{n=0}^{1} n|\langle K_n|\nu_e(t)\rangle|^2 = P_{e\mu}$$

Similarly, for state $|\nu_\mu(t)\rangle = (A_{\mu e}(t), A_{\mu\mu}(t))^T$ (with $\{|K_n\rangle\} = \{|\nu_\mu(0)\rangle, |\nu_e(0)\rangle\}$)

$$\chi_\mu = P_{\mu e}$$

- The more the oscillation probability of neutrino flavor, the more complex the evolution of the neutrino flavor state.
- Since $P_{e\mu} = P_{\mu e}$ for standard vacuum oscillations, the complexity embedded in this system comes out to be same for both cases of initial flavor, *i.e.,* complexity of the system doesn't depend on the initial flavor of neutrino.

Spread complexity in three flavor neutrino oscillations

We have three types of initial states as $|\nu_e\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $|\nu_\mu\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix}$, $|\nu_\tau\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}$ with Hamiltonian $H_f = UH_mU^{-1}$, $H_m = diag(0, \Delta m^2_{21}, \Delta m^2_{31})$ and U $\rightarrow 3 \times 3$ PMNS mixing matrix

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{13}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Here, Krylov basis $\neq$ flavor basis.

- For initial $|\nu_e\rangle$ state $|K_0\rangle \equiv |\nu_e\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}$, other states spanning the Krylov basis take the form

$$|K_1\rangle = N_1 \begin{pmatrix}0\\a_1\\a_2\end{pmatrix} = N_1 \begin{pmatrix} 0 \\ \left(\frac{\Delta m^2_{21}}{2E}\right) U^*_{e2}U_{\mu 2} + \left(\frac{\Delta m^2_{31}}{2E}\right) U^*_{e3}U_{\mu 3} \\ \left(\frac{\Delta m^2_{21}}{2E}\right) U^*_{e2}U_{\tau 2} + \left(\frac{\Delta m^2_{31}}{2E}\right) U^*_{e3}U_{\tau 3} \end{pmatrix},$$

$$|K_2\rangle = N_2 \begin{pmatrix}0\\b_1\\b_2\end{pmatrix} = N_2 \begin{pmatrix} 0 \\ \left(\frac{\Delta m^2_{21}}{2E}\right)\left(\frac{\Delta m^2_{21}}{2E} - A\right) U^*_{e2}U_{\mu 2} + \left(\frac{\Delta m^2_{31}}{2E}\right)\left(\frac{\Delta m^2_{31}}{2E} - A\right) U^*_{e3}U_{\mu 3} \\ \left(\frac{\Delta m^2_{21}}{2E}\right)\left(\frac{\Delta m^2_{21}}{2E} - A\right) U^*_{e2}U_{\tau 2} + \left(\frac{\Delta m^2_{31}}{2E}\right)\left(\frac{\Delta m^2_{31}}{2E} - A\right) U^*_{e3}U_{\tau 3} \end{pmatrix}$$

Spread complexity in three flavor neutrino oscillations

$$\boxed{\begin{aligned}\chi_e = P_{e\mu}(t)(N_1^2|a_1|^2 + 2N_2^2|b_1|^2) + P_{e\tau}(t)(N_1^2|a_2|^2 + 2N_2^2|b_2|^2) + 2\Re(N_1^2 a_1^* a_2 A_{e\mu}(t)A_{e\tau}(t)^*) \\ + 4\Re(N_2^2 b_1^* b_2 A_{e\mu}(t)A_{e\tau}(t)^*)\end{aligned}}$$

with

$$A = \frac{\begin{pmatrix} \left(\Delta m_{21}^2\right)^3 |U_{\alpha 2}|^2(1-|U_{\alpha 2}|^2) + \left(\Delta m_{31}^2\right)^3 |U_{\alpha 3}|^2(1-|U_{\alpha 3}|^2) \\ - \left(\Delta m_{21}^2\right)\left(\Delta m_{31}^2\right)|U_{\alpha 2}|^2|U_{\alpha 3}|^2\left(\Delta m_{21}^2 + \Delta m_{31}^2\right)\end{pmatrix}}{\left(\Delta m_{21}^2\right)^2 |U_{\alpha 2}|^2(1-|U_{\alpha 2}|^2) + \left(\Delta m_{31}^2\right)^2 |U_{\alpha 3}|^2(1-|U_{\alpha 3}|^2) - 2\left(\Delta m_{21}^2\right)\left(\Delta m_{31}^2\right)|U_{\alpha 2}|^2|U_{\alpha 3}|^2},$$

Spread complexity in neutrino oscillations

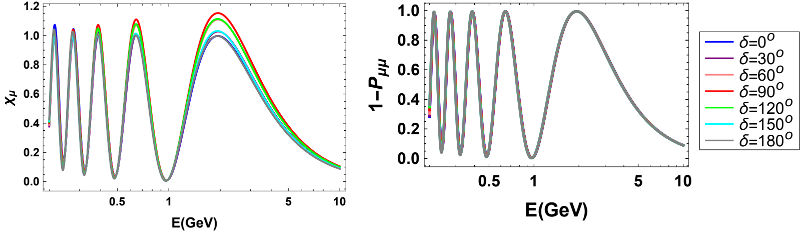


Figure: Cost function χ_μ and 1-$P_{\mu\mu}$ with respect to energy E. Here, $L = 1000$ km and mixing parameters $\theta_{12} = 33.64^o$, $\theta_{13} = 8.53^o$, $\theta_{23} = 47.63^o$, $\Delta m^2_{21} = 7.53 \times 10^{-5}\ eV^2$ and $\Delta m^2_{31} = 2.45 \times 10^{-3}\ eV^2$ are considered.

Spread complexity in neutrino oscillations

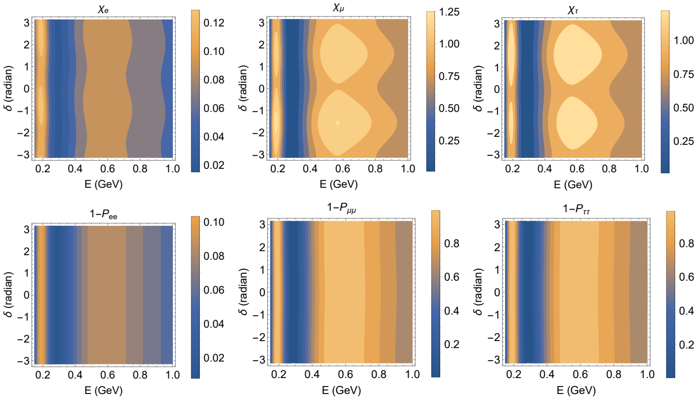


Figure: T2K: Cost function (upper panel) and 1-$P_{\alpha\alpha}$ (lower panel) in the plane of $E-\delta$ in case of initial flavor ν_e (left), ν_μ (middle) and ν_τ (right). Here, $L = 295$ km and mixing parameters $\theta_{12} = 33.64^o$, $\theta_{13} = 8.53^o$, $\theta_{23} = 47.63^o$, $\Delta m^2_{21} = 7.53 \times 10^{-5}\ eV^2$ and $\Delta m^2_{31} = 2.45 \times 10^{-3}\ eV^2$ are considered.

Spread complexity in neutrino oscillations

Matter effects on complexity

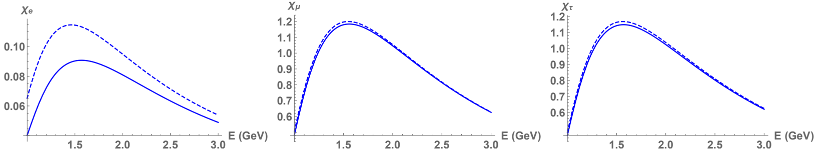


Figure: Cost function χ_e (left), χ_μ (middle) and χ_τ (right) w. r. t. neutrino-energy E is shown. Here, $L = 810$ km, $\delta = -90^o$ and higher octant of θ_{23} is considered. Solid and dashed curves represent the case of vacuum and matter oscillations, respectively.

Spread complexity in neutrino oscillations

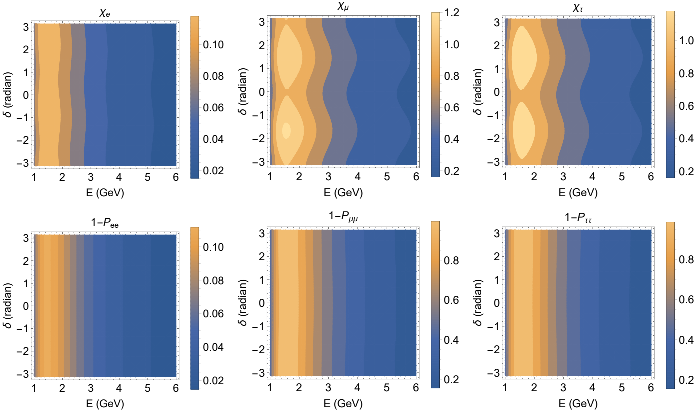


Figure: NOνA: Cost function (upper panel) and 1-$P_{\alpha\alpha}$ (lower panel) in the plane of $E - \delta$ in case of initial flavor ν_e (left), ν_μ (middle) and ν_τ (right). Here, $L = 810$ km, and higher octant of θ_{23} (47.63^o) is considered.

Summary & Conclusions

- Quantum correlations show sensitivity to the neutrino mass ordering, i.e. the sign of Δ_{31}. It is a general feature displayed by all the correlations that the sensitivity to the mass ordering becomes more prominent for the high energy and long baseline experiment like DUNE compared to NOνA and T2K.
- In order to probe various measures of nonclassicality in neutrino sector, one must use neutrino beam for the positive sign of Δ_{31} and an antineutrino beam otherwise.
- Coherence parameter shows more deviation from its SM value due to NSI effects in comparison to the probabilities, both in case of normal and inverted mass ordering. Hence, measurement of coherence and other correlation features can also be used to probe new physics in neutrino sector.
- We formulated and derived the quantum spread complexity embedded in the neutrino flavor states both for two and three flavor oscillation scenario.
- The complexity is found to be sensitive to the CP-violation phase hence it can provide some important information regarding its preferred value by nature.

THANK YOU