Massive white dwarfs and neutron stars in noncommutative geometry

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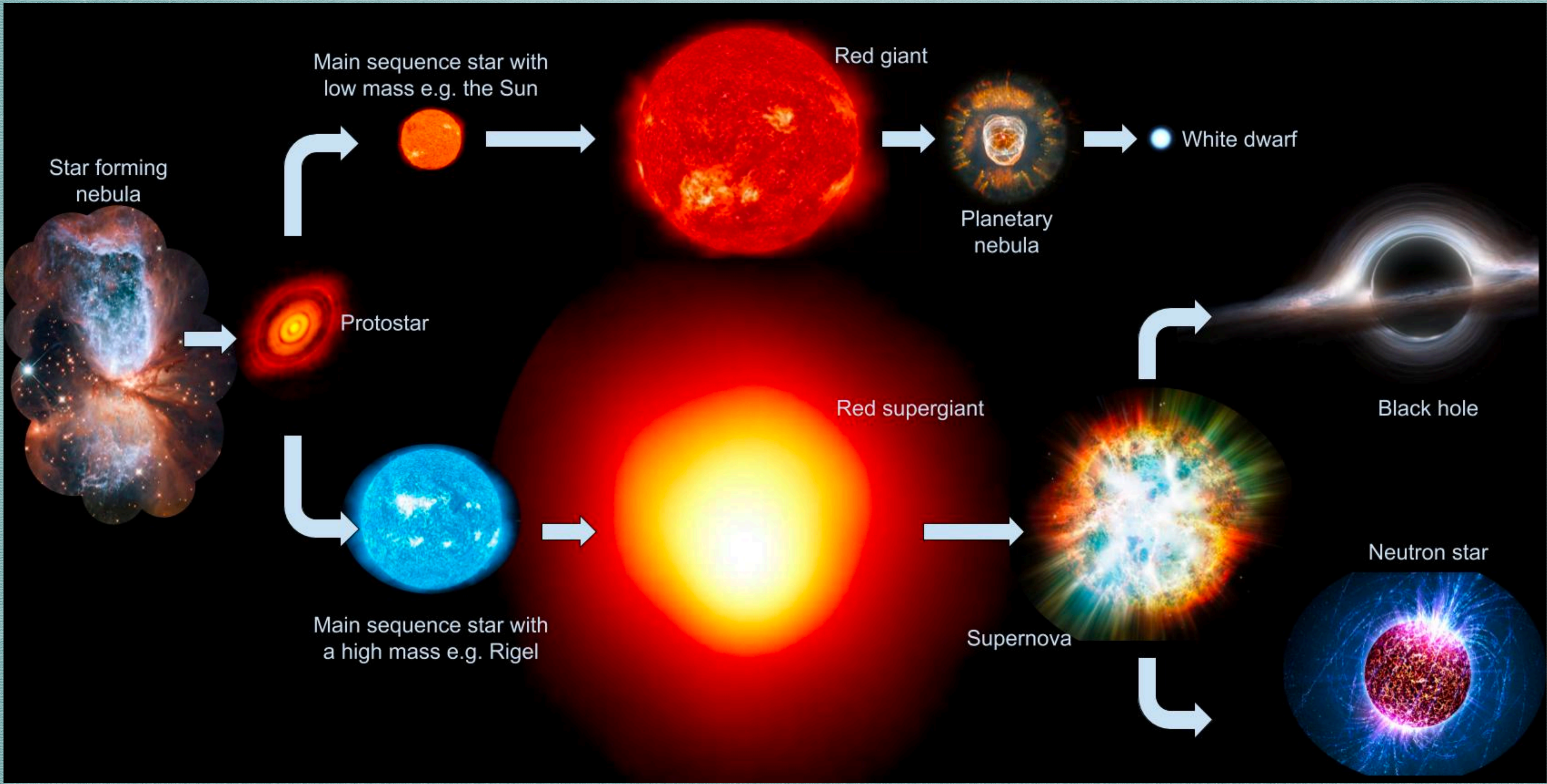
UCT workshop, 26 April 2023

Life cycle of a star



Properties of compact objects

	White dwarf	Neutron star	Black hole (Stellar)	Black hole (Supermassive)
Mass	$\sim M_\odot$	$\sim M_\odot$	$\sim 3 - 100\, M_\odot$	$> 10^6\, M_\odot$
Progenitor mass	$\lesssim 10\, M_\odot$	$< 10 - 25\, M_\odot$	$> 25\, M_\odot$	-
Radius	$\gtrsim 1000\,\mathrm{km}$	$\sim 10\,\mathrm{km}$	$2GM/c^2$	$2GM/c^2$
Central density	$\lesssim 2 \times 10^{10}\,\mathrm{g\,cm^{-3}}$	$\lesssim 10^{14} - 10^{15}\,\mathrm{g\,cm^{-3}}$	-	-
Surface Temperature	$\sim 10^4 - 10^5\,\mathrm{K}$	$\sim 10^6 - 10^7\,\mathrm{K}$	-	-

Degenerate electron gas

- Electrons become degenerate when all the energy states below the Fermi level are filled.
- Electrons are Fermions.
- Fermi-Dirac distribution: (Assume $k_B T \ll E_F$)

$$f(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1} \approx \begin{cases} 1 & E < E_F \\ 0 & E > E_F \end{cases}$$

Degenerate electron gas

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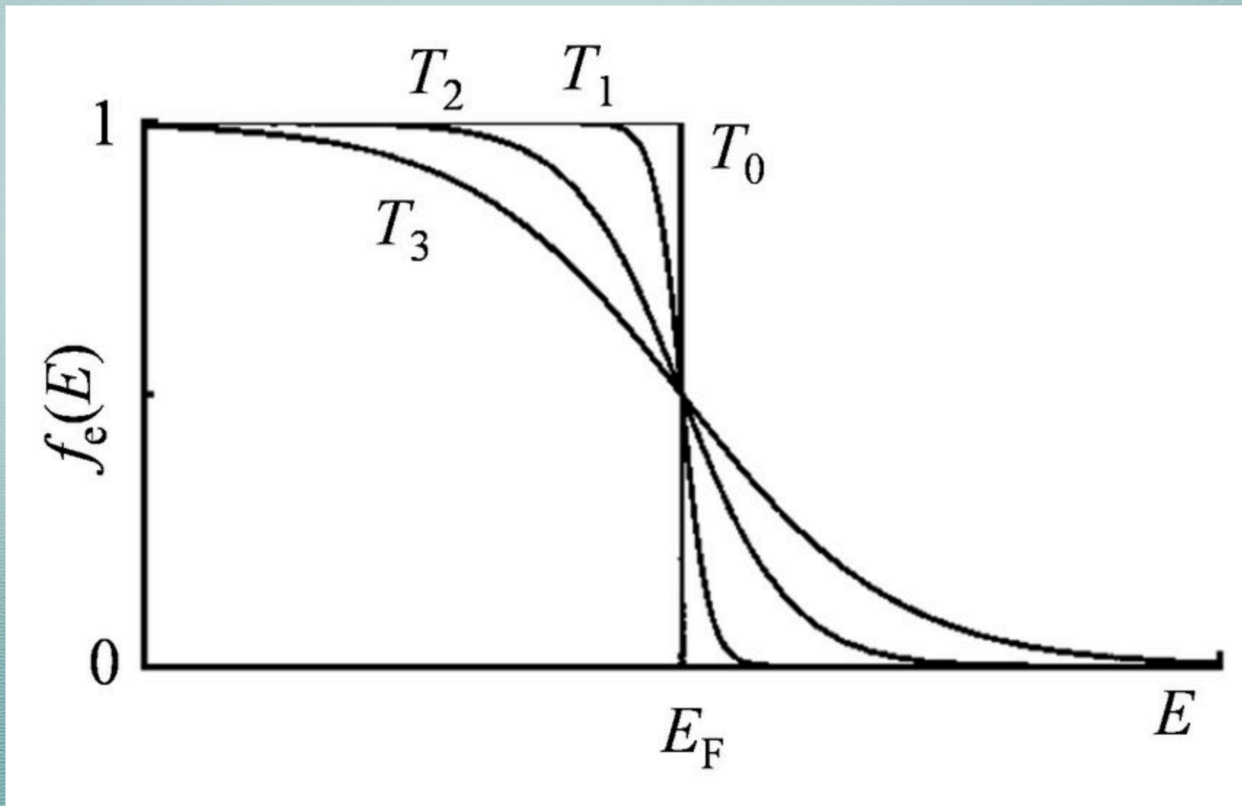
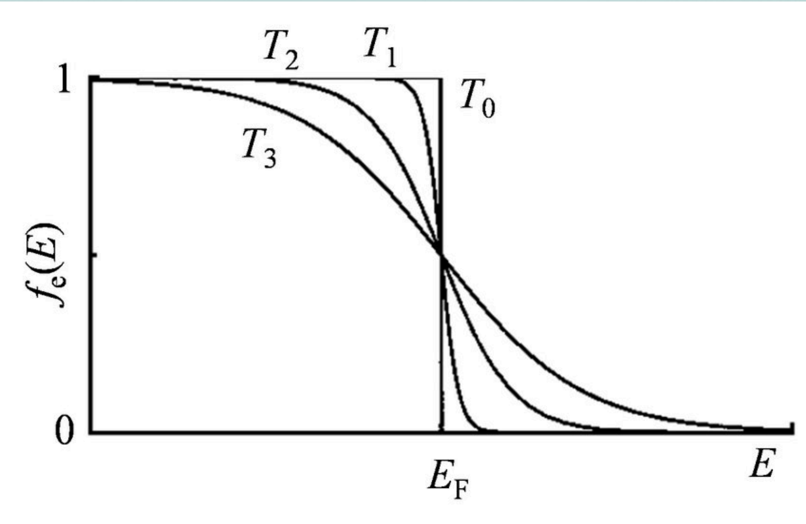


Degenerate electron gas

$$f(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1} \approx \begin{cases} 1 & E < E_F \\ 0 & E > E_F \end{cases}$$

* White dwarfs are assumed to be cold.
* Zero temperature calculation is a good assumption.



Degenerate electron gas

* Number density of electrons: $n_e = \int_0^{p_{\rm F}} \frac{8\pi}{h^3} p^2 dp = \frac{8\pi}{3h^3} p_{\rm F}^3.$

* Degenerate pressure: $P = \frac{8\pi}{3h^3} \int_0^{p_{\rm F}} v p^3 dp = \frac{8\pi}{3h^3} \int_0^{p_{\rm F}} \frac{p^4 c^2}{\sqrt{p^2c^2 + m_{\rm e}^2 c^4}} dp$

Non-relativistic electron gas: $\sqrt{p^2c^2 + m_{\rm e}^2c^4} \approx m_{\rm e}c^2$ — $P \propto \rho^{5/3}$

Relativistic electron gas: $\sqrt{p^2c^2 + m_{\rm e}^2c^4} \approx pc$ — $P \propto \rho^{4/3}$

Chandrasekhar equation of state

$$P = \frac{\pi m_e^4 c^5}{3h^3}\left[x(2x^2-3)\sqrt{x^2+1} + 3\sinh^{-1} x\right]$$

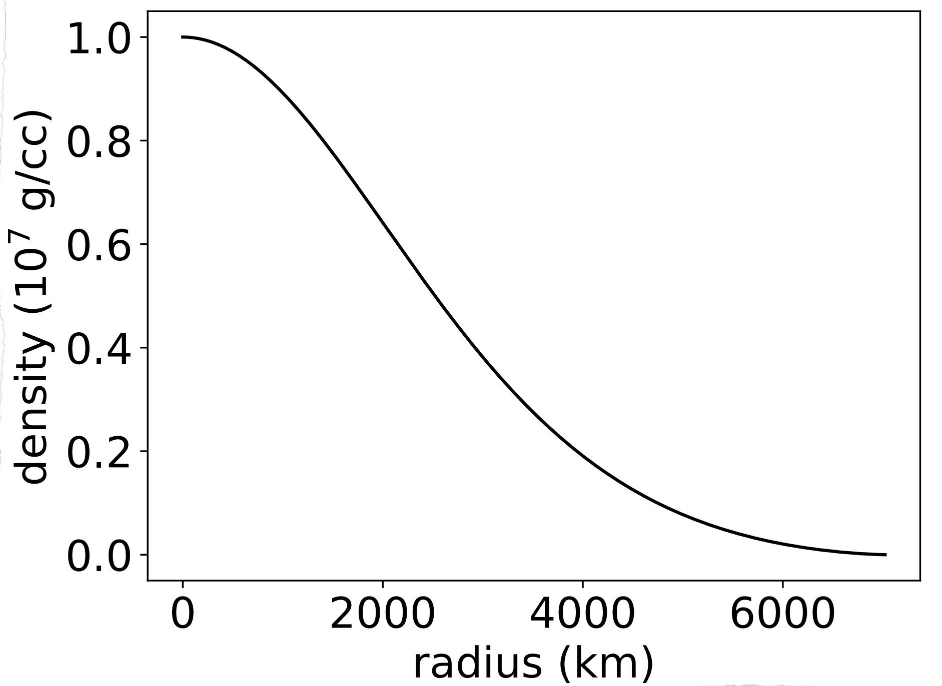
$$\rho = \frac{8\pi\mu_e m_p (m_e c)^3}{3h^3} x^3, \qquad x = \frac{p_F}{m_e c}$$

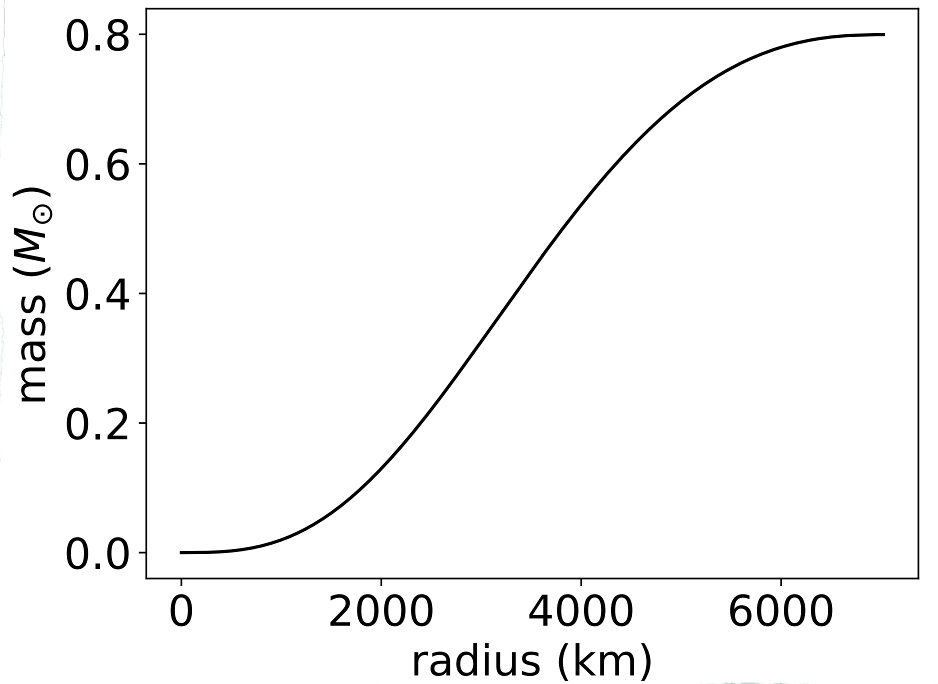
- Pressure balance (Newtonian): $\dfrac{dP}{dr} = -\dfrac{GM\rho}{r^2}$.
- Pressure balance (GR): $\dfrac{dP}{dr} = -\dfrac{G}{r^2}\dfrac{(P+\rho c^2)}{1-\frac{2GM}{c^2 r}}\left(\dfrac{4\pi r^3 P}{c^4} + \dfrac{M}{c^2}\right)$.
- Mass estimate: $\dfrac{dM}{dr} = 4\pi r^2 \rho$.

White dwarf

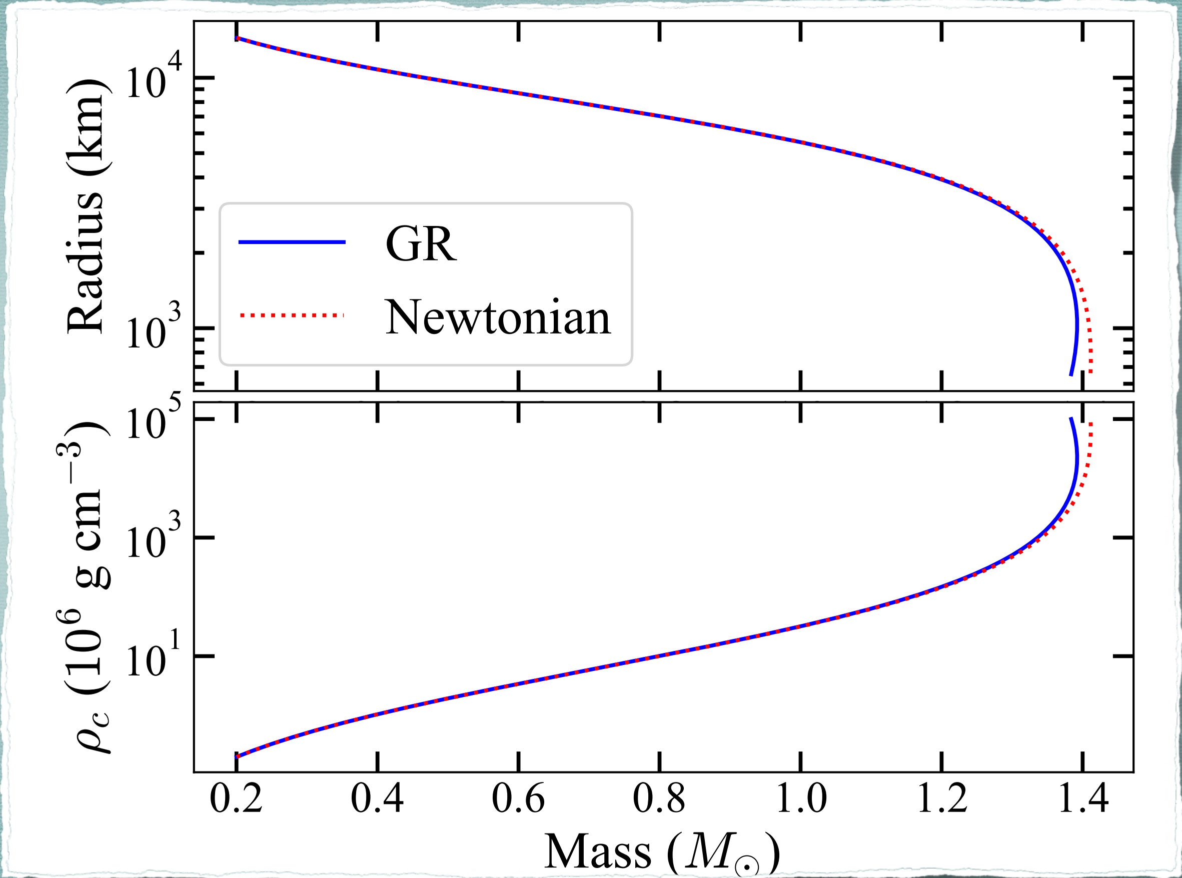
- Boundary conditions:

$$M(r=0)=0, \quad M(r=R_*)=M_*$$
$$\rho(r=0)=\rho_c, \quad P(r=0)=P_c$$
$$\rho(r=R_*)=0, \quad P(r=R_*)=0$$





Structure of WD/NS

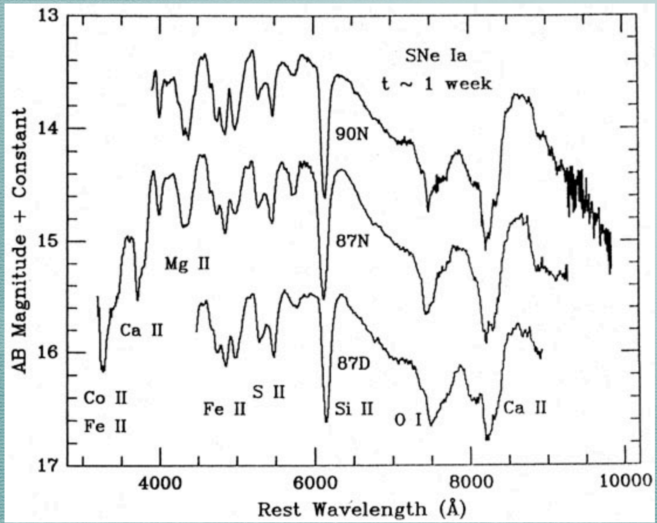


S. Chandrasekhar (1935)

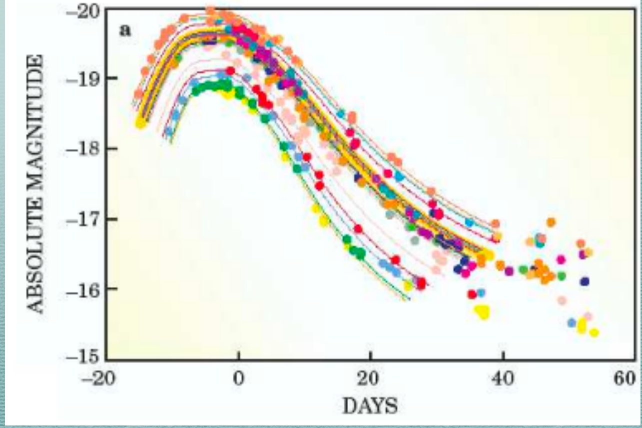
$$M_{\rm Ch} = \frac{\sqrt{6}}{32\pi}\left(\frac{hc}{G}\right)^{3/2}\left(\frac{2}{\mu_{\rm e}}\right)^2 \frac{2.018}{m_{\rm p}^2}$$

Type Ia supernovae

- If a WD has a binary partner, it starts pulling out matter.
- At Chandrasekhar mass-limit, it bursts out to produce type Ia supernova.
- Type Ia supernovae have similar behaviours, therefore they are used as standard candles in cosmology.

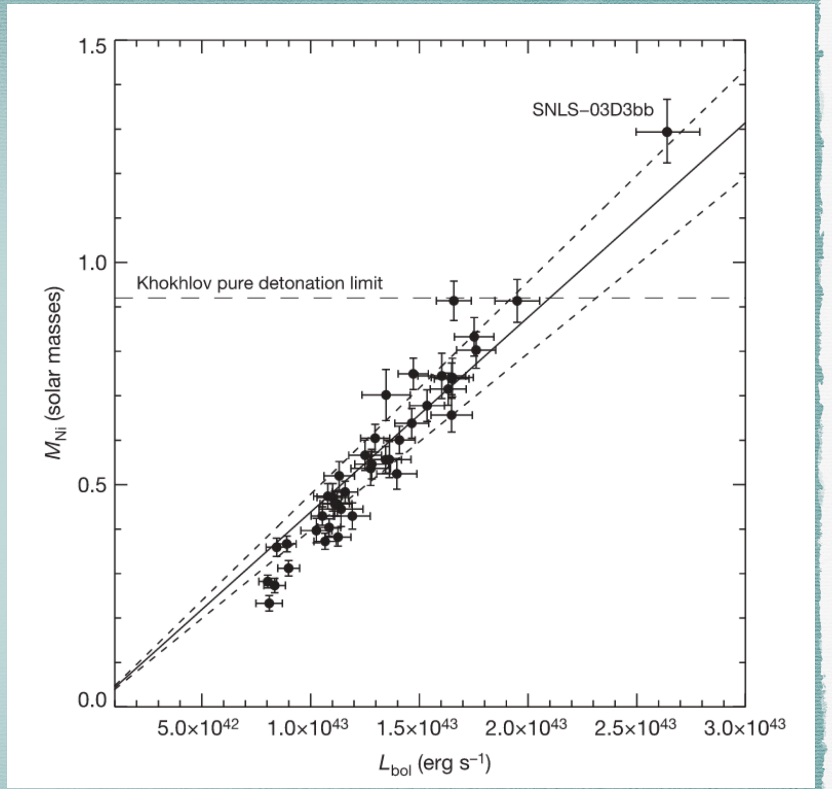


- Absence of hydrogen.
- Strong ionised silicon absorption line.



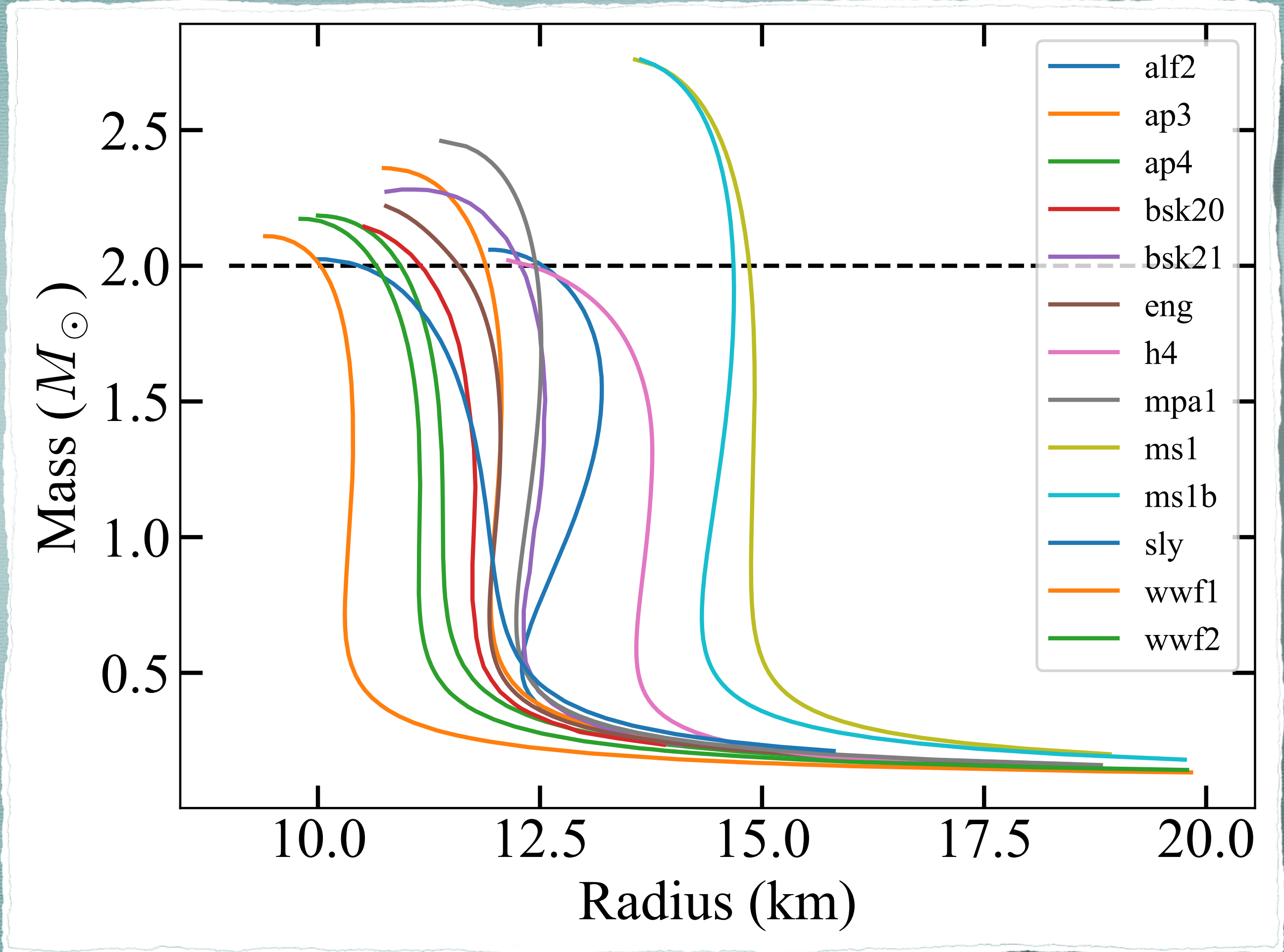
Peculiar SNeIa

- Recent observations show some peculiar SNeIa with extremely high luminosity.
- Their light curves also show different trend.
- $L \propto M_{\rm WD}c^2 + mv^2 \implies M_{\rm WD} \approx 2.1 - 2.8 M_\odot$
- Chandrasekhar mass-limit is violated.



Howell *et al.* Nature 443 (2006) 308

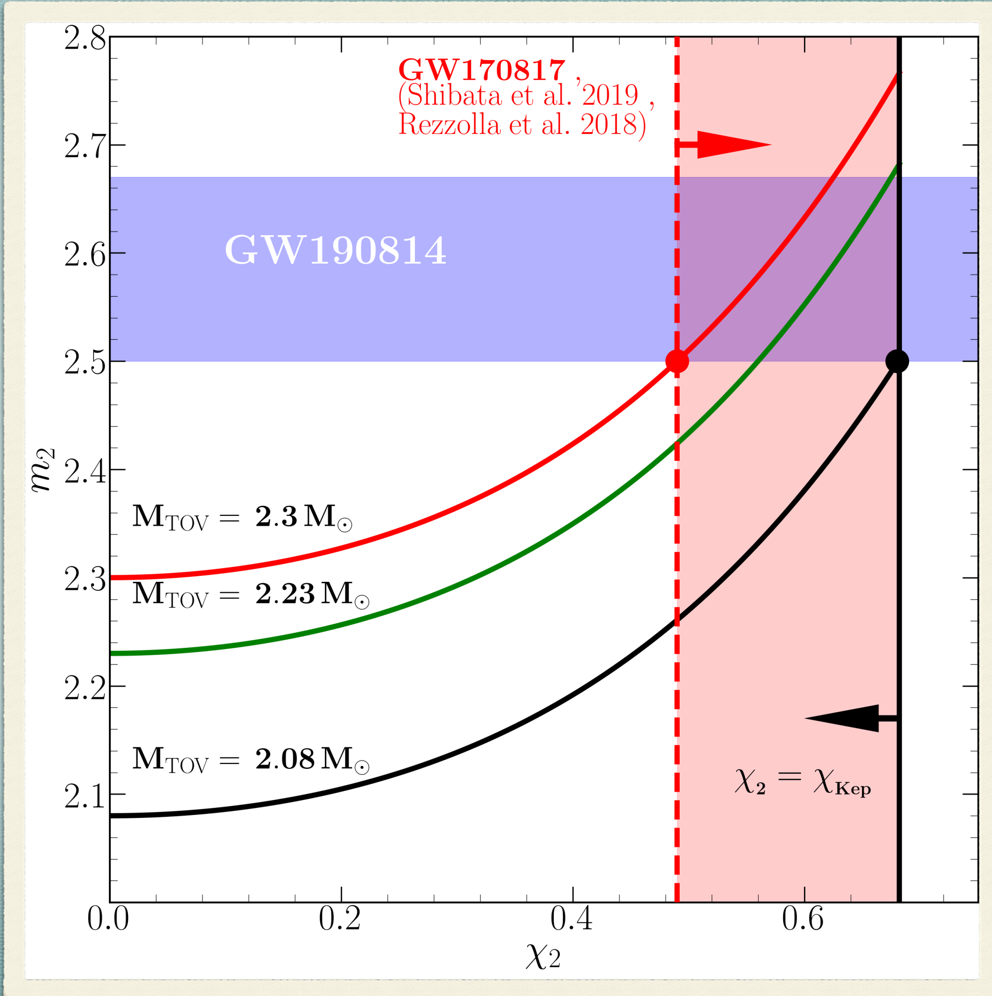
Neutron star



Massive neutron star

* GW190817 event: Merger of a 23 $M_\odot$ black hole with a 2.5 – 2.67 $M_\odot$ object.
* The secondary object is the highest measured massive NS or the lightest black hole.
* Assuming it to be a NS, lower bound on maximum NS has to be $\sim 2.1\, M_\odot$.

Most, Papenfort, Weih & Rezzolla, MNRAS-Letters 499 (2020) L82

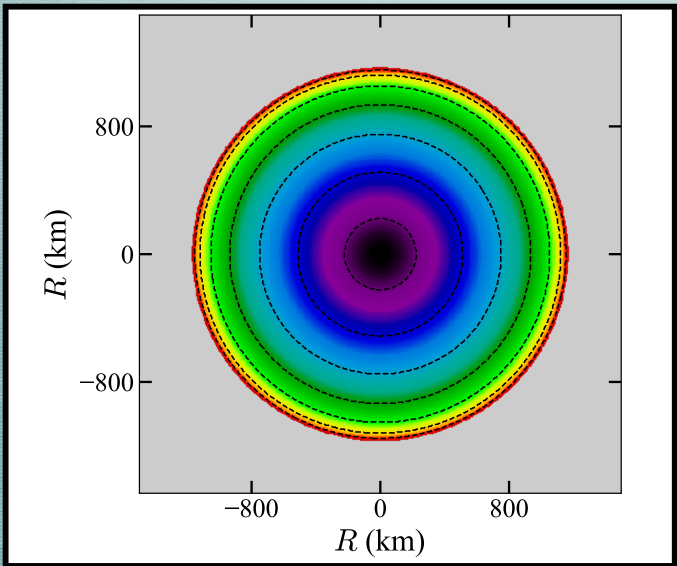


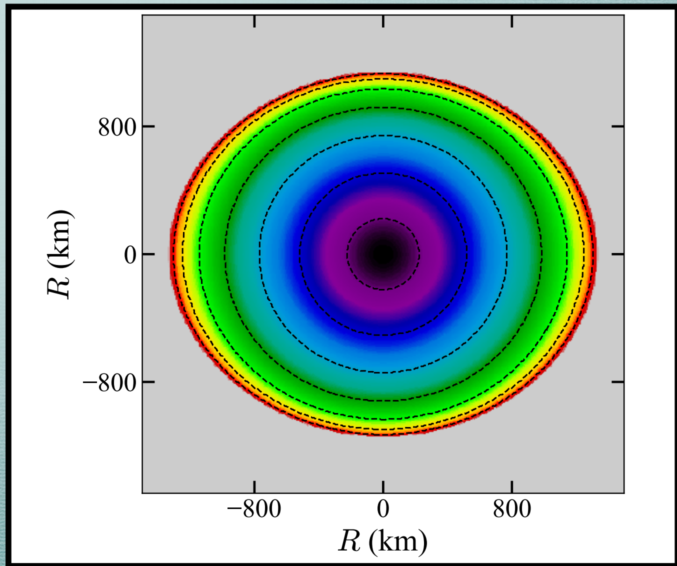
Massive WD and NS

* Rotation, magnetic fields, modified theory of general relativity, noncommutative geometry, etc.
* Each theory gives different mass-radius relation.
* GW astronomy in the future can be relevant to single out the theories.

Rotating WD/NS

- Rotation can increase the mass of a WD/NS.
- Ostriker & Hartwick in 1968 showed that rotation alone can increase the mass of a WD up to $\sim 1.8 M_{\odot}$.
- Rezzolla and collaborators showed that rotation can increase the mass of a NS upto 1.2 times of its original value.
- Rotation turns a spherical WD/NS to an oblate shaped WD/NS.

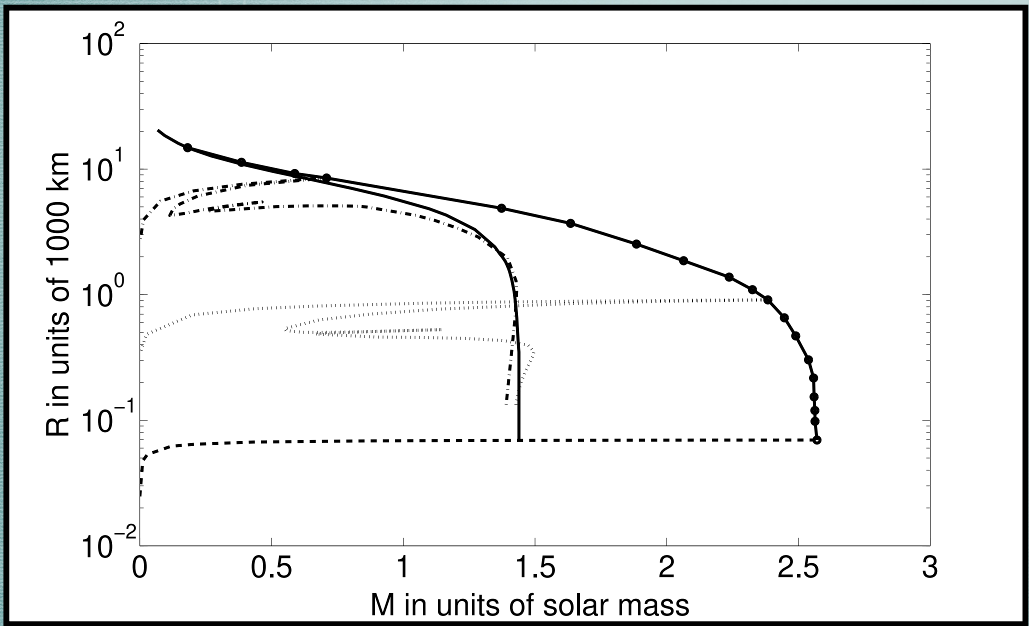


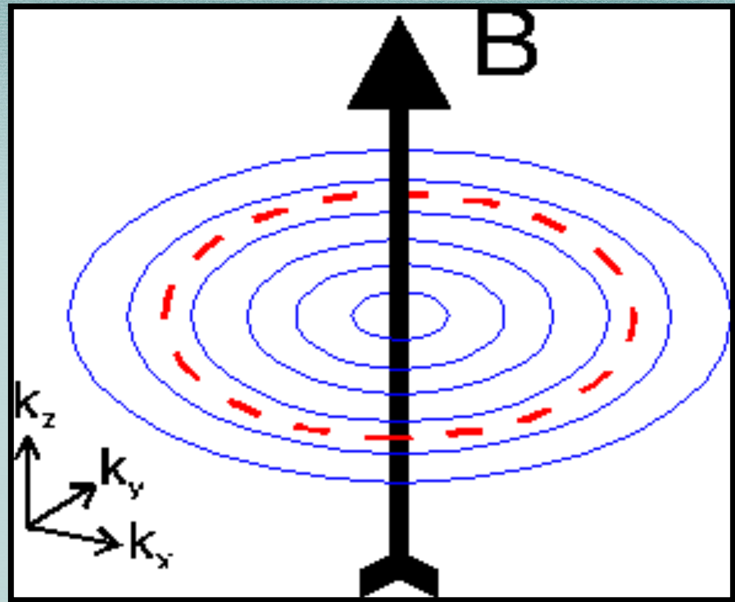


Magnetic fields

* Microscopic effect: Formation of Landau levels.

* $$E^2 = p_z^2 c^2 + m_e^2 c^4 \left(1 + 2\nu \frac{B}{B_c}\right), \quad B_c = 4.414 \times 10^{13}\,\mathrm{G}$$

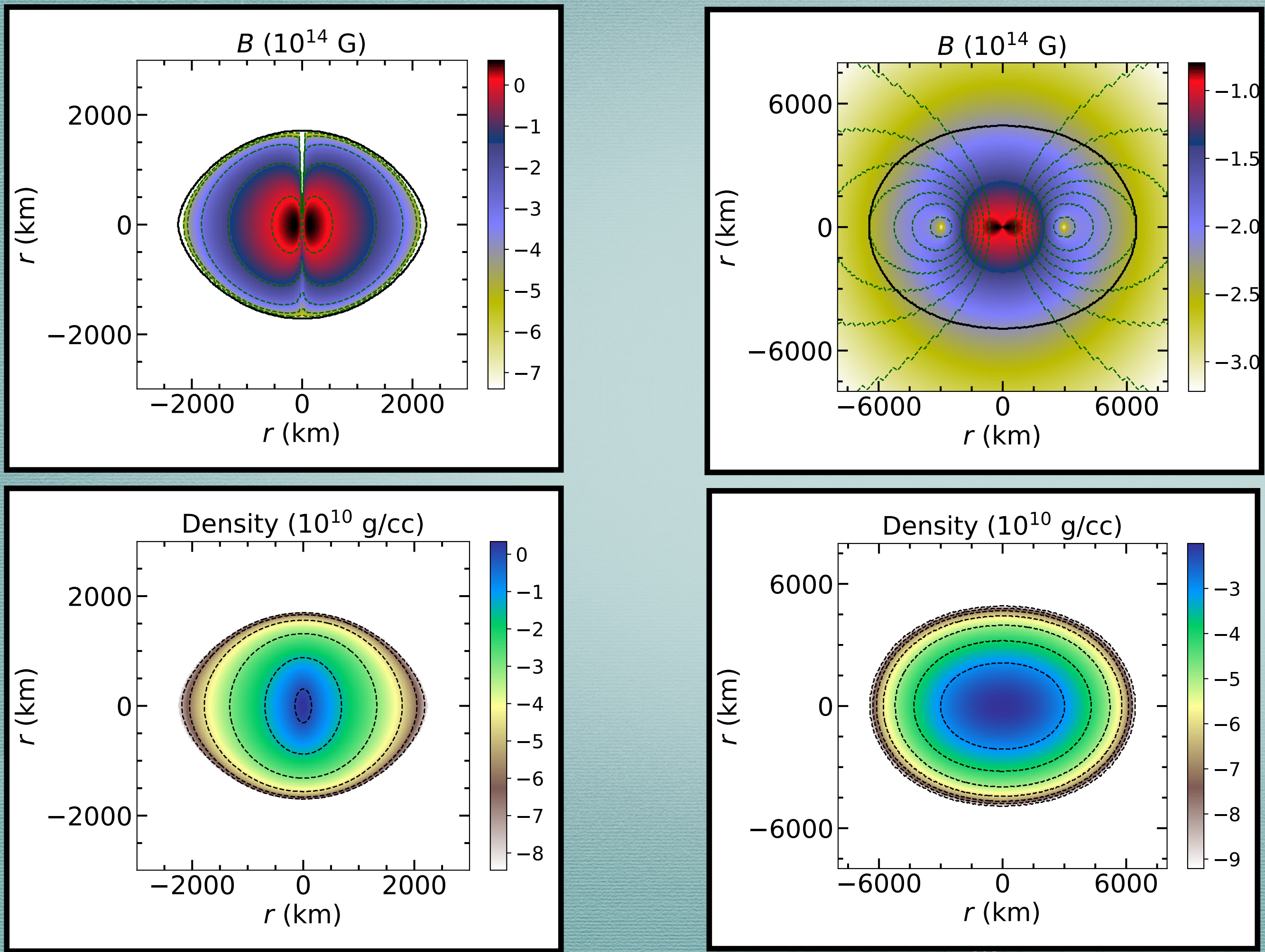




Das & Mukhopadhyay (2013)
PRL 110, 071102

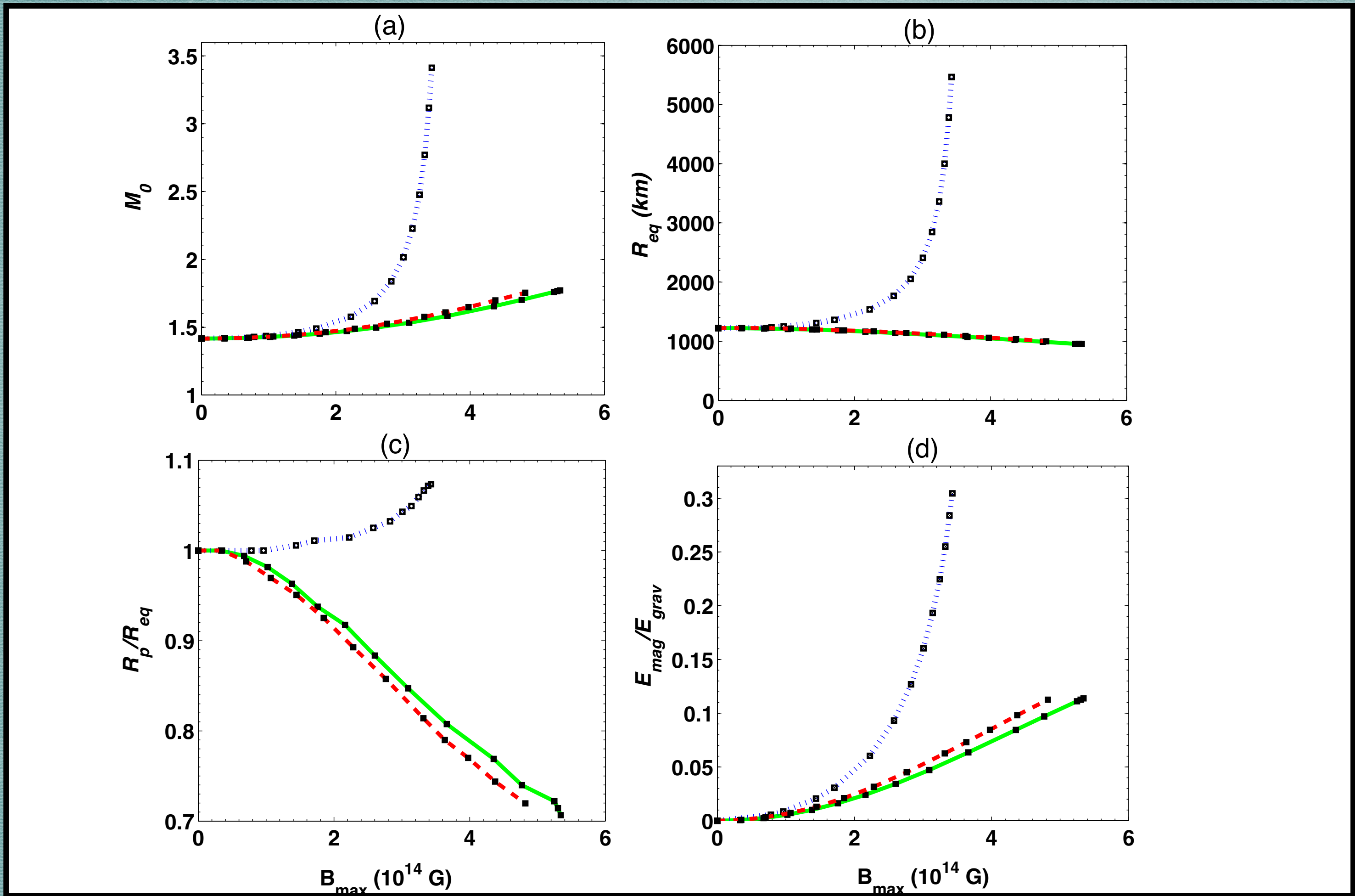
* Macroscopic effect: Shape, size, etc.

Magnetized WD/NS



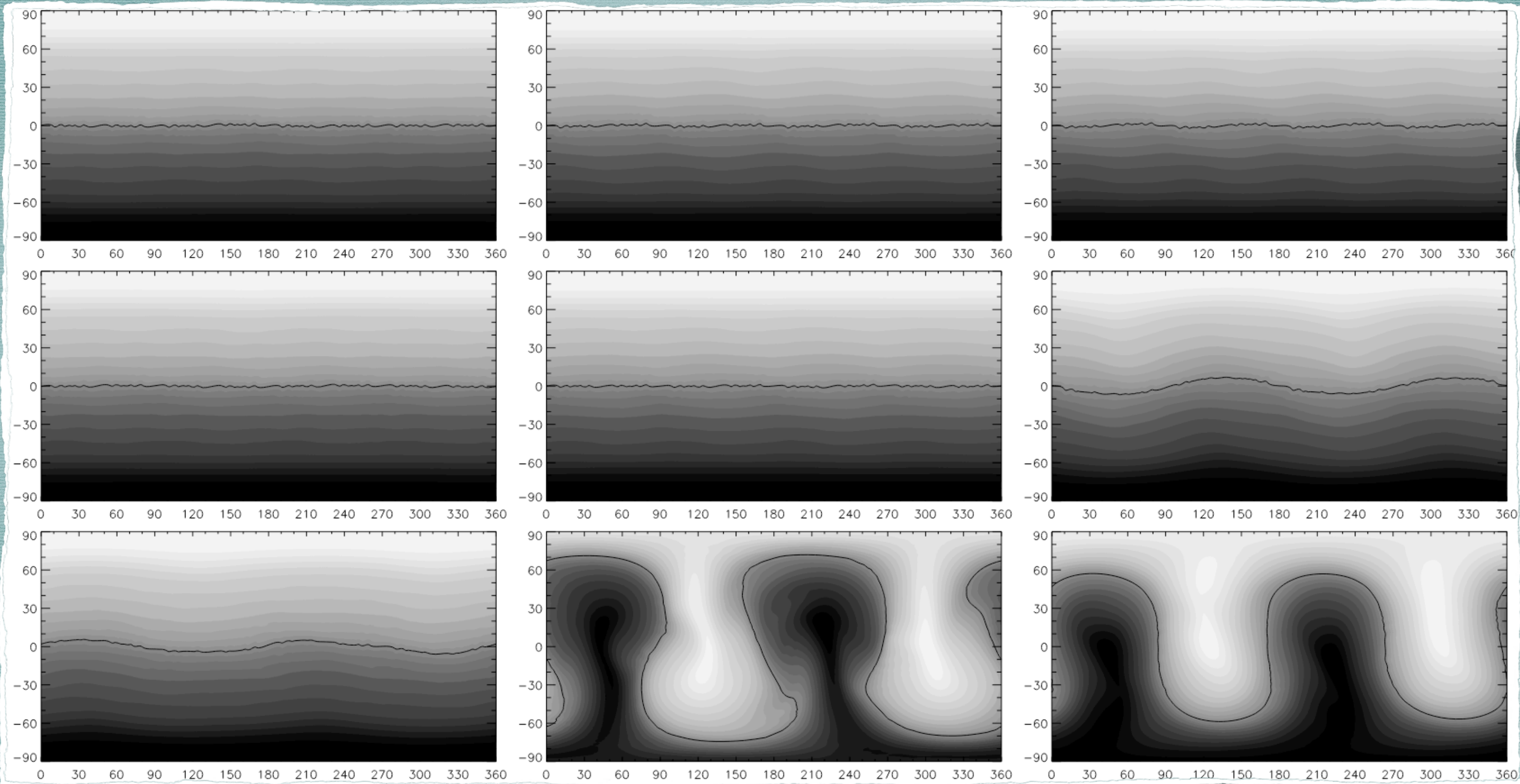
Kalita & Mukhopadhyay
MNRAS 490 (2019) 2692

Magnetized WD/NS



Das & Mukhopadhyay
JCAP 05 (2015) 016

Instability in high magnetic fields

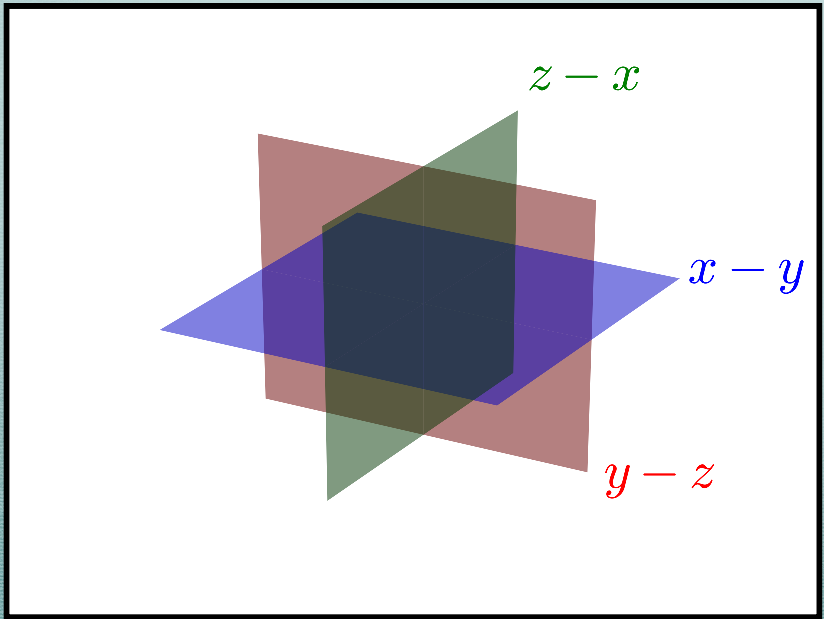


J. Braithwaite, MNRAS 397 (2009) 763

Noncommutative geometry

Ordinary quantum mechanics: (Heisenberg algebra)

$$[p_\mu, p_\nu] = 0, \quad [x_\mu, x_\nu] = 0, \quad [x_\mu, p_\nu] = i\hbar\delta_{\mu\nu}$$



Noncommutative geometry:

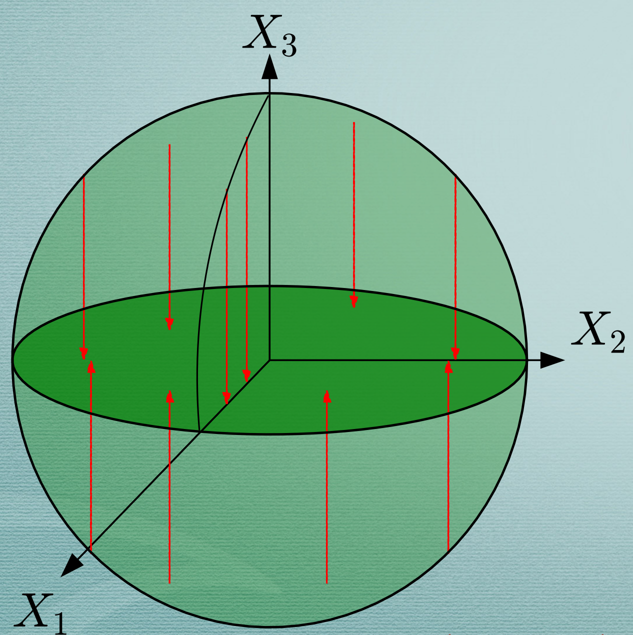
$$[\hat{p}_x, \hat{p}_y] = i\theta,$$

$$[\hat{x}, \hat{y}] = i\frac{\theta\eta^2}{4\hbar^2}$$

Fuzzy sphere NC

Angular momentum algebra

$$[J_i, J_j] = i\hbar\epsilon_{ijk}J_k$$



Fuzzy sphere algebra

$$J_i \to X_i = \frac{k}{r}J_i$$

$$[X_i, X_j] = i\frac{k\hbar}{r}\epsilon_{ijk}X_k$$

Madore, CGQ 9 (1992) 69

Squashed fuzzy sphere algebra

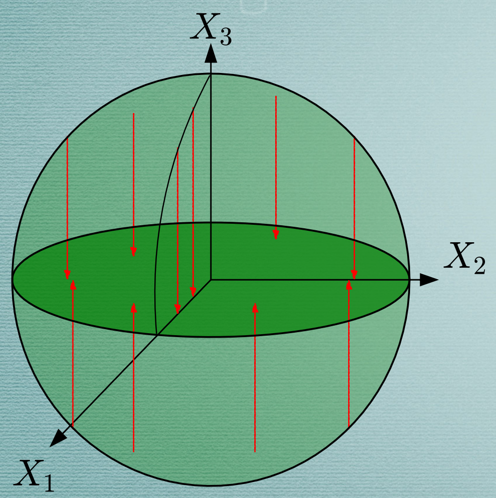
$$[X_1, X_2] = \pm i\frac{k\hbar}{r}\sqrt{r^2 - X_1^2 - X_2^2}$$

Andronache & Steinacker, Journal of Physics A, 48 (2015) 295401

Energy dispersion relation in NC

$$J_1^2 + J_2^2 + J_3^2 = \frac{\hbar^2}{4}\left(N^2 - 1\right)\mathbb{I}_{N\times N}$$

$$k = \frac{2r^2}{\hbar\sqrt{N^2 - 1}}$$



Dirac operator, $D = \frac{c}{k}\left(\sigma_1 \otimes [X_1, .\,] + \sigma_2 \otimes [X_2, .\,]\right)$

Energy eigenvalues, $E_{l,m}^2 = \frac{2\hbar c^2}{k\sqrt{N^2 - 1}}\left\{l(l+1) - m(m \pm 1)\right\}$

Energy dispersion relation in NC

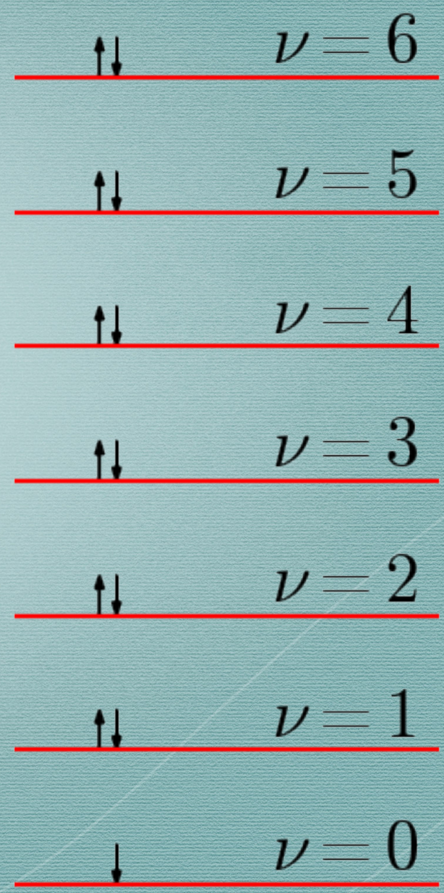
$$E^2 = p_r^2 c^2 + m_{\mathrm{e}}^2 c^4 \left[1 + \{ l(l+1) - m(m \pm 1) \} \frac{\hbar^2}{m_{\mathrm{e}}^2 c^2 r^2} \right]$$

In the limit $N \gg 1$

$$E^2 = p_r^2 c^2 + m_{\mathrm{e}}^2 c^4 \left(1 + 2\nu \frac{2\hbar}{m_{\mathrm{e}}^2 c^2 k} \right), \qquad \nu \in \mathbb{Z}^{0+}$$

Landau levels in magnetic field in z-direction

$$E^2 = p_z^2 c^2 + m_{\mathrm{e}}^2 c^4 \left(1 + 2\nu \frac{B}{B_{\mathrm{c}}} \right), \quad B_{\mathrm{c}} = \frac{m_{\mathrm{e}}^2 c^3}{\hbar e} = 4.414 \times 10^{13}\,\mathrm{G}$$



EoS in the presence of NC

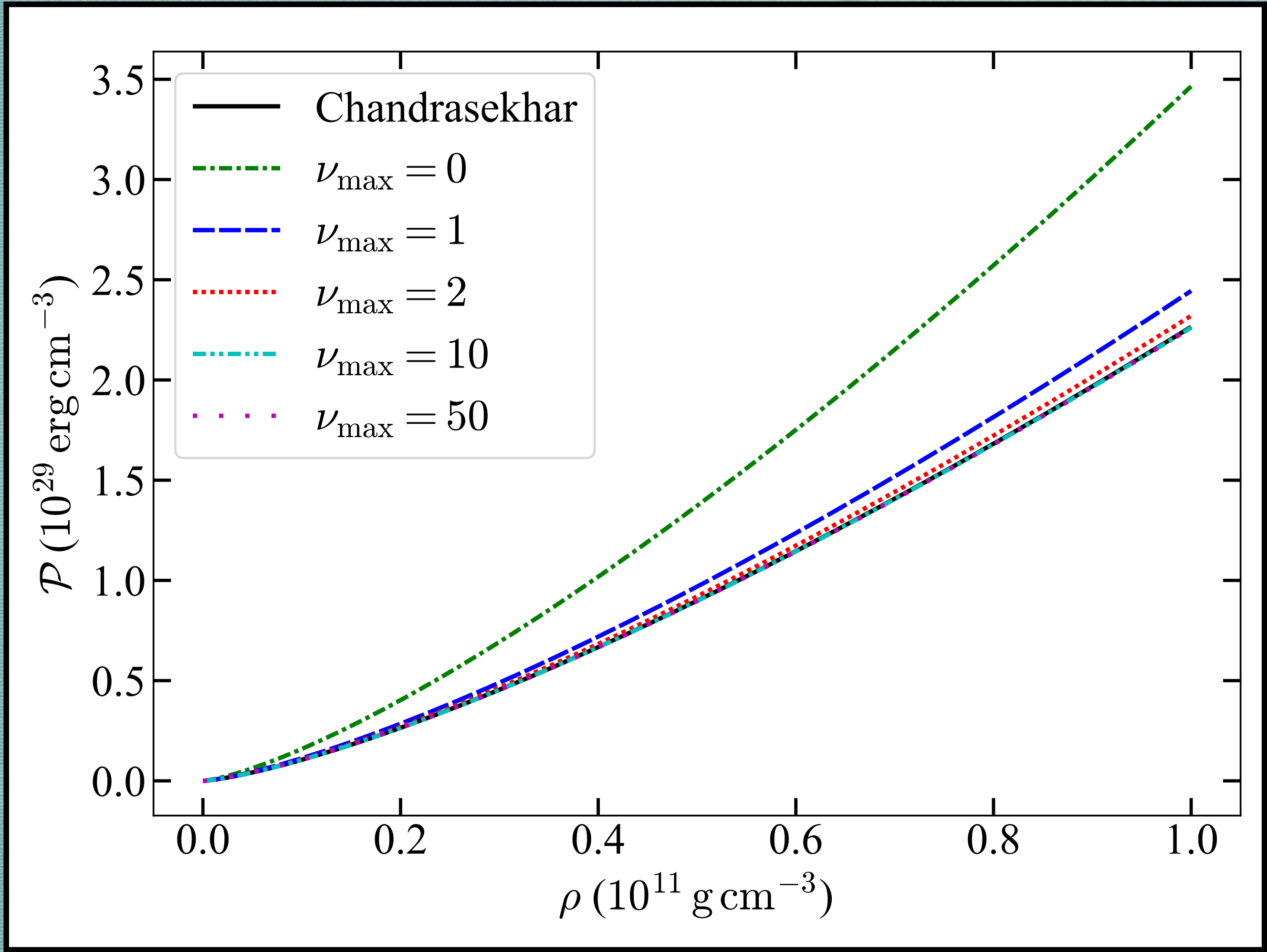
$$\frac{2}{h^3}\int \mathrm{d}^3p \quad \rightarrow \quad \sum_{\nu} g_\nu \int \frac{4\pi\theta}{h^3}\,\mathrm{d}p_z$$

$$n_\mathrm{e} = \sum_{\nu=0}^{\nu_\mathrm{max}} \frac{4\pi m_\mathrm{e}^3 c^3 \theta_\mathrm{D}}{h^3} g_\nu x_\mathrm{F}(\nu), \qquad \theta_\mathrm{D} = 2\hbar/m_\mathrm{e}^2 c^2 k, \quad x_\mathrm{F} = \frac{p_\mathrm{F}}{m_\mathrm{e} c}$$

$$P = \sum_{\nu=0}^{\nu_\mathrm{max}} \frac{2\pi m_\mathrm{e}^4 c^5 \theta_\mathrm{D}}{h^3} g_\nu \left[\epsilon_\mathrm{F} x_\mathrm{F}(\nu) - \left(1 + 2\nu\theta_\mathrm{D}\right) \log\left(\frac{\epsilon_\mathrm{F} + x_\mathrm{F}(\nu)}{\sqrt{1 + 2\nu\theta_\mathrm{D}}} \right) \right]$$

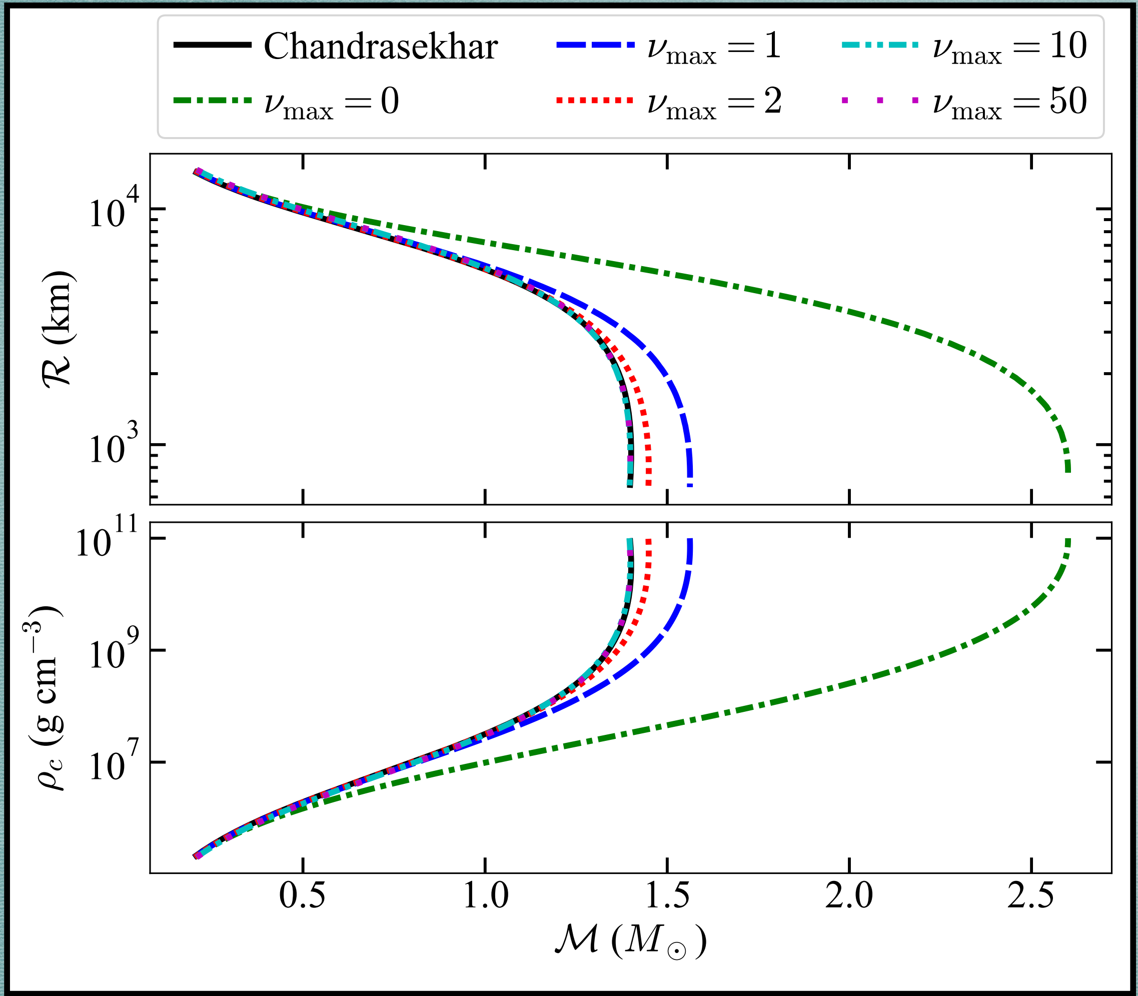
$$\rho = \mu_\mathrm{e} m_\mathrm{p} n_\mathrm{e}$$

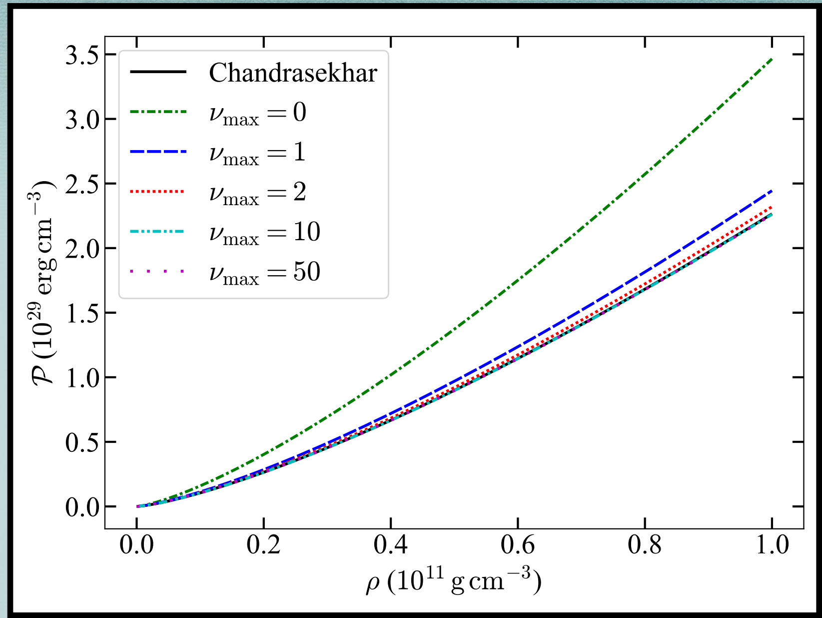
EoS in the presence of NC



Kalita et al. (2021)
IJMPD 30, 13 (2021) 2150101

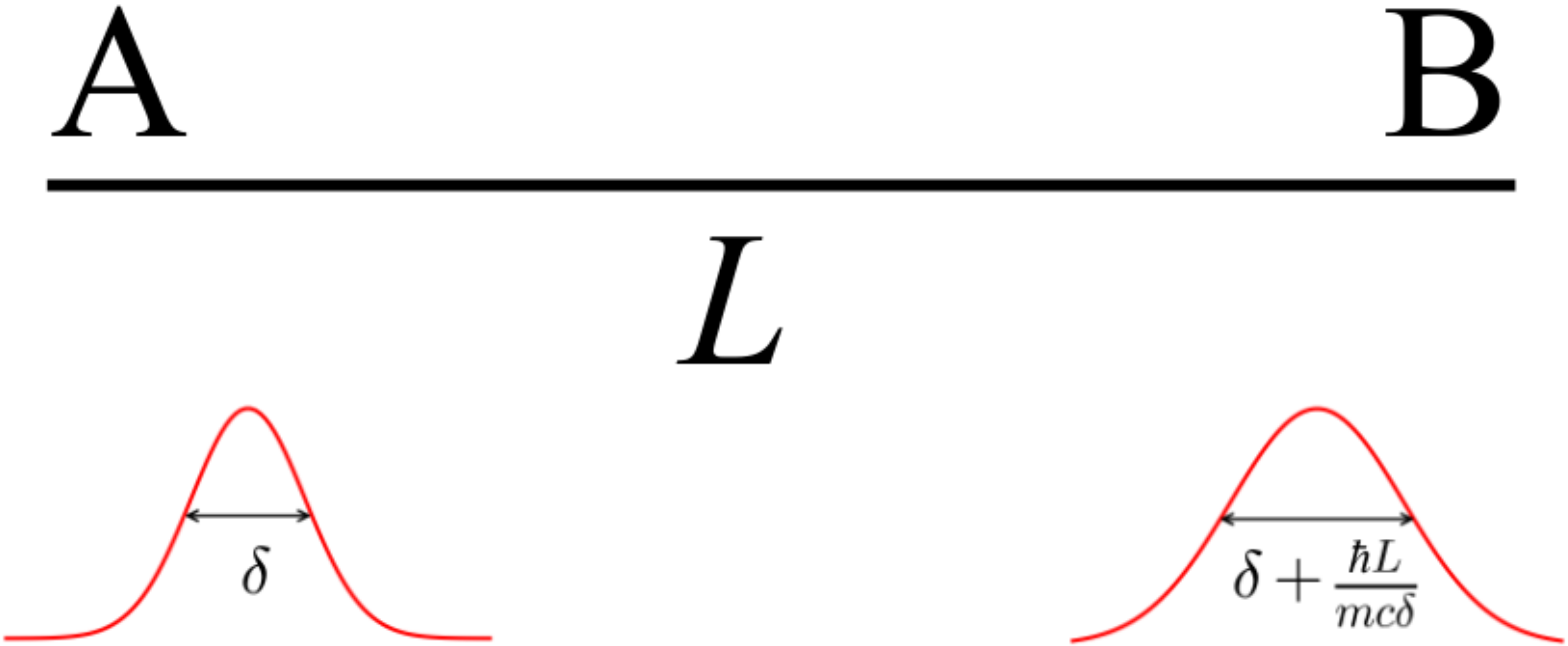
WD mass-radius relation in NC





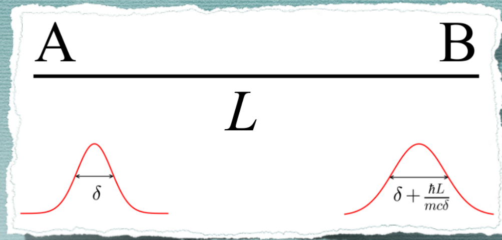
Kalita et al. (2021)
IJMPD 30, 13 (2021) 2150101

Scales of NC



Salecker & Wigner, Physical Review, 109 (1958) 571

Scales of NC



- $\delta \to \delta + \dfrac{\hbar L}{mc\delta}$
- It has a minimum at $\delta = \left(\dfrac{\hbar L}{mc}\right)^{1/2}$, implying $\delta^2 \gtrsim \dfrac{\hbar L}{mc}$
- From GR, we know $\delta \gtrsim \dfrac{Gm}{c^2}$
- Combining them, we have $\delta^3 \gtrsim \dfrac{L\hbar G}{c^3} \implies \delta \gtrsim \left(LL_{\mathrm{P}}^2\right)^{1/3}$

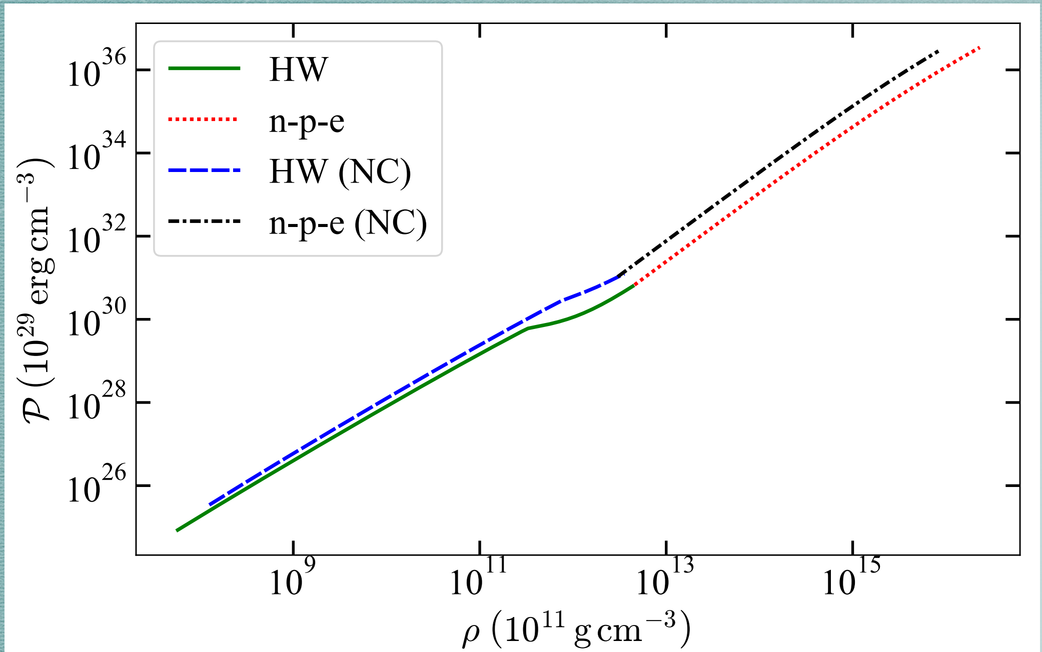
Salecker & Wigner, Physical Review, 109 (1958) 571

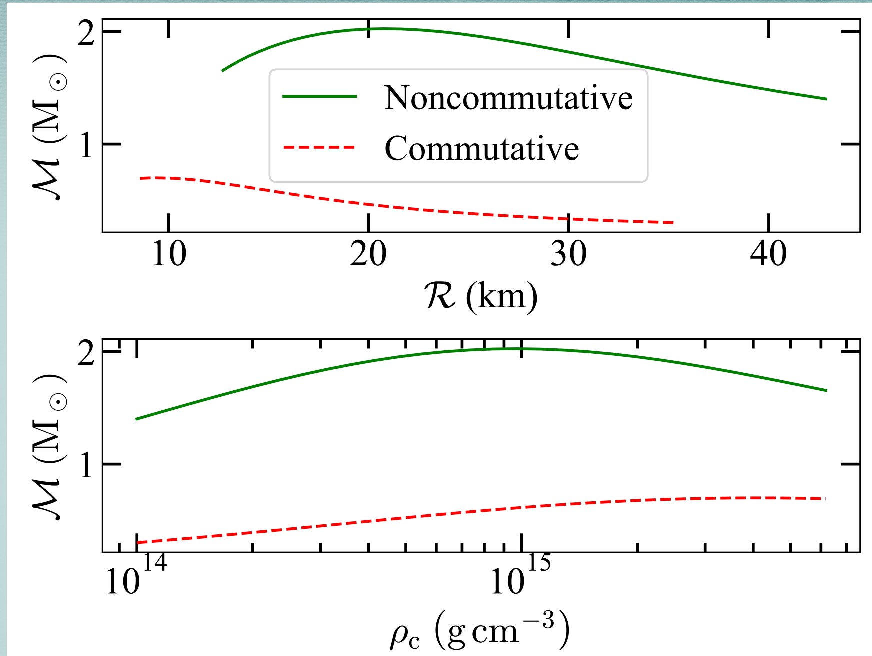
Scales of NC

$$E^2 = p_r^2 c^2 + m_{\mathrm{e}}^2 c^4 \left(1 + 2\nu \frac{\lambda_{\mathrm{e}}^2}{2\pi^2 \hbar k} \right), \qquad k = \xi \frac{\mu_{\mathrm{e}}^{2/3} m_{\mathrm{p}}^{2/3}}{h \rho^{2/3}}$$

* NC is prominent if $\lambda_{\mathrm{e}}^2 \gtrsim 2\pi^2 \hbar k$.
* $L \lesssim \lambda_{\mathrm{e}} / \sqrt{\pi \xi} = L_{\mathrm{eff}}$
* (General thought) NC is prominent only at the Planck scale.
* New uncertainty in length scale $\delta \sim \left(L L_{\mathrm{P}}^2 \right)^{1/3}$, where L_{P} is the Planck length.
* Hence, uncertainty in length scale is $\delta \lesssim \left(\lambda_{\mathrm{e}} L_{\mathrm{P}}^2 \right)^{1/3}$.

NC in NS





Kalita & Mukhopadhyay (2022)
Universe 8, 388

Conclusions

- NC behaves as an internal magnetic fields.
- Systems's length scale is important for the prominence of NC.
- For WDs, $\delta \lesssim \left(\lambda_e L_P^2\right)^{1/3}$.
- Massive WDs and NSs can be explained through NC.
- Magnetic field, rotation, modified gravity, etc. are some other physics to explain massive WDs and NSs.
- GW observations in the future can single out these theories.

Thanks