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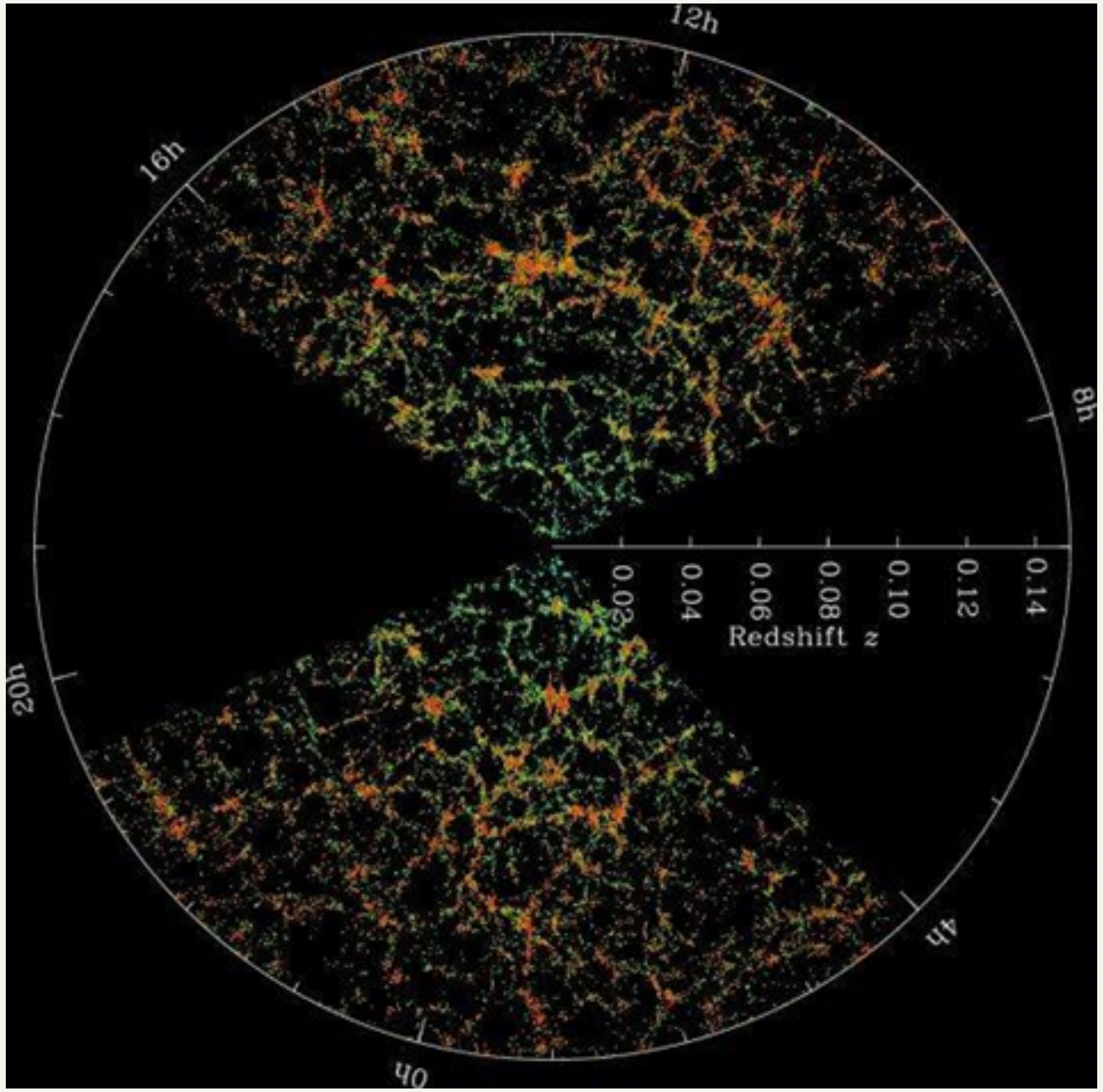
Particle-like solutions in the generalized SU(2) Proca theory

Introduction: why modified gravity?

Einstein's theory is very successful, but...

- Dark matter,
- Dark energy,
- Hubble tension,
- Singularities,
- Renormalization...



Generalized SU(2) Proca theory

A. Gallego et. al., Phys. Rev. D.,2020.
Y. Rodríguez et al., Phys. Dark Univ.,2018.
https://bit.ly/2Z5m6XT.

Generalized SU(2) Proca theory (GSU2P)

Aditional vectorial degrees of freedom

- Second order equations of motion
- Correct number of propagating degrees of freedom
- Global internal SU(2) symmetry,
- Early inflation: constant roll,
- Late acceleration: dark energy.

J. C. Garnica et. al., Ann. Phys. (Berl.), 2022.

Lagrangian GSU2P

$$\mathcal{L} = \frac{1}{16\pi}\Big[R - F^a_{\mu\nu}F_a^{\mu\nu} - 2\mu^2 B_a^\mu B^a_\mu$$
$$+ \alpha_1\left(\mathcal{L}^1_{4,2} - 2\mathcal{L}^4_{4,2} - \frac{20}{3}\mathcal{L}^5_{4,2} + 5\mathcal{L}^7_2\right)$$
$$+ \alpha_3\left(2\mathcal{L}^2_{4,2} + \mathcal{L}^3_{4,2} + \frac{7}{20}\mathcal{L}^4_{4,2} + \frac{14}{3}\mathcal{L}^5_{4,2} - 8\mathcal{L}^6_{4,2} + \mathcal{L}^7_2\right)$$
$$+ \chi_1\mathcal{L}^1_2 + \chi_2\mathcal{L}^2_2$$
$$+ \chi_4\left(\mathcal{L}^4_2 - \frac{\mathcal{L}^7_2}{2}\right) + \chi_5\mathcal{L}^5_2 + \chi_6\left(\mathcal{L}^6_2 - 3\mathcal{L}^7_2\right)\Big],$$

J. C. Garnica et. al., Ann. Phys. (Berl.), 2022.

Lagrangian GSU2P

$$\mathcal{L}_2^1 \equiv (B^a \cdot B_a)(B^b \cdot B_b)\,, \qquad \mathcal{L}_2^6 \equiv (B^b \cdot B_b) A_{\mu\nu a} A^{\mu\nu a}\,,$$

$$\mathcal{L}_2^2 \equiv (B^a \cdot B_b)(B^b \cdot B_a)\,, \qquad \mathcal{L}_2^7 \equiv (B^b \cdot B_a) A_{\mu\nu b} A^{\mu\nu a}\,,$$

$$\mathcal{L}_2^3 \equiv B_\mu^b B_{\rho b} A^{\mu\nu a} A^\rho{}_{\nu a}\,, \qquad A_{\mu\nu}^a \equiv \nabla_\mu B_\nu^a - \nabla_\nu B_\mu^a,$$

$$\mathcal{L}_2^4 \equiv B_\mu^b B_{\rho a} A^{\mu\nu a} A^\rho{}_{\nu b}\,, \qquad S_{\mu\nu}^a \equiv \nabla_\mu B_\nu^a + \nabla_\nu B_\mu^a,$$

$$\mathcal{L}_2^5 \equiv B_{\mu a} B_\rho^b A^{\mu\nu a} A^\rho{}_{\nu b}\,,$$

J. C. Garnica et. al., Ann. Phys. (Berl.), 2022.

Lagrangian GSU2P

$$\mathcal{L}^1_{4,2} \equiv \left(B_b \cdot B^b\right) \left[S^{\mu a}_{\mu} S^{\nu}_{\nu a} - S^{\mu a}_{\nu} S^{\nu}_{\mu a}\right] + 2\left(B_a \cdot B_b\right) \left[S^{\mu a}_{\mu} S^{\nu b}_{\nu} - S^{\mu a}_{\nu} S^{\nu b}_{\mu}\right] \ ,$$

$$\mathcal{L}^2_{4,2} \equiv A^a_{\mu\nu} S^{\mu b}_{\sigma} B^{\nu}_a B^{\sigma}_b - A^a_{\mu\nu} S^{\mu b}_{\sigma} B^{\nu}_b B^{\sigma}_a + A^a_{\mu\nu} S^{\sigma b}_{\sigma} B^{\mu}_a B^{\nu}_b \ ,$$

$$\mathcal{L}^3_{4,2} \equiv B^{\mu a} R^{\alpha}{}_{\sigma\rho\mu} B_{\alpha a} B^{\rho c} B^{\sigma}_c + \frac{3}{4}\left(B^a \cdot B_a\right)\left(B_b \cdot B^b\right) R \, ,$$

$$\mathcal{L}^4_{4,2} \equiv \Big[\left(B^a \cdot B_a\right)\left(B^b \cdot B_b\right) + 2\left(B^a \cdot B^b\right)\left(B_a \cdot B_b\right)\Big] R \, ,$$

$$\mathcal{L}^5_{4,2} \equiv G_{\mu\nu} B^{\mu a} B^{\nu}_a \left(B^b \cdot B_b\right) \ ,$$

$$\mathcal{L}^6_{4,2} \equiv G_{\mu\nu} B^{\mu a} B^{\nu b} \left(B_a \cdot B_b\right) \, ,$$

J. C. Garnica et. al., Ann. Phys. (Berl.), 2022.

Einstein + Yang-Mills vector fields

- Static, localized solutions, asymptotically flat: soliton, particlelike
- Attractive (gravity) and repulsive (Yang-Mills) interactions.
- EYM Good: particlelike and BHs. Bad: unstable
- Einstein + Skyrme: Particlelike, BHs. Stable solutions
- Are there particlelike, BH solutions in GSU2P theory?

R. Bartnik et. al., Phys. Rev. Lett., 1988.

Stationary and spherical symmetric solution

- Regular solutions: all algebraic curvature invariants, energy density and pressure are finite.

$$ds^2 = -e^{2\Phi}dt^2 + (1-2m/r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

$$e^{2\Phi} \equiv e^{-2\delta}(1-2m/r),$$

- Asymptotic flatness: when $r \to \infty, m \to M = \text{const.}, \delta \to 1,$

- Vector fields configuration:

$$\mathbf{B} = \frac{\tau^i}{g_c}\Big[A_0\frac{x_i}{r}dt + A_1\frac{x_ix_j}{r^2}dx^j + \frac{\phi_1}{r}\left(\delta_{ij} - \frac{x_ix_j}{r^2}\right)dx^j - \epsilon_{ijk}x^j\frac{(1-w)}{r^2}dx^k,$$

where τ_i are the Pauli matrices.

J. Martinez et. al., JCAP, 2023.

Field equations

- Variation with respect to metric,

$$G_{\mu\nu} = 8\pi T^{\text{effec}}_{\mu\nu},$$

$$T^{\text{effec}}_{\mu\nu} \equiv -\frac{1}{8\pi\sqrt{-g}}\frac{\delta(\mathcal{L}^{\text{effec}}\sqrt{-g})}{\delta g^{\mu\nu}}, \qquad \mathcal{L}^{\text{effec}} \equiv \mathcal{L} - R,$$

- Variation with respect vector field,

$$\frac{\delta\mathcal{L}}{\delta B^{a\mu}} = 0,$$

- Individually the only configuration consistent with asymptotic flatness: 't Hooft-Polyakov monopole

$$A_0 = A_1 = \phi_1 = 0, w \neq 0,$$

- Normalized variables by $\mathrm{g}_c, r \to r\mathrm{g}_c, m \to m\mathrm{g}_c.$

Analytic solution for χ_1, χ_2
Schwarzschild and Reissnar Nordstrom black hole

- We found an analytic solution for the parameters $\chi_{12} = 2\chi_1 + \chi_2, w = w_c = \text{const.},$

- Black holes:

$$w_c = w_{I,II}, \quad m = M - \frac{Q_{I,II}^2}{2r}, \quad \Phi = \frac{1}{2}\ln\left(1 - \frac{2M}{r} + \frac{Q_{I,II}^2}{r^2}\right),$$

$$w_{I,II} = \frac{1 + 2\chi_{12} \pm \sqrt{1 + 8\chi_{12}}}{2 - 2\chi_{12}}, \qquad Q_{I,II}^2 = \frac{1 - 4\chi_{12}(5 + 2\chi_{12}) \mp (1 + 8\chi_{12})\sqrt{1 + 8\chi_{12}}}{2(1 - \chi_{12})^3},$$

with $\chi_{12} > -1/8.$

- For the solution I the charge can be imaginary in $\chi_{12} \in (0, 1).$

Asymptotic solutions when $r \to 0$

Power series expansion:

$$m = a_0 + a_1 r + a_2 r^2 + a_3 r^3 + a_4 r^4 + a_5 r^5 + \mathcal{O}(r^6),$$
$$w = b_0 + b_1 r + b_2 r^2 + b_3 r^3 + b_4 r^4 + \mathcal{O}(r^5),$$
$$\Phi = c_1 r + c_2 r^2 + c_3 r^3 + c_4 r^4 + \mathcal{O}(r^5)\,.$$

- Curvature invariants are finite at $r = 0$ when $a_1 = a_2 = c_1 = 0.$
- Effective energy density and pressure are finite when $b_0 = -1, b_1 = 0.$

$$a_3 = 2b_2^2\,,$$
$$a_5 = \frac{3}{5}\mu^2 b_2^2 - \frac{8b_2^3}{5} + \frac{172\alpha_1 b_2^4}{3} + \frac{7\alpha_3 b_2^4}{15} - 4\chi_6 b_2^4\,,$$
$$b_4 = \frac{\mu^2 b_2}{10} - \frac{3b_2^2}{10} + \frac{4b_2^3}{5} + \alpha_1 b_2^3 + \frac{7\alpha_3 b_2^3}{10} + \frac{\chi_5 b_2^3}{5} - \chi_6 b_2^3\,,$$
$$c_2 = 2b_2^2\,,$$
$$c_4 = \frac{\mu^2 b_2^2}{5} - \frac{4b_2^3}{5} + \frac{12b_2^4}{5} - 8\alpha_1 b_2^4 + \frac{9\alpha_3 b_2^4}{10} - \frac{2\chi_5 b_2^4}{5} - 2\chi_6 b_2^4\,,$$

Asymptotic solutions when $r \to \infty$

Globally neutral and charged solutions

- SU(2) global symmetry breaks $w \to -w.$
- We obtained globally neutral solutions (EYM generalizations) and new globally charge solutions.
- Solutions as a series of inverse powers of r,

$$
\begin{aligned}
m =& M + \frac{\tilde{a}_1}{r} + \frac{\tilde{a}_2}{r^2} + \frac{\tilde{a}_3}{r^3} + \mathcal{O}\left(\frac{1}{r^4}\right) ,\\
w =& w_\infty + \frac{\tilde{b}_1}{r} + \frac{\tilde{b}_2}{r^2} + \frac{\tilde{b}_3}{r^3} + \mathcal{O}\left(\frac{1}{r^4}\right) ,\\
\Phi =& \Phi_\infty + \frac{\tilde{c}_1}{r} + \frac{\tilde{c}_2}{r^2} + \frac{\tilde{c}_3}{r^3} + \mathcal{O}\left(\frac{1}{r^4}\right) .
\end{aligned}
$$

Asymptotic solutions when $r \to \infty$

	$w_\infty = -1$	$w_\infty = 1$	$w_\infty = w_{\mathrm{I,II}}$
$\tilde{a}_1$	0	0	$-Q_{\mathrm{I,II}}^2/2$
$\tilde{a}_2$	0	0	0
$\tilde{a}_3$	$-\tilde{b}_1^2$	$64(25\alpha_1 - 36\alpha_3 + 5\chi_6)/15 - \tilde{b}_1^2$	$\frac{2}{3}\tilde{b}_2(w_\infty + 1)[\chi_{12} + (\chi_{12} - 1)w_\infty^2 + 2\chi_{12}w_\infty + w_\infty] + \frac{4}{15}(w_\infty + 1)^4(25\alpha_1 - 36\alpha_3 + 5\chi_6)$
$\tilde{b}_1$	Free	Free	0
$\tilde{b}_2$	$3(2M - \tilde{b}_1)\tilde{b}_1/4$	$16\alpha_1 + 6\alpha_3 - 16\chi_6 + 3(2M + \tilde{b}_1)\tilde{b}_1/4$	$\frac{(w_\infty + 1)^3(8\alpha_1 + 3\alpha_3 - 8\chi_6)}{3\chi_{12}(w_\infty + 1)^2 - 3w_\infty^2 + 7}$
$\tilde{b}_3$	$\tilde{b}_1[48M^2 - 42M\tilde{b}_1 + (11 - 2\chi_{12})\tilde{b}_1^2]/20$	$(512\alpha_1 M + 96\alpha_3 - 512\chi_6 M)/20$ $+16\tilde{b}_1(26\alpha_1 + 13\alpha_3 + 2\chi_5 - 26\chi_6)/20$ $+(48M^2 - 42M\tilde{b}_1 + 11\tilde{b}_1^2)/20$	$\frac{2M\left(8\tilde{b}_2 - 3\alpha_3(w_\infty + 1)^3\right)}{3\chi_{12}(w_\infty + 1)^2 - 3w_\infty^2 + 13}$
$\tilde{c}_1$	$-M$	$-M$	$-M$
$\tilde{c}_2$	$-M^2$	$-M^2$	$-M^2 + Q_{\mathrm{I,II}}^2/2$
$\tilde{c}_3$	$-4M^3/3$	$-4M^3/3$	$-4M^3/3 + Q_{\mathrm{I,II}}^2 M/2$

Asymptotic charged solution

Case χ_{12}

- Series of inverse powers of r with noninteger exponent:

$$m = M - \frac{Q_{\rm I}^2}{2r} - \frac{d^2 D}{r^{2\beta+1}} + \mathcal{O}\left(\frac{1}{r^{3\beta+1}} + \frac{1}{r^{2\beta+2}}\right),$$

$$w = w_{\rm I} + \frac{d}{r^{\beta}} + \mathcal{O}\left(\frac{1}{r^{\beta+1}} + \frac{1}{r^{2\beta}}\right),$$

$$\delta = -\frac{d^2\beta^2}{(1+\beta)}\frac{1}{r^{2\beta+2}} + \mathcal{O}\left(\frac{1}{r^{2\beta+3}} + \frac{1}{r^{3\beta+2}}\right),$$

where

$$\beta \equiv -\frac{1}{2}\left(1 - \sqrt{\frac{3 + 15\chi_{12} + 6\sqrt{1+8\chi_{12}}}{1-\chi_{12}}}\right),$$

$$D \equiv \frac{1 + 8\chi_{12} + \sqrt{1+8\chi_{12}} - 2\beta^2(\chi_{12}-1)}{2(1+2\beta)(\chi_{12}-1)}.$$

Effective charge and topological charge

- Effective charge is defined by comparing with the RN solution

$$Q^2 \equiv 2r(M - m),$$

- The topological charge is defined from the Bianchi identity:

$$d({}^*\mathbf{F}) = 0,$$

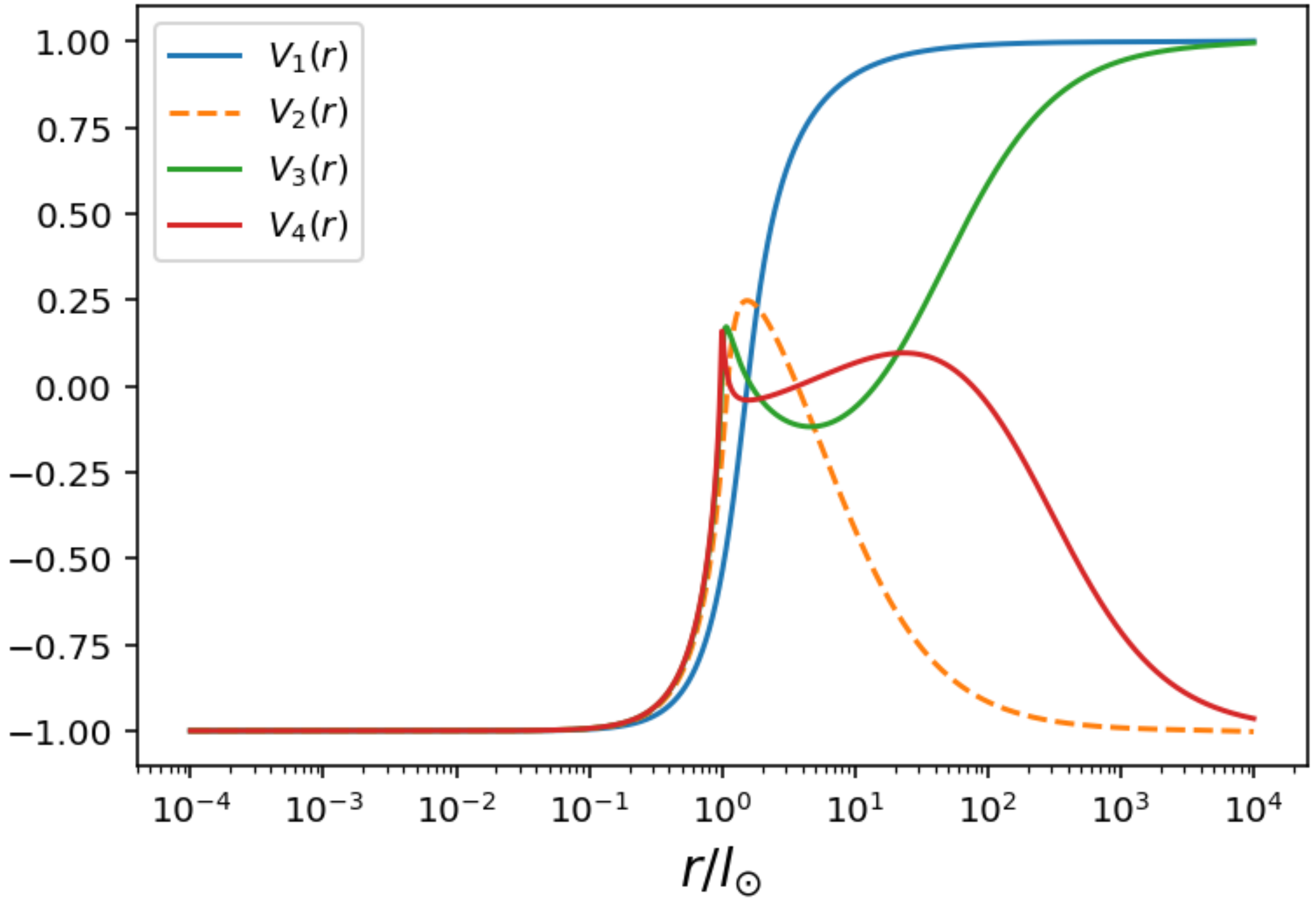
- The conserved topological charge is,

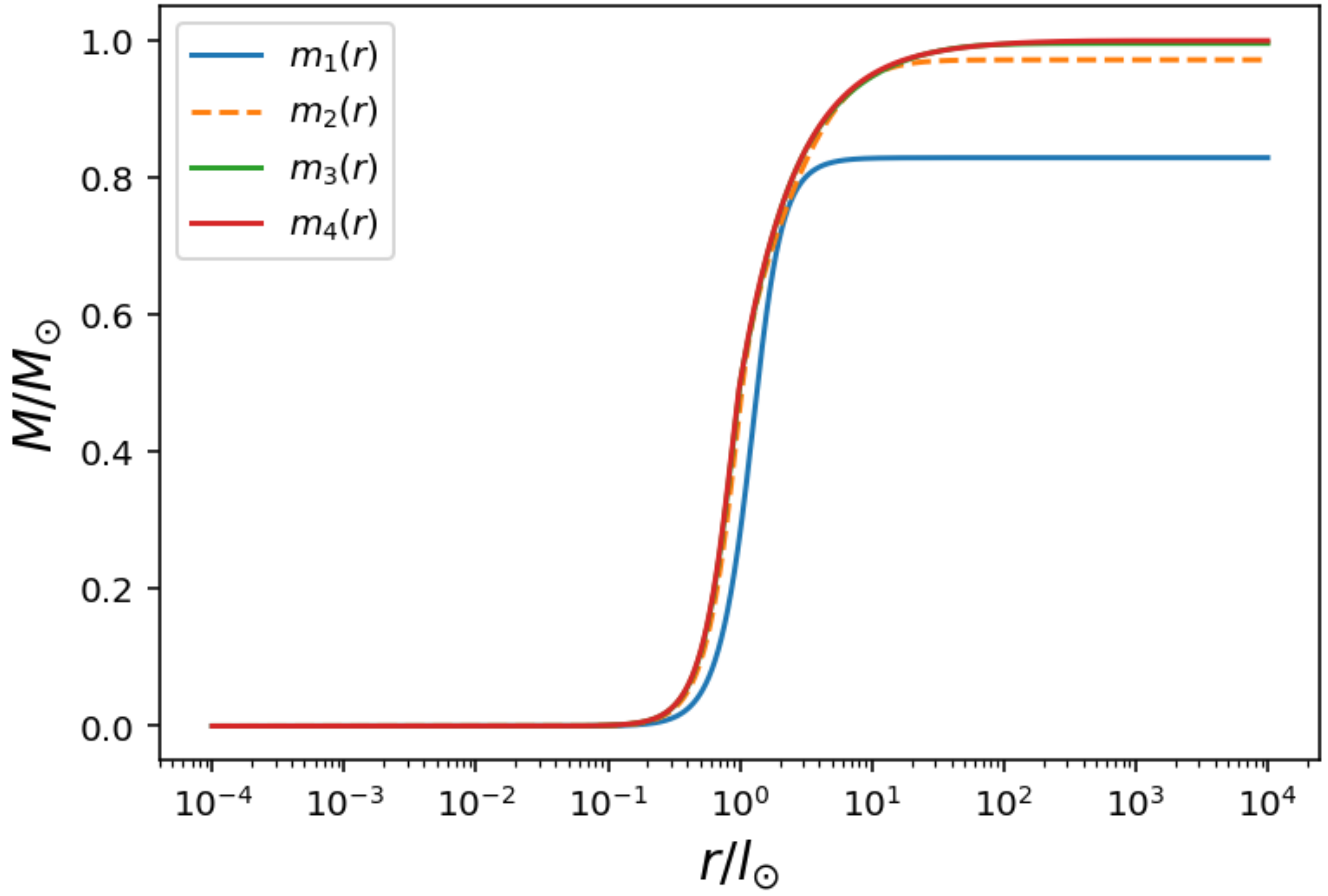
$$Q^a_M = \int \nabla_\mu ({}^*\mathbf{F}^{a\mu 0})\sqrt{-g} d^3x = \int \partial_\mu (\epsilon^{\mu 0 \alpha\beta} \mathbf{F}^a{}_{\alpha\beta}) d^3x = \int dS_k \sqrt{-g} ({}^*\mathbf{F}^{k0}),$$

- The topological charge depends on the value of the value of the fields at spatial infinity

$$Q^a_M \propto (1 - w^2_\infty),$$
$$Q = Q^a_M + q.$$

EYM



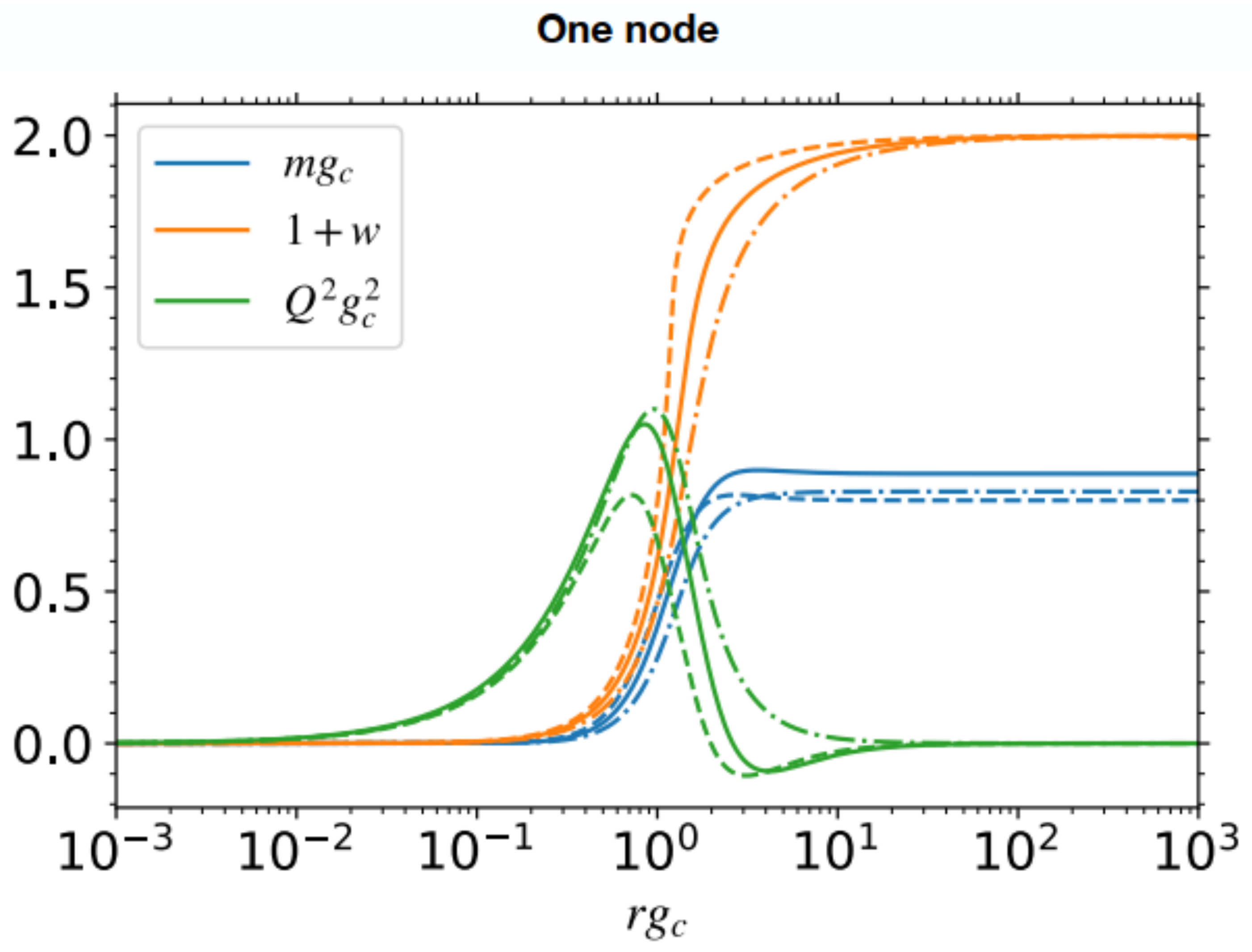


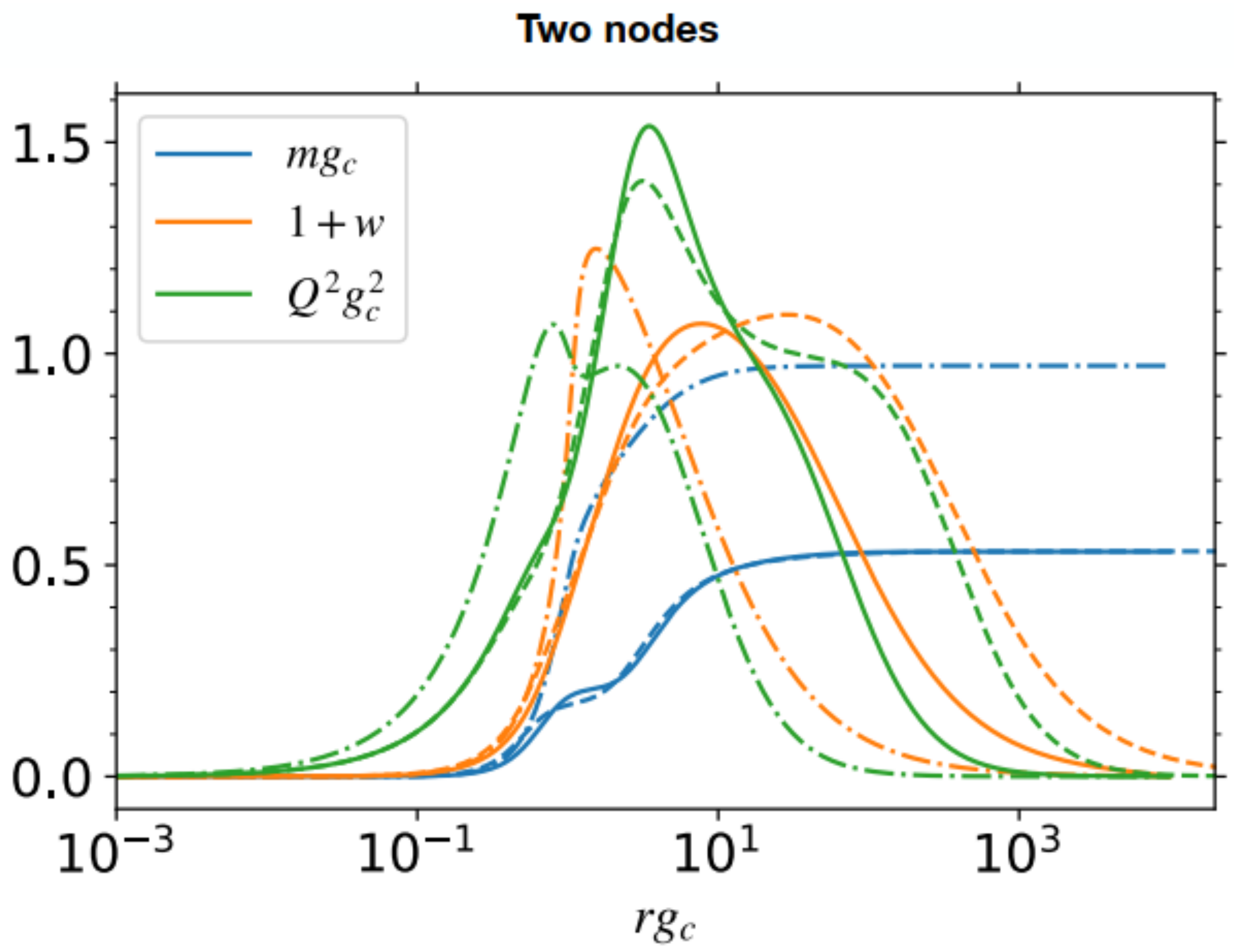
$$b_2 \in [0, 0.706).$$

R. Bartnik et. al., Phys. Rev. Lett., 1988.

Numerical solution

Case α_1



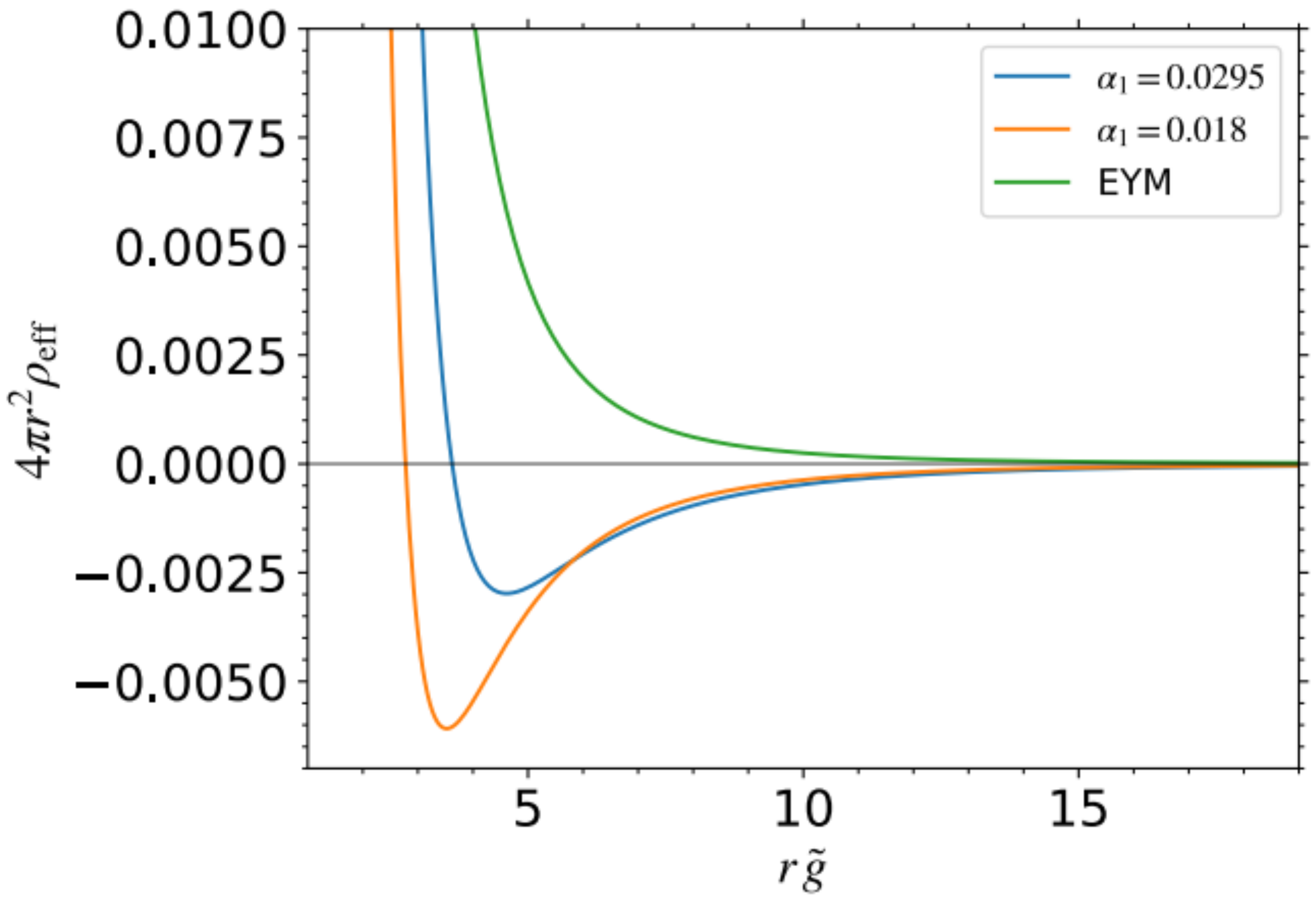


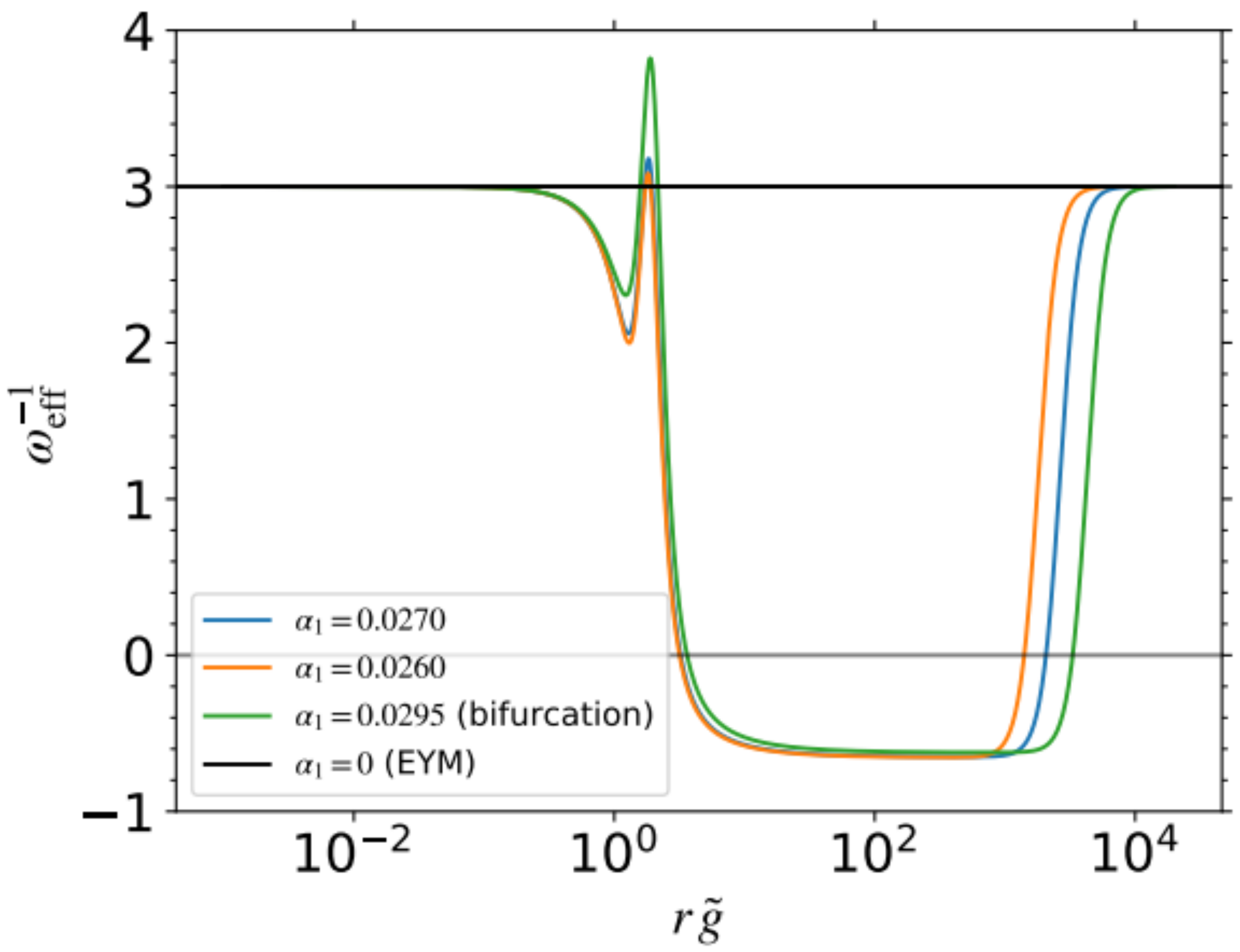
Continuous $\alpha_1 = 0.029519$, dashed $\alpha_1 = 0.017$ dotted-dashed EYM.

Continuous and dashed $\alpha_1 = -1$, dotted-dashed EYM.

Energy density

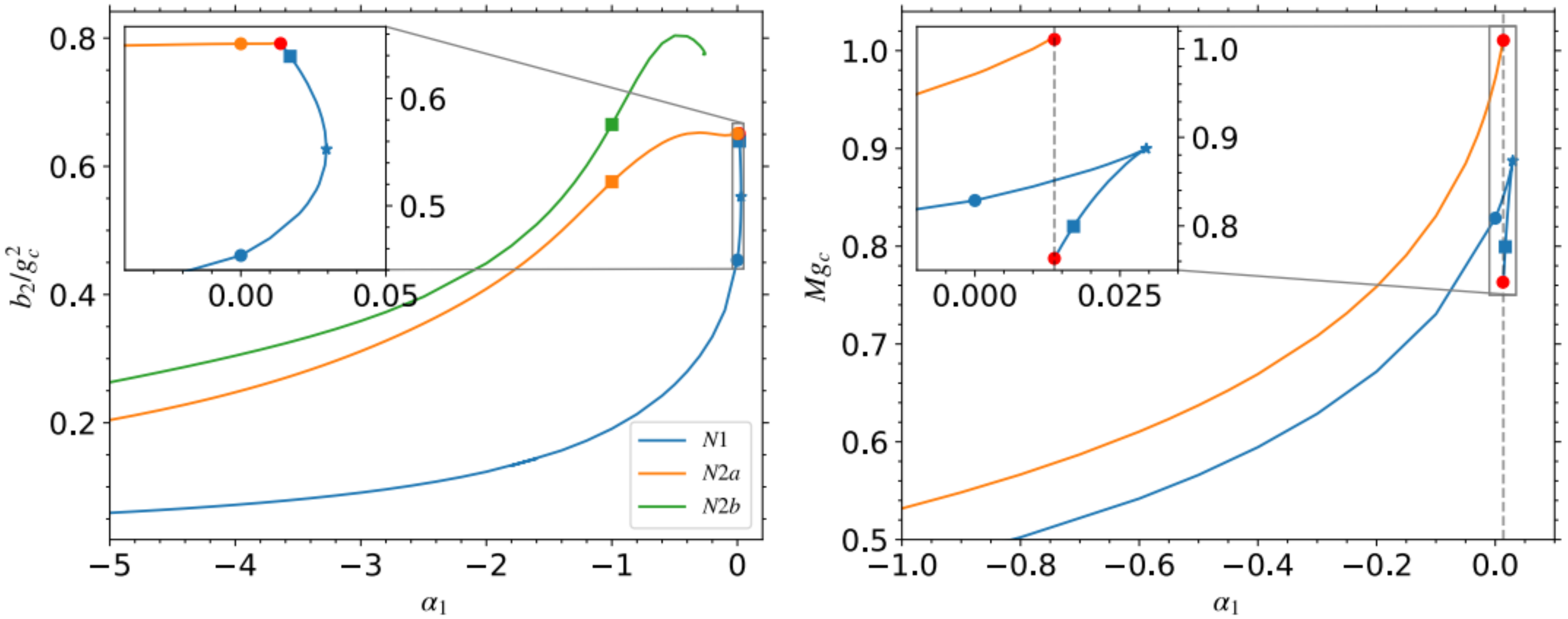
Case α_1





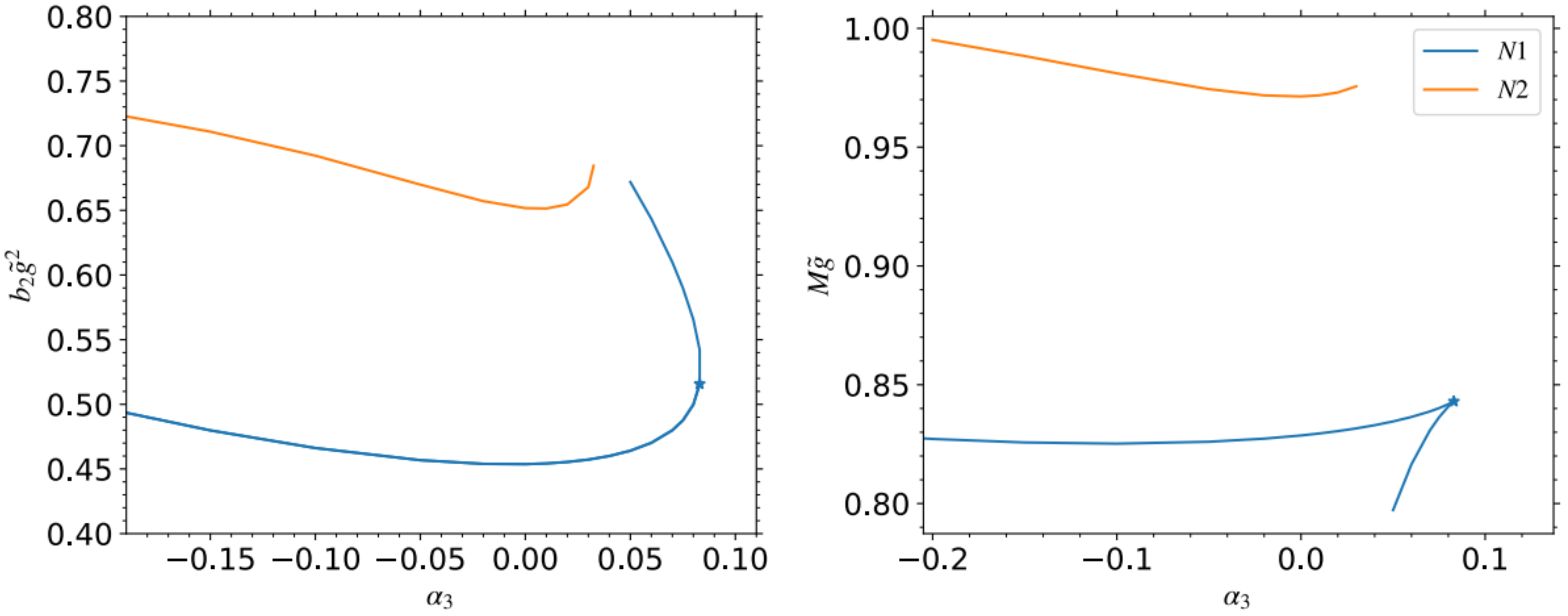
Equilibrium sequence

Case α_1



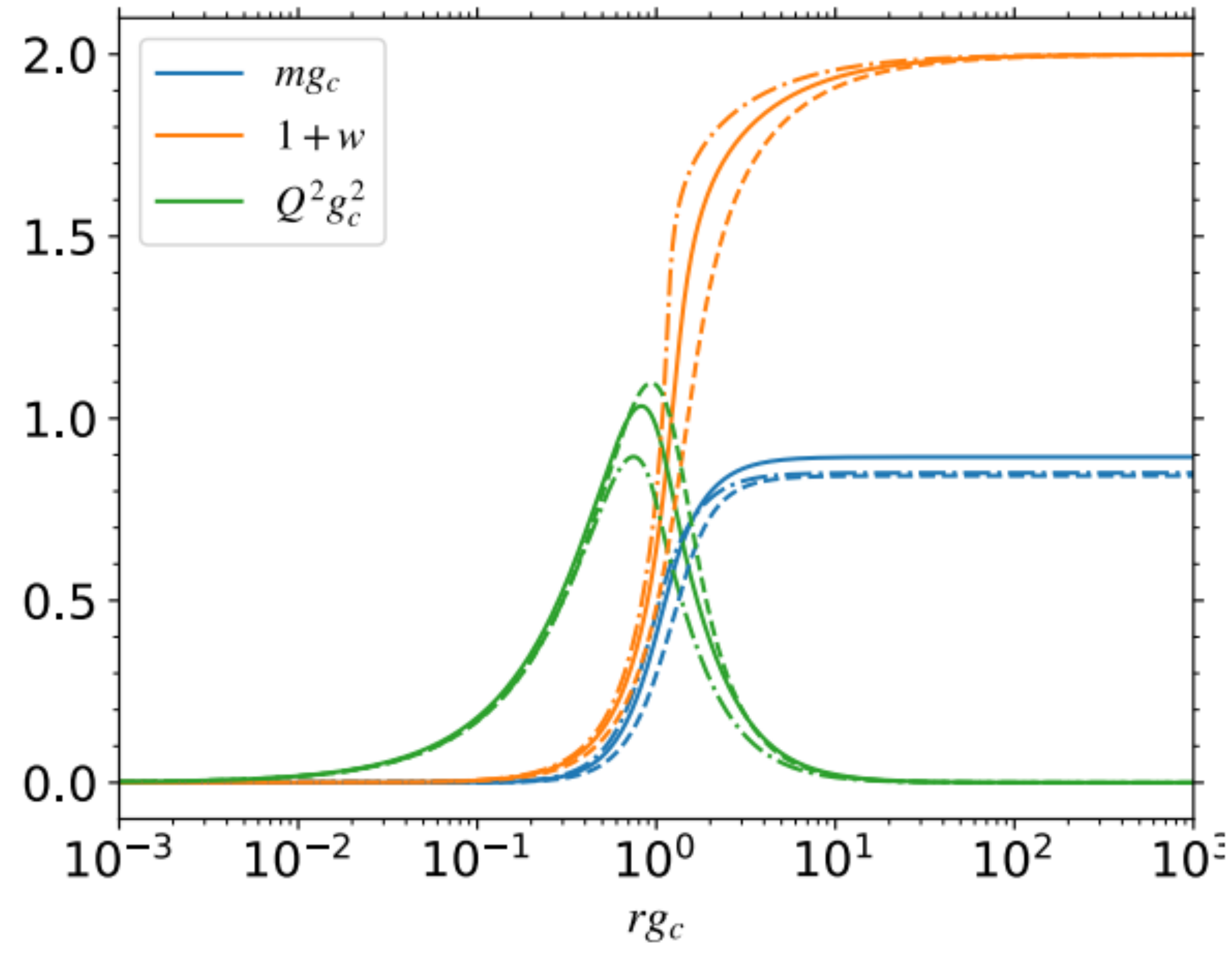
Equilibrium sequence: One node solutions

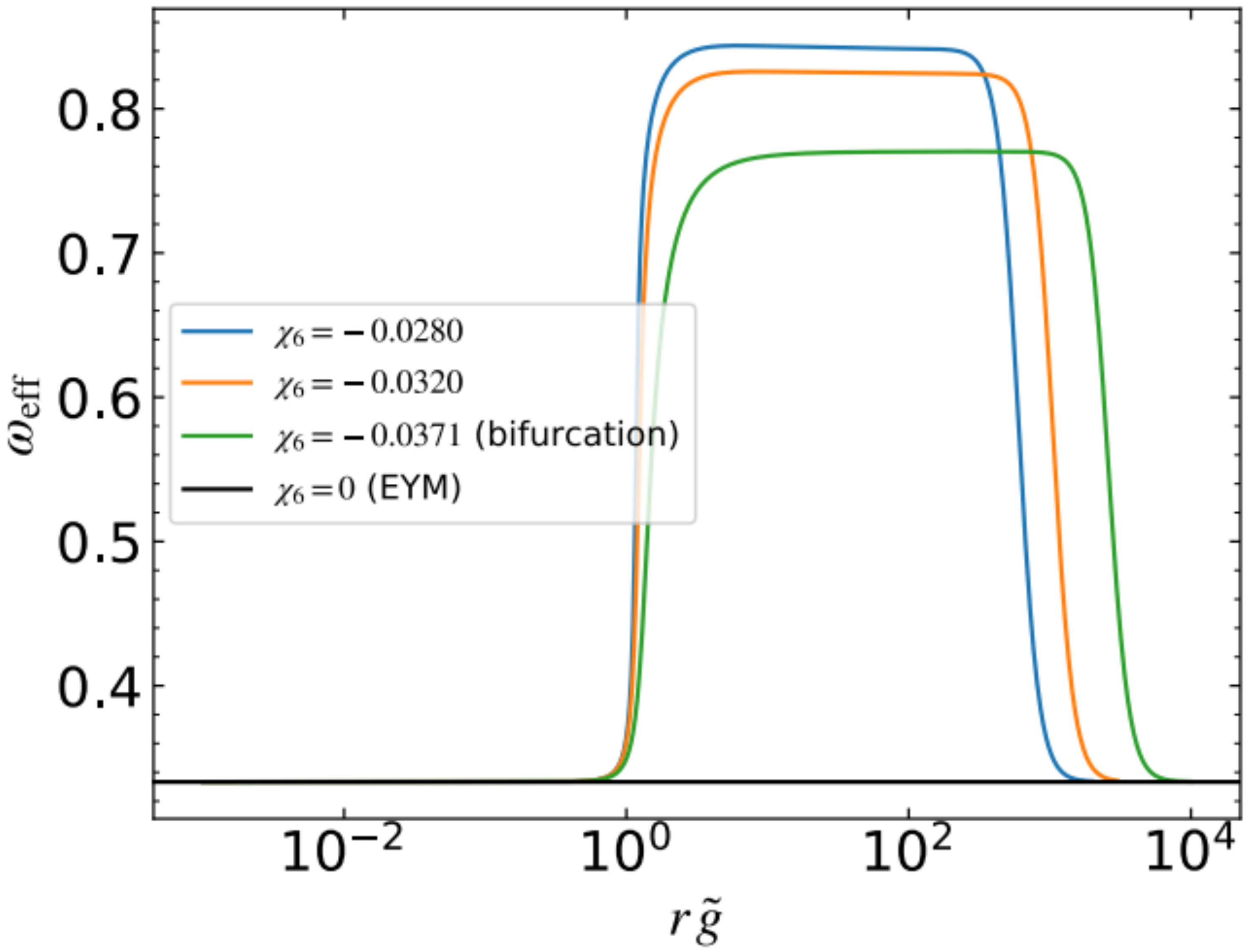
Case α_3



Numerical solution

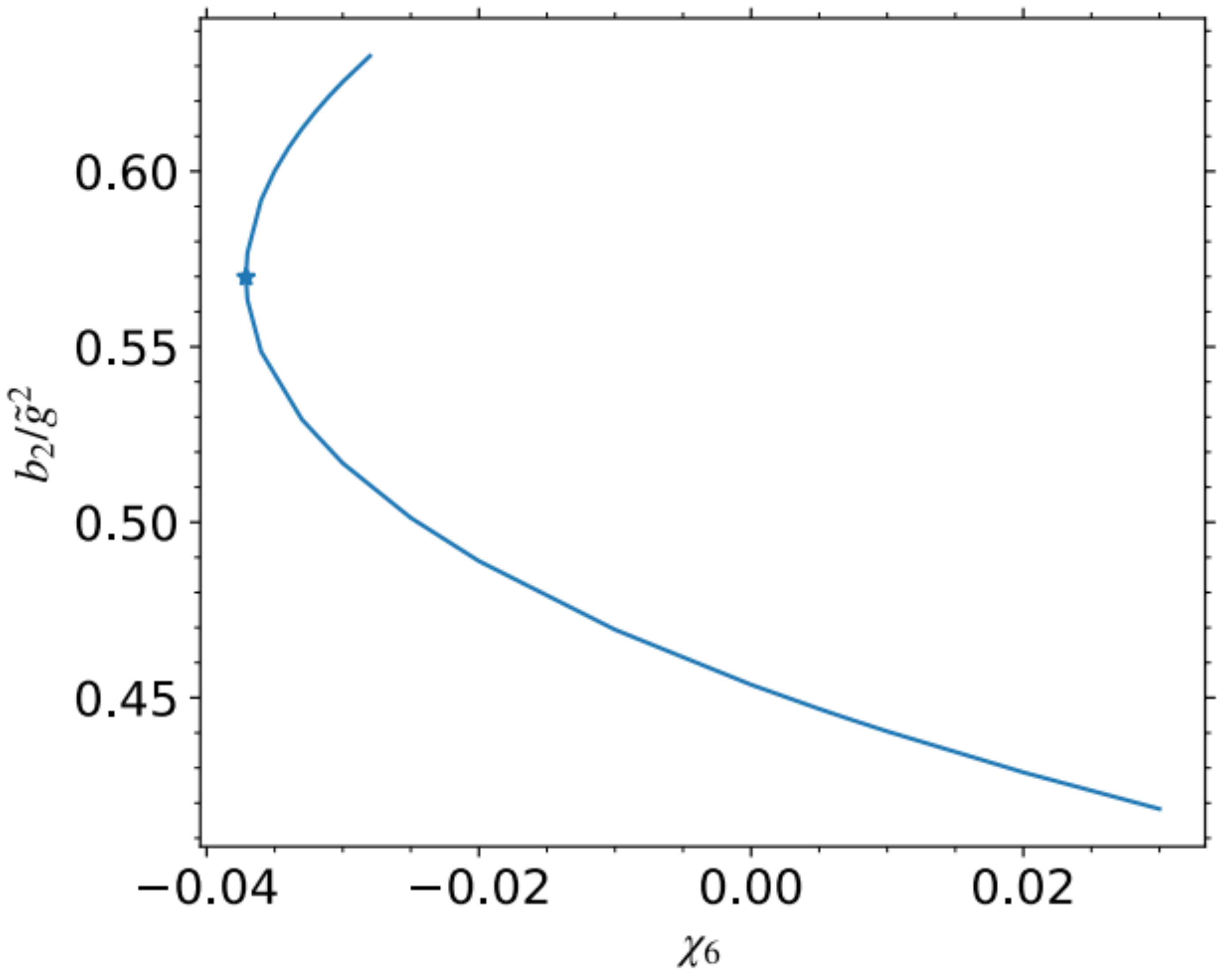
Case χ_6

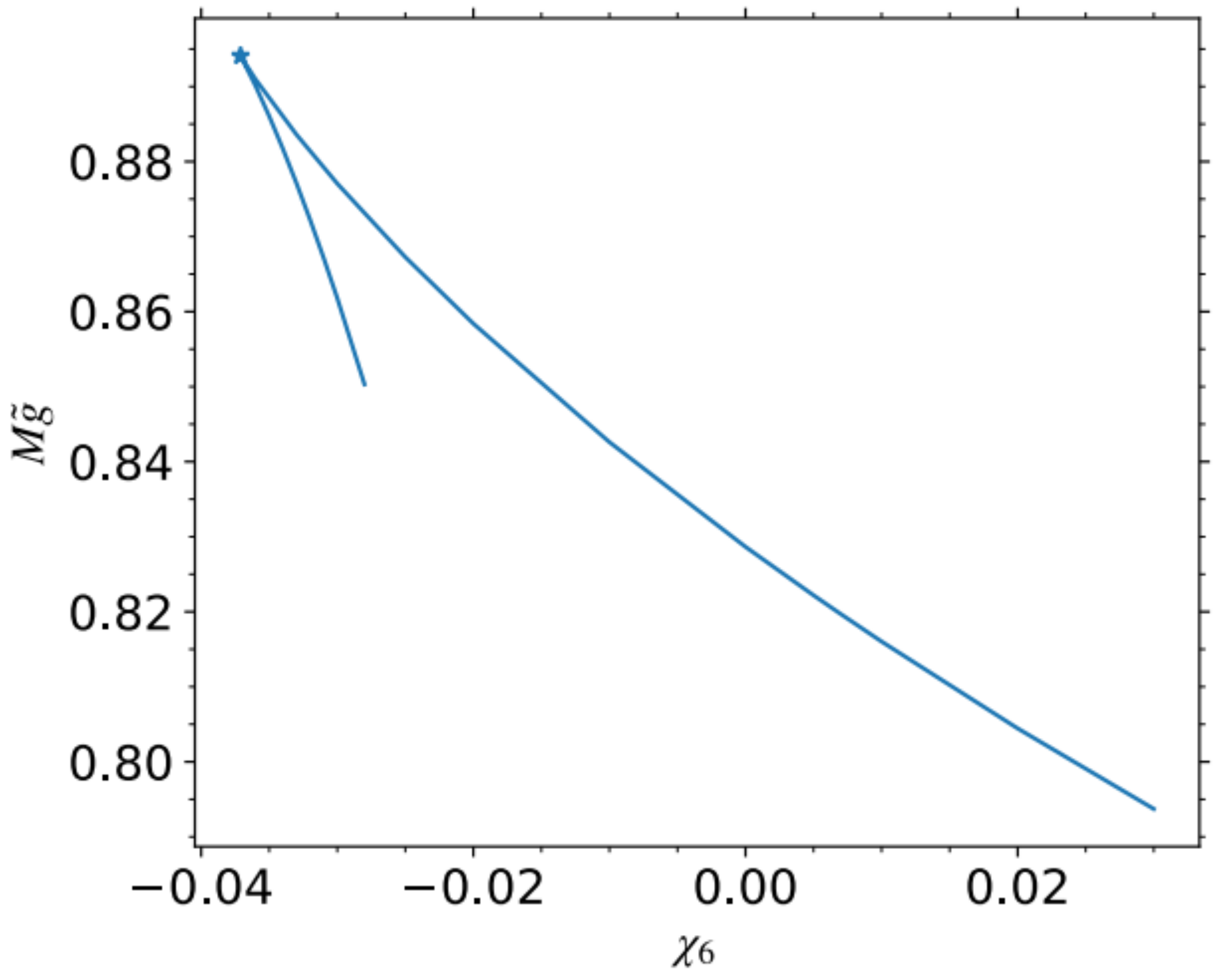




Equilibrium sequence: One node solutions

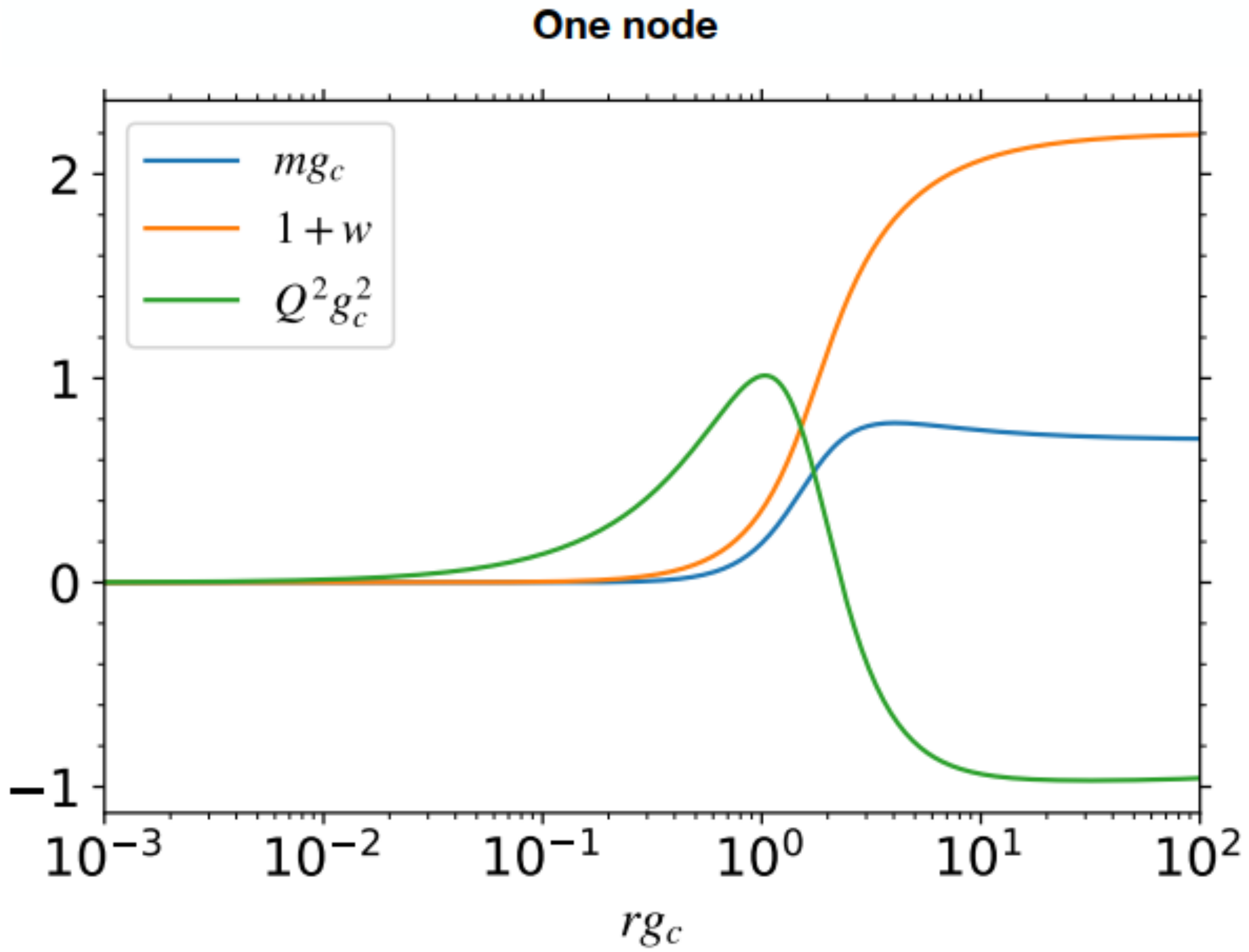
Case χ_6



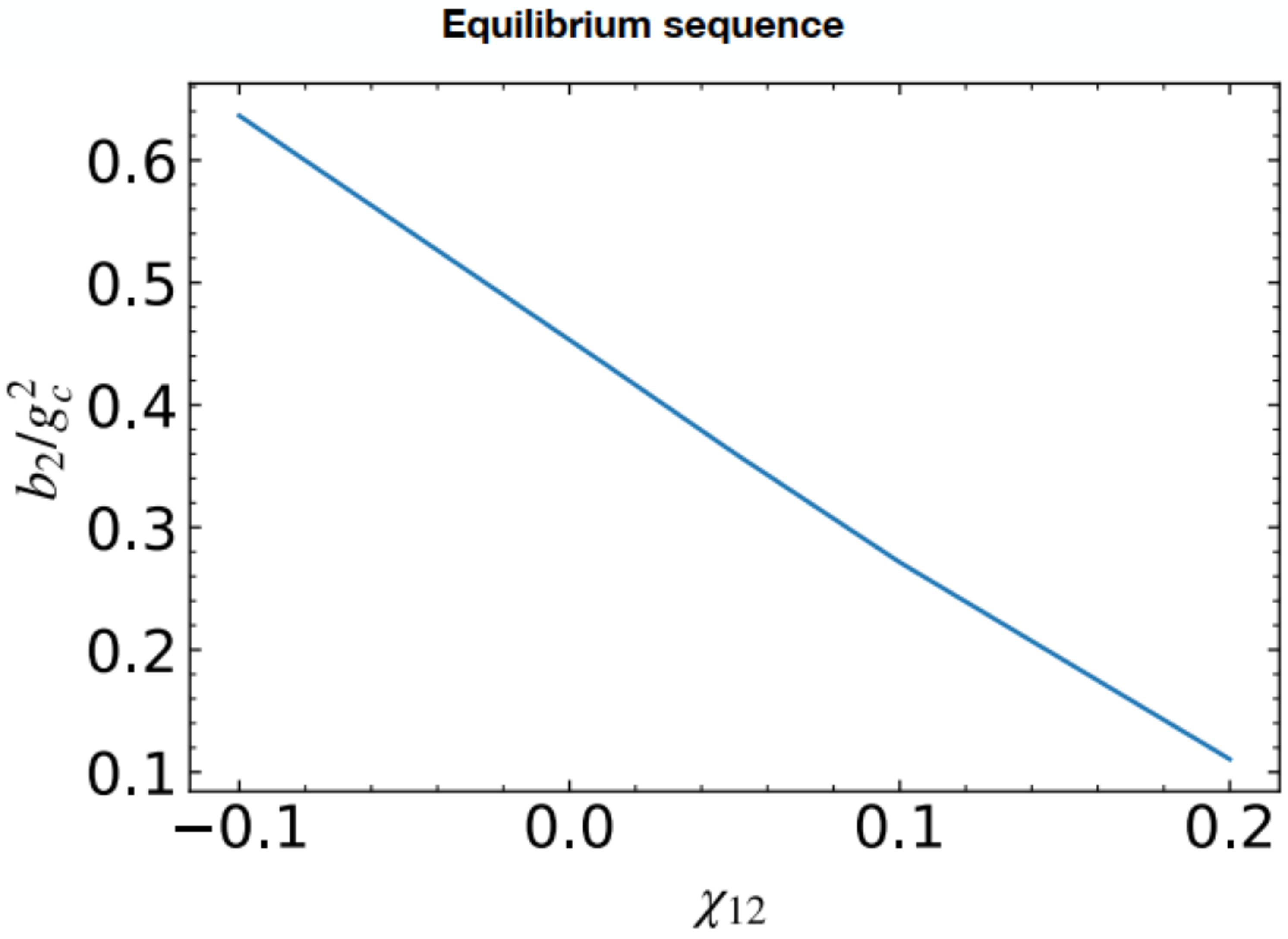


Charged solutions

Case χ_{12}



$$\chi_{12} = -1$$

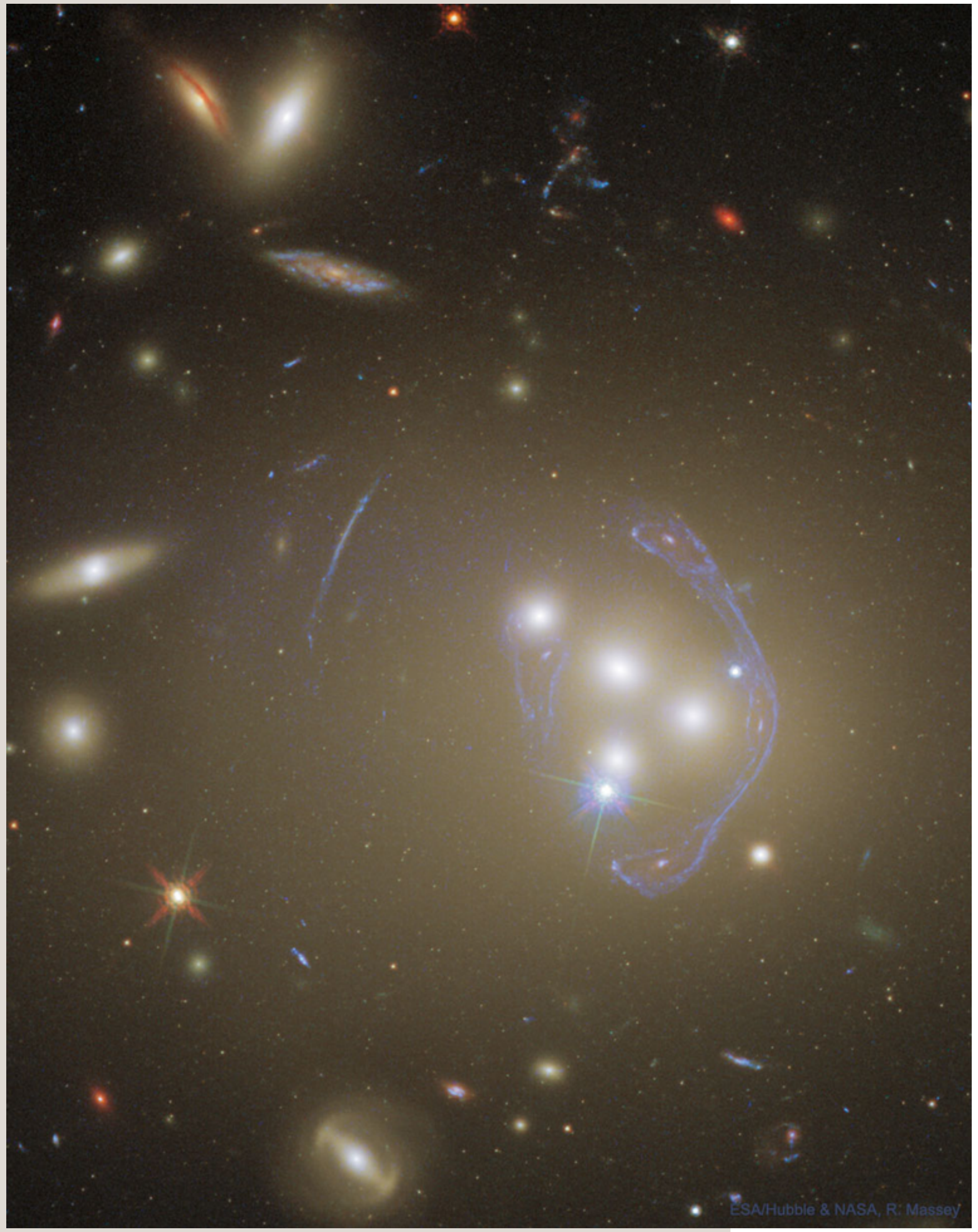


No bifurcations, no change in stability (?)

Results

- We have found particlelike solutions in the GSU2P theory,
- Generalization of the EYM case: charged solutions with negative energy density regions,
- We have constructed equilibrium sequences and found bifurcations points for in the cases $\alpha_1, \alpha_3, \chi_6,$ which hints towards the existence of stable solutions,
- Highly compact objects with photon sphere,
- Black hole solutions and solutions with matter.

J. Martinez et. al., JCAP, 2023.



Thanks a lot!

https://cutt.ly/KnxiZaU