Peccei-Quinn mechanism and axion interactions in the 3-3-1 model with Cosmological Inflation

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Abstract

- Based on the Peccei-Quinn (PQ) charge assignment, the PQ charge operator in the 3-3-1 model with Cosmological Inflation is constructed in terms of diagonal generators T_3 and T_8 of the $SU(3)_L$ subgroup.
- The formula shows that difference of PQ $/(Q_A)$ charges of up and down quarks is 2, i.e., $\Delta Q_A = 2$, while for electric charge, are assumed to be equal, while the (Q_A) are opposite.
- PQ charge of neutral scalars equal ± 2, while for charged scalars, it vanishes.
-To have correct kinetic term for the axion, $f_a = v_\phi$.
- The axion is an electric charge-phobia (unlike charged) scalar and gauge bosons.
- The axion has doubly derivative coupling with scalar playing the role of inflaton.
- The new effects mainly happen in the energy region from 10^7 GeV to 10^{11} GeV.
- The chiral effective Lagrangian as usually provides axion mass consistent with model-independent prediction.

Contents

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I. Strong CP problem

Yang-Mills theories, such as QCD generally break CP symmetry via [G.'tHooft, Phys. Rev. D **14**, 3432 (1976)]

$$\mathcal{L}_\theta \sim \tilde{\theta} G \tilde{G} \,, \tag{1}$$

where G is gluon field strength, $\tilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} G^{\alpha\beta}$ is dual tensor. No electric dipole moment of neutron follows [NEDMCollab., C. Abel et al, PRL, 124 (2020) 081803]

$$\tilde{\theta} \leq 10^{-10}$$

This tiny value leads to strong CP problem .

I. Strong CP problem

Then the electric and magnetic fields are:

$$
\begin{aligned}
W^a_{0\nu} &\equiv E^a_\nu\,, \qquad \nu = 1,2,3 = i\,;\ \text{(space index)};\ a = 1,2,\cdots 8 \\
B^a_k &\equiv \epsilon_{ijk}\,W^a_{ij} \qquad i \neq j \neq k, (j,k = 1,2,3)
\end{aligned} \tag{2}
$$

Then the Lagrangian of gauge bosons are following

$$
\begin{aligned}
-4\,\mathcal{L}_{gauge} &\supset a(x)\,W_{a\mu\nu}\,W^{a\mu\nu} = 2a(x)(\,W_{a0\nu}\,W^{a0\nu} + \,W_{aij}\,W^{aij}) \\
&= 2a(x)\,(E_{ai}E^{ai} + B_{ak}B^{ak}) = 2a(x)(\mathbf{E}^2 + \mathbf{B}^2)\,.
\end{aligned} \tag{3}
$$

Now we consider a part with the dual tensor

$$
\begin{aligned}
\mathcal{L}_\theta &\supset \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\,W_{a\mu\nu}\,W^a_{\alpha\beta} = 4!\,\epsilon^{0i\alpha\beta}\,W_{a0i}\,W^a_{\alpha\beta} \\
&= 4!\,\epsilon^{0ijk}\,W_{a0i}\,W^a_{jk} = 4!\,(E_{ai}B^{ai}) = 4!\,(\mathbf{E}.\mathbf{B})\,.
\end{aligned} \tag{4}
$$

II. Peccei-Quinn mechanism

The main element - scalar singlet
i) In cartesian coordinate system

$$z = x + iy \Rightarrow z = \rho e^{ia} \,. \tag{5}$$

In space-time coordinate [R. D. Peccei and H. Quinn, Phys.Rev. D.**16**,1791(1977)]

$$\Phi(x) = R(x) e^{i\theta(x)}. \tag{6}$$

Here θ is responsible for neutron electric diplole moment (EDM).
ii) $\theta = 0$ energetically favourable.
Potential $V = V(\Phi)$ then it will have a minimum $R = v \gg 0$. Strong CP problem, $\theta = 0$. Put neutron EDM to zero. V harmonic form $V(\theta) \sim \theta^2$.
iii) Redefinition [Georgi,Kaplan and Randall, Phys. Lett. **B 169** (1986) 73]

$$\Phi = R(x) \, e^{i \frac{a(x)}{f_a} c_\phi} \,, \tag{7}$$

where $f_a \geq 10^{10}$ GeV. The factor a/f_a ensures a solution of strong CP puzzle.

II. Peccei-Quinn mechanism

According to [R. D. Peccei and H. Quinn, Phys.Rev. D **16** (1977) 1791]:

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\bar{\psi}\gamma_\mu D^\mu\psi + L_{Yukawa} - V(\phi)\,, \tag{8}$$

with transformation

$$\psi \to e^{i\sigma\gamma_5}\psi\,, \quad \bar{\psi} \to \bar{\psi}e^{i\sigma\gamma_5} \quad \phi \to e^{-2i\sigma}\phi\,. \tag{9}$$

Then

$$\partial^\mu J^5_\mu = \frac{g^2}{16\pi^2}F^a_{\mu\nu}\tilde{F}^{a\mu\nu} \tag{10}$$

Then one has

$$\mathcal{L}_{ag} = \frac{\alpha_s}{4\pi}\frac{a}{2f_{pq}}G\tilde{G} + \frac{\alpha_2}{4\pi}\frac{a}{2f_{pq}}W\tilde{W} + \frac{\alpha_1}{4\pi}\frac{a}{2f_{pq}}B\tilde{B}\,. \tag{11}$$

III. The 3-3-1 with Inflation

[C. A. de S. Pires, *et al*, Phys. Lett. B **771** (2017) 199;
V. H. Binh, *et al*, Phys. Rev. D **107** (2023)]
Particle content: -Leptons in triplet

$$\begin{aligned} f^a_L &= \begin{pmatrix} \nu^a_L \\ l^a_L \\ \nu^{ca}_L \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, -\mathbf{1}/\mathbf{3}), l^a_R \sim (1, 1, -1), \\ N_{aR} &\sim (1, 1, 0) \quad \text{Majorana RH neutrino} \end{aligned} \tag{12}$$

where $a = 1, 2, 3$
- Two quark generations in antitriplets and one in triplet

$$\begin{aligned} Q_{\alpha L} &= \begin{pmatrix} d_{\alpha L} \\ -u_{\alpha L} \\ D_{\alpha L} \end{pmatrix} \sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{0}), \ \alpha = 1, 2\,, \ D_{\alpha R} \sim (3, 1, -1/3), \\ Q_{3L} &= \begin{pmatrix} t \\ b \\ T \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}, \mathbf{1}/\mathbf{3}), T_R \sim (3, 1, 2/3), \end{aligned} \tag{13}$$

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III. The 3-3-1 with Inflation

For SSB, we need three Higgs triplets and one singlet

$$\rho = \begin{pmatrix} \rho_1^+ \\ \rho_2^0 \\ \rho_3^+ \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, \mathbf{2/3}), \quad \eta = \begin{pmatrix} \eta_1^0 \\ \eta_2^- \\ \eta_3^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, -\mathbf{1/3}),$$
$$\chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \chi_3^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, -\mathbf{1/3}), \quad \phi \sim (1, 1, 0) \tag{14}$$

Note that η and χ have the same quantum number, but the difference is that their components have different VEV structure:

$$\langle\rho\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}, \quad \langle\eta\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ 0 \\ 0 \end{pmatrix}, \quad \langle\chi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix}. \tag{15}$$
$$\langle\phi\rangle = \frac{1}{\sqrt{2}} v_\phi \,. \tag{16}$$

III. The 3-3-1 with Inflation

Since lepton and anti-lepton lie in the same triplet, the lepton number in these models are not conserved. The new conserved value should have the form [D. Chang & HNL, Phys. Rev. D (2006)]

$$L = \alpha T_3 + \beta T_8 + \mathcal{L}. \tag{17}$$

Applying for the lepton triplet, we get

$$L = \frac{4}{\sqrt{3}}\lambda_8 + \mathcal{L}$$

The fields with non-zero lepton number are presented below

Fields	N_R	ν^{ca}_L	l_L	l_R	ρ_3^+	η_3^0	χ_1^0	χ_2^-	ϕ	$D_{\alpha L}$	$D_{\beta L}$	T_L	T_R
L	-1	-1	1	1	-2	-2	2	2	2	2	-2	-2	-2

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IIIa. Neutrino masses

The Yukawa couplings for quarks are [C. A. de S. Pires, *et al*, Phys. Lett. B (2017); V. H. Binh, *et al*, Phys. Rev. D (2023)]

$$\begin{aligned}-\mathcal{L}_q^Y &= y_1 \bar{Q}_{3L} T_R \chi + \sum_{n,m=1}^{2} (y_2)_{n,m} \bar{Q}_{nL} D_{mR} \chi^* \\ &+ \sum_{a=1}^{3} (y_3)_{3a} \bar{Q}_{3L} u_{aR} \eta + \sum_{n=1}^{2}\sum_{a=1}^{3} (y_4)_{na} \bar{Q}_{nL} d_{aR} \eta^* \\ &+ \sum_{a=1}^{3} (y_5)_{3a} \bar{Q}_{3L} d_{aR} \rho + \sum_{n=1}^{2}\sum_{a=1}^{3} (y_6)_{na} \bar{Q}_{nL} u_{aR} \rho^* + H.c. \end{aligned} \quad (18)$$

and for the lepton [V. H. Binh, *et al*, Phys. Rev. D (2023)]

$$\begin{aligned}-\mathcal{L}_l^Y &= \sum_{a=1}^{3}\sum_{b=1}^{3} g_{ab} \bar{\psi}_{aL} l_{bR} \rho + \sum_{a=1}^{3}\sum_{b=1}^{3} \left(y_\nu^D\right)_{ab} \bar{\psi}_{aL} \eta N_{bR} \\ &+ \sum_{a=1}^{3}\sum_{b=1}^{3} (y_N)_{ab} \phi \bar{N}_{aR}^C N_{bR} + H.c. \end{aligned} \quad (19)$$

IIIa. Neutrino masses

The last two terms in (19) contains related to neutrino mass

$$-\mathcal{L}_l^Y \supset \left(y_\nu^D\right)_{ab} \bar{\psi}_{aL}\eta N_{bR} + \left(y_N\right)_{ab} \bar{N}_{aR}^C N_{bR}\,\phi + H.c. \tag{20}$$

The Dirac neutrino mass term arises from v_η, while the Majorana mass term arises from v_ϕ.
From (20), it follows that the light active neutrinos are generated from a type I seesaw mechanism mediated by right handed Majorana neutrinos.
Thus implying that the resulting light active neutrino mass matrix has the form [V. H. Binh, *et al*, Phys. Rev. D (2023)]

$$M_\nu = M_\nu^D M_N^{-1} \left(M_\nu^D\right)^T, \qquad M_\nu^D = \frac{v_\eta}{\sqrt{2}}\, y_\nu^D, \qquad M_N = \sqrt{2}\, v_\phi\, y_N. \tag{21}$$

The standard neutrinos get mass at eV scale, while $M_R \sim 10^7$ GeV [C. A. de S. Pires, *et al*, Phys. Lett. B (2017)].

Note that the similar papers containing axion, inflaton and seesaw exist in Refs. below:

- A. Salvio, Phys. Lett. B 743 (2015) 428.
- G. Ballesteros, et al, JCAP 08 (2017) 001.

IIIb. Higgs sector

The full potential of the model under consideration has the form [C. A. de S. Pires, *et al*, Phys. Lett. B (2017)]

$$\begin{aligned} V_{tot} &= \mu_\chi^2\chi^\dagger\chi + \mu_\rho^2\rho^\dagger\rho + \mu_\eta^2\eta^\dagger\eta + \mu_\phi^2\phi^*\phi + \lambda_1(\chi^\dagger\chi)^2 + \lambda_2(\eta^\dagger\eta)^2 \\ &+ \lambda_3(\rho^\dagger\rho)^2 + \lambda_4(\chi^\dagger\chi)(\eta^\dagger\eta) + \lambda_5(\chi^\dagger\chi)(\rho^\dagger\rho) + \lambda_6(\eta^\dagger\eta)(\rho^\dagger\rho) \\ &+ \lambda_7(\chi^\dagger\eta)(\eta^\dagger\chi) + \lambda_8(\chi^\dagger\rho)(\rho^\dagger\chi) + \lambda_9(\eta^\dagger\rho)(\rho^\dagger\eta) \\ &+ \lambda_{10}(\phi^*\phi)^2 + \lambda_{11}(\phi^*\phi)(\chi^\dagger\chi) + \lambda_{12}(\phi^*\phi)(\rho^\dagger\rho) + \lambda_{13}(\phi^*\phi)(\eta^\dagger\eta) \\ &+ \left(\lambda_\phi\epsilon^{ijk}\eta_i\rho_j\chi_k\phi^* + H.c.\right) \end{aligned} \tag{22}$$

Expansions of the scalar fields

$$\begin{aligned} \chi^T &= (\chi_1^0, \chi_2^-, \chi_3^0) \sim \left(1, 3, -\frac{1}{3}\right), \; \eta^T = (\eta_1^0, \eta_2^-, \eta_3^0) \sim \left(1, 3, -\frac{1}{3}\right), \\ \rho^T &= (\rho_1^+, \rho_2^0, \rho_3^+) \sim \left(1, 3, \frac{2}{3}\right), \qquad \phi = \frac{1}{2}(v_\phi + R_\phi)e^{i\frac{a}{f_a}} \sim (1, 1, 0), \end{aligned} \tag{23}$$

where $\tan\theta_{PQ} = \frac{I_\phi}{v_\phi + R_\phi}$

IIIb. Higgs sector

The VEV v_ϕ is responsible for the PQ symmetry breaking resulting (see below).

Then VEV v_χ breaks $SU(3)_L \times U(1)_N$ to the SM group. Two others v_ρ, v_η are needed for the usual $U(1)_Q$ symmetry. Hence, it follows $v_\phi \gg v_\chi \gg v_\rho, v_\eta$.

Result of [V. H. Binh, *et al*, Phys. Rev. D (2023)] showed

1. One heavy field with mass in the range of 10^{11} GeV and associated with singlet ϕ is identified to inflaton Φ.
2. One SM-like Higgs boson h with mass ~ 125 GeV.
3. Two remain fields include one heavy with mass at TeV scale (H_χ) and another with mass at EW scale (h_5).

IIIb. Higgs sector

In the limit $v_\phi \gg v_\chi \gg v_\rho \gg v_\eta$, one has

$$\begin{aligned}
\eta &\simeq \begin{pmatrix} \frac{1}{\sqrt{2}}\left(u + h_5 + iA_5\right) \\ H_1^- \\ G_{X^0} \end{pmatrix}, \ \chi \simeq \begin{pmatrix} \chi_1^0 \\ G_{Y^-} \\ \frac{1}{\sqrt{2}}\left(v_\chi + H_\chi + iG_{Z'}\right) \end{pmatrix}, \\
\rho &\simeq \begin{pmatrix} G_{W^+} \\ \frac{1}{\sqrt{2}}\left(v + h + iG_Z\right) \\ H_2^+ \end{pmatrix}, \\
\phi &= \frac{1}{\sqrt{2}}\left(v_\phi + \Phi\right) e^{-i\frac{a}{f_a}} . \qquad (24)
\end{aligned}$$

Notice:

- A_5 new CP - odd scalar
- χ_1^0 - bilepton DM [C. A. de S. Pires, P. S. Rodrigues da Silva, JCAP 0712:012 (2007)]

IIIc. PQ charge in the 3-3-1 model with Inflation

The PQ transformations are as follows

$$\begin{aligned} f &\to f' = e^{i\left(\frac{c_f}{2f_a}\right)\gamma_5\, a} f\,, \qquad \bar{f} \to \bar{f}' = \bar{f}\, e^{i\left(\frac{c_f}{2f_a}\right)\gamma_5\, a}\,, \\ \varphi &\to \varphi' = e^{i\left(\frac{c_\varphi}{2f_a}\right)a} \varphi\,, \end{aligned} \tag{25}$$

For chiral fermions

$$\begin{aligned} f_L &\to f'_L = e^{-i\left(\frac{c_f}{2f_a}\right)a} f_L\,, \qquad \bar{f}_L \to \bar{f}'_L = \bar{f}_L\, e^{i\left(\frac{c_f}{2f_a}\right)a}\,, \\ f_R &\to f'_R = e^{i\left(\frac{c_f}{2f_a}\right)a} f_R\,, \qquad \bar{f}_R \to \bar{f}'_R = \bar{f}_R\, e^{-i\left(\frac{c_f}{2f_a}\right)a}\,, \end{aligned} \tag{26}$$

where c_f is PQ charge of fermion and $f_a \sim 10^{11}$ GeV is axion decay constant relating to the scale of symmetry breaking of $U(1)_{PQ} \sim U(1)_{Q_A}$ global group.

The PQ charges of fermions are given as [[C. A. de S. Pires, *et al*, Phys. Lett. B (2017)]

$$c_u = c_T = -c_d = -c_D = c_l = -c_{lR} = -c_\nu = c_{\nu_R} = -c_N \equiv R\,. \tag{27}$$

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IIIc. PQ charge in the 3-3-1 model with Inflation

	Q_{nL}	Q_{3L}	u_{aR}	d_{aR}	T_{3R}	D_{nR}	ψ_{aL}	l_{aR}	N_{aR}	η	χ	ρ	
$SU(3)_C$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	
$SU(3)_L$	$\overline{\mathbf{3}}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	
$U(1)_X$	0	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	-1	0	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	
Z_{11}	ω_4^{-1}	ω_0	ω_5	ω_2	ω_3	ω_4	ω_1	ω_3	ω_5^{-1}	ω_5^{-1}	ω_3^{-1}	ω_2^{-1}	ω
Z_2	1	1	-1	-1	1	1	1	-1	-1	-1	1	-1	

$SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes Z_{11} \otimes Z_2$ charge assignments of the particle content of the model.

The PQ charges of fermions are given as [C. A. de S. Pires, *et al*, Phys. Lett. B (2017)]

$$\begin{aligned} c_u &= c_T = -c_d = -c_D = c_l = -c_{lR} \\ &= -c_\nu = c_{\nu_R} = -c_N \equiv R\,. \end{aligned} \tag{28}$$

R is non-zero integer. $|R| = 1 \Rightarrow R = 1.$

	u	d	T	D_α	l	ν	ν_R	N_R	η_1^0	η_3^0	χ_1^0	χ_3^0	ρ^0	ϕ	η_2^-	χ
$U(1)_{Q_A}$	1	-1	1	-1	1	-1	1	-1	2	2	2	2	-2	2	0	

$U(1)_{PQ}$ charge assignments of the particle content of the model. Here $c_F = c_{F_L} = -c_{F_R}$

Notice:

- Charged scalars do not have PQ charge
- Singlet ϕ must carry PQ charge

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IIId. Particles and Q_A

[C. A. de S. Pires, *et al*, Phys. Lett. B (2017);
V. H. Binh, et al, Phys. Rev. D 107 (2023) 095030]

Multiplets and Q_A charges in the model

Particle$\|$ Q_A	Particle$\|$ Q_A	Particle$\|$ Q_A
$\begin{pmatrix} t \\ b \\ T \end{pmatrix}_L \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \sim 3$	$\begin{pmatrix} d_\alpha \\ -u_\alpha \\ D_\alpha \end{pmatrix}_L \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \sim \tilde{3}$	$\begin{pmatrix} \nu_a \\ l_a \\ (\nu^c_R)^a \end{pmatrix}_L \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \sim 3$
$\begin{pmatrix} \chi^0_1 \\ \chi^-_2 \\ \chi^0_3 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} \sim 3$	$\begin{pmatrix} \eta^0_1 \\ \eta^-_2 \\ \eta^0_3 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} \sim 3$	$\begin{pmatrix} \rho^+_1 \\ \rho^0_2 \\ \rho^+_3 \end{pmatrix} \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} \sim 3$
$N_{aR} \sim (1,1,0,-1)$	$d_{aR}, D_{\alpha R} \sim (3,1,-\frac{1}{3},1)$	$t_R, T_R \sim (3,1,\frac{2}{3},-1)$
	$\phi \sim (1,1,0,2)$	

IIIe. Formula of PQ charge operator

The PQ charges given in Table allows us to write some nice formula as generalized lepton number.
Let us write PQ charge operator in diagonal operators as follows (for left-handed fermions sitting in non-singlets).

$$Q_A = \alpha\, T_3 + \beta\, T_8 + \delta\, \mathcal{X}_{pq}\,, \tag{29}$$

Applying for Q_3, one gets

$$\alpha = +2, \qquad \beta = -\frac{2}{\sqrt{3}}\,, \qquad \delta\, \mathcal{X}_{pq}(Q_3) = +\frac{1}{3} \tag{30}$$

Assuming $\delta = 1$, one gets $\mathcal{X}_{pq}(Q_3) = +\frac{1}{3}$. Hence

$$Q_A = 2\, T_3 - \frac{2}{\sqrt{3}}\, T_8 + \mathcal{X}_{pq}\,. \tag{31}$$

For all fermion triplets, one has $\mathcal{X}_{pq}(\mathbf{3}) = \mathbf{\frac{1}{3}}$.
Note that the above formula is applicable for left-handed ferions, for right-handed fermions just take opposite

IIIe. Formula of PQ charge operator

For scalars

$$\mathcal{X}_{pq}(\chi,\eta) = \frac{4}{3}\,, \qquad \mathcal{X}_{pq}(\rho) = -\frac{2}{3}\,. \tag{32}$$

The PQ charge of the singlet ϕ follows from Yukawa coupling and equal - 2.
Let us connect PQ charge with electric one being as follows

$$Q = T_3 - \frac{1}{\sqrt{3}}\,T_8 + N\,, \tag{33}$$

From Eqs. (31) and (33), it follows

$$Q_A = 2\,Q + \mathcal{X}_{pq} - 2\,N\,. \tag{34}$$

For singlets, their values are given from Yukawa interactions as follows

$$\mathcal{X}_{pq}(N_R) = 1\,, \qquad \mathcal{X}_{pq}(f_R) = -Q_A(f_L)\,, \qquad \mathcal{X}_{pq}(\phi) = 2\,. \tag{35}$$

For minimal 3-3-1 model, PQ symmetry may be not suitable because of the Landau pole around 5 TeV .

Note that electric charges of up and down elements differ by one unit, while PQ charges of the above elements differ by two. That is why the factor 2 in (32).

IIIe. Formula of PQ charge operator

If the singlet scalar has VEV, it means that PQ symmetry is broken, because

$$Q_A \langle \phi \rangle = \frac{2}{\sqrt{2}} v_\phi \neq 0\,. \quad (36)$$

Similarity between electric and PQ charges is showed in Table below

	$U(1)_Q$	$U(1)_{Q_A}$
$\begin{pmatrix} f_u \\ f_d \end{pmatrix}$	$\Delta Q = 1$	$\Delta Q_A = 2$
$\begin{pmatrix} \varphi^0 \\ \varphi^- \end{pmatrix}$	$\begin{pmatrix} Q(\varphi^0) = 0 \\ Q(\varphi^-) = -1 \end{pmatrix}$	$\begin{pmatrix} Q_A(\varphi^0) = 2 \\ Q_A(\varphi^-) = 0 \end{pmatrix}$
Chiral fermion f	$Q(f_R) = Q(f_L)$	$Q_A(f_R) = -Q_A(f_L)$

SSB is in three steps by following scheme:

$$SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes Z_{11} \otimes Z_2 \otimes U(1)_{Q_A}$$
$$\downarrow v_\phi$$
$$SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes Z_2$$
$$\downarrow v_\chi$$
$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$
$$\downarrow v_\eta, v_\rho$$
$$SU(3)_C \otimes U(1)_Q.$$

IV. Axion couplings

(i) Axion-gauge boson couplings = Anomalous couplings
To understand these couplings, let us denote

$$\begin{aligned} G^a_{0\nu} &\equiv E^a_\nu\,, \qquad \nu = 1,2,3 = i \\ B^a_k &= \epsilon_{ijk}\, G^a_{ij} \qquad i \neq j \neq k, (j,k = 1,2,3) \end{aligned} \tag{37}$$

Then the Lagrangian of gauge bosons are follows

$$\begin{aligned} -4\,\mathcal{L}_{gauge} &\supset W_{a\mu\nu}\, W^{a\mu\nu} = 2\, W_{a0\nu}\, W^{a0\nu} + 2\, W_{aij}\, W^{aij} \\ &= 2\,(E_{ai} E^{ai} + B_{ak} B^{ak}) = 2(\mathbf{E}^2 + \mathbf{B}^2)\,. \end{aligned} \tag{38}$$

Now we consider a part with the dual tensor

$$\begin{aligned} \mathcal{L}_\theta &\supset \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\, W_{a\mu\nu}\, W^a_{\alpha\beta} = 4!\, \epsilon^{0i\alpha\beta}\, W_{a0i}\, W^a_{\alpha\beta} \\ &= 4!(\mathbf{E}.\mathbf{B}) \qquad (39) \\ &= 4!\, \epsilon^{0ijk}[\partial_0\, W^a_i - \partial_i\, W^a_0 + g f_{abc}\, W^b_0\, W^c_i] \\ &\times [\partial_j\, W^a_k - \partial_k\, W^a_j + g f_{ade}\, W^d_j\, W^e_k] \qquad (40) \end{aligned}$$

IV. Axion couplings

(ii) Axion-fermion couplings
Let us start from kinetic term of fermion as follows

$$\mathcal{L}_f^0 = \frac{i}{2}\bar{f}\gamma^\mu \overleftrightarrow{\partial_\mu} f = \frac{i}{2}(\bar{f}\gamma^\mu\partial_\mu f - \partial_\mu\bar{f}\gamma^\mu f)\,. \qquad (41)$$

For the gauge version, one has

$$\begin{aligned}\mathcal{L}_f &= \frac{i}{2}(\bar{f}\gamma^\mu D_\mu f - D_\mu\bar{f}\gamma^\mu f)\\ &= \frac{i}{2}[\bar{f}\gamma^\mu(\partial_\mu - iP_\mu^f)f - (\partial_\mu\bar{f} + i\bar{f}P_\mu^f)\gamma^\mu f]\\ &= \frac{i}{2}\bar{f}\gamma^\mu \overleftrightarrow{\partial_\mu} f + \bar{f}P_\mu^f\gamma^\mu f \equiv I + II \qquad (42)\end{aligned}$$

Here $P_\mu = g_S G_\mu - gA_\mu - g' B_\mu$ and G_μ, A_μ, B_μ are matrices of gluon, $SU(3)_L$ and $U(1)_X$ bosons, respectively.

IV. Axion couplings

For any fermion f, one has:

$$\mathcal{L}_{(f-a)} = \left(\frac{c_f}{2f_{pq}}\right) \partial_\mu a \bar{f} \gamma_5 \gamma^\mu f \tag{43}$$

For up quarks with $c_f = 1$, one has derivative coupling

$$\mathcal{L}_{(u-a)} = \left(\frac{1}{2f_{pq}}\right) \partial_\mu\, a\, \bar{u} \gamma^\mu \gamma_5 u \tag{44}$$

In summary, we have

$$\begin{aligned} \mathcal{L}_{(f-a)} = & + \left(\frac{1}{f_{pq}}\right) \partial_\mu\, a\, \Big[\bar{d}\, \mathbf{c}_d\, \gamma^\mu \gamma_5 d + \bar{u}\, \mathbf{c}_u\, \gamma^\mu \gamma_5 u + \bar{T}\, \mathbf{c}_T\, \gamma^\mu \gamma_5 T \\ & + \bar{D}_\alpha\, \mathbf{c}_{D_\alpha}\, \gamma^\mu \gamma_5 D_\alpha \\ & + \bar{l}\, \mathbf{c}_l \gamma^\mu \gamma_5 l + I_\nu \bar{\nu}_a\, \mathbf{c}_\nu\, \gamma^\mu \gamma_5 \nu_a + \frac{1}{2} \bar{N}_a\, \mathbf{c}_{N_a}\, \gamma^\mu P_R N_a \Big] . \end{aligned} \tag{45}$$

in which the number of color, flavor indexes and PQ charge have to be counted.

IV. Axion couplings

(iii) Axion-scalar boson couplings
Let us write for arbitrary scalar boson φ

$$\begin{aligned}
\mathcal{L}_\varphi &= (D^\mu\varphi)^\dagger D_\mu\varphi = [(\partial_\mu - iP^\varphi_\mu)\varphi]^\dagger(\partial^\mu - iP^{\varphi\mu})\varphi \\
&= \partial^\mu\varphi^\dagger\partial_\mu\varphi - i\partial_\mu\varphi^\dagger P^{\varphi\mu}\varphi + i\varphi^\dagger P^{\varphi\mu}\partial_\mu\varphi + \varphi^\dagger P^\varphi_\mu P^{\varphi\mu}\varphi \\
&\equiv \mathbf{A1} + \mathbf{A2} + \mathbf{A3}\,.
\end{aligned} \tag{46}$$

Within PQ transformation, one has

$$\begin{aligned}
\mathbf{A1} &= \partial^\mu\left(\varphi^\dagger e^{i\left(\frac{a}{2f_a}\right)Q_A}\right)\partial_\mu\left(e^{-i\left(\frac{a}{2f_{pq}}\right)Q_A}\varphi\right) \\
&= \partial^\mu\left(\varphi^\dagger e^{i\left(\frac{c_\varphi}{2f_a}\right)a}\right)\partial_\mu\left(e^{-i\left(\frac{c_\varphi}{2f_{pq}}\right)a}\varphi\right) \\
&= \left[\partial_\mu\varphi^\dagger . e^{i\left(\frac{c_\varphi}{2f_{pq}}\right)a} + \varphi^\dagger e^{i\left(\frac{c_\varphi}{2f_{pq}}\right)a}(i)\left(\frac{c_\varphi}{2f_{pq}}\right)\partial_\mu a\right] \\
&\times \left[i\left(\frac{c_\varphi}{2f_{pq}}\right)e^{-i\left(\frac{c_\varphi}{2f_{pq}}\right)a}\partial^\mu a\,\varphi + e^{-i\left(\frac{c_\varphi}{2f_{pq}}\right)a}\partial^\mu\varphi\right] \\
&= \partial_\mu\varphi^\dagger\partial^\mu\varphi + \left(\frac{c_\varphi}{2f_{pq}}\right)^2\partial_\mu a\partial^\mu a\,\varphi^\dagger\varphi - i\left(\frac{c_\varphi}{2f_{pq}}\right)\partial_\mu a(\partial^\mu\varphi^\dagger\varphi - \varphi^\dagger\partial^\mu\varphi)
\end{aligned} \tag{47}$$

IVa. Interactions of axion to scalar and gauge bosons

Notice

$$\mathcal{L}_\varphi \supset \frac{1}{2}\left(\frac{1}{f_a}\right)^2 \partial_\mu a \partial^\mu a \ (v_\eta^2 + v_\rho^2 + v_\chi^2 + v_\phi^2), \Rightarrow f_a^2 = v_\eta^2 + v_\rho^2 + v_\chi^2 + v_\phi^2$$

and

$$\begin{aligned}\mathcal{L}_\varphi \supset\ & 2\left(\frac{g}{f_a}\right)\partial^\mu a \Bigg\{\left(W_{3\mu} + \frac{1}{\sqrt{3}}W_{8\mu} - \frac{1}{3}t\sqrt{\frac{2}{3}}B_\mu\right)v_\eta^2 - \left(W_{3\mu} - \frac{1}{\sqrt{3}}W_{8\mu} - \frac{2}{3}t\sqrt{\frac{2}{3}}B_\mu\right)v_\rho^2 \\ & - \left(\frac{2}{\sqrt{3}}W_{8\mu} + \frac{1}{3}t\sqrt{\frac{2}{3}}B_\mu\right)v_\chi^2\Bigg\} + 2\left(\frac{g}{f_a}\right)\partial^\mu a \Bigg\{\left(W_{3\mu} + \frac{1}{\sqrt{3}}W_{8\mu} - \frac{1}{3}t\sqrt{\frac{2}{3}}B_\mu\right) \\ & \times [2v_\eta R_\eta^1 + (R_\eta^1)^2 + (I_\eta^1)^2] + \left(-W_{3\mu} + \frac{1}{\sqrt{3}}W_{8\mu} + \frac{2}{3}t\sqrt{\frac{2}{3}}B_\mu\right)[2v_\rho R_\rho^2 + (R_\rho)^2 + (I_\rho)^2] \\ & + \left(-\frac{2}{\sqrt{3}}W_{8\mu} - \frac{1}{3}t\sqrt{\frac{2}{3}}B_\mu\right)[2v_\chi R_\chi^3 + (R_\chi^3)^2 + (I_\chi^3)^2] \\ & + \left(-\frac{2}{\sqrt{3}}W_{8\mu} - \frac{1}{3}t\sqrt{\frac{2}{3}}B_\mu\right)\eta_3^{0*}\eta_3^0 + \left(W_{3\mu} + \frac{1}{\sqrt{3}}W_{8\mu} - \frac{1}{3}t\sqrt{\frac{2}{3}}B_\mu\right)\chi_1^{0*}\chi_1^0\Bigg\} \qquad (48)\end{aligned}$$

V. Total axion Lagrangian

The total part concerned to axion is given below

$$\begin{aligned}
\mathcal{L}_a &= \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2} m_{ao}^2\, a^2 + c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{2f_a} G\tilde{G} + c_{WW}\frac{\alpha_2}{4\pi}\frac{a}{2f_a} W\tilde{W} + c_{BB}\frac{\alpha_1}{4\pi}\frac{a}{2f_a} B\tilde{B} && (49)\\
&+ \frac{\partial^\mu a}{2f_a}\left(\sum_{f=u,d,T,D}^{l,\nu} \bar{\psi}_f c_f \gamma_\mu \gamma_5 \psi + \frac{1}{2}\bar{N}_a \mathbf{c}_{N_a} \gamma^\mu P_R N_a\right) - (\bar{q}_L\, M_a\, q_R + H.c.) && (50)\\
&+ \left(\frac{1}{f_a}\right)^2 \partial_\mu a \partial^\mu a \left(\sum_{H=\eta_1^0,\eta_3^0,\rho_2^0}^{\chi_1^0,\chi_3^0,\phi} H^* H\right) \quad \Leftarrow \text{kinetic term} && (51)\\
&- i\left(\frac{c_\varphi}{2f_a}\right)\partial^\mu a \sum_{D=\eta,\chi,\phi}^{K=\rho^0}\left[D^* \overleftrightarrow{\partial_\mu} D - K^* \overleftrightarrow{\partial_\mu} K\right] && (52)\\
&+ 2\left(\frac{c_\varphi}{2f_a}\right)\partial^\mu a \sum_{H=\eta_1^0,\eta_3^0,\rho_2^0}^{\chi_1^0,\chi_3^0,\phi} H^\dagger P_\mu^H H \quad \Leftarrow \text{axion - GB mixing}
\end{aligned}$$

where $H^* H = \frac{1}{2}[(v_H + R_H)^2 + I_H^2]$.
It is worth emphasizing that in (52), the matrices M_q are diagonal, i.e., $M_q = \mathrm{diag}(m_u, m_d, \cdots)$

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Va. Elimination of mixing between axion and Goldstone bosons G_Z and $G_{Z'}$

There are terms mixing axion with Goldstone bosons G_Z and $G_{Z'}$. To avoid this trouble [H. Georgi, D. B. Kaplan and L. Randall, Phys. Lett. **B 169** (1986) 73], we rotate triplets of scalars by opposite $U(1)_{PQ}$ as follows

$$\begin{aligned} \chi &\rightarrow \chi' = e^{-i\left(\frac{a}{2f_{pq}}\right)}\chi\,, \\ \eta &\rightarrow \eta' = e^{-i\left(\frac{a}{2f_{pq}}\right)}\eta\,, \\ \rho &\rightarrow \rho' = e^{i\left(\frac{a}{2f_{pq}}\right)}\rho\,. \end{aligned} \tag{53}$$

Of course, the quarks have to be changed to keep Yukawa interactions invariant.

Va. Elimination of mixing between axion and Goldstone bosons G_Z and $G_{Z'}$

Note that the singlet ϕ is unchanged. Then, Eq. (52) becomes *final* Lagrangian

$$\begin{aligned}\mathcal{L}_a &= \frac{1}{2}\partial_\mu a\partial^\mu a - \frac{1}{2}m_{ao}^2 + c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{2f_{pq}}G\tilde{G} + c_{WW}\frac{\alpha_2}{4\pi}\frac{a}{2f_{pq}}W\tilde{W} \\ &\quad + c_{BB}\frac{\alpha_1}{4\pi}\frac{a}{2f_{pq}}B\tilde{B} \\ &+ \frac{\partial^\mu a}{2f_{pq}}\left(\sum_{f=u,d,T,D}^{l,\nu}\bar{\psi}_f c_f\gamma_\mu\gamma_5\psi + \frac{1}{2}\bar{N}_a\,\mathbf{c}_{N_a}\gamma^\mu P_R N_a\right) - \bar{q}_L M_a q_R \\ &+ \left(\frac{1}{2\,f_{pq}}\right)^2\partial_\mu a\partial^\mu a\left(v_\phi^2 + 2v_\phi R_\phi + R_\phi^2\right) \Leftarrow \text{New Physics}\end{aligned} \quad (54)$$

Here $M_a = e^{-i\left(\frac{a}{2f_a}\right)Q_A}M_q e^{-i\left(\frac{a}{2f_a}\right)Q_A}$.

Va. Elimination of mixing between axion and Goldstone bosons G_Z and $G_{Z'}$

To have correct kinetic term of the axion, Eq. (54) leads to condition

$$f_a = v_\phi \,. \tag{55}$$

The point is worth noting:

1. the terms in last line (doubly derivative coupling of axion to inflaton) are characteristic for the model under consideration.
2. As mentioned in [H. Georgi, D. B. Kaplan and L. Randall, Phys. Lett. **B 169** (1986) 73], the axion has only two kinds of couplings: derivative couplings with fermions and anomalous couplings with gauge bosons $aG\tilde{G}$.
3. Note that the new effects mainly happen in the energy region from 10^7 GeV to 10^{11} GeV.

As written in [H. Georgi, D. B. Kaplan and L. Randall, Phys. Lett. **B 169** (1986) 73]
" If it (invisible axion) exists at all, it is interloper in our low energy world, a renegade from unknown physics at some large energy scale, conventionally called f, constrained to lie in the range 10^7 GeV $\leq f \leq 10^{12}$ GeV."

VI. Chiral perturbative QCD

In the frameworks of chiral perturbative Lagrangian in the energy scale ≤ 1 GeV , the axion's mass was derived. For two quark flavors, the leading order chiral Lagrangian is given by [L. Di Luzio, M. Giannotti, E. Nardi and L. Visinelli, Phys. Rept. 870 (2020) 1]

$$\begin{aligned} \mathcal{L}_a^{\chi(LO)} &= +\frac{f_\pi^2}{4}\mathrm{Tr}\left[(D^\mu U)^\dagger D_\mu U\right] + \frac{\partial_\mu a}{2f_a}\frac{1}{2}\mathrm{Tr}\left[c_q\sigma^a\right] J^a_{A,\,\mu} \\ & +\frac{f_\pi^2}{2}B_0\mathrm{Tr}[M_a U^\dagger + UM_a^\dagger)] \,. \end{aligned} \tag{56}$$

The terms in the first line connected with kinetic term of fields, while the term in the second line provides mass of the fields.
Hence we continue with this term.

$$\begin{aligned} UM_a^\dagger + M_a U^\dagger &= UM_q + M_q U^\dagger + i\frac{a}{2f_a}\{Q_A, M_q\}U^\dagger \\ &- 2\left(\frac{a}{2f_a}\right)^2 U(Q_A^2 M_q + 2Q_A M_q Q_A + M_q Q_A^2) + \cdots \,, \end{aligned}$$

where $M_q = \mathrm{diag}(m_u, m_d)$.

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VI. Chiral perturbative QCD

Using

$$U = e^{i\frac{\pi^a\sigma^a}{f_\pi}} = \mathbf{1}\cos\left(\frac{\pi}{f_\pi}\right) + i\frac{\pi^a\sigma^a}{\pi}\sin\left(\frac{\pi}{f_\pi}\right), \tag{57}$$

where $\pi = \sqrt{(\pi^0)^2 + 2\pi^+\pi^-}$ and $Q_A = \frac{M_q^{-1}}{\mathrm{Tr}(M_q^{-1})}$, one gets

$$\mathcal{L}_{amass} = \frac{f_\pi^2}{2}B_0\mathrm{Tr}[M_a U^\dagger + UM_a^\dagger)] = f_\pi^2 m_\pi^2\cos\left(\frac{\pi}{f_\pi}\right) - \frac{m_a^2}{2}. \tag{58}$$

Here, the axion mass is given by

$$m_a^2 = \frac{m_\pi^2 f_\pi^2}{f_a^2}\frac{m_u m_d}{(m_u+m_d)^2}\cos\left(\frac{\pi}{f_\pi}\right) \simeq \frac{m_\pi^2 f_\pi^2}{f_a^2}\frac{m_u m_d}{(m_u+m_d)^2}. \tag{59}$$

A consequence is as follows

$$m_a \simeq 5.7\left(\frac{10^{12}\ \mathrm{GeV}}{f_a}\right)\mu\mathrm{eV}. \tag{60}$$

VI. Chiral perturbative QCD

In a similar way, we can deal with three flavour quarks (with s quark). The result is given by

$$m_a^2 = \frac{1}{2} m_\pi^2 \left(\frac{f_\pi}{f_a}\right)^2 \frac{1}{(m_u + m_d)} \left(\frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s}\right) . \quad (61)$$

This result coincides with one given in Ref. [M. Srednicki, Nucl. Phys. B 260 (1985) 689]

$$\begin{aligned} m_a &= 4\frac{f_\pi m_\pi}{f_a/N} \left[\frac{m_u m_d m_s}{(m_u m_d + m_u m_s + m_d m_s)(m_u + m_d)}\right]^{\frac{1}{2}} \\ &\simeq (1.2 \times 10^{-5}\,\text{eV}) \left(\frac{10^{12}\ \text{GeV}}{f_a/N}\right) , \end{aligned} \quad (62)$$

VII. Conclusions

1. Realization PQ formalism in the 3-3-1 model with Inflation.
2. All necessary elements are obtained: PQ transformation, charges,
3. PQ charge operator Q_A in terms of diagonal generators.
4. Axion couplings with gauge bosons, fermions, scalars.
5. Axion is an electric charge-phobia (unlike charged) scalar and gauge bosons.
6. Total axion Lagrangian contains new couplings, especially to inflaton.
7. Axion's mass determined by chiral perturbative QCD.

Thanks for your attention!

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