Inflation beyond slow roll and primordial black holes

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Hurghada, Egypt

KOGAKUIN UNIVERSITY

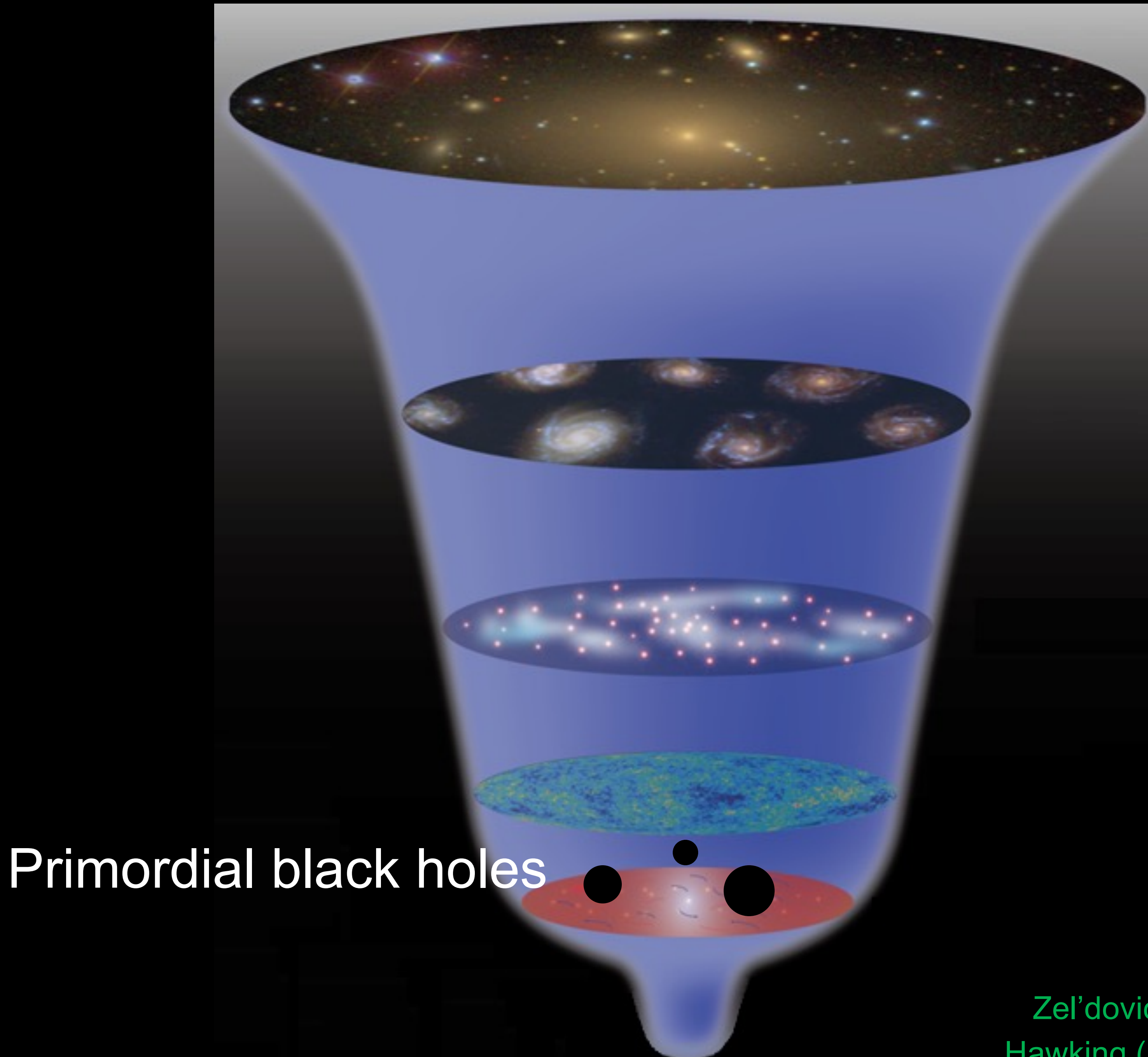
科研費
KAKENHI

No go of slow roll for PBH production

Simple model:

Constant-roll inflation

Bispectrum and
one-loop corrections



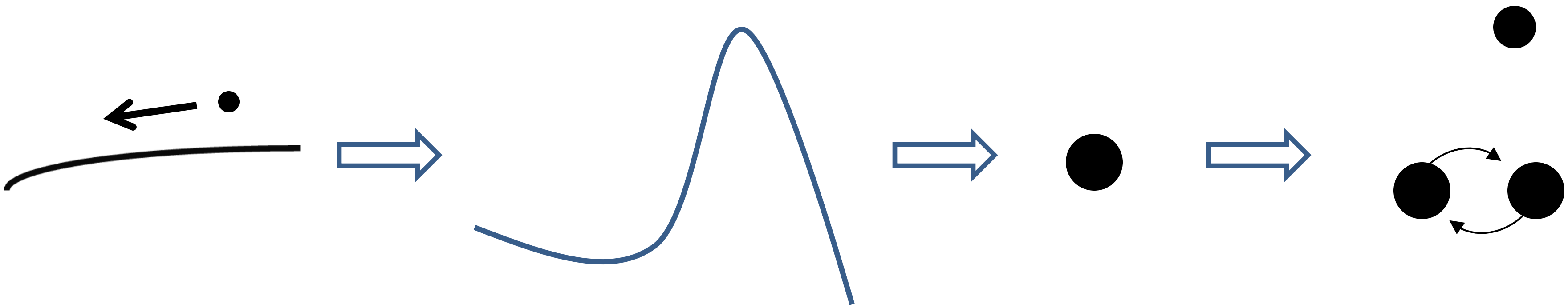
Zel'dovich, Novikov (1967)
Hawking (1971), Carr (1974)

Inflation

Primordial curvature

PBH

Binary



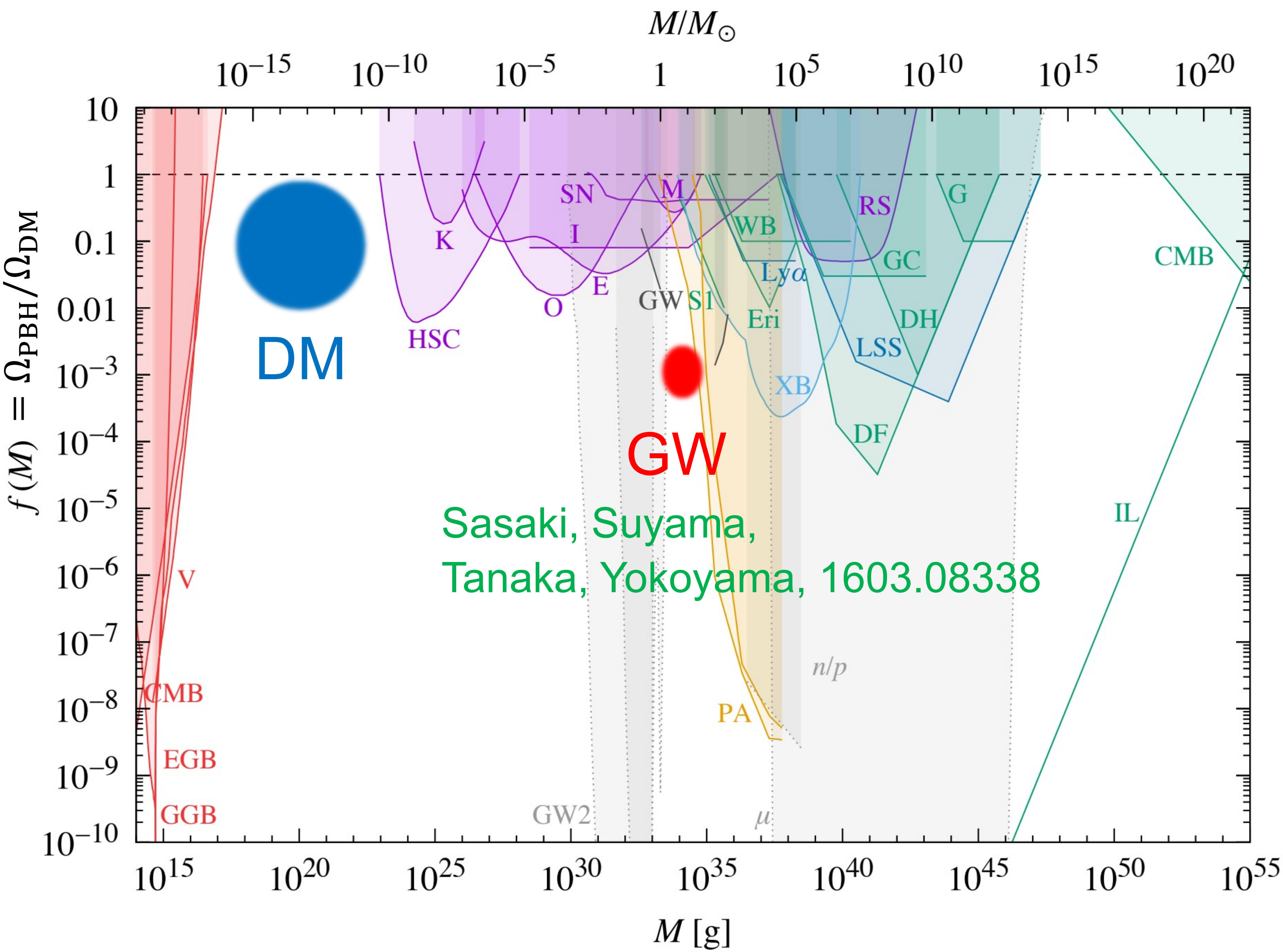
- Model building

- Induced GWs

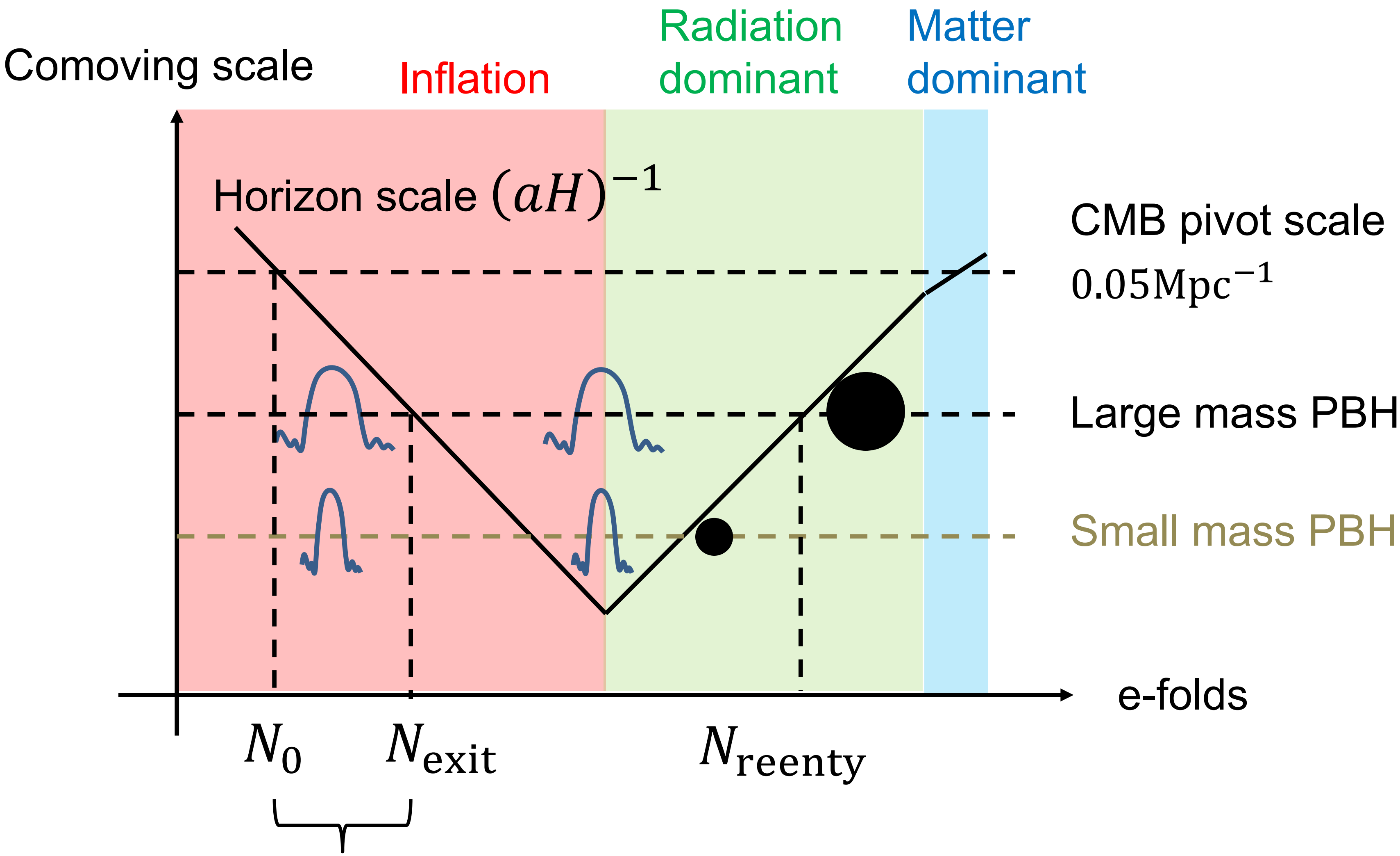
- DM
- Lensing
- Accretion
- Evaporation

- GWs

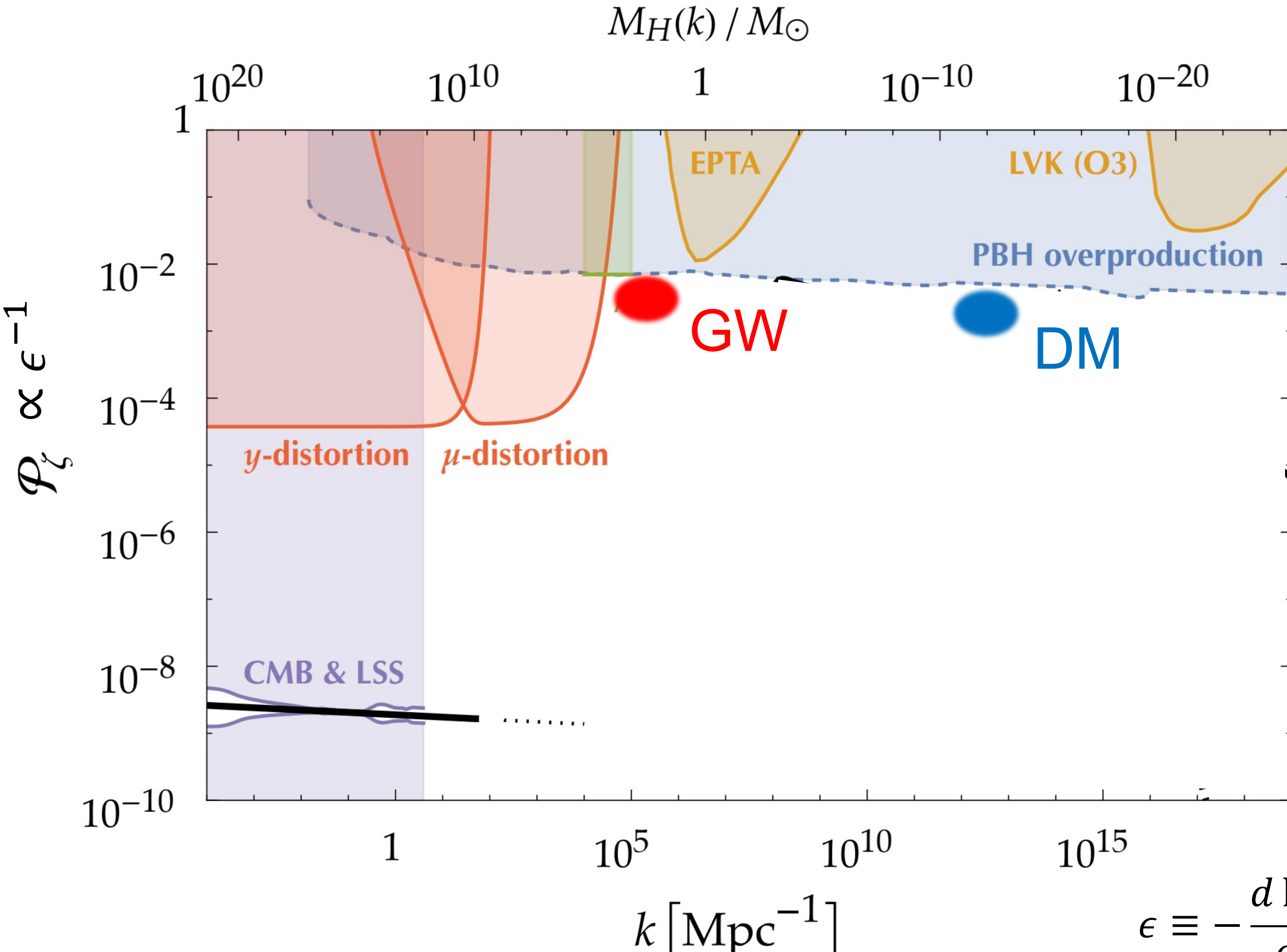
Recent reviews:
Khlopov 0801.0116, Sasaki+ 1801.05235, Carr+ 2002.12778, 2006.02838,
Green+ 2007.10722, Escriva+ 2211.05767, Ozsoy+ 2301.03600



Carr, Kohri, Sendouda, Yokoyama, 2002.12778

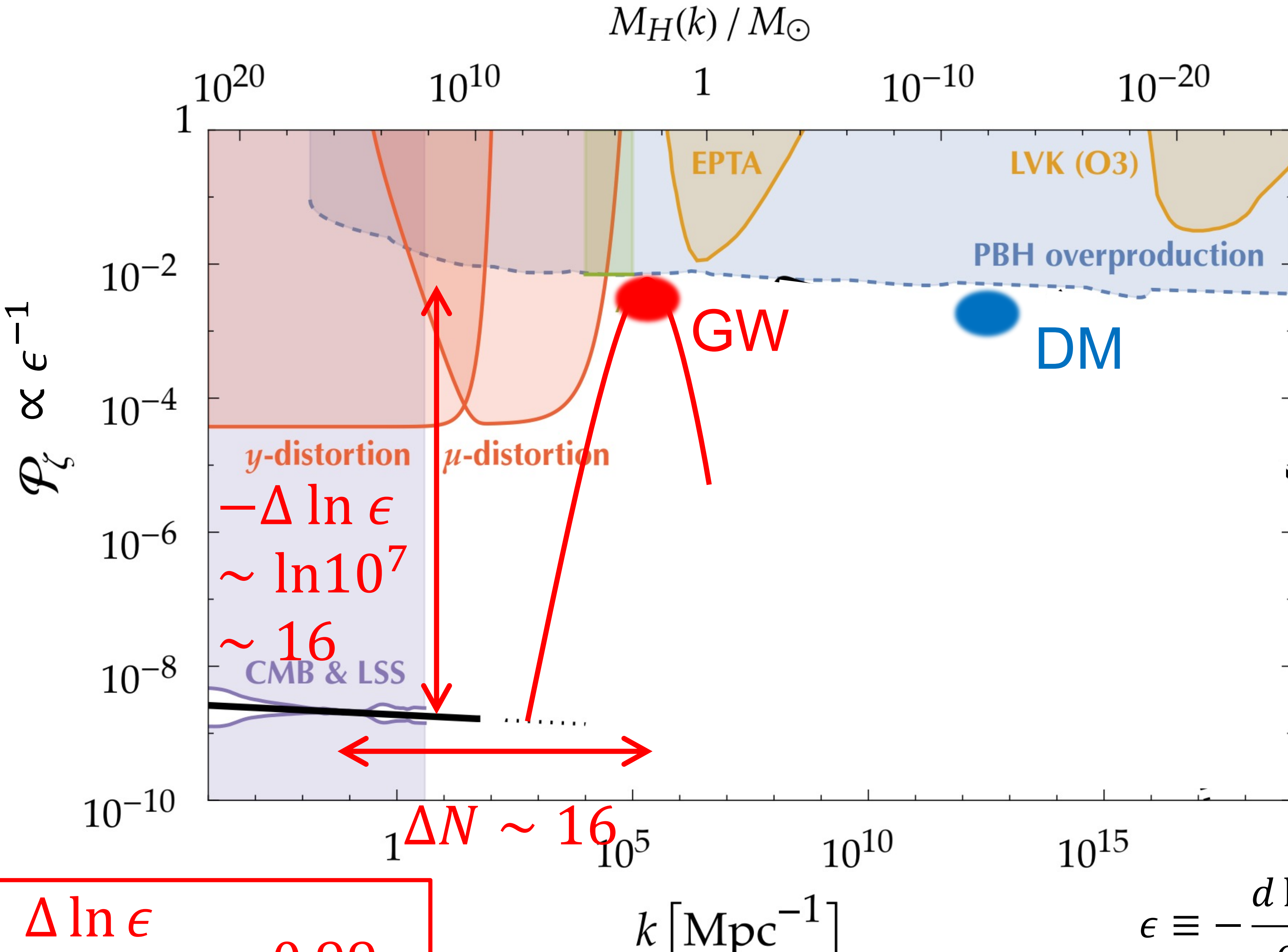


$$\Delta N \approx 18 - \frac{1}{2}\ln\frac{M_{\mathrm{PBH}}}{M_{\odot}} < 60$$



$$\epsilon \equiv -\frac{d\ln H}{dN}$$

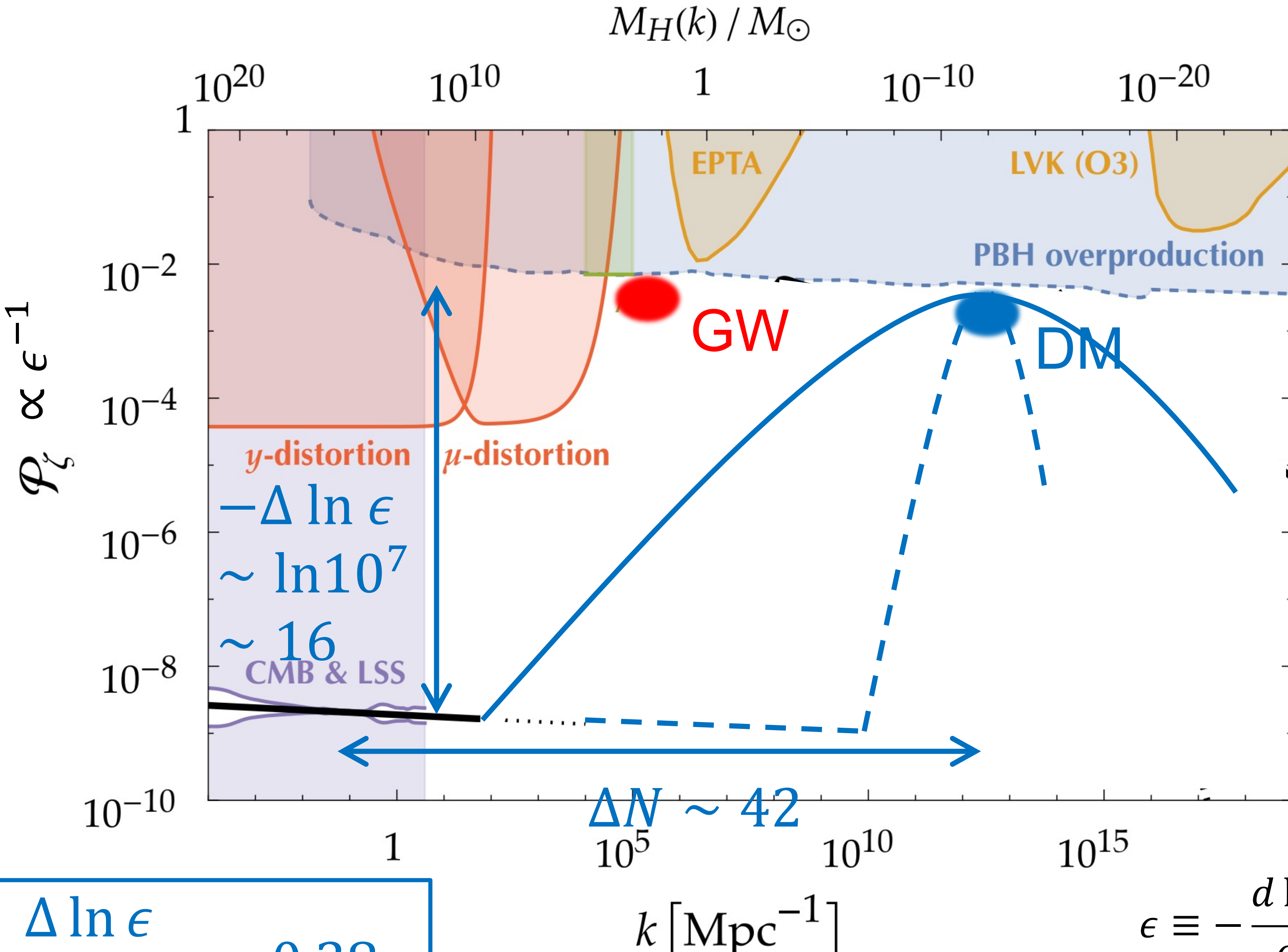
Escriva, Kuhnel, Tada, 2211.05767



$$\boxed{\frac{\Delta \ln \epsilon}{\Delta N} \sim -0.99}$$

$$\epsilon \equiv -\frac{d \ln H}{dN}$$

Escriva, Kuhnel, Tada, 2211.05767



$$\boxed{\frac{\Delta \ln \epsilon}{\Delta N} \sim -0.38}$$

$$\epsilon \equiv -\frac{d \ln H}{dN}$$

Escriva, Kuhnel, Tada, 2211.05767

No go of slow roll for PBH production

HM, Hu, 1706.06784

$$\boxed{\frac{\Delta \ln \epsilon}{\Delta N} \lesssim -0.4}$$

$$\epsilon \equiv -\frac{d \ln H}{dN}, \ \eta \equiv \frac{d \ln \epsilon}{dN}$$

Slow-roll condition ($\epsilon \ll 1, |\eta| \ll 1$)

must be violated transiently for

PBH production from canonical single-field inflation.

Potentials with $\boxed{\eta \lesssim -0.4}$

$\eta \neq \text{const.}$

$\eta = -6 \quad \subset \quad \eta = \text{const.}$

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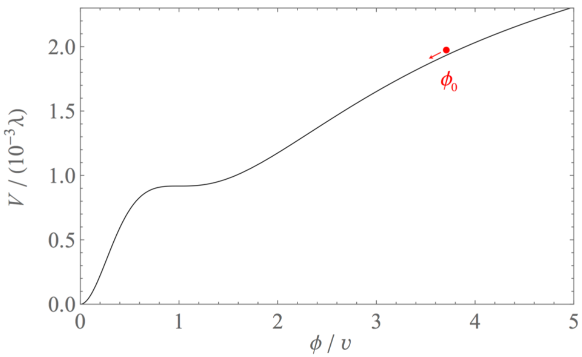
Ultra slow roll /
Inflection model

$$\eta = -6 \quad \subset \quad \eta = \text{const.} \qquad\qquad \eta \neq \text{const.}$$

- Extreme model
- Technically cumbersome



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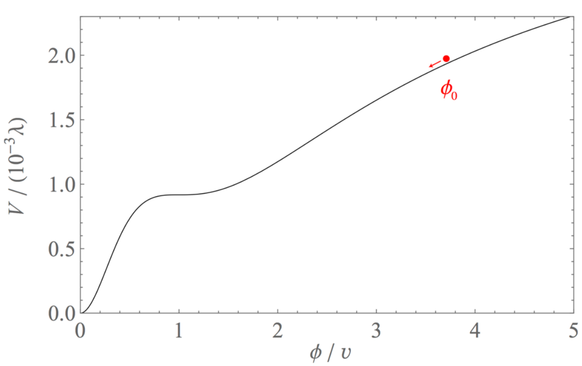
✓ General model

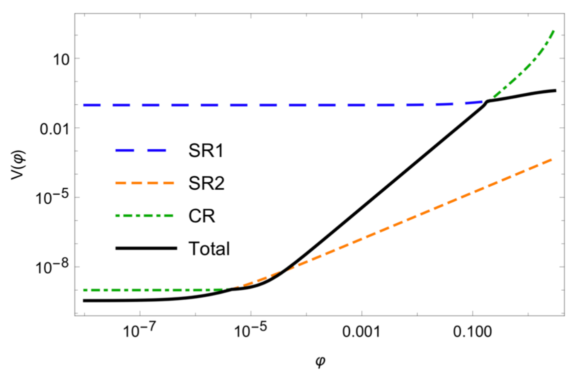
✓ Easy to handle

✓ Exact solution!

Martin, Suyama, HM 1211.0083

HM+ 1411.5021, 1910.13235





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Others

(Running mass, resonance, bump, etc)

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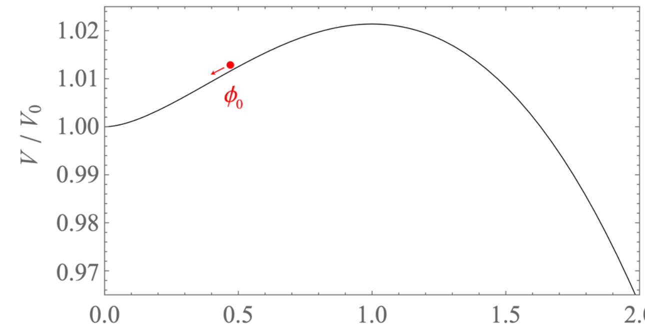
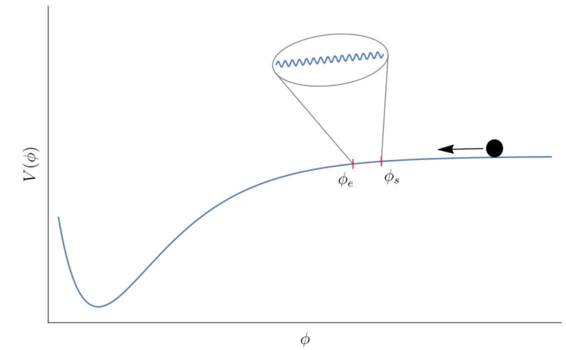
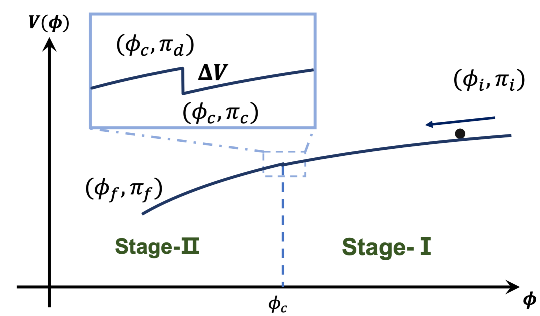
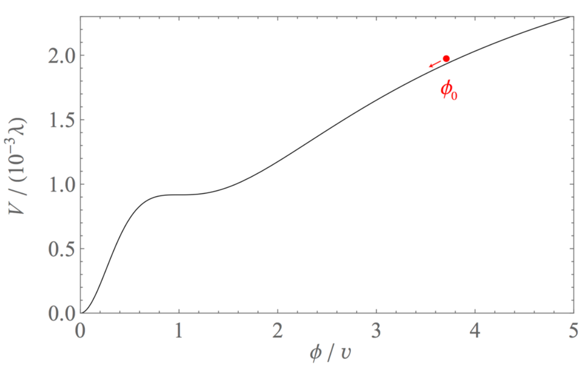
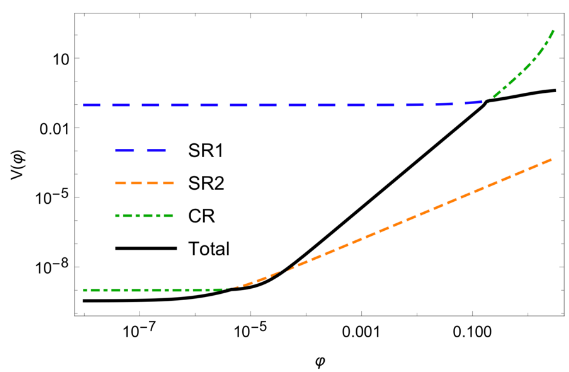










No go of slow roll for PBH production:

$$\boxed{\eta \lesssim -0.4}$$ Slow roll violation

$\Downarrow$

Simple model:
Constant-roll inflation

$$\eta = 2\beta = \text{const.}$$

$\Rightarrow$

Bispectrum and
one-loop corrections

Constant-roll inflation

$$\eta = 2\beta = \text{const.}$$

$$\eta \equiv \frac{\mathrm{d}\ln \epsilon_H}{\mathrm{d}N} \simeq \frac{2\ddot{\phi}}{H\dot{\phi}}$$

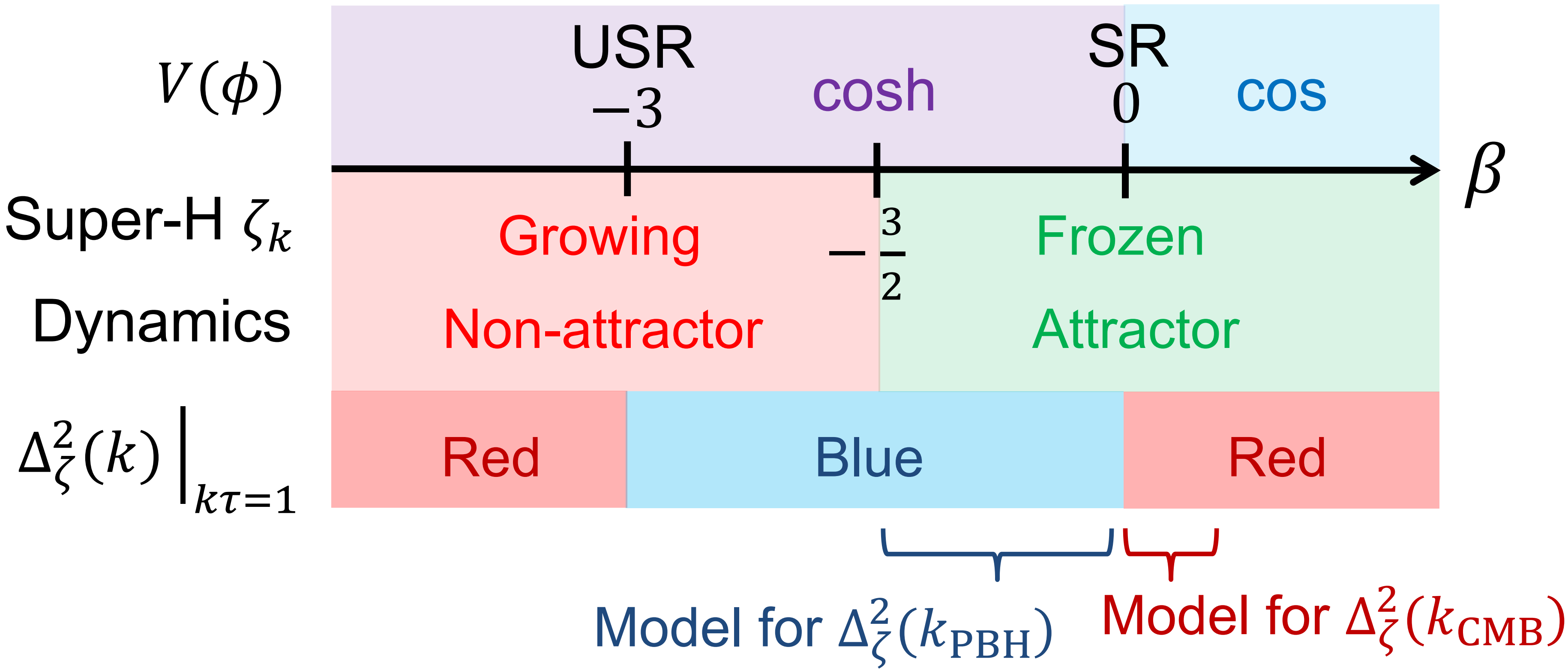
✓ $V(\phi) =$ simple functions ($\cos$ / $\cosh$)

✓ Exact solution for $\phi(t), H(t), a(t)$

✓ Can create red spectrum (for CMB) and blue spectrum (for PBH) by adjusting β

Model	HM, Starobinsky, Yokoyama, 1411.5021
CMB	HM, Starobinsky, 1702.05847
PBH	HM, Mukohyama, Oliosi, 1910.13235
	HM, Tada, 2303.16035

$$\ddot{\phi}/(H\dot{\phi}) = \beta \text{ (constant)}$$



HM, Mukohyama, Oliosi, 1910.13235

$\beta \approx 0.015$

HM, Starobinsky, Yokoyama, 1411.5021

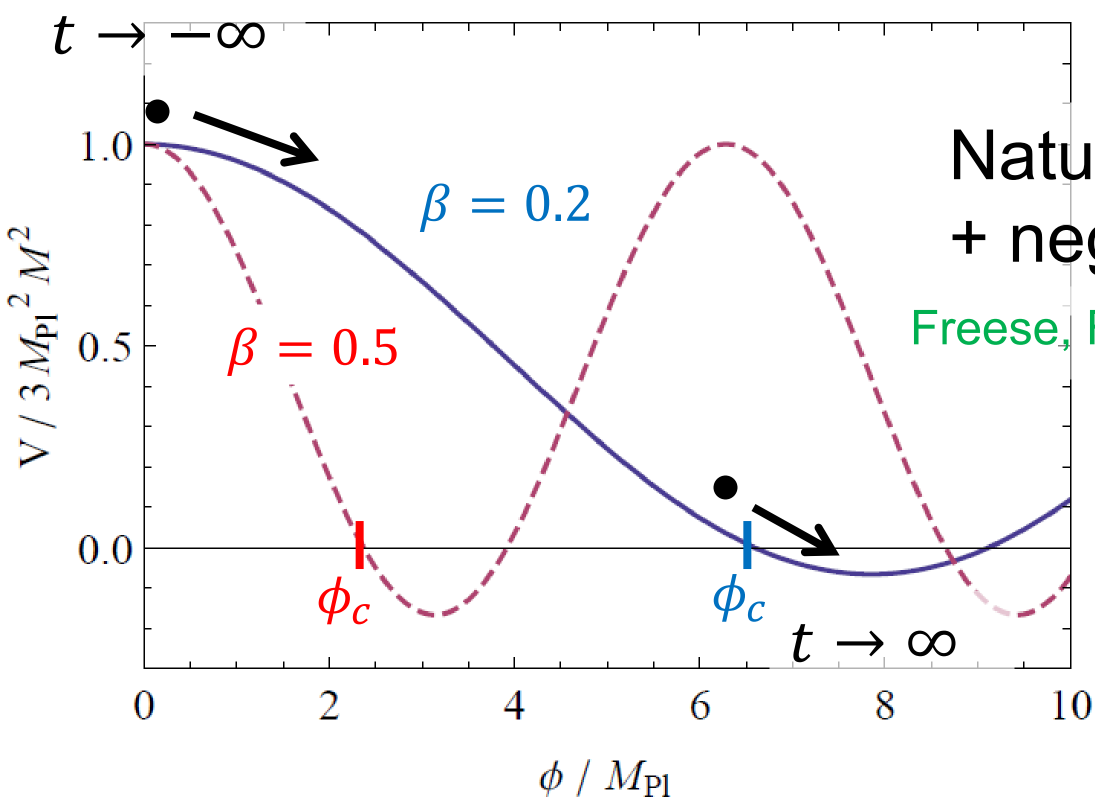
HM, Starobinsky, 1702.05847

$$n_s - 1\big|_{k\tau=1} = 3 - |2\beta + 3|$$

$\beta > 0$: cos potential

$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cos\left(\sqrt{2\beta}\,\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

Assume a transition to reheating at $\phi < \phi_c$.

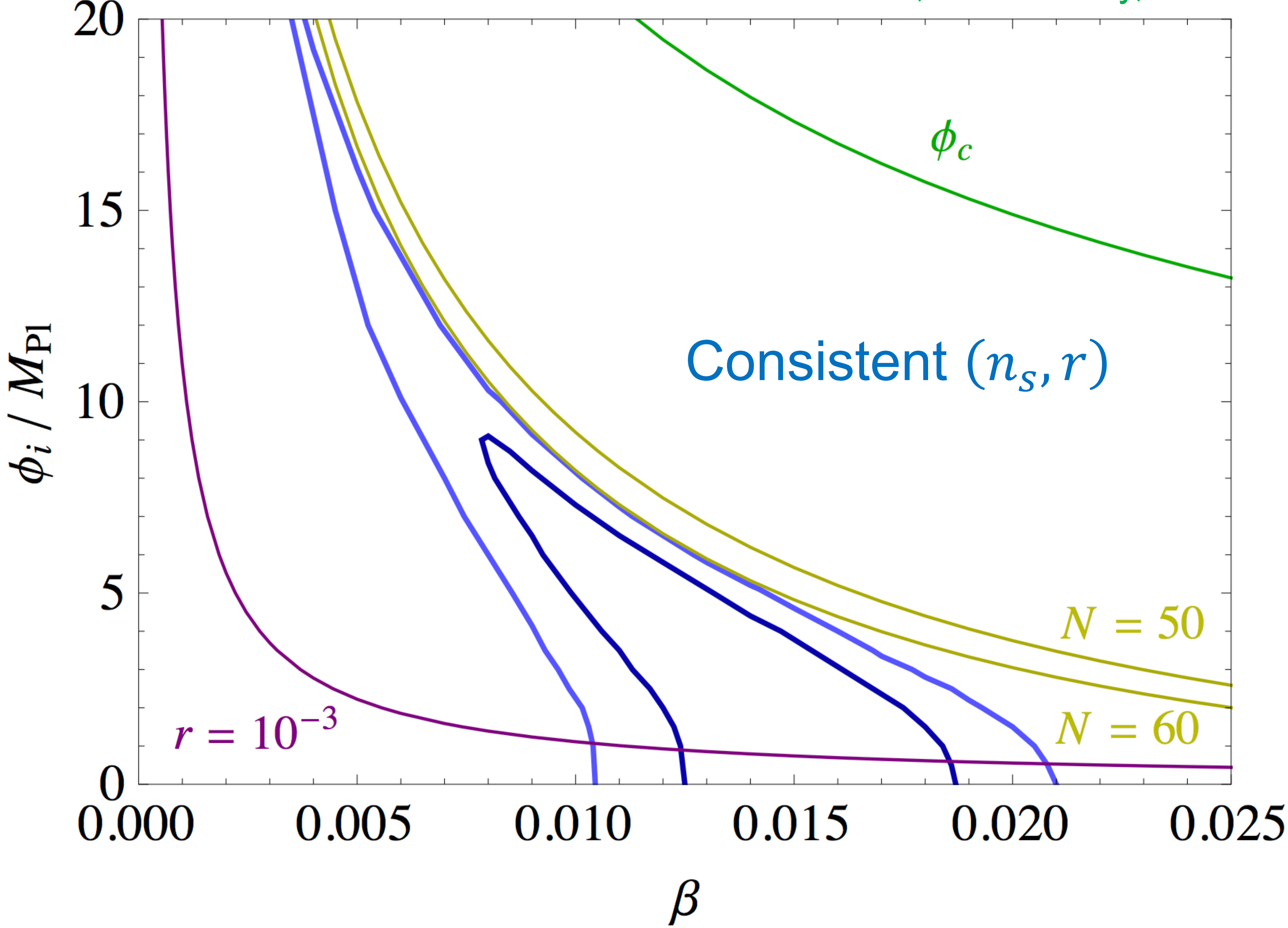


Natural inflation
+ negative CC

Freese, Frieman, Olinto (1990)

CMB constraint on constant-roll inflation

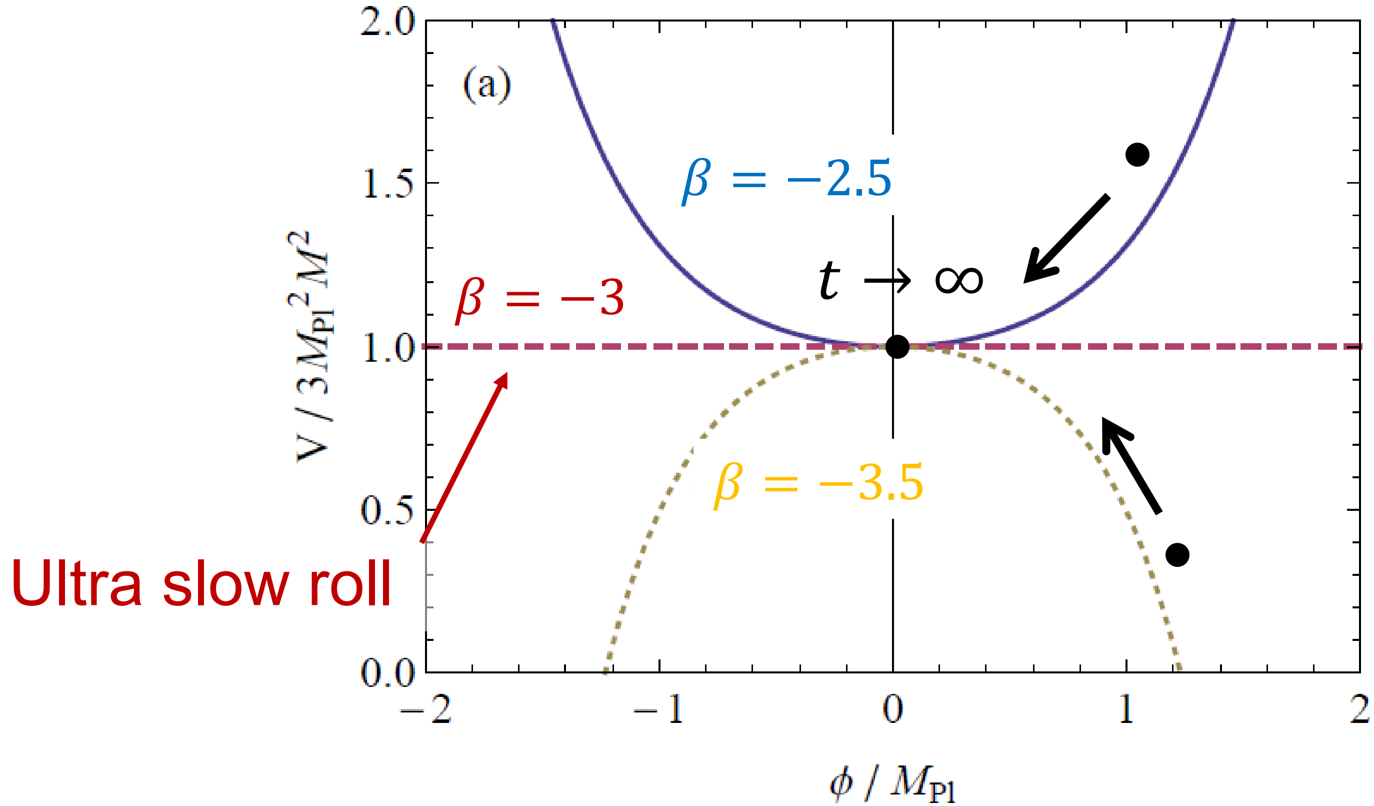
HM, Starobinsky, 1702.05847



$\beta < 0$: cosh potential

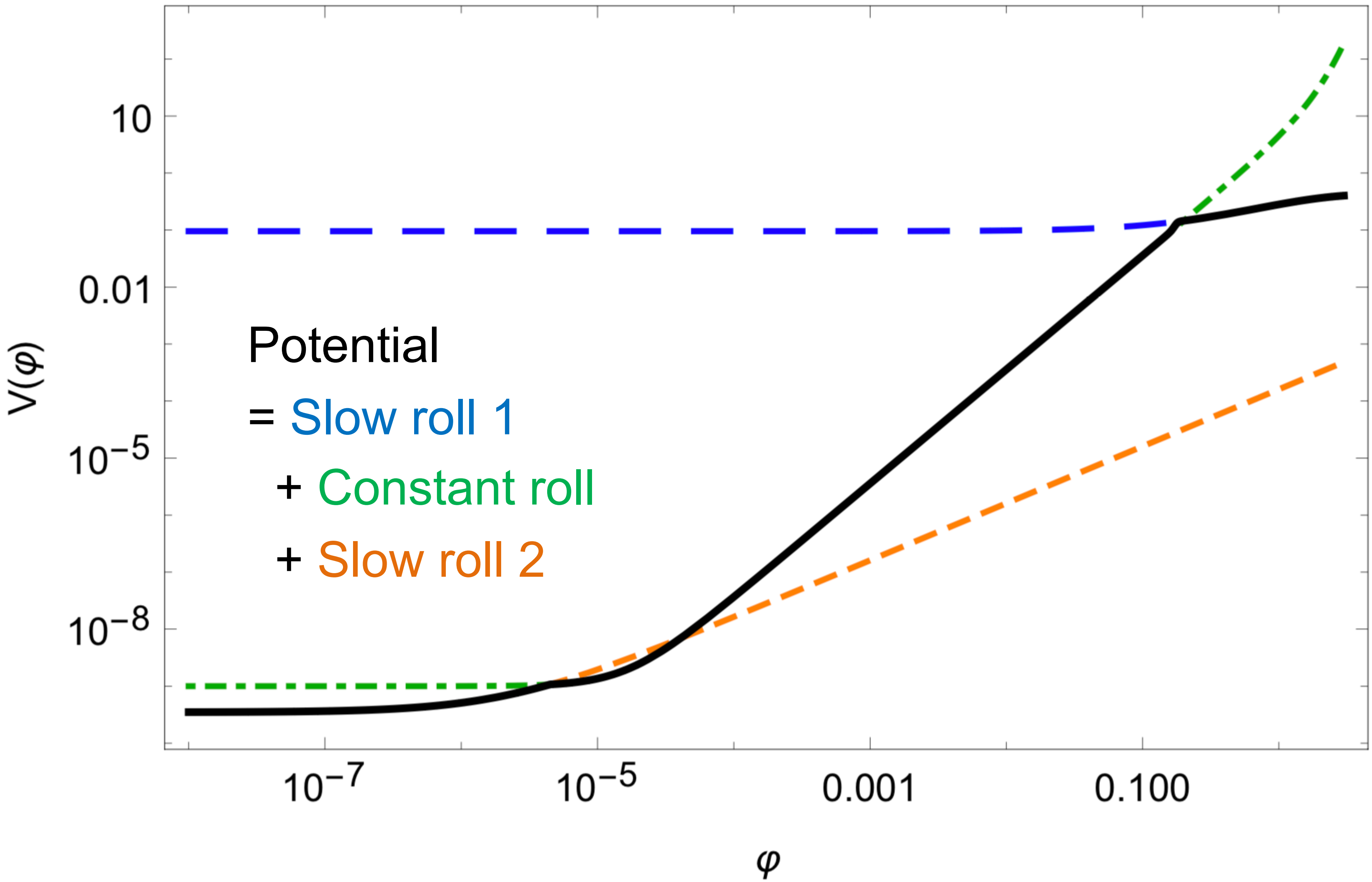
$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cosh\left(\sqrt{-2\beta}\,\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

Assume a transition to reheating at $\phi > 0$.



PBH from constant-roll inflation

HM, Mukohyama, Oliosi, 1910.13235



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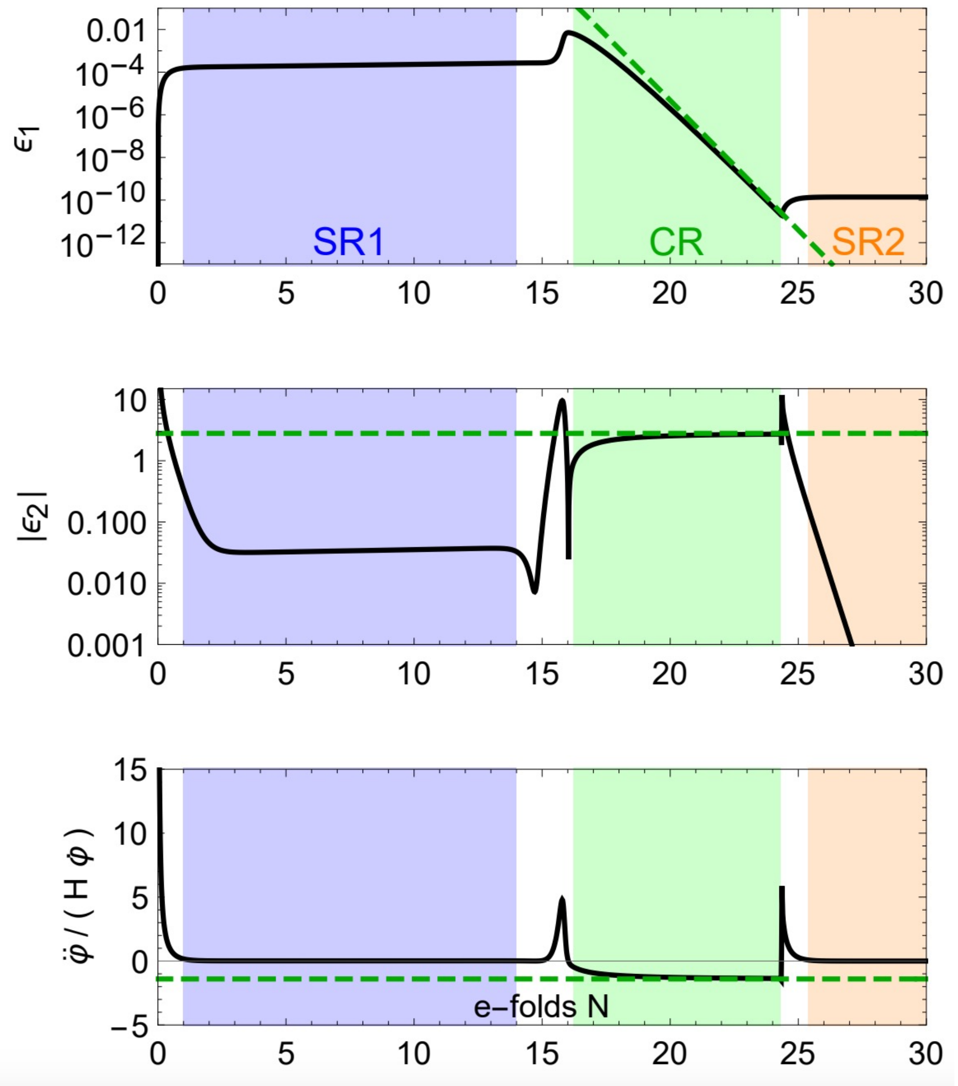

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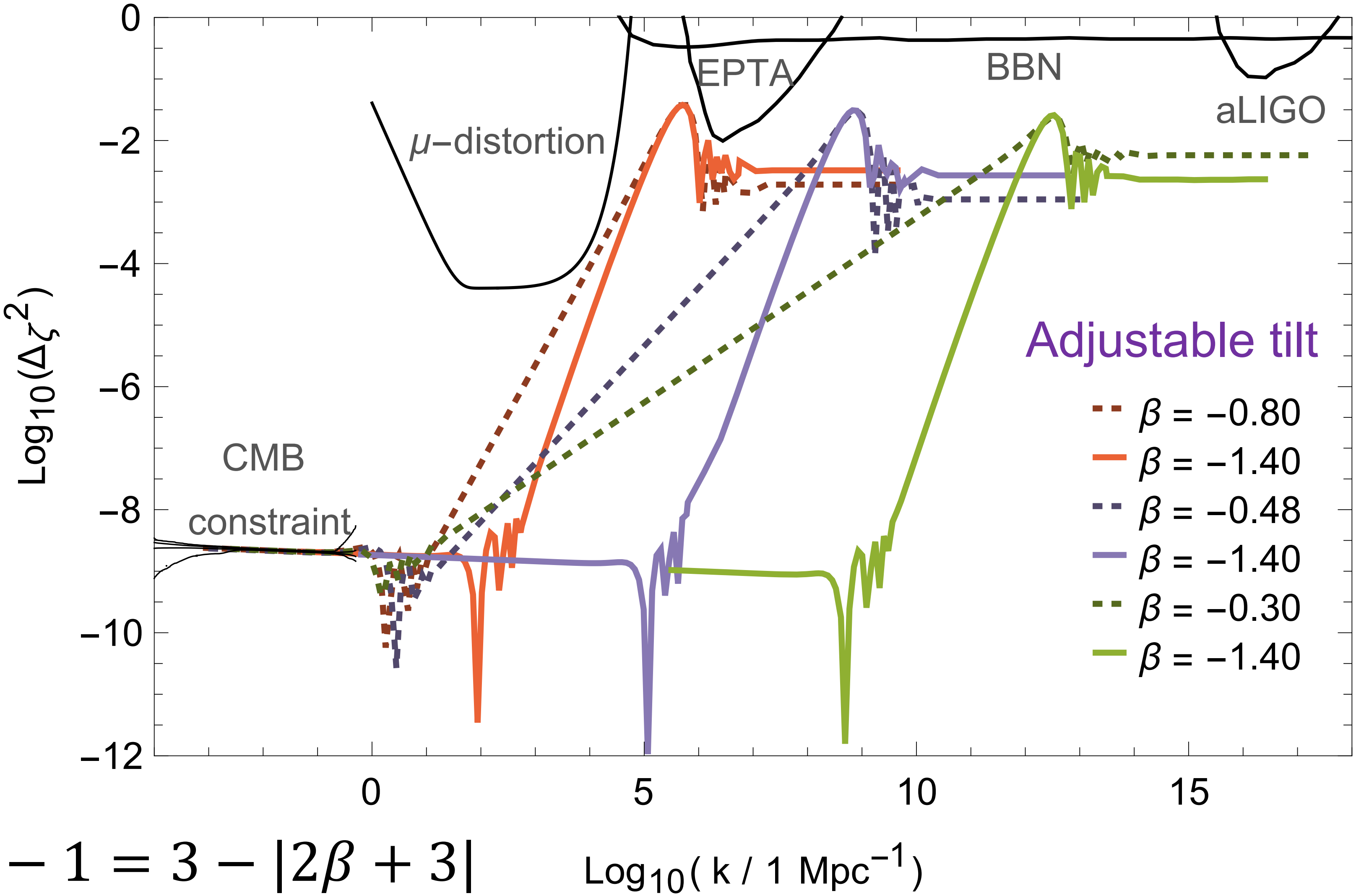

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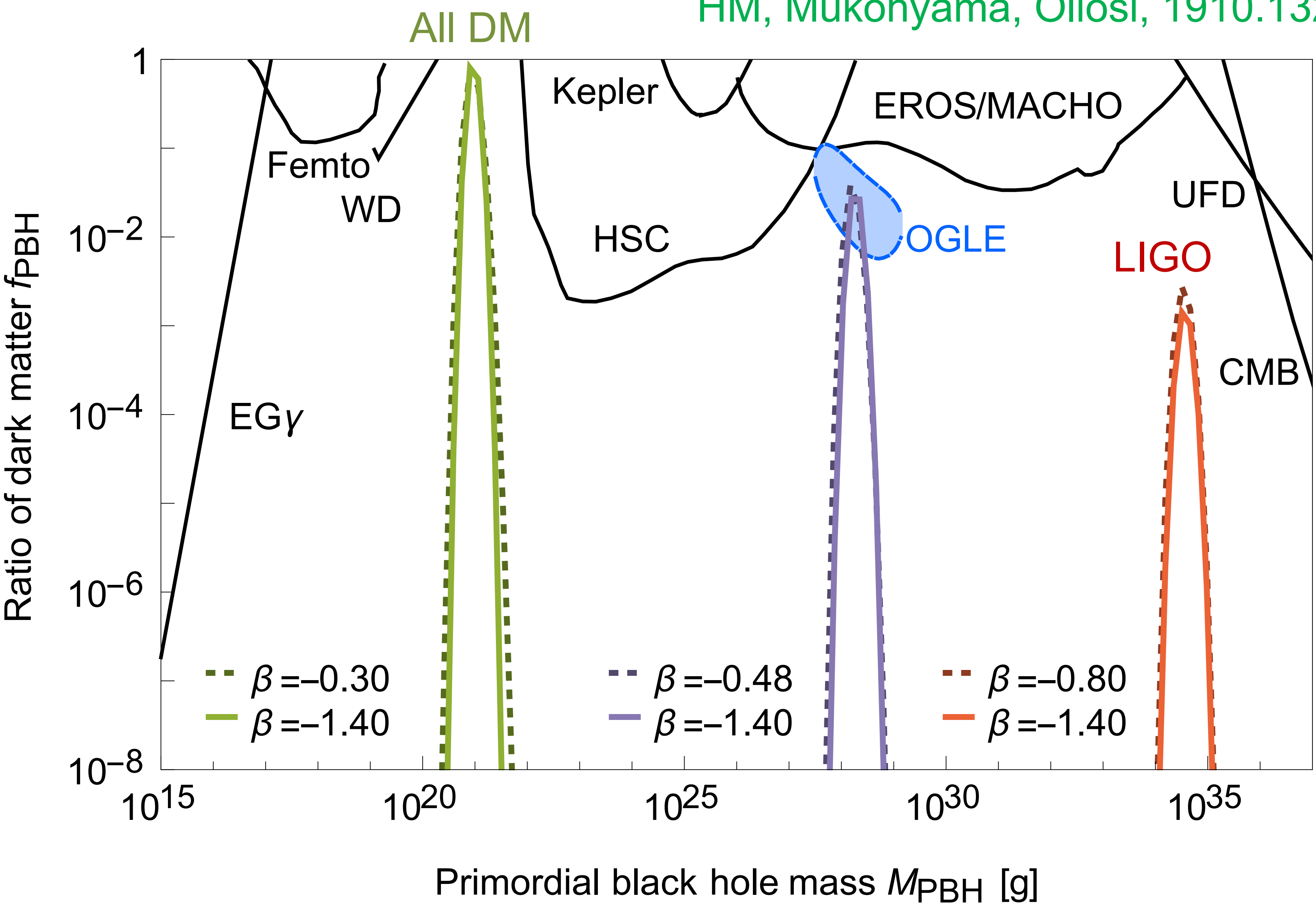

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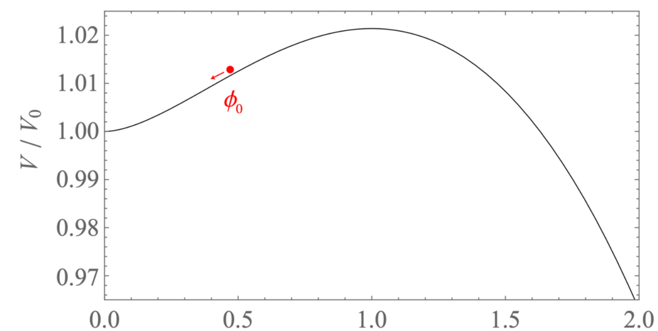

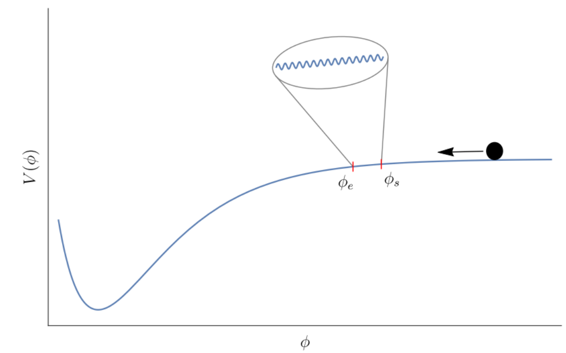

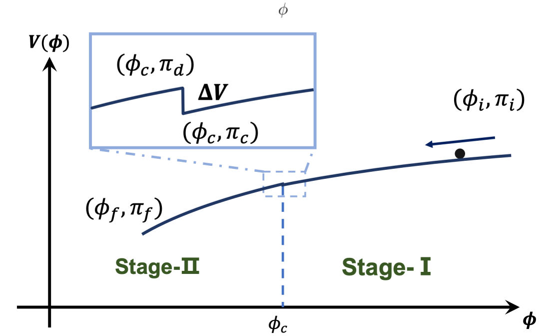

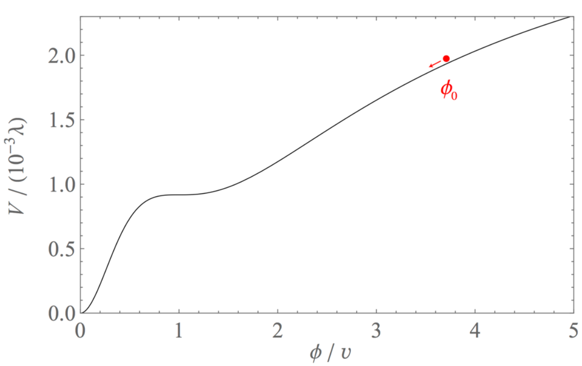

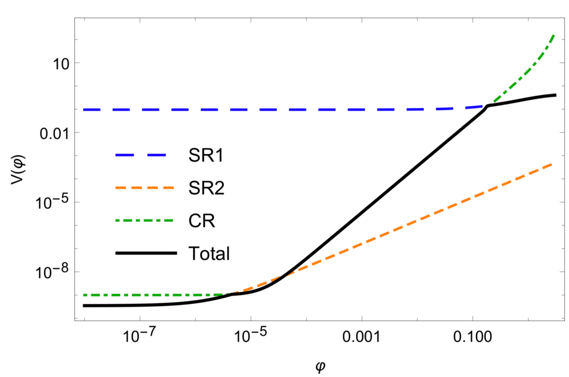

Recent arguments:

PBH formation from single-field inflation is "ruled out" due to $\mathcal{P}_{\mathrm{1-loop}} \gg \mathcal{P}_{\mathrm{tree}}$?

Kristiano, Yokoyama, 2211.03395

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- SR-USR-SR model
- Instantaneous transitions
- Considering only $\dot{\eta}$ term in H_{int}

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Too general conclusions from a specific setup? 🤔

* Breakdown of perturbation theory does not necessarily mean the model is ruled out.

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Too general conclusions from a specific setup? 🤔

$\Longrightarrow$ Counterexample satisfying $\mathcal{P}_{1-\mathrm{loop}} \ll \mathcal{P}_{\mathrm{tree}}$

HM, Tada, 2303.16035

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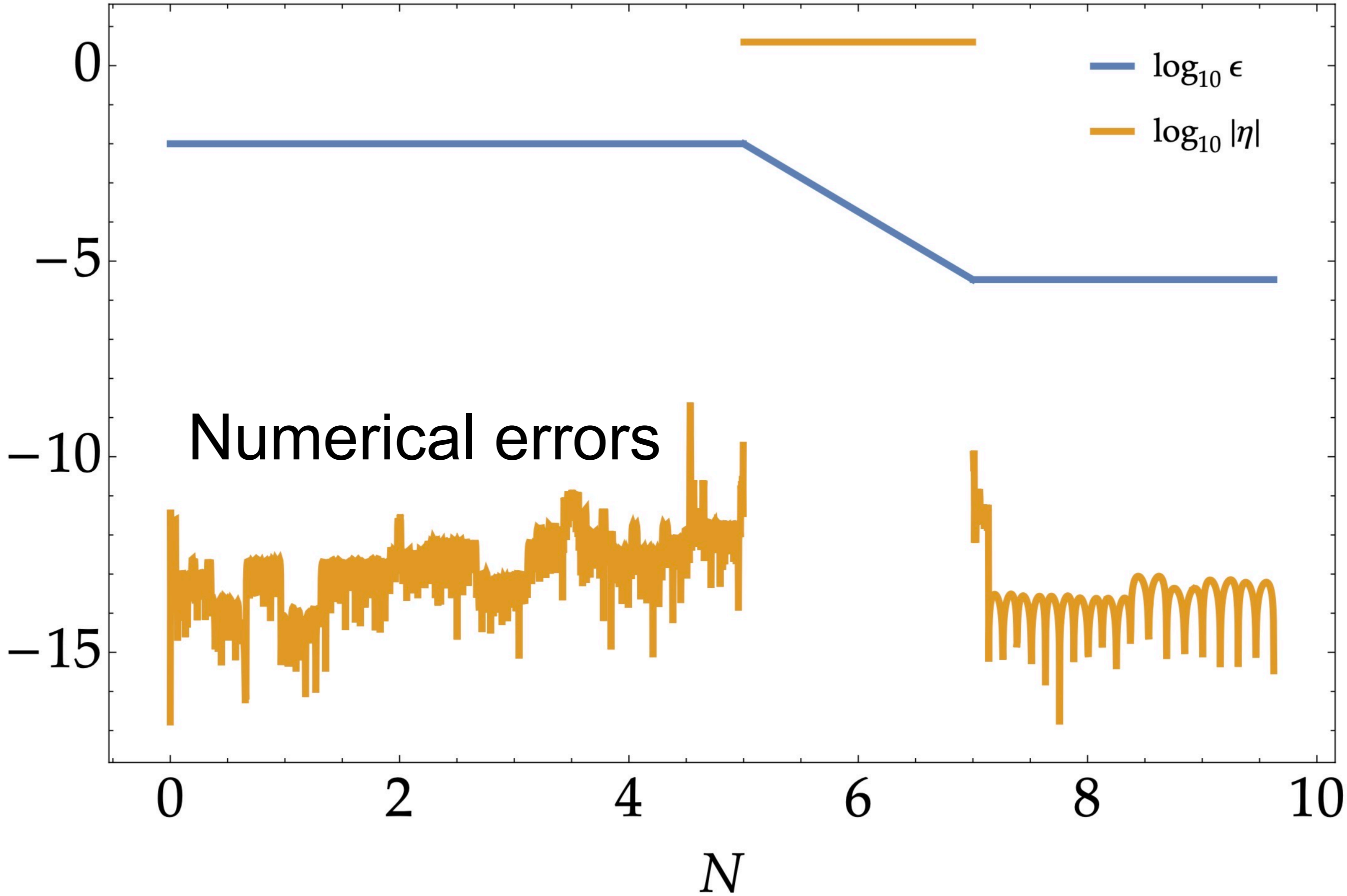
- SR-CR-SR model
- Instantaneous transitions

$$\eta = \begin{cases} 0, & \tau < \tau_s \\ 2\beta, & \tau_s \leq \tau < \tau_e \\ 0, & \tau_e \leq \tau \end{cases}$$

We have derived a potential to realize transient constant-roll evolution.



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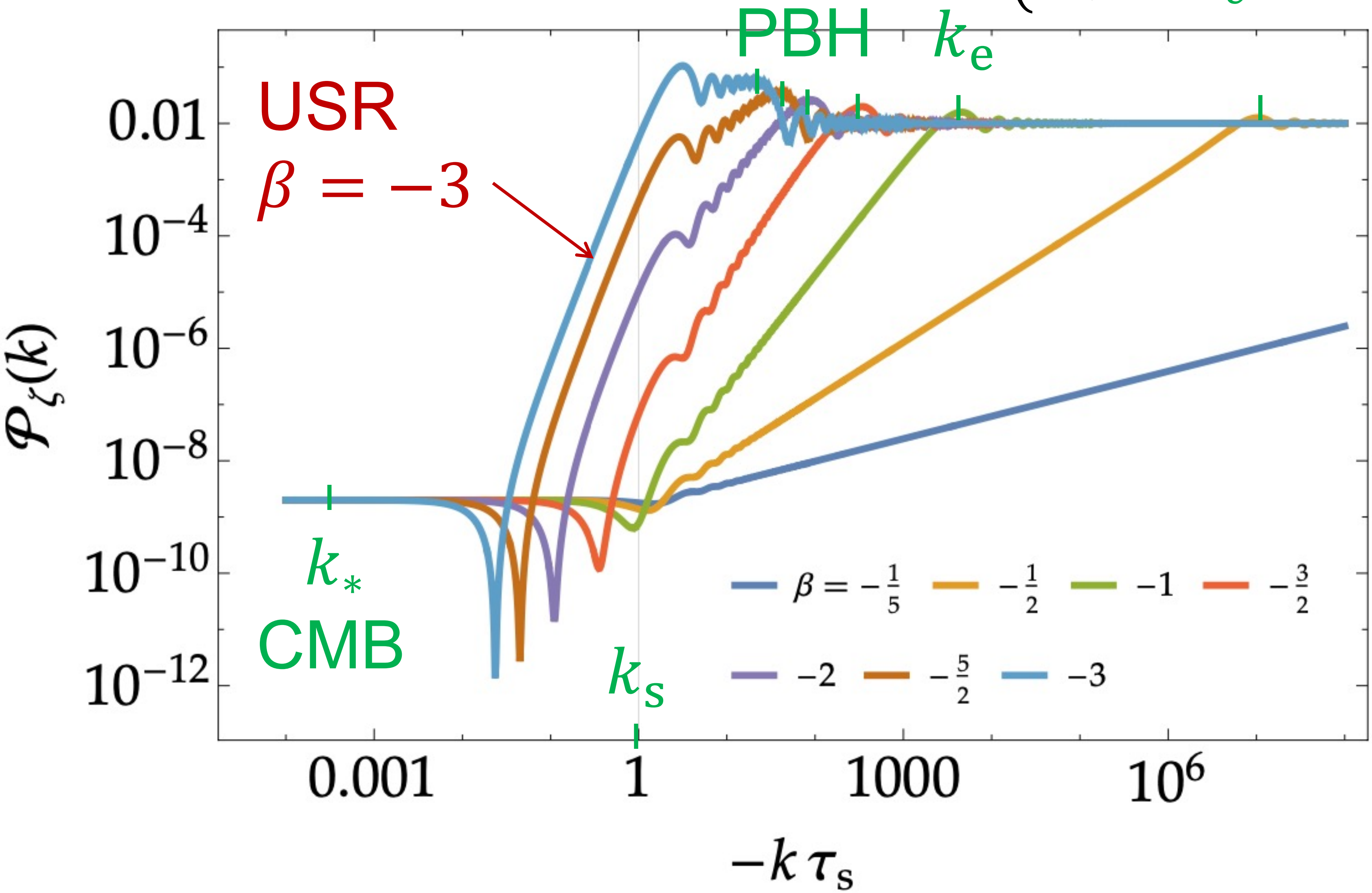


Squeezed bispectrum

Non-Gaussianity parameter for $k_{\mathrm{L}} \ll k_{\mathrm{S}}$

$$f_{\mathrm{NL}}(k_{\mathrm{L}}, k_{\mathrm{S}}) = \frac{5}{12} \frac{B(k_{\mathrm{L}}, k_{\mathrm{S}}, k_{\mathrm{S}})}{P(k_{\mathrm{L}}) P(k_{\mathrm{S}})}$$

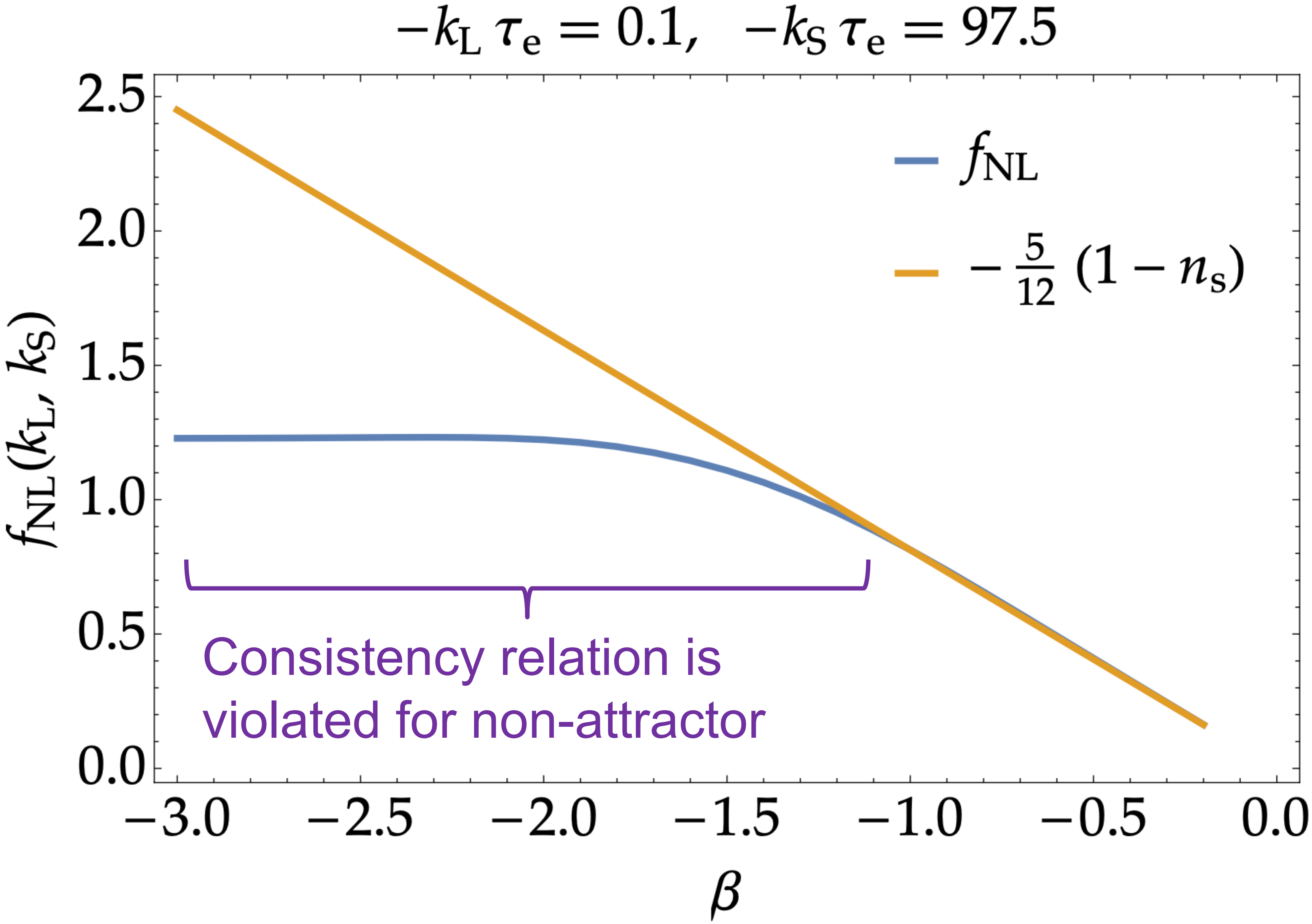
Consistency relation

$$f_{\mathrm{NL}}(k_{\mathrm{L}}, k_{\mathrm{S}}) = \frac{5}{12}(n_s - 1)$$

Maldacena, astro-ph/0210603

Squeezed bispectrum

HM, Tada, 2303.16035



- SR-CR-SR model
- Instantaneous transitions
- In-in formalism
- Considering only $\dot{\eta}$ term in H_{int} at $\tau = \tau_{\mathrm{e}}$

$$\eta = \begin{cases} 0, & \tau < \tau_{\mathrm{s}} \\ 2\beta, & \tau_{\mathrm{s}} \leq \tau < \tau_{\mathrm{e}} \\ 0, & \tau_{\mathrm{e}} \leq \tau \end{cases}$$

Perturbativity requirement:

$$\frac{\mathcal{P}_{(1)}(k_*)}{\mathcal{P}_{\mathrm{SR}}(k_*)} \approx \beta^2 \mathcal{P}_{\mathrm{SR}}(k_*) I\left(\beta, \frac{k_{\mathrm{e}}}{k_{\mathrm{s}}}\right) \ll 1$$

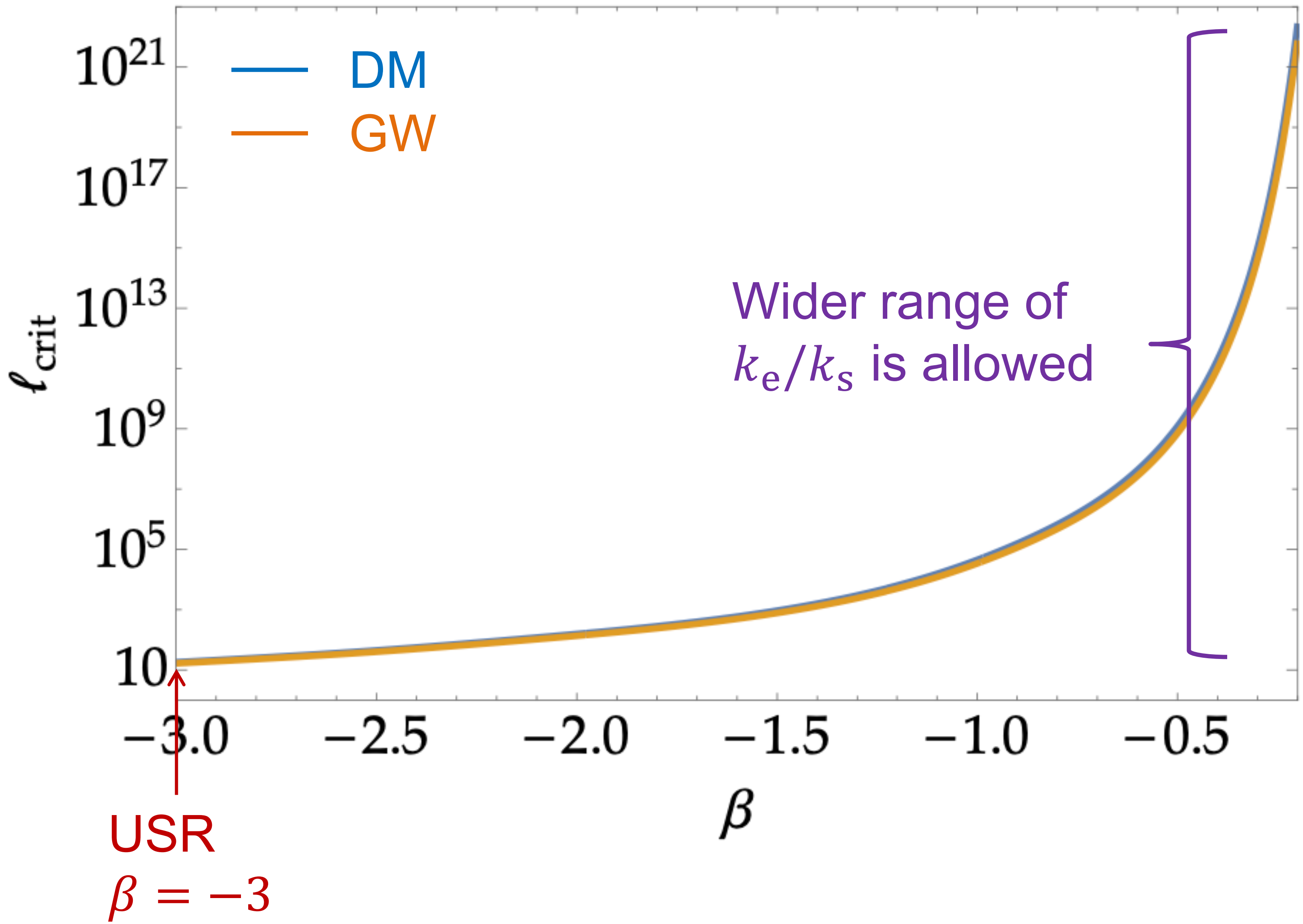
$$\Longrightarrow \quad \frac{k_{\mathrm{e}}}{k_{\mathrm{s}}} \ll \ell_{\mathrm{crit}}(\beta)$$

$$\Longrightarrow \quad \mathcal{P}_{\mathrm{PBH}} \ll \mathcal{P}_{\mathrm{crit}}(\beta)$$

If $\mathcal{P}_{\mathrm{crit}} = O(10^{-2})$, PBH formation is not compatible with perturbativity.
If $\mathcal{P}_{\mathrm{crit}} > 1$, there is no constraint from perturbativity.

Perturbativity requirement

HM, Tada, 2303.16035



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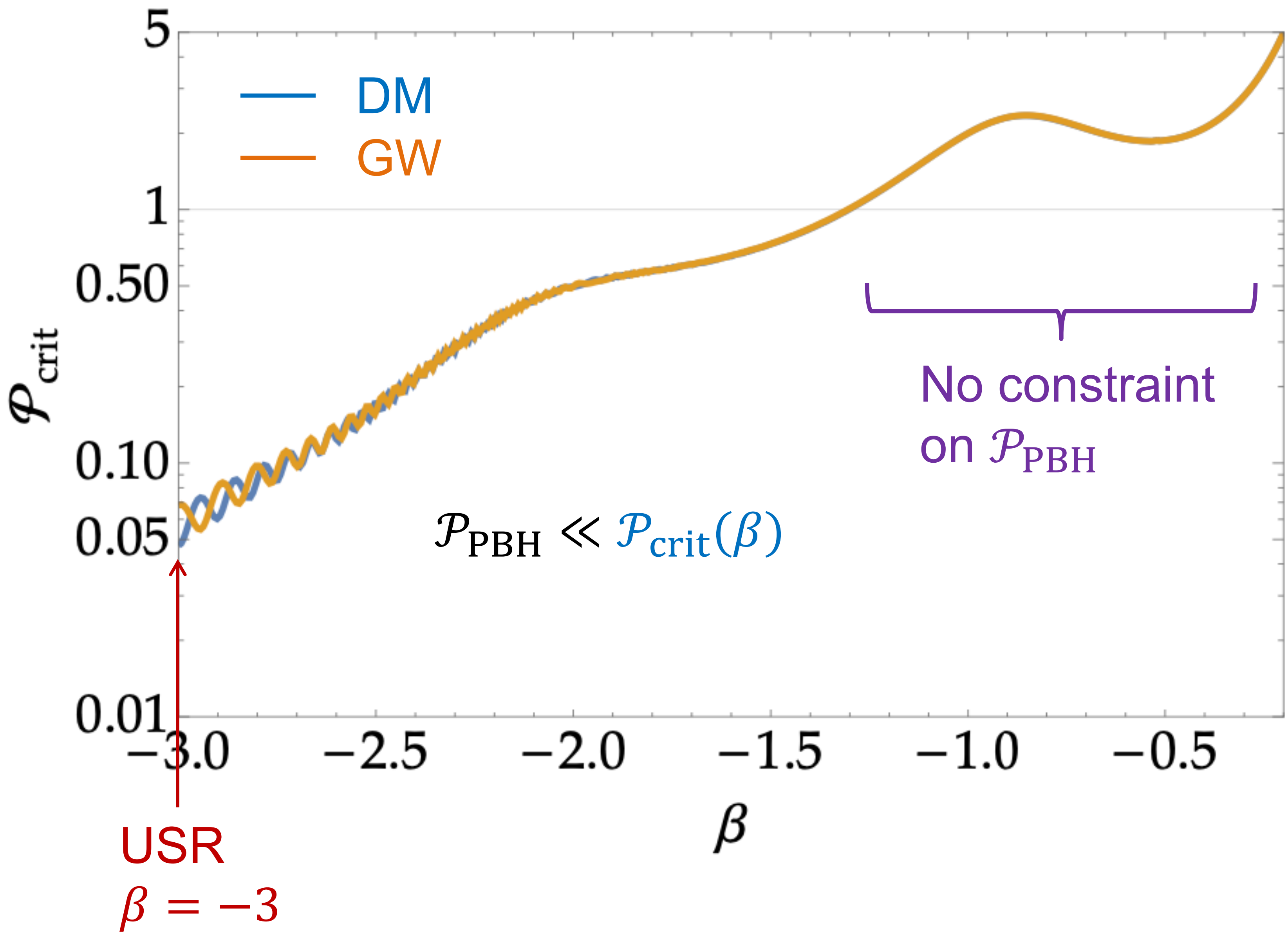

PBH production from single-field inflation

One-loop corrections seem to be large if we focus on:

- SR-USR-SR model
- Instantaneous transitions
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They are actually subdominant if we relax one of the assumptions:

- SR-CR-SR model

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They are actually subdominant if we relax one of the assumptions:

- SR-CR-SR model — HM, Tada, 2303.16035
- Smooth transitions — Firouzjahi, Riotto, 2304.07801
- Considering all terms in $H_{\rm int}$ — Fumagalli, 2305.19263

Tada, Terada, Tokuda, 2308.04732

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PBH production is compatible with perturbativity