Conformal Field Theory Dual To f(T) Black Holes

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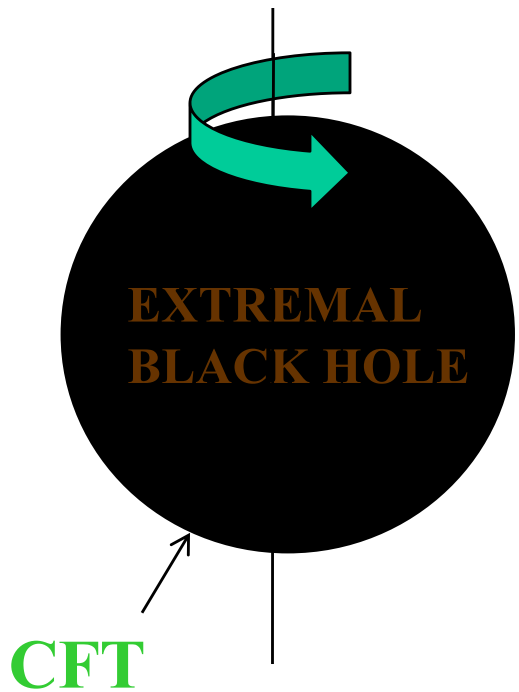
KERR/CFT CORRESPONDENCE



The near-horizon states of extremal rotating BH (4D or higher) could be identified with a certain chiral CFT under the assumption that the central charges from non-gravitational fields vanish (Example: Kerr-Sen Black Hole with three non-gravitational fields)

The String Theory (Heterotic $E_8 \times E_8$) Effective Action in 4D

$$S = -\int d^4x \sqrt{-\det G}\, \underbrace{e^{-\Phi}}_{\text{Dilaton}} \left(-R + \frac{1}{12}H^2 - G^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + \frac{1}{8}F^2\right),$$

$$H^2 = H_{\mu\nu\rho}H^{\mu\nu\rho}$$

Antisymmetric Tensor Field

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu} - \frac{1}{4}(A_\mu F_{\nu\rho} + A_\nu F_{\rho\mu} + A_\rho F_{\mu\nu})$$

$$F^2 = F_{\mu\nu}F^{\mu\nu} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

U(1) Gauge Field

$$U(1) \subset E_8 \times E_8$$

KERR-SEN BLACK HOLE

$$
\begin{aligned}
ds^2 &= -(1-\frac{2M\tilde{r}}{\rho^2})dt^2 + \rho^2(\frac{d\tilde{r}^2}{\Delta} + d\theta^2) \\
&- \frac{4M\tilde{r}a}{\rho^2}\sin^2\theta d\tilde{t}d\tilde{\phi} + \{\tilde{r}(\tilde{r}+\varrho) + a^2 + \frac{2M\tilde{r}a^2\sin^2\theta}{\rho^2}\}\sin^2\theta d\tilde{\phi}^2
\end{aligned}
$$

$$
\begin{aligned}
\Phi &= -\ln\frac{r^2 + a^2\cos^2(\theta) + 2mr\sinh^2(\alpha/2)}{r^2 + a^2\cos^2(\theta)} \\
A_t &= \frac{2mr\sinh(\alpha)}{r^2 + a^2\cos^2(\theta) + 2mr\sinh^2(\alpha/2)} \\
A_\phi &= \frac{2mra\sinh(\alpha)\sin^2(\theta)}{r^2 + a^2\cos^2(\theta) + 2mr\sinh^2(\alpha/2)}
\end{aligned}
$$

$$B_{t\phi} = \frac{2mra\sinh^2(\alpha/2)\sin^2(\theta)}{r^2 + a^2\cos^2(\theta) + 2mr\sinh^2(\alpha/2)}$$

Mass of BH

$$M = m\cosh^2(\alpha/2)$$

Charge of BH

$$Q = \tfrac{m}{\sqrt{2}}\sinh\alpha$$

Angular Velocity of Horizon

$$\Omega_H = \frac{a}{m(m+\sqrt{m^2-a^2})(1+\cosh(\alpha))}$$

$$T_H = \frac{\sqrt{m^2-a^2}}{2\pi m(m+\sqrt{m^2-a^2})(1+\cosh(\alpha))}$$

Hawking Temperature

Ang. Mom. of BH

$$J = ma\cosh^2(\alpha/2)$$

Horizon

$$r_H = M - \frac{\varrho}{2} + \frac{1}{2}\sqrt{(2M-\varrho)^2 - 4a^2}$$

$$\varrho = 2m\sinh^2(\alpha/2) = Q^2/M$$

To avoid any naked singularities $\longrightarrow$ $|J| \leq M^2 - \frac{1}{2}Q^2$

$$T_H = \frac{\sqrt{(2M^2 - Q^2)^2 - 4J^2}}{4\pi M(2M^2 - Q^2 + \sqrt{(2M^2 - Q^2)^2 - 4J^2})} \qquad \Omega_H = \frac{J}{M(2M^2 - Q^2 + \sqrt{(2M^2 - Q^2)^2 - 4J^2})}$$

Entropy of BH $\rightarrow$ $S = 2\pi M(M - \frac{Q^2}{2M} + \sqrt{(M - \frac{Q^2}{2M})^2 - \frac{J^2}{M^2}})$

Extremal Black Hole $J = M^2 - \frac{1}{2}Q^2$

$$S = 2\pi J$$

Could we obtain any of these results (especially entropy) from quantum theory of gravity? YES

Kerr, Kerr-Newman, Kerr-Bolt, Kerr-Bolt-(A)dS, Kerr-Sen, five and higher dimensional rotating black holes such as BMPV black hole in 5D N = 2 supergravity, ….

Near Horizon Geometry of Extremal BH with Horizon at $M - \frac{\varrho}{2}$

Scaling parameter

$$
\begin{aligned}
\tilde{r} &= (M - \frac{\varrho}{2})(1 + \frac{\lambda}{y}) \\
\tilde{t} &= \frac{2M}{\lambda} t \\
\tilde{\phi} &= \phi + t/\lambda
\end{aligned}
$$

$$
\begin{aligned}
ds^2 &= \frac{(2M-\varrho)\{\frac{1}{2}\varrho \sin^2\theta + M(1+\cos^2\theta)\}^2}{2M(1+\cos^2\theta) + \varrho\sin^2\theta}(\frac{-dt^2 + dy^2}{y^2}) \\
&+ \{M^2(1+\cos^2\theta) + \frac{1}{4}(-\varrho^2\sin^2\theta - 4\varrho M\cos^2\theta)\}d\theta^2 \\
&+ \frac{4(2M-\varrho)M^2\sin^2\theta}{2M(1+\cos^2\theta)+\varrho\sin^2\theta}(d\phi + \frac{dt}{y})^2
\end{aligned}
$$

or

AdS$_2$

$$
\begin{aligned}
ds^2 &= \{M^2(1+\cos^2\theta) + \frac{1}{4}(-\varrho^2\sin^2\theta - 4\varrho M\cos^2\theta)\}\{\frac{-dt^2+dy^2}{y^2} + d\theta^2 + \\
&+ \frac{4M^2\sin^2\theta}{(\frac{1}{2}\varrho\sin^2\theta + M(1+\cos^2\theta))^2}(d\phi + \frac{dt}{y})^2\}
\end{aligned}
$$

Near-horizon Dilaton (in local coordinates)

$$\Phi = \ln \frac{(2M^2 - Q^2)(1+\cos^2\theta)}{Q^2 \sin^2\theta + 2M^2(1+\cos^2\theta)}$$

Near Horizon Gauge Field

$$A = -\frac{2\sqrt{2}Q(2M^2 - Q^2)\sin^2\theta}{(Q^2\sin^2\theta + 2M^2(1+\cos^2\theta))}(d\phi + \frac{dt}{y})$$

Near Horizon 3-Form Field Strength

$$H = \{\mathcal{H}\frac{dy}{y^2} - \frac{1}{y}\mathcal{H}' d\theta\} \wedge dt \wedge d\phi$$

$$\mathcal{H} = \frac{2(2M^2 - Q^2)^2 Q^2 \sin^4\theta}{\{Q^2\sin^2\theta + 2M^2(1+\cos^2\theta)\}^2}$$

$\Rightarrow$

$$H = d\mathcal{B}.$$

$$\mathcal{B} = -\frac{\mathcal{H}(\theta)}{y} dt \wedge d\phi$$

Global Coordinates

$$
\begin{aligned}
y &= \frac{1}{\cos\tau\sqrt{1+r^2}+r} \\
t &= y\sin\tau\sqrt{1+r^2} \\
\phi &= \varphi + \ln\left(\frac{\cos\tau + r\sin\tau}{1+\sin\tau\sqrt{1+r^2}}\right)
\end{aligned}
$$

The Global Near Horizon Metric

$$
\begin{aligned}
ds^2 &= \{M^2(1+\cos^2\theta) + \frac{1}{4}(-\varrho^2\sin^2\theta - 4\varrho M\cos^2\theta)\}\{-(1+r^2)d\tau^2 + \frac{dr^2}{1+r^2} + d\theta^2 + \\
&+ \frac{4M^2\sin^2\theta}{(\frac{1}{2}\varrho\sin^2\theta + M(1+\cos^2\theta))^2}(d\varphi + rd\tau)^2\}
\end{aligned}
$$

$\mathbf{AdS_2}$

a S^1 bundle over $\mathbf{AdS_2}$

This geometry has a SL(2,R) isometry as well as a rotational U(1) isometry generated by the Killing vector ∂_φ

The Global Near Horizon Gauge Field

$$A = -\frac{2\sqrt{2}Q(2M^2 - Q^2)\sin^2\theta}{(Q^2\sin^2\theta + 2M^2(1+\cos^2\theta))}(d\varphi + r d\tau)$$

The Global Near Horizon 3-Form Field Strength

$$\begin{aligned}
H_{\tau r\varphi} &= \mathcal{H} \\
H_{\tau\theta\varphi} &= \cos(\tau)\sqrt{1+r^2}\mathcal{H}' \\
H_{\tau r\theta} &= -\frac{\sin(\tau)}{\sqrt{1+r^2}}\mathcal{H}' \\
H_{r\theta\varphi} &= \frac{r\sin(\tau)}{\sqrt{1+r^2}}\mathcal{H}'
\end{aligned}$$

Asymptotic Symmetries of the Action includes diffeomorphisms ξ

$$
\begin{aligned}
\delta_\xi \Phi &= \mathcal{L}_\xi \Phi = \xi_\mu \nabla^\mu \Phi \\
\delta_\xi A_\mu &= \mathcal{L}_\xi A_\mu = \xi^\nu F_{\mu\nu} + \nabla_\mu (A_\nu \xi^\nu) \\
\delta_\xi g_{\mu\nu} &= \mathcal{L}_\xi g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu \\
\delta_\xi \mathcal{B}_{\mu\nu} &= \mathcal{L}_\xi \mathcal{B}_{\mu\nu} = B_{\mu\rho}\partial_\nu \xi^\rho + B_{\rho\nu}\partial_\mu \xi^\rho + \xi^\rho \partial_\rho B_{\mu\nu}
\end{aligned}
$$

and gauge transformations

$$
\begin{aligned}
\delta_\Lambda A_\mu &= \partial_\mu \Lambda \\
\delta_\Psi \mathcal{B}_{\mu\nu} &= \partial_\mu \Psi_\nu - \partial_\nu \Psi_\mu
\end{aligned}
$$

The near-horizon geometry is not asymptotically flat (the asymptotic flatness has been lost in taking the near-horizon limit). As a result we should choose proper boundary conditions at infinity.

Corresponding to every consistent set of boundary conditions there should be an associated asymptotic symmetry group. The boundary conditions should be quite proper such that the generators of asymptotic symmetry group be well defined and not diverge at the boundary.

The most general diffeomorphisms that preserve a specific set of boundary conditions are given by

$$\varsigma_n = -e^{-in\varphi}(\partial_\varphi + inr\partial_r)$$

$$n = 0, \pm 1, \pm 2, \cdots$$

$n = 0 \longrightarrow$ **u(1) rotational isometry subalgebra of Virasoro algebra**

$$i[\varsigma_m, \varsigma_n]_{L.B.} = (m-n)\varsigma_{m+n}$$

Enhancement of U(1) to Virasoro !

Boundary Conditions

$$h_{\mu\nu} \sim \mathcal{O}\begin{pmatrix} r^2 & 1 & 1/r & 1/r^2 \\ & 1 & 1/r & 1/r \\ & & 1/r & 1/r^2 \\ & & & 1/r^3 \end{pmatrix}$$

$$\phi \sim \mathcal{O}(1)$$

$$a_\mu \sim \mathcal{O}(r, 1/r^2, 1, 1/r)$$

$$b_{\mu\nu} \sim \mathcal{O}\begin{pmatrix} & 1 & 1/r & 1/r^2 \\ & & 1/r & 1/r \\ & & & 1/r^2 \\ & & & \end{pmatrix}$$

The generator of diffeomorphism ς and gauge transformations is a conserved charge

$$Q_{\zeta,\Lambda,\Psi} = \frac{1}{8\pi}\int_{\partial\Sigma}(k^g_\zeta[h;g] + k^\Phi_\zeta[h,\phi;g,\Phi] + k^A_{\zeta,\Lambda}[h,a;g,A] + k^{\mathcal{B}}_{\zeta,\Psi}[h,b;g,\mathcal{B}])$$

$$k^g_\zeta[h;g] \;=\; -\delta\mathbf{Q}^g_\zeta + \mathbf{Q}^g_{\delta\zeta} + i_\zeta\mathbf{\Theta}[h] - \mathbf{E}_{\mathcal{L}}[\mathcal{L}_\zeta g, h]$$

$$k^\Phi_\zeta[h,\phi;g,\Phi] \;=\; -i_\zeta\mathbf{\Theta}_\Phi$$

$$\mathbf{\Theta}_\Phi = *(\phi d\Phi),\ \mathbf{\Theta}[h] = \star\{(D^\beta h_{\alpha\beta} - g^{\mu\nu}D_\alpha h_{\mu\nu})dx^\alpha\}$$

$$\mathbf{E}_{\mathcal{L}}[\mathcal{L}_\zeta g, h] = \star\{\frac{1}{2}h_{\alpha\gamma}(D^\gamma\zeta_\beta + D_\beta\zeta^\alpha)dx^\alpha\wedge dx^\beta\}$$

$$\mathbf{Q}^g_\zeta = \frac{1}{2}\star(D_\mu\xi_\nu - D_\nu\xi_\mu)dx^\mu\wedge dx^\nu$$

Koumar two-form

for a $\hat{p}$-form P with the associated $(\hat{p}+1)$-form field strength R

$$k^P_{\zeta,\Pi}[h,p;g,P] = -\delta \mathbf{Q}^P_{\zeta,\Pi} + \mathbf{Q}^P_{\delta\zeta,\delta\Pi} - i_\zeta \mathbf{\Theta^P} - \mathbf{E}^P_{\mathcal{L}}[\mathcal{L}_\zeta P + d\Pi, p]$$

$$\begin{aligned}
\mathbf{\Theta^P} &= p \wedge \star R \\
\mathbf{E}^P_{\mathcal{L}}[\mathcal{L}_\zeta P + d\Pi, p] &= \star\{\frac{1}{2(\hat{p}-1)!} p_{\mu\rho_1\cdots\rho_{\hat{p}-1}} (\mathcal{L}_\zeta P + d\Pi)_\nu^{\rho_1\cdots\rho_{\hat{p}-1}} dx^\mu \wedge dx^\nu\}
\end{aligned}$$

$$\mathbf{Q}^P_{\zeta,\Pi} = (i_\zeta P + \Pi) \wedge \star R$$

For antisymmetric tensor field

$$\begin{aligned}
k^{\mathcal{B}}_{\zeta,\Psi}[h,b;g,\mathcal{B}] &= \frac{1}{12}\{\zeta^\lambda(\epsilon_{\mu\nu\rho\beta} b_{\lambda\alpha} + \epsilon_{\mu\nu\rho\alpha} b_{\beta\lambda} + \epsilon_{\mu\nu\rho\lambda} b_{\alpha\beta}) H^{\mu\nu\rho}\} dx^\alpha \wedge dx^\beta \\
&- \frac{1}{6}\epsilon_{\mu\nu\rho\sigma}\{\frac{1}{2}\xi^\lambda b_{\nu\lambda} H^{\mu\nu\rho} + (\frac{1}{2}\xi^\lambda \mathcal{B}_{\nu\lambda} + \Psi_\nu)(\delta H^{\mu\nu\rho} + \frac{1}{2} h H^{\mu\nu\rho})\} dx^\nu \wedge dx^\sigma \\
&+ \frac{1}{8}\epsilon^{\mu\nu} b^\alpha_\mu (\mathcal{L}_\zeta \mathcal{B} + d\Psi)_{\nu\alpha}
\end{aligned}$$

For gauge field

$$\begin{aligned} k^{A}_{\zeta,\Lambda}[h,a;g,A] &= \frac{1}{8\pi}\epsilon_{\alpha\beta\mu\nu}\{(-\frac{1}{2}hF^{\mu\nu}+2F^{\mu\rho}h^{\nu}_{\rho}-\delta F^{\mu\nu})(\zeta^{\sigma}A_{\sigma}+\Lambda)-F^{\mu\nu}\zeta^{\sigma}a_{\sigma}-2F^{\sigma\mu}\zeta^{\nu}a_{\sigma}\}dx^{\alpha}\wedge dx^{\beta} \\ &- \frac{1}{8}\epsilon^{\mu\nu}_{\alpha\beta}a_{\mu}(\mathcal{L}_{\zeta}A_{\nu}+\partial_{\nu}\Lambda)dx^{\alpha}\wedge dx^{\beta}. \end{aligned}$$

For gravity

$$\begin{aligned} k^{g}_{\zeta}[h;g] &= -\frac{1}{4}\epsilon_{\mu\nu\rho\sigma}\{\zeta^{\sigma}\nabla^{\rho}h-\zeta^{\sigma}\nabla_{\lambda}h^{\rho\lambda}+\zeta_{\lambda}\nabla^{\sigma}h^{\rho\lambda}+\frac{1}{2}h\nabla^{\sigma}\zeta^{\rho} \\ &- h^{\sigma\lambda}\nabla_{\lambda}\zeta^{\rho}+\frac{1}{2}h^{\lambda\sigma}(\nabla^{\rho}\zeta_{\lambda}+\nabla_{\lambda}\zeta^{\rho})\}dx^{\mu}\wedge dx^{\nu}. \end{aligned}$$

For dilaton

$$k^{\Phi}_{\zeta}[h,\phi;g,\Phi]=-\frac{1}{6}\phi\epsilon^{\nu}_{\rho\sigma\lambda}\zeta^{\rho}\partial_{\nu}\Phi dx^{\sigma}\wedge dx^{\lambda}.$$

Charge algebra

$$
\begin{aligned}
\{Q_{\zeta,\Lambda,\Psi}, Q_{\tilde{\zeta},\tilde{\Lambda},\tilde{\Psi}}\}_{D.B.} &= (\delta_{\tilde{\zeta}} + \delta_{\tilde{\Lambda}} + \delta_{\tilde{\Psi}}) Q_{\zeta,\Lambda,\Psi} \\
&= \frac{1}{8\pi} \int_{\partial\Sigma} (k^g_\zeta[\mathcal{L}_{\tilde{\zeta}} g; g] + k^\Phi_\zeta[\mathcal{L}_{\tilde{\zeta}} g, \mathcal{L}_{\tilde{\zeta}} \Phi; g, \Phi] \\
&+ k^A_{\zeta,\Lambda}[\mathcal{L}_{\tilde{\zeta}} g, \mathcal{L}_{\tilde{\zeta}} A + d\tilde{\Lambda}; g, A] + k^{\mathcal{B}}_{\zeta,\Psi}[\mathcal{L}_{\tilde{\zeta}} g, \mathcal{L}_{\tilde{\zeta}} \mathcal{B} + d\tilde{\Psi}; g, \mathcal{B}])
\end{aligned}
$$

$$
\begin{aligned}
&= Q_{[(\zeta,\Lambda,\Psi),(\tilde{\zeta},\tilde{\Lambda},\tilde{\Psi})]} + \frac{1}{8\pi} \int_{\partial\Sigma} (k^g_\zeta[\mathcal{L}_{\tilde{\zeta}} \hat{g}; \hat{g}] + k^\Phi_\zeta[\mathcal{L}_{\tilde{\zeta}} \hat{g}, \mathcal{L}_{\tilde{\zeta}} \hat{\Phi}; \hat{g}, \hat{\Phi}] \\
&+ k^A_{\zeta,\Lambda}[\mathcal{L}_{\tilde{\zeta}} \hat{g}, \mathcal{L}_{\tilde{\zeta}} \hat{A} + d\tilde{\Lambda}; \hat{g}, \hat{A}] + k^{\mathcal{B}}_{\zeta,\Psi}[\mathcal{L}_{\tilde{\zeta}} \hat{g}, \mathcal{L}_{\tilde{\zeta}} \hat{\mathcal{B}} + d\tilde{\Psi}; \hat{g}, \hat{\mathcal{B}}])
\end{aligned}
$$

Central terms

$$\{.,.\}_{D.B.} \to -\frac{i}{\hbar}[.,.]$$

Charge Algebra

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{\hbar}m(m^2-1)\delta_{m+n,0}$$

Virasoro Algebra with central charge c, for the 2D dual CFT corresponding to KERR-SEN BH

$$c = c_g + c_\Phi + c_A + c_{\mathcal{B}}$$

Explicit Calculations show

$$c_\Phi = 0 \qquad c_A = 0 \qquad c_{\mathcal{B}} = 0$$

$$\Rightarrow \quad c = \frac{12J}{\hbar}$$

The Cardy formula gives the entropy of the two dimensional CFT

$$\boxed{S = 2\pi\sqrt{\frac{cL}{6}}}$$

Energy

Central charge

First Law of Thermodynamics

$$\boxed{dL = TdS} \to \boxed{dS = \pi\sqrt{\frac{c}{6L}}TdS} \to \boxed{\sqrt{L} = \pi\sqrt{\frac{c}{6}}T} \Rightarrow \boxed{S = \frac{\pi^2}{3}cT}$$

Frolov-Thorne temperature of the near horizon region ~Temp. of left moving CFT

$$\boxed{T_{F.T.} \equiv T_L = \frac{1}{2\pi}}$$

$$\Rightarrow \boxed{S_{\text{microscopic}} = 2\pi J} \checkmark$$

This is exactly equal to the macroscopic Bekenstein-Hawking entropy

$$S_{BH} = 2\pi J$$

How about generic non-extremal Black Holes?

If Kerr/CFT correspondence is correct, then energy excitations of CFT should correspond to generic non-extremal black hole.

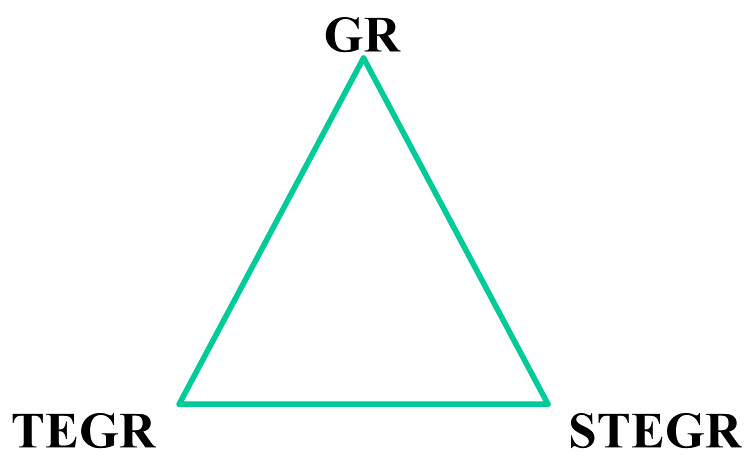
Problem:
Away from the extremality, there is no AdS structure for the near horizon geometry.
In fact the near horizon geometry is Rindler space with no known associated CFT.

Solution: Existence of conformal invariance in a near-horizon geometry is not a necessary condition for the interactions to exhibit conformal invariance.

Instead the existence of a local conformal invariance (known as hidden conformal symmetry) in the solution space of the wave equation for the propagating field is sufficient to ensure a dual CFT description.

This hidden conformal symmetry is a sufficient condition the scattering amplitudes exhibit conformal invariance though the space on which the field propagates doesn't have the conformal symmetry

Gravitational Trinity



$$\mathcal{S}_{GR} \equiv \frac{\kappa}{2}\int d^4x\,\sqrt{-g}\,R \qquad \mathcal{S}_{TEGR} \equiv \frac{\kappa}{2}\int d^4x\,\sqrt{-g}\,T \qquad \mathcal{S}_{STEGR} \equiv \frac{\kappa}{2}\int d^4x\,\sqrt{-g}\,Q$$

No Trinity

$$f(R)$$

$$f(T) \qquad f(Q)$$

$f(R)$	**Extension of GR-Dark energy and matter addressed as curvature effects on Astrophysical and cosmological scales - Explain well the acceleration of the universe - Explain galaxy rotation curves without dark matter/energy**
$f(T)$	**Extension of TEGR (torsion as a result of Weitzenbock connection, instead of Levi-Civita connection)**
$f(Q)$	**Extension of STEGR (non metricity which implies the covariant derivative of the metric does not vanish)**

f(T) Gravity

$$\mathcal{S} = \frac{1}{2\mathfrak{K}} \int d^4x \, |e| \left(f\left(T\right) - 2\Lambda - F \wedge^* F \right)$$

$$W^{\alpha}{}_{\mu\nu} = e_a{}^{\alpha} \partial_\nu e^a{}_\mu = -e^a{}_\mu \partial_\nu e_a{}^{\alpha}$$

Weitzenbock connection, which is curvature free, but has a non zero torsion

$$T^{\alpha}{}_{\mu\nu} = W^{\alpha}{}_{\nu\mu} - W^{\alpha}{}_{\mu\nu} = e_i{}^{\alpha} \left(\partial_\mu e^i{}_\nu - \partial_\nu e^i{}_\mu \right)$$

Scalar torsion

$$T = T^{\alpha}{}_{\mu\nu} S_{\alpha}{}^{\mu\nu}$$

$$S_{\alpha}{}^{\mu\nu} = \frac{1}{2} \left(K^{\mu\nu}{}_{\alpha} + \delta^{\mu}_{\alpha} T^{\beta\nu}{}_{\beta} - \delta^{\nu}_{\alpha} T^{\beta\mu}{}_{\beta} \right)$$

Contortion tensor

$$K_{\alpha\mu\nu} = \frac{1}{2} \left(T_{\nu\alpha\mu} + T_{\alpha\mu\nu} - T_{\mu\alpha\nu} \right)$$

Rotating Charged AdS Black Holes in $f\left(T\right)=T+\alpha T^{2}$ Gravity

Gravity field equations

$$S_{\mu}{}^{\rho\nu}\partial_{\rho}T f''\left(T\right)+\left[e^{-1}e^{a}{}_{\mu}\partial_{\rho}\left(ee_{\alpha}{}^{\alpha}S_{\alpha}{}^{\rho\nu}\right)-T^{\alpha}{}_{\lambda\mu}S_{\alpha}{}^{\nu\lambda}\right]f'\left(T\right)-\frac{\delta_{\mu}^{\nu}}{4}\left(f\left(T\right)+\frac{6}{l^{2}}\right)$$

$$=-\frac{\mathfrak{K}}{2}\mathcal{T}_{\mathrm{em}}{}^{\nu}{}_{\mu}$$

Maxwell's field equations $$\partial_{\nu}\left(\sqrt{-g}F^{\mu\nu}\right)=0$$

Black hole solution

$$ds^{2}=-A(r)(\Xi dt-\Omega d\phi)^{2}+\frac{dr^{2}}{B(r)}+\frac{r^{2}}{l^{4}}\left(\Omega dt-\Xi l^{2}d\phi\right)^{2}+\frac{r^{2}}{l^{2}}dz^{2}$$

$$ds^2 = -A(r)(\Xi dt - \Omega d\phi)^2 + \frac{dr^2}{B(r)} + \frac{r^2}{l^4}\left(\Omega dt - \Xi l^2 d\phi\right)^2 + \frac{r^2}{l^2}dz^2$$

Charge parameter

$$A(r) = r^2\Lambda_{eff} - \frac{M}{r} + \frac{3Q^2}{2r^2} + \frac{2Q^3\sqrt{6\,|\alpha|}}{6r^4}$$

The black hole may have 6 horizons!

$$B(r) = A(r)\beta(r)$$

Mass parameter

$$\beta(r) = \left(1 + Q\sqrt{6\,|\alpha|}/r^2\right)^{-2}$$

Rotation parameter

$$\Xi = \sqrt{1 + \frac{\Omega^2}{l^2}}$$

Cosmological constant parameter

$$\Lambda_{eff} = \frac{1}{36|\alpha|}$$

Gauge potential $$\tilde{\Phi}(r) = -\Phi(r)\left(\Omega d\phi - \Xi dt\right)$$

$$\Phi(r) = \frac{Q}{r} + \frac{Q^2\sqrt{6|\alpha|}}{3r^3}$$

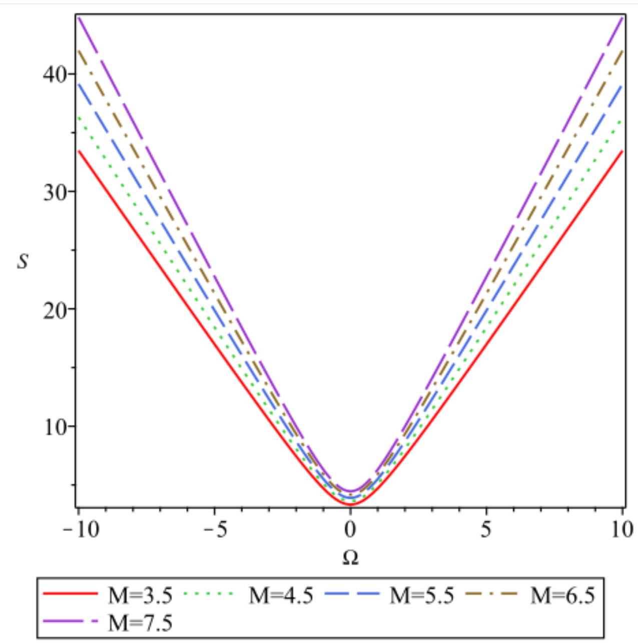
Black hole torsion $$T(r) = \frac{4A'(r)B(r)}{rA(r)} + \frac{2B(r)}{r^2}$$

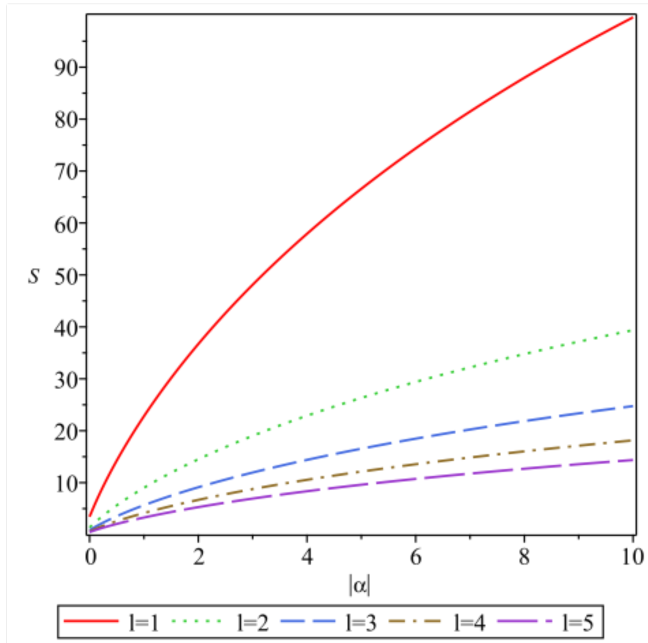
Entropy of black hole

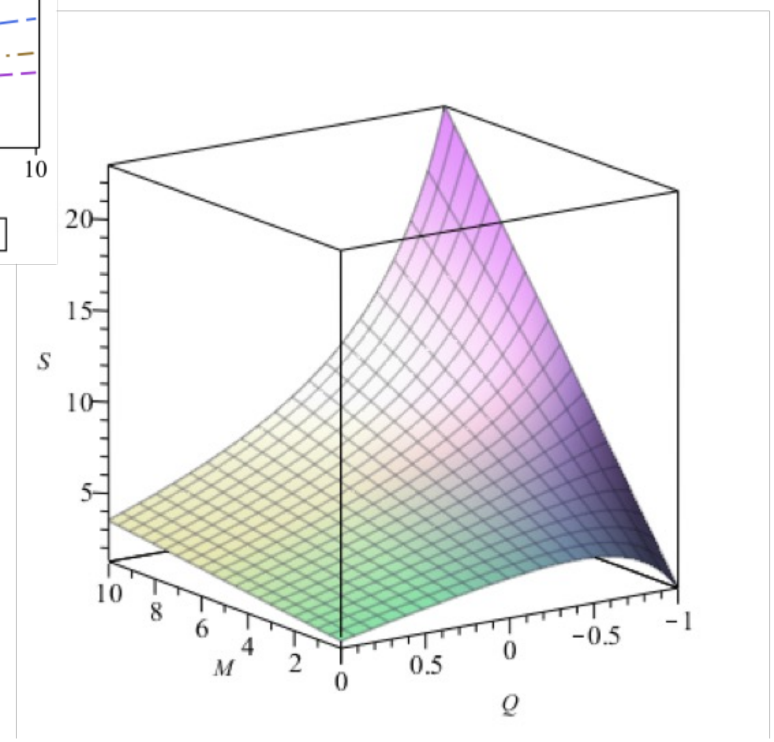
$$S = \frac{f'(T)\mathscr{A}}{4}$$

Outer event horizon area $$\mathscr{A} = \int_0^{2\pi} d\phi \int_0^L dz \sqrt{-g|_{dt=dr=0}} = \frac{2\pi r_+^2 \Xi L}{l}$$

$$S = \frac{\pi \Xi L \left(7{r_+}^6 + 9\sqrt{6\,|\alpha|}\,Q{r_+}^4 + 18M\,|\alpha|\,{r_+}^3 - 54Q^2\,|\alpha|\,{r_+}^2 - 42\sqrt{6|\alpha|^3}\,Q^3\right)}{9l\left(Q\sqrt{6\,|\alpha|} + {r_+}^2\right)^2}$$







Holography for the Rotating Charged AdS Black Holes in $f(T) = T + \alpha T^2$ Gravity

Consider a massless scalar probe in the background of black hole

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\psi\right)=0$$

Thanks to three Killing vectors $\psi(t,r,z,\phi)=e^{-i\omega t+ikz+im\phi}R(r)$

$$B(r)\frac{d^2R(r)}{dr^2}+\left(rB(r)\frac{dA(r)}{dr}+rA(r)\frac{dB(r)}{dr}+4A(r)B(r)\right)\frac{dR(r)}{dr}+V(r)R(r)=0$$

$$V(r)=\frac{r^2\left(\Xi l^2\omega-\Omega m\right)^2-A(r)l^2\left\{k^2l^4\Xi^4+k^2\Omega^4+l^2\left[m^2\Xi^2-2m\Xi\Omega\omega+\Omega^2\left(\omega^2-2\Xi^2k^2\right)\right]\right\}}{A(r)r^2(\Xi^2l^2-\Omega^2)^2}$$

In near horizon region: $A(r) \simeq K\left(r-r_{+}\right)\left(r-r_{*}\right)$

$$K = 15{r_+}^4\Lambda_{eff} - 3Mr_+ + \frac{3Q^2}{2} \qquad r_* = r_+ - \frac{2r_+\left(2{r_+}^4\Lambda_{eff} - Mr_+ + Q^2\right)}{10{r_+}^4\Lambda_{eff} - 2Mr_+ + Q^2}$$

Considering low energy scalar probe and closeness of r_+ **to** r_*

$$\frac{d}{dr}\left\{\left(r-r_{+}\right)\left(r-r_{*}\right)\frac{d}{dr}R\left(r\right)\right\} + \left[\left(\frac{r_+-r_*}{r-r_+}\right)\mathcal{A} + \left(\frac{r_+-r_*}{r-r_*}\right)\mathcal{B} + \mathcal{C}\right]R\left(r\right) = 0$$

$$\mathcal{A} = \frac{\mathcal{D}m^2 + \mathcal{E}m\omega}{K^2{r_+}^2{r_*}^3(\Xi^2l^2-\Omega^2)^2(r_+-r_*)^2\beta} + \frac{\mathcal{F}\omega^2}{K{r_+}^2(\Xi^2l^2-\Omega^2)^2(r_+-r_*)^2\beta} - C_1$$

$$\mathcal{B} = \frac{\mathcal{G}m^2 + \mathcal{I}m\omega}{K^2{r_+}^3r_*(\Xi^2l^2-\Omega^2)^2(r_+-r_*)^2\beta} + \frac{\mathcal{J}\omega^2}{K{r_+}^2(\Xi^2l^2-\Omega^2)^2(r_+-r_*)^2\beta} + C_2$$

Generators of CFT

$$H_1 = i\partial_+ \qquad \bar{H}_1 = i\partial_-$$

$$H_0 = i\left(\omega^+\partial_+ + \frac{1}{2}y\partial_y\right) \qquad \bar{H}_0 = i\left(\omega^-\partial_- + \frac{1}{2}y\partial_y\right)$$

$$H_{-1} = i(\omega^{+2}\partial_+ + \omega^+ y\partial_y - y^2\partial_-) \qquad \bar{H}_{-1} = i(\omega^{-2}\partial_- + \omega^- y\partial_y - y^2\partial_+)$$

$$SL(2,R)_L \times SL(2,R)_R$$

$$[H_0, H_{\pm 1}] = \mp i H_{\pm 1}, \quad [H_{-1}, H_1] = -2iH_0$$

$$[\bar{H}_0, \bar{H}_{\pm 1}] = \mp i \bar{H}_{\pm 1}, \quad [\bar{H}_{-1}, \bar{H}_1] = -2i\bar{H}_0$$

The Casimir operators of the $SL(2,R)_L \times SL(2,R)_R$

$$\mathcal{H}^2 = \bar{\mathcal{H}}^2 = -H_0^2 + \frac{1}{2}(H_1 H_{-1} + H_{-1} H_1) = \frac{1}{4}(y^2\partial_y^2 - y\partial_y) + y^2\partial_+\partial_-$$

Choosing the conformal coordinates

$$\omega^+ = \sqrt{\frac{r-r_+}{r-r_*}}e^{2\pi T_R\phi+2n_Rt}$$

$$\omega^- = \sqrt{\frac{r-r_+}{r-r_*}}e^{2\pi T_L\phi+2n_Lt}$$

$$y = \sqrt{\frac{r_+-r_*}{r-r_*}}e^{\pi(T_L+T_R)\phi+(n_L+n_R)t}$$

We find

$$\mathcal{H}^2 = (r-r_+)(r-r_*)\frac{\partial^2}{\partial r^2} + (2r-r_+-r_*)\frac{\partial}{\partial r} + \left(\frac{r_+-r_*}{r-r_*}\right)\left[\left(\frac{n_L-n_R}{4\pi G}\partial_\phi - \frac{T_L-T_R}{4G}\partial_t\right)^2 + C_2\right]$$
$$-\left(\frac{r_+-r_*}{r-r_+}\right)\left[\left(\frac{n_L+n_R}{4\pi G}\partial_\phi - \frac{T_L+T_R}{4G}\partial_t\right)^2 - C_1\right]$$

Temperatures of CFT

$$T_R = \frac{r_+ K \left(r_+ - r_*\right) \left(\Xi^2 l^2 - \Omega^2\right) \sqrt{\beta r_+ r_* \delta}}{4\pi\delta}$$

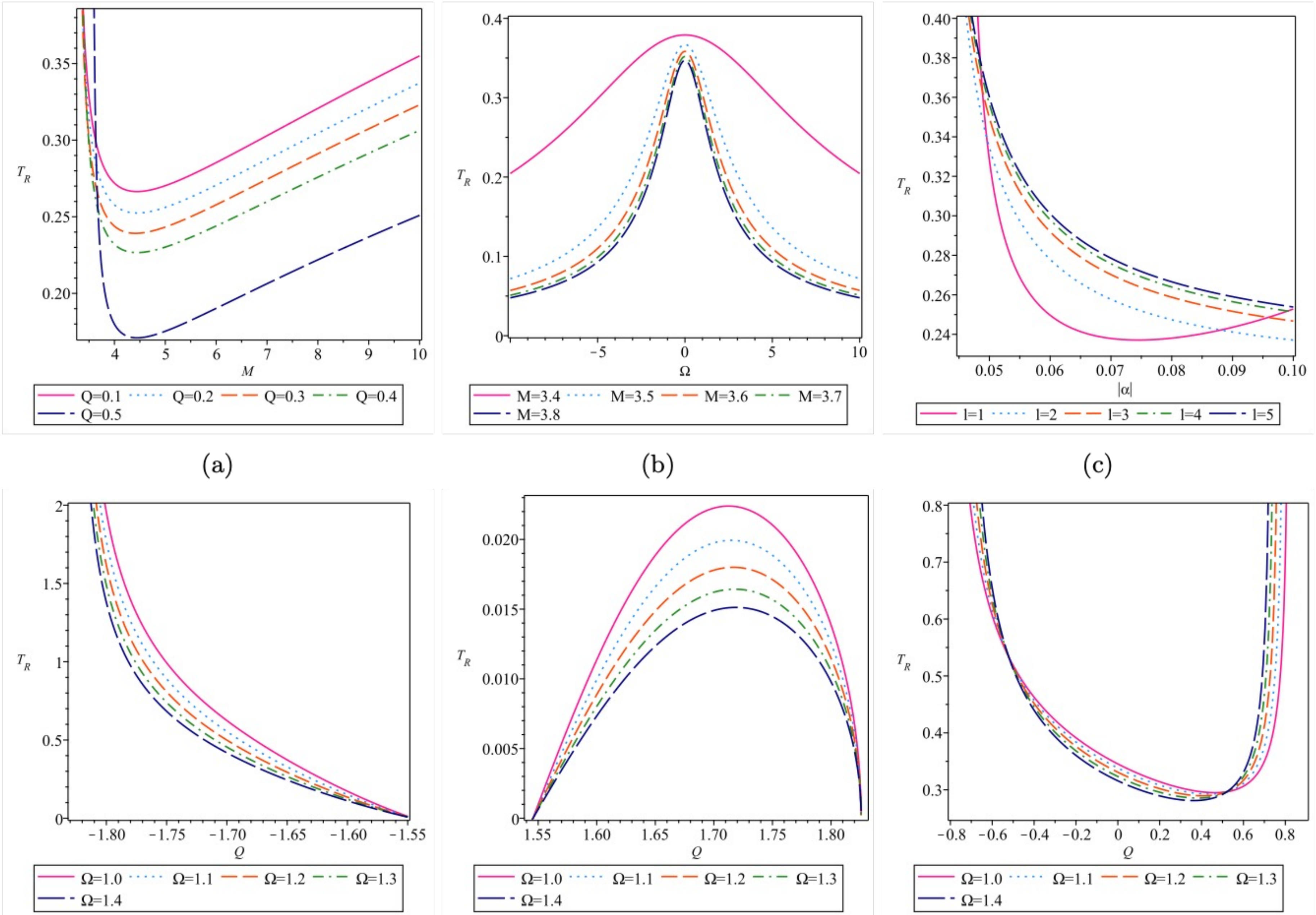
$$T_L = \frac{r_+ K \left(\Xi^2 l^2 - \Omega^2\right) \left[{r_+}^4 + 2{r_+}^3 r_* + 6{r_+}^2{r_*}^2 - 2{r_*}^3 r_+ \left(K l^2 - 1\right) + {r_*}^4\right] \sqrt{\beta r_+ r_* \delta}}{4\pi (r_+ + r_*)^3 \delta}$$

Mode numbers of CFT

$$n_R = 0$$

$$n_L = \frac{{r_*}^2 r_+ K \left(\Omega^2 - \Xi^2 l^2\right) \sqrt{\beta r_+ r_* \delta}}{2\Omega l^2 \Xi (r_+ + r_*)^3}$$

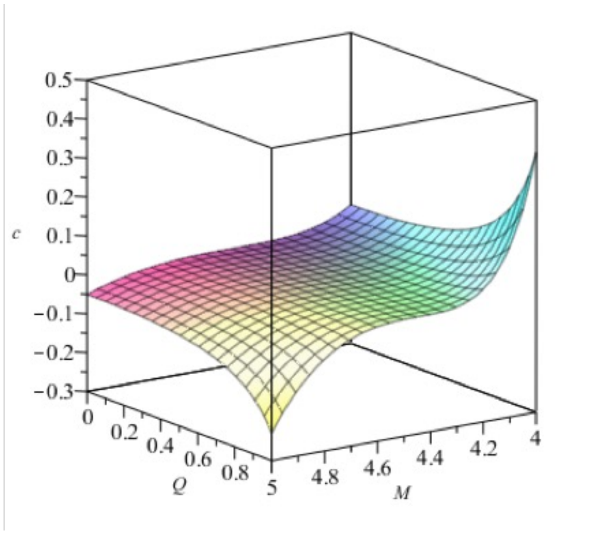
Cardy entropy for the CFT $$S_{CFT} = \frac{\pi^2}{3}\left(c_L T_L + c_R T_R\right)$$



(a)
T_R
M
Q=0.1
Q=0.2
Q=0.3
Q=0.4
Q=0.5
(b)
T_R
Ω
M=3.4
M=3.5
M=3.6
M=3.7
M=3.8
(c)
T_R
|α|
l=1
l=2
l=3
l=4
l=5
T_R
Q
Ω=1.0
Ω=1.1
Ω=1.2
Ω=1.3
Ω=1.4
T_R
Q
Ω=1.0
Ω=1.1
Ω=1.2
Ω=1.3
Ω=1.4
T_R
Q
Ω=1.0
Ω=1.1
Ω=1.2
Ω=1.3
Ω=1.4

Choosing the central charges for the CFT

$$c \equiv c_L = c_R = \frac{12\Xi\delta L\varpi(r_+ + r_*)^3}{lKr_+{}^2\left(\Xi^2 l^2 - \Omega^2\right)\left(r_+{}^3 + 2r_+{}^2 r_* + 3r_+ r_*{}^2 - l^2 K r_*{}^3\right)\left(Q\sqrt{6\,|\alpha|} + r_+{}^2\right)^2\sqrt{\beta r_+ r_* \delta}}$$



We find the Cardy entropy is exactly the same as the entropy of black hole

$$S = \frac{\pi\Xi L\left(7r_+{}^6 + 9\sqrt{6\,|\alpha|}Qr_+{}^4 + 18M\,|\alpha|\,r_+{}^3 - 54Q^2\,|\alpha|\,r_+{}^2 - 42\sqrt{6|\alpha|^3}Q^3\right)}{9l\left(Q\sqrt{6\,|\alpha|} + r_+{}^2\right)^2}$$