Dynamical Inflation Stimulated Cogenesis

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Outline

- Cosmic Inflation → why?
  - How?
    - Inflation Parameters
- Particle Physics Paradigm : Dynamical Inflection Point Inflation
- Matter Asymmetry
- Model
- Conclusion

Cosmic Inflation

- Why do we need this?
  - Horizon Problem
  - Flatness Problem

    - Solution via Inflation

Inflation parameters

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inflaton

Given a inflaton potential $V(\phi)$. We need to define the slow roll parameters

$$\epsilon_V = \frac{M_{Pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)\Bigg|_{\phi=\phi_i} \qquad \eta_V(\phi) = M_{Pl}^2\left(\frac{V_{,\phi\phi}}{V}\right)\Bigg|_{\phi=\phi_i} \qquad \xi_V^2(\phi) = M_{Pl}^4\,\frac{V_{,\phi\phi\phi}\,V_{,\phi}}{V^2(\phi)}\Bigg|_{\phi=\phi_i}$$

$$V_{,\phi} = \frac{dV}{d\phi}$$

$\therefore$ in the slow-roll regime $\epsilon_V, \eta_V, \xi_V^2 \ll 1$

Inflation parameters

$n_s = 1 - 6\epsilon_V + 2\eta_V$:= Scalar spectral Index $= 0.9691 \pm 0.0041$

$r = 16\,\epsilon_V$:= Tensor to scalar amplitude < 0.0067

$\alpha_s = 16\,\epsilon_V \eta_V - 24\epsilon_V^2 - 2\,\xi_V^2 \equiv \dfrac{d n_s}{d \ln k}$:= Running of the scalar spectral index

$= 0.0023 \pm 0.0063$

$A_s = \dfrac{1}{12\pi^2} \dfrac{V^3}{V'^2}$:= scalar amplitude $= (2.189 \pm 0.053) \times 10^{-9}$

and the number of e-foldings before the inflation ends

$$N_e \simeq \int_{\phi_e}^{\phi_i} \frac{V(\phi)}{V'(\phi)}\, d\phi \simeq 55 - 60$$

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- Taking Coleman-Weinberg (CW) inflaton potential which is well motivated from wide range of field-theory models.

$$V(\phi) = \lambda \left\{ \phi^4 \left[\ln(\phi/f) - 1/4 \right] + f^4/4 \right\}$$ f := minimum of potential

⇒ For accommodating n_s & the constraint of r for $N_e \sim 60$ $\phi_i \simeq 23\, M_{Pl}$

(large field model)

→ $\phi_i \ll M_{Pl}$

⇒ For small fields the spectral index $n_s \sim 1 - 3/N_e = 1 - 6\epsilon_V + 2\eta_V$

so, for $N_e = 60$ $n_s \sim 0.95$ which is far off from the observational value.

$\sim 5\sigma$ away

- In small field models, n_s is controlled by the second derivative of the potential i.e $V_{,\phi\phi}$

⇒ One needs to find smaller value to fit the data

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Small field Problem

$$V(\phi) = \lambda \{ \phi^4 [\ln(\phi/f) - 1/4] + f^4/4 \}$$

for $f < M_{Pl}$

$$\Rightarrow N_e \sim f^4/\phi_i^2$$

$$\epsilon_V \sim f^4/N_e^3 \quad \& \quad \eta_V \sim 1/N_e$$

$$\epsilon_V \ll \eta_V$$

$$n_s \sim 1 + 2\eta_V$$

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- To have $|V_{,\phi\phi}| \simeq 0$ at some field point close to the initial inflation field value, one basically requires that the potential contains an inflection point

- To realize a viable small-field inflation potential with an inflection point one has to generalize the CW with Renormalization Group (RG) improved version.

$$V(\phi) = \frac{1}{4}\lambda(\phi)\phi^4 + V_0$$

- The first and second derivative

$$V'(\phi) = \left(\lambda + \frac{1}{4}\beta_\lambda\right)\phi^3$$

$$V''(\phi) = \frac{1}{4}\left(12\lambda + 7\beta_\lambda + \tilde{\beta}_\lambda'\right)$$

2-loop order in a weakly coupled theory and thus ignored

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- Now, the extrema of the potential

$$\lambda(\phi_{ext}) = -\frac{1}{4}\beta_\lambda(\phi_{ext}) \;;\; V'(\phi) = \left(\lambda + \frac{1}{4}\beta_\lambda\right)\phi^3$$

& the inflection points ϕ_{inf}

$$\lambda(\phi_{inf}) \approx -\frac{7}{12}\beta_\lambda(\phi_{inf}) \;;\; V''(\phi) \simeq \frac{1}{4}\left(12\lambda + 7\beta_\lambda\right)$$

- With only scalar field, $\beta_\lambda \sim \frac{1}{16\pi^2}\lambda^2$ the above conditions are not possible to satisfy in a perturbative theory.

However, more complicated models where other couplings are g then

$$\beta_\lambda \sim \frac{1}{16\pi^2}\left(g^4 + g^2\lambda + \lambda^2\right)$$

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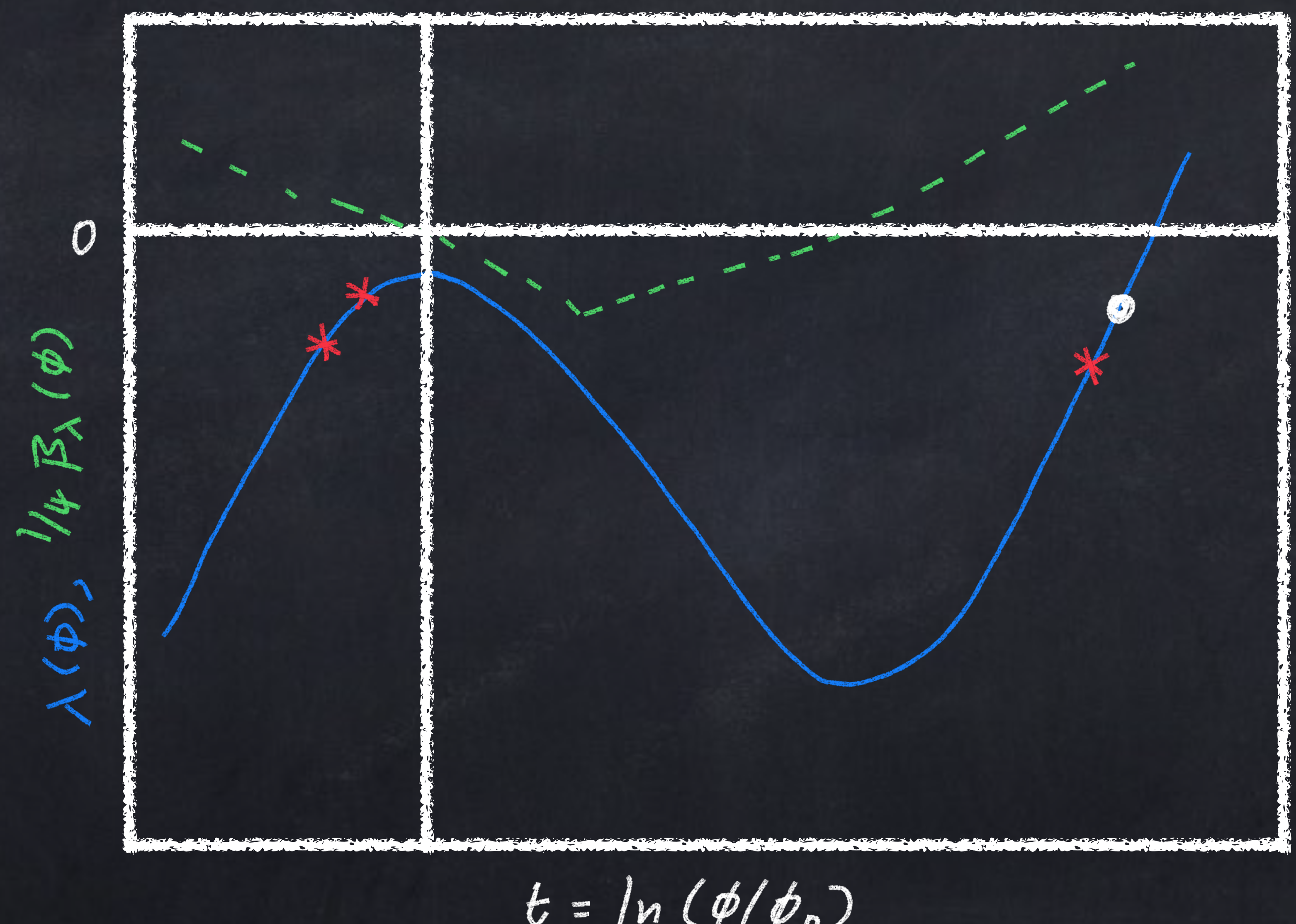
- This can be solved if there are two locations in field space where $\beta_\lambda = 0$ that are parametrically seperated.



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Feild theoretic Realization

Taking $SU(N_c)$ gauge theory

$$\mathcal{L} = -\frac{1}{4} G^{a\mu\nu} G^{a}_{\mu\nu} + (D_\mu \Phi)^\dagger (D_\mu \Phi) + i\, \bar{\chi} \not{\partial} \chi + i\, \bar{\Psi} \not{D} \Psi - \sum_{i=N_{\bar{\psi}R}+1}^{N_w} M \bar{\Psi}_i \Psi_i$$

$$- \frac{\lambda}{4} (\Phi^\dagger \Phi)^2 - (\Phi \bar{\Psi} \mathbb{Y} \chi + h.c.)$$

$$\kappa = \frac{1}{16\pi^2}$$

$$\beta_g = -\kappa g^3 \left(\frac{11}{3} N_c - \frac{1}{6} - \frac{2 n_f}{3} \right)$$

$$\beta_{\mathbb{Y}} = \kappa \left(\frac{3}{2} \mathbb{Y} \mathbb{Y}^\dagger \mathbb{Y} + \mathbb{Y}\, \mathrm{tr}(\mathbb{Y}^\dagger \mathbb{Y}) - 3 \frac{N_c^2 - 1}{2N_c} g^2 \mathbb{Y} \right)$$

$$\beta_\lambda = \kappa \left(\frac{3(N_c - 1)(N_c^2 + 2N_c - 2)}{4N_c^2} g^4 - 2\, \mathrm{tr}(\mathbb{Y}^\dagger \mathbb{Y} \mathbb{Y}^\dagger \mathbb{Y}) - 6 \frac{(N_c^2 - 1)}{N_c} \lambda g^2 + 4 \lambda\, \mathrm{tr}(\mathbb{Y}^\dagger \mathbb{Y}) \right.$$

$$\left. + 4 (N_c + 4) \lambda^2 \right)$$

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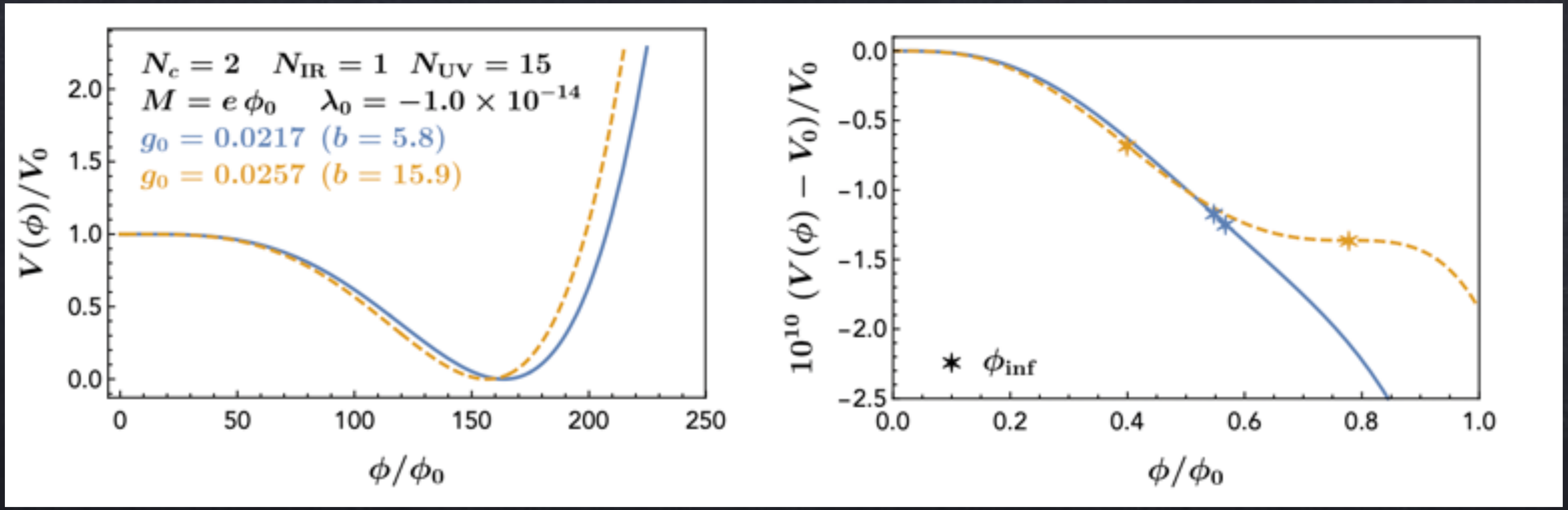
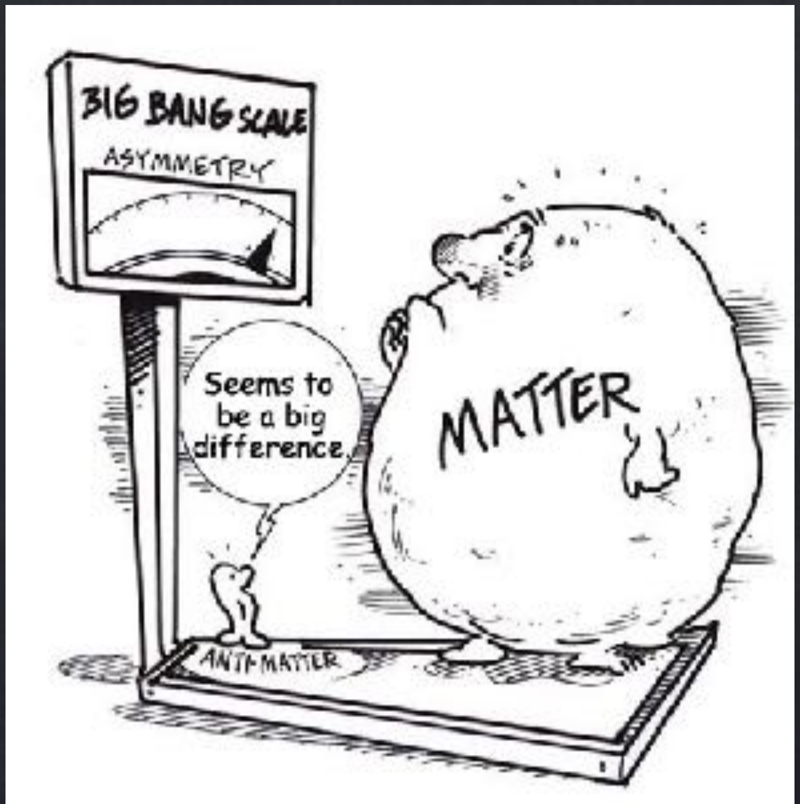
Feild theoretic Realization

$$V(\phi) = -\frac{a}{4}\left(1 + b\ln^2\left(\frac{\phi}{\phi_0}\right) + c\ln^2\left(\frac{\phi}{M}\right)\theta(\phi - M)\right)$$



Baryon Asymmetry of the Universe

- The Standard Model does not explain the present asymmetry.
  1. The CP violation coming from Jarlskog invarient is of the order 10^{-20}.
  2. The experimental lower bound on the Higgs mass implies the transition is not strongly first order.



- The observed BAU is often quoted in terms of baryon to photon ratio

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = 6.04 \pm 0.08 \times 10^{-10}$$

Kinds of Mechanism in generating Asymmetry

- Baryogenesis from Decay/Scattering
- Baryogenesis from Electroweak Phase Transitions
- Spontaneous Baryogenesis
- ... (Affleck-Dine, Gravitational Baryogenesis, etc.)

Sakharov's Conditions

- The three basic ingredients necessary to generate a net baryon asymmetry from an initially baryons symmetric Universe (Sakharov 1967):

1) Baryon Number (B) violation $X \to Y + B$

2) C and CP violation

$$\Gamma(X \to Y + B) \neq \Gamma(\bar{X} \to \bar{Y} + \bar{B})$$

3) Deperature from thermal Equilibrium.

CP violation from the Amplitude

- Now, in order to get a non-zero CP violation we start need at-least two distinct amplitude for a particular process

- In order to understand the above claim we start with the amplitude of a B-violating process ($X \to b$)

$$i\mathcal{M} = (\mathcal{C}_1 \mathcal{A}_1 + \mathcal{C}_2 \mathcal{A}_2) f$$

Similarly, for anti-particle

$$i\bar{\mathcal{M}} = (\mathcal{C}_1^* \mathcal{A}_1 + \mathcal{C}_2^* \mathcal{A}_2) f^*$$

$\longrightarrow$ spinor wave function

- The difference between the two processes comes out to be

$$\delta = 4 \, \underbrace{\mathrm{Im}[c_1^* c_2]}_{\text{Purely from coupling}} \, \underbrace{\mathrm{Im}[A_1^* A_2]}_{\text{Purely from Amplitudes}} |f|^2$$

- Now, originally the imaginary part comprises of 1-loop

  $\Rightarrow$ Which was due to the fact it was a 2-body decay

- But if we consider a 2-2 process or a 3-body decay the imaginary part comes from the imaginary part of the amplitude.

Model

- The particle content is given by

	$(2s+1)$	n_f	$SU(2)_D$	$Z_3 \times Z_2$
ψ_{i_L}	2	N_ψ	2	$(1,-1)$
ψ_{i_R}	2	N_ψ	2	$(1,-1)$
ψ_{1_L}	2	1	2	$(\omega,1)$
ψ_{1R}	2	1	2	$(\omega^2,1)$
χ_{iL}	2	2	1	$(\omega^2,1)$
χ_{iR}	2	2	2	$(\omega,1)$
Φ	1	1	2	$(1,1)$

Model

$$\mathcal{L} \supset \left(\sum_{i=1}^{8} y_i\, \bar{\psi}_i\, \Phi\, \chi + \sum_{i=9}^{16} y_i\, \bar{\psi}_i\, \tilde{\Phi}\, \chi + \sum_{i=2}^{N_\psi+1} M\, \overline{\psi_{iL}}\, \psi_{iR} + h.c. \right) - \frac{1}{4} G^{a\mu\nu} G^{a}_{\mu\nu}$$

$$+ \frac{\lambda_\Phi}{4} |\Phi|^4 + \frac{\lambda_H}{4} |H|^4 + \lambda_{\Phi H} |\Phi|^2 |H|^2$$

$$+ \left((Y_D)_{\alpha i}\, \bar{L}_\alpha\, \tilde{H}\, \chi_{iR} + (Y_L)_{\alpha i}\, \overline{L^c}_\alpha\, H\, \chi_{iL} + \frac{1}{2} \mu_L\, \overline{\psi^c_{1L}}\, \psi_{1L} + \frac{1}{2} \mu_R\, \overline{\psi^c_{1R}}\, \psi_{1R} + h.c \right)$$

Model

- $\mathcal{L} \supset \left(\sum_{i=1}^{8} y_i \bar{\psi}_i \bar{\Phi} \chi + \sum_{i=9}^{16} y_i \bar{\psi}_i \bar{\tilde{\Phi}} \chi + \sum_{i=2}^{N_\psi+1} M \bar{\psi}_{iL} \psi_{iR} + h.c. \right) - \frac{1}{4} G^{a\mu\nu} G^{a}_{\mu\nu}$

$+ \frac{\lambda_\Phi}{4} |\Phi|^4 + \frac{\lambda_H}{4} |H|^4 + \lambda_{\Phi H} |\Phi|^2 |H|^2$

$+ \left((Y_D)_{\alpha i} \bar{L}_\alpha \tilde{H} \chi_{iR} + (Y_L)_{\alpha i} \bar{L}^c_\alpha H \chi_{iL} + \frac{1}{2} \mu_L \bar{\psi}^c_{1L} \psi_{1L} + \frac{1}{2} \mu_R \bar{\psi}^c_{1R} \psi_{1R} + h.c \right)$

$\Rightarrow$ *Inflation*

Model

- $\mathcal{L} \supset \left(\sum_{i=1}^{8} y_i \, \bar{\Psi}_i \bar{\Phi} \chi + \sum_{i=9}^{16} y_i \, \bar{\Psi}_i \bar{\tilde{\Phi}} \chi + \sum_{i=2}^{N_F+1} M \, \overline{\Psi_{iL}} \, \Psi_{iR} + h.c. \right) - \frac{1}{4} G^{a\mu\nu} G^{a}_{\mu\nu}$

$$+ \frac{\lambda_\Phi}{4} |\Phi|^4 + \frac{\lambda_H}{4} |H|^4 + \lambda_{\Phi H} |\Phi|^2 |H|^2$$

$$+ \left((Y_D)_{\alpha i} \, \bar{L}_\alpha \tilde{H} \chi_{iR} + (Y_L)_{\alpha i} \, \bar{L}^c_\alpha H \chi_{iL} + \frac{1}{2} \mu_L \overline{\Psi^c_{1L}} \Psi_{1L} + \frac{1}{2} \mu_R \overline{\Psi^c_{1R}} \Psi_{1R} + h.c \right)$$

$\Rightarrow$ **Inflation**

$\Rightarrow$ **Freeze-in Dark Matter**

Model

- $\mathcal{L} \supset \left(\sum_{i=1}^{8} y_i \bar{\psi}_i \bar{\Phi} \chi + \sum_{i=9}^{16} y_i \bar{\psi}_i \tilde{\bar{\Phi}} \chi + \sum_{i=2}^{N_f+1} M \overline{\psi_{iL}} \psi_{iR} + h.c. \right) - \frac{1}{4} G^{a\mu\nu} G^{a}_{\mu\nu}$

$+ \frac{\lambda_\Phi}{4} |\Phi|^4 + \frac{\lambda_H}{4} |H|^4 + \lambda_{\Phi H} |\Phi|^2 |H|^2$

$+\left((Y_D)_{\alpha i} \bar{L}_\alpha \tilde{H} \chi_{iR} + (Y_L)_{\alpha i} \bar{L}^c_\alpha H \chi_{iL} + \frac{1}{2} \mu_L \overline{\psi^c_{iL}} \psi_{iL} + \frac{1}{2} \mu_R \overline{\psi^c_{1R}} \psi_{1R} + h.c. \right)$

$\Rightarrow$ Inflation

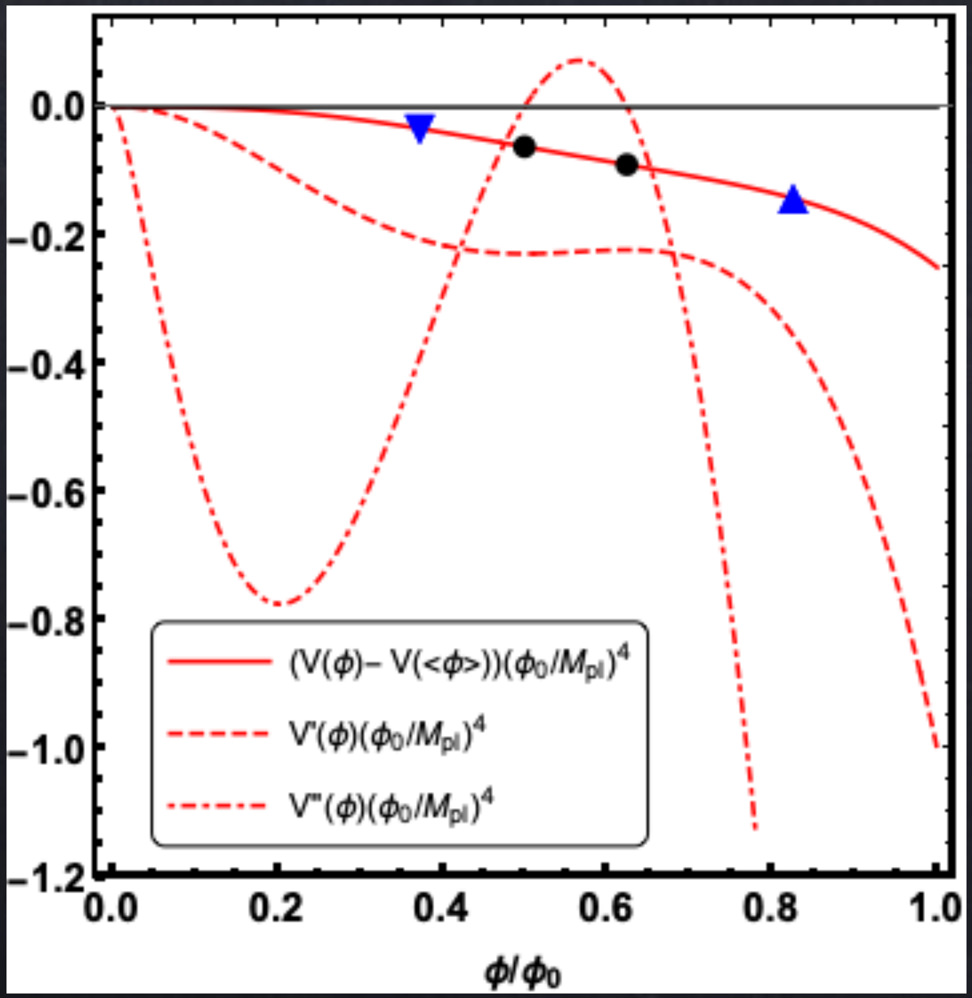
$\Rightarrow$ Freeze-in Dark Matter

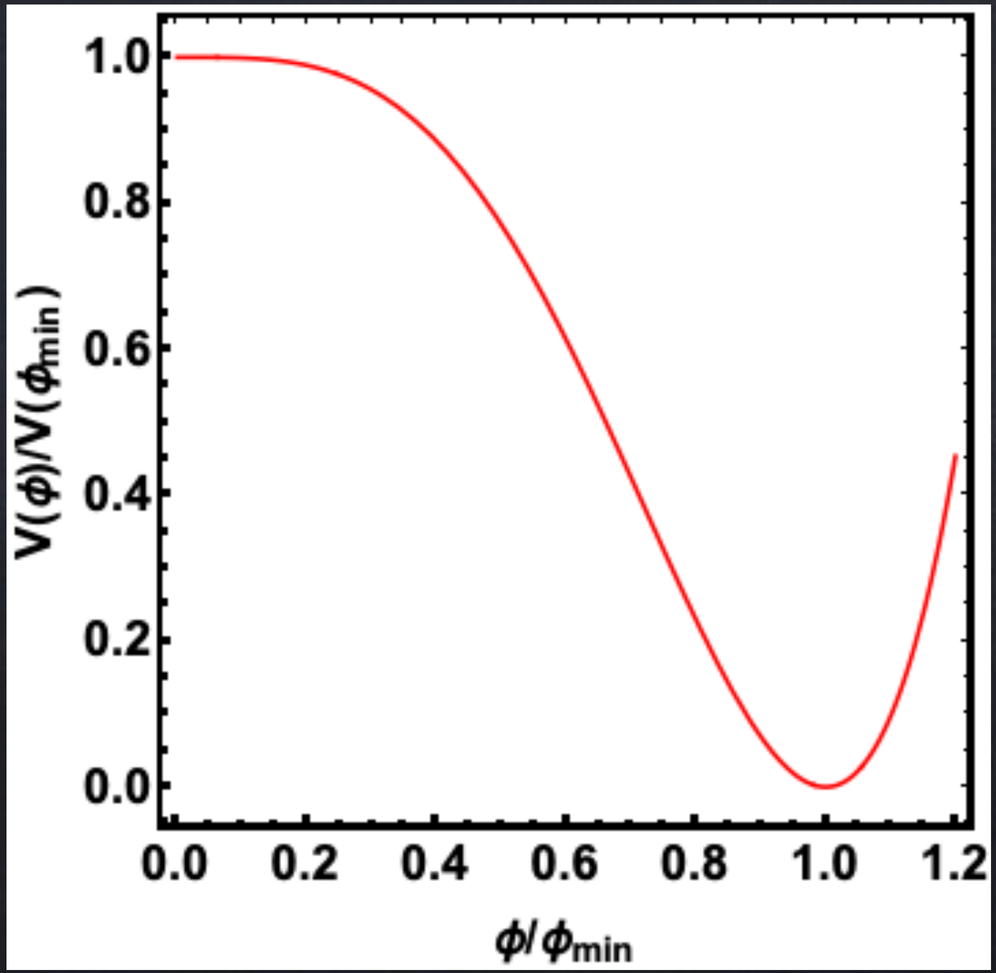
$\Rightarrow$ Inverse See-Saw Neutrino Mass & Leptogenesis

Inflation

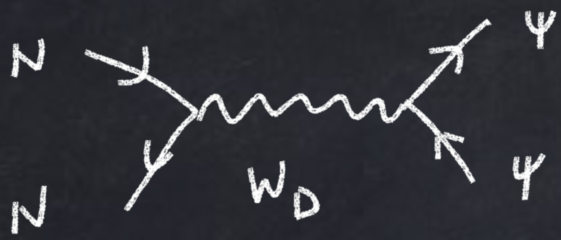
- The potential

$$V(\phi) = -\frac{a}{4}\left(1 + b\log^2(\phi/\phi_0) + c\log^2(\phi/M)\,\theta(\phi - M)\right) + aV_0$$



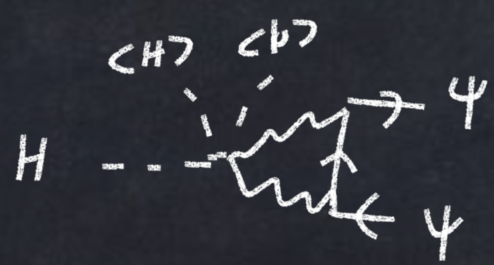


Freeze-in Dark Matter



$T > T_{EWSB}$

$T < T_{EWSB}$



Inverse See-Saw Neutrino Mass & Leptogenesis

▣ Inverse See-saw Neutrino Mass

Mass Matrix

$$M_f = \begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M_X \\ 0 & M_X & \mu \end{pmatrix}$$

$m_D \dashrightarrow Y_D \langle H \rangle \oplus Y_L \langle H \rangle$

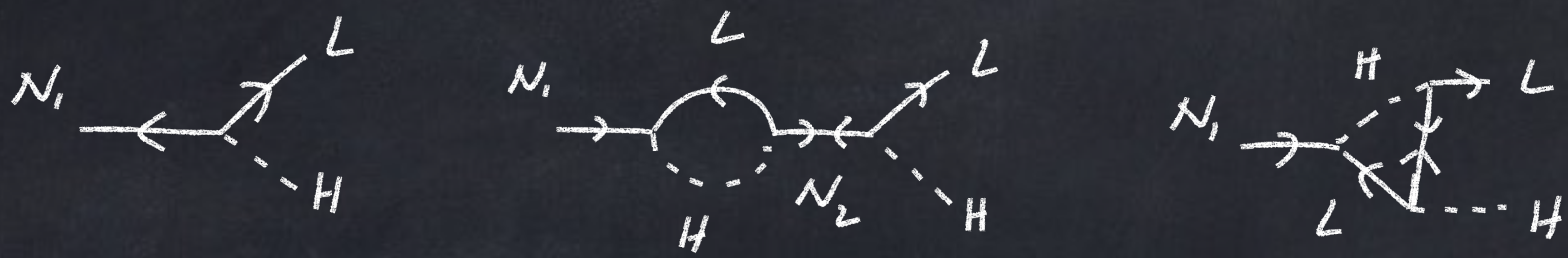
$M_X \dashrightarrow y_i \langle \Phi \rangle$

$\mu \dashrightarrow \mu_L \oplus \mu_R$

$$M_\nu \approx \frac{m^2 \mu}{M^2}$$

Inverse See-Saw Neutrino Mass & Leptogenesis

Leptogenesis



Boltzmann Equation

- The coupled Boltzmann Equation for the reheating

$$\dot{\rho}_\phi = -3\mathcal{H}\rho_\phi - (\Gamma_{\phi\to NN} + \Gamma_{\phi\to\psi\psi})\rho_\phi$$

$$\dot{\rho}_N = -(3+\theta(3T-M_N))\mathcal{H}\rho_N + \Gamma_{\phi\to NN}\rho_\phi - \Gamma_N\rho_N$$

$$\dot{n}_{B-L} = -3\mathcal{H}n_{B-L} + \epsilon\frac{\Gamma_N}{\langle E_N\rangle}(\rho_N - \rho_N^{eq}) - \Gamma_{ID}n_{B-L}$$

$$\dot{\rho}_R = -4\mathcal{H}\rho_R + \Gamma_N\rho_N$$

$$\dot{\rho}_\psi = -(3+\theta(3T-M_\psi))\mathcal{H}\rho_\psi + 2\frac{\langle\sigma v\rangle}{\langle E_N\rangle}\rho_N^2 + \Gamma_{\phi\to\psi\psi}\rho_\phi$$

Boltzmann Equation

- Now re-writing the above coupled differential equation

$$\tilde{\rho}_\phi = \rho_\phi a^3 ; \quad \tilde{\rho}_N = \rho_N a^3 ; \quad \tilde{N}_{B-L} = n_{B-L} a^3 ; \quad \tilde{\rho}_\psi = \rho_\psi a^3 ; \quad \tilde{\rho}_R = \rho_R a^4$$

& $\xi = a/a_I$; $\langle \bar{E}_f \rangle = \rho_f^{eq}/n_f^{eq}$; $T = \left(\dfrac{30 \, \rho_R}{\pi^2 g_*(T)} \right)^{1/4}$

$$\mathcal{H} = \left[\frac{8\pi}{3 M_{Pl}^2} \frac{\tilde{\rho}_\phi a_I \xi + \tilde{\rho}_N a_I \xi + \tilde{\rho}_R}{a_I^4 \xi^4} \right]^{1/2}$$

Boltzmann Equation

- Re-write the above boltzmann equation

$$\tilde{\rho}_\phi' = \frac{-1}{\mathcal{H}\xi}\left(\Gamma_{\phi\to NN} + \Gamma_{\phi\to\psi\psi}\right)\tilde{\rho}_\phi$$

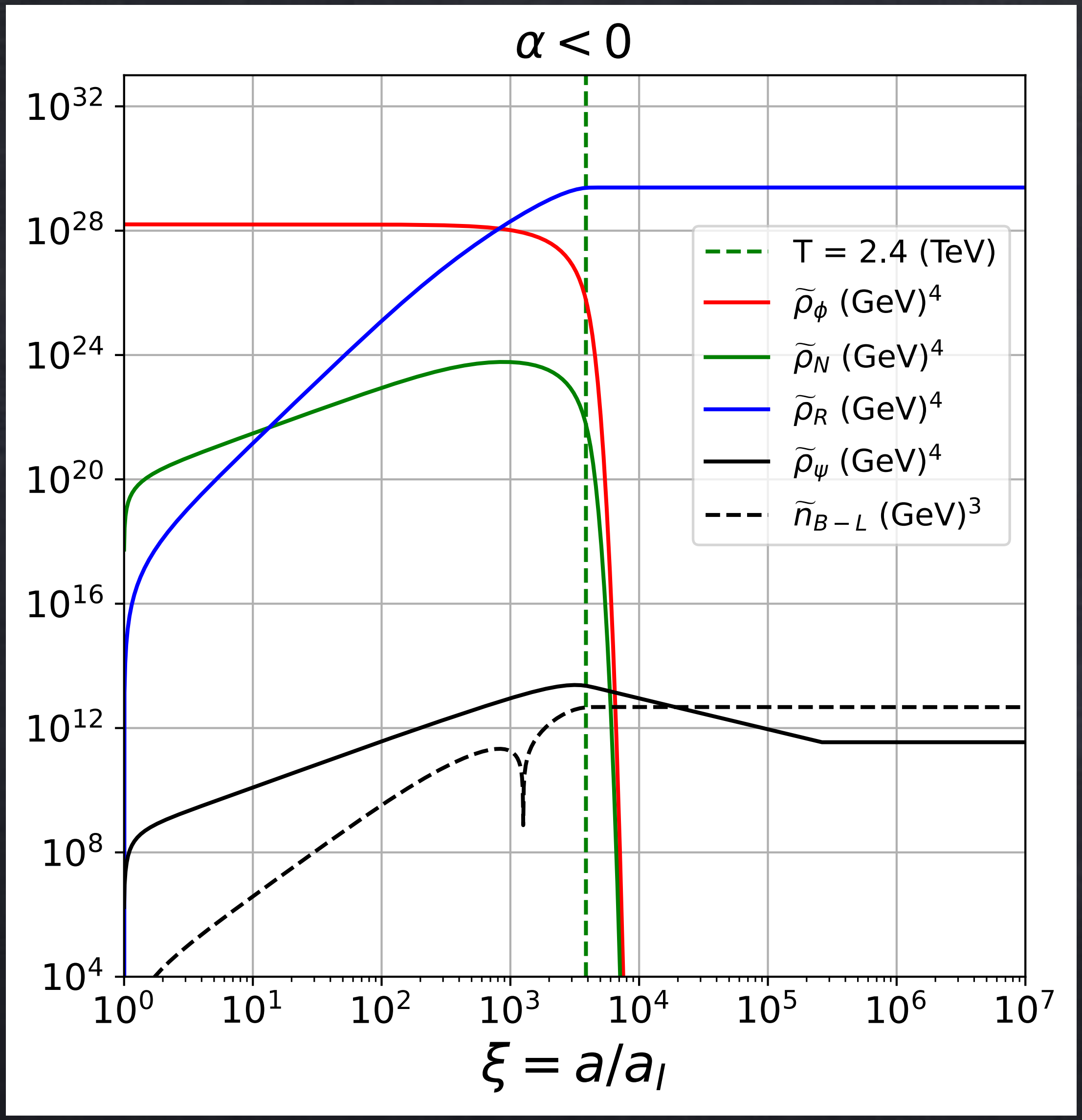
$$\tilde{\rho}_N' = -\frac{\theta(3T - M_N)}{\xi}\tilde{\rho}_N + \frac{1}{\mathcal{H}\xi}\left(\Gamma_{\phi\to NN}\tilde{\rho}_\phi - \Gamma_N\tilde{\rho}_N\right)$$

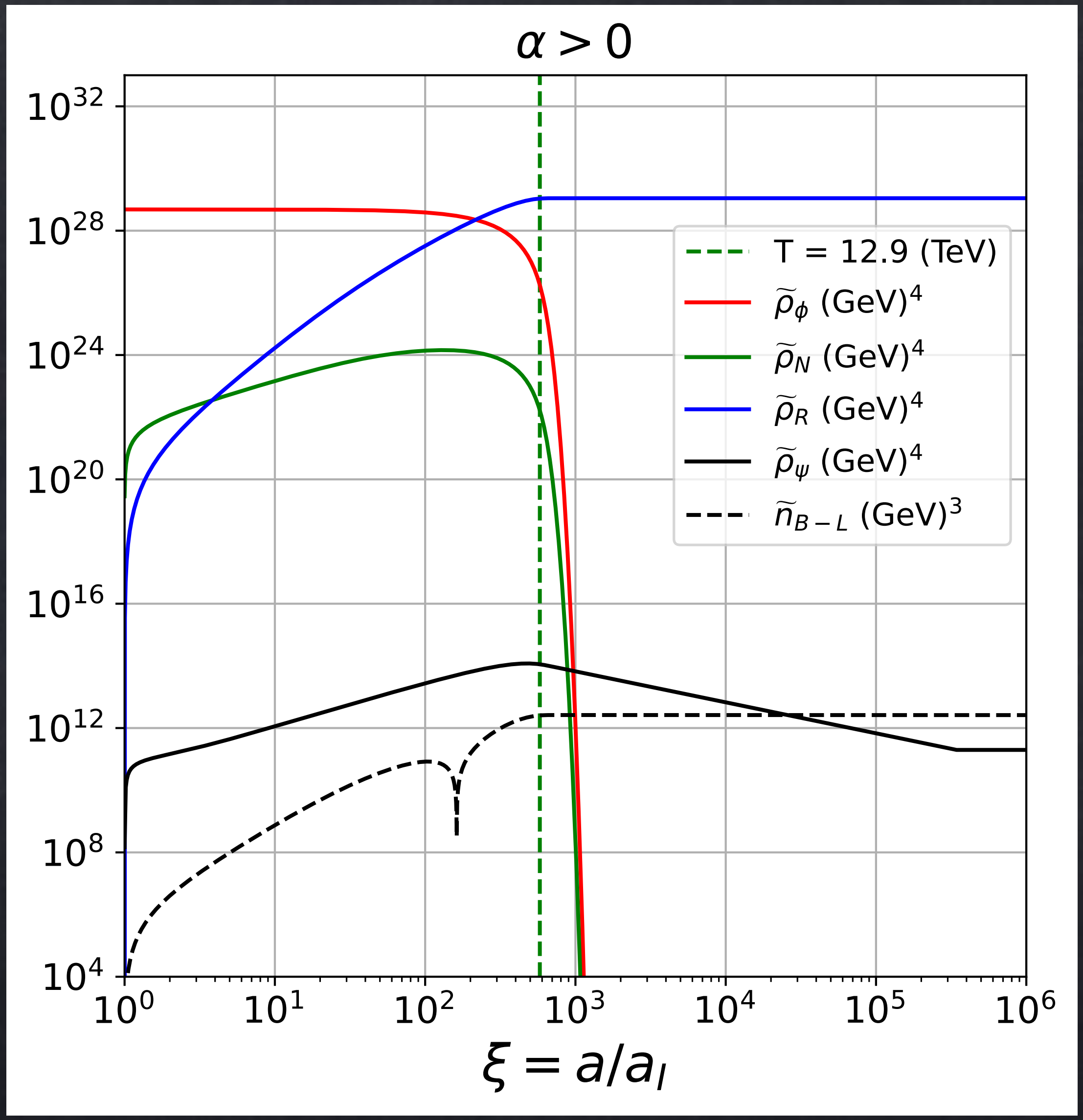
$$\tilde{N}_{B-L}' = \frac{1}{\mathcal{H}\xi}\left(\epsilon\frac{\Gamma_N}{\langle \bar{E}_N\rangle}\left(\rho_N - \rho_N^{eq}\right) - \Gamma_{ID}\tilde{N}_{B-L}\right)$$

$$\tilde{\rho}_R' = \frac{\Gamma_N}{\mathcal{H}}\tilde{\rho}_N$$

$$\tilde{\rho}_\psi' = -\frac{\theta(3T - M_\psi)}{\xi}\tilde{\rho}_\psi + \frac{1}{\mathcal{H}\xi}\left(2\frac{\langle\sigma v\rangle}{\langle\bar{E}_N\rangle\xi^3}\tilde{\rho}_N^2 + \Gamma_{\phi\to\psi\psi}\tilde{\rho}_\phi\right)$$

Bench Mark Points





Bench Mark Points

	BP1	BP2
M/ϕ_0	6.46	6.37
ϕ_0/M_{Pl}	1.39×10^{-18}	8.85×10^{-19}
ϕ_i (GeV)	14.	4.04
λ_0	1.94×10^{-14}	6.88×10^{-13}
m_ϕ (TeV)	10	33.1
$M=m_\psi$ (GeV)	110	70
M_χ (GeV)	1.1	2.7
T_{reheat} (GeV)	2.4	12.9
α_s	-8.54×10^{-4}	2.348×10^{-3}
γ	4.6×10^{-45}	1.3×10^{-44}

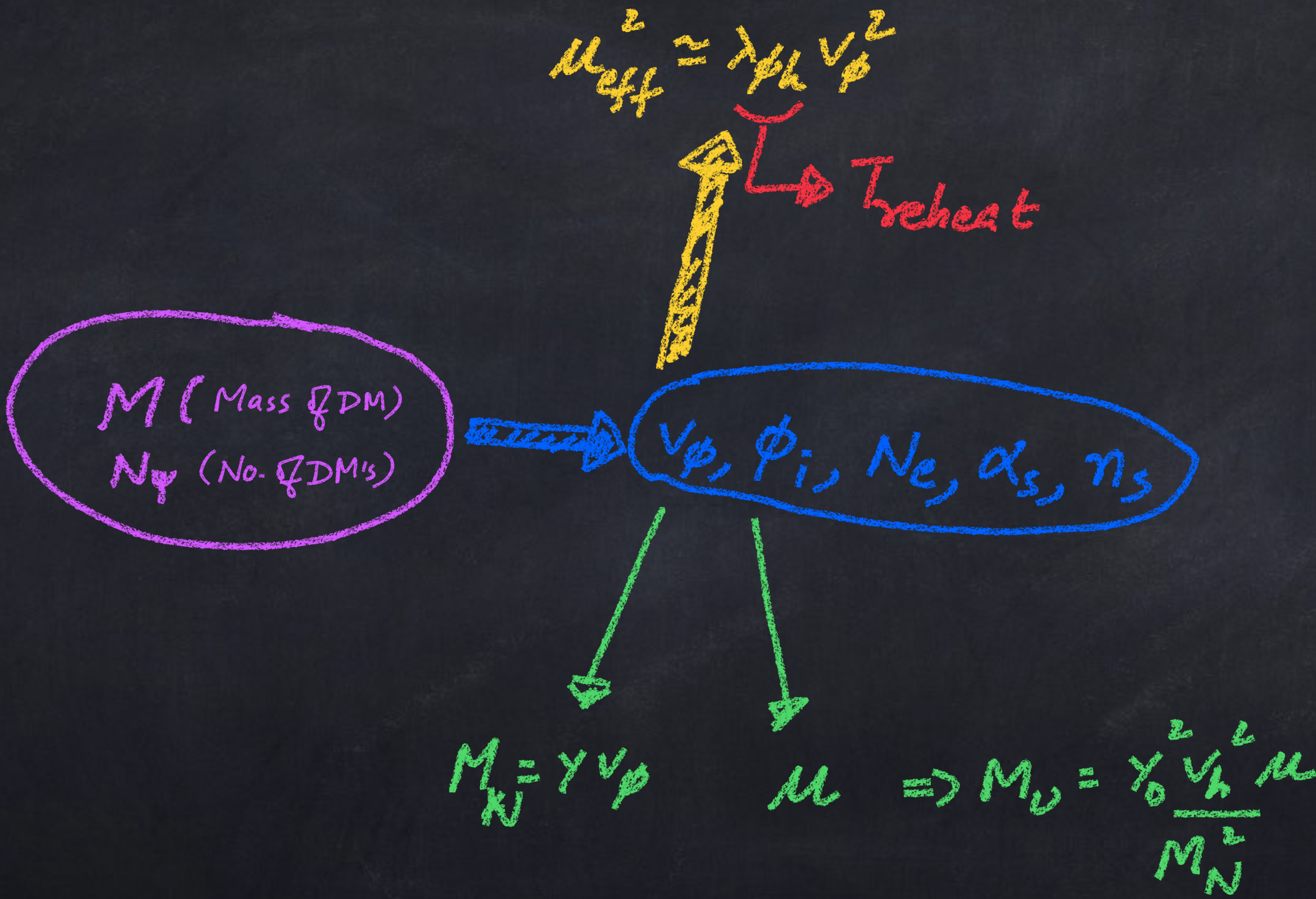
$n_s = 0.9691$

$N_\psi = 13$

$\epsilon = 4.5\times10^{-10}$

$N_e = 60$

Schematic Depedence



Conclusion

- We have shown a minimal extension of Standard Model (SM) wherein we address
  - Inflation (Small field)
  - **Dark Matter**
  - Neutrino Mass & Leptogenesis

- In this minimal setup the only two parameters are
  - a) Number of DM Candidate
  - b) Mass of the DM

Thank You

Backup slides

Particle Physics Paradigm

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Feild theoretic Realization

Taking $SU(N_c)$ gauge theory

$$\mathcal{L} = -\frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + (D_\mu \Phi)^\dagger (D_\mu \Phi) + i\, \bar{\chi} \not{D} \chi + i\, \bar{\Psi} \not{D} \Psi - \sum_{i=N_{IR}+1}^{N_{UV}} M \bar{\Psi}_i \Psi_i$$

$$-\frac{\lambda}{4} (\Phi^\dagger \Phi)^2 - (\Phi\, \bar{\Psi}\, Y \chi + h.c.)$$

$$\kappa = \frac{1}{16\pi^2}$$

$$\beta_g = -\kappa g^3 \left(\frac{11}{3} N_c - \frac{1}{6} - \frac{2 n_f}{3} \right)$$

$$\beta_Y = \kappa \left(\frac{3}{2} Y Y^\dagger Y + Y\, \mathrm{tr}(Y^\dagger Y) - 3\, \frac{N_c^2 - 1}{2N_c} g^2 Y \right)$$

$$\beta_\lambda = \kappa \left(\frac{3(N_c-1)(N_c^2+2N_c-2)}{4N_c^2} g^4 - 2\,\mathrm{tr}(Y^\dagger Y Y^\dagger Y) - 6\, \frac{(N_c^2-1)}{N_c} \lambda g^2 + 4 \lambda\, \mathrm{tr}(Y^\dagger Y) + 4(N_c+4)\lambda^2 \right)$$

Particle Physics Paradigm

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- Assuming $Y^4 \equiv \mathrm{tr}(Y^\dagger Y Y^\dagger Y) = \frac{3(N_c-1)(N_c^2+2N_c-2)}{8N_c^2} g^4$

$$g(t) = g_0 - K g_0^3 \left[\left(\frac{11}{3} N_c - \frac{2}{3} N_{IR} - \frac{1}{6} \right) t + \frac{2}{3} (N_{IR} - N_{UV})(t - t_M)\,\Theta(t - t_M) \right]$$

$$Y(t) = Y_0 + K t \left[\frac{3}{2} Y_0 Y_0^\dagger Y_0 + Y_0\, \mathrm{tr}(Y_0^\dagger Y_0) - 3\, \frac{N_c^2-1}{2N_c} g_0^2 Y_0 \right]$$

$$\lambda(t) = \lambda_0 - K^2 t^2 \left[g_0^6\, \frac{3(N_c-1)(N_c^2+2N_c-2)}{2N_c^2} \left(\frac{11}{3} N_c - \frac{2}{3} N_{IR} - \frac{1}{6} \right) + 4\, \mathrm{tr}(Y_0^\dagger Y_0 Y_0^\dagger Y_0)\, \mathrm{tr}(Y_0^\dagger Y_0) \right.$$

$$\left. + 6\, \mathrm{tr}(Y_0^\dagger Y_0 Y_0^\dagger Y_0 Y_0^\dagger Y_0) - 6\, \frac{N_c^2-1}{N_c} g_0^2\, \mathrm{tr}(Y_0^\dagger Y_0 Y_0^\dagger Y_0) \right]$$

$$+ \frac{(N_c-1)(N_c^2+2N_c-2)}{N_c^2} K^2 g_0^6 (N_{UV} - N_{IR})(t - t_M)^2\, \Theta(t - t_M)$$

Particle Physics Paradigm

2008.09639

$$V(\phi) = \frac{\lambda_0}{4}\phi^4\Bigg[1 - \frac{k^2 t^2}{\lambda_0}\Bigg[g_0^6\,\frac{3(N_c-1)(N_c^2+2N_c-2)}{2N_c^2}\left(\frac{11}{3}N_c - \frac{2}{3}N_{IR} - \frac{1}{6}\right)$$

$$+4\,\mathrm{tr}(Y_0^\dagger Y_0 Y_0^\dagger Y_0)\,\mathrm{tr}(Y_0^\dagger Y_0) + 6\,\mathrm{tr}(Y_0^\dagger Y_0 Y_0^\dagger Y_0 Y_0^\dagger Y_0)$$

$$-6\,\frac{N_c^2-1}{N_c}\,g_0^2\,\mathrm{tr}(Y_0^\dagger Y_0 Y_0^\dagger Y_0)\Bigg]$$

$$+\frac{(N_c-1)(N_c^2+2N_c-2)}{N_c^2}\,\frac{k^2 g_0^6}{\lambda_0}(N_{UV}-N_{IR})(t-t_M)^2\,\Theta(t-t_M) + V_0$$

$$= \frac{-a}{4}\phi^4\left[1 + b\ln^2\left(\frac{\phi}{\phi_0}\right) - c\ln^2\left(\frac{\phi}{M}\right)\Theta(\phi-M)\right] + a\tilde{V}_0$$

Particle Physics Paradigm

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- Imposing restrictions on this potential
  1) The potential does not develop any local minima so that the inflaton rolls smoothly towards the global minimum:
  $$V'(\phi) < 0 \text{ for } \phi < \phi_{min}$$
  2) Some field value $\phi \sim \phi_0 < M$ the potential develops an inflection point: $V''(\phi) = 0$

$\Rightarrow$ $b < 16$ ($V'(\phi) < 0$ for $\phi < \phi_{min}$)

$b \geqslant 144/25$ (existence of inflection points for $\phi < M$)
~ 5.7

Particle Physics Paradigm

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- The global minimum of the potential ϕ_{min}

$$\ln\left(\frac{\phi_{min}}{M}\right) = \frac{-1}{4} + \frac{4b\ln(M/\phi_0) + \sqrt{(c-b)(16+c-b)+16bc\ln^2(M/\phi_0)}}{4(c-b)}$$

$$\text{for } \phi_{min} > M$$