Probing the origin of primordial black holes through novel GW spectrum*

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Beyond Standard Model: From Theory to Experiment
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* Based on I. K. Banerjee, UKD, JCAP 07 (2023) 024

Outline

- Introduction
- Phase transition as origin of GW and PBH
- Interactions: PBH-PBH & PBH-ABH
- Results
- Summary and Conclusion

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- Primordial Black Holes (PBH) were formed in the early Universe through non-stellar way
- First proposed in 60's by Zel'dovich & Novikov and later refined by Hawking and Carr in 70's
- Discovery of gravitational waves originated from the black hole mergers by the Laser Interferometer Gravitational-Wave Observatory (LIGO) and Virgo resurrected new interests in PBH
- PBHs can be viable dark matter candidate making up some percentages of the total amount of DM
- Not only DM but PBHs can also explain many of the microlensing observations; masses, spins and coalescence rates for the black holes found by LIGO/Virgo, non-observation of ultrafaint dwarf galaxies below some critical radius etc.
- Many formation mechanisms: collapse from inhomogeneities during radiation- and matter-dominated era, critical collapse, collapse in single and multi-field inflationary models, collapse of cosmic string loops, collapse during a first order phase transition (FOPT)

- We will focus on the FOPT origin of PBHs
- Moreover, FOPTs are famous for generating gravitational waves detectable in future GW detectors
- But, PBHs themselves can be an originator of GW
- We will consider the combined GW spectrum, sourced from (FOPT + PBH)
- In doing so, we try to examine the possibility of exploring the class of PBHs which were produced in a FOPT by means of characteristic GW spectrum

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Lightning review of FOPT

- During the evolution of the Universe, it underwent several phase transitions

- Usually they are characterised by the potential of the order parameter that drives the transition

- Depending on the shape of the potential the transition can be first order or second order

- If it has a barrier between the true vacuum (newly generated global minima) and the false vacuum (pre-existing yet local minima), then it is a first order phase transition, otherwise, it is a second order

- The temperature, below which the pre-existing minima becomes a local minima and a new global minima is created, is called the critical temperature

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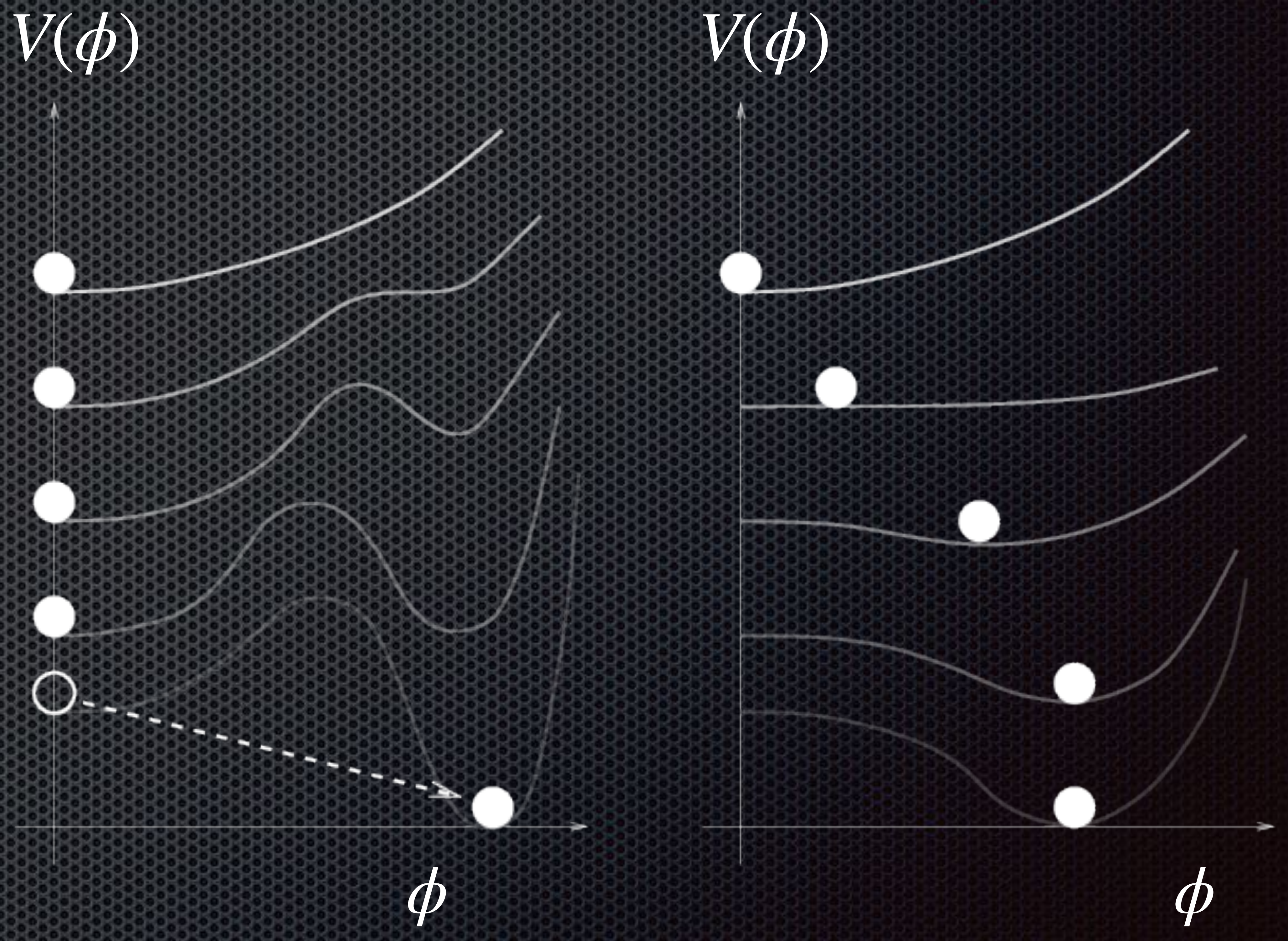

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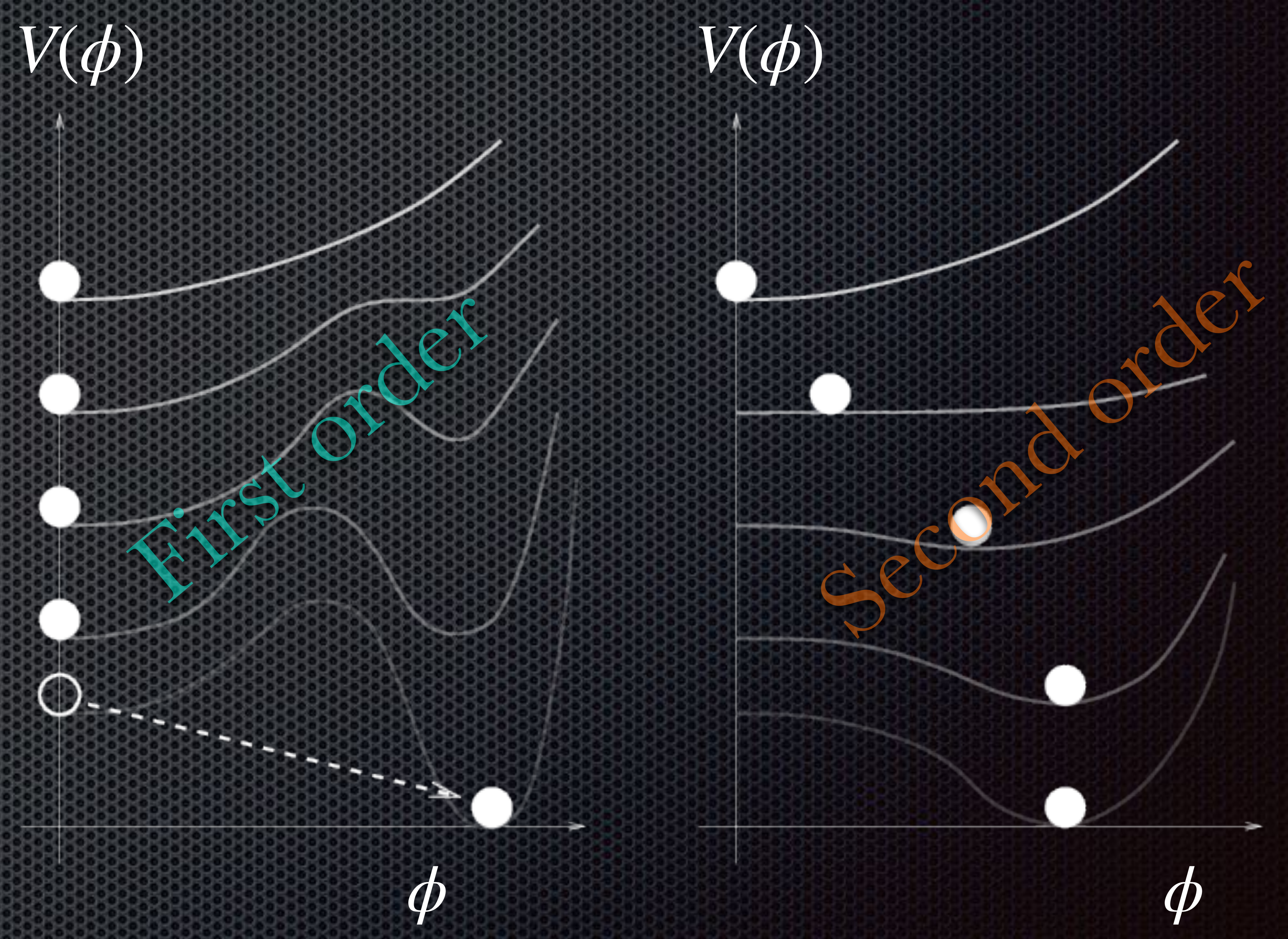
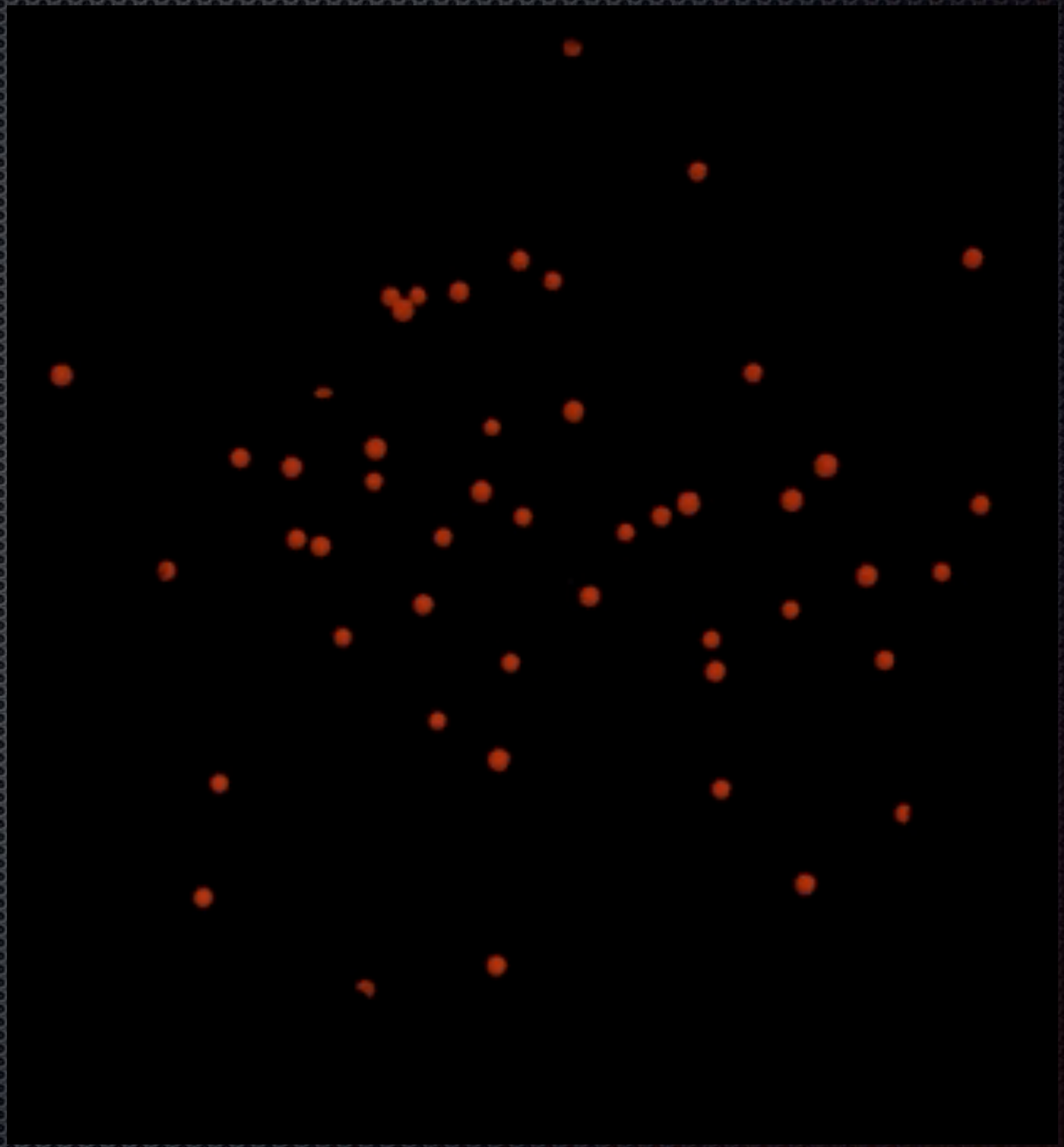


Lightning review of FOPT

- A first order phase transition occurs through the nucleation of true vacuum bubbles of the scalar field which is driving the transition
- The bubbles then expand releasing energy to the bubble walls and the surrounding plasma
- More technically, this excites the plasma and creates a 'sound shell' around the wall of plasma moving with non-zero outward radial velocity
- Various fluid-dynamical quantities are estimated using various approximations



(Credit: James Mertens YT Channel @jbmertens)

Generating GW from FOPT

- Production of gravitational waves at a first-order phase transition can be separated into three stages:

  (i) Initial collision of the scalar field shells i.e., the collision of the bubble walls; motion of the bubble walls as a propagation of spherical, infinitely thin shapes (envelope approximation) (Ω_{env})

  (ii) After the bubble merging, the wave of fluid kinetic energy in the plasma continues to propagate outwards into the broken phase. As the shells of kinetic energy from different bubbles overlap, gravitational waves are produced (Ω_{sw})

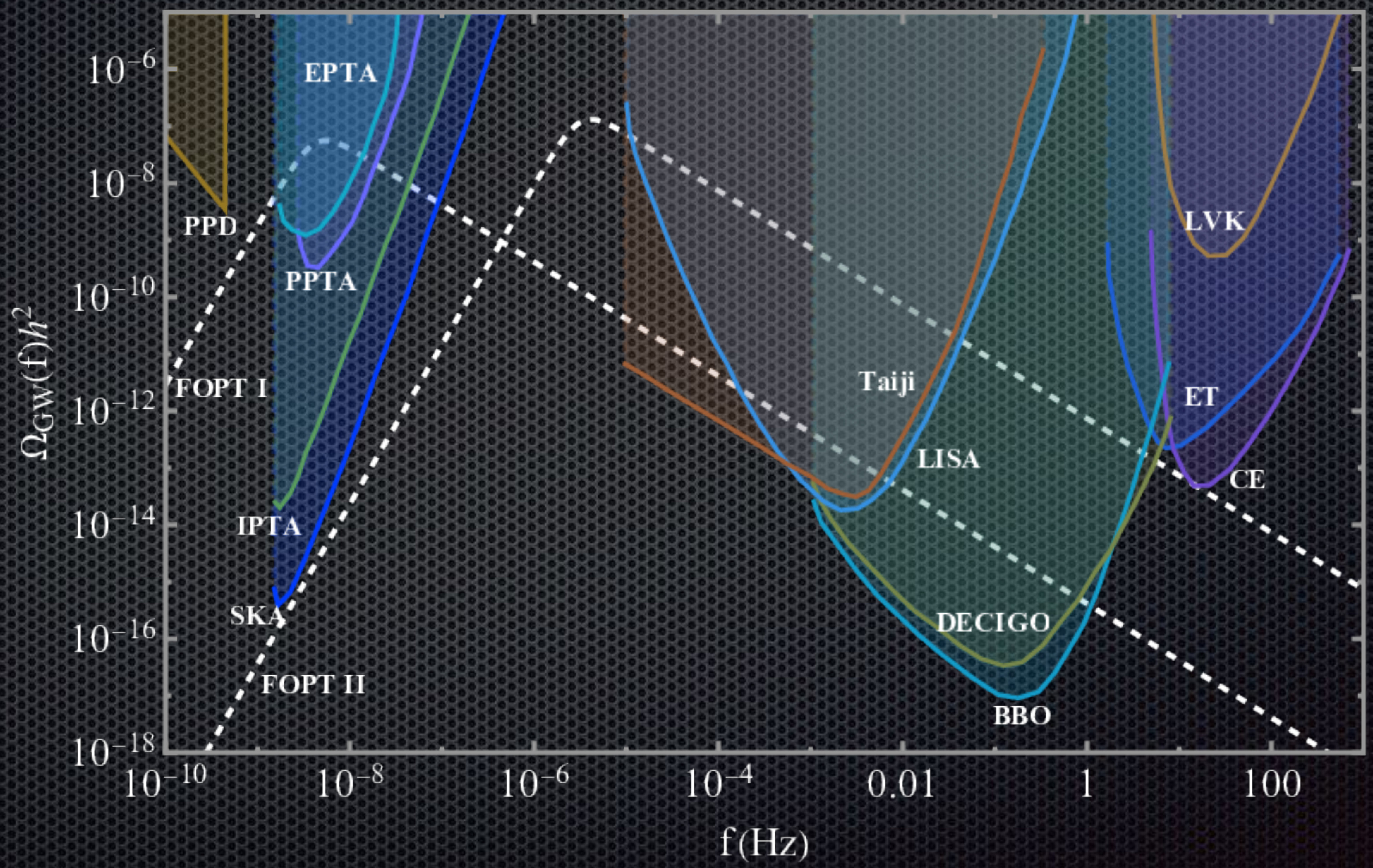
  (iii) Acoustic phase may give way to shocks and a turbulent regime ⇒ MHD turbulence, which generates GWs through the anisotropic stresses of the chaotic fluid motions and of the magnetic field (Ω_{mhd})

- In our case it is the bubble collision which is going to source the GW dominantly

Generating GW from FOPT

- The SGWB spectrum due to FOPT can be expressed as,

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln f} = 1.67\times 10^{-5}\left(\frac{H}{\beta}\right)^2\left(\frac{\kappa\alpha}{1+\alpha}\right)^2\frac{0.11v_w^2}{0.42+v_w^3}\left(\frac{100}{g_*}\right)^{1/3}\frac{3.8(f/f_p)^{2.8}}{1+2.8(f/f_p)^{3.8}}$$



Generating PBH from FOPT

- As the universe evolves, before a phase transition reaches the critical temperature, the average fraction of false vacuum is unity

- With the progress of the transition, the average fraction of false vacuum reduces depending of the specific dynamics of the phase transition

- Nucleation of true vacuum bubble is a probabilistic process, in some Hubble volume, the creation of a true vacuum bubble might be delayed

- Time passes and the universe expands, the radiation and the wall energy density in the false vacuum region falls as $a^{-4}(t)$ though the vacuum energy density remains constant

- In the *false vacuum* → vacuum energy starts decaying into the radiation and wall energy density

- In the *true vacuum* → since the vacuum energy density is much less than that of the false vacuum region, the ratio of energy density inside and outside the false vacuum region keeps increasing and reaches a maximum value when the entire vacuum energy inside the false vacuum region has been decayed

Generating PBH from FOPT

- Once it reaches a critical value the false vacuum region collapses to form a PBH

- Basically, it is the collapse of overdense regions which are a consequence of delayed nucleation of true vacuum bubbles

- Slight differences in the specifics of the creation mechanisms, i.e., the contribution of the nucleation in the past light-cone which leads to slightly different abundance of PBHs for the same parameter values

- However, mass of the PBHs are dependent on the nucleation temperature and therefore is of the same order for all the cases

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$$M(t) = \gamma \frac{4\pi}{3} \bar{\rho}(t) H^{-3}(t), \quad \text{where,} \quad H^2(t) = \bar{\rho}(t)/3M_{\mathrm{P}}^2$$

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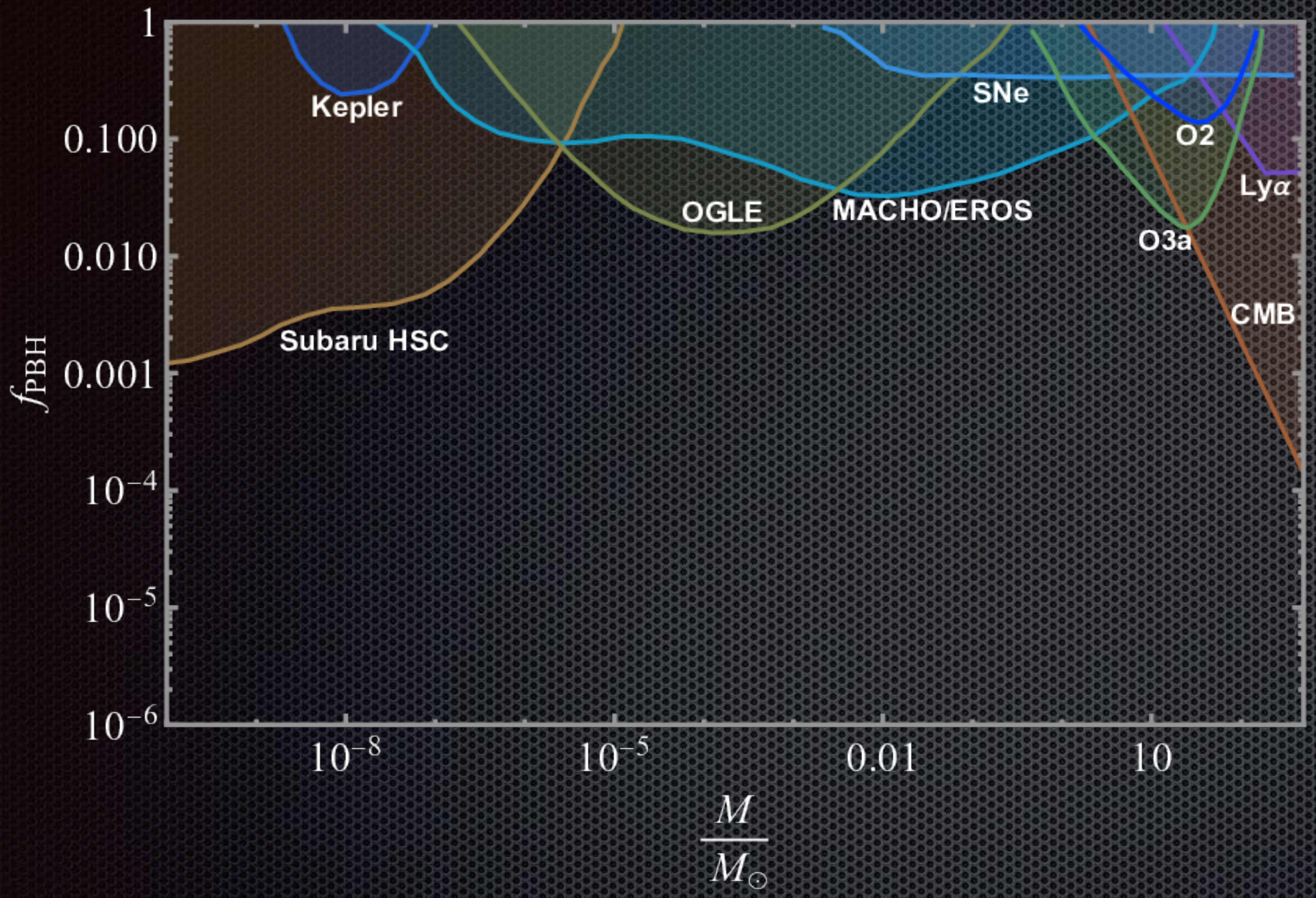
- Assuming that most of the PBHs which are formed during a FOPT have a mass, $M_{\mathrm{PBH}} = M(t_{1.45})$ where $\rho_{\mathrm{in}}(t_{1.45})/\rho_{\mathrm{out}}(t_{1.45}) = 1.45$

Constraints on PBH and Choice of Benchmark Parameters



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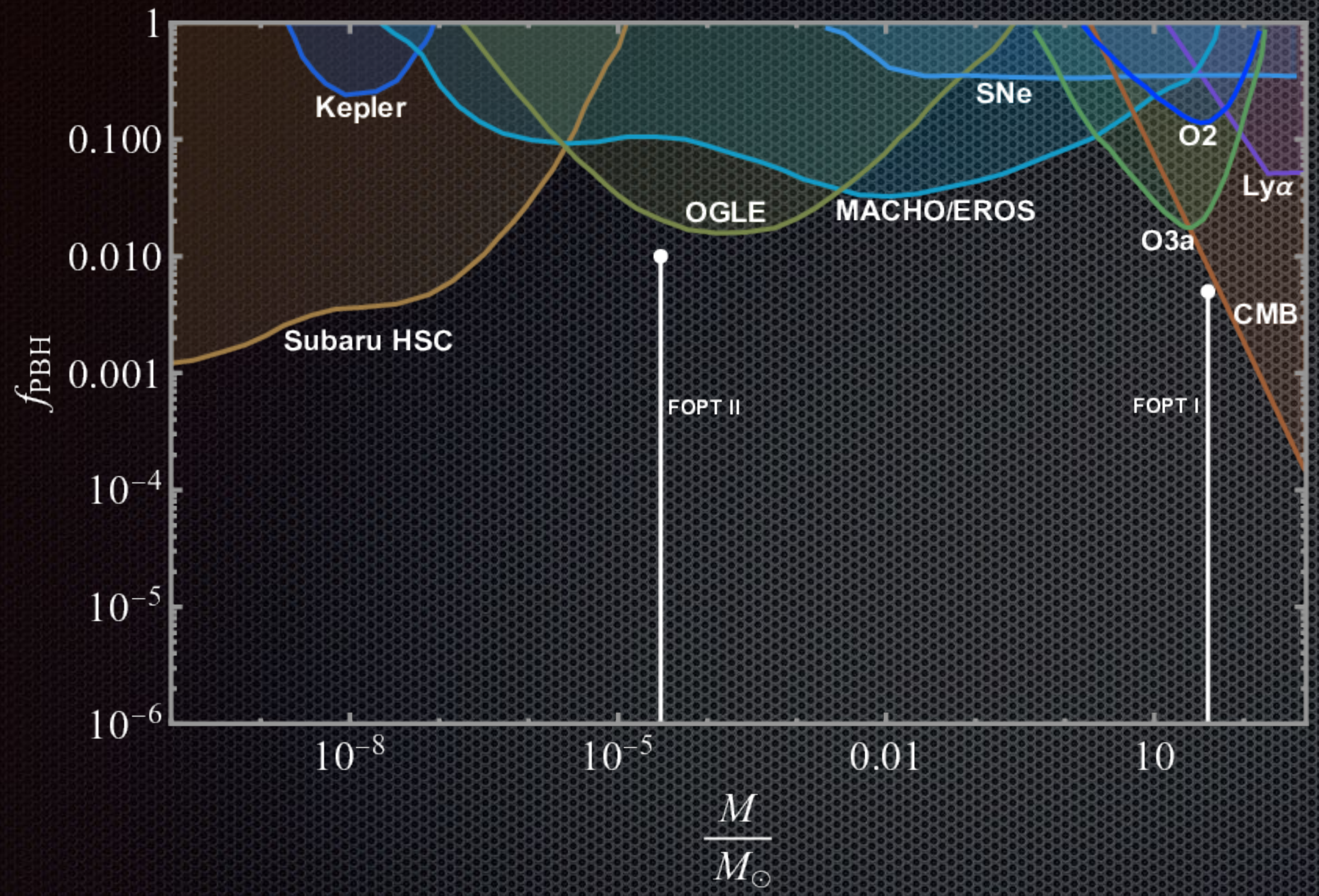

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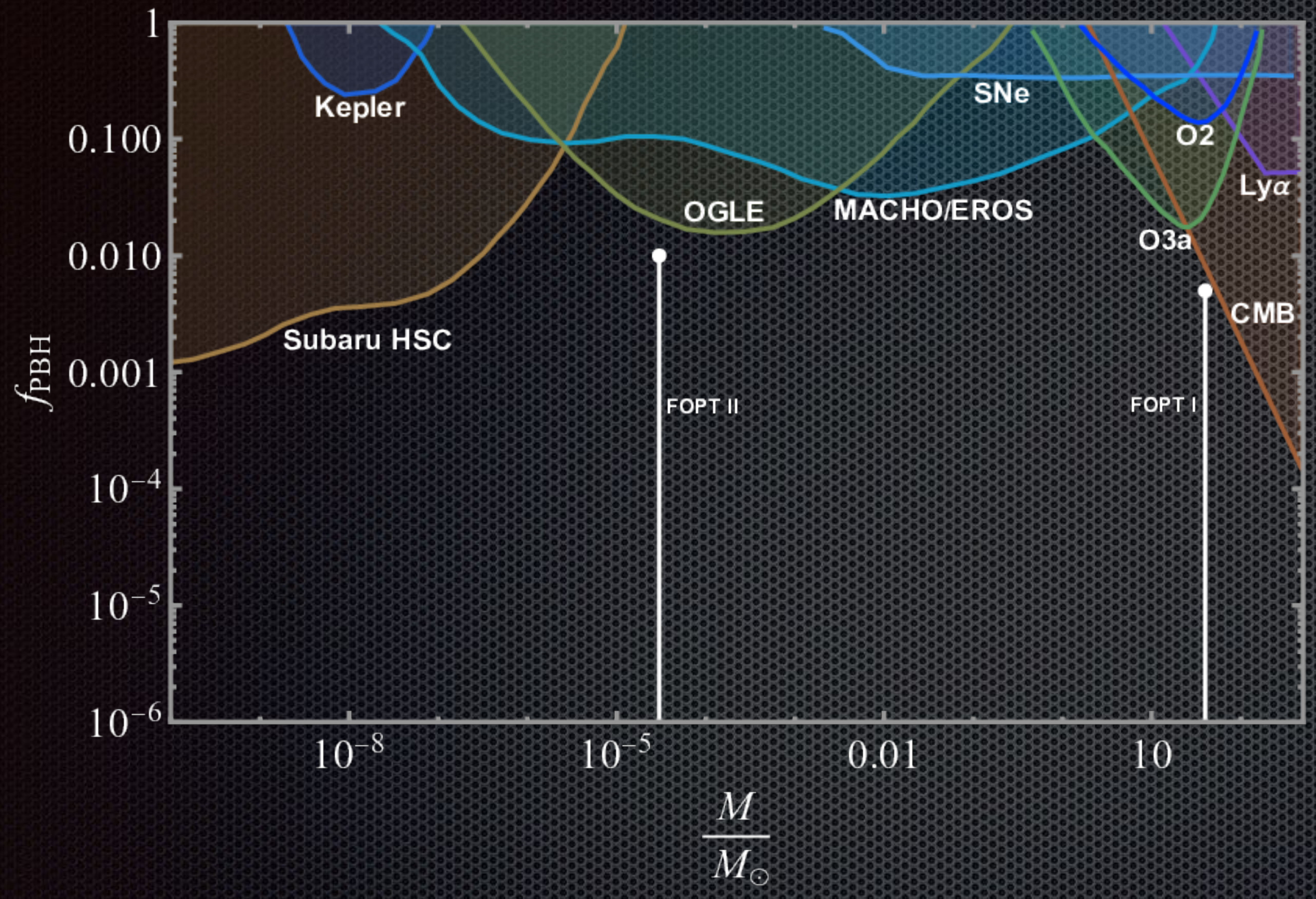

Parameters	I	II
T (in GeV)	0.06	45
α	1	1.5
β/H	3.5	2.5
κ	0.5	1

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Interactions: PBH-PBH

- After being formed during the FOPT, PBHs are free to interact among themselves
- Two black holes can form a binary BH pair by a close encounter if the energy loss ($|\Delta E|$) due to GW emission exceeds the kinetic energy

$$E_\infty = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2} v_0^2$$

- So, BBH is formed if $E_\infty \leq |\Delta E|$
- There can be an alternate situation (mostly at the inner part of the BH cluster), where the above condition is not met
- In such a case the two BHs just pass each other and such event will be called close hyperbolic encounters
- These events also generate GWs

Interactions: PBH-PBH

- After being formed during the FOPT, PBHs are free to interact among themselves
- Depending on the initial conditions two PBHs can either form a bound binary system or there could be a single scattering event
- As opposed to the SGWB spectrum from FOPT, these hese individual events can be considered to be point-like sources of GW and in that case the GW spectra has to consider the effect of redshift etc.

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln f} = \frac{1}{\rho_c}\int_0^\infty dz \frac{N(z)}{1+z}\frac{dE_{\rm GW}}{d\ln f_s}$$

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$$\frac{\tau(z)}{(1+z)H(z)}$$

$$(1+z)f$$

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$$f_s = (1+z)f$$

$$N(z) = \frac{\tau(z)}{(1+z)H(z)}$$

$$\tau(z) = \int\int \frac{dm_1}{m_1}\frac{dm_2}{m_2}\frac{d\tau}{d\ln m_1 d\ln m_2}$$

PBH-PBH: Binary System

- If the PBHs are clustered in dense halos, then their number density can be expressed as a combination of their mass function, the critical density of the universe, their masses and the local density contrast

- In case of PBH binary systems, the event rate per logarithmic mass interval is given by,

$$\frac{d\tau_{\mathrm{BBH}}}{d\ln m_1 d\ln m_2} = 14.8\ \mathrm{yr}^{-1}\mathrm{Gpc}^{-3} f(m_1) f(m_2) \frac{M^{10/7}}{(m_1 m_2)^{5/7}} \left(\frac{\Omega_{\mathrm{DM}}}{0.25}\right)^2 \left(\frac{\delta_{\mathrm{loc}}}{10^8}\right) \left(\frac{v_0}{10\ \mathrm{km/s}}\right)^{-11/7}$$

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Logarithmic mass function, $\int f(m)dm/m = f_{\rm PBH}$

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Logarithmic mass function, $\int f(m)dm/m = f_{\rm PBH}$

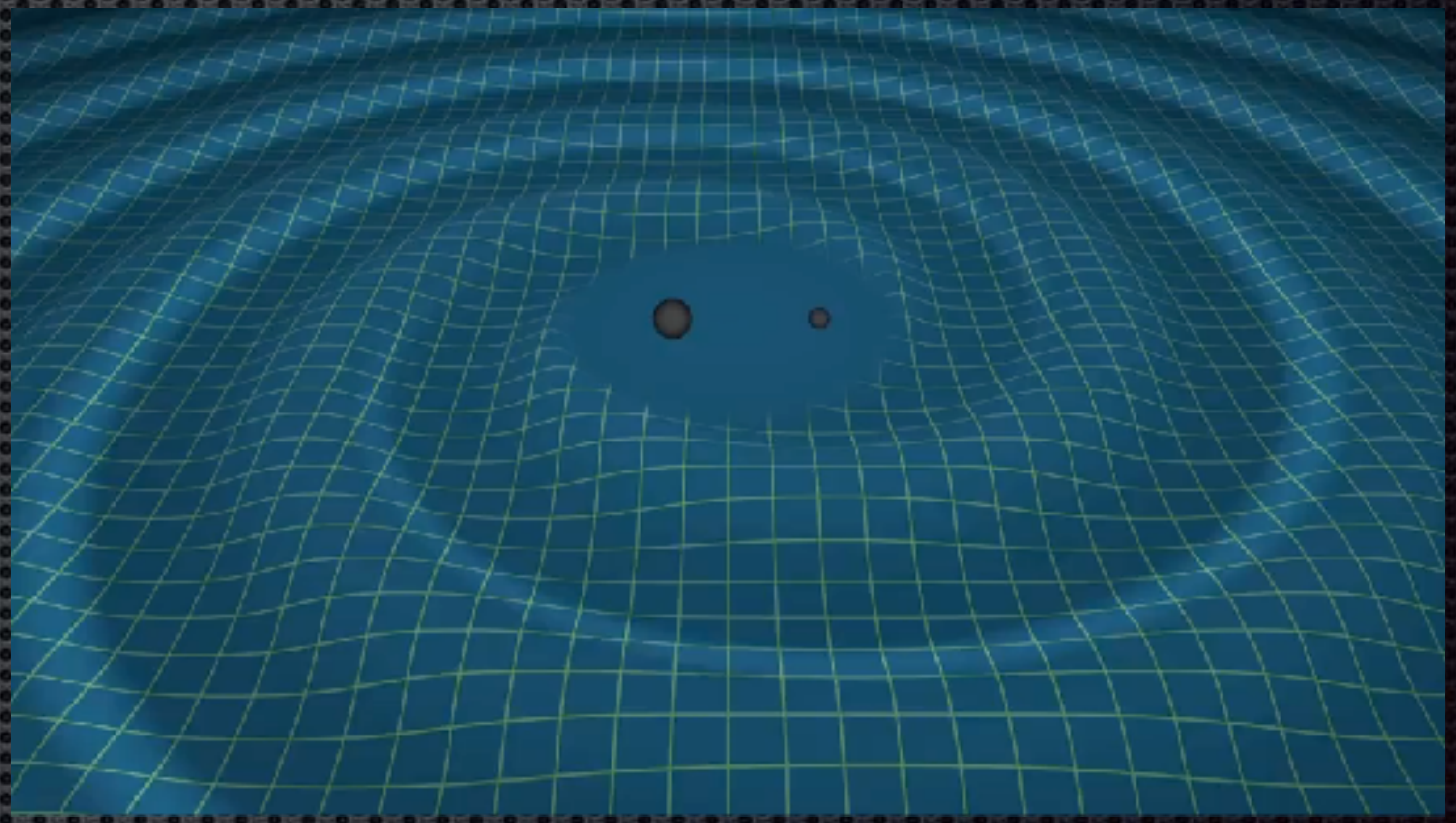
$\propto \delta(m - m')$; assume, all the PBHs are of same mass

- Now, a PBH binary contributes to the GW spectrum in three different phases, the inspiral, the merger and the ringdown

PBH-PBH: Binary System

- The energy emitted as GW per frequency interval can be given as,

$$\frac{dE_{\mathrm{GW}}}{df_s} = \frac{(G\pi)^{2/3} M_c^{5/3}}{3} \begin{cases} f_s^{-1/3}, & f_s \leq f_{\mathrm{merg}} \\ f_{\mathrm{merg}}^{-1} f_s^{2/3}, & f_{\mathrm{merg}} \leq f_s \leq f_{\mathrm{ring}} \\ f_{\mathrm{merg}}^{-1} f_{\mathrm{ring}}^{-4/3} f_s^{2} \left[\left(\frac{f_s - f_{\mathrm{ring}}}{\sigma/2} \right)^2 + 1 \right]^{-2} & , f_{\mathrm{ring}} \leq f_s \leq f_{\mathrm{cut}} \\ 0, & f_s > f_{\mathrm{cut}} \end{cases}$$



(Credit: YT channel @georgiatech)

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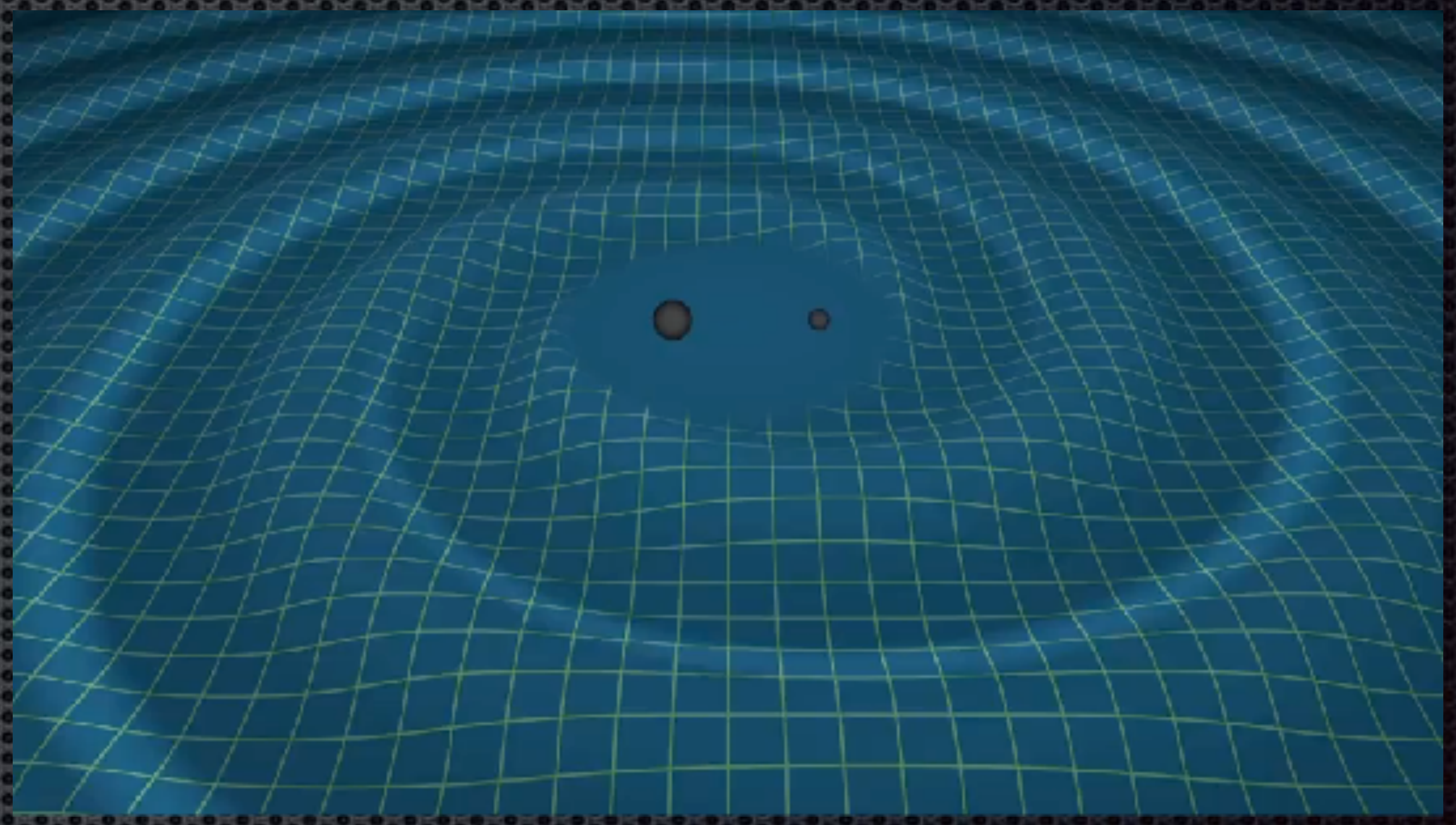

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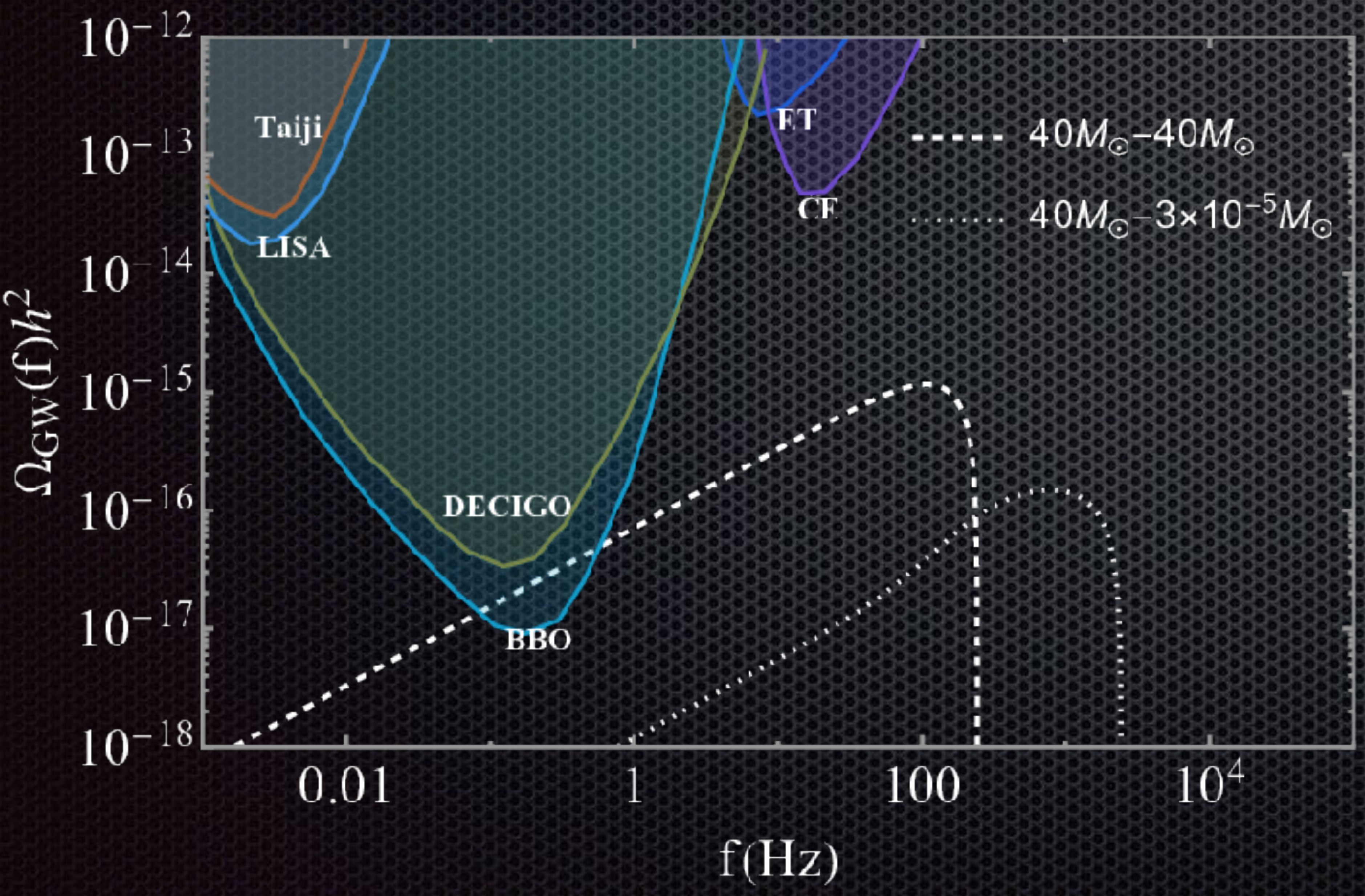


- Though inspiral, merger and ringdown, all three of them contribute to the GW spectrum, almost all of the contributions come from the inspiral phase
- Also, $f_{\rm peak} \approx 0.7 \times f_{\rm merg}$

PBH-PBH: Close Hyperbolic Encounters

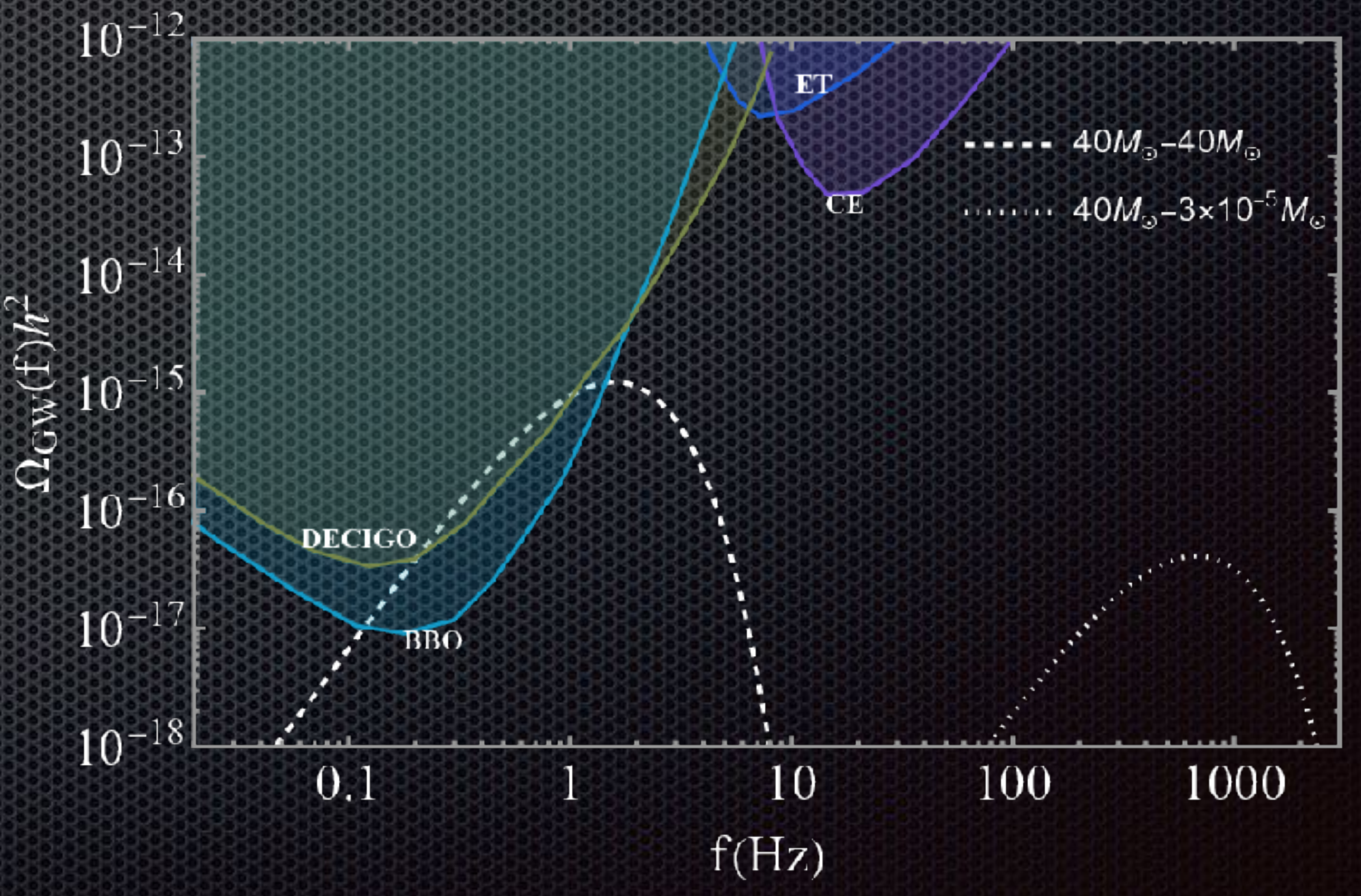
- PBHs can take part in close hyperbolic encounters (CHE) which can also give rise to specific type of GW spectrum

- The event rate in this case can be given as,

$$\frac{\tau_{\rm CHE}}{d\ln m_1 d\ln m_2} = 25.4\times 10^{-8}\left(\frac{\Omega_{\rm DM}}{0.25}\right)^2\left(\frac{\delta_{\rm loc}}{10^8}\right) f(m_1)f(m_2)\frac{M^2}{m_1 m_2}\frac{e^2-1}{(v_0/c)^3}$$

- Emitted energy in the form of GW —

$$\frac{dE_{\rm GW}}{d\ln f_s} = \frac{4\pi}{45}\frac{G^3 m_1^2 m_2^2}{a^2 c^5 \nu_0}\nu^5 F_e(\nu)$$



Interactions: PBH-ABH

- If PBHs exist, then there should necessarily be the PBH-ABH interactions
- Like the previous case, there are two possibilities:

  (i) PBH-ABH binary systems

  (ii) PBH-ABH close hyperbolic encounters
- Averaged number of ABH per solar mass of stellar objects turns out to be $\sim 2 \times 10^{-3}$

PBH-ABH: Binary System

- Merger rate for PBH-ABH binaries per logarithmic mass interval is given by,

$$\frac{d\tau_{\mathrm{BBH}}}{d\ln m_1 d\ln m_2} = \frac{1}{\delta_{\mathrm{loc}}}\sigma_{\mathrm{BBH}} v_{\mathrm{BH}} n_A(m_A) n_P(m_P)$$

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ABH number density

PBH number density

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Relative velocity

ABH number density

PBH number density

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Cross section of PBH-ABH interaction

Relative velocity

ABH number density

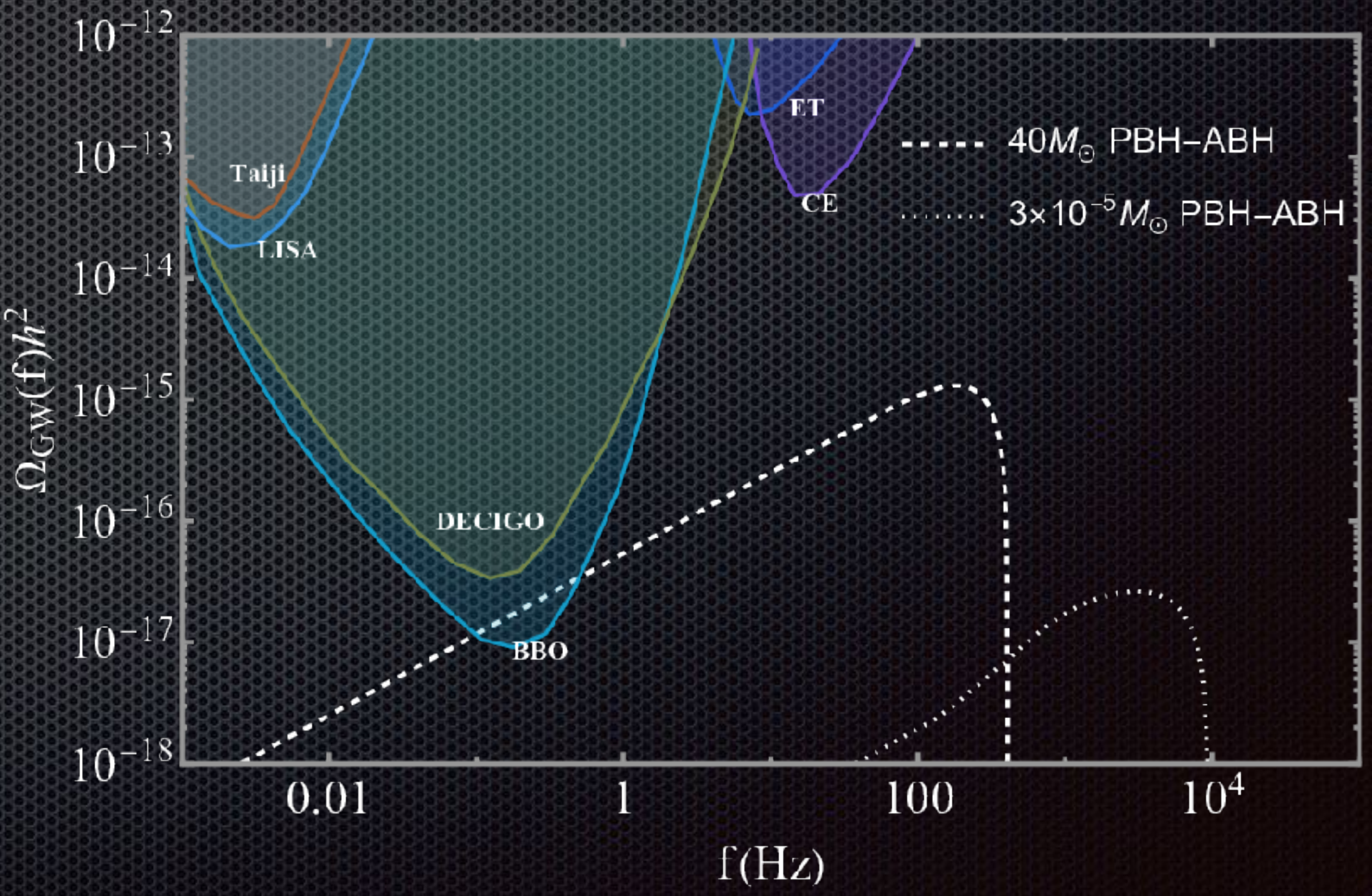
PBH number density

$$\sigma_{\rm BBH} = \pi \left(\frac{340\pi}{3}\right)^{2/7} \frac{G^2 M^{10/7} (m_1 m_2)^{2/7}}{c^{10/7} v_0^{18/7}}$$

PBH-ABH: Binary System

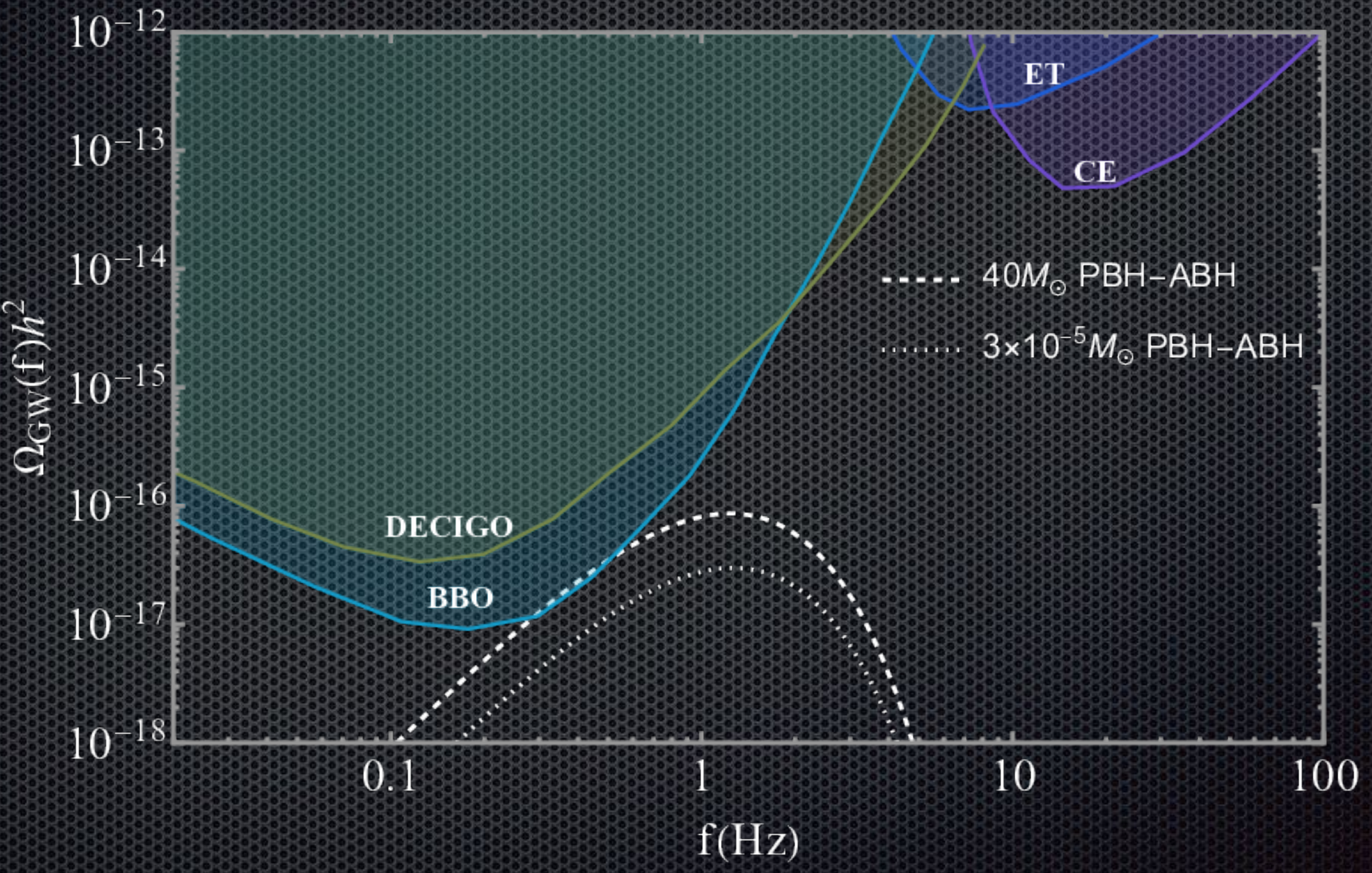
- For numerical purposes assume ABH mass range from $5M_\odot$ to tens of $M_\odot$
- Also such binaries can be formed only after the ABHs are formed
- For our purposes we have considered the upper limit of the redshift for the PBH-ABH interactions to be 5
- Thus the GW spectrum takes the form

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\int_0^5 dz \frac{N(z)}{1+z}\frac{dE_{\rm GW}}{d\ln f_s}$$



PBH-ABH: Close Hyperbolic Encounters

- For this we follow the similar prescription as the PBH-PBH CHE
- The number density has to taken accordingly



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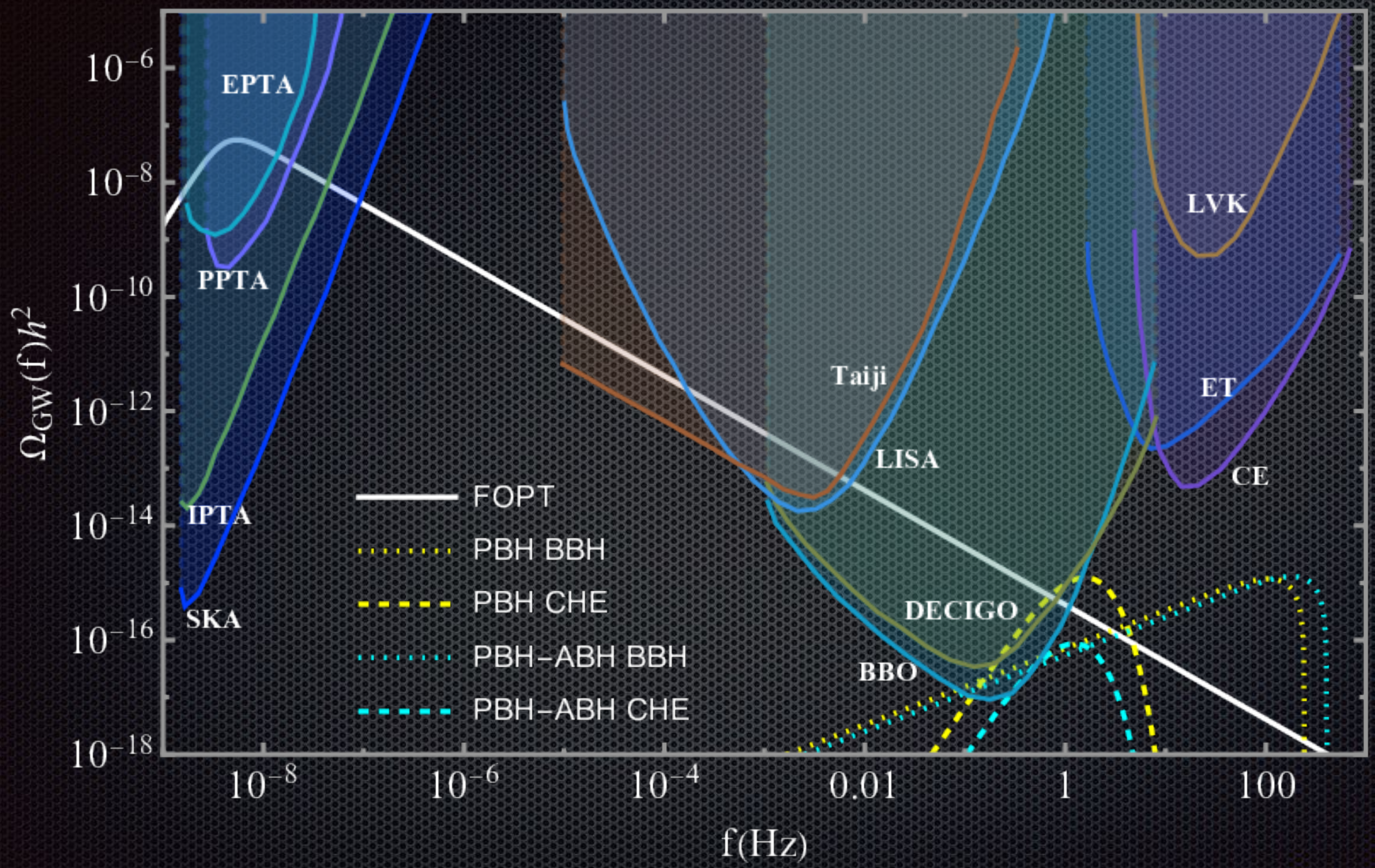
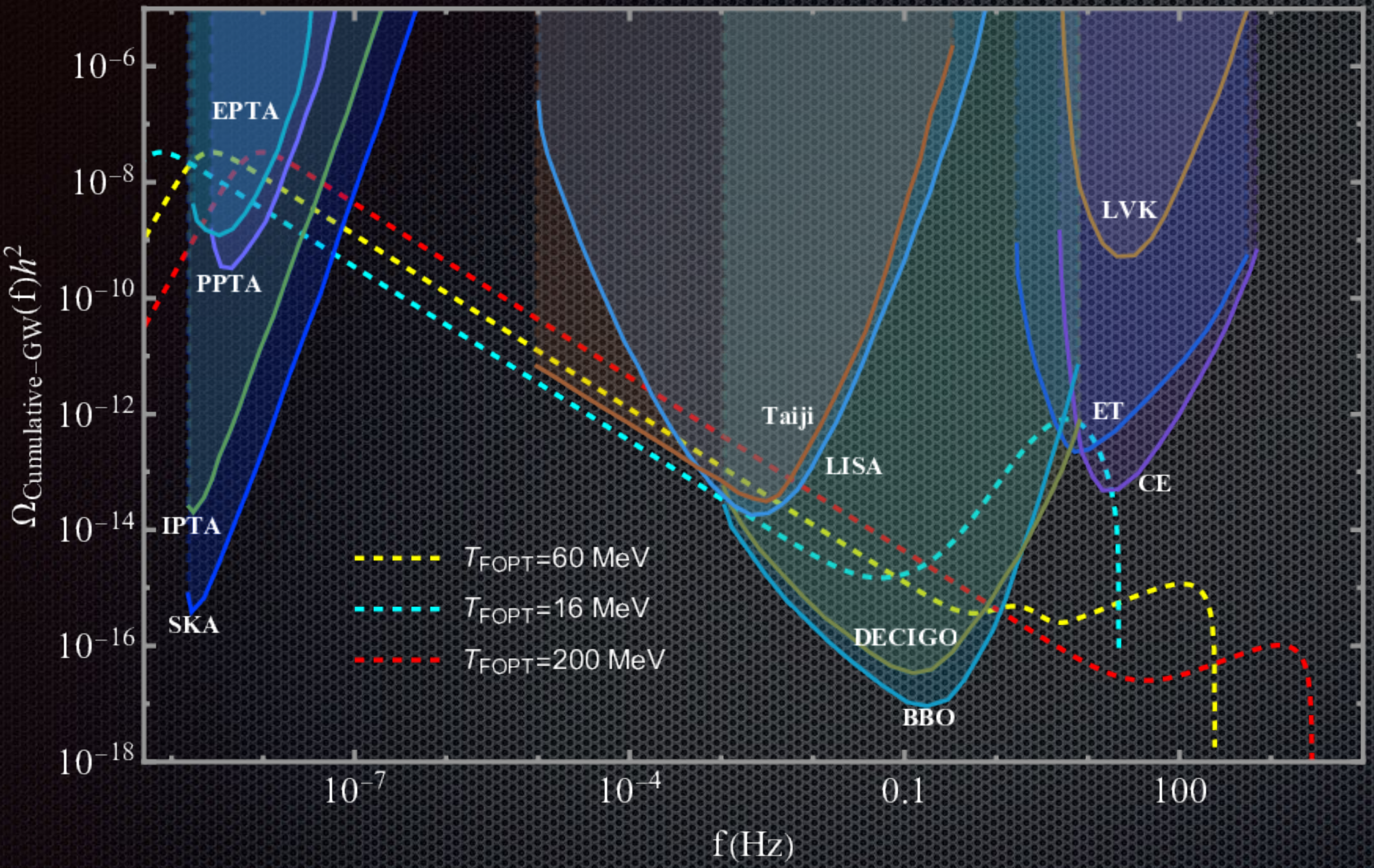
Cumulative SGWB



$T = 60$ MeV,
$m_{\text{PBH}} = 40 M_{\odot}$

Dependence on Transition Temperatures



- Cumulative SGWB spectrum show, first peak around $\mathcal{O}(10^{-9}$ Hz), the second peak occurs around $\mathcal{O}(1$ Hz)
- Other two cases as well, the peaks are separated by $\mathcal{O}(10^{10,11}$ Hz)
- High separation between the peaks makes this signal unique as compared to other exotic SGWB signals with multiple peaks arising from different mechanisms

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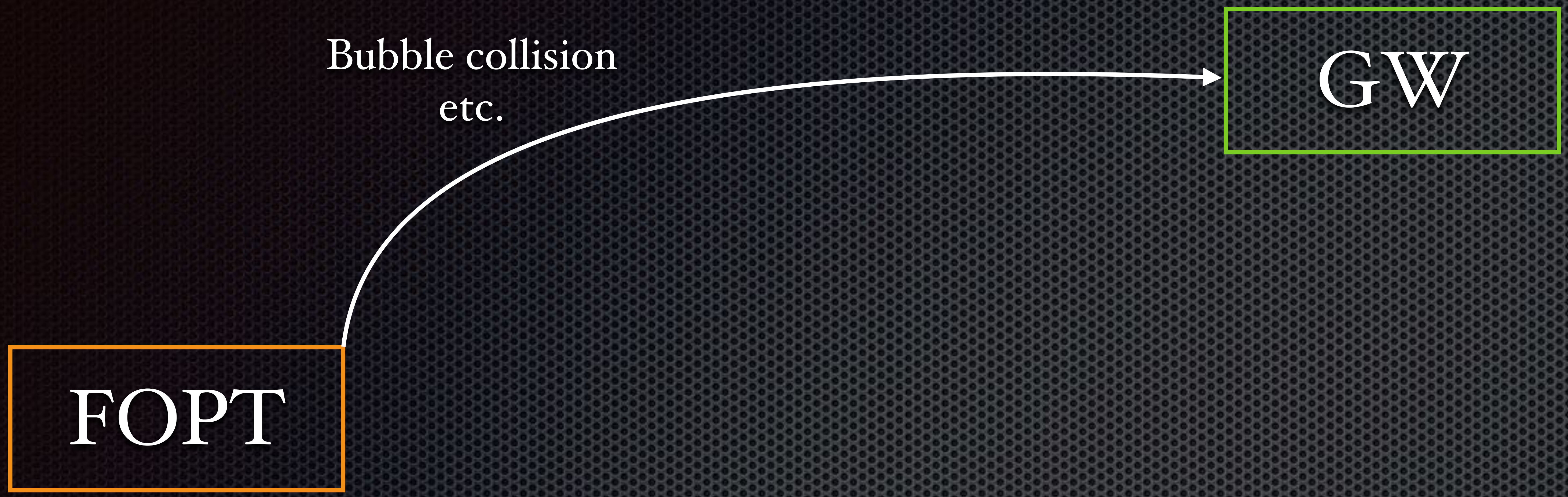
FOPT

Summary and Conclusion



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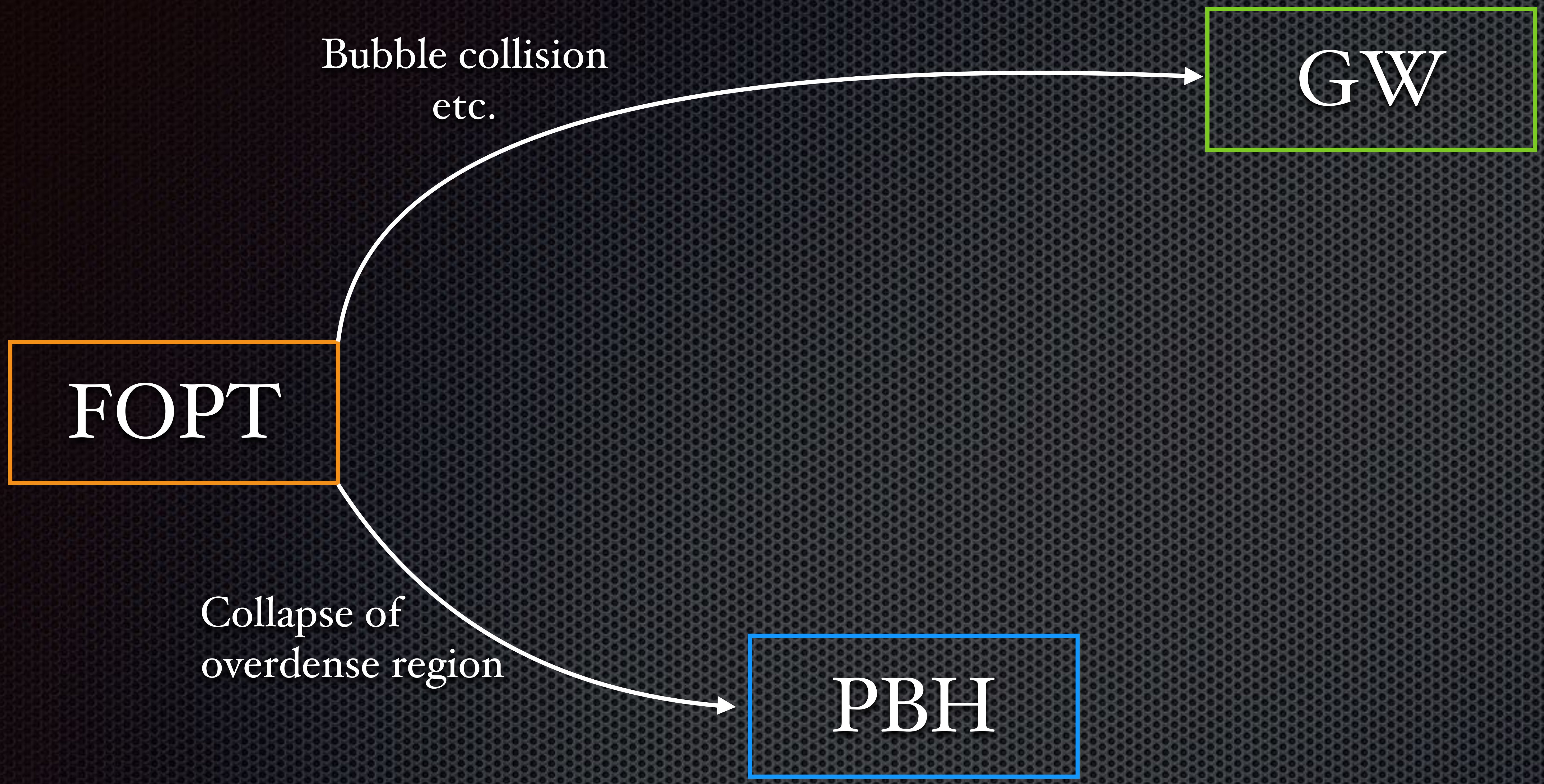

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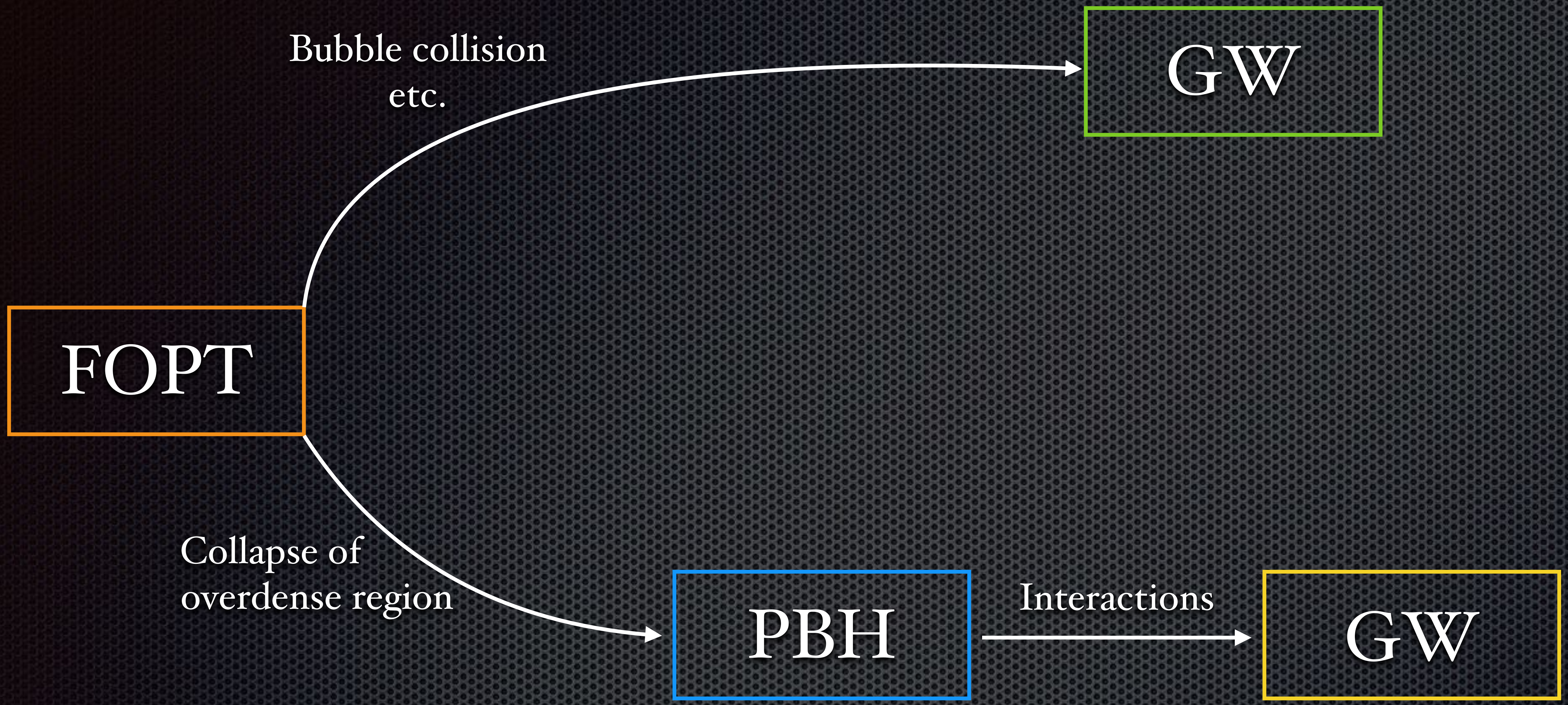

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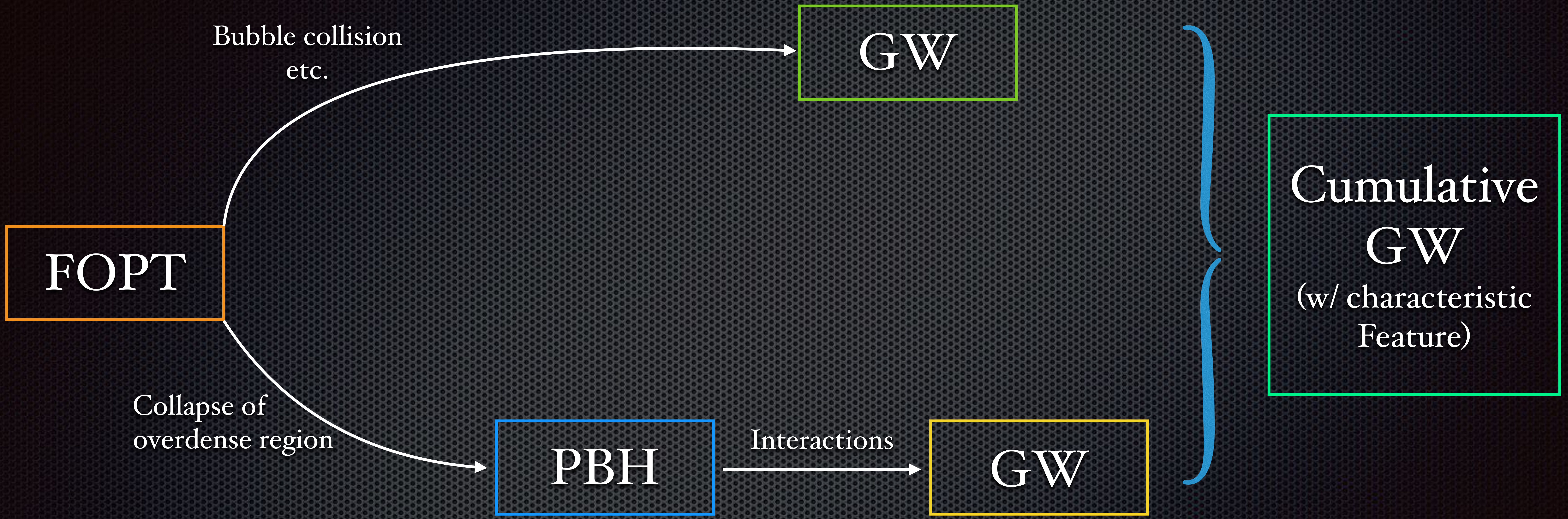

Thank you