Tri-bimaximal-Cabibbo mixing: flavour violations in the charged lepton sector

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- Quark-Lepton unification suggests Cabibbo sized effects in lepton mixings (Phys.Lett.B620:42-51,2005, Nucl.Phys. B866 (2013) 255-269)

- Tri-bimaximal mixing ansatz can be modified as,

$$U \approx \begin{pmatrix} \sqrt{\frac{2}{3}}(1-\frac{1}{2}s) & \frac{1}{\sqrt{3}}(1+s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+s-a+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s-a-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{\sqrt{6}}(1+s+a-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1-\frac{1}{2}s+a+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1-a) \end{pmatrix} P.$$

- The data is consistent with $s_{13} = \frac{sin\theta_c}{\sqrt{2}} = \frac{\lambda}{\sqrt{2}}$, in terms of the TB deviations $r = \lambda, s = a = 0$

In general the mixing in the lepton sector (U_{pmns}) can be written as, $U_{pmns} = U_e^{L\dagger} U_\nu^L$. Where U_e^L is the mixing in the charged lepton sector and U_ν^L is the mixing in the neutrino sector.

$$U_e^{L\dagger} = \begin{pmatrix} c_{12}^e & s_{12}^e e^{-i\delta_{12}^e} & 0 \\ -s_{12}^e e^{i\delta_{12}^e} & c_{12}^e & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

$$U_\nu^L = U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

where $c_{12}^e = \cos\theta_{12}^e$ and $s_{12}^e = \sin\theta_{12}^e$.

The mixing in the first two generations of charged leptons are, in general, strongly constrained from charge flavour violation decays of mesons and muon. To illustrate this, consider a scalar operator in the mass basis, given by,

$$\mathcal{O}^S_{mnij} = \frac{1}{\Lambda^2} J^Q_{mn} J^L_{ij},$$

where,

$$J^Q_{mn} = \mathcal{C}^Q_{mn} \bar{q}_m P_R q_n$$
$$J^L_{ij} = \mathcal{C}^L_{ij} \bar{\ell}_i P_R \ell_j$$

Here, Λ is the energy scale of the NP. In the above currents, ℓ represents a leptonic field and q represents a quark field. C^Q and C^L are 3×3 matrix, with m, n representing quark generation and i, j representing lepton generation. Rotating to the flavour basis,

$$\ell'_{Li} \rightarrow (U^L_{e/\nu})_{ik} \ell_{Lk}; \quad q'_{Li} \rightarrow (U^L_q)_{ik} q_{Lk}$$

$$J^Q_{mn} = \mathcal{C}'^Q_{mn} \bar{q}'_m P_R q'_n$$
$$J^L_{ij} = \mathcal{C}'^L_{ij} \bar{\ell}'_i P_R \ell_j'$$

$$\mathcal{C}^L_{ij} = \sum_k \mathcal{C}'^L_{kj} (U^L_{e/\nu})^\dagger_{ik}$$

Scalar and dipole operators where the effects of this mixing are prominent are given in Table 1.

Table 1 Scalar and dipole operators in which we can realize the TBC mixing, where $\ell = \{e, \mu\}$, $\nu = \{\nu_e, \nu_\mu, \nu_\tau\}$ and $q = \{u, c, t\}/\{d, s, b\}$

Scalar	Dipole
$\mathcal{O}^{SRR}_{\ell\ell qq} = (\bar{\ell}_L\, \ell_R)(\bar{q}_L\, q_R)$	$\mathcal{O}^{TLR}_{\ell\ell} = (\bar{\ell}_L \sigma^{\mu\nu} \ell_R) F_{\mu\nu}$
$\mathcal{O}^{SRL}_{\ell\ell qq} = (\bar{\ell}_L\, \ell_R)(\bar{q}_R\, q_L)$	$\mathcal{O}^{TLR}_{\nu\ell} = (\bar{\nu}_L \sigma^{\mu\nu} \ell_R) F_{\mu\nu}$
$\mathcal{O}^{SRR}_{\nu\ell qq} = (\bar{\nu}_L\, \ell_R)(\bar{q}_L\, q_R)$	
$\mathcal{O}^{SRL}_{\nu\ell qq} = (\bar{\nu}_L\, \ell_R)(\bar{q}_R\, q_L)$	

The relation between Wilson coefficients in mass basis and flavour basis is,

$$\mathcal{C}^{SRR}_{ijqq} = \sum_k \mathcal{C'}^{SRR}_{kjqq} (U^L_{e/\nu})^\dagger_{ik}$$

$$\mathcal{C}^{SRL}_{ijqq} = \sum_k \mathcal{C'}^{SRL}_{kjqq} (U^L_{e/\nu})^\dagger_{ik}$$

$$\mathcal{C}^{TLR}_{ij} = \sum_k \mathcal{C'}^{TLR}_{kj} (U^L_{e/\nu})^\dagger_{ik}$$

$$
\begin{aligned}
\bar{u}_{iLw}u_{iRt} &= -2B_0\Big\{\frac{1}{4}F_0^2U_{tw} + L_4\langle D_\mu U^\dagger D^\mu U\rangle U_{tw} \\
&+ L_5(UD_\mu U^\dagger D^\mu U)_{tw} + 2L_6\langle U^\dagger\chi + \chi^\dagger U\rangle U_{tw} - 2L_7\langle U^\dagger\chi - \chi^\dagger U\rangle U_{tw} \\
&+ 2L_8(U\chi^\dagger U)_{tw} + H_2\,\chi_{tw}\Big\} + \mathcal{O}(p^6)\ ,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{O}^{SRR}_{eu_i} &= (\bar{\mu}_L e_R)(\bar{u}_{iLw}u_{iRt}) \\
&= (\bar{\mu}_L e_R)\Big[-2B_0\frac{1}{4}F_0^2U_{tw}\Big] + \mathcal{O}(p^4),
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{O}^{SRL}_{eu_i} &= (\bar{\mu}_L e_R)(\bar{u}_{iRw}u_{iLt}) \\
&= (\bar{\mu}_L e_R)\Big[-2B_0\frac{1}{4}F_0^2U^\dagger_{tw}\Big] + \mathcal{O}(p^4),
\end{aligned}
$$

Operators $\mathcal{O}^{SRR}_{\ell\ell qq}$, $\mathcal{O}^{SRL}_{\ell\ell qq}$ and $\mathcal{O}^{TLR}_{\ell\ell}$ are sensitive to U^L_e, where as operator $\mathcal{O}^{SRR}_{\nu\ell qq}$, $\mathcal{O}^{SRL}_{\nu\ell qq}$ and $\mathcal{O}^{TRR}_{\nu\ell}$ are sensitive to U^L_ν. Hence, the constraints on the operators $\mathcal{O}^{SRR}_{\ell\ell qq}$, $\mathcal{O}^{SRL}_{\ell\ell qq}$ and $\mathcal{O}^{TLR}_{\ell\ell}$ are best studied using flavour violating decays like $\pi^0 \to e^+\mu^-$, $K_L \to \mu^+e^-$, $\mu \to e\gamma$ and muon conversion ($\mu N \to eN$). In fact, if we assume that U^L_e is the source of LFV, current experimental bounds from $K_L \to \mu^+e^-$, $\pi^0 \to e^+\mu^-$ and $\mu \to e\gamma$ rules out the TBC mixing.

$$
U(x) \equiv \exp\left[i\frac{\sqrt{2}}{F_0}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}\right]
$$

$$
\begin{aligned}
&\frac{\Gamma(K_L \to \mu^+e^-)}{\Gamma(K_L \to \mu^+\mu^-)} \\
&= |(U^L_e)_{e\mu}|^2\frac{|p_\mu|_{K_L\mu e}(m^2_{K_L} - (m_\mu + m_e)^2)}{|p_\mu|_{K_L\mu\mu}(m^2_{K_L} - (m_\mu + m_\mu)^2)} \\
&= 6.14 \times 10^{-2}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Br}(\mu \to e\gamma) &= \frac{\tau_\mu\alpha m^3_\mu}{4}\left(|\mathcal{C}^{TLR}_{e\mu}|^2 + |\mathcal{C}^{TRL}_{e\mu}|^2\right) \\
&= \frac{2\tau_\mu\alpha m^3_\mu}{4}|U^L_{e\mu}|^2|\mathcal{C}^{TLR}_{\mu\mu}|^2
\end{aligned}
$$

The above value is much higher than the current upper bound $\mathrm{BR}(\mu \to e\gamma) < 4.2 \times 10^{-13}$

Minimal flavour violation with TBC mixing

The Minimal Flavour Violation (MFV) hypothesis assumes that the Standard Model (SM) Yukawa couplings are the only source of flavour symmetry breaking . This means all the higher dimensional operators should be constructed out of the SM Yukawa couplings, satisfying the flavour symmetry $\mathcal{G}_F : SU(3)_Q \times SU(3)_u \times SU(3)_d \times SU(3)_L \times SU(3)_e$. The Yukawa couplings are considered as non-dynamical fields (spurions) which transform under the flavour symmetry $\mathcal{G}_F : \mathcal{G}_{QF} \times \mathcal{G}_{LF}$ ($\mathcal{G}_{QF} : SU(3)_Q \times SU(3)_u \times SU(3)_d$, $\mathcal{G}_{LF} : SU(3)_L \times SU(3)_e$) as,

$$Y_u \sim (3, \bar{3}, 1), \quad Y_d \sim (3, 1, \bar{3}), \quad Y_e \sim (\bar{3}, 3)$$

$$Y_e = \frac{m_\ell}{v} = \frac{1}{v}\,\mathrm{diag}(m_e, m_\mu, m_\tau) \ ,$$

$$g_\nu = \frac{\Lambda_{\mathrm{LN}}}{v^2}\,\hat{U}^* m_\nu \hat{U}^\dagger = \frac{\Lambda_{\mathrm{LN}}}{v^2}\,\hat{U}^* \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\hat{U}^\dagger$$

$$e_L = U_{e_L} e'_L \ , \qquad e_R = U_{e_R} e'_R \ , \qquad \nu_L = U_{\nu_L} \nu'_L$$

$$\Delta = g_\nu^\dagger g_\nu = \frac{\Lambda_{\mathrm{LN}}^2}{v^4}\,\hat{U} m_\nu^2 \hat{U}^\dagger$$

Minimal field content

$$\mathcal{L} = -vY_e^{ij}\bar{e}_R^i e_L^j - \frac{v^2}{2\Lambda_{LN}} g_\nu^{ij} \overline{\nu^{ci}}_L \nu_L^j + h.c$$

$$L_L \to V_L L_L, \quad e_R \to V_R e_R$$

In the above expression L_L and e_R represent $SU(2)$ doublet and singlet leptonic field. In order to keep the Lagrangian invariant under $\mathcal{G}_{LF}$, Y_e and g_ν transform as:

$$Y_e \to V_R Y_e V_L^\dagger, \quad g_\nu \to V_L^* g_\nu V_L^\dagger$$

Assuming TBC mixing ansatz, the basis for MFV could be chosen as,

$$Y_e = D_e U_e^{L\dagger}, \quad g_\nu = \frac{\Lambda_{LN}}{v^2} U_\nu^{L*} m_\nu U_\nu^{L\dagger}$$

where $D_e = \frac{1}{v} diag(m_e, m_\mu, m_\tau)$ and $m_\nu = diag(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$. Since this is different from the usual MFV where there is no mixing in the charged lepton sector, we call this scenario as Modified Minimal Flavour Violation (MMFV). A spurion that transforms as (8, 1) under the group $\mathcal{G}_{LF}$ can be constructed as $\Delta = g_\nu^\dagger g_\nu = \frac{\Lambda_{LN}^2}{v^4} U_\nu^L m_\nu^2 U_\nu^{L\dagger}$.

Extended field content

$$\mathcal{L} = -vY_e^{ij}\bar{e}_R^i e_L^j - vY_\nu^{ij}\bar{\nu}_R^i \nu_L^j - \frac{1}{2} M_\nu^{ij} \bar{\nu}_R^{ci} \nu_R^j + h.c$$

The right-handed neutrino mass term breaks $SU(3)_{\nu_R}$ symmetry to $O(3)_{\nu_R}$ and they are assumed to be in their mass basis, that is $M_\nu^{ij} = M_\nu \delta^{ij}$. The Lagrangian remains invariant under the flavour symmetry $\mathcal{G}_{LF} \times O(3)_{\nu_R}$ if the field and spurions transform as,

$$L_L \to V_L L_L, \quad e_R \to V_R e_R, \quad \nu_R \to O_R \nu_R$$
$$Y_e \to V_R Y_e V_L^\dagger, \quad Y_\nu \to O_R Y_\nu V_L^\dagger.$$

Generating an effective left-handed Majorana mass matrix by integrating out the right-handed neutrinos, we get,

$$\frac{v^2}{\Lambda_{LFV}} g_\nu = \frac{v^2}{M_\nu} Y_\nu^T Y_\nu$$

If we take $M_\nu = \Lambda_{LN}$ then, $g_\nu = Y_\nu^T Y_\nu$. Using $\mathcal{G}_{LF} \times O(3)_{\nu_R}$ symmetry, we rotate the fields such that there is mixing in the charged lepton sector. In this basis,

$$Y_e = D_e U_e^{L\dagger}, \quad Y_\nu^T Y_\nu = \frac{\Lambda_{LN}}{v^2} U_\nu^L m_\nu U_\nu^{L\dagger}$$

and one can construct $\Delta = Y_\nu^\dagger Y_\nu = \frac{\Lambda_{LN}}{v^2} U_\nu^L m_\nu U_\nu^{L\dagger}$

The operators that we consider are listed in Table Here we have kept only the dominant operators which are proportional to Δ and $Y_e\Delta$, and have neglected the operators that go as $Y_e Y_e^\dagger$ or higher orders of Y_e.

Operators satisfying flavour symmetry . Q_L and L_L represents $SU(2)$ doublet quark and lepton. u_R, d_R and e_R represent $SU(2)$ singlet up quark, down quark and charged lepton respectively. H is the SM Higgs field

Scalar	Dipole	Vector
$\mathcal{O}^{S1} = (\bar{L}_L \Delta^\dagger Y_e^\dagger e_R)(\bar{d}_R \lambda^{S1} Q_L)$	$\mathcal{O}^{T1} = H(\bar{L}_L \sigma^{\mu\nu} \Delta^\dagger Y_e^\dagger e_R) F_{\mu\nu}$	$\mathcal{O}^{V1} = (\bar{L}_L \gamma^\mu \Delta L_L)(\bar{Q}_L \lambda^{V1} \gamma_\mu Q_L)$
$\mathcal{O}^{S2} = -(\bar{L}_L \Delta^\dagger Y_e^\dagger e_R)(\bar{Q}_L \lambda^{S2} i\tau^2 u_R)$		$\mathcal{O}^{V2} = (\bar{L}_L \gamma^\mu \Delta L_L)(\bar{L}_L \lambda^{V2} \gamma_\mu L_L)$
$\mathcal{O}^{S3} = (\bar{L}_L \Delta^\dagger Y_e^\dagger e_R)(\bar{e}_R \lambda^{S3} L_L)$		$\mathcal{O}^{V1} = (\bar{L}_L \gamma^\mu \Delta L_L)(\bar{u}_R \lambda^{V3} \gamma_\mu u_R)$
		$\mathcal{O}^{V4} = (\bar{L}_L \gamma^\mu \Delta L_L)(\bar{d}_R \lambda^{V4} \gamma_\mu d_R)$

MMFV operators in minimal field content scenario

MMFV operators in minimal field content scenario	
$\mathcal{O}^{S1}$	$\frac{\Lambda_{LN}^2}{v^4}\ \{(\bar{\nu}_L m_\nu^2 U_{pmns}^\dagger D_e e_R)(\bar{d}_R \lambda^{S1} u_L) + (\bar{e}_L U_{pmns} m_\nu^2 U_{pmns}^\dagger D_e e_R)(\bar{d}_R \lambda^{S1} d_L)\}$
$\mathcal{O}^{T1}$	$\frac{\Lambda_{LN}^2}{v^4}\ (\bar{e}_L \sigma^{\mu\nu} v U_{pmns} m_\nu^2 U_{pmns}^\dagger D_e e_R) F_{\mu\nu}$
$\mathcal{O}^{S2}$	$\frac{\Lambda_{LN}^2}{v^4}\ \{(\bar{\nu}_L m_\nu^2 U_{pmns}^\dagger D_e e_R)(\bar{d}_L \lambda^{S2} u_R) - (\bar{e}_L U_{pmns} m_\nu^2 U_{pmns}^\dagger D_e e_R)(\bar{u}_L \lambda^{S2} u_R)\}$
$\mathcal{O}^{S3}$	$\frac{\Lambda_{LN}^2}{v^4}\ \{(\bar{\nu}_L m_\nu^2 U_{pmns}^\dagger D_e e_R)(\bar{e}_R \lambda^{S3} \nu_L) + (\bar{e}_L U_{pmns} m_\nu^2 U_{pmns}^\dagger D_e e_R)(\bar{e}_R \lambda^{S3} e_L)\}$
$\mathcal{O}^{V1}$	$\frac{\Lambda_{LN}^2}{v^4}\ (\bar{\nu}_L \gamma^\mu m_\nu^2 \nu_L + \bar{e}_L \gamma^\mu U_{pmns} m_\nu^2 U_{pmns}^\dagger e_L)(\bar{u}_L \lambda^{V1} \gamma_\mu u_L + \bar{d}_L \gamma_\mu \lambda^{V1} d_L)$
$\mathcal{O}^{V2}$	$\frac{\Lambda_{LN}^2}{v^4}\ (\bar{\nu}_L \gamma^\mu m_\nu^2 \nu_L + \bar{e}_L \gamma^\mu U_{pmns} m_\nu^2 U_{pmns}^\dagger e_L)(\bar{\nu}_L \lambda^{V2} \gamma_\mu \nu_L + \bar{e}_L \lambda^{V2} \gamma_\mu e_L)$
$\mathcal{O}^{V3}$	$\frac{\Lambda_{LN}^2}{v^4}\ (\bar{\nu}_L \gamma^\mu m_\nu^2 \nu_L + \bar{e}_L \gamma^\mu U_{pmns} m_\nu^2 U_{pmns}^\dagger e_L)(\bar{u}_R \lambda^{V3} \gamma_\mu u_R)$
$\mathcal{O}^{V4}$	$\frac{\Lambda_{LN}^2}{v^4}\ (\bar{\nu}_L \gamma^\mu m_\nu^2 \nu_L + \bar{e}_L \gamma^\mu U_{pmns} m_\nu^2 U_{pmns}^\dagger e_L)(\bar{d}_R \lambda^{V4} \gamma_\mu d_R)$

MMFV operators in extended field content scenario

MMFV operators in extended field content scenario.	
$\mathcal{O}^{S1}$	$\frac{\Lambda_{LN}}{v^2}\ \{(\bar{\nu}_L m_\nu U_{pmns}^\dagger D_e e_R)(\bar{d}_R \lambda^{S1} u_L) + (\bar{e}_L U_{pmns} m_\nu U_{pmns}^\dagger D_e e_R)(\bar{d}_R \lambda^{S1} d_L)\}$
$\mathcal{O}^{T1}$	$\frac{\Lambda_{LN}}{v^2}\ (\bar{e}_L \sigma^{\mu\nu} v U_{pmns} m_\nu U_{pmns}^\dagger D_e e_R) F_{\mu\nu}$
$\mathcal{O}^{S2}$	$\frac{\Lambda_{LN}}{v^2}\ \{(\bar{\nu}_L m_\nu U_{pmns}^\dagger D_e e_R)(\bar{d}_L \lambda^{S2} u_R) - (\bar{e}_L U_{pmns} m_\nu U_{pmns}^\dagger D_e e_R)(\bar{u}_L \lambda^{S2} u_R)\}$
$\mathcal{O}^{S3}$	$\frac{\Lambda_{LN}}{v^2}\ \{(\bar{\nu}_L m_\nu U_{pmns}^\dagger D_e e_R)(\bar{e}_R \lambda^{S3} \nu_L) + (\bar{e}_L U_{pmns} m_\nu U_{pmns}^\dagger D_e e_R)(\bar{e}_R \lambda^{S3} e_L)\}$
$\mathcal{O}^{V1}$	$\frac{\Lambda_{LN}}{v^2}\ (\bar{\nu}_L \gamma^\mu m_\nu \nu_L + \bar{e}_L \gamma^\mu U_{pmns} m_\nu U_{pmns}^\dagger e_L)(\bar{u}_L \lambda^{V1} \gamma_\mu u_L + \bar{d}_L \gamma_\mu \lambda^{V1} d_L)$
$\mathcal{O}^{V2}$	$\frac{\Lambda_{LN}}{v^2}\ (\bar{\nu}_L \gamma^\mu m_\nu \nu_L + \bar{e}_L \gamma^\mu U_{pmns} m_\nu U_{pmns}^\dagger e_L)(\bar{\nu}_L \lambda^{V2} \gamma_\mu \nu_L + \bar{e}_L \lambda^{V2} \gamma_\mu e_L)$
$\mathcal{O}^{V3}$	$\frac{\Lambda_{LN}}{v^2}\ (\bar{\nu}_L \gamma^\mu m_\nu \nu_L + \bar{e}_L \gamma^\mu U_{pmns} m_\nu U_{pmns}^\dagger e_L)(\bar{u}_R \lambda^{V3} \gamma_\mu u_R)$
$\mathcal{O}^{V4}$	$\frac{\Lambda_{LN}}{v^2}\ (\bar{\nu}_L \gamma^\mu m_\nu \nu_L + \bar{e}_L \gamma^\mu U_{pmns} m_\nu U_{pmns}^\dagger e_L)(\bar{d}_R \lambda^{V4} \gamma_\mu d_R)$

The effective Lagrangian that we consider in our analysis is of the form,

$$\mathcal{L} = \frac{1}{\Lambda_{LFV}^2}\mathcal{C}\mathcal{O}$$

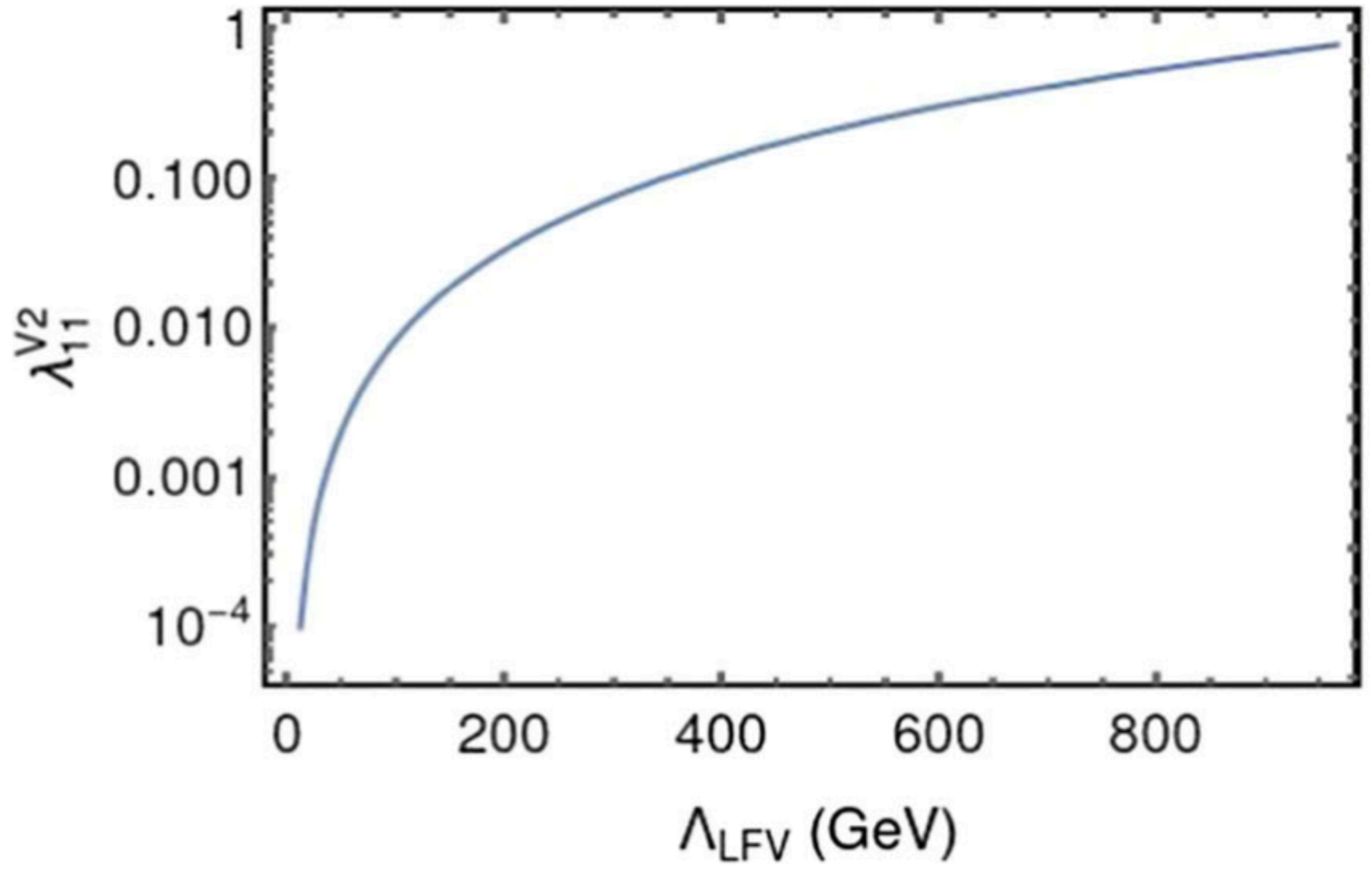
$\underline{\mu^- \to e^- e^+ e^-}$: The operators that contribute to $\mu^- \to e^- e^+ e^-$ are:

$$\mathcal{C}^{S3}_{e\mu ee}\mathcal{O}^{S3}_{e\mu ee} = \Big(\bar{e}_L \, \{\Delta^\dagger Y_e^\dagger\}_{12} \, \mu_R\Big)(\bar{e}_R \, \lambda_{11}^{S3} \, e_L)$$

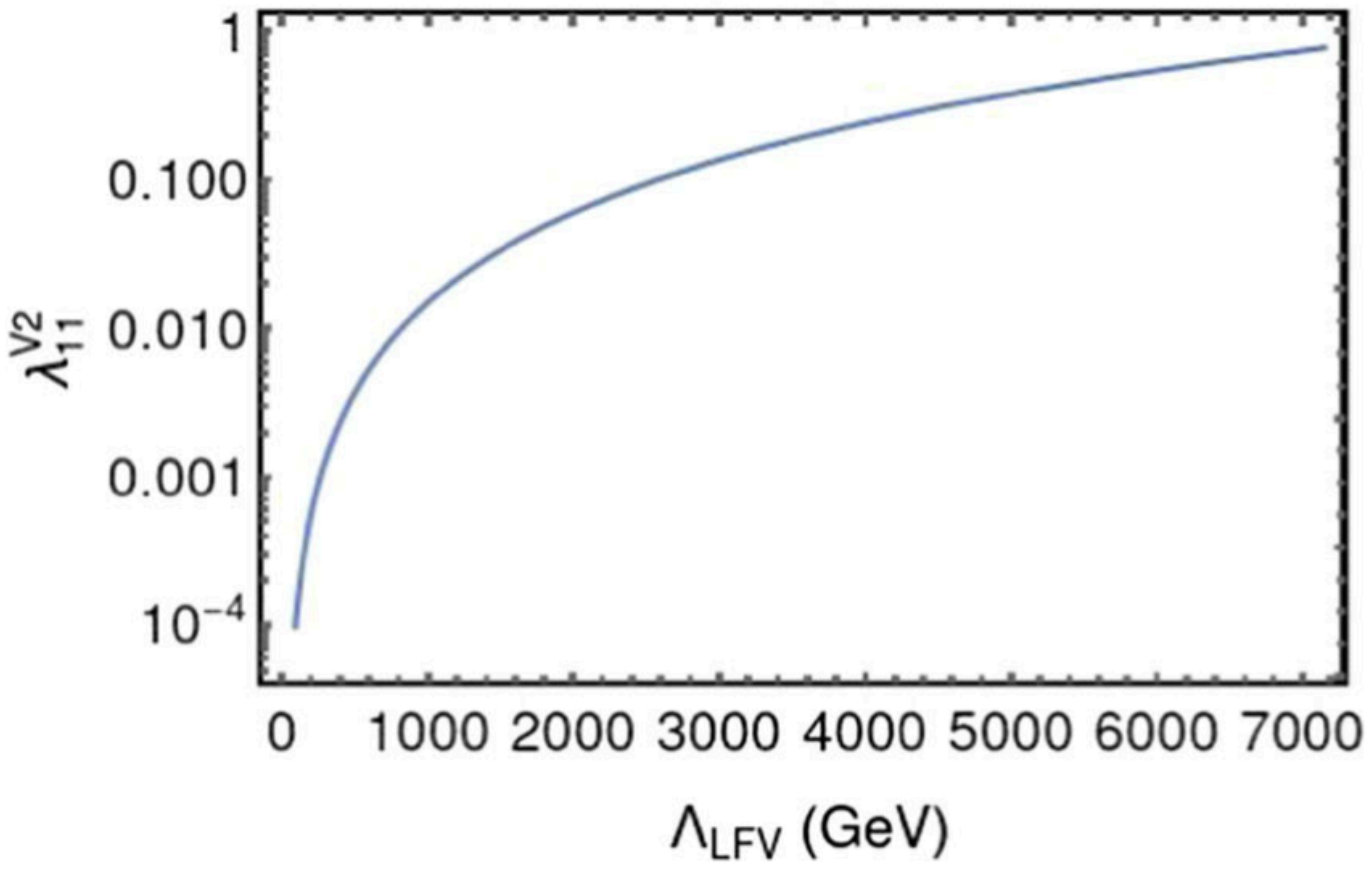
$$\mathcal{C}^{V2}_{e\mu ee}\mathcal{O}^{V2}_{e\mu ee} = \Big(\bar{e}_L \, \gamma^\mu \Delta_{12} \, \mu_L\Big)(\bar{e}_L \, \gamma_\mu \lambda_{11}^{V2} \, e_L)$$

The BR($\mu^- \to e^- e^+ e^-$) measured by SINDRUM collaboration gives an upper limit of $< 1 \times 10^{-12}$. Branching ratio of $\mu^- \to e^- e^+ e^-$ in case of a scalar operator becomes,

$$\mathrm{BR}(\mu^- \to e^- e^+ e^-) = \frac{1}{12}\frac{m_\mu^5 \tau_\mu}{512\pi^3}\frac{|\mathcal{C}^{S3}_{e\mu ee}|^2}{\Lambda_{LFV}^4} \,,$$



(a) Minimal field content scenario



(b) Extended field content scenario

$\underline{\pi^0 \to e^+\mu^-}$: The operators that contribute to $\pi^0 \to e^+\mu^-$ are:

$$\mathcal{C}^{S1}_{\mu edd}\mathcal{O}^{S1}_{\mu edd} = \Big(\bar{\mu}_L \, \{\Delta^\dagger Y_e^\dagger\}_{21} \, e_R\Big)(\bar{d}_R \, \lambda^{S1}_{11} \, d_L)$$

$$\mathcal{C}^{S2}_{\mu euu}\mathcal{O}^{S2}_{\mu euu} = \Big(\bar{\mu}_L \, \{\Delta^\dagger Y_e^\dagger\}_{21} \, e_R\Big)(\bar{u}_R \, \lambda^{S2}_{11} \, u_L)$$

$$\mathcal{C}^{V1}_{\mu eqq}\mathcal{O}^{V1}_{\mu eqq} = \Big(\bar{\mu}_L \gamma^\mu \Delta_{21} \, e_L\Big) \times (\bar{u}_L \, \gamma_\mu \lambda^{V1}_{11} \, u_L + \bar{d}_L \, \gamma_\mu \lambda^{V1}_{11} \, d_L)$$

$$\mathcal{C}^{V3}_{\mu euu}\mathcal{O}^{V3}_{\mu euu} = \Big(\bar{\mu}_L \, \gamma^\mu \Delta_{21} \, e_L\Big)(\bar{u}_R \, \gamma_\mu \lambda^{V3}_{11} \, u_R)$$

$$\mathcal{C}^{V4}_{\mu edd}\mathcal{O}^{V4}_{\mu edd} = \Big(\bar{\mu}_L \, \gamma^\mu \Delta_{21} \, e_L\Big)(\bar{d}_R \, \gamma_\mu \lambda^{V4}_{11} \, d_R)$$

$$\mathrm{BR}(\pi^0 \to e^+\mu^-) = \frac{|p_\mu|_{\pi^0 \mu e}}{8\pi m_\pi^2} \frac{\tau_{\pi^0} B_0^2 F_0^2}{4} \times \frac{|\mathcal{C}^{S1}_{\mu edd} + \mathcal{C}^{S2}_{\mu euu}|^2}{\Lambda^4_{LFV}} (m_\pi^2 - (m_e + m_\mu)^2)$$

$$\mathrm{BR}(\pi^0 \to e^+\mu^-) = \frac{|p_\mu|_{\pi^0 \mu e}}{8\pi m_\pi^2} \frac{\tau_{\pi^0} F_0^2}{4} \times \frac{|\mathcal{C}^{V3}_{\mu euu} - \mathcal{C}^{V4}_{\mu edd} + \mathcal{C}^{V1}_{\mu edd} - \mathcal{C}^{V1}_{\mu euu}|^2}{\Lambda^4_{LFV}} \times (m_\pi^2 (m_\mu^2 + m_e^2) - (m_\mu^2 - m_e^2)^2) \, ,$$

$\underline{K_L \to \mu^+ e^-}$: The operators that contribute to $K_L \to \mu^+ e^-$ are:

$$\mathcal{C}^{S1}_{e\mu ds}\mathcal{O}^{S1}_{e\mu ds} = \left(\bar{e}_L \, \{\Delta^\dagger Y_e^\dagger\}_{12} \, \mu_R\right)(\bar{d}_R \, \lambda^{S1}_{12} \, s_L)$$

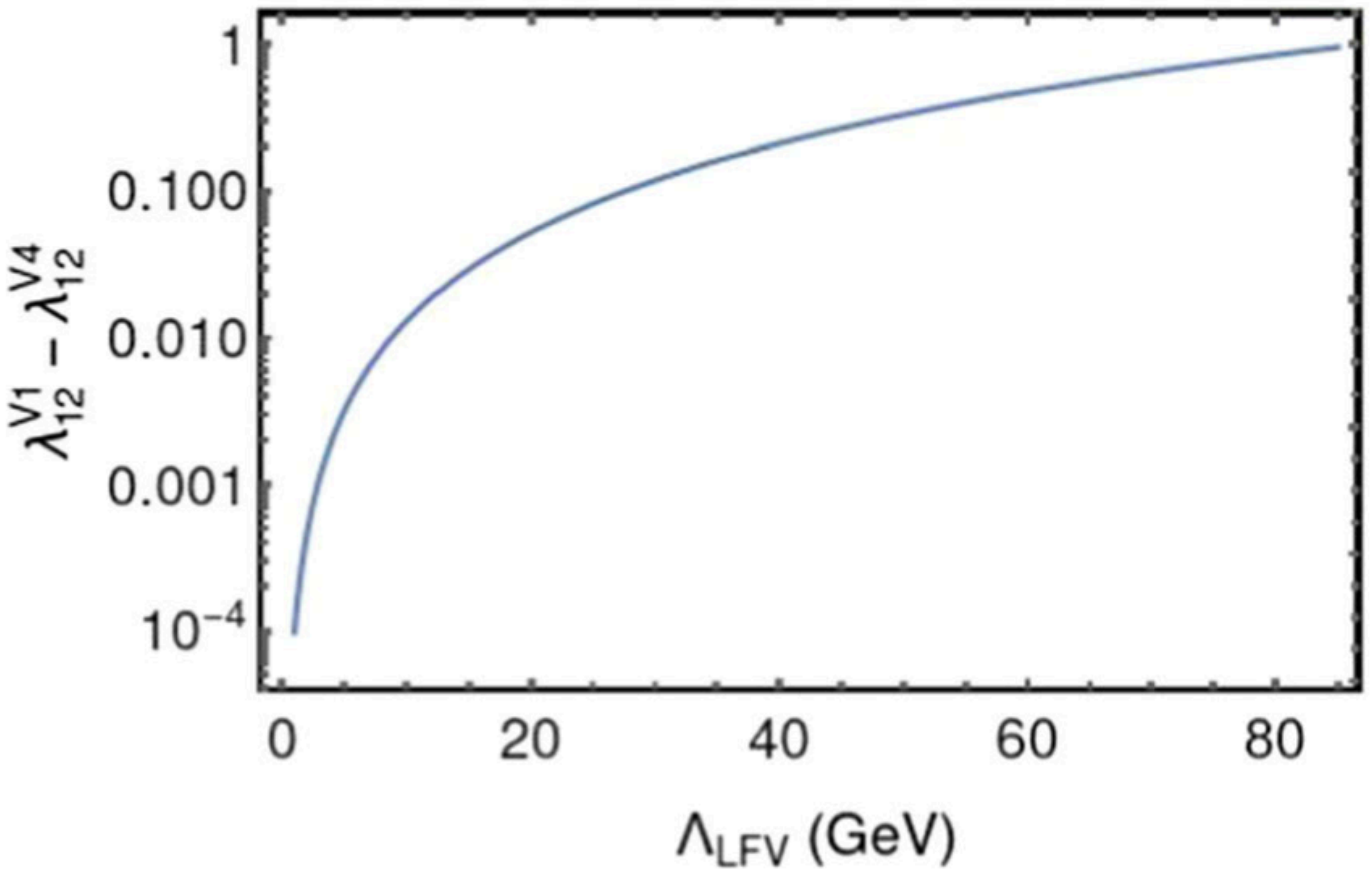
$$\mathcal{C}^{S1}_{e\mu sd}\mathcal{O}^{S1}_{e\mu sd} = \left(\bar{e}_L \, \{\Delta^\dagger Y_e^\dagger\}_{12} \, \mu_R\right)(\bar{s}_R \, \lambda^{S1}_{21} \, d_L)$$

$$\mathcal{C}^{V1}_{e\mu qq}\mathcal{O}^{V1}_{e\mu qq} = \left(\bar{e}_L \, \gamma^\mu \Delta_{12} \, \mu_L\right) \times (\bar{d}_L \, \gamma_\mu \lambda^{V1}_{12} \, s_L + \bar{s}_L \, \gamma_\mu \lambda^{V1}_{21} \, d_L)$$

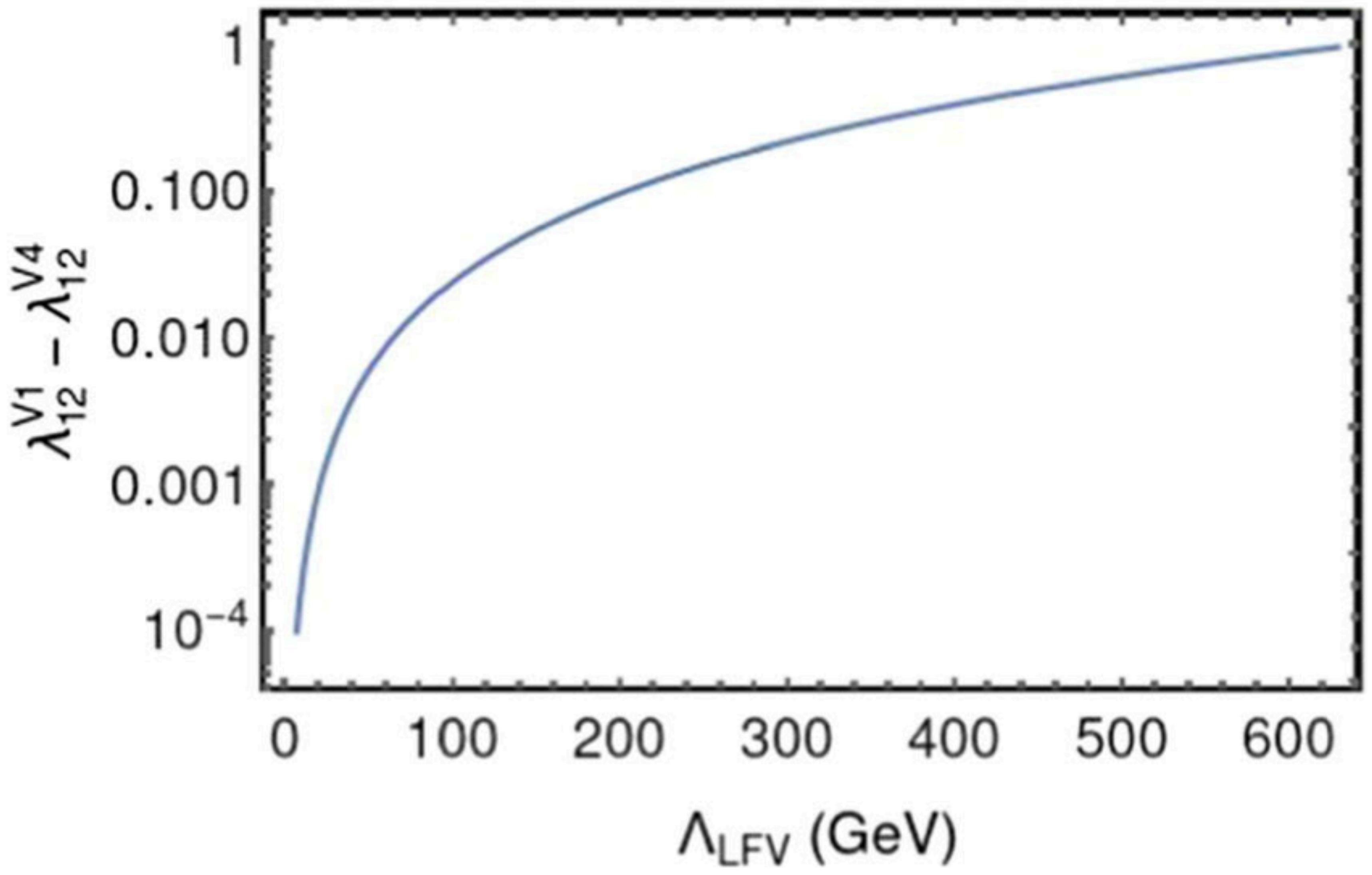
$$\mathcal{C}^{V4}_{e\mu qq} O^{V4}_{e\mu qq} = \left(\bar{e}_L \, \gamma^\mu \Delta_{12} \, \mu_L\right) \times (\bar{d}_L \, \gamma_\mu \lambda^{V4}_{12} \, s_L + \bar{s}_L \, \gamma_\mu \lambda^{V4}_{21} \, d_L)$$

$$\mathrm{BR}(K_L \to \mu^+ e^-) = \frac{|p_\mu|_{K_L\mu e}}{8\pi m^2_{K^0_L}} \frac{\tau_{K_L} B_0^2 F_0^2}{4} \times \left|\left(\frac{\mathcal{C}^{S1}_{e\mu ds}}{\Lambda^2_{LFV}\alpha} - \frac{\mathcal{C}^{S1}_{e\mu sd}}{\Lambda^2_{LFV}\beta}\right)\right|^2 \times (m^2_{K^0_L} - (m_\mu + m_e)^2) \, ,$$

$$\mathrm{BR}(K_L \to e^+ \mu^-) = \frac{|p_\mu|_{K_L\mu e}}{8\pi m^2_{K^0_L}} \frac{\tau_{K_L} F_0^2}{4} \times \left|\left(-\frac{\mathcal{C}^{V1}_{e\mu ds}}{\Lambda^2_{LFV}\alpha} + \frac{\mathcal{C}^{V1}_{e\mu sd}}{\Lambda^2_{LFV}\beta} + \frac{\mathcal{C}^{V4}_{e\mu ds}}{\Lambda^2_{LFV}\alpha} - \frac{\mathcal{C}^{V4}_{e\mu sd}}{\Lambda^2_{LFV}\beta}\right)\right|^2 \times (m^2_{K^0_L}(m_\mu^2 + m_e^2) - (m_\mu^2 - m_e^2)^2) \, ,$$



(c) Vector operator minimal field content scenario



(d) Vector operator extended field content scenario

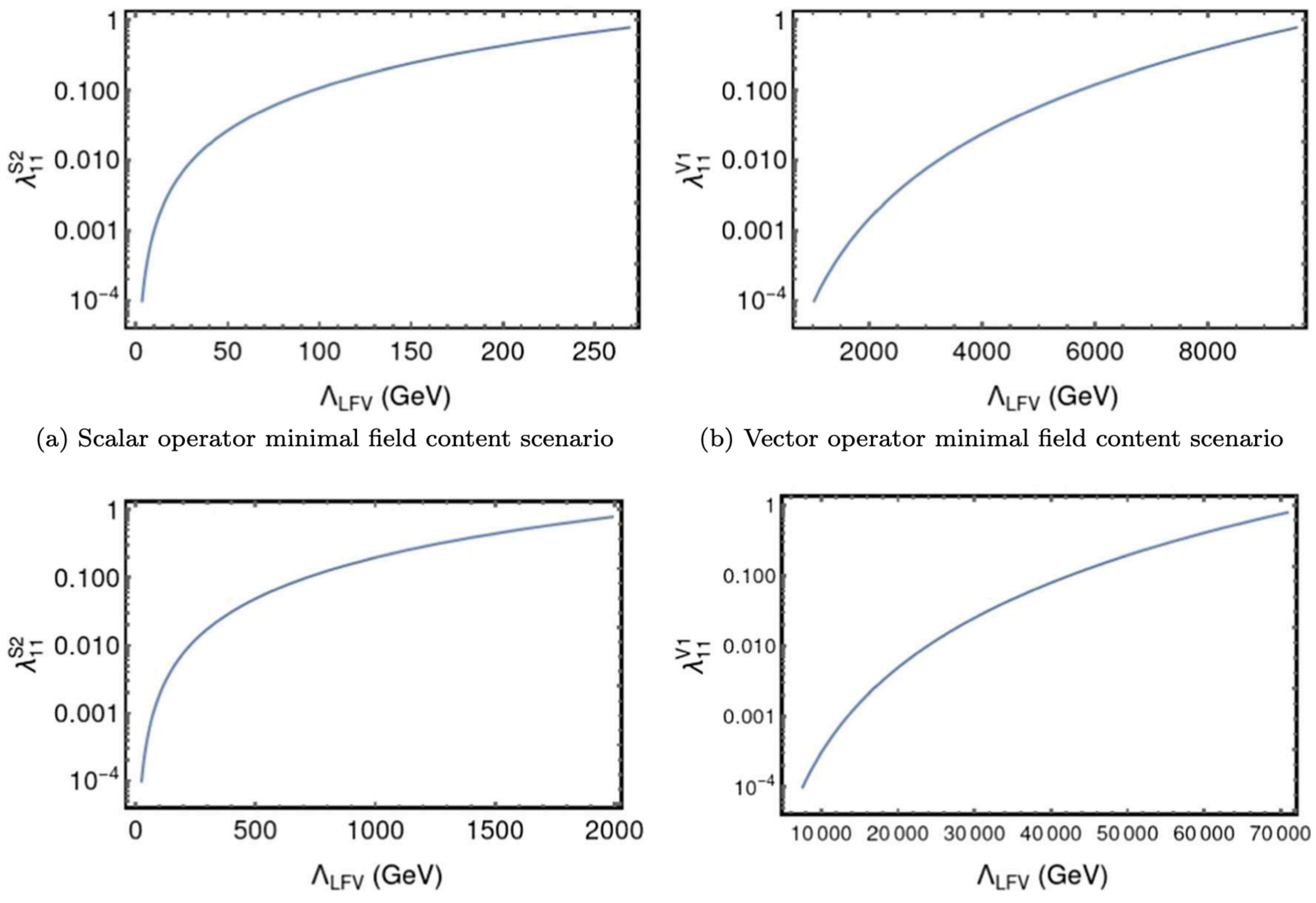
$\mu N \to eN$: The operators that contribute to $\mu N \to eN$ are:

$$\mathcal{C}^{S1}_{e\mu dd}\mathcal{O}^{S1}_{e\mu dd} = \left(\bar{e}_L \{\Delta^\dagger Y_e^\dagger\}_{12}\, \mu_R\right)(\bar{d}_R\, \lambda^{S1}_{11}\, d_L)$$
$$\mathcal{C}^{S2}_{e\mu uu}\mathcal{O}^{S2}_{e\mu uu} = \left(\bar{e}_L \{\Delta^\dagger Y_e^\dagger\}_{12}\, \mu_R\right)(\bar{u}_R\, \lambda^{S2}_{11}\, u_L)$$
$$\mathcal{C}^{V1}_{e\mu qq}\mathcal{O}^{V1}_{e\mu qq} = \left(\bar{e}_L\, \gamma^\mu \Delta_{12}\, \mu_L\right) \times (\bar{u}_L\, \gamma_\mu \lambda^{V1}_{11}\, u_L + \bar{d}_L\, \gamma_\mu \lambda^{V1}_{11}\, d_L)$$
$$\mathcal{C}^{V3}_{e\mu uu}\mathcal{O}^{V3}_{e\mu uu} = \left(\bar{e}_L\, \gamma^\mu \Delta_{12}\, \mu_L\right)(\bar{u}_R\, \gamma_\mu \lambda^{V3}_{11}\, u_R)$$
$$\mathcal{C}^{V4}_{e\mu dd}\mathcal{O}^{V4}_{e\mu dd} = \left(\bar{e}_L\, \gamma^\mu \Delta_{12}\, \mu_L\right)(\bar{d}_R\, \gamma_\mu \lambda^{V4}_{11}\, d_R)$$

$$\mathrm{BR}(\mu N \to eN) = \frac{2G_F^2}{w_{capt}}\Big(|\tilde{g}^{(p)}_{LS} S^{(p)} + \tilde{g}^{(n)}_{LS} S^{(n)} + \tilde{g}^{(p)}_{LV} V^{(p)} + \tilde{g}^{(n)}_{LV} V^{(n)}|^2 + |\tilde{g}^{(p)}_{RS} S^{(p)} + \tilde{g}^{(n)}_{RS} S^{(n)} + \tilde{g}^{(p)}_{RV} V^{(p)} + \tilde{g}^{(n)}_{RV} V^{(n)}|^2\Big) \quad (43)$$

$$\mathrm{BR}(\mu Ti \to eTi) = \frac{16}{w_{capt}}(G_S^{(d,p)} S^{(p)} + G_S^{(d,n)} S^{(n)})^2 \times \frac{|\mathcal{C}^{S2}_{e\mu dd}|^2}{\Lambda^4_{LFV}},$$

$$\mathrm{BR}(\mu Ti \to eTi) = \frac{36}{w_{capt}}(V^{(p)} + V^{(n)})^2 \frac{|\mathcal{C}^{V1}_{e\mu dd}|^2}{\Lambda^4_{LFV}}$$



(a) Scalar operator minimal field content scenario

(b) Vector operator minimal field content scenario

(c) Scalar operator extended field content scenario

(d) Vector operator extended field content scenario

Table 7 Limit on Λ_{LFV} from different lepton flavour violating decays in minimal field content scenario

Observables	Scenario	Limit on Λ_{LFV} (TeV)	Scenario	Limit on Λ_{LFV} (TeV)
$BR(\pi^0 \to e^+\mu^-)$	$\lambda_{11}^{S1} = 1$ $\lambda_{11}^{S2} = 0.1$	1×10^{-5}	$\lambda_{11}^{S1} = \frac{m_d}{v}$ $\lambda_{11}^{S2} = \frac{m_u}{v}$	4.2×10^{-8}
	$\lambda_{11}^{V3} = 1$ $\lambda_{11}^{V4} = 0.1$	1.2×10^{-3}	$\lambda_{11}^{V3} = \frac{m_u}{v}$ $\lambda_{11}^{V4} = \frac{m_d}{v}$	4.9×10^{-6}
$BR(\mu^- \to e^-e^+e^-)$	$\lambda_{11}^{S3} = 1$	1.58×10^{-2}	$\lambda_{11}^{S3} = c_{12}^e$	1.56×10^{-2}
	$\lambda_{11}^{V2} = 1$	1.083	$\lambda_{11}^{V2} = c_{12}^e$	1.069
BR $(K_L \to \mu^+e^-)$	$\lambda_{12}^{S1} = 1$	0.01	$\lambda_{12}^{S1} = \frac{m_s}{v}$	0.24×10^{-3}
	$\lambda_{12}^{V1} = 1$ $\lambda_{12}^{V4} = 0.1$	0.274	$\lambda_{12}^{V1} = \frac{m_s}{v}$ $\lambda_{12}^{V4} = \frac{m_s}{v}$	8.6×10^{-2}
$BR(\mu Ti \to eTi)$	$\lambda_{11}^{S2} = 1$	0.301	$\lambda_{11}^{S2} = \frac{m_d}{v}$	1.6×10^{-3}
	$\lambda_{11}^{V1} = 1$	10.126	$\lambda_{11}^{V1} = \frac{m_d}{v}$	0.728
$BR(\mu \to e\gamma)$	–	4.17	–	4.17

Table 8 Limit on Λ_{LFV} from different lepton flavour violating decays in extended field content scenario

Observables	Scenario	Limit on Λ_{LFV} (TeV)	Scenario	Limit on Λ_{LFV} (TeV)
$BR(\pi^0 \to e^+\mu^-)$	$\lambda_{11}^{S1} = 1$ $\lambda_{11}^{S2} = 0.1$	7.8×10^{-5}	$\lambda_{11}^{S1} = \frac{m_d}{v}$ $\lambda_{11}^{S2} = \frac{m_u}{v}$	3.1×10^{-7}
	$\lambda_{11}^{S1} = 1$ $\lambda_{11}^{S2} = 0.1$	9.21×10^{-3}	$\lambda_{11}^{V3} = \frac{m_u}{v}$ $\lambda_{11}^{V4} \frac{m_d}{v}$	3.6×10^{-5}
$BR(\mu^- \to e^-e^+e^-)$	$\lambda_{11}^{S3} = 1$	0.117	$\lambda_{11}^{S3} = c_{12}^e$	0.115
	$\lambda_{11}^{V2} = 1$	8.019	$\lambda_{11}^{V2} = c_{12}^e$	7.915
BR $(K_L \to \mu^+e^-)$	$\lambda_{12}^{S1} = 1$	0.078	$\lambda_{12}^{S1} = \frac{m_s}{v}$	1.8×10^{-3}
	$\lambda_{12}^{V1} = 1$ $\lambda_{12}^{V4} = 0.1$	2	$\lambda_{12}^{V1} = \frac{m_s}{v}$ $\lambda_{12}^{V4} = \frac{m_s}{v}$	0.636
$BR(\mu Ti \to eTi)$	$\lambda_{11}^{S2} = 1$	1.577	$\lambda_{11}^{S2} = \frac{m_d}{v}$	8.1×10^{-3}
	$\lambda_{11}^{V1} = 1$	74.97	$\lambda_{11}^{V1} = \frac{m_d}{v}$	5.397
$BR(\mu \to e\gamma)$	–	30.94	–	30.94

- Tri-bimaximal_cabibbo mixing ansatz is a possible venue for new physics that will contribute to charged lepton flavour violation

- Here we investigated cLF violating decays like $\mu \to eee, \mu \to e\gamma, \pi^0 \to \mu e, K_L \to \mu e$ and $\mu N \to eN$

- The MFV hypothesis protects TBC mixing from these lepton flavour violating decays.

- Within MFV hypothesis, the minimal field content has lower bounds, while on including the right handed neutrino the bounds become very large.