Non-perturbative effects across dark matter models: from the relic density to indirect detection

Simone Biondini

Department of Physics - University of Basel

BSM 2023 - from Theory to Experiment

6–9 Nov 2023, Hurghada Egypt

in collaboration with S. Vogl and J. Bollig (2308.14594)

FNSNF
Swiss National Science Foundation

University of Basel

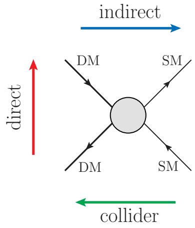
Particle interpretation of DM and freeze-out

- DM from many compelling (gravitational) observations

- DM as a particle: many candidates (Bertone and Hooper [1605.04909])
- Any model has to comply with

$$\Omega_{\rm DM} h^2(M_{\rm DM}, M_{\rm DM'}, \alpha_{\rm DM}, \alpha_{\rm SM}) = 0.1200 \pm 0.0012$$

◇ from CMB anisotropies with ΛCDM *Planck Collab. Results 2018*



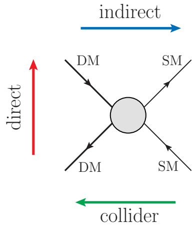
Particle interpretation of DM and freeze-out

- DM from many compelling (gravitational) observations

- DM as a particle: many candidates (Bertone and Hooper [1605.04909])
- Any model has to comply with

$$\Omega_{\rm DM} h^2(M_{\rm DM}, M_{\rm DM'}, \alpha_{\rm DM}, \alpha_{\rm SM}) = 0.1200 \pm 0.0012$$

◇ from CMB anisotropies with ΛCDM *Planck Collab. Results 2018*



Thermal freeze-out Gondolo and Gelmini (1991)

- Boltzmann equation for DM (χ)

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{\chi,\rm eq}^2)$$

- relevant processes $\chi\chi \leftrightarrow \mathrm{SM\,SM}$, $\chi\chi \leftrightarrow \chi'\chi'$
- decoupling from $H \sim n_{\rm eq} \langle \sigma_{\rm ann} v_{\rm rel} \rangle$

$$H \simeq \frac{T^2}{M_{\rm Pl}}, \quad \langle \sigma_{\rm ann} v_{\rm rel} \rangle \simeq \frac{\alpha^2}{M^2}, \quad \frac{T}{M} \approx \frac{1}{25}$$

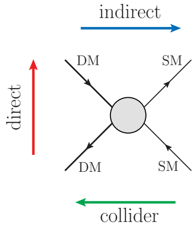
Particle interpretation of DM and freeze-out

- DM from many compelling (gravitational) observations

- DM as a particle: many candidates (Bertone and Hooper [1605.04909])
- Any model has to comply with

$$\Omega_{\rm DM} h^2(M_{\rm DM}, M_{\rm DM'}, \alpha_{\rm DM}, \alpha_{\rm SM}) = 0.1200 \pm 0.0012$$

◇ from CMB anisotropies with ΛCDM *Planck Collab. Results 2018*



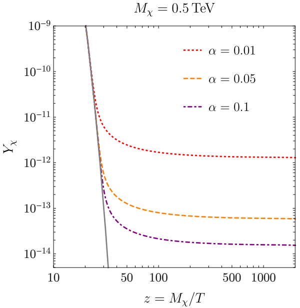
Thermal freeze-out Gondolo and Gelmini (1991)

- Boltzmann equation for DM (χ)

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{\chi,\rm eq}^2)$$

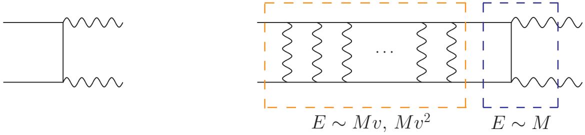
- relevant processes $\chi\chi \leftrightarrow \mathrm{SM\,SM}$, $\chi\chi \leftrightarrow \chi'\chi'$
- decoupling from $H \sim n_{\rm eq} \langle \sigma_{\rm ann} v_{\rm rel} \rangle$

$$H \simeq \frac{T^2}{M_{\rm Pl}}, \quad \langle \sigma_{\rm ann} v_{\rm rel} \rangle \simeq \frac{\alpha^2}{M^2}, \quad \frac{T}{M} \approx \frac{1}{25}$$



DM models with mediators

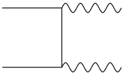
- DM and/or coannihilating partners interact with gauge bosons and scalars

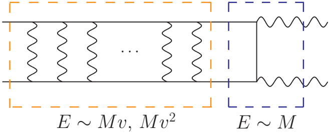


- repeated soft interactions: Sommerfeld enhancement and bound states Hisano, Matsumoto, Nojiri [hep-ph/0212022], [hep-ph/0307216]; B. von Harling and K. Petraki [1407.7874]; Beneke, Hellmann, Ruiz-Femenia [1411.6924] ...

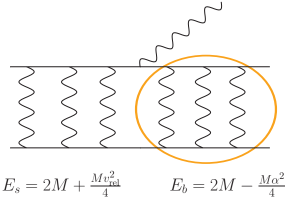
DM models with mediators

- DM and/or coannihilating partners interact with gauge bosons and scalars





- repeated soft interactions: Sommerfeld enhancement and bound states Hisano, Matsumoto, Nojiri [hep-ph/0212022], [hep-ph/0307216]; B. von Harling and K. Petraki [1407.7874]; Beneke, Hellmann, Ruiz-Femenia [1411.6924] ...



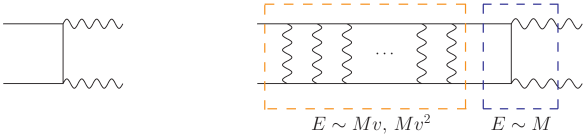
$$\frac{dn_X}{dt} + 3Hn_X = -\langle \sigma_{\rm eff}\, v_{\rm rel}\rangle (n_X^2 - n_{X,\rm eq}^2)\ ,$$

$$\langle \sigma_{\rm eff}\, v_{\rm rel}\rangle = \langle \sigma_{\rm ann}\, v_{\rm rel}\rangle + \sum_n \langle \sigma^n_{\rm bsf}\, v_{\rm rel}\rangle \frac{\Gamma^n_{\rm ann}}{\Gamma^n_{\rm ann} + \Gamma^n_{\rm bsd}}$$

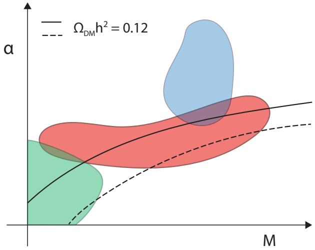
J. Ellis, F. Luo, and K. A. Olive [1503.07142], K. Petraki, M. Postma and J. de Vries [1611.01394], J. Harz and K. Petraki [1805.0120], M.Garny and J.Heisig [2112.01499]...

DM models with mediators

- DM and/or coannihilating partners interact with gauge bosons and scalars



- repeated soft interactions: Sommerfeld enhancement and bound states Hisano, Matsumoto, Nojiri [hep-ph/0212022], [hep-ph/0307216]; B. von Harling and K. Petraki [1407.7874]; Beneke, Hellmann, Ruiz-Femenia [1411.6924] ...



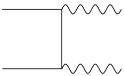
$$\frac{dn_X}{dt} + 3Hn_X = -\langle \sigma_{\rm eff}\, v_{\rm rel}\rangle (n_X^2 - n_{X,\rm eq}^2) \,,$$

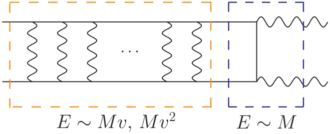
$$\langle \sigma_{\rm eff}\, v_{\rm rel}\rangle = \langle \sigma_{\rm ann}\, v_{\rm rel}\rangle + \sum_n \langle \sigma^n_{\rm bsf}\, v_{\rm rel}\rangle \frac{\Gamma^n_{\rm ann}}{\Gamma^n_{\rm ann} + \Gamma^n_{\rm bsd}}$$

J. Ellis, F. Luo, and K. A. Olive [1503.07142], K. Petraki, M. Postma and J. de Vries [1611.01394], J. Harz and K. Petraki [1805.0120], M.Garny and J.Heisig [2112.01499]...

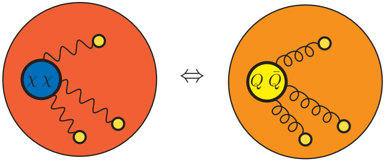
DM models with mediators

- DM and/or coannihilating partners interact with gauge bosons and scalars





- repeated soft interactions: Sommerfeld enhancement and bound states Hisano, Matsumoto, Nojiri [hep-ph/0212022], [hep-ph/0307216]; B. von Harling and K. Petraki [1407.7874]; Beneke, Hellmann, Ruiz-Femenia [1411.6924] ...



Heavy Quarkonium in QGP

- $(Q\bar{Q})$ dynamics in a thermal environment
- NRQCD and pNRQCD

  Matsui and Satz (1986); Laine, Philipsen, Romatschke and Tassler [hep-ph/0611300]; Brambilla, Ghiglieri, Petreczky, Vairo [0804.0993]

pNREFT for (pseudo)scalar mediators

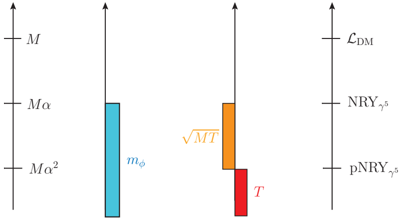
Simplified model

$$\mathcal{L} = \bar{X}(i\partial\!\!\!/ - M)X + \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 - \bar{X}(g + ig_5\gamma_5)X\phi + \mathcal{L}_{\text{portal}}\,,$$

M. B. Wise and Y. Zhang [1407.4121]; K. Kainulainen, K. Tuominen and V. Vaskonen [1507.04931]

- g_5 allows for pair annihilations $X\bar{X} \to \phi\phi$ at $\mathcal{O}(v^0)$

- pair annihilations $\to \Omega_{\rm DM} h^2$ and signals for indirect detection



- Derive $\sigma v_{\rm rel}$ and Γ in the unified framework of NREFTs
- Include near-threshold effects for $\Omega_{\rm DM} h^2$

$$M \gg M\alpha \gg \pi T \gtrsim M\alpha^2\,, \quad M\alpha \gtrsim m_\phi$$

- Include near-threshold effects for **indirect detection** [prospects for **CTA**]

NREFTs for pseudo-scalar mediators

- integrate out the hard scale $E \sim M$ [$T = 0$ matching] M. E. Luke and A. V. Manohar (hep-ph/9610534)

$$\mathcal{L}_{\text{NRY}_{\gamma_5}} = \mathcal{L}_{\psi}^{\text{bilinear}} + \mathcal{L}_{\chi}^{\text{bilinear}} + \mathcal{L}_{\text{4-fermions}} + \mathcal{L}_{\text{scalar}}$$

- interactions between NR fermions/antifermions and scalar mediators

$$\mathcal{L}_{\psi}^{\text{bilinear}} = \psi^{\dagger} \left(i\partial_0 - g\phi + g_5 \frac{\sigma \cdot [\boldsymbol{\nabla}\phi]}{2M} - g_5^2 \frac{\phi^2}{2M} + \frac{\boldsymbol{\nabla}^2}{2M} \right) \psi$$

NREFTs for pseudo-scalar mediators

- integrate out the hard scale $E \sim M$ [$T=0$ matching] M. E. Luke and A. V. Manohar (hep-ph/9610534)

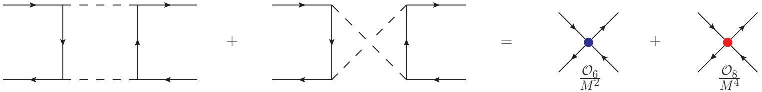
$$\mathcal{L}_{\text{NRY}_{\gamma_5}} = \mathcal{L}_\psi^{\text{bilinear}} + \mathcal{L}_\chi^{\text{bilinear}} + \mathcal{L}_{\text{4-fermions}} + \mathcal{L}_{\text{scalar}}$$

- interactions between NR fermions/antifermions and scalar mediators

$$\mathcal{L}_\psi^{\text{bilinear}} = \psi^\dagger \left(i\partial_0 - g\phi + g_5 \frac{\sigma \cdot [\boldsymbol{\nabla}\phi]}{2M} - g_5^2 \frac{\phi^2}{2M} + \frac{\boldsymbol{\nabla}^2}{2M} \right) \psi$$

- four-fermion interactions encode the annihilations into light degrees of freedom $[^{2S+1}L_J]$

G. T. Bodwin, E. Braaten and G. P. Lepage (hep-ph/9407339)



$$(\mathcal{L}_{\text{4-fermions}})_{d=6} = \frac{f(^1S_0)}{M^2}\,\mathcal{O}(^1S_0) + \frac{f(^3S_1)}{M^2}\,\mathcal{O}(^3S_1)\,, \quad \mathcal{O}(^1S_0) = \psi^\dagger\chi\,\chi^\dagger\psi\,, \quad \mathcal{O}(^3S_1) = \psi^\dagger\,\boldsymbol{\sigma}\,\chi\cdot\chi^\dagger\,\boldsymbol{\sigma}\,\psi$$

$$\text{Im}[f(^1S_0)] = 2\pi\alpha\alpha_5\mathcal{F}(m_\phi/M)\,, \quad \mathcal{F}(m_\phi/M) = \sqrt{1-\frac{m_\phi^2}{M^2}}\left(1-\frac{m_\phi^2}{2M^2}\right)^{-2}\,, \quad \text{Im}[f(^1S_0)] = 0$$

pNREFT for (pseudo)scalar mediators

- integrating out the scale $M\alpha$ and m_ϕ: EFT for DM pairs and ultrasoft/thermal scalar mediators

  SB and V. Shtabovenko (2106.06472 and 2112.10145)

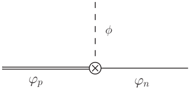
$$\mathcal{L}_{\rm pNRY_{\gamma_5}} = \int d^3\boldsymbol{r}\, \varphi^\dagger(\boldsymbol{r},\boldsymbol{R},t)\Big\{ i\partial_0 - H(\boldsymbol{r},\boldsymbol{p},\boldsymbol{P},\boldsymbol{S}_1,\boldsymbol{S}_2) - 2g\phi(\boldsymbol{R},t) - g\,\frac{r^i r^j}{4}\left[\nabla^i_R\nabla^j_R\,\phi(\boldsymbol{R},t)\right]$$
$$-g\phi(\boldsymbol{R},t)\frac{\boldsymbol{\nabla}^2_{\boldsymbol{r}}}{M^2}\Big\}\varphi(\boldsymbol{r},\boldsymbol{R},t) + \tfrac{1}{2}(\partial^\mu\phi(\boldsymbol{R},t))^2 - \tfrac{m^2}{2}\phi(\boldsymbol{R},t)^2 - \tfrac{\lambda}{4!}\phi(\boldsymbol{R},t)^4$$

- soft and ultrasoft pseudoscalar interactions are suppressed; Hamitonian and potential

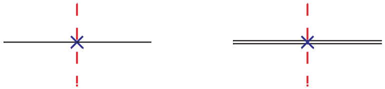
$$H(\boldsymbol{r},\boldsymbol{p},\boldsymbol{P},\boldsymbol{S}_1,\boldsymbol{S}_2) = 2M + \frac{\boldsymbol{p}^2}{M} + \frac{\boldsymbol{P}^2}{4M} - \frac{\boldsymbol{p}^4}{4M^3} + V(\boldsymbol{r},\boldsymbol{p},\boldsymbol{P},\boldsymbol{S}_1,\boldsymbol{S}_2) + \dots$$
$$V(\boldsymbol{r},\boldsymbol{p},\boldsymbol{P},\boldsymbol{S}_1,\boldsymbol{S}_2) = V^{(0)} + \frac{V^{(1)}}{M} + \frac{V^{(2)}}{M^2} + \dots\,, \qquad \boxed{V^{(0)} = -\frac{\alpha}{r}e^{-m_\phi r}}$$

- account for above-threshold and below-threshold states (for heavy quarkonium X. Yao and T. Mehen [1811.07027])

$$\varphi_{ij}(t,\boldsymbol{r},\boldsymbol{R}) = \int\frac{d^3\boldsymbol{P}}{(2\pi)^3}\Big[\sum_n e^{-iE_nt+i\boldsymbol{P}\cdot\boldsymbol{R}}\,\Psi_n(\boldsymbol{r})\,S_{ij}\,\varphi_n(\boldsymbol{P})$$
$$+\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}e^{-iE_pt+i\boldsymbol{P}\cdot\boldsymbol{R}}\,\Psi_p(\boldsymbol{r})\,S_{ij}\,\varphi_p(\boldsymbol{P})\Big]$$



Annihilations and decays



$$\mathrm{Im}[f(^1S_0)] = 2\pi\alpha\alpha_5\mathcal{F}(m_\phi/M)$$

Annihilation scattering states

$$(\sigma_{\mathrm{ann}} v_{\mathrm{rel}})(\boldsymbol{p}) = \frac{1}{M^2}\left\{\left(\mathrm{Im}[f(^1S_0)] + \mathrm{Im}[g(^1S_0)]\frac{v_{\mathrm{rel}}^2}{4}\right) S^0_{\mathrm{ann}}(\zeta,\xi) + \frac{\mathrm{Im}[f(^3P_0)] + 5\mathrm{Im}[f(^3P_2)]}{12} v_{\mathrm{rel}}^2 S^1_{\mathrm{ann}}(\zeta,\xi)\right\}$$

Bound-state decay

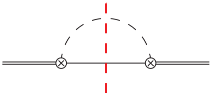
$$\Gamma^{nS}_{\mathrm{ann}} = \frac{|R_{nS}(0)|^2}{\pi M^2}\mathrm{Im}[f(^1S_0)]\,, \quad \Gamma^{nP_J}_{\mathrm{ann}} = \frac{|R'_{nP}(0)|^2}{\pi M^4}\mathrm{Im}[f(^3P_J)]\,, \quad E_{nl} = -\gamma^2_{nl}(\xi)\frac{M\alpha^2}{4n^2}$$

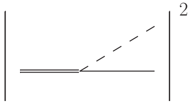
- $\zeta = \frac{\alpha}{v_{\mathrm{rel}}}$, $\xi = \frac{M\alpha}{2m_\phi}$; let us keep in mind the dependence of $\Gamma^{1S}_{\mathrm{ann}} \propto \alpha_5\alpha^4$

Bound-state formation and dissociation

Bound-state formation

$$\sigma^n_{\rm BSF} v_{\rm rel} = \frac{\alpha}{120}\left[(\Delta E^p_n)^2 - m^2_\phi\right]^{\frac{5}{2}}\left[|\langle \boldsymbol{p}|\boldsymbol{r}^2|n\rangle|^2 + 2|\langle \boldsymbol{p}|r^i r^j|n\rangle|^2\right] + 2\alpha\left[(\Delta E^p_n)^2 - m^2_\phi\right]^{\frac{1}{2}}\left|\left\langle \boldsymbol{p}\left|\frac{\nabla^2_{\boldsymbol{r}}}{M^2}\right|n\right\rangle\right|^2$$
$$-\frac{\alpha}{3}\left[(\Delta E^p_n)^2 - m^2_\phi\right]^{\frac{3}{2}}\,\mathrm{Re}\left[\left\langle \boldsymbol{p}\left|\frac{\nabla^2_{\boldsymbol{r}}}{M^2}\right|n\right\rangle\langle n|\boldsymbol{r}^2|p\rangle\right]\left[1 + n_B(\Delta E^p_n)\right]$$





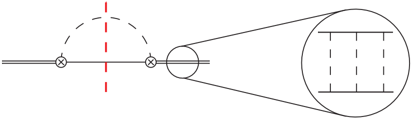
Bound-state dissiation

$$\Gamma^n_{\rm BSD} = \int_{|\boldsymbol{k}|>k_{\rm min}} \frac{d^3k}{(2\pi)^3}\, n_B(|\boldsymbol{k}|)\,\sigma^n_{\rm ion}(|\boldsymbol{k}|)\,, \quad \sigma^n_{\rm ion}(|\boldsymbol{k}|) = \frac{g^2_X}{g_\phi\, g_n}\frac{M^2 v_{\rm rel}}{4|\boldsymbol{k}|^2}\sigma^n_{\rm BSF}$$

Bound-state formation and dissociation

Bound-state formation

$$\sigma^n_{\rm BSF} v_{\rm rel} = \frac{\alpha}{120}\left[(\Delta E^p_n)^2 - m^2_\phi\right]^{\frac{5}{2}}\left[|\langle \boldsymbol{p}|\boldsymbol{r}^2|n\rangle|^2 + 2|\langle \boldsymbol{p}|r^i r^j|n\rangle|^2\right] + 2\alpha\left[(\Delta E^p_n)^2 - m^2_\phi\right]^{\frac{1}{2}}\left|\left\langle \boldsymbol{p}\left|\frac{\nabla^2_{\boldsymbol{r}}}{M^2}\right|n\right\rangle\right|^2$$
$$-\frac{\alpha}{3}\left[(\Delta E^p_n)^2 - m^2_\phi\right]^{\frac{3}{2}} \mathrm{Re}\left[\left\langle \boldsymbol{p}\left|\frac{\nabla^2_{\boldsymbol{r}}}{M^2}\right|n\right\rangle\langle n|\boldsymbol{r}^2|p\rangle\right]\left[1 + n_B(\Delta E^p_n)\right]$$

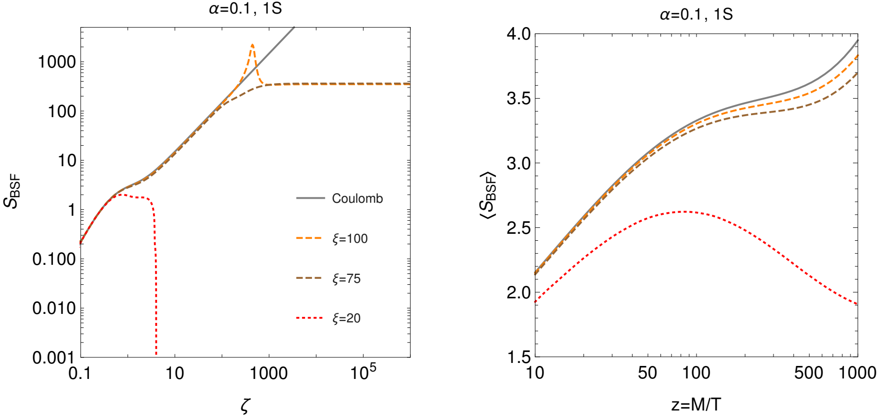


Bound-state dissiation

$$\Gamma^n_{\rm BSD} = \int_{|\boldsymbol{k}|>k_{\rm min}} \frac{d^3k}{(2\pi)^3} n_B(|\boldsymbol{k}|)\sigma^n_{\rm ion}(|\boldsymbol{k}|)\,, \quad \sigma^n_{\rm ion}(|\boldsymbol{k}|) = \frac{g^2_X}{g_\phi\, g_n}\frac{M^2 v_{\rm rel}}{4|\boldsymbol{k}|^2}\sigma^n_{\rm BSF}$$

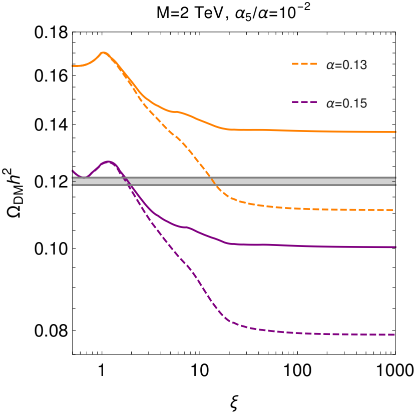
Thermal bound-state formation

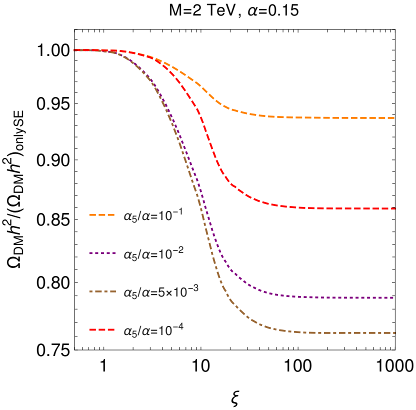
$$\zeta = \frac{\alpha}{v_{\rm rel}}\,, \quad \xi = \frac{M\alpha}{2m_\phi}\,, \quad \sigma^{1S}_{\rm BSF} v_{\rm rel} \equiv \underbrace{\frac{\pi\alpha^4}{M^2}}_{\sigma_0} S^{1S}_{\rm BSF}(\zeta,\xi)\,, \quad \langle \sigma^{1S}_{\rm BSF} v_{\rm rel}\rangle = \frac{2\sigma_0}{\sqrt{\pi}}\int_0^\infty du S^{1S}_{\rm BSF}\left(\alpha\sqrt{\frac{z}{4u}}\right)\frac{\sqrt{u}e^{\frac{|E_b|}{T}}}{e^{u+\frac{|E_b|}{T}}-1}$$



Relic density

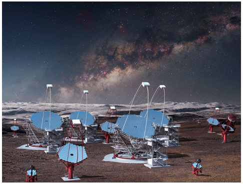
$$\frac{dn_X}{dt} + 3Hn_X = -\frac{1}{2}\langle\sigma_{\rm eff}\, v_{\rm rel}\rangle(n_X^2 - n_{X,{\rm eq}}^2)\,, \quad \langle\sigma_{\rm eff}\, v_{\rm rel}\rangle = \langle\sigma_{\rm ann}\, v_{\rm rel}\rangle + \sum_n \langle\sigma^n_{\rm BSF}\, v_{\rm rel}\rangle \frac{\Gamma^n_{\rm ann}}{\Gamma^n_{\rm ann} + \Gamma^n_{\rm BSD}}$$

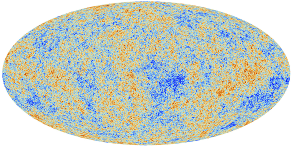


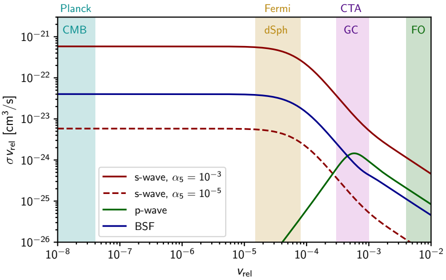


Indirect detection

- look for **DM residual annihilations** on the flux of cosmic γ**-rays**







- photons from cascade decays

$$\mathcal{L}_{\rm portal} = -\mu_{\phi h}\phi \left(H^\dagger H - \frac{v^2}{2}\right) - \frac{1}{2}\lambda_{\phi h}\phi^2 \left(H^\dagger H - \frac{v^2}{2}\right),$$

- $(X\bar{X} \to \phi\phi) \times (\phi \to \mathrm{SMSM}')$
- [left plot] $M = 1$ TeV, $m_\phi = 1$ GeV, $\alpha = 0.1$

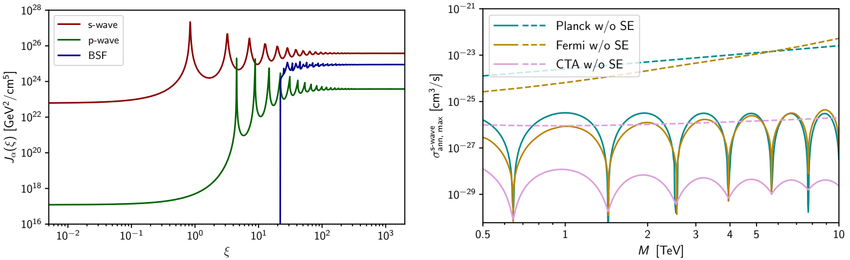
GENERALIZED J FACTORS

- The contribution of DM annihilation to the differential photon flux is F. Ferrer and D. R. Hunter (1306.6586)

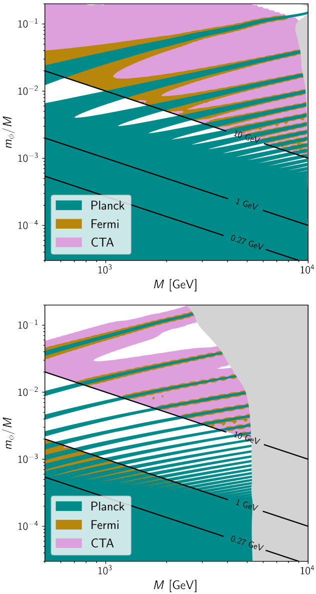
$$\frac{\mathrm{d}\Phi_\gamma}{\mathrm{d}E_\gamma} = \frac{1}{16\pi M^2}\frac{\mathrm{d}N_\gamma}{\mathrm{d}E_\gamma}\int_{\Delta\Omega}\mathrm{d}\Omega\int_0^\infty \mathrm{d}\psi \int d^3v_1 \int d^3v_2 f_X(r(\psi,\Omega),\vec{v}_1) f_X(r(\psi,\Omega),\vec{v}_2)\,\sigma v_{\mathrm{rel}}\,,$$

- $\sigma^{\text{s-wave}}_{\text{ann}} \equiv \sigma^{(0,0)}_{\text{ann}} = \mathrm{Im}[f(^1S_0)]/M^2$ and $\sigma^{\text{p-wave}}_{\text{ann}} \equiv \sigma^{(1,0)}_{\text{ann}} = (\mathrm{Im}[f(^3P_0)] + 5\mathrm{Im}[f(^3P_2)])/12M^2$

$$\frac{\mathrm{d}\Phi_\gamma}{\mathrm{d}E_\gamma} = \frac{1}{16\pi M^2}\left(\sum_f B_f \frac{\mathrm{d}N_\gamma^{(f)}}{\mathrm{d}E_\gamma}\right)\left(\sigma^{\text{s-wave}}_{\text{ann}} J^{(0,0)}_{\text{ann},\alpha}(\xi) + \sigma^{\text{p-wave}}_{\text{ann}} J^{(1,0)}_{\text{ann},\alpha}(\xi) + \sigma_{\text{BSF}} J_{\text{BSF},\alpha}(\xi)\right).$$

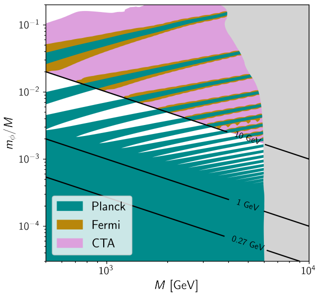


Exclusion limits for $\alpha_5/\alpha = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}$



Conclusions

- DM particles and thermal freeze-out in the early universe:
  non-relativistic particles in a thermal environment
- Focus on models with scalar force carriers between DM particles
  $\Rightarrow$ develop NREFTs similar to heavy quarkonium at $T = 0$ and finite temperature



- Developed NREFTs and pNREFT techniques for determining $\sigma_i v_{\rm rel}$ and Γ_i
- Include the leading effects of bound-state formation for both $\Omega_{\rm DM} h^2$ and **indirect detection**
- J-factors for Sommerfeld effects and BSF
- Assessed **CTA** prospects for the model

- Outlook: include **excited states** and inspect their effects on $\Omega_{\rm DM} h^2$ and **indirect detection**

[see T.Binder et al (2308.01336) for vector-mediator models and excited bound states on the thermal freeze-out]

Matching coefficients

$$\mathrm{Im}[f({}^1S_0)] = 2\pi\alpha\alpha_5\mathcal{F}(\tilde{r})\,,$$

$$\mathrm{Im}[g({}^1S_0)] = -\frac{8\pi\alpha\alpha_5}{3}\frac{\left(1-\frac{13}{8}\tilde{r}^2+\frac{5}{8}\tilde{r}^4-\frac{3}{32}\tilde{r}^6\right)}{(1-\tilde{r}^2/2)^2(1-\tilde{r}^2)}\mathcal{F}(\tilde{r})$$

$$\mathrm{Im}[f({}^3P_0)] = \frac{\pi}{6}\left[3\alpha\left(2-\frac{\mathcal{G}(\tilde{r})}{3}\right)-\alpha_5\mathcal{G}(\tilde{r})\right]^2\mathcal{F}(\tilde{r})\,,$$

$$\mathrm{Im}[f({}^3P_2)] = \frac{\pi}{15}(\alpha+\alpha_5)^2\mathcal{G}(\tilde{r})^2\mathcal{F}(\tilde{r})$$

- auxiliary functions with $\tilde{r} \equiv m_\phi/M$

$$\mathcal{F}(\tilde{r}) = \frac{\sqrt{1-\tilde{r}^2}}{\left(1-\frac{\tilde{r}^2}{2}\right)^2}\,, \quad \mathcal{G}(\tilde{r}) = \frac{1-\tilde{r}^2}{1-\frac{\tilde{r}^2}{2}}\,.$$

J FACTORS

$$\int d^3 v f_X(\vec{r},\vec{v}) = \rho_X(\vec{r}) \quad J_0 = \int d\Omega \int d\psi \, \rho_X(\vec{r}(\psi,\Omega))^2$$

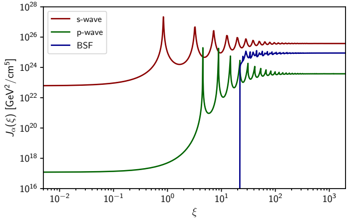
- In the following, we assume that we can split up the cross section as

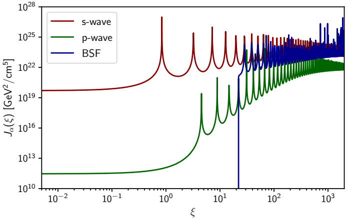
$$\sigma v_{\rm rel} = \sum_{l,j=0}^{\infty} \sigma_{\rm ann}^{(l,j)} v_{\rm rel}^{2(l+j)} S_{\rm ann}^{(l)}(\zeta,\xi) + \sum_{n>l} \sigma_{\rm BSF}^{(n,l)} S_{\rm BSF}^{(n,l)}(\zeta,\xi)\,.$$

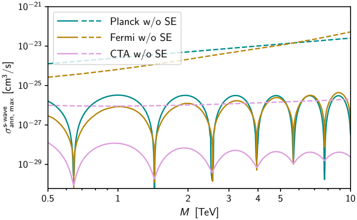
- This motivates the definition of a generalized J-factor

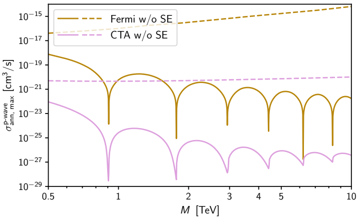
$$J_{{\rm ann},\alpha}^{(l,j)}(\xi) \equiv \int_{\Delta\Omega} d\Omega \int_0^\infty d\psi \int d^3 v_1 f_X(r(\psi,\Omega),\vec{v}_1) \int d^3 v \, v_2 f_X(r(\psi,\Omega),\vec{v}_2) v_{\rm rel}^{2(l+j)} S_{\rm ann}^{(l)}(\alpha/v_{\rm rel},\xi)$$

- for f_X we use methods for the analysis of galactic dynamics that hold in collisionless systems in a quasi-equilibrium state J. Binney and S. Tremaine (Galactic Dynamics 2008)









- [UP] Galactic center and Dwarf Spheroidal Galaxies
- [DOWN] Limits on the s-wave and p-wave dominated DM annihilation scenarios, $m_\phi = 10$ GeV, $\alpha = 0.1$

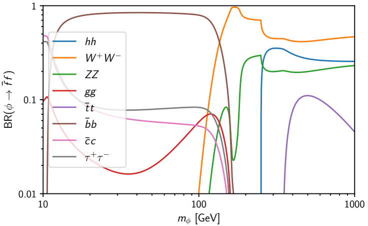
Photon spectrum

- $\frac{dN_\gamma}{dE_\gamma}$ number of photons produced by annihialting dark matter per photon energy

1) Br$_\phi$ into various SM final states
2) γ produces by a given SM final state (decay or hadronization)
3) prescription for the boost of the decaying ϕ in the galactic rest frame

- high-mediator mass range: $m_\phi \geq 10$ GeV; $\phi \to \{hh, W^+W^-, ZZ, gg, \bar{t}t, \bar{b}b, \bar{c}c, \tau^+\tau^-\}$

A. Djouadi (hep-ph/0503172)



- low-mediator mass range $2m_{\pi^0} \leq m_\phi \leq 1$ GeV: decays into pion pairs play a large role; $\bar{X} + X \to 2\phi \to 4\pi_0 \to 8\gamma$ [we use here M.W. Winkler (1809.01876)]