LOCAL PHOTONS AND THE ORIGIN OF THE CASIMIR EFFECT

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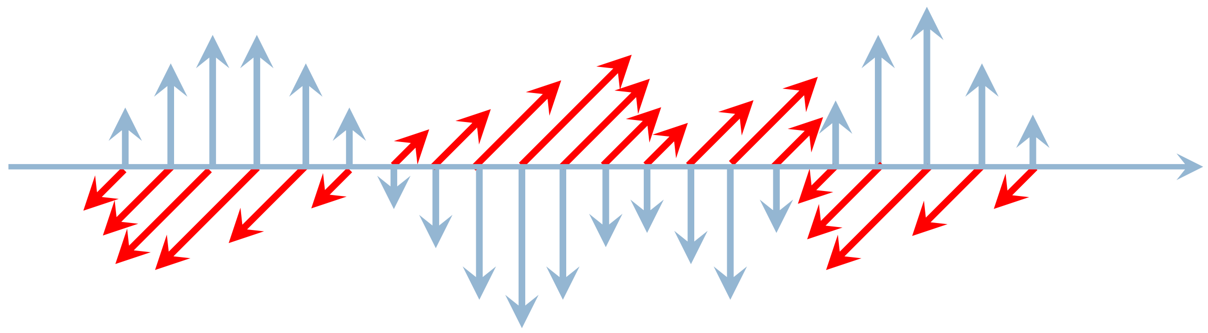
Part I: Background

Representations of the EM field

Electromagnetic waves have an electric component E and a magnetic component B.



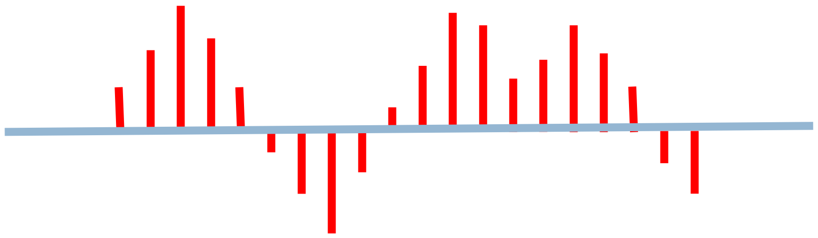
The fields have both a magnitude and an orientation perpendicular to each other and the direction of propagation.

A representation of the field is the chosen set of parameters that specify the properties of the field. Different representations have various advantages and disadvantages.

Position and momentum

The position representation

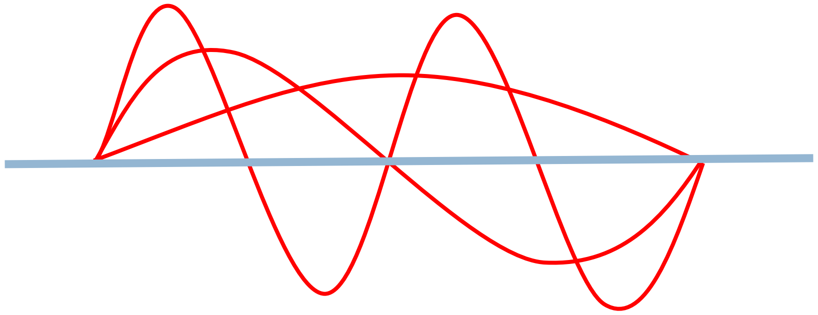
The field is built up of localised parts. The magnitude of each part is specified by its position. Useful for modelling local dynamics.



$E(x,t)$ and $B(x,t)$

The momentum representation

The field is built up of non-localised harmonics. The magnitude is specified by the frequency of each part. Useful for modelling stationary systems.



$\tilde{E}(k,t)$ and $\tilde{B}(k,t)$

Quantising the EM field

Quantisation is usually in the momentum representation via canonical quantisation.

- Annihilation and creation operators $a_\lambda(k,t)$ and $a_\lambda^\dagger(k,t)$ annihilate and create excitations of a single frequency $|k|c$ and polarisation λ.
- Bosonic commutation relation.

$$\left[a_\lambda(k,t), a_{\lambda'}^\dagger(k',t)\right] = \delta_{\lambda\lambda'}\,\delta(k-k')$$

Orthogonal excitations can be used as a basis for single-photon wave packets:

$$|1\rangle = \sum_\lambda \int_{-\infty}^{\infty} dk\;\psi_\lambda(k)\,a_\lambda^\dagger(k,t)|0\rangle$$

$|1\rangle$ - single-photon state $|0\rangle$ - Vacuum state $\psi_\lambda(k)$ - normalised momentum wave function

Field Observables

- In the momentum representation there is a simple relationship between the excitations and the fields.

$$\tilde{\mathcal{E}}(k,t) = \sqrt{\frac{\hbar|k|}{2A\varepsilon}}\,[a_H(k,t)\hat{\mathbf{y}} + a_V(k,t)\hat{\mathbf{z}}]$$

$$\widetilde{\mathcal{B}}(k,t) = \sqrt{\frac{\hbar|k|}{2A\varepsilon}}\,[-a_V(k,t)\hat{\mathbf{y}} + a_H(k,t)\hat{\mathbf{z}}]$$

- A states containing $a_\lambda(k,t)$ excitations generates a field with a wave vector k and polarisation λ.

Motivations

- Although quantisation possible in momentum representation, current descriptions are insufficient in some cases (Mirror Hamiltonians).
- Problems arise when we attempt to quantise the field in the position representation:
    - Causality violations
    - Non-locality of wave function
    - No spherical symmetry (3D only)

Question: Is there an analogous and complementary description in position space?

Yes, there is!

Part II: Local photons

Localised wave packets

Starting point: Maxwell's equations in free space.

$$\left[\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right] O(x,t) = 0 \quad \Rightarrow \quad \left[\frac{\partial}{\partial x} - \frac{1}{c}\frac{\partial}{\partial t}\right]\left[\frac{\partial}{\partial x} + \frac{1}{c}\frac{\partial}{\partial t}\right] O(x,t) = 0$$

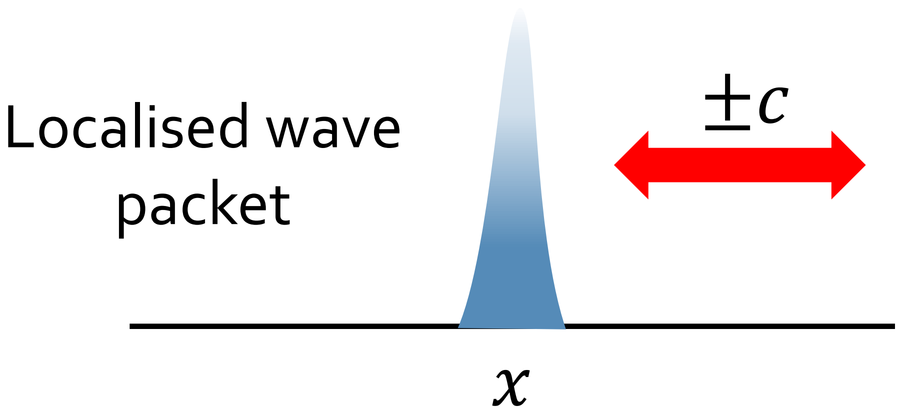
$O = E, B$ $\quad c =$ speed of light $\quad$ 1D fields depends on x coordinate only

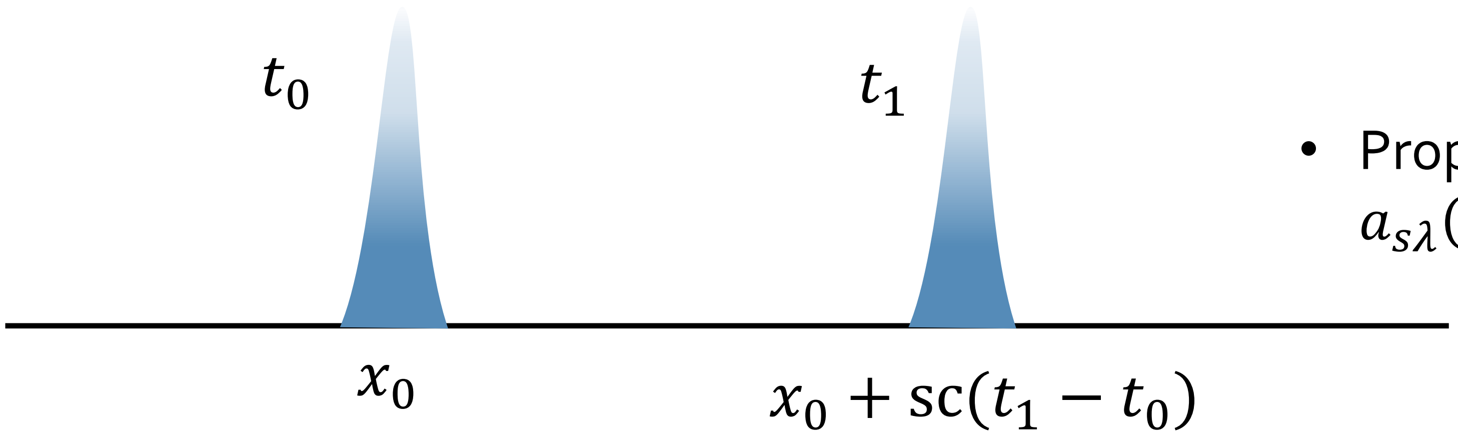
There is a simple solution in the position representation

$$O(x,t) = \sum_{s=\pm 1} O_s(x - sct)$$

- Wave has any shape
- Propagates without dispersion
- $s = \pm 1$ indicate propagation to the left and the right

Local photons





- Introduce localised basis of excitations characterised by
  - Position x at a given time
  - Polarisation λ
  - Direction of propagation s

- Creation and annihilation operators: $a_{s\lambda}(x,t)$ and $a^{\dagger}_{s\lambda}(x,t)$

- Propagation at speed of light: $a_{s\lambda}(x,t) = a_{s\lambda}(x - sct, 0)$

New parameters

- To ensure a localised basis, we need a local commutation relation:

$$\left[a_{s\lambda}(x,t), a^{\dagger}_{s',\lambda'}(x',t)\right] = \delta_{ss'}\,\delta_{\lambda\lambda'}\,\delta(x-x').$$

- Single-photon wave packets constructed as in momentum space:

$$|1\rangle = \sum_{\lambda} \int_{-\infty}^{\infty} dk\ \psi_{s\lambda}(x)\, a^{\dagger}_{s\lambda}(x,t)|0\rangle$$

$\psi_{s\lambda}(x)$ - normalised position wave function

- Relationship to momentum operators:

$$a_{s\lambda}(x,t) = \int_{-\infty}^{\infty} \frac{dk}{\sqrt{2\pi}} e^{iskx}\, a_{s\lambda}(k,t)$$

- Localisation requires contributions from all wave vectors.
- Localisation is possible for each s independently.
- s is a new parameter unrelated to k
- Number of degrees of freedom are doubled

Dynamical Hamiltonian

Additional degrees of freedom correspond to negative frequency states

$$\left[\frac{\partial}{\partial x} - \frac{s}{c}\frac{\partial}{\partial t}\right] a_{s\lambda}(x,t) = 0 \quad \Longrightarrow \quad H_{dyn}(t) = -i \sum_{s,\lambda} \int_{-\infty}^{\infty} dx\, \hbar s c\, a^{\dagger}_{s\lambda}(x,t) \frac{\partial}{\partial x} a_{s\lambda}(x,t)$$

- Overcome causality problems (Hegerfeldt 1974, Malament 1996)
- A state with a negative frequency -ω has the same positive energy as a state with the opposite frequency ω.

$\boxed{H_{dyn}(t) \neq \text{energy}}$

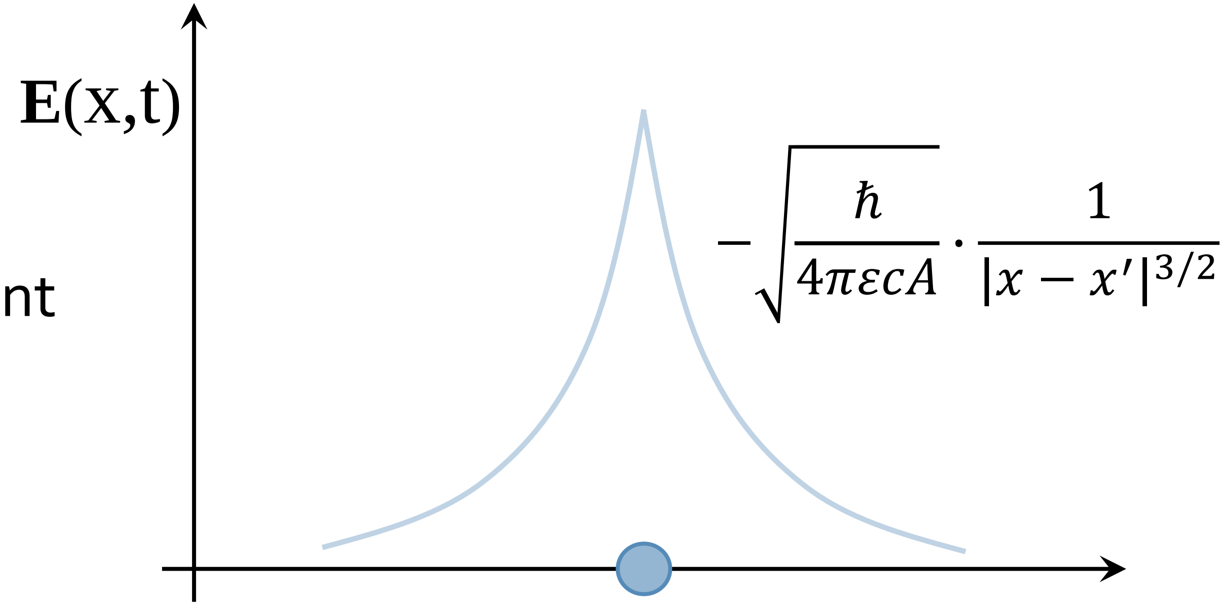
Field observables

$$\mathrm{E}(x,t) = \sum_{s=\pm 1} \int_{-\infty}^{\infty} dx' \; c\, \Re(x-x')[a_{sH}(x',t)\,\hat{\mathbf{y}} + a_{sV}(x',t)\,\hat{\mathbf{z}}] + \mathrm{H.c}$$

$$\mathrm{B}(x,t) = \sum_{s=\pm 1} \int_{-\infty}^{\infty} dx' \; s\, \Re(x-x')[-a_{sV}(x',t)\,\hat{\mathbf{y}} + a_{sH}(x',t)\,\hat{\mathbf{z}}] + \mathrm{H.c}$$

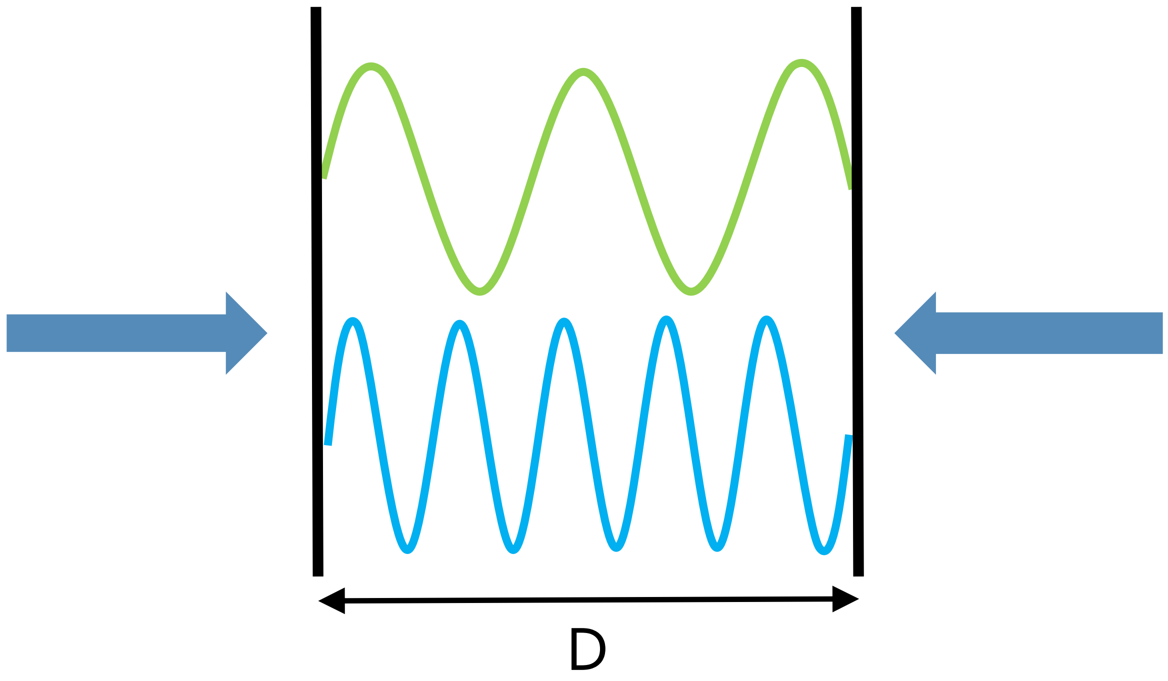
$\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$ - unit polarisation vectors

- $\Re(x-x')$ determines how the field responds to local excitations at different positions.
- $\Re(x-x')$ is determined from the energy.
- The fields display non-locality.



Part III: The Casimir effect

The Casimir effect

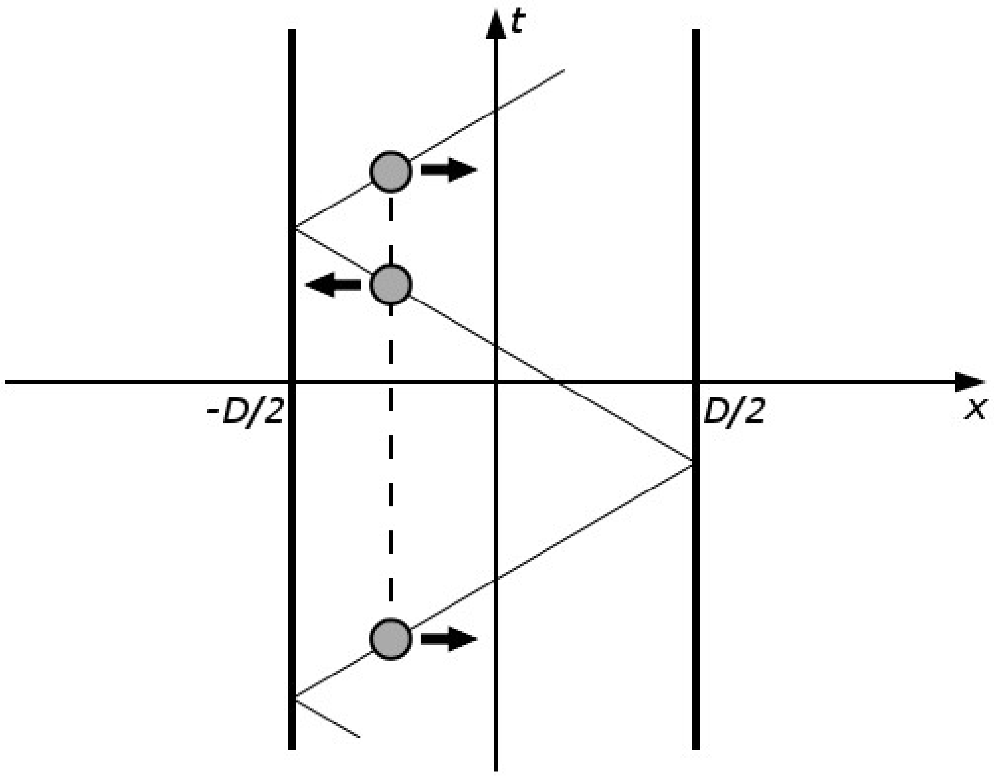


- Predicts a small attractive force between the walls of a cavity with flat reflecting sides. In 1D $F \propto D^{-2}$.
- Force generated by changes in the zero-point energy of the field inside the cavity.
- Calculations usually based on standing wave modes.

Questions: Can an alternative derivation be found in terms of local excitations? Does this method have any advantages?

Local photons inside a cavity

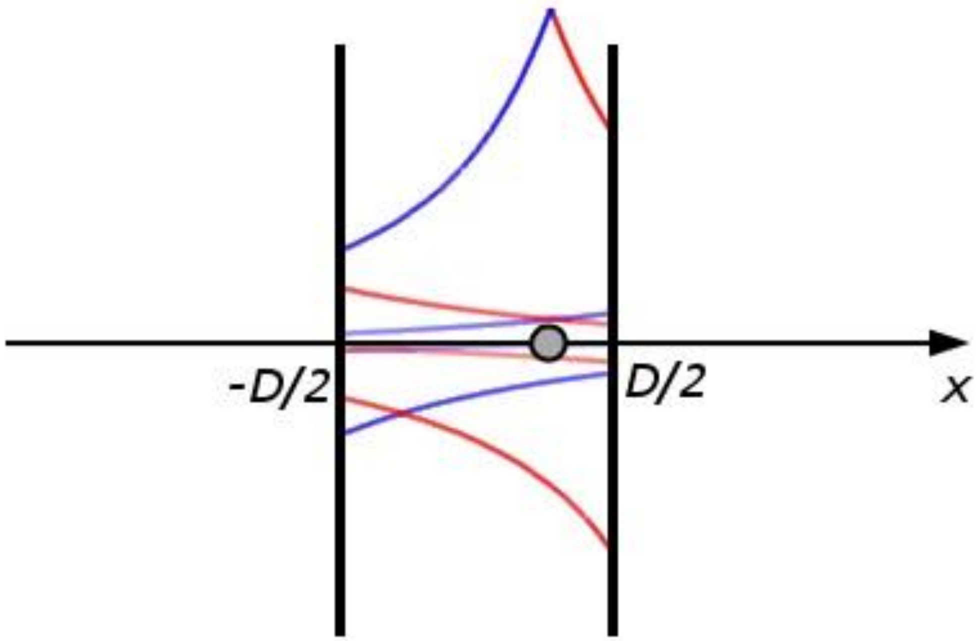
- Local photons interact only locally with their surroundings.
- At the mirror surfaces, local excitations are completely reflected resulting in a change of direction.
- When not in contact with the mirror, local excitations behave as if in free space



This motion impacts the propagation of the field near the cavity. This produces a change in the vacuum energy of the field.

Cavity field observables

- The field depends non-locally on the position of the local excitations.
- The contribution to the field observables are also reflected at the boundaries. excitations outside cannot contribute to the field inside.
- Due to non-locality, the same field can contribute several times to the total field at the same position.



Inside cavity:

Either $\boldsymbol{E}(\boldsymbol{x},\boldsymbol{t})$ or $\boldsymbol{B}(\boldsymbol{x},\boldsymbol{t})$

$$O^{\mathrm{Cav}}(x,t) = \sum_{n=-\infty}^{\infty} \left[O^{\mathrm{free}}(x+2nD,t) \pm O^{\mathrm{free}}(-x+(2n-1)D,t) \right]$$

Zero-point energies

- The zero-point energy $\langle 0 | H_{\text{energy}} | 0 \rangle$ is the vacuum expectation value of the expression

$$H_{\text{energy}} = \sum_{s\lambda} \int_{-D/2}^{D/2} dx \, \frac{\varepsilon A}{2} \left[E^{\text{cav}}(x,t)^2 + \, c^2 B^{\text{cav}}(x,t)^2 \right]$$

- Using the expressions for the fields in a cavity and the blip commutators we find that

$$H_{\text{ZPE}} = \frac{\hbar c}{4\pi} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} dx \int_{-D/2}^{D/2} dx' \; |(x - x')(x - x' + 2nD)|^{-3/2}$$

$$\boxed{= -\frac{\hbar c}{2\pi D} \sum_{n=-\infty}^{\infty} \frac{1}{n^2}}$$

n is related to the difference in the number of reflections in the overlap.

The Casimir force

- The expression for the zero-point energy is infinite. Nevertheless, the correction to the free ZPE can be determined.

- The $n = 0$ term is the complete overlap between two excitations of the EM field.
- This is the free field contribution to the zero-point energy.
- Outside the cavity the zero-point energy is unchanged
- The difference is a finite correction.
- This leads to a finite and attractive force.

$$H_{\mathrm{ZPE}}^{\mathrm{free}} = \frac{\varepsilon A}{4} \sum_{s\lambda} \int_{-D/2}^{D/2} dx \left\langle 1_{s\lambda}^{\mathrm{field}}(x,t) \middle| 1_{s\lambda}^{\mathrm{field}}(x,t) \right\rangle$$

$$H_{\mathrm{ZPE}}^{\mathrm{corr}} = -\frac{\hbar c}{\pi D} \sum_{n=1}^{\infty} \frac{1}{n^2} = -\frac{\hbar c \pi}{6D}$$

$$\boxed{F = -\frac{\partial H_{\mathrm{ZPE}}^{\mathrm{corr}}}{\partial D} = -\frac{\hbar c \pi}{6D^2}}$$

Comments and conclusions

Local Photons

- We introduced a notation for orthogonal excitations in the position representation.
- By specifying the direction of propagation, we double the usual number of degrees of freedom. There are now negative as well as positive frequency states.
- A consequence of expanding the Hilbert space is that $H_{\text{dyn}} \neq H_{\text{energy}}$.
- There is a non-local relationship between the particles and the fields. This property is responsible for the Casimir effect.

Casimir effect

- By modelling the local interactions between the mirrors and the local excitations, new field observables can be derived leading to a modified vacuum energy.
- The resulting force is correct up to an overall factor of 2. The result is dependent on the phase picked up by the fields at the mirror surface.
- This method does not require any form of regularisation to make the result finite. Infinities cancel naturally.

Thank you for your attention