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One-loop Fierz identities

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Outline

1. Introduction
2. Fierz transformations
3. Applications
4. Summary

based on: 2208.10513, 2211.01379, 2306.16449 in collaboration with Marko Pesut and Zach Polonsky

Outline

Motivation

EFT computations

Matching and running

Different bases

Simplified calculation

Combine results

Basis changes

Outline

Fierz transformations

Four-fermion (4F) operators

$\mathcal{O}_{4F} = \mathcal{F}\mathcal{O}_{4F}$

Example

$(\overline{q}_1^\alpha \gamma_\mu P_L q_2^\beta)(\overline{q}_3^\beta \gamma^\mu P_R q_4^\alpha) = -2(\overline{q}_1 P_R q_4)(\overline{q}_3 P_L q_2)$

$\boldsymbol{d = 4 - 2\epsilon}$

$\mathcal{O}_{4F} = \mathcal{F}\mathcal{O}_{4F} + E_{\mathcal{O}}$

Evanescent operators

Definition

$E_{\mathcal{O}} = \mathcal{O}_{4F} - \mathcal{F}\mathcal{O}_{4F}$

Evanescent

$E_{\mathcal{O}} \stackrel{d \to 4}{\to} 0$

Basis

$\{\mathcal{O}_j, E_i\}$

Evanescent operators: Complication

E_i

Finite contributions from one-loop insertions

Scheme dependence

ADMs and matching

Solution

Interpret finite contributions as one-loop shifts in $\mathcal{F}$

Traditional way

Basis

$\{\mathcal{O}_j, E_i\}$

Fierz identities

$\mathcal{O}_i = \mathcal{F}\mathcal{O}_i + E_i$

Finite contributions

Resulting from E_i

Novel way: One-loop Fierz identities

JA/Pesut: 2208.10513

Basis

$\{\mathcal{O}_j\}$

Fierz identities

$\mathcal{O}_i = \mathcal{F}\mathcal{O}_i + \frac{\alpha_s}{4\pi}\sum_j a_j \mathcal{O}_j$

One-loop shifts

Expressed in physical basis

Computation

One-loop corrections

$L\mathcal{O}, \quad L\mathcal{F}\mathcal{O}$

Taking difference

$L\mathcal{O} - L\mathcal{F}\mathcal{O} = LE$

Finite shifts

$LE = \frac{\alpha_s}{4\pi} \sum_i a_i \mathcal{O}_i$

One-loop computation

Diagrams

Vertex corrections, penguins

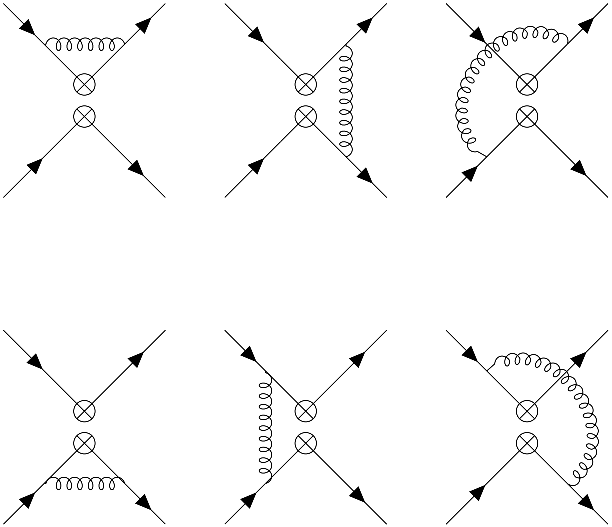
Divergent parts

since $E \sim \mathcal{O}(\epsilon)$

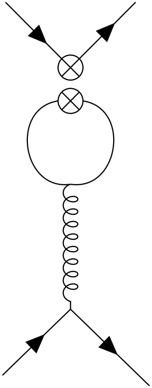
Matrix elements

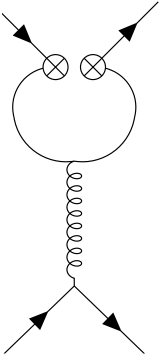
tree-level

Vertex corrections



Penguin diagrams





Operators

Four-fermion operators

4q, SL, 4l

Dirac structures

vector, scalar, tensor

Colour

singlet and crossed

Four-fermi operators

JA/Pesut: 2208.10513

Basis

$\gamma_\mu P_A \otimes \gamma^\mu P_B\,, \quad P_A \otimes P_B\,, \quad \sigma_{\mu\nu} P_A \otimes \sigma^{\mu\nu} P_A \qquad A, B = L, R$

Contributions

QCD, QED

General scheme

$\gamma_\mu \gamma_\nu \gamma_\rho P_L \otimes \gamma^\mu \gamma^\nu \gamma^\rho P_L = 4(4 - a_1 \epsilon) \gamma_\mu P_L \otimes \gamma^\mu P_L$

Dipole operators

JA/Pesut/Polonsky: 2211.01379

Mass effects

Penguin diagrams

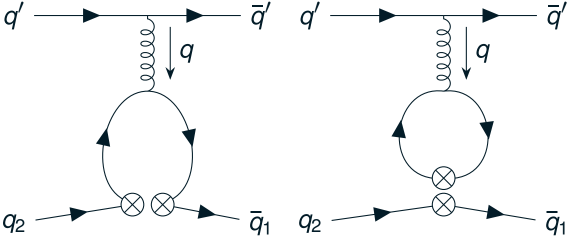
Contributions

QCD, QED

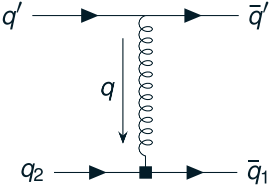
Amplitude level

Dipoles generated through Fierzing

Penguin diagrams



Dipole diagram



Four-fermi operators

JA/Pesut/Polonsky: 2211.01379

Basis

$\gamma_\mu P_A \otimes \gamma^\mu P_B\,, \quad P_A \otimes P_B\,, \quad \sigma_{\mu\nu} P_A \otimes \sigma^{\mu\nu} P_A \qquad A, B = L, R$

Contributions

QCD, QED

Dipoles

$D^B_{q_1 q_2 G} = \frac{1}{g_s} m_q (\bar{q}_1 \sigma^{\mu\nu} P_B T^A q_2) G^A_{\mu\nu}$

$D^B_{f_1 f_2 \gamma} = \frac{1}{e} m_f (\bar{f}_1 \sigma^{\mu\nu} P_B f_2) F_{\mu\nu}$

Outline

LQ matching

Scalar leptoquark

$\mathcal{L}_{q\ell}^{LQ} = \bar{q}\left(\Gamma_L^S P_L + \Gamma_R^S P_R\right)\ell\,\Phi^* + \mathrm{h.c.}$

Matching

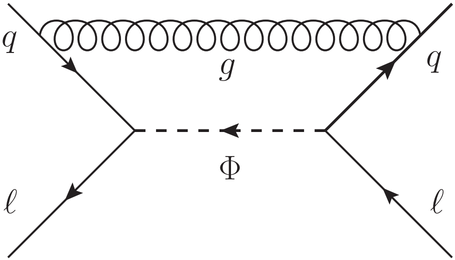
SL operators

QCD corrections

$\sim$ 10 % corrections

JA/Crivellin/Greub: 1811.08907

Matching: Example



Issue

Matching: LQ basis

$$\widetilde{O}_S^{AB} = (\overline{q} P_A \ell)(\overline{\ell} P_B q)$$
$$\widetilde{O}_T^{A} = (\overline{q} \sigma_{\mu\nu} P_A \ell)(\overline{\ell} \sigma^{\mu\nu} P_A q)$$

Running: SM basis

$$O_S^{AB} = (\overline{q} P_A q)(\overline{\ell} P_B \ell)$$
$$O_T^{A} = (\overline{q} \sigma_{\mu\nu} P_A q)(\overline{\ell} \sigma^{\mu\nu} P_A \ell)$$

Combine results

One-loop Fierz

Basis change

Tree-level Fierz

$$R_0 = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{8} \\ -6 & \frac{1}{2} \end{pmatrix}$$

One-loop Fierz

$$R_1 = \begin{pmatrix} 0 & \frac{N_c^2-1}{16N_c} \\ \frac{7-7N_c^2}{N_c} & 0 \end{pmatrix}$$

$$\begin{pmatrix} O_S^{AA} \\ O_T^{A} \end{pmatrix} = R_0 \begin{pmatrix} \widetilde{O}_S^{AA} \\ \widetilde{O}_T^{A} \end{pmatrix} + \frac{\alpha_s}{4\pi} R_1 \begin{pmatrix} O_S^{AA} \\ O_T^{A} \end{pmatrix}$$

Wilson coefficients

$$
\begin{aligned}
\begin{pmatrix} C_S^{AA} \\ C_T^{A} \end{pmatrix} &= \left[R_0 + \frac{\alpha_s}{4\pi} R_1 R_0\right]^{-T} \begin{pmatrix} \widetilde{C}_S^{AA} \\ \widetilde{C}_T^{A} \end{pmatrix} \\
&= \left[R_0^{-T} - \frac{\alpha_s}{4\pi} R_1^T R_0^{-T}\right] \begin{pmatrix} \widetilde{C}_S^{AA} \\ \widetilde{C}_T^{A} \end{pmatrix} \\
&= \left[\begin{pmatrix} -\frac{1}{2} & -6 \\ -\frac{1}{8} & \frac{1}{2} \end{pmatrix} + \frac{\alpha_s}{4\pi} \begin{pmatrix} -\frac{7}{4} C_F & 0 \\ \frac{1}{16} C_F & 0 \end{pmatrix}\right] \begin{pmatrix} \widetilde{C}_S^{AA} \\ \widetilde{C}_T^{A} \end{pmatrix}
\end{aligned}
$$

Problematic fermion traces

NDR

$Tr[\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_5]$ not well defined

Ways out

Other scheme, other basis

Example

$(g-2)_\mu$: Tensor mixing into dipole

Example: $(g-2)_\mu$

Tensor

$O^{T,RR}_{\mu\mu cc} = (\overline{\mu}\sigma_{\alpha\beta}P_R\mu)(\overline{c}\,\sigma^{\alpha\beta}P_Rc)$

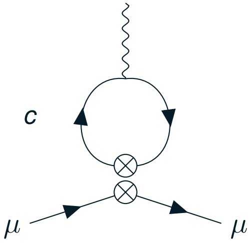
Dipole

$D^R_\mu = (\overline{\mu}\,\sigma_{\alpha\beta}P_R\mu)F^{\alpha\beta}$

Problematic trace

Tensor mixing into dipole

Closed penguin



$$O^{T,RR}_{\mu\mu cc} = (\overline{\mu}\sigma_{\alpha\beta}P_R\mu)(\overline{c}\,\sigma^{\alpha\beta}P_R c)$$

Solution

Fierzed basis

$O^{T,RR}_{\mu cc\mu} = (\overline{\mu}\sigma_{\alpha\beta}P_R c)(\overline{c}\,\sigma^{\alpha\beta}P_R\mu)$

Open penguin

Use NDR

One-loop shift

$O^{T,RR}_{\mu\mu cc} = O^{T,RR}_{\mu cc\mu} + \text{shift}$

Computation

$O^{T,RR}_{\mu cc\mu}$

$$-i\frac{e}{16\pi^2}N_c m_c Q_c(\overline{\mu}\not{q}\gamma_\mu P_R\mu)\varepsilon^\mu(q)\left[1+8\log\left(\frac{m_c^2}{\mu^2}\right)\right]L^{T,RR}_{\substack{eu\\ \mu cc\mu}}$$

Shift

JA/Pesut/Polonsky: 2211.01379

$$T^{RR}_{\mu\mu cc}=\mathcal{F}\,T^{RR}_{\mu cc\mu}+N_c\,Q_c\,m_c\,\mathcal{D}^{\mu\mu\gamma}_L$$

Final contribution

JA//Dekens/Jenkins/Manohar/Sengupta/Stoffer: 2102.08954

$$a^{\mu\mu cc}_\mu=-m_\mu\frac{N_c Q_c m_c}{Q_\mu\pi^2}\log\left(\frac{\mu^2}{m_c^2}\right)L^{T,RR}_{\substack{eu\\ \mu\mu cc}}$$

Outline

Summary

Evanescent operators

Shifts in Fierz transformations

Shifts

One-loop QCD, QED

Applications

LQ matching, problematic traces