From flavour to dark matter

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The flavour problem

- Why are the masses of fermions are hierarchical among and within three families with the charged lepton masses being of the same order as down-type quark masses?
- What is the origin of quark-mixing?
- What is the origin of neutrino masses and mixing?
- This decade (2012-2022) is being dominated by flavour anomalies in b-quark transitions ($b \to sll$ and $b \to c\nu l$).

The Froggart-Nielson mechanism

Solution

- An abelian flavour symmetry $U(1)_F$ is added to the SM in such a way that only top quark acquires its mass through renormalized operator. Froggatt and Nielson1978
- Thus masses of fermions are recovered through higher order effective operators having the following structure :

$$\mathcal{O} = y(\frac{\chi}{\Lambda})^{(\theta_i+\theta_j)}\bar{\psi}_i\varphi\psi_j,$$

where y is the coupling constant, and χ is the flavon field.

The $\mathcal{Z}_{\rm N} \times \mathcal{Z}_{\rm M}$ flavour symmetry

A new solution

- The $\mathcal{Z}_{\rm N} \times \mathcal{Z}_{\rm M}$ flavour symmetry provides a realization of the Froggatt-Nielson mechanism where one does not need to impose a continuous $U(1)_F$ symmetry.

  Int.J.Mod.Phys.A 36 (2021) 2150090, G. Abbas

- For achieving this goal, we need a complex singlet scalar field χ which behaves under the SM symmetry as,

$$\chi : (1, 1, 0),$$

  and impose the $\mathcal{Z}_{\rm N} \times \mathcal{Z}_{\rm M}$ flavour symmetry on the SM.

- The masses of the three fermionic families appear in terms of the expansion parameter $\epsilon = \langle \chi \rangle / \Lambda$.

The $\mathcal{Z}_{\rm N} \times \mathcal{Z}_{\rm M}$ flavour symmetry

The following Lagrangian provides masses to the charged fermions of the SM,

$$\begin{aligned} -\mathcal{L} &= \left[\frac{\chi}{\Lambda}\right]^{n_{ij}^u} y_{ij}^u \bar{\psi}_{L_i}^q \tilde{\varphi} \psi_{R_j}^u + \left[\frac{\chi}{\Lambda}\right]^{n_{ij}^d} y_{ij}^d \bar{\psi}_{L_i}^q \varphi \psi_{R_j}^d \\ &+ \left[\frac{\chi}{\Lambda}\right]^{n_{ij}^\ell} y_{ij}^\ell \bar{\psi}_{L_i}^\ell \varphi \psi_{R_j}^\ell + \mathrm{H.c.}, \\ &= Y_{ij}^u \bar{\psi}_{L_i}^q \tilde{\varphi} \psi_{R_j}^u + Y_{ij}^d \bar{\psi}_{L_i}^q \varphi \psi_{R_j}^d + Y_{ij}^\ell \bar{\psi}_{L_i}^\ell \varphi \psi_{R_j}^\ell + \text{H.c.}, \end{aligned} \tag{1}$$

where the couplings Y_{ij} are the effective Yukawa couplings given by, $Y_{ij} = y_{ij}\epsilon^{n_{ij}}$, and $\frac{\langle\chi\rangle}{\Lambda} = \frac{f}{\sqrt{2}\Lambda} = \epsilon << 1$.

The $\mathcal{Z}_N \times \mathcal{Z}_M$ flavour symmetries

The minimal $\mathcal{Z}_N \times \mathcal{Z}_M$ symmetry

Fields	$\mathcal{Z}_2$	$\mathcal{Z}_5$
u_R, c_R, t_R	+	ω^2
$d_R, s_R, b_R, e_R, \mu_R, \tau_R$	-	ω
$\nu_{e_R}, \nu_{\mu_R}, \nu_{\tau_R}$	-	ω^3
ψ_L^1	+	ω
ψ_L^2	+	ω^4
ψ_L^3	+	ω^2
χ	-	ω
φ	+	1

Table: The charges of left and right-handed fermions of three families of the SM, right-handed neutrinos, Higgs, and singlet scalar fields under $\mathcal{Z}_2$ and $\mathcal{Z}_5$ symmetries, where ω is the fifth root of unity.

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The $\mathcal{Z}_N \times \mathcal{Z}_M$ flavour symmetries

The minimal $\mathcal{Z}_2 \times \mathcal{Z}_5$ symmetry

In terms of expansion parameter $\frac{\langle\chi\rangle}{\Lambda} = \frac{f}{\sqrt{2}\Lambda} = \epsilon$, the fermionic mass matrices are,

$$\mathcal{M}_{\mathcal{U}} = \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^u \epsilon^4 & y_{12}^u \epsilon^4 & y_{13}^u \epsilon^4 \\ y_{21}^u \epsilon^2 & y_{22}^u \epsilon^2 & y_{23}^u \epsilon^2 \\ y_{31}^u & y_{32}^u & y_{33}^u \end{pmatrix}, \mathcal{M}_{\mathcal{D}} = \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^d \epsilon^5 & y_{12}^d \epsilon^5 & y_{13}^d \epsilon^5 \\ y_{21}^d \epsilon^3 & y_{22}^d \epsilon^3 & y_{23}^d \epsilon^3 \\ y_{31}^d \epsilon & y_{32}^d \epsilon & y_{33}^d \epsilon \end{pmatrix}.$$

$$\mathcal{M}_{\ell} = \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^\ell \epsilon^5 & y_{12}^\ell \epsilon^5 & y_{13}^\ell \epsilon^5 \\ y_{21}^\ell \epsilon^3 & y_{22}^\ell \epsilon^3 & y_{23}^\ell \epsilon^3 \\ y_{31}^\ell \epsilon & y_{32}^\ell \epsilon & y_{33}^\ell \epsilon \end{pmatrix}.$$

where $\epsilon = 0.1$.

The $\mathcal{Z}_{\rm N} \times \mathcal{Z}_{\rm M}$ flavour symmetries

The minimal $\mathcal{Z}_2 \times \mathcal{Z}_5$ symmetry

The masses of fermions approximately are,

$$\{m_t, m_c, m_u\} \simeq \{|y^u_{33}|, \ \left|y^u_{22}\right| \epsilon^2, \left|y^u_{11}\right| \epsilon^4\} v/\sqrt{2},$$

$$\{m_b, m_s, m_d\} \simeq \{|y^d_{33}|\epsilon, \ \left|y^d_{22}\right| \epsilon^3, \left|y^d_{11}\right| \epsilon^5\} v/\sqrt{2},$$

$$\{m_\tau, m_\mu, m_e\} \simeq \{|y^l_{33}|\epsilon, \ \left|y^l_{22}\right| \epsilon^3, \left|y^l_{11}\right| \epsilon^5\} v/\sqrt{2},$$

The $\mathcal{Z}_{\rm N} \times \mathcal{Z}_{\rm M}$ flavour symmetries

The minimal $\mathcal{Z}_2 \times \mathcal{Z}_5$ symmetry

The quark mixing angles approximately are,

$$\begin{aligned}
\sin\theta_{12} \simeq |V_{us}| &\simeq \left|\frac{y^d_{12}}{y^d_{22}} - \frac{y^u_{12}}{y^u_{22}}\right| \epsilon^2, \\
\sin\theta_{23} \simeq |V_{cb}| &\simeq \left|\frac{y^d_{23}}{y^d_{33}} - \frac{y^u_{23}}{y^u_{33}}\right| \epsilon^2, \\
\sin\theta_{13} \simeq |V_{ub}| &\simeq \left|\frac{y^d_{13}}{y^d_{33}} - \frac{y^u_{12} y^d_{23}}{y^u_{22} y^d_{33}} - \frac{y^u_{13}}{y^u_{33}}\right| \epsilon^4.
\end{aligned}$$

The $\mathcal{Z}_{\rm N} \times \mathcal{Z}_{\rm M}$ flavour symmetries

A non-minimal $\mathcal{Z}_{\rm N} \times \mathcal{Z}_{\rm M}$ symmetry

Fields	$\mathcal{Z}_2$	$\mathcal{Z}_9$
u_R, t_R	+	1
c_R	+	ω^4
$d_R, s_R, b_R, e_R, \mu_R, \tau_R$	-	ω^3
ν_{e_R}, ν_{μ_R}	+	ω^6
ν_{τ_R}	+	ω^7
ψ_L^1	+	ω
ψ_L^2	+	ω^8
ψ_L^3	+	1
χ	-	ω
φ	+	1

Eur.Phys.J.C 83 (2023) 4, 305, arXiv: 2208.03733, V. Singh, N. Singh, R. Sain and G. Abbas.

The $\mathcal{Z}_{\mathrm{N}} \times \mathcal{Z}_{\mathrm{M}}$ flavour symmetries

A non-minimal $\mathcal{Z}_{\mathrm{N}} \times \mathcal{Z}_{\mathrm{M}}$ symmetry

The mass matrices for up and down-type quarks and charged leptons turn out to be,

$$\begin{aligned}
\mathcal{M}_u &= \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^u \epsilon^8 & y_{12}^u \epsilon^6 & y_{13}^u \epsilon^8 \\ y_{21}^u \epsilon^8 & y_{22}^u \epsilon^4 & y_{23}^u \epsilon^8 \\ y_{31}^u & y_{32}^u \epsilon^4 & y_{33}^u \end{pmatrix}, \mathcal{M}_d = \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^d \epsilon^7 & y_{12}^d \epsilon^7 & y_{13}^d \epsilon^7 \\ y_{21}^d \epsilon^5 & y_{22}^d \epsilon^5 & y_{23}^d \epsilon^5 \\ y_{31}^d \epsilon^3 & y_{32}^d \epsilon^3 & y_{33}^d \epsilon^3 \end{pmatrix}, \\
\mathcal{M}_\ell &= \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^\ell \epsilon^7 & y_{12}^\ell \epsilon^7 & y_{13}^\ell \epsilon^7 \\ y_{21}^\ell \epsilon^5 & y_{22}^\ell \epsilon^5 & y_{23}^\ell \epsilon^5 \\ y_{31}^\ell \epsilon^3 & y_{32}^\ell \epsilon^3 & y_{33}^\ell \epsilon^3 \end{pmatrix}.
\end{aligned}$$

The $\mathcal{Z}_{\rm N} \times \mathcal{Z}_{\rm M}$ flavour symmetries

A non-minimal $\mathcal{Z}_{\rm N} \times \mathcal{Z}_{\rm M}$ symmetry

The masses of fermions are given by,

$$\{m_t, m_c, m_u\} \simeq \{|y^u_{33}|,\ \left|y^u_{22}\epsilon^4\right|,\ \left|y^u_{11}\epsilon^8\right|\}v/\sqrt{2},$$

$$\{m_b, m_s, m_d\} \simeq \{|y^d_{33}|\epsilon^3,\ \left|y^d_{22}\right|\epsilon^5,\ \left|y^d_{11}\right|\epsilon^7\}v/\sqrt{2},$$

$$\{m_\tau, m_\mu, m_e\} \simeq \{|y^l_{33}|\epsilon^3,\ \left|y^l_{22}\right|\epsilon^5,\ \left|y^l_{11}\right|\epsilon^7\}v/\sqrt{2},$$

where $\epsilon = 0.225$.
The quark mixing angles are identical to that of the minimal symmetry.

The scalar potential

The scalar potential of the model can be written in the following form,

$$V = -\mu^2 \varphi^\dagger \varphi + \lambda(\varphi^\dagger \varphi)^2 - \mu_\chi^2 \chi^* \chi + \lambda_\chi (\chi^* \chi)^2 + (\rho\, \chi^2 + \mathrm{H.c.}) + \lambda_{\varphi\chi}(\chi^* \chi)(\varphi^\dagger \varphi).$$

We can parametrize the flavon field by excitations around its VEV,

$$\chi(x) = \frac{f + s(x) + i\, a(x)}{\sqrt{2}}. \tag{2}$$

The minimization conditions can be written in terms of the scalar and pseudo-scalar components having the following masses:

$$m_s = \sqrt{\mu_\chi - 2\rho} = \sqrt{\lambda_\chi} f \qquad \text{and} \qquad m_a = \sqrt{-2\rho}. \tag{3}$$

Flavour observables

$$C_{\epsilon_K} = \frac{\mathrm{Im}\langle K^0|\mathcal{H}_{\rm eff}^{\Delta F=2}|\bar{K}^0\rangle}{\mathrm{Im}\langle K^0|\mathcal{H}_{\rm SM}^{\Delta F=2}|\bar{K}^0\rangle} = 1.12^{+0.27}_{-0.25},\ C_{\Delta m_K} = \frac{\mathrm{Re}\langle K^0|\mathcal{H}_{\rm eff}^{\Delta F=2}|\bar{K}^0\rangle}{\mathrm{Re}\langle K^0|\mathcal{H}_{\rm SM}^{\Delta F=2}|\bar{K}^0\rangle} = 0.93^{+1.14}_{-0.42}.$$

M. Bona et al. [UTfit Collaboration], 2008

The new physics contributions to neutral meson mixing can be written as,

$$M_{12}^{d,s,K} = (M_{12}^{d,s,K})_{\rm SM}\left(1 + h_{d,s,K}e^{2i\sigma_{d,s,K}}\right). \tag{4}$$

Observables	Phase I	Phase II	Ref.
h_d	$0 - 0.04$	$0 - 0.028$	PRD 102, 056023, 2020
h_s	$0 - 0.036$	$0 - 0.025$	PRD 102, 056023, 2020
h_K	$0 - 0.3$	$-$	PRD 102, 056023, 2020

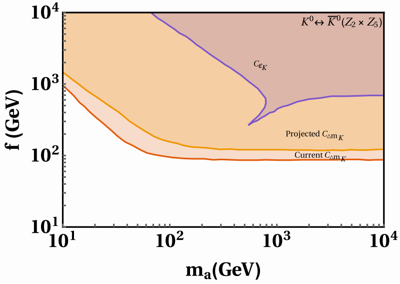
Observables	Current	LHCb-I	LHCb-II	CMS	ATLAS
$\mathcal{R}_{\mu\mu}$	$\sim 70\%$	$\sim 34\%$	$\sim 10\%$	$\sim 21\%$	$-$

where

$$\mathcal{R}_{\mu\mu} = \frac{\mathrm{BR}(B_d \to \mu^+\mu^-)}{\mathrm{BR}(B_s \to \mu^+\mu^-)} = 0.039^{+0.030+0.006}_{-0.024-0.004}. \tag{5}$$

LHCb 21

Bounds on the flavour scale



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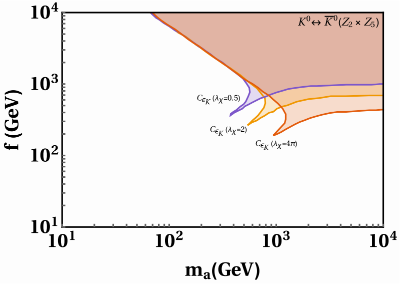
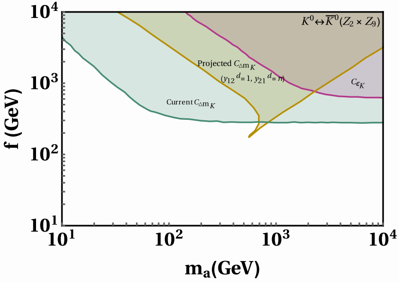


(b)

Figure

Bounds on the flavour scale



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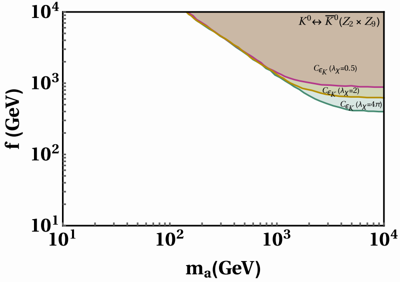

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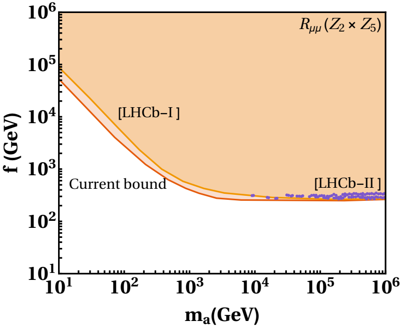
Bounds on the flavour scale



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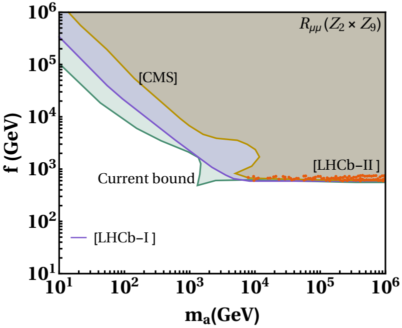


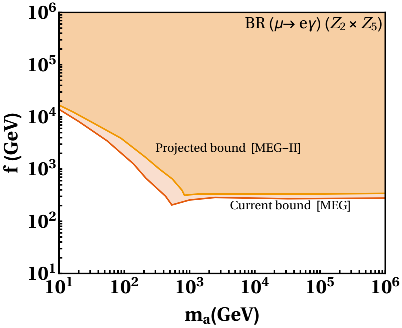
(b)

Bounds on the flavour scale

Observables	Current sensitivity	Ref.	Future projection	Ref.
BR($\mu \to e\gamma$)	$< 4.2 \times 10^{-13}$	MEG	6×10^{-14}	MEGII
BR ($\mu \to e)^{\mathrm{Au}}$	$< 7 \times 10^{-13}$	SINDRUM II	–	–
BR ($\mu \to e)^{\mathrm{Al}}$	–	–	3×10^{-15}	COMET Phase-I
BR ($\mu \to e)^{\mathrm{Al}}$	–	–	6×10^{-17}	COMET Phase-II
BR ($\mu \to e)^{\mathrm{Al}}$	–	–	6×10^{-17}	Mu2e
BR ($\mu \to e)^{\mathrm{Al}}$	–	–	3×10^{-18}	Mu2e II
BR ($\mu \to e)^{\mathrm{Si}}$	–	–	2×10^{-14}	DeeMe
BR ($\mu \to e)^{\mathrm{Ti}}$			$\sim 10^{-20} - 10^{-18}$	PRISM/PRIME

Table: Experimental upper limits on various Leptonic flavour violation (LFV) processes.

Bounds on the flavour scale



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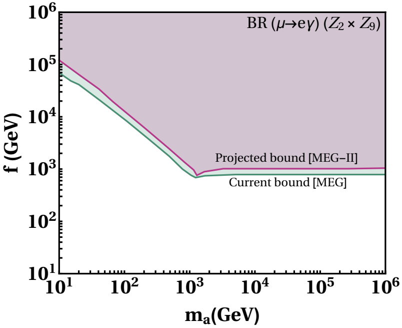

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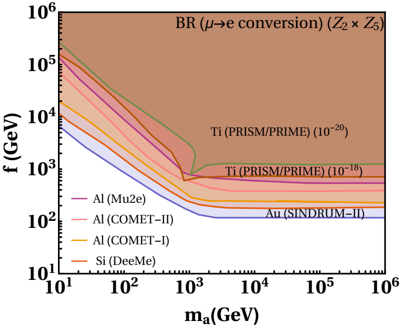
Bounds on the flavour scale



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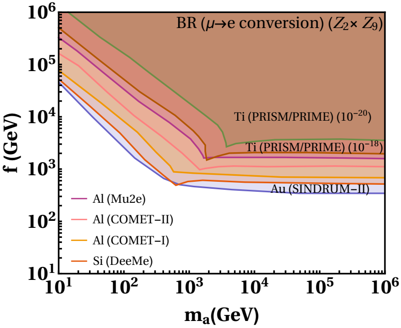

(b)

From flavour to dark matter

The $\mathcal{Z}_N \times \mathcal{Z}_M$ flavour symmetry can be used to show an entirely novel scenario where a solution to the flavour problem and the dark matter have the same origin by writing the potential,

$$V = -\lambda \frac{\chi^{\tilde{N}}}{\Lambda^{\tilde{N}-4}} + \text{H.c.}, \tag{6}$$

where $\tilde{N}$ is the least common multiple of N and M.
The masses of pseudo-scalar particles are now given by,

$$m_a^2 = \frac{1}{8}|\lambda|\tilde{N}^2 \epsilon^{\tilde{N}-4} f^2. \tag{7}$$

For getting the right dark matter density, the mass of the flavonic dark matter is given by,

$$m_a = 0.88 \times 10^{16} \left(\epsilon^{\tilde{N}-4} \tilde{N}^4 \frac{|\lambda|}{a_0^2} \right)^{2/5} \text{eV}. \tag{8}$$

arXiv: 2303.10125 [hep-ph] A. Adhikari, E. J. Chun and G. Abbas

From flavour to dark matter

For instance, we can use a model based on the $\mathcal{Z}_8 \times \mathcal{Z}_{22}$ flavour symmetry, where the mass of the top quark is forbidden at tree level.

Fields	$\mathcal{Z}_8$	$\mathcal{Z}_{22}$	Fields	$\mathcal{Z}_8$	$\mathcal{Z}_{22}$	Fields	$\mathcal{Z}_8$	$\mathcal{Z}_{22}$	Fields	$\mathcal{Z}_8$	$\mathcal{Z}_{22}$	Fields	$\mathcal{Z}_8$	$\mathcal{Z}_{22}$
u_R	ω^2	ω^2	c_R	ω^5	ω^5	t_R	ω^6	ω^6	d_R	ω^3	ω^3	s_R	ω^4	ω^4
b_R	ω^4	ω^4	$\psi^q_{L,1}$	ω^2	ω^{10}	$\psi^q_{L,2}$	ω	ω^9	$\psi^q_{L,3}$	ω^7	ω^7	$\psi^\ell_{L,1}$	ω^3	ω^3
$\psi^\ell_{L,2}$	ω^2	ω^2	$\psi^\ell_{L,3}$	ω^2	ω^2	e_R	ω^4	ω^{12}	μ_R	ω^7	ω^7	τ_R	ω^7	ω^{21}
ν_{e_R}	ω^2	1	ν_{μ_R}	ω^5	ω^3	ν_{τ_R}	ω^6	ω^4	χ	ω	ω	H	1	1

Table: The charges of the SM and the flavon fields under the $\mathcal{Z}_8 \times \mathcal{Z}_{22}$ symmetry, where ω is the 8th and 22nd root of unity.

arXiv: 2303.10125 [hep-ph] A. Adhikari, E. J. Chun and G. Abbas

From flavour to dark matter

The masses of charged fermions approximately read,

$$\begin{aligned}
\{m_t, m_c, m_u\} &\simeq \{|y^u_{33}|\epsilon, \ \left|y^u_{22}\right| \epsilon^4, \ \left|y^u_{11}\right| \epsilon^8\} v/\sqrt{2}, \\
\{m_b, m_s, m_d\} &\simeq \{|y^d_{33}|\epsilon^3, \ \left|y^d_{22}\right| \epsilon^5, \left|y^d_{11}\right| \epsilon^7\} v/\sqrt{2}, \\
\{m_\tau, m_\mu, m_e\} &\simeq \{|y^l_{33}|\epsilon^3, \ \left|y^l_{22}\right| \epsilon^5, \ \left|y^l_{11}\right| \epsilon^9\} v/\sqrt{2}.
\end{aligned} \tag{9}$$

The mixing angles of quarks are given by,

$$\begin{aligned}
\sin\theta_{12} \simeq |V_{us}| &\simeq \left|\frac{y^d_{12}}{y^d_{22}} - \frac{y^u_{12}}{y^u_{22}}\right| \epsilon, \ \sin\theta_{23} \simeq |V_{cb}| \simeq \left|\frac{y^d_{23}}{y^d_{33}} - \frac{y^u_{23}}{y^u_{33}}\right| \epsilon^2, \quad (10) \\
\sin\theta_{13} \simeq |V_{ub}| &\simeq \left|\frac{y^d_{13}}{y^d_{33}} - \frac{y^u_{12} y^d_{23}}{y^u_{22} y^d_{33}} - \frac{y^u_{13}}{y^u_{33}}\right| \epsilon^3.
\end{aligned}$$

where $\epsilon = 0.225$.

From flavour to dark matter

- For the flavour symmetry $\mathcal{Z}_8 \times \mathcal{Z}_{22}$, we have $\tilde{N} = 88$ leading to

$$f \approx 1.0 \times 10^{14}\,\mathrm{GeV}, \quad \text{and} \quad \mathrm{m_a} \approx 1.9 \times 10^{-3}\,\mathrm{eV}, \tag{11}$$

  considering $\epsilon = 0.225$ with $|\lambda| = 1$ and $a_0 = 1$.
- For the longevity of the flavonic DM, its decay to electrons has to be forbidden, that is, $m_a < 2m_e$ which requires

$$\tilde{N} > 53, \quad \text{and} \quad f > 4 \times 10^{11}\,\mathrm{GeV}. \tag{12}$$

- The most stringent bound on the flavon scale f comes from the FCNC process $K^+ \to \pi^+ a$ Bjorkeroth et al 2018

$$f \gtrsim 7 \times 10^{11}\, V_{21}^{d}\,\mathrm{GeV}, \tag{13}$$

  where we have $V_{21}^{d} \approx \epsilon$.

From flavour to dark matter

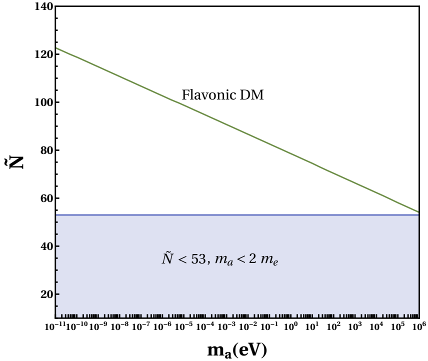

Figure: The allowed mass range of flavonic dark matter for a range of $\tilde{N}$. The light-blue region is ruled out by the constraint $m_a < 2m_e$.

arXiv: 2308.14811 [hep-ph] A. Adhikari, E. J. Chun N. Singh and G. Abbas

From flavour to dark matter

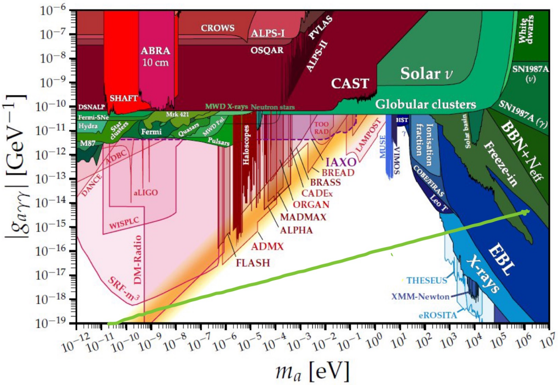

Figure: The prediction of flavonic dark matter (thick green line) and axion-like particle searches taken from C. Antel, et al 2023.

arXiv: 2303.10125 [hep-ph] A. Adhikari, E. J. Chun and G. Abbas

Summary

- The flavour problem can be addressed in the framework of the $\mathcal{Z}_{\rm N} \times \mathcal{Z}_{\rm M}$ symmetry symmetry.
- So far two flavour physics of two proto-type symmetries ($\mathcal{Z}_2 \times \mathcal{Z}_{5,9}$) is investigated.
- The flavonic dark matter model predicts specific axial flavon coupling to photons which is mostly far below the standard QCD axion DM region, and limited by X-ray searches to $m_\varphi \lesssim 1$ keV and $v_F \gtrsim 4 \times 10^{12}$ GeV.
- Thus, there appear no observable consequences in flavour phenomenology.
- Only a limited region of parameter space around $m_\varphi \sim$ meV could be probed by the future radio searches.