Non-Renormalizable Grand Unification Utilizing the Leptoquark Mechanism of Neutrino Mass

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Motivation

Leptoquarks are interesting, because they might potentially explain some of the puzzles particle physicists are faced with such as

- f_{D_s} puzzle, discrepancy between experimental and lattice results for decays of D_s mesons,
- B-physics anomalies, in particular, ratios $R_{K^{(*)}}$ and $R_{D^{(*)}}$ regarding lepton flavor universality,

to name a couple.

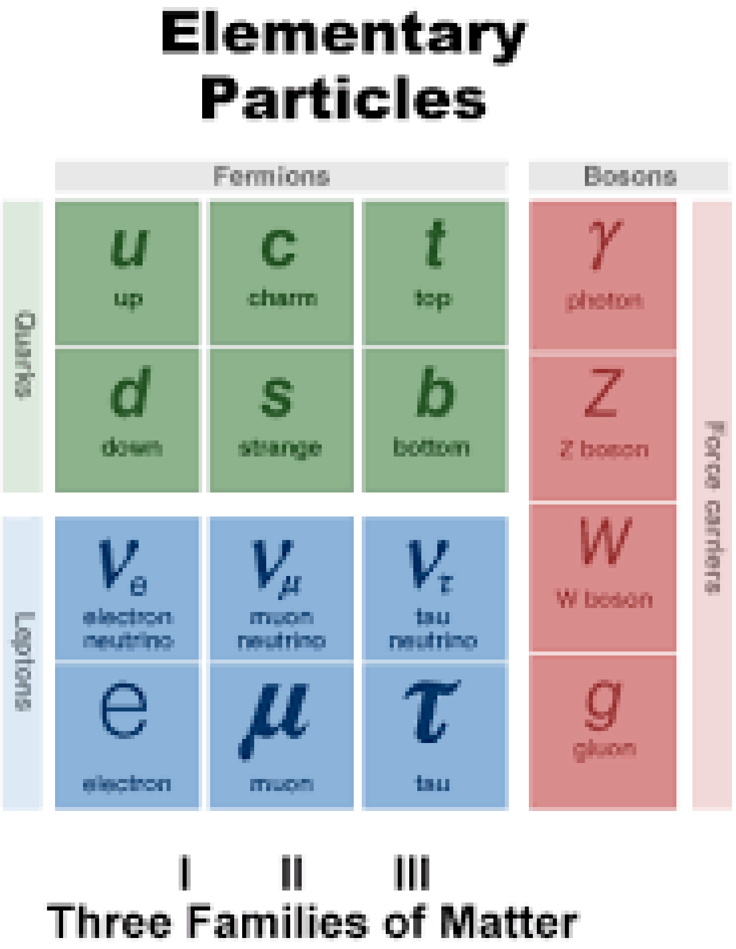
Standard Model of Particle Physics

- Strong, weak and electromagnetic interactions comprise the Standard Model of particle physics,
- Mathematically, it is a renormalizable gauge theory based on the direct product of semi-simple groups given below:

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

- Symmetry is broken to that of Color and Electromagnetism due to Spontaneous Symmetry Breaking (SSB) at the weak scale.

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \overset{SSB}{\rightarrow} SU(3)_c \otimes U(1)_{e.m.}$$



Left-handed particles of SM

$$\left(\bar{3}, 1, +\frac{1}{3}\right) \oplus \left(1,2,-\frac{1}{2}\right) \oplus \left(3,2,+\frac{1}{6}\right) \oplus \left(\bar{3}, 1, -\frac{2}{3}\right) \oplus (1,1,+1)$$

and right-handed anti-particles

in each of the 3 families (or generations)

Gauge bosons of SM

$$\underbrace{(8,1,0)}_{gluons} \oplus \underbrace{(1,3,0)}_{W^{\pm}, A} \oplus \underbrace{(1,1,0)}_{B}$$

Georgi-Glashow (GG) Unification Paradigm

- Matter fields, that is fermions, of the Standard Model are placed in the following representations of SU(5):

$$(\bar{\mathbf{5}} \oplus \mathbf{10})_L \oplus (\mathbf{5} \oplus \overline{\mathbf{10}})_R$$

where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$

-(+) sign indicates left- (right-) handed projection operator

There is one copy of the above fermions for each of the 3 generations!

- Gauge bosons of SU(5) reside in the adjoint representation $\mathbf{24}$. Gauge bosons with quantum numbers of the SM gauge bosons remain massless, whereas those mediating proton decay eat the Goldstone bosons in adjoint $\mathbf{24}_H$ to obtain mass after GUT scale spontaneous symmetry breaking!

- Decomposition of the scalars in $\mathbf{24}_H$ under the Standard Model gauge group is:

$$\mathbf{24}_H = \underbrace{(8,1,0) \oplus (1,3,0) \oplus (1,1,0)}_{\substack{scalars\ corresponding\ to \\ unbroken\ symmetry \\ generators}} \oplus \underbrace{(3,2,-5/6) \oplus (\bar{3},2,+5/6)}_{\substack{Goldstone\ bosons\ corresponding \\ to\ broken\ symmetry\ generators}}$$

GUT Scale Symmetry Breaking

- At $M_{GUT} \sim 10^{16}$ GeV, adjoint scalars $\mathbf{24}_H$ acquire a Vacuum Expectation Value (VEV) in the SM singlet direction, that is,
- $$< \mathbf{24}_H >= \frac{v_{24}}{\sqrt{30}} \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix}$$
- $M_{GUT} = v_{24}\sqrt{\frac{5\pi\alpha_{GUT}}{3}}$ proton lifetime $\tau_{proton} \propto \left(\frac{M_{GUT}}{\sqrt{\alpha_{GUT}}}\right)^4$
- $M_{GUT} \geq 5.5 \times 10^{15}$ GeV

Electroweak Symmetry Breaking (EWSB)

- At $M_Z \sim 10^2$ GeV, color singlet, (electric charge) neutral component of fundamental scalars $\mathbf{5}_H$ acquires a V.E.V. as given below:
- $< \mathbf{5}_H >= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v_5/\sqrt{2} \end{pmatrix}$
- Standard Model particles obtain mass as a result of this electroweak or low-energy spontaneous symmetry breaking.
- To summarize, $SU(5) \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ at M_{GUT}, then $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_{e.m.}$ at M_Z.

Drawbacks of GG Model

- Coupling constants don't unify when measured values of physical observables (Weinberg angle, electromagnetic and strong fine-structure constants) are used,
- Neutrinos are massless. This could easily be overcomed by adding at least two right-handed SU(5) singlet neutrinos, however, it wouldn't explain why the neutrino masses are so small!
- Disparity between the predicted and observed values of the ratios of charged leptons and d-type quarks in the lighter two generations at the weak scale!

Objectives

- Unify gauge fine-structure constants, $\alpha_i \ (i = 1{,}2{,}3)$,
- Achieve a unification scale, M_{GUT}, of at least 5.5x10^{15} GeV,
- Generate the Majorana neutrino mass matrix that gives the observed masses and mixing angles of the 3 flavors of neutrinos through the leptoquark mechanism,
- Obtain the mass matrices of charged fermions, i.e., u- and d-type quarks, and charged leptons, which correctly reproduce the observed masses of particles at low energies,
- Accomplish these objectives with the minimal number of additional particles, i.e., in the most economical way possible!

Proposed Model

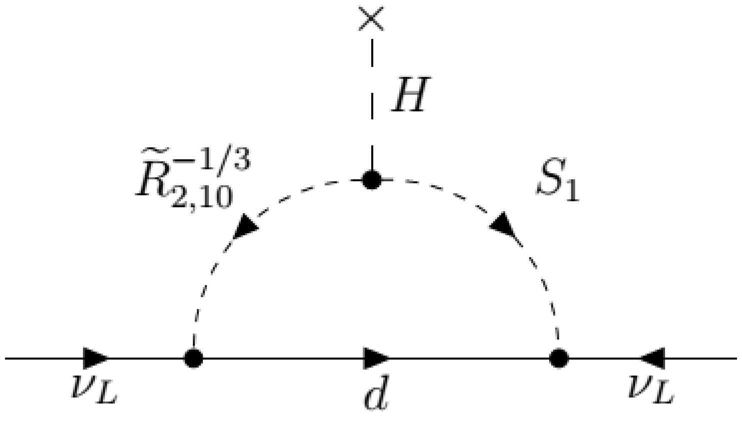
- In addition to the particle content of the Georgi-Glashow model, there are scalars in the 10
- $\mathbf{10}_S \equiv \left(\phi_1,\ \tilde{R}_{2,10}\ ,\ \bar{S}_1\right) = (1,1,1) \oplus \left(3,2,\frac{1}{6}\right) \oplus \left(\bar{3},1,-\frac{2}{3}\right)$

and 40 representations of SU(5), but no fermions beyond those of SM,

- $\mathbf{40}_S \equiv \left(\eta_1, \eta_2, \tilde{R}^*_{2,40}\ , \eta_4, \eta_5, \eta_6\right) = \left(1,2,\frac{3}{2}\right) \oplus \left(3,1,\frac{2}{3}\right) \oplus \left(\bar{3},2,-\frac{1}{6}\right) \oplus \left(3,3,\frac{2}{3}\right) \oplus \left(6,2,-\frac{1}{6}\right) \oplus (8,1,-1)$
- However, only the following split multiplets are not decoupled from the theory below the GUT scale and contribute to running.

  $S_1^*; \tilde{R}_2\ ;\ \eta_4, \eta_5$ The first two are (scalar) leptoquarks!

Majorana Neutrino Mass Matrix



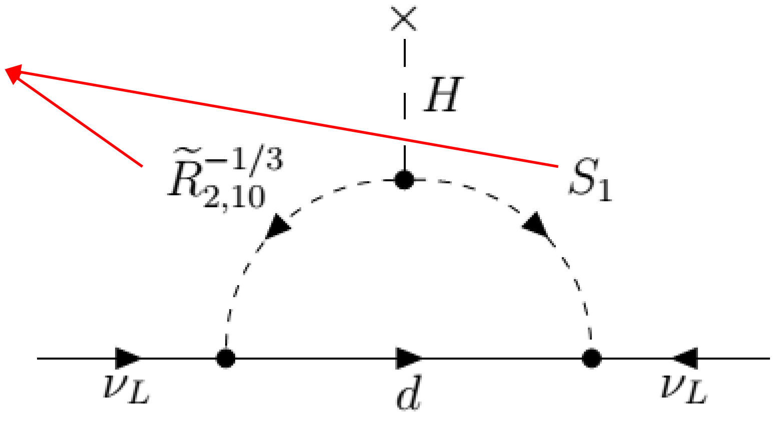
Feynmann diagram for Majorana neutrino mass

$$-\frac{1}{2} M_{ab} \overline{\nu^c_{L}}_a \nu_{Lb}$$

- $\overline{d_R} L^\alpha_L \tilde{R}_{2,10\alpha}$ originates from $\mathbf{10}_S^{ij} \overline{\mathbf{5}}_{Fai} \overline{\mathbf{5}}_{Fbj}$, $\mathbf{10}_S^{ij} \overline{\mathbf{5}}_{Fai} \overline{\mathbf{5}}_{Fbk} \mathbf{24}^k_{Hj}$;
- $H^\alpha i\tau_2 \tilde{R}^*_{2,10} S^*_1$ originates from $\mathbf{10}_S^{ij} \mathbf{5}^*_{Hi} \mathbf{5}^*_{Hk} \mathbf{24}^k_{Hj}$;
- $\overline{Q^c_L}^\alpha \epsilon^{\alpha\beta} L^\beta_L S_1$ originates from $\mathbf{10}_{Fa}^{ij} \overline{\mathbf{5}}_{Fbi} \mathbf{5}^*_{Hj}$, $\mathbf{10}_{Fa}^{ik} \overline{\mathbf{5}}_{Fbi} \mathbf{5}^*_{Hj} \mathbf{24}^j_{Hk}$, $\mathbf{10}_{Fa}^{kj} \overline{\mathbf{5}}_{Fbi} \mathbf{5}^*_{Hj} \mathbf{24}^i_{Hk}$.

Majorana Neutrino Mass Matrix

(Scalar) leptoquarks with electric charge -1/3



Feynmann diagram for Majorana neutrino mass

$-\frac{1}{2} M_{ab} \overline{\psi_a^c} \psi_b$

- $\overline{d_R} L_L^{\alpha} \tilde{R}_{2,10\alpha}$ originates from $\mathbf{10}_S^{ij} \overline{\mathbf{5}}_{Fai} \overline{\mathbf{5}}_{Fbj}$, $\mathbf{10}_S^{ij} \overline{\mathbf{5}}_{Fai} \overline{\mathbf{5}}_{Fbk} < \mathbf{24}_{Hj}^{k} >$;
- $H^{\alpha} i\tau_2 \tilde{R}_{2,10}^{*} S_1^{*}$ originates from $\mathbf{10}_S^{ij} \mathbf{5}_{Hi}^{*} < \mathbf{5}_{Hk}^{*} >< \mathbf{24}_{Hj}^{k} >$;
- $\overline{Q_L^c}^{\alpha} \epsilon^{\alpha\beta} L_L^{\beta} S_1$ originates from $\mathbf{10}_{Fa}^{ij} \overline{\mathbf{5}}_{Fbi} \mathbf{5}_{Hj}^{*}$, $\mathbf{10}_{Fa}^{ik} \overline{\mathbf{5}}_{Fbi} \mathbf{5}_{Hj}^{*} < \mathbf{24}_{Hk}^{j} >$, $\mathbf{10}_{Fa}^{kj} \overline{\mathbf{5}}_{Fbi} \mathbf{5}_{Hj}^{*} < \mathbf{24}_{Hk}^{i} >$.

- A neutrino that is its own anti-particle is called Majorana. If it's anti-particle is distinct, then it is a Dirac neutrino.

- Charge conjugation operation of a fermion field is defined by

$$\psi^c \equiv i\gamma^2\psi^*$$

- In the previous slide, a,b are generation (or family or flavor) indices, whereas i,j,k are group indices, in our case those of SU(5).

  a,b=1,2,3 while i,j,k=1,2,3,4,5.

- By calculating the Feynmann diagram we will have found the neutrino Majorana mass matrix, M_{ab}.

Mixing due to Electroweak Symmetry Breaking (EWSB)

- Changes in good quantum numbers after EWSB
- $S_1^* \left(3,1,-\frac{1}{3}\right) \rightarrow \left\{\left(3,-\frac{1}{3}\right)\right.$
- $\tilde{R}_2 \left(3,2,+\frac{1}{6}\right) \rightarrow \begin{cases}\left(3,+\frac{2}{3}\right)\\ \left(3,-\frac{1}{3}\right)\end{cases}$
- $\tilde{R}_{2,decoup} \left(3,2,+\frac{1}{6}\right) \rightarrow \begin{cases}\left(3,+\frac{2}{3}\right)\\ \left(3,-\frac{1}{3}\right)\end{cases}$
- $\bar{S}_1^* \left(3,1,+\frac{2}{3}\right) \rightarrow \left\{\left(3,+\frac{2}{3}\right)\right.$
- $\eta_2 \left(3,1,+\frac{2}{3}\right) \rightarrow \left\{\left(3,+\frac{2}{3}\right)\right.$
- $\eta_4 \left(3,3,+\frac{2}{3}\right) \rightarrow \begin{cases}\left(3,+\frac{5}{3}\right)\\ \left(3,+\frac{2}{3}\right)\\ \left(3,-\frac{1}{3}\right)\end{cases}$

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- $\tilde{R}_2 \left(3,2,+\frac{1}{6}\right) \rightarrow \begin{cases} \left(3,+\frac{2}{3}\right) \\ \left(3,-\frac{1}{3}\right) \end{cases}$
- $\tilde{R}_{2,decoup} \left(3,2,+\frac{1}{6}\right) \rightarrow \begin{cases} \left(3,+\frac{2}{3}\right) \\ \left(3,-\frac{1}{3}\right) \end{cases}$
- $\bar{S}_1^* \left(3,1,+\frac{2}{3}\right) \rightarrow \left\{\left(3,+\frac{2}{3}\right)\right.$
- $\eta_2 \left(3,1,+\frac{2}{3}\right) \rightarrow \left\{\left(3,+\frac{2}{3}\right)\right.$
- $\eta_4 \left(3,3,+\frac{2}{3}\right) \rightarrow \begin{cases} \left(3,+\frac{5}{3}\right) \\ \left(3,+\frac{2}{3}\right) \\ \left(3,-\frac{1}{3}\right) \end{cases}$

- $R^{-1/3} = \begin{pmatrix} 1 & \frac{\bar{M}_{12}^2}{\left(m_{S_1^*}^2 - m_{\tilde{R}_2}^2\right)} & 0 & 0 \\ -\frac{\bar{M}_{12}^2}{\left(m_{S_1^*}^2 - m_{\tilde{R}_2}^2\right)} & 1 & 0 & \frac{\bar{M}_{24}^2}{\left(m_{\tilde{R}_2}^2 - m_{\eta_4}^2\right)} \\ 0 & 0 & 1 & \frac{\bar{M}_{34}^2}{\left(m_{\tilde{R}_{2,decoup}}^2 - m_{\eta_4}^2\right)} \\ 0 & -\frac{\bar{M}_{24}^2}{\left(m_{\tilde{R}_2}^2 - m_{\eta_4}^2\right)} & -\frac{\bar{M}_{34}^2}{\left(m_{\tilde{R}_{2,decoup}}^2 - m_{\eta_4}^2\right)} & 1 \end{pmatrix}$

where $\bar{M}_{12}^2 = -\frac{5}{2\sqrt{30}}\ v_5 v_{24} \lambda_{5-10}$. This is the rotation matrix that diagonalizes the squared mass matrix ($\beta_{GUT} = 0$ for simplicity).

Majorana Mass Matrix of Neutrinos

- $\mathcal{M}^{\nu} = \frac{3 v_5 \lambda_{5-10}}{64\pi^2\sqrt{2\pi}} \frac{M_{GUT}}{\sqrt{\alpha_{GUT}}} \cos\beta_{GUT} \frac{\log\left(m^2_{S_1^*}/m^2_{\tilde{R}_2}\right)}{\left(m^2_{S_1^*} - m^2_{\tilde{R}_2}\right)}$

$$\times \left[\left(2Y^{\nu} - \frac{M_{GUT}}{\Lambda\sqrt{2\pi\alpha_{GUT}}} \tilde{Y}^{\nu} \right) m \left(Y^D + \frac{M_{GUT}}{\Lambda\sqrt{25\pi\alpha_{GUT}/2}} \tilde{Y}^D - \frac{M_{GUT}}{\Lambda\sqrt{50\pi\alpha_{GUT}/9}} \bar{Y}^D \right)^T + transpose \right]$$

where m is a diagonal matrix with elements m_d, m_s, m_b along the diagonal.

- Unification scenario determines M_{GUT}, α_{GUT}, $m_{S_1^*}$, and $m_{\tilde{R}_2}$.
- The cutoff scale Λ is in the interval $M_{GUT} \ll \Lambda \leq M_{Pl} \approx 10^{19}$ GeV, and $v_5 = 246$ GeV.

In the basis where M^E , the mass matrix of charged leptons, is diagonal, U_{PMNS} , Pontecorvo-Maki-Nakagawa-Sakata matrix, has the following form:

$$U_{PMNS} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $c_{23} \equiv cos\theta_{23}$, etc.

$U_{PMNS}^T = U_{PMNS}^{-1}$ because the CP violating phase is taken to be $\delta = 0, \pi$

$$M^\nu = U_{PMNS} M_{diag}^\nu U_{PMNS}^T$$

Input parameters:

$\Delta m^2_{21} = (7.57 \pm 0.18) \times 10^{-5}\ eV^2$ (a.k.a. $\Delta m^2_{\odot}$)
$\Delta m^2_{31} = (2.50 \pm 0.03) \times 10^{-3}\ eV^2$ (a.k.a. Δm^2_{atm})

One of the neutrinos is assumed to be massless for simplicity, so

$$m_1 = 0, m_2 = 8.70 \times 10^{-3}\ eV, m_3 = 5.00 \times 10^{-2}\ eV$$

$sin^2\theta_{12} = 0.322 \pm 0.018$ (a.k.a. $sin^2\theta_{\odot}$)
$sin^2\theta_{23} = 0.542 \pm 0.025$ (a.k.a. $sin^2\theta_{atm}$)
$sin^2\theta_{13} = (2.219 \pm 0.075) \times 10^{-2}$

CP violating phase is taken to be $\delta = \pi$ and the Majorana phases are assumed to be zero.

- β_{GUT} is the rotation angle that relates the interaction eigenstates $\tilde{R}_{2,10}$ and $\tilde{R}_{2,40}$ to mass eigenstates $\tilde{R}_2$ and $\tilde{R}_{2,decoup}$.

$$\left|\tilde{R}_2 >\equiv cos\beta_{GUT}\left|\tilde{R}_{2,10} > -sin\beta_{GUT}\right|\tilde{R}_{2,40} >\right.$$

$$\left|\tilde{R}_{2,decoup} >\equiv sin\beta_{GUT}\left|\tilde{R}_{2,10} > +cos\beta_{GUT}\right|\tilde{R}_{2,40} >\right.$$

- The coupling constant λ_{5-10} is a free parameter,whereras (elements of matrix) coupling constants $Y^{\nu}, \tilde{Y}^{\nu}; Y^{D}, \tilde{Y}^{D}, \bar{Y}^{D}$ must reproduce the observed values of charged lepton mass matrices.
- Coupling constants should not exceed $\sqrt{4\pi}$ for our calculation to be consistent. This is the so-called perturbativity constraint.

Bounds on Leptoquark Masses

- We can use the Majorana neutrino mass matrix to place upper bounds on the leptoquark masses as shown below:

$$m_{S_1^*}, m_{\tilde{R}_2} \leq 2.5 \times 10^{15} \text{ GeV}$$

- The mass of the scalar leptoquark S_1^* is constrained from below due to the lower bound on the lifetime of the proton to be

$$m_{S_1^*} \geq 2.8 \times 10^{11} \text{ GeV}$$

- The mass of the scalar leptoquark $\tilde{R}_2$, on the other hand, is restricted to be above the lower bound given below due to destabilization by EWSB.

$$m_{\tilde{R}_2} \geq 2.4 \times 10^{9} \text{ GeV}$$

$\tilde{R}_2$: EW interaction eigenstate

$\hat{\tilde{R}}_2^{-1/3}$: mass eigenstate after EWSB with electric charge -1/3

$$m^2_{\tilde{R}_2} - \frac{\bar{M}^4_{12}}{\left(m^2_{S_1^*} - m^2_{\hat{\tilde{R}}_2^{-1/3}}\right)} - \frac{\bar{M}^4_{24}}{\left(m^2_{\eta_4} - m^2_{\hat{\tilde{R}}_2^{-1/3}}\right)} = m^2_{\hat{\tilde{R}}_2^{-1/3}}$$

provided $m_{\hat{\tilde{R}}_2^{-1/3}} \leq m_{\tilde{R}_2} \ll m_{\eta_4}, m_{S_1^*}$

$$m_{\tilde{R}_2} \geq \sqrt{\frac{\bar{M}^4_{12}}{m^2_{S_1^*}} + \frac{\bar{M}^4_{24}}{m^2_{\eta_4}}} \approx \frac{\bar{M}^2_{24}}{m_{\eta_4}} \sim \frac{v_5 v_{24}}{m_{\eta_4}}$$

Unification of Gauge Couplings

- Fine-structure constants evolve with energy as follows:

$$\frac{d\alpha_i(\mu)}{dlog(\mu)} = \frac{b_i}{2\pi}\alpha_i^2(\mu) + \frac{1}{8\pi^2}\alpha_i^2(\mu)\sum_{j=1}^{3} b_{ij}\,\alpha_j(\mu) + \frac{1}{32\pi^3}\alpha_i^2(\mu)\sum_{l=U,D,E} C_{il}Tr\left[\left(Y^l\right)^\dagger Y^l\right]$$

- (Matrix) coefficients b_i, b_{ij}, C_{il}, and (matrix) functions $Y^l(\mu)$ are known. The coupled differential equations above are solved to find $\alpha_i(\mu)$ for i=1,2,3. No summation over i!
- At two-loop order, quartic couplings don't conribute!

$$B_i = b_i + \sum_I b_{iI} \frac{log(M_{GUT}/M_I)}{log(M_{GUT}/m_Z)}$$

intermediate mass scales satisfy $m_Z \leq M_I \leq M_{GUT}$

$$\frac{d\alpha_i^U}{dlog\mu} = \frac{\alpha_i^U}{2\pi}\left(\bar{T} - \bar{G}^U + \frac{3}{2}\alpha_i^U - \frac{3}{2}\sum_j \left|V_{ij}^{CKM}\right|^2 \alpha_j^D\right),$$

$$\frac{d\alpha_i^D}{dlog\mu} = \frac{\alpha_i^D}{2\pi}\left(\bar{T} - \bar{G}^D + \frac{3}{2}\alpha_i^D - \frac{3}{2}\sum_j \left|V_{ji}^{CKM}\right|^2 \alpha_j^U\right),$$

$$\frac{d\alpha_i^E}{dlog\mu} = \frac{\alpha_i^E}{2\pi}\left(\bar{T} - \bar{G}^E + \frac{3}{2}\alpha_i^E\right).$$

$$\bar{T} \equiv T/(4\pi), \bar{G}^l \equiv G^l/(4\pi), \alpha_i^l \equiv \left(Y_i^l\right)^2/(4\pi)$$

l=U,D,E whereas i=1,2,3.

$$T = Tr\left[(Y^E)^\dagger Y^E + 3(Y^U)^\dagger Y^U + 3(Y^D)^\dagger Y^D\right]$$

$$\begin{pmatrix} G^U \\ G^D \\ G^E \end{pmatrix} = \begin{pmatrix} \frac{17}{20} & \frac{9}{4} & 8 \\ \frac{1}{4} & \frac{9}{4} & 8 \\ \frac{9}{4} & \frac{9}{4} & 0 \end{pmatrix} \begin{pmatrix} g_1^2 \\ g_2^2 \\ g_3^2 \end{pmatrix}$$

Masses of Split Multiplets in $\mathbf{10}_S$ Representation

$$m_{\phi_1}^2 = M_{10}^2 - \frac{3}{\sqrt{30}} g_{10} v_{24} + \frac{3}{10} \lambda_{g,10} v_{24}^2 + \frac{3}{10} \lambda_{10} v_{24}^2 \,,$$

$$m_{\tilde{R}_{2.10}}^2 = M_{10}^2 - \frac{1}{2\sqrt{30}} g_{10} v_{24} + \frac{13}{60} \lambda_{g,10} v_{24}^2 - \frac{1}{5} \lambda_{10} v_{24}^2 \,,$$

$$m_{\phi_3}^2 = M_{10}^2 + \frac{2}{\sqrt{30}} g_{10} v_{24} + \frac{2}{15} \lambda_{g,10} v_{24}^2 + \frac{2}{15} \lambda_{10} v_{24}^2 \,.$$

$$M_{10}^2 \equiv m_{10}^2 + \lambda_{m^2,10} v_{24}^2$$

Masses of Split Multiplets in $\mathbf{40}_s$ Representation

$$m_{\eta_1}^2 = M_{40}^2 - \frac{3}{\sqrt{30}} g_{40} v_{24} + \frac{3}{10} \lambda_{g,40} v_{24}^2 - \frac{3}{\sqrt{30}} \tilde{g}_{40} v_{24} + \frac{3}{10} \lambda_{\tilde{g},40} v_{24}^2 + \frac{3}{10} \tilde{\lambda}_{40} v_{24}^2 + \frac{3}{10} \lambda_{40} v_{24}^2 \,,$$

$$m_{\eta_2}^2 = M_{40}^2 + \frac{1}{3\sqrt{30}} g_{40} v_{24} + \frac{17}{90} \lambda_{g,40} v_{24}^2 - \frac{13}{6\sqrt{30}} \tilde{g}_{40} v_{24} + \frac{49}{180} \lambda_{\tilde{g},40} v_{24}^2 + \frac{1}{6} \tilde{\lambda}_{40} v_{24}^2 - \frac{7}{60} \lambda_{40} v_{24}^2 \,,$$

$$m_{\tilde{R}_{2.40}}^2 = M_{40}^2 - \frac{4}{3\sqrt{30}} g_{40} v_{24} + \frac{22}{90} \lambda_{g,40} v_{24}^2 + \frac{7}{6\sqrt{30}} \tilde{g}_{40} v_{24} + \frac{29}{180} \lambda_{\tilde{g},40} v_{24}^2 + \frac{1}{18} \tilde{\lambda}_{40} v_{24}^2 - \frac{13}{90} \lambda_{40} v_{24}^2 \,,$$

$$m_{\eta_4}^2 = M_{40}^2 - \frac{3}{\sqrt{30}} g_{40} v_{24} + \frac{3}{10} \lambda_{g,40} v_{24}^2 - \frac{1}{2\sqrt{30}} \tilde{g}_{40} v_{24} + \frac{13}{60} \lambda_{\tilde{g},40} v_{24}^2 - \frac{1}{5} \tilde{\lambda}_{40} v_{24}^2 + \frac{1}{20} \lambda_{40} v_{24}^2 \,,$$

$$m_{\eta_5}^2 = M_{40}^2 + \frac{2}{\sqrt{30}} g_{40} v_{24} + \frac{2}{15} \lambda_{g,40} v_{24}^2 - \frac{1}{2\sqrt{30}} \tilde{g}_{40} v_{24} + \frac{13}{60} \lambda_{\tilde{g},40} v_{24}^2 - \frac{1}{5} \tilde{\lambda}_{40} v_{24}^2 - \frac{1}{30} \lambda_{40} v_{24}^2 \,,$$

$$m_{\eta_6}^2 = M_{40}^2 + \frac{2}{\sqrt{30}} g_{40} v_{24} + \frac{2}{15} \lambda_{g,40} v_{24}^2 + \frac{2}{\sqrt{30}} \tilde{g}_{40} v_{24} + \frac{2}{15} \lambda_{\tilde{g},40} v_{24}^2 + \frac{2}{15} \tilde{\lambda}_{40} v_{24}^2 + \frac{2}{15} \lambda_{40} v_{24}^2 \,.$$

$$M_{40}^2 \equiv m_{40}^2 + \lambda_{m^2,40} v_{24}^2$$

in the MS scheme,

$$sin^2\theta_W(m_Z) = 0.23120 \pm 0.00015\ ,$$
$$\alpha_{em}^{-1}(m_Z) = 127.906 \pm 0.019\ ,$$
$$\alpha_s(m_Z) = 0.1187 \pm 0.002\ .$$

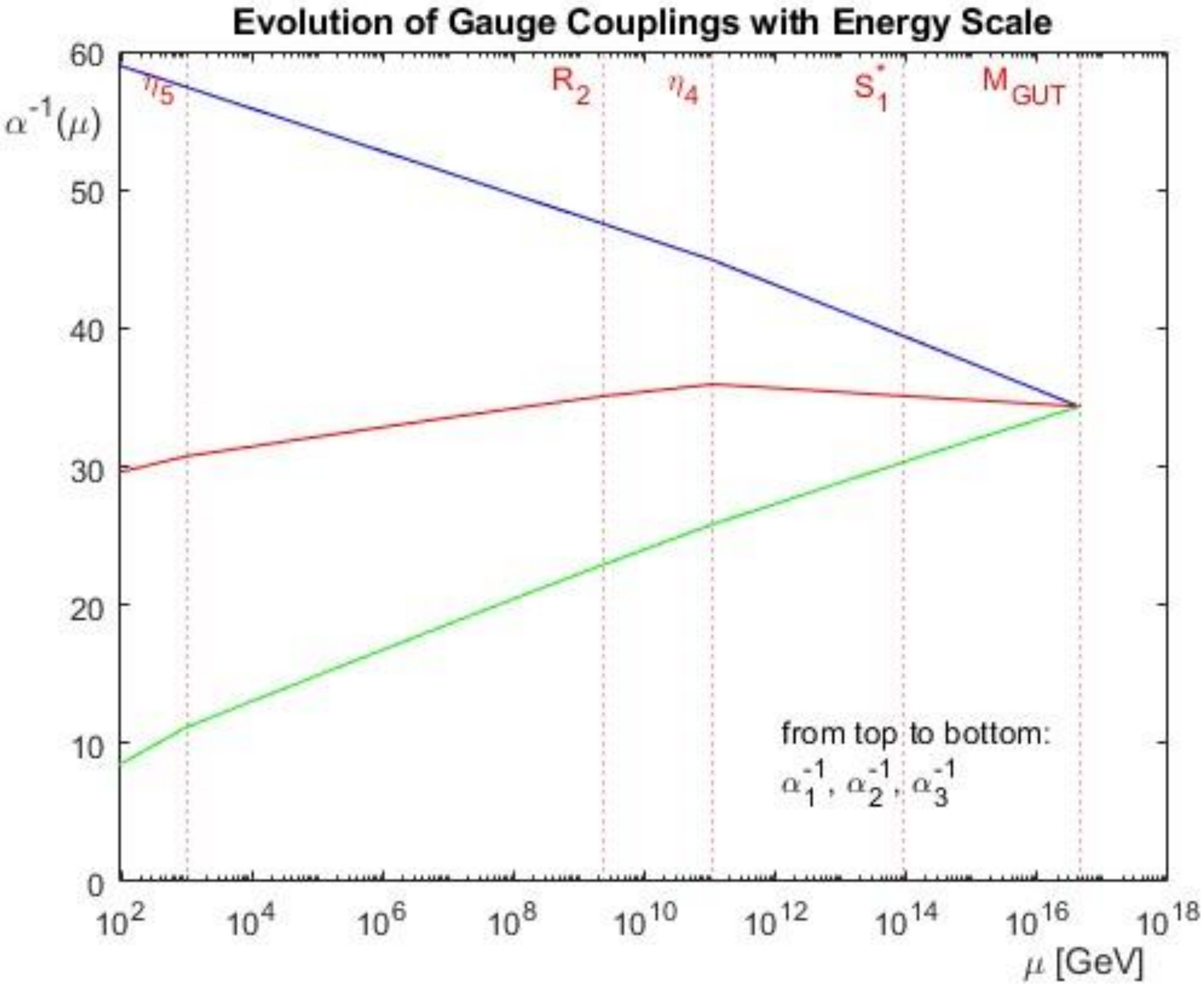
boundary conditions at the GUT scale,

$$\alpha_{GUT}^{-1}(M_{GUT}) = \alpha_1^{-1}(M_{GUT}) + \frac{5}{12\pi} = \alpha_2^{-1}(M_{GUT}) + \frac{3}{12\pi} = \alpha_3^{-1}(M_{GUT}) + \frac{2}{12\pi}$$

After the masses of leptoquarks S_1^*, $\tilde{R}_2$ are set within the allowed ranges, and the mass of η_5 is set, M_{GUT}, α_{GUT}, and the mass of η_4 is solved for.

$$M_{GUT} = 8.30 \times 10^{15} \left(\frac{m_{\eta_5}}{10\ TeV}\right)^{-22/127} \left(\frac{m_{S_1^*}}{1.2 \times 10^{14}\ GeV}\right)^{-1/127} \left(\frac{m_{\tilde{R}_2}}{2.5 \times 10^{15}\ GeV}\right)^{-5/127}$$

$$m_{\eta_4} = 4.76 \times 10^{7} \left(\frac{m_{\eta_5}}{10\ TeV}\right)^{-36/381} \left(\frac{m_{S_1^*}}{1.2 \times 10^{14}\ GeV}\right)^{+33/381} \left(\frac{m_{\tilde{R}_2}}{2.5 \times 10^{15}\ GeV}\right)^{-89/381}$$



$\alpha_{GUT} \cong 1/35$

Charged Fermion Masses

- Mass matrix of down-type quarks,

$$M^D = \left[\frac{1}{2}Y^D - \frac{3}{2}\left(\frac{v_{24}}{\Lambda\sqrt{30}}\right)\left(\tilde{Y}^D - \frac{2}{3}\bar{Y}^D\right)\right]v_5$$

- Mass matrix of charged leptons,

$$(M^E)^T = \left[\frac{1}{2}Y^D - \frac{3}{2}\left(\frac{v_{24}}{\Lambda\sqrt{30}}\right)(\tilde{Y}^D + \bar{Y}^D)\right]v_5$$

- Mass matrix of up-type quarks,

$$M^U = \sqrt{2}[Y^U + (Y^U)^T]v_5$$

Conclusion

- Achieved unification of coupling constants at GUT scales exceeding 1.4×10^{16} GeV in all scenarios,
- Predicts color sextet, weak isodoublet scalar particle with mass 1 TeV (10 TeV),
- Majorana neutrino mass matrix is compatible with the observed masses and mixing angles,
- Degeneracy between the mass matrices of down-type quarks and charged leptons lifted by non-renormalizable operators,
- Leptoquarks can NOT be made light due to mixing!!!

Thank you for your attention!