Primordial gravitational waves in generalized Palatini gravity

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November 7, 2023

Beyond Standard Model: From Theory to Experiment (BSM- 2023)

Overview

Scalar-Vector-Tensor Decomposition

In a universe largely dominated by isotropy and homogeneity, small density fluctuations induces small deviation from FRW

$$ds^2 = ({}^{(0)}g_{\mu\nu} + \delta g_{\mu\nu})dx^\mu dx^\nu$$

${}^{(0)}g_{\mu\nu} \equiv \eta_{\mu\nu} a(\eta)$ and $\delta g_{\mu\nu} \ll^{(0)} g_{\mu\nu}$. Perturbations can be conveniently split into

$$\begin{aligned}
\delta g_{00} &= 2a^2\phi \\
\delta g_{0i} &= a^2(B_{,i} - S_i) \\
\delta g_{ij} &= a^2(2\psi\delta_{ij} + E_{,ij} + 2F_{(i,j)} + h_{ij})
\end{aligned}$$

1. Scalars: 4 DOF encoded in $\{\phi, B, \psi, E\}$ (2 are physical).
2. Vectors: 4 DOF encoded in two divergence-free vectors $\{F_i, S_i\}$ (2 are physical).
3. Tensor: 2 DOF encoded in the traceless, transverse h_{ij}

Gauge fixing

After fixing the coordinates, four DOF can be eliminated i.e. in longitudinal gauge $B = E = 0$ and $V_i = S_i - \dot{F}_i$;

$$
\begin{aligned}
g_{00} &= -a^2(t)(1 + 2\psi^{(1)} + \psi^{(2)}) \\
g_{0i} &= 0 \\
g_{ij} &= a^2(t)\Big[(1 - 2\phi^{(1)} - \phi^{(2)})\delta_{ij} + \frac{1}{2}(\partial_i \chi_j^{(2)} + \partial_j \chi_i^{(2)} + \chi_{ij}^{(2)})\Big]
\end{aligned}
$$

the goal is to solve Einstein field equations resulting from these small perturbations order by order;

$$\delta G_{\mu\nu} = \delta T_{\mu\nu}$$

First-order perturbations

1. Scalar perturbations: they are related to density perturbations and explain structure formation and CMB anisotropy.

$$\begin{aligned} \partial_i \partial^i \psi - 3H(\dot{\psi} + H\phi) &= 2a^2 \delta^{(1)} T_0^0 \\ (\dot{\psi} + H\phi)_{,i} &= 2a^2 \delta^{(1)} T_i^0 \\ \left[\ddot{\psi} + H(2\dot{\psi} + \dot{\phi}) + (2\dot{H} + H^2)\phi + \frac{1}{2}\partial^i \partial_i (\phi - \psi)\right]\delta_{ij} &= -2a^2 \delta^{(1)} T_j^i \end{aligned}$$

2. Vector perturbations:
$(V_{i,j} + V_{j,i})' + 2H(V_{i,j} + V_{j,i}) = -2a^2 \delta T_{ij}$
3. Tensor perturbations: they carry the fundamental DOF of the graviton and usually do not receive a contribution from linear perturbations. They obey a *wave-like* equation

$$h_{ij}'' + 2Hh_{ij}' - \Delta h_{ij} = 2a^2 \delta T^i_{j(T)}$$

Why cosmological perturbation theory?

- Providing the only link between cosmology of the early universe and current observational data.
- Study larger structures from small early density fluctuations.
- Test astrophysical models for compact objects.
- A link between classical and quantum physics and a possible insight to a quantum theory of gravity.
- Study models of particle physics at energy scales beyond the reach of Earthly accelerators.
- Study modified theories of gravity.
- Study models for inflation.

General structure for a theory of gravity

the Poincaré group global as the isometry group of M_4;

$$x^\mu \to x^\mu + \xi^\mu(X)$$
$$\xi^\mu(X) = \epsilon^\mu + \omega^\mu{}_\nu x^\nu$$

generator of Lie group ($\phi' = U(\Lambda, \epsilon)\phi$) are the translation and rotation generators P^μ, $M_{\mu\nu}$;

$$U(\Lambda, \epsilon) \simeq \exp(\frac{1}{2}\omega_{\mu\nu} M^{\mu\nu} + \epsilon_\mu P^\mu)$$

satisfying the algebra

$$[M^{\mu\nu}, M^{\rho\sigma}] = \eta^{\nu\rho} M^{\mu\sigma} - \eta^{\mu\rho} M^{\nu\sigma} + \eta^{\mu\sigma} M^{\nu\rho} - \eta^{\nu\sigma} M^{\mu\rho}$$
$$[M^{\mu\nu}, P^\sigma] = \eta^{\nu\sigma} P^\mu - \eta^{\mu\sigma} P^\nu$$
$$[P^\mu, P^\nu] = 0$$

transformation of the field derivative picks up a derivative term;

$$(\phi)'_{,\mu} = (U\phi)_{,\mu} = U\phi_{,\mu} + U_{,\mu}\phi \neq U\phi_{,\mu}$$
$$\delta_0\phi_{,\mu} = (\frac{1}{2}\omega^{\alpha\beta}M_{\alpha\beta} + \epsilon_\alpha P^\alpha)\phi_{,\mu} - \xi^\nu{}_{,\mu}\phi_{,\nu} + \frac{1}{2}\omega^{\alpha\beta}{}_{,\mu}\Sigma_{\alpha\beta}\phi$$

We define a new "covariant derivative" that includes gauge fields which transform to offset the undesired effect;

$$(\phi)'_{;\mu} \equiv U\phi_{;\mu} \Rightarrow \delta_0\phi_{;\mu} \approx (\frac{1}{2}\omega^{\alpha\beta}M_{\alpha\beta} + \epsilon_\alpha P^\alpha)\phi_{;\mu} + \omega_\mu{}^\alpha\phi_{;\alpha}$$

to eliminate the inhomogeneous term, we simply add a set of compensating gauge fields $A^{ij}{}_\lambda$ such that;

$$\phi_{|\mu} \equiv (\partial_\mu + \frac{1}{2}A^{ij}{}_\mu\Sigma_{ij})\phi$$
$$\delta_0\phi_{|\mu} = (\frac{1}{2}\omega^{\alpha\beta}M_{\alpha\beta} + \epsilon_\alpha P^\alpha)\phi_{|\mu} - \xi^\nu{}_{,\mu}\phi_{|\nu}$$
$$\Rightarrow \delta_0 A^{ij}{}_\lambda = \omega^i{}_\rho A^{\rho j}{}_\lambda + \omega^j{}_\rho A^{i\rho}{}_\lambda - \xi^\rho_{,\lambda}A^{ij}{}_\rho - \xi^\rho A^{ij}{}_{\lambda,\rho} - \omega^{ij}{}_{,\lambda}$$

to eliminate the homogeneous term, we introduce a set of fields ${h_k}^\nu$;

$$\begin{aligned}
\phi_{;k} &\equiv {h_k}^\nu \phi_{|\nu} \\
\delta_0 \phi_{;k} &= (\frac{1}{2}\omega^{\alpha\beta} M_{\alpha\beta} + \epsilon_\alpha P^\alpha)\phi_{;k} + {\omega_k}^\alpha \phi_{;\alpha} \\
\Rightarrow \delta_0 {h_k}^\nu &= {\xi^\nu}_{,\rho} {h_k}^\rho - \xi^\rho {h_k}^\nu{}_{,\rho} + {\omega_k}^\rho {h_\rho}^\nu
\end{aligned}$$

Having done all that, we end up with a Lagrangian for which $\delta\mathscr{L} = 0$, However, we have an extra term in the invariance condition

$$\delta S = 0 \Rightarrow \underbrace{\delta\mathscr{L}}_{=0} + \mathscr{L}(\delta x^\mu)_{,\mu} = 0$$

Owing to the conformal invariance of the Lagrangian, we can define a new Lagrangian $\mathcal{L}' = \mathfrak{L}\mathscr{L}$ and we look for the $\mathfrak{L}$ such that;

$$\begin{gathered}
\Delta\mathscr{L}' = 0 \Rightarrow \mathfrak{L}\delta\mathscr{L} + \mathscr{L}\delta\mathfrak{L} + {\xi^\mu}_{,\mu}\mathfrak{L}\mathscr{L} = 0 \\
\Rightarrow \delta\mathfrak{L} + {\xi^\mu}_{,\mu}\mathfrak{L} = 0
\end{gathered}$$

the only function of the new field variables satisfying this condition can be shown to be shown to be $\mathfrak{L} = [det({h_\mu}^\nu)]^{-1}$.

The covariant derivative allowed for a 40 new gauge fields

$$L(\phi, \phi_{,\mu}) \to \mathscr{L}(\phi, \phi_{,\mu}, h_k{}^{\nu}, A^{ij}{}_{\mu}) \equiv \mathfrak{L}L(\phi, \phi_{;k})$$

to find invariant quantities, we look at the commutator of two covariant derivative

$$\begin{aligned} [D_k, D_l]\phi &= h_k{}^{\lambda}(\partial_\lambda + \frac{1}{2}A^{ij}{}_{\lambda}\Sigma_{ij})h_l{}^{\rho}(\partial_\rho + \frac{1}{2}A^{ij}{}_{\rho}\Sigma_{ij})\phi - (k \leftrightarrow l) \\ &= \frac{1}{2}R^{ij}{}_{kl}\Sigma_{ij}\phi - S^{i}{}_{kl}\phi_{;i} \\ R^{ij}{}_{kl} &\equiv h_k{}^{\lambda}h_l{}^{\rho}(A^{ij}{}_{\rho,\lambda} - A^{ij}{}_{\lambda,\rho} + A^{i}{}_{k\lambda}A^{kj}{}_{\rho} - A^{i}{}_{m\rho}A^{mj}{}_{\lambda}) \\ S^{i}{}_{kl} &\equiv h_k{}^{\lambda}h_l{}^{\rho}(b^{i}{}_{\rho|\lambda} - b^{i}{}_{\lambda|\rho}) \end{aligned}$$

Geometric interpretation

theories of gravity follow from the geometric interpretation of the gauge fields. GR is based on Riemannian geometry, which satisfies the following constraints:

1. $\nabla_\alpha g_{\mu\nu} = 0$.
2. $\Gamma^\lambda_{\ \ \mu\nu}$. These two conditions lead to the Levi-Civita connection

$$^g\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} \left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} \right)$$

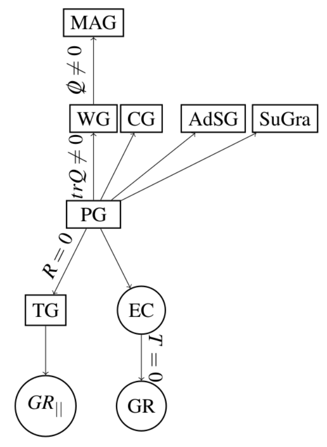

Figure: Reproduced from Milutin Blagojevic and Friedrich W. Hehl. Gauge Theories of Gravitation. 10 2012.

Palatini formalism

By Define nonmetricity tensor $\nabla_\rho g^{\alpha\beta} \equiv Q_\rho{}^{\alpha\beta}$ and torsion tensor $S_{\mu\nu}{}^{\alpha} = \Gamma^{\alpha}{}_{[\mu\nu]}$. The Affine connection can be written as

$$
\begin{aligned}
\Gamma^{\lambda}{}_{\alpha\mu} &= \underbrace{\frac{1}{2}g^{\lambda\nu}(\partial_\mu g_{\nu\alpha} + \partial_\alpha g_{\nu\mu} - \partial_\nu g_{\alpha\mu})}_{\tilde{\Gamma}^{\lambda}{}_{\alpha\mu}} \\
&\quad + \underbrace{S^{\lambda}{}_{\alpha\mu} + S_{\alpha}{}^{\lambda}{}_{\mu} + S_{\mu}{}^{\lambda}{}_{\alpha}}_{K^{\lambda}{}_{\alpha\mu}} + \underbrace{\frac{1}{2}g^{\lambda\nu}(Q_{\mu\nu\alpha} + Q_{\alpha\mu\nu} - Q_{\nu\alpha\mu})}_{N^{\lambda}{}_{\alpha\mu}}
\end{aligned}
$$

Reimmann tensor is still used to construct invariant quantities
Define the covariant derivative of a tensorial field to measure the parallel transport. For a vector field u^μ parallel transported along two paths;

$$
\begin{aligned}
[\nabla_\alpha, \nabla_\beta]u^\mu &= R^{\mu}{}_{\nu\alpha\beta}u^\nu + 2S_{\alpha\beta}{}^{\nu}\nabla_\nu u^\mu \\
R^{\mu}{}_{\nu\alpha\beta} &\equiv 2\partial_{[\alpha}\Gamma^{\mu}{}_{|\nu|\beta]} + 2\Gamma^{\mu}_{\lambda[\alpha}\Gamma^{\lambda}{}_{|\nu|\beta]} \\
S_{\alpha\beta}{}^{\nu} &\equiv \Gamma^{\nu}{}_{[\alpha\beta]}
\end{aligned}
$$

Field equations in GR

Simplest invariant quantity can be constructed from the contraction $R = g^{\mu\nu} R^{\alpha}_{\mu\alpha\nu}$

$$
\begin{aligned}
S &= \int d^4x \left(\frac{1}{2\kappa} R(g) + \mathscr{L}_m\right)\sqrt{-g} \\
\delta_g S = 0 \Rightarrow G_{\mu\nu}(\Gamma) &= 8\pi G_N T_{\mu\nu} \\
\delta_S = 0 \Rightarrow S^{\alpha}{}_{\mu\nu} &= 0 \\
\delta_Q S = 0 \Rightarrow Q^{\alpha}{}_{\mu\nu} &= 0
\end{aligned}
$$

where $G_{\mu\nu}(\Gamma) \equiv R_{\mu\nu}(\Gamma) - \frac{1}{2} R(\Gamma) g_{\mu\nu}$ is the Einstein tensor. While reproducing GR, the structure allows in general for a non-vanishing torsion and non-metricity provided that a matter field can couple with these geometric fields.

R2 theories

another scalar can be constructed from Riemann tensor known as *homoethic curvature*

$$\hat{R}_{\alpha\beta} = \partial_\alpha \Gamma^{\nu}{}_{\nu\beta} - \partial_\beta \Gamma^{\nu}{}_{\nu\alpha} = \frac{1}{2}\partial_\alpha Q_\beta - \frac{1}{2}\partial_\beta Q_\alpha$$

note that $g^{\mu\nu}\hat{R}_{\alpha\beta} = 0$ so the simplest invariant quantity must be quadratic;

$$S(g,\Gamma) = \int d^4x\sqrt{-g}(g^{\mu\nu}R_{\mu\nu} + \xi \hat{R}_{\mu\nu}\hat{R}^{\mu\nu} + \mathscr{L}_m)$$

variation with respect to the metric leads to

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{\xi}{2}g_{\mu\nu}\hat{R}_{\alpha\beta}\hat{R}^{\alpha\beta} + \xi g^{\lambda\rho}(\hat{R}_{\alpha\mu}\hat{R}_{\beta\nu} + \hat{R}_{\mu\alpha}\hat{R}_{\nu\beta}) &= T_{\mu\nu} \\ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= T_{\mu\nu} + T^{eff}_{\mu\nu} \end{aligned}$$

Einstein-Proca field

variation wrt Γ reveals that non-metricity can be expressed in terms of one vectorial DOF

$$Q_{\lambda\mu\nu} = 2g_{\mu\nu}Q_\lambda - 2g_{\lambda\mu}Q_\nu - 2g_{\lambda\nu}Q_\mu$$

vector is scaled Q_μ obeys Proca equation

$$\begin{aligned}
R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} &= \underbrace{F_{\alpha\mu}F_{\beta\nu}g^{\alpha\beta} - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta}g_{\mu\nu} + m^2A_\mu A_\nu - \frac{1}{2}m^2A^\alpha A_\alpha g_{\mu\nu}}_{T^{eff}_{\mu\nu}} \\
\tilde{\nabla}_\mu F^{\mu\nu} - m^2A^\nu &= 0 \\
F_{\mu\nu} &\equiv \partial_\mu A_\nu - \partial_\nu A_\mu
\end{aligned}$$

The theory has advantage of being quantum mechanically stable and free of ghosts c.f. $R_{\mu\nu}R^{\nu\mu}$.

Zeroth order

We assume an FRW background with cosmological constant

$$\begin{aligned} ds^2 &= -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \\ \Rightarrow G^{(0)}_{\mu\nu} + \Lambda g^{(0)}_{\mu\nu} &= 0 \\ \Rightarrow a(t) &= a(t_0)e^{\sqrt{\frac{\Lambda}{3}}t} \end{aligned}$$

Our mail assumption is that A_μ is a first order perturbation to the geometry in essence so it would exhibit no influence on the zeroth order. We are only interested in the footprint of this field not the inflaton itself so we adopt a general exponentially expanding universe.

First order

As the effective EM is second order in nature, $\delta^{(1)} T_{\mu\nu} = 0$ and the first order field equations $\delta^{(1)} G_{\mu\nu} = \delta^{(1)} T_{\mu\nu}$

$$\begin{aligned}
\delta^{(1)} G^0{}_0 a^2 &= 6H\partial_t\psi^{(1)} + 6H^2\phi^{(1)} - 2\partial_{ii}\psi^{(1)} \\
\delta^{(1)} G^0{}_i a^2 &= 2H\partial_i\phi^{(1)} + 2\partial_t\partial_i\psi^{(1)} \\
\delta^{(1)} G^i{}_i a^2 &= -2\partial_{tt}\psi^{(1)} - 2H\partial_t\phi^{(1)} - 4H\partial_t\psi^{(1)} - 4\phi^{(1)}\dot{H} - 2H^2\phi^{(1)} \\
&+ \Delta(\psi^{(1)} - \phi^{(1)}) - (\psi^{(1)} - \phi^{(1)})_{,ii} \\
\delta^{(1)} G^i{}_j a^2 &= \delta^{(1)} G^2{}_1 = (\Phi - \Psi)_{,ij}, \quad i \neq j
\end{aligned}$$

The last equation gives $\psi = \phi$. The third equation give the time evolution of $\psi^{(1)}$ as;

$$\phi^{(1)} = C_1 f(t) + C_2 g(t)$$

The second equation then leads to $(a\phi^{(1)})_{,it} = 0$ which implies $C_1(r,\theta,\phi) = C_1$ and $C_2(r,\theta,\phi) = C_2$. Finally, the first equation implies $C_1 = C_2 = 0$. So the solution is

$$\phi^{(1)} = \psi^{(1)} = 0$$

Second order

An explicit solution of the Proca equation reveals

$$Y_0^{(1)}(\eta) = 0$$
$$Y_i^{(1)}(\eta) = C_{i+}\eta^{\frac{1}{2}+\sqrt{1+\frac{m^2}{H^2}}} + C_{i-}\eta^{\frac{1}{2}-\sqrt{1+\frac{m^2}{H^2}}},$$

From ${G_0}^0$, $\Delta^{-1}\partial_i {G_0}^i$ and ${G_k}^k$

$$2\partial_i\partial^i\psi^{(2)} - 6H\dot{\psi}^{(2)} - 6H^2\phi^{(2)} = m^2 Q_k Q^k - \frac{s}{a^2}\dot{Q}_i\dot{Q}^i$$
$$\dot{\psi}^{(2)} + H\phi^{(2)} = 0$$
$$\Delta\phi^{(2)} + 3H\dot{\phi}^{(2)} + 6H^2\phi^{(2)} - \Delta\psi^{(2)} + 6H\dot{\psi}^{(2)} + 3\ddot{\psi}^{(2)} = -\frac{m^2}{2}Q_k Q^k + \frac{s}{2a^2}\dot{Q}$$

Using the second equation ϕ can be eliminated in the first and third revealing that ψ is not dynamical

$$\Delta\psi^{(2)} = \frac{m^2}{2}Q_k Q^k - \frac{s}{2a^2}\dot{Q}_i\dot{Q}^i$$
$$-\frac{1}{H}\Delta\dot{\psi}^{(2)} - \Delta\psi^{(2)} = -\frac{m^2}{2}Q_k Q^k + \frac{s}{2a^2}\dot{Q}_k\dot{Q}^k$$

Tensor perturbations

The projection operator can be used to eliminate the scalar and vector parts of the second-order Einstein field equations as

$$\Pi_{ij}^{lm} G_{lm}^{(2)} = \Pi_{ij}^{lm} T_{lm}^{(2)}$$

this leads then to the following wave equation

$$\begin{aligned} h''_{ij} + 2Hh'_{ij} - \nabla^2 h_{ij} &= -4\Pi_{ij}{}^{lm}\mathcal{S}_{lm} \\ \mathcal{S}_{lm}(\eta) &\equiv 8\pi G_N \left(m^2 Y_l Y_m - \frac{1}{a^2} Y'_l Y'_m \right) \\ &- 8\pi G_N \left(m^2 Y_k Y_k - \frac{1}{a^2} Y'_k Y'_k \right) \delta_{lm} \end{aligned}$$

By Fourier-transforming from $(\eta, \mathbf{x})$ to $(\eta, \mathbf{k})$ space one gets

$$h''^{(\lambda)}(\mathbf{k}, \eta) + 2Hh'^{(\lambda)}(\mathbf{k}, \eta) + k^2 h^{(\lambda)}(\mathbf{k}, \eta) = \mathcal{S}^{(\lambda)}(\mathbf{k}, \eta),$$

Green's function

Equation can be written as

$$\frac{1}{a}\left(\ddot{v}_{\mathbf{k}} + (k^2 - \frac{\ddot{a}}{a})v_{\mathbf{k}}\right) \quad = \quad q^{ik}(\mathbf{k})S_{ij}(t)\delta^3(\mathbf{k}) \equiv \mathcal{S}(\mathbf{k},t)$$

Where $h_k = \frac{v_k}{a}$. We resort to Green's function method to solve this equation, writing it as a second order differential equation $\hat{\mathcal{L}}G(t,\tilde{t}) = \delta(t-\tilde{t})$ where

$$G(t;\tilde{t}) \equiv \frac{v_1(t)v_2(\tilde{t}) - v_1(\tilde{t})v_2(t)}{v_1'(\tilde{t})v_2(\tilde{t}) - v_1(\tilde{t})v_2'(\tilde{t})}$$

where $v_1(t),\ v_2(t)$ being two linearly independent solutions to the homogenous equation $\hat{\mathcal{L}}v(t) = 0$. For a De-Sitter universe $H(t) \equiv H$

$$\begin{aligned} v_\pm(t) &= e^{(-H\pm\sqrt{H^2-k^2})t} \\ G(t;\tilde{t}) &= \frac{1}{2\sqrt{H^2-k^2}}\left(e^{(-H+\sqrt{H^2-k^2})(t-\tilde{t})} - e^{(-H-\sqrt{H^2-k^2})(t-\tilde{t})}\right) \\ &= \frac{1}{\sqrt{H^2-k^2}}\sinh(\sqrt{H^2-k^2}(\eta-\tilde{\eta})) \end{aligned}$$

Power spectrum

The particular solution for the gravitational waves is then given by an integral over the Greens function

$$\begin{aligned} h_{\mathbf{k}}(t) &= \int_{t_0}^{t} G_k(t;\tilde{t})\mathcal{S}(\mathbf{k},\tilde{t})d\tilde{t}, \\ \text{or,} \quad h_{\mathbf{k}}(t) &= \frac{1}{a(t)}\int_{t_0}^{t} \bar{G}_k(t;\tilde{t})a(\tilde{t})S(\mathbf{k},\tilde{t})d\tilde{t} \\ \bar{G}_k(t;\tilde{t}) &= \frac{1}{\sqrt{H^2-k^2}}\sinh(\sqrt{H^2-k^2}(t-\tilde{t})) \end{aligned}$$

The power spectrum is then defined by;

$$\begin{aligned} \langle h_{\mathbf{k}_1}(t)h_{\mathbf{k}_2}(t)\rangle &= \frac{2\pi^2}{k^3}\delta(\mathbf{k}_1-\mathbf{k}_2)\mathcal{P}_h(k,t) \\ \mathcal{P}_h(k,t) &= k^3\frac{1}{a(t)^2}\int_{t_0}^{t}\int_{t_0}^{t} \bar{G}_k(t;\tilde{t}_1)a(\tilde{t}_1)S(\tilde{t}_1)\bar{G}_k(t;\tilde{t}_2)a(\tilde{t}_2)S(\tilde{t}_2)d\tilde{t}_1d\tilde{t}_2 \\ \Omega_{GW}(k,\eta) &\equiv \frac{\rho_{GW}(k,\eta)}{\rho_{cr}(\eta)} = \frac{1}{24}\left(\frac{k}{H}\right)^2\overline{\mathcal{P}_h(k,\eta)} \end{aligned}$$

Sensitivity curves

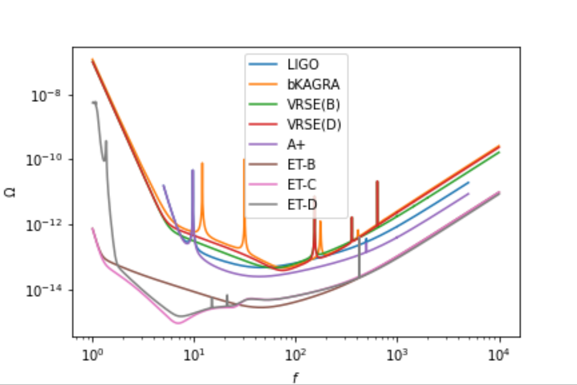

Figure: Sensitivity curves for several ground-based interferometers. Compiled from publicly available data.

Dependence on Q modes

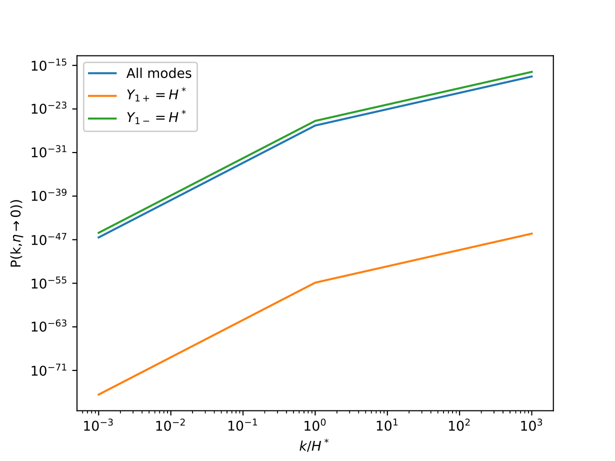

Figure: Dependance of P of Q modes. Y_{i-} modes dominate the power spectrum because these modes grow rapidly towards the end of inflation.

We assume high-scale inflation $H^\star = 10^{14}\ \mathrm{GeV}$ compatible with the observational upper bound.

Dependence on mass

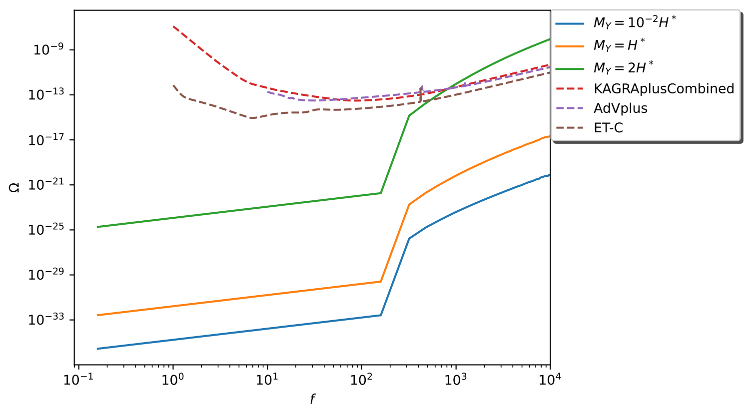

Figure: The GW energy density $\Omega_{GW}(k,\eta)$ defined in (**??**) as a function of the GW frequency f and the geometric Proca mass M_Y. Superimposed on the plots are the sensitivity curves.

The main result reveals that for slightly larger masses, a detectable stochastic signal can be within reach at higher frequencies.

Conclusions and prospects

- Geometric Proca field significantly enhances the gravitational wave signal to detection level of future upgrades of GW detectors into third-generation (3G).
- If these findings are confirmed through observation, it will provide evidence that gravity is non-Riemannian in nature
- A lack of observational evidence would hugely constraint this class of theories so it would be disfavored direction of research. This could be a similar scenario to scalar-tensor theories after GW170817.
- A confirmation on non-Riemannian gravity would have far-reaching implications for our understanding of the universe and its evolution, including shedding light on the origin of PGWs and the nature of dark matter.
- Further possible directions include studying the effects of the geometric Proca field on other astrophysical phenomena, such as the formation of galaxies and the evolution of black holes.
- Study of BSM with fields of geometric origin.