Interpreting the Cosmic History of the Universe Through Five-Dimensional Supergravity

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CFP at Zewail City, Egypt

BSM: From Theory to Experiment
November 6-9, 2023. Hurghada-Egypt

Outline

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Motivation

Constructing a model that describes the whole cosmic evolution

of the universe since the big bang till the late time acceleration

Vital questions in cosmology:

1. What causes the inflation in the early universe?
2. What is the dark energy?

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1 $\mathcal{N} = 2$ $\mathcal{D} = 5$ supergravity has been derived from the dimensional reduction of $\mathcal{D} = 11$ $\mathcal{N} = 1$ supergravity over Calabi-Yau 3-fold.

2 Matter content: The *universal hypermultiplet*: Composed of $\left(\phi, \sigma, \zeta^0, \tilde{\zeta}_0\right)$; where ϕ is the universal axion, and the dilaton σ is proportional to the volume of the underlying Calabi-Yau manifold $\mathcal{M}$. The remaining hypermultiplet scalars are $\left(z^i, z^{\bar{i}}, \zeta^i, \tilde{\zeta}_i : i = 1, \ldots, h_{2,1}\right)$, where the z's are the complex structure moduli of $\mathcal{M}$, and $h_{2,1}$ is the Hodge number determining the dimensions of the manifold $\mathcal{M}_C$ of the Calabi-Yau's complex structure moduli. The fields $\left(\zeta^I, \tilde{\zeta}_I : I = 0, \ldots, h_{2,1}\right)$ are the axions. The axions are represented by the symplectic representation: $\langle \Xi \mid \Xi \rangle = \zeta^I \tilde{\zeta}_I - \tilde{\zeta}_I \zeta^I$.

Moataz H. Emam and Safinaz Salem, Class. Quant. Grav. **40** 9, 095001 , (2023) [arXiv: 2303.12625 [gr-qc]]

$$S_5 = \int_5 \Big[\mathcal{R} \star \mathbf{1} - \frac{1}{2} d\sigma \wedge \star d\sigma - G_{i\bar{j}} dz^i \wedge \star dz^{\bar{j}} + e^{\sigma} \langle d\Xi | \underset{\wedge}{\mathbf{\Lambda}} | \star d\Xi \rangle$$

$$- \frac{1}{2} e^{2\sigma} \left[d\phi + \langle \Xi \mid d\Xi \rangle \right] \wedge \star \left[d\phi + \langle \Xi \mid d\Xi \rangle \right] \Big] .$$

$$T^{Bulk}_{\mu\nu} = G_{i\bar{j}} \left(\partial_\mu z^i \right) \left(\partial_\nu z^{\bar{j}} \right) - \frac{1}{2} g_{\mu\nu} G_{i\bar{j}} \left(\partial_\alpha z^i \right) \left(\partial^\alpha z^{\bar{j}} \right) .$$

Constructing a 3-brane as a flat Robertson-Walker universe embedded in five dimensions:

$$ds^2 = g_{MN}\ dx^M\ dx^N = -dt^2 + a^2(t) \left(dr^2 + r^2 d\Omega^2 \right) + b^2(t)\, dy^2 .$$

$$T^{Brane}_{\mu\nu} = (\rho + p) U_\mu U_\nu + g_{\mu\nu} p .$$

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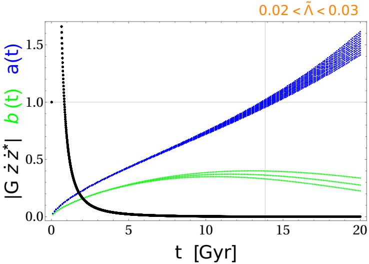
Substitute in the Einstein's equations $G_{MN} + \Lambda g_{MN} = T_{MN}$, yields:

$$\left[\left(\frac{\dot{a}}{a}\right)^2 + \left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{b}}{b}\right)\right] = G_{i\bar{j}}\dot{z}^i\dot{z}^{\bar{j}} + H_0^2\left(\frac{\Omega_{m0}}{a^3} + \frac{\Omega_{r0}}{a^4}\right)$$

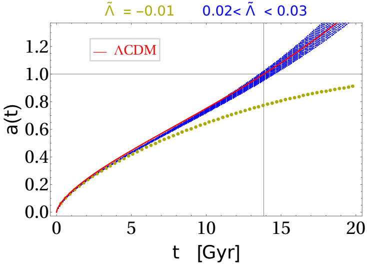
$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\ddot{b}}{b} + 2\left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{b}}{b}\right) = -H_0^2\frac{\Omega_{r0}}{a^4} - G_{i\bar{j}}\dot{z}^i\dot{z}^{\bar{j}}$$

$$3\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right] = \tilde{\Lambda} - G_{i\bar{j}}\dot{z}^i\dot{z}^{\bar{j}}.$$

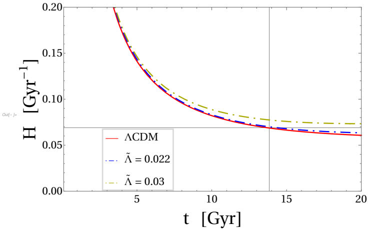
In our model, we consider the dark energy density parameter $\Omega_\Lambda = 0$, and the current matter density parameter $\Omega_{m0} \sim 0.99$. According to ΛCDM model based on CMB observations $\Omega_{m0} = 0.3111$, $\Omega_{r0} = 8.2 \times 10^{-5}$, and $\Omega_{\Lambda 0} = 0.6889$. $H_0 = 0.0686751$ [Gyr^{-1}] $\sim 2.176 \times 10^{-18}$ [sec^{-1}]. The age of the universe is $t_0 = 13.842$ [Gyr] [1].



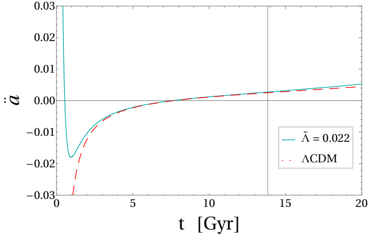
(a) The brane's scale factor a (blue), the bulk's scale factor b (green), and $\left|G_{i\bar{j}}\dot{z}^i\dot{z}^{\bar{j}}\right|$ (black) are plotted versus time for $0.02 < \tilde{\Lambda} < 0.03$ [Gyr^{-2}]. $a_0 = 1$, and $t_0 = 13.842$ [Gyr].



(b) The brane's scale factor (blue) fits the scale factor of the ΛCDM model (red) for the range $0.02 < \tilde{\Lambda} < 0.03$. The yellow curve shows the brane's scale factor for $\tilde{\Lambda} = -0.01$.



(a) The Hubble parameter of ΛCDM (red), the brane's Hubble parameters for $\tilde{\Lambda} = 0.022$ [Gyr^{-2}] (blue dashed) and $\tilde{\Lambda} = 0.03$ [Gyr^{-2}] (yellow dashed) are plotted versus time. $H_0 = 0.0689751$ [Gyr^{-1}].



(b) The acceleration of the brane's scale factor (blue) and the acceleration of the scale factor of ΛCDM (red dashed) are plotted versus time. ($\ddot{a} > 0$) inflationary behavior. Around $t = 10$ [Gyr] the accelerated expansion has been initiated.

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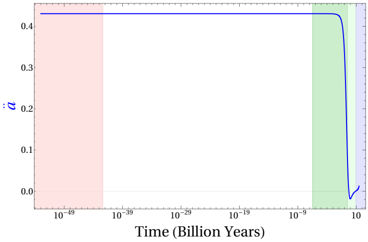
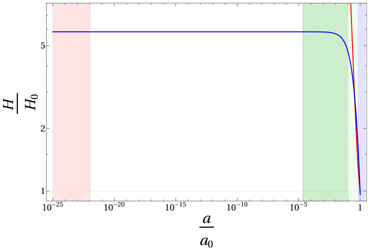
The cosmic history



(a) The acceleration of the brane's scale factor is plotted versus time on a logarithmic scale ($\tilde{\Lambda} = 0.022.$). The pink area corresponds to the inflation era $t \sim (10^{-53} - 10^{-43})$ [Gyr] the dark green area when the CMB happened after the big bang by $t_{CMB} = 380.000$ years, the light green area when the deceleration expansion happened, and the light blue area corresponds to the late-time acceleration era happens after $(9 - 10)$ billion years of the big bang till time beings. This is according to BOSS data based on the baryon acoustic oscillation (BAO) [2].



(b) $H(a)/H_0$ for the brane-universe (blue) and the ΛCDM model (red) are plotted versus the expansion scale factor. The pink area is a high regime era where $H >> H_0$, which corresponds to the inflation era, and the dark green area is when the CMB happens. Then H/H_0 starts to get close to unity as the brane starts deceleration expansion through the light green area. The light blue area when the accelerated expansion started at $a \sim 0.7$ till the time beings.

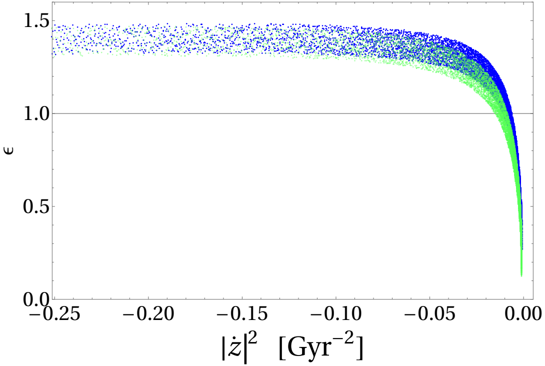


Figure: The slow-roll parameters are plotted against the moduli's flow velocity $(G_{i\bar{j}}\dot{z}^i\dot{z}^{\bar{j}})$ at $\tilde{\Lambda} = 0.022$ [Gyr^{-2}] (blue) and $\tilde{\Lambda} = 0.03$ [Gyr^{-2}] (green).

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Conclusion

According to $\mathcal{D} = 5 - \mathcal{N} = 2$ supergravity

1. The cosmic evolution of the scale factor of a 3-brane embedded in a five-dimensional bulk coincides with our universe's scale factor according to ΛCDM model and BOSS data depending on the bulk's cosmological constant and the strong correlation between the complex structure moduli of the Calabi-Yau manifold and the brane's scale factor.
2. The inflationary epoch of the brane-universe appears when the acceleration of the brane's scale factor $(\ddot{a} > 0)$. In the early universe the complex structure moduli can play the role of the inflaton, where the inflation's slow-roll parameter $(\epsilon < 1)$ when the moduli's flow velocity is large.
3. The present accelerated expansion of the brane-universe is induced by the effects of a de Sitter extra fifth dimension.

References

1 Planck Collaboration, Y. Akrami, et al., "Planck 2018 results. VI. Cosmological parameters," Astron. Astrophys. **641** (2020) A6, arXiv:astro-ph/1807.06209.

2 BOSS Collaboration, Timothée Delubac, et al., "Baryon acoustic oscillations in the Lyα forest of BOSS quasars," Astron.Astrophys. **574** (2015) A59, arXiv:astro-ph.CO/1404.1801.

Thank you!