جامعة الإمارات العربية المتحدة
United Arab Emirates University

Neutrino phenomenology in a flavored NMSSM without domain wall problem

Mohamed Amin Loualidi

Department of physics, United Arab Emirates University, Al-Ain, UAE

Based on 1) Phys. Rev. D,102.115023, in collaboration with M.A.Ouahid, R.Ahl.Laamara and E.H.Saidi
2) Second work to appear soon, in collaboration with A. Jueid, S. Nasri, and M.A. Ouahid

Beyond Standard Model: From Theory to Experiment (BSM- 2023)
November 6 - 9, 2023, Hurghada – Egypt

Plan of the talk

I- Introduction and motivations

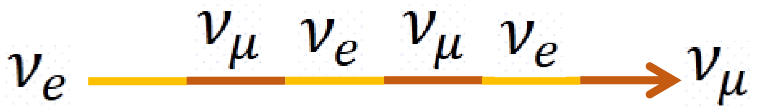
II- Neutrino phenomenology in a A4-flavored NMSSM

III- Domain walls formation in the charged lepton and neutrino sectors

IV- Summary and conclusion

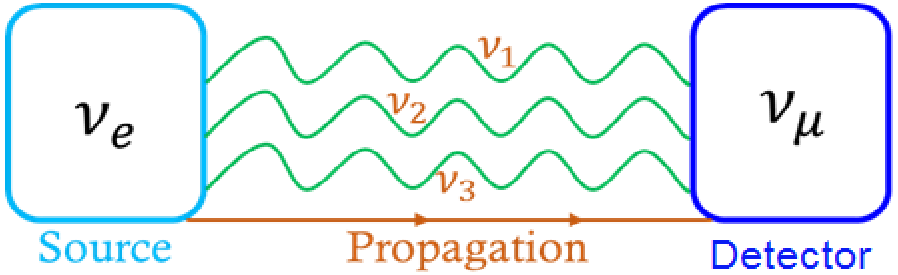
I-Introduction and motivations

- The SuperKamiokande (1998) and SNO (2002) collaborations discovered that neutrinos oscillate



This phenomenon is only possible if neutrinos have different masses.

- In 1969, Pontecorvo and Gribov proposed the reason for the oscillation as being the mixing of neutrinos



I-Introduction and motivations

- The SuperKamiokande (1998) and SNO (2002) collaborations discovered that neutrinos oscillate

$$\nu_e \xrightarrow{\;\nu_\mu \;\; \nu_e \;\; \nu_\mu \;\; \nu_e\;} \nu_\mu \Longrightarrow$$

This phenomenon is only possible if neutrinos have different masses.

- In 1969, Pontecorvo and Gribov proposed the reason for the oscillation as being the mixing of neutrinos

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

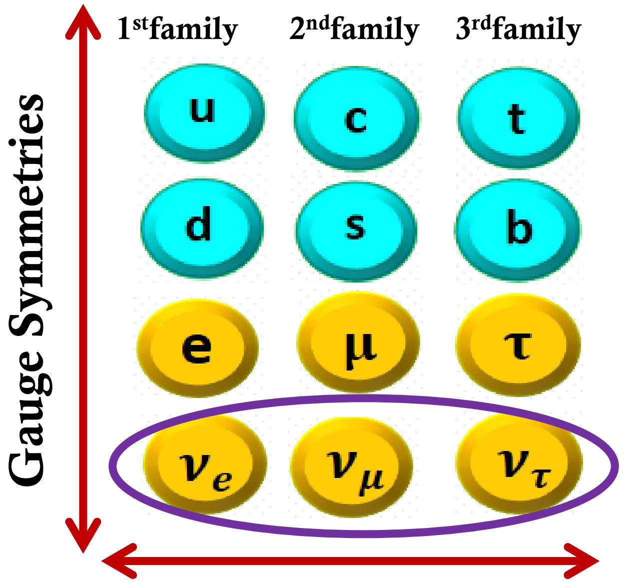
- In the case of the three flavors, the mixing matrix is expressed in terms of 3 mixing angles and a CP violation phase.

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & e^{-i\delta}s_{13} \\ -s_{12}c_{23} - e^{-i\delta}c_{12}s_{23}s_{13} & c_{12}c_{23} - e^{-i\delta}s_{12}s_{13}s_{23} & s_{23}c_{13} \\ s_{13}s_{23} - e^{-i\delta}c_{12}c_{23}s_{13} & e^{-i\delta}c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{23}c_{13} \end{pmatrix} \quad \text{with} \quad \begin{cases} s_{ij} = \sin\theta_{ij} \\ c_{ij} = \cos\theta_{ij} \end{cases}$$

- Among the six oscillation parameters, there are two that are still unknown: $\boldsymbol{\delta}$ and the sign of $\boldsymbol{\Delta m^2_{31}}$

I-Introduction and motivations

Family Symmetries



Family Symmetries

S_3, S_4 , A_4 , A_5, D_4 …

$$SU(2)\begin{pmatrix} u \\ d \end{pmatrix}_L \xleftrightarrow{SU(3)} \begin{pmatrix} u \\ d \end{pmatrix}_L$$

- A family symmetry is a symmetry that connects the three generations of the fermions of the SM

- Non-Abelian Discrete symmetries are motivated by large mixing angles measured by neutrino oscillation data

Octant: $\theta_{23} > \pi/4$ **or** $\theta_{23} < \pi/4$

Normal or inverted neutrino mass hierarchy

$\Delta m^2_{31} < 0$ **or** $\Delta m^2_{31} > 0$

CP violation $\longrightarrow$ $\delta \neq \{0, \pi\}$

II- Neutrino phenomenology in A4-flavored NMSSM

Implementation of the A_4 discrete symmetry in the NMSSM

Fields	L_i	e^c	μ^c	τ^c	N_i^c	H_u	H_d	Φ	Ω	χ	S
A_4	$3_{(-1,0)}$	$1_{(1,\omega^2)}$	$1_{(1,\omega)}$	$1_{(1,1)}$	$3_{(-1,0)}$	$1_{(1,1)}$	$1_{(1,\omega)}$	$3_{(-1,0)}$	$3_{(-1,0)}$	$1_{(1,1)}$	$1_{(1,\omega^2)}$
Z_3	ω	ω	ω	ω	ω^2	1	ω	1	ω^2	ω^2	ω^2

Type-I Seesaw

Charged leptons

TBM mixing

Deviation from TBM

The chiral superpotential for neutrino Yukawa couplings respecting gauge and A_4 symmetries are given by

$$W_D = Tr_{A_4}(Y^{ij} L_i N_j^c H_u) \longrightarrow \underbrace{3 \times 3 \times 1}_{1 = 11 + 23 + 32} \longrightarrow m_D = Y_0 \upsilon_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \langle H_u \rangle = \upsilon_u$$

For Majorana neutrinos, the couplings are given by: $W_R = TrA_4(\lambda\, \chi\, N^c N^c + \lambda' \Omega\, N^c N^c + \lambda''\, S\, N^c N^c)$

$$\left.\begin{matrix} \langle \Omega_1 \rangle = \langle \Omega_2 \rangle = \langle \Omega_3 \rangle = \upsilon_\Omega \\ \langle S \rangle = \upsilon_S \quad , \quad \langle \chi \rangle = \upsilon_\chi \end{matrix}\right\} \longrightarrow M_R = \begin{pmatrix} a + (2b/3) & -(b/3) + \epsilon & -(b/3) \\ -(b/3) + \epsilon & 2b/3 & a - (b/3) \\ -(b/3) & a - (b/3) & (2b/3) + \epsilon \end{pmatrix} \text{ with } \begin{cases} a = 2\lambda \upsilon_\chi \\ b = 2\lambda' \upsilon_\Omega \\ \epsilon = 2\lambda'' \upsilon_S \end{cases}$$

II- Neutrino phenomenology in A4-flavored NMSSM

The light neutrino mass matrix is obtained using type I seesaw mechanism formula $\boldsymbol{m_\nu = m_D^T M_R^{-1} m_D}$

$\boldsymbol{M_R}$ exhibit the "magic symmetry" and thus, it is diagonalized by the well known trimaximal mixing matrix U_{TM2}.

For the inverse Majorana neutrino mass, we have: $M_R^{-1} = U^*_{\text{TM}_2}[\text{diag}(M_1, M_2, M_3)]^{-1}(U^*_{\text{TM}_2})^T$

with
$$\begin{cases} M_1 = a + b - (\epsilon/2) \\ M_2 = a + \epsilon \\ M_3 = b - a + (\epsilon/2) \end{cases}$$

$$U_{TM_2} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin\theta\, e^{-i\sigma} \\ -\frac{\cos\theta}{\sqrt{6}} - \frac{\sin\theta}{\sqrt{2}} e^{i\sigma} & \frac{1}{\sqrt{3}} & \frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{6}} e^{-i\sigma} \\ -\frac{\cos\theta}{\sqrt{6}} + \frac{\sin\theta}{\sqrt{2}} e^{i\sigma} & \frac{1}{\sqrt{3}} & -\frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{6}} e^{-i\sigma} \end{pmatrix} .U_P$$

$\boldsymbol{\theta \to 0}$ corresponds to TBM mixing

Due to the form of $\boldsymbol{m_D}$, the diagonalization of $\boldsymbol{m_\nu}$ remains of trimaximal form. The light neutrino masses are

$$\boxed{(\tilde{U}_{TM_2})^T m_\nu \tilde{U}_{TM_2} = diag(m_1, m_2, m_3) \quad \text{where} \quad m_i = (Y_0^2 v_u^2)/M_i \quad \text{and} \quad \tilde{U}_{TM_2} = \frac{m_D}{Y_0 v_u} U_{TM_2}}$$

In the case of trimaximal mixing, the three neutrino mixing angles are expressed as follows

$$\boxed{\sin^2\theta_{13} = \frac{2}{3}\sin^2\theta \quad , \quad \sin^2\theta_{12} = \frac{1}{3-2\sin^2\theta} \quad , \quad \sin^2\theta_{23} = \frac{1}{2} - \frac{\sqrt{3}\sin 2\theta}{2(3-\sin^2\theta)}}$$

II- Neutrino phenomenology in A4-flavored NMSSM

What is the absolute mass scale of neutrinos?

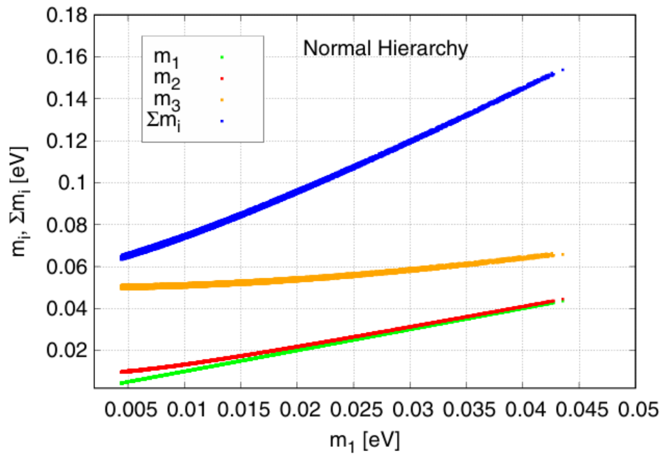
- Constraints from cosmological observations

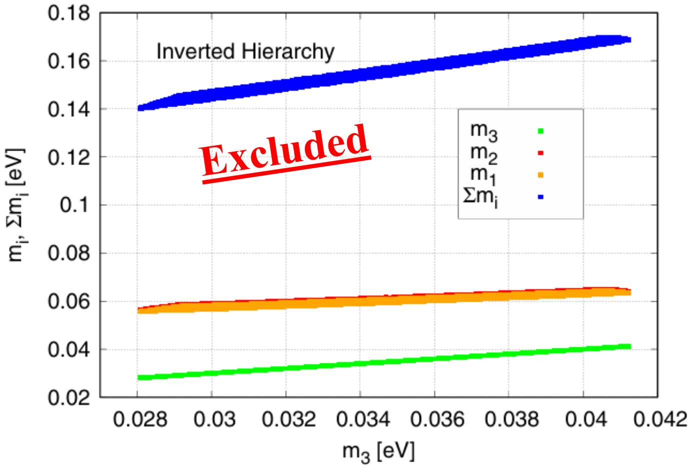
$$\Sigma m_i = m_1 + m_2 + m_3$$

Current bound $\Sigma m_i < 0.12$ eV (Planck)

The predictions for our FNMSSM model are

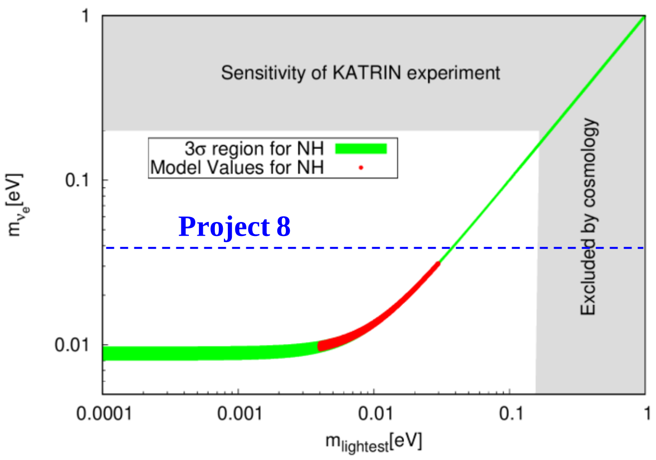
$$\left.\begin{array}{l} m_1 \simeq 0.0055eV \\ m_2 \simeq 0.0102eV \\ m_3 \simeq 0.0503eV \\ \Sigma m_i = 0.06614eV \end{array}\right\} \text{NH} \qquad \left.\begin{array}{l} m_1 \simeq 0.0606eV \\ m_2 \simeq 0.0612eV \\ m_3 \simeq 0.035eV \\ \Sigma m_i = 0.1573eV \end{array}\right\} \text{IH}$$

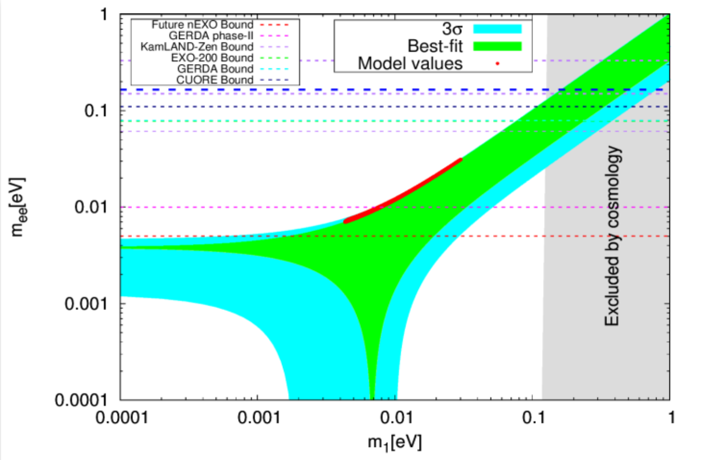




- Effective electron antineutrino mass from beta decay experiments

$$m_{\nu_e} = \sqrt{\frac{1}{3}\left(2m_1^2\cos^2\theta + m_2^2 + 2m_3^2\sin^2\theta\right)}$$





- Effective Majorana mass from $0\nu\beta\beta$ searches

$$|m_{ee}| = \frac{2}{3}\left|m_1\cos^2\theta + \frac{m_2}{2}e^{i\alpha_{21}} + m_3\sin^2\theta\, e^{i(\alpha_{31}-2\sigma)}\right|$$

$$\left.\begin{array}{l} m_{\nu_e}(eV) \in [0.0095 - 0.033] \\ |m_{ee}(eV)| \in [0.0072 - 0.0420] \end{array}\right\} \text{NH}$$

III- Domain walls formation in the charged lepton and neutrino sectors

Domain walls (DWs) arise from spontaneous breaking of a discrete symmetry, manifesting as surface-like topological defects that separate distinct regions of space characterized by equivalent ground states.

The A_4 group is isomorphic to, $Z_2 \times Z_2' \rtimes Z_3$ and it has two generators $\boldsymbol{S}$ and $\boldsymbol{T}$ that can be expressed as

$$S = \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \quad , \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \bar{\omega} & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

$\boldsymbol{S}$ generates Z_2 and $\boldsymbol{T}$ generates Z_3 while Z_2' is generated by $\boldsymbol{S}' = \boldsymbol{T}\boldsymbol{S}\boldsymbol{T}^{-1}$

- **In the charged lepton sector**, the A_4 invariant superpotential is given by: $W_{lep^+} = \frac{y_i}{\Lambda} H_d \left(L \otimes \Phi \big|_{\mathbf{1}_{(1,\omega^{i-1})}} \right) E_i^c$

  The charged leptons obtain their masses via the breaking $A_4 \to Z_3$ which is realized when Φ acquires its VEV along the direction: $\langle \Phi \rangle = \upsilon_\Phi (1,0,0)^T$ (To verify: $T\langle \Phi \rangle = \langle \Phi \rangle$, $S\langle \Phi \rangle \neq \langle \Phi \rangle$, $S'\langle \Phi \rangle \neq \langle \Phi \rangle$)

  This VEV direction leads to a diagonal charged lepton mass matrix with mass eigenvalues are given as follows

$$\boxed{m_e = y_e \frac{\upsilon_d \upsilon_\Phi}{\Lambda} \quad , \quad m_\mu = y_\mu \frac{\upsilon_d \upsilon_\Phi}{\Lambda} \quad , \quad m_\tau = y_\tau \frac{\upsilon_d \upsilon_\Phi}{\Lambda}}$$

III- Domain walls formation in the charged lepton and neutrino sectors

Graphical representations of the domain wall networks

We establish a flavon space by introducing a vector space capable of accommodating the two possible dimensions of A_4 representations:

$$X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix}$$

Since these flavons are in general complex: $X \in \mathbb{C}^4 \sim \mathbb{R}^8$

We think of the flavon triplets and singlets as follows:

$$\boldsymbol{\Phi} \sim \Phi_i X_i \quad , \quad \boldsymbol{\Omega} \sim \Omega_i X_i \quad , \quad \boldsymbol{S} = S X_4 \qquad \text{with } i = 1,2,3$$

To use graphical representation for DWs, we need to find a real representation for the flavon fields which are complex fields. Let us split the complex flavon fields and the X vector like

$$\Phi_i = Re(\Phi_i) + i\, Im(\Phi_i) \qquad \text{and} \qquad X_i = Re(U_i) + i\, Im(V_i)$$

where U_i and V_i are 3D vectors with $X_1 = \begin{pmatrix} U_1 \\ V_1 \end{pmatrix}, X_2 = \begin{pmatrix} U_2 \\ V_2 \end{pmatrix}, \ \ldots$

$$\longrightarrow \quad \boldsymbol{\Phi} \sim \textstyle\sum_i Re(\Phi_i) U_i + Im(\Phi_i) V_i \in \mathbb{R}^6$$

For example:

$$\Phi = v_\Phi (1,0,0)^T \longrightarrow \Phi = v_\Phi (1,0,0,0,0,0)^T$$

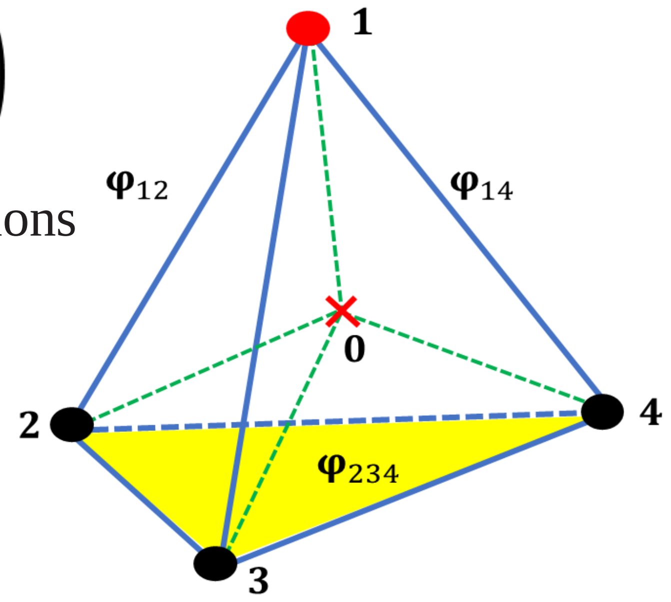
$$\Phi = v_\Phi (1,\omega,\omega^2)^T \longrightarrow \Phi = v_\Phi (1,0,-1/2,\sqrt{3}/2,-1/2,-\sqrt{3}/2)^T$$

III- Domain walls formation in the charged lepton and neutrino sectors

The breaking of A_4 driven by the flavon triplet Φ may be expressed as: $A_4 \cong V_4 \rtimes Z_3 \xrightarrow{\langle\Phi\rangle} Z_3$

The number of degenerate vacua is equal to the order of the broken part, therefore, the flavon vacua $\langle\Phi\rangle$ sit in four degenerate points which we denote as $\varphi_1, \varphi_2, \varphi_3$ and φ_4 in the flavon space.

$$\varphi_1 = v_\Phi \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \varphi_2 = \frac{v_\Phi}{3} \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}, \quad \varphi_3 = \frac{v_\Phi}{3} \begin{pmatrix} -1 \\ 2\bar{\omega} \\ 2\omega \end{pmatrix}, \quad \varphi_4 = \frac{v_\Phi}{3} \begin{pmatrix} -1 \\ 2\omega \\ 2\bar{\omega} \end{pmatrix}$$



Being associated with V_4, these vacua are related to each other by $Z_2 \times Z_2'$ transformations

$$\varphi_1 = \langle\Phi\rangle \quad , \quad \varphi_2 = S\varphi_1 \quad , \quad \varphi_3 = S'\varphi_1 \quad , \quad \varphi_4 = SS'\varphi_1$$

These four vacua correspond to specific vectors within the complex three (real six) dimensional space. They exhibit the constraint $\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4 = 0$

→ **they define a tetrahedron with Kahler modulus given by $Re\, v_\Phi$.**

We have $m_\tau = y_\tau \frac{v_d v_\Phi}{\Lambda}$, and from the PDG, $m_\tau = 1776.86\, MeV$. For $y_\tau v_d \lesssim 246\, GeV$, we obtain $\left(\frac{v_\Phi}{\Lambda} > 0.07\right)$

Assuming that the cutoff scale $\Lambda \sim M_{GUT} \sim 2 \times 10^{16} GeV$, we find a lower bound of the flavon VEV

$$v_\Phi > 1.4 \times 10^{14} GeV$$

The A_4 breaking scale is higher than the inflationary scale → **DWs are inflated away**

III- Domain walls formation in the charged lepton and neutrino sectors

- **In the neutrino sector**, the breaking $A_4 \longrightarrow Z_2$ is realized when Ω acquires a VEV along the direction:

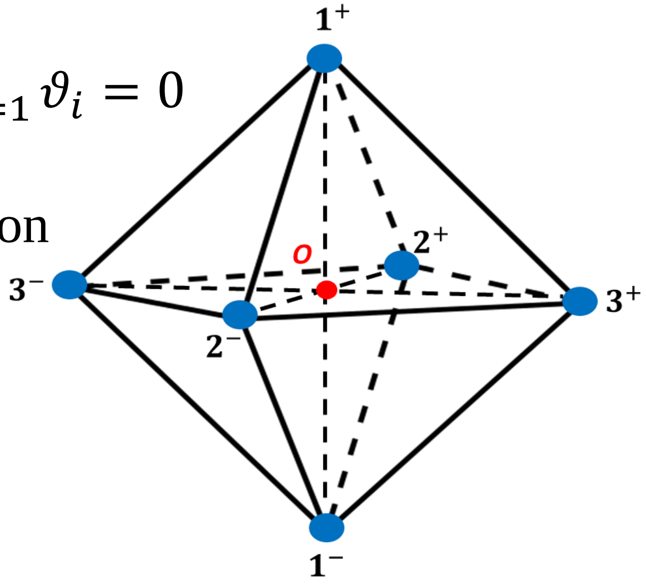
$$\langle\Omega\rangle = \upsilon_\Omega(1,1,1)^T$$

This breaking may be expressed $Z_2 \times \underbrace{Z_2' \rtimes Z_3}_{\text{Broken part}} \xrightarrow{\langle\Omega\rangle} Z_2$ ⟶ $S\langle\Omega\rangle = \langle\Omega\rangle \;\; , S'\langle\Omega\rangle \neq \langle\Omega\rangle \;\; , T\langle\Omega\rangle \neq \langle\Omega\rangle$

Broken part ⟶ **Six degenerate vacua**

$$\vartheta_1^\pm = \pm\upsilon_\Omega\begin{pmatrix}1\\1\\1\end{pmatrix} \;,\quad \vartheta_2^\pm = \pm\upsilon_\Omega\begin{pmatrix}1\\\overline{\omega}\\\omega\end{pmatrix} \;,\quad \vartheta_3^\pm = \pm\upsilon_\Omega\begin{pmatrix}1\\\omega\\\overline{\omega}\end{pmatrix}$$

with the property $\sum_{i=1}^6 \vartheta_i = 0$



In the flavon space, these six vacua define the six vertices of a homogenous octahedron

The superpotential for the Majorana neutrino is given by

$$\boldsymbol{W_R = TrA_4(\lambda\,\chi\, N^cN^c + \lambda'\Omega\, N^cN^c + \lambda''\, S\, N^cN^c)} \quad \text{where} \quad \upsilon_S < \upsilon_\chi \lesssim \upsilon_\Omega$$

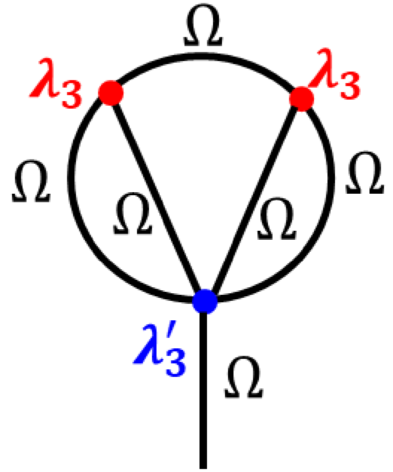
For neutrino masses compatible with the data, $\{\upsilon_\chi\,, \upsilon_\Omega\} \sim [10^7 \longrightarrow 10^{10}]\text{GeV}$

⟶ The DWs in the neutrino sector are created below the inflationary scale; they are stable and thus, they are inconsistent with standard cosmology.

III- Domain walls formation in the charged lepton and neutrino sectors

Solution to the DW problem

One way to solve the DW problem is to make the discrete symmetry only approximate.



Here the leading operators that break explicitly the discrete symmetry via higher dimensional operators—Planck scale operators—are of order five such as

$$W_{NR} = \frac{\lambda'_3}{M_{Pl}^2}(\Omega)^5\Big|_{(1,\omega)} \longrightarrow V_{soft} \supset \varepsilon M_W^3 \phi_\Omega + h.c. \quad \text{where} \quad \varepsilon = \frac{\lambda_3^2 \lambda'_3}{(16\pi^2)^3}$$

Since the triplet ϕ_Ω transforms nontrivially under $A_4 \times \boldsymbol{Z}_3$ flavor symmetry, the term in V_{soft} breaks explicitly $A_4 \times \boldsymbol{Z}_3$ down to Z_2.

This contribution creates an energy gap among the degenerate vacua of Ω--$\vartheta_1^{\pm}, \vartheta_2^{\pm}, \vartheta_3^{\pm}$--where the region in space of the energetically dominant vacuum, say ϑ_1^{+}, start to expand and thus pushing the walls away.

This holds true as long as $\varepsilon > \left(\frac{v_\Omega}{M_{pl}}\right) \longrightarrow \varepsilon > [5 \times 10^{-13} \to 5 \times 10^{-10}]$

VII - Summary and conclusion

- The origin of fermion mass hierarchies and mixings is one of the unresolved and most difficult problems in high-energy physics.

- Predictions and correlations among the neutrino masses, mixing angles, and *CP* phases frequently emerge from models based on non-Abelian discrete flavor symmetries, rendering them intriguing and insightful.

  - Future neutrino experiments could reveal neutrino nature, mass hierarchy, and *CP* phase.
  - The lack of a clear preferred direction in “model space”. Too many phenomenologically equivalent models !

- To explain the observed masses and mixing angles, the non abelian flavor symmetry must be broken.

  - Profound implications on cosmology usually ignored in lepton model building: Formation of domain walls
  - Domain walls must be unstable and annihilated in order to avoid cosmological problems.

Thanks for your

ATTENTION