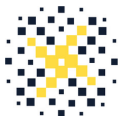


Anomaly aware machine learning for dark matter direct detection at DARWIN

Andre Scaffidi and Roberto Trotta for the DARWIN collaboration.

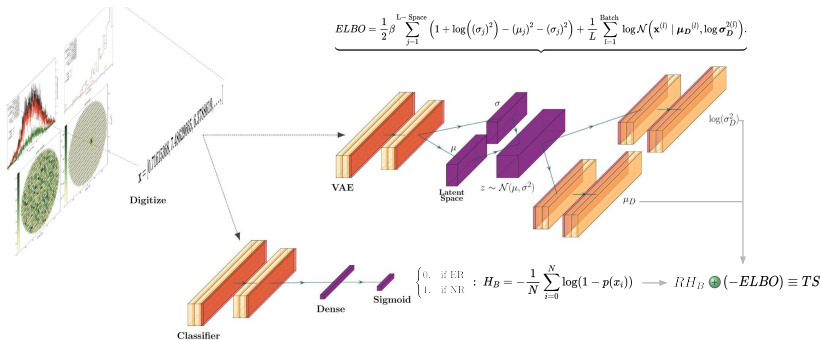


SISSA
DATA SCIENCE
Machine Learning for the Natural Sciences



Overview

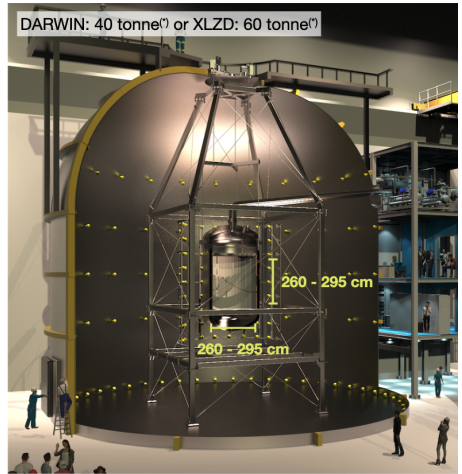
- \Rightarrow New deep learning pipeline to improve upon traditional likelihood approaches.
- Can improve sensitivity over standard approach.
- Methods directly applicable to any detector!



DARWIN collaboration: Proposal



~ 200 members



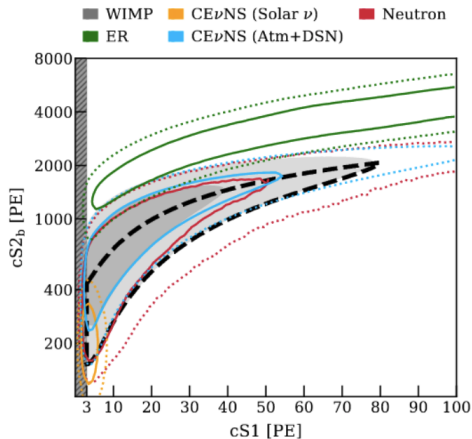
DARWIN collaboration



Direct detection: Traditional likelihood-based analysis

$$\prod_{i=1}^n \frac{d(N_s + N_b)}{dE} (E_i | \theta) \rightarrow \text{2D pdf derived from 'templates'}$$

Relies heavily on high-level
summary statistics $cS1, cS2$:
 $\Rightarrow \mathbf{E} = \mathbf{g}(cS1, cS2)$



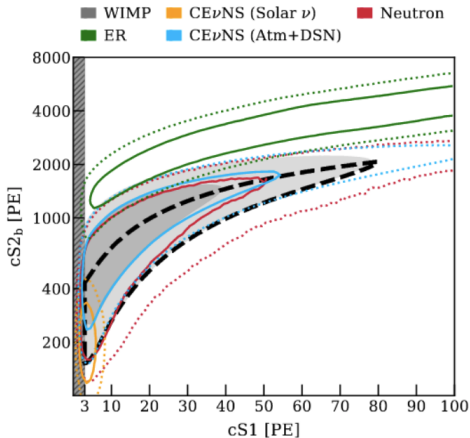
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↳ Fitted analytically



Simulation based inference (SBI)

Benefits of SBI

Bypass need for high level stats. Do inference directly from data.

- Can handle complex models with intractable likelihoods.
- Use deep neural nets to learn underlying features of simulated data/summary stats.
- Once a simulator has been established, possible to include arbitrarily complicated simulations into analysis: prompt readouts \rightarrow high level summary stats.
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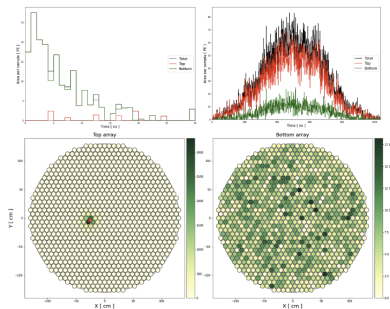
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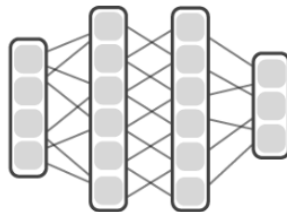
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Simulation-Based Inference with Neural Nets

We have a variety of data/summary stats available to us.

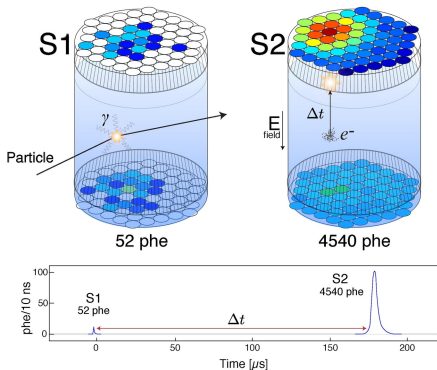


Neural net ‘learns’
underlying likelihood
function directly from data.



Analysis pipeline 1: Classification of recoil events

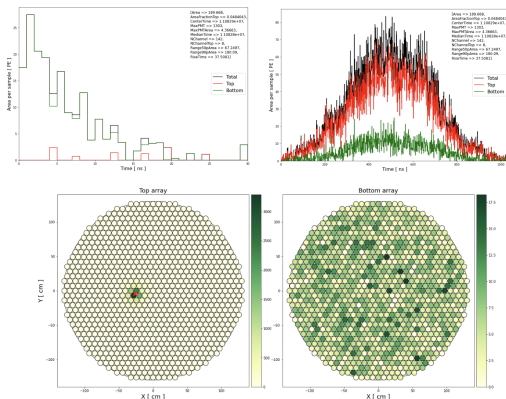
Underground TPCs: Two types of events



- First primary objective in an analysis is to veto the dominant ER \Rightarrow Binary classification!
- Previous ML studies Sanz et. al, Herrero-Garcia et. al
arXiv:1911.09210, 2110.12248 for XENONnT.

Training data: Simulations

RAW event output S1, S2 PMT deposits (4-fold coincidence, 200 ns):

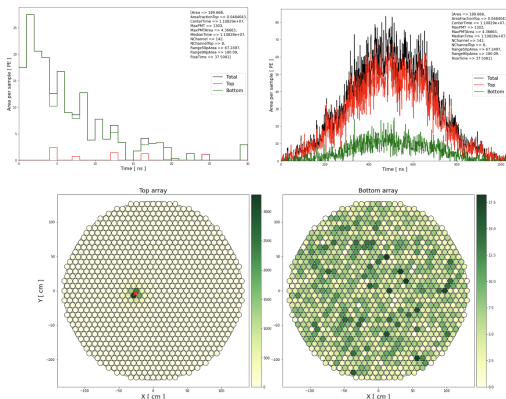


$$\Rightarrow \mathbf{x} = [\text{S1WaveformTotal}, \text{S2WaveformTotal}, \text{S2Pattern}]$$

Two distinct quanta: **Electron Recoil (ER)** and **Nuclear Recoil (NR)**

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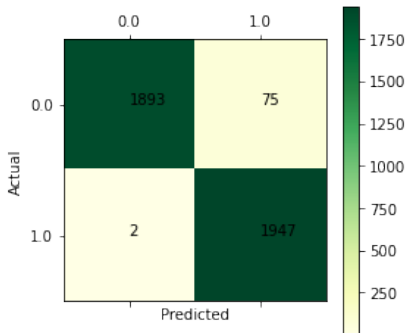


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Classification: ER vs. NR Results

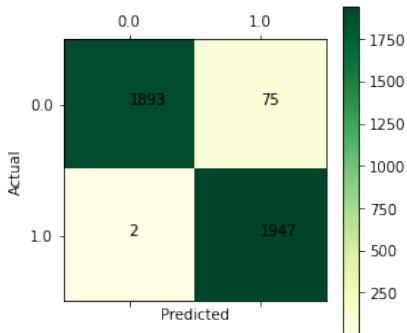
- Train on ~ 40000 ER/NR events with $E \in [0, 100]$ keV.
- Check performance \rightarrow confusion matrix:



- Takeaway \Rightarrow **98.03% accuracy**. (Recall = 98.07%, Precision = 96.39%)
- **This works regardless WIMP properties: NR/ER are what matter.** (Originally thought not i.e Sanz 1911.0921)

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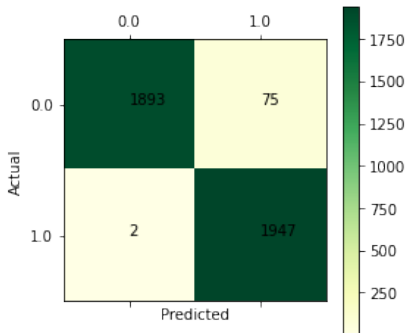
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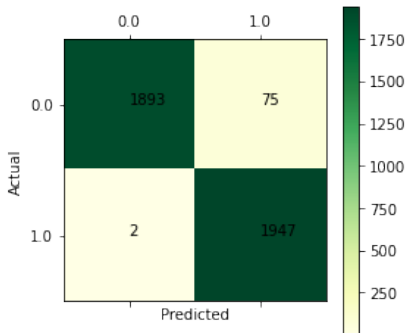
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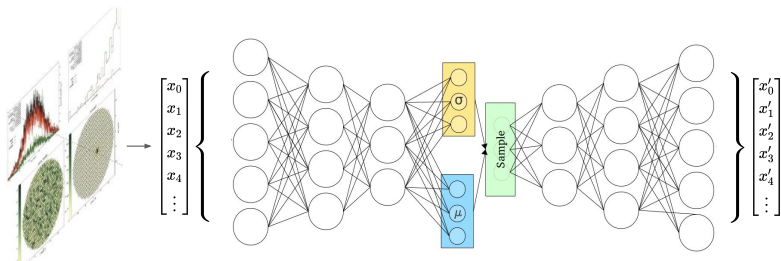
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Analysis pipeline 2: Unsupervised approach

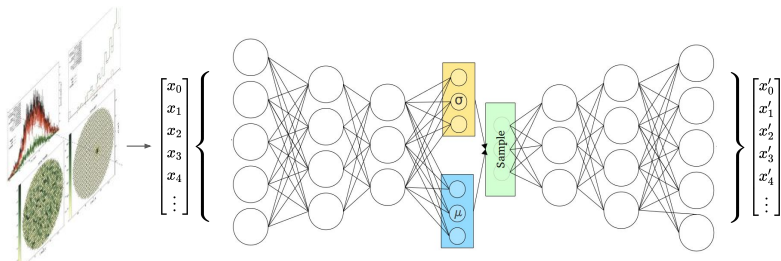
Generative Deep Learning: The Variational Auto-Encoder

- Variety of studies in HEP use these for anomaly detection tasks.
- Goal: Learn low dimensional representation (encoding) of data via dimensional reduction.
- Latent space (bottleneck) layer is a bunch of normal distributions parameterized by some μ and σ .
- Network should return accurate representations of the input.



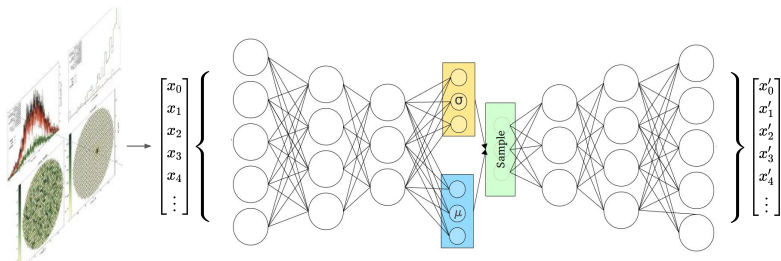
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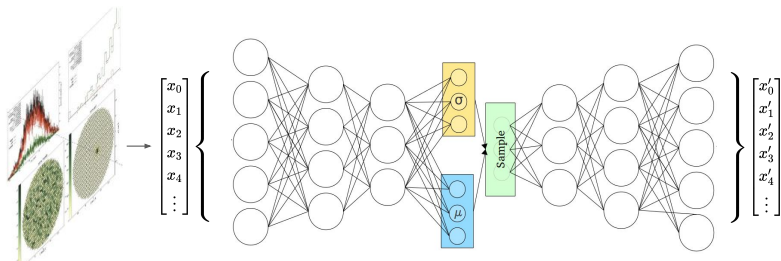
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Variational-Auto-Encoder: Training

- Train by maximising evidence lower bound (ELBO):

$$\begin{aligned}\log p(x) \geq \text{ELBO} &= \mathbb{E}_{q(z|x)} \left[\log \frac{p(x, z)}{q(z|x)} \right] \\ &= E[\log p(x|z)] - \beta D_{KL}(q(z|x)||p(z))\end{aligned}$$

x = Input

z = Latent vector

β = Regularization parameter

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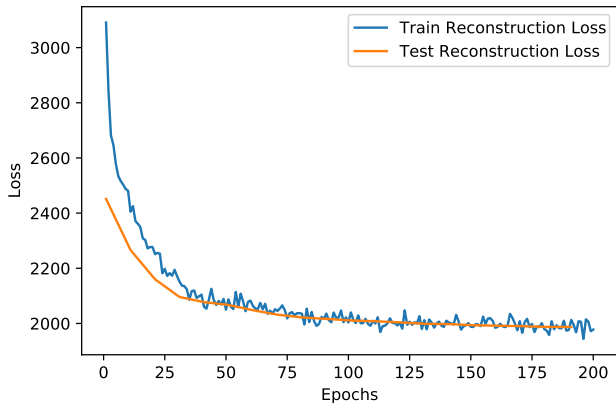
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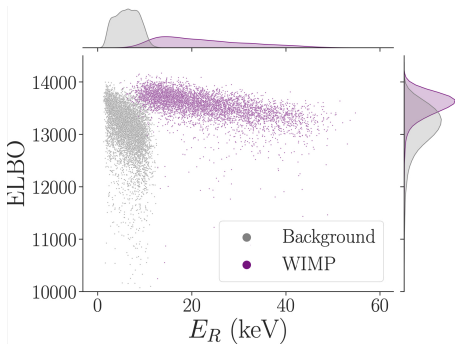
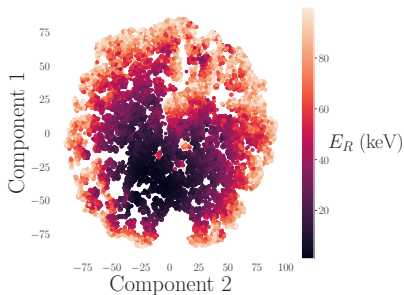
VAE: Training

- Train the network for 200 epochs.

$$Loss \equiv -ELBO$$



VAE trained on ER realisations: Spectral information



- Auto-encoder can learn underlying spectral information of events
⇒ Sensitivity to WIMP mass.
- Can we also just fully reconstruct the energy of an event straight from the data? Yes! See later.

Anomaly detection (Looking
for non background-like
events)

- Accept/Reject

$$\mathcal{H}_0 : TS \sim \mathcal{P}(x \mid \text{No signal})$$

\mathcal{P} : Conditional process generating some statistic TS , under the assumption that no WIMP signal is present.

- \mathcal{P} intractable: Use neural networks to derive optimal TS .

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What is a suitable TS ?

- If VAE has learned the underlying properties of ER events, any **non-ER** events will in general have **higher loss** (smaller ELBO).
- Furthermore, non trivial spectral info learned \Rightarrow ELBO **distribution** shape can further inform about NR background.
- Loss distribution of anomalous data (new physics) will show as an **excess** over background only loss distribution.
- Try distribution of $= ELBO$

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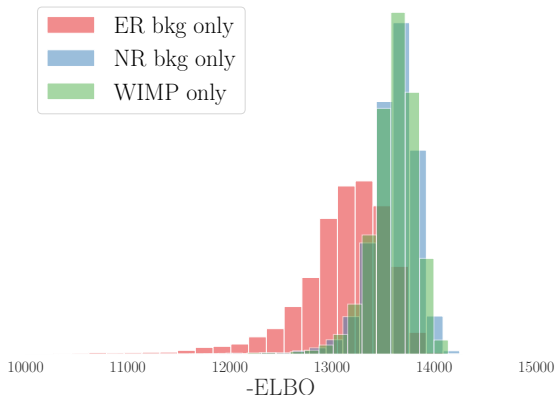
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Anomaly detection



- Background loss distributions + WIMP loss distribution.
- Any* anomalous signal will show up as statistical deviation in (pseudo)data loss vs. (known) background loss.

Semi-supervised anomaly detection: New distance metric

Cool. But...

- A bit rubbish: Can we get greater separation (anomaly awareness) between these distributions?
- New ‘anomaly score’ that utilizes pre-trained supervised NN classifier:

$$TS = -ELBO + R H_B ,$$

where

- $H_B = -\frac{1}{N} \sum_{i=0}^N \log(1 - p(x_i))$ (Binary cross-entropy.)
- R scales the contribution of the cross-entropy term \rightarrow makes it more/less supervised.

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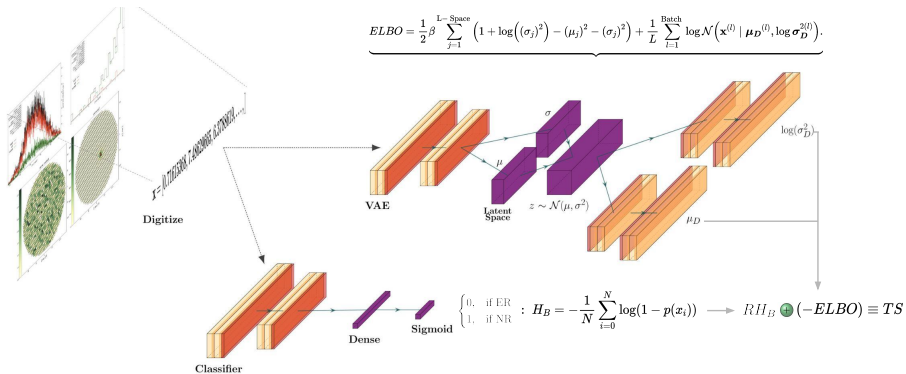
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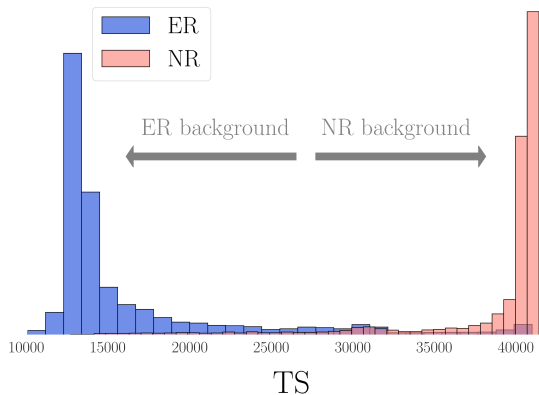
Semi-supervised anomaly detection: Full pipeline



Semi-supervised anomaly detection: New distance metric

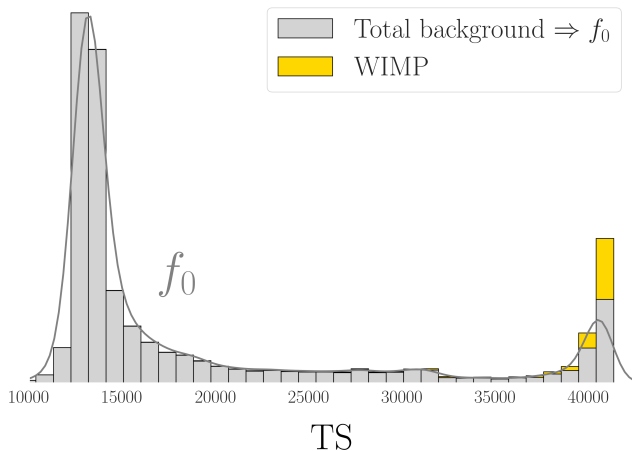
$$TS = (-ELBO) + RH_B ,$$

⇒ Semi-supervised. Much greater anomaly awareness!



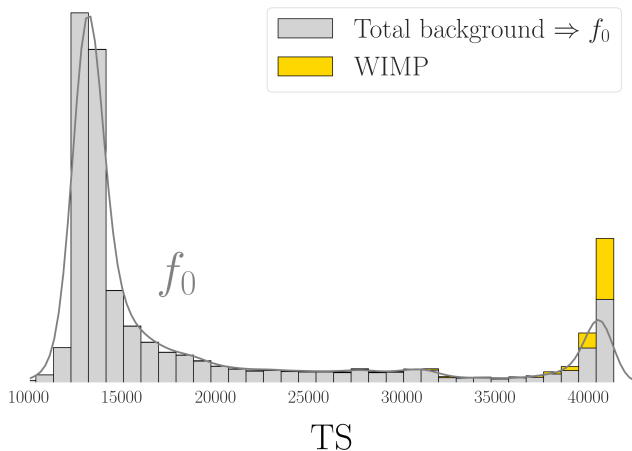
Pseudo-data sets

Re-weight anomaly score distributions TS according to expected ER+NR backgrounds and inject some WIMP signal. ER [2-10] keVee, NR [5-35] keVnr



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Dimensionally reduced two-sample test

- \Rightarrow 1D analysis in TS space: Accept/reject \mathcal{H}_0 .

$$\mathcal{P}(x \mid \text{No signal}) \equiv \mathcal{L}(\mathbf{TS} \mid \mathcal{H}_0) \propto e^{-B} \prod_{i=1}^N (B f_0(TS_i))$$

- Unbinned.
- Parametrically independent on WIMP model.
- No auxiliary terms/nuisance parameters required *assuming* simulations have suitably descriptive coverage.
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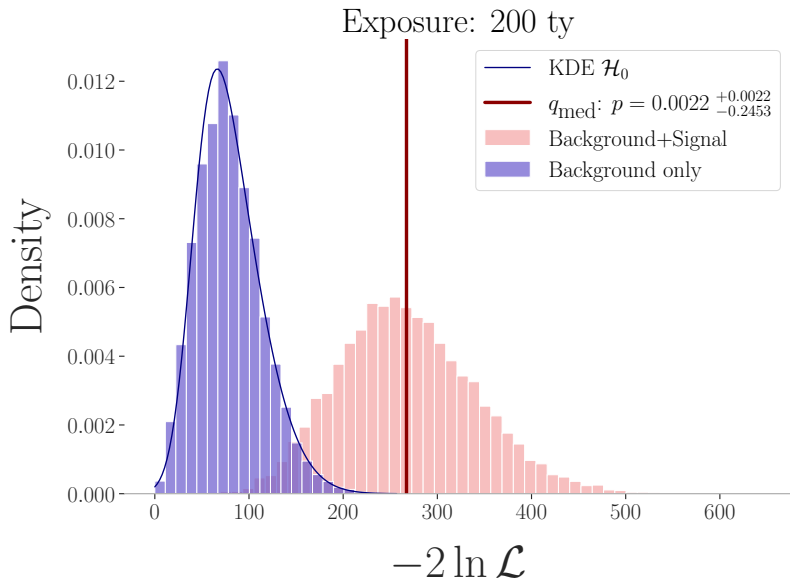
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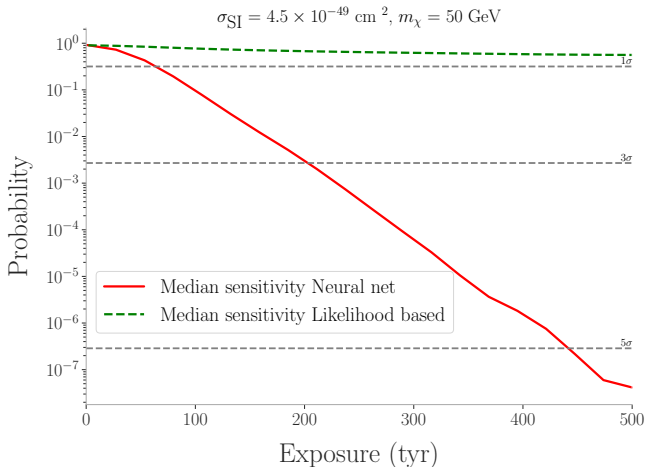
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Median Sensitivity (50 GeV, $\sigma_{SI} = 4.9 \times 10^{-49} \text{ cm}^2$)

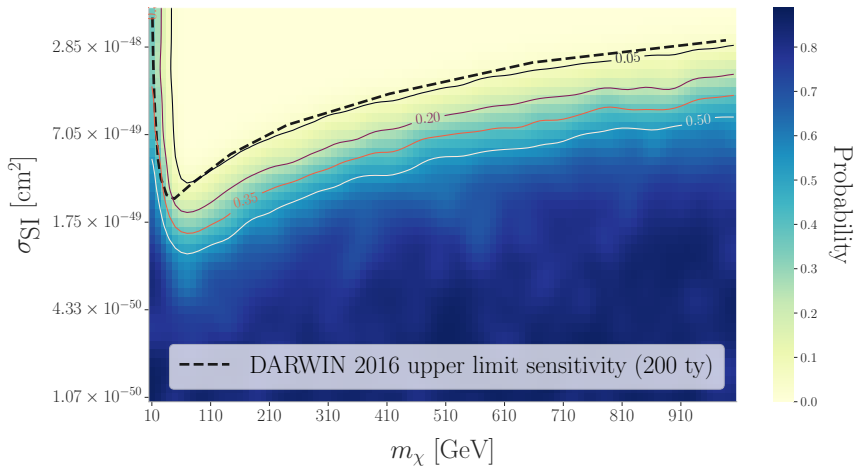


As a function of exposure

- Neural net
- Binned likelihood based: Median sensitivity [30% NR acceptance, 99% ER rejection]



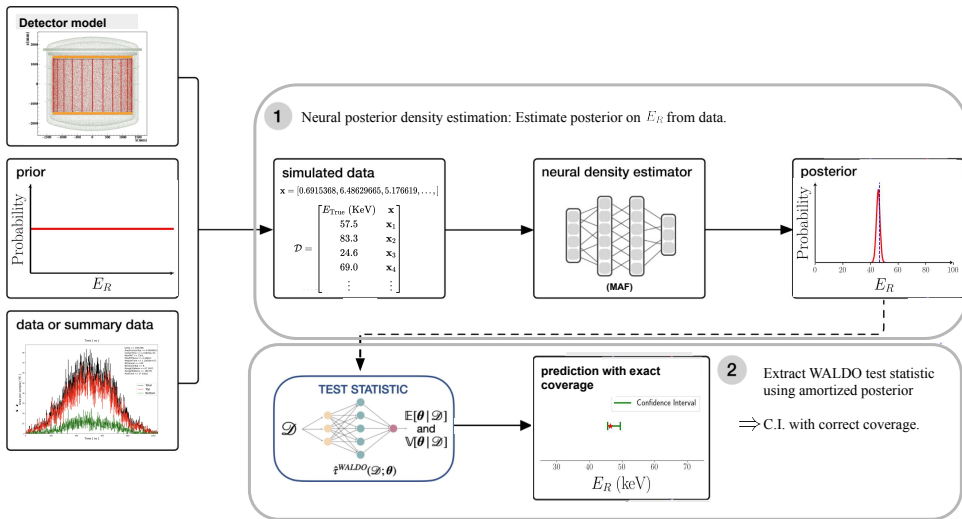
Full sensitivity (Preliminary)



Caution: 90% C.L upper limit is model dependent \rightarrow 'weaker' test.

Energy reconstruction

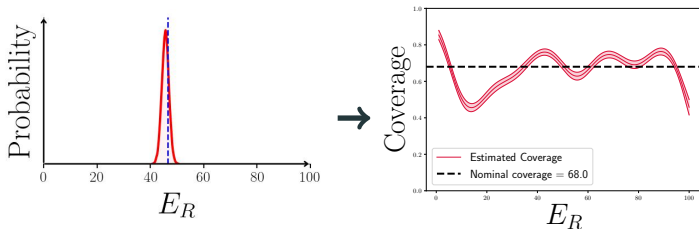
Energy reconstruction SBI with masked autoregressive flows



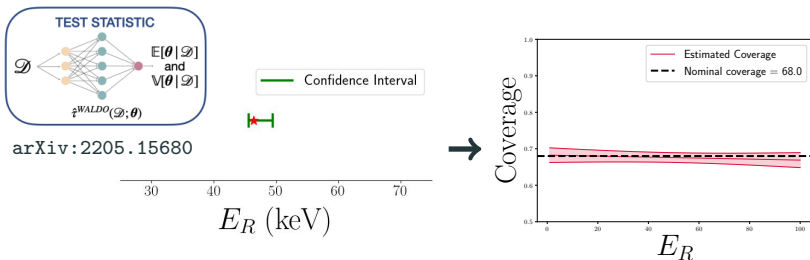
$$\tau^{\text{WALDO}}(\mathcal{D}; \theta_0) = (\mathbb{E}[\theta | \mathcal{D}] - \theta_0)^T \mathbb{V}[\theta | \mathcal{D}]^{-1} (\mathbb{E}[\theta | \mathcal{D}] - \theta_0)$$

Follow up work: E reconstruction

Neural posterior density estimation (Masked auto-regressive flows)



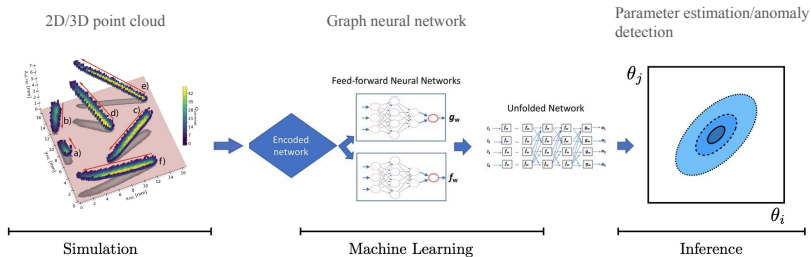
Neural posterior density estimation + WALDO



On directional detection

On directional detection

- SBI methods directly applicable.
- Depending on data:
 - 3D Structured: MLP, Unstructured Point clouds \rightarrow PointNet, GNN's



Thank you!

Backup Slides

Simulation-Based Inference (in a nutshell)

Simulation-based inference is a statistical technique that allows us to make inferences about a population or process based on simulated/calibrated data. It involves the following steps:

1. Generate simulated data.
2. Use deep neural nets to learn underlying features of simulated data.
3. Use trained models to inference.

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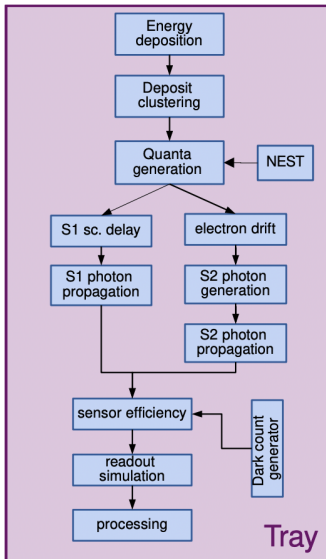
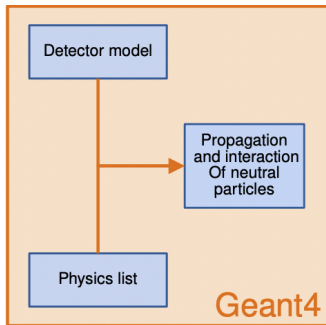
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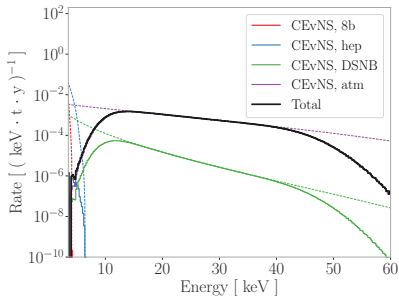
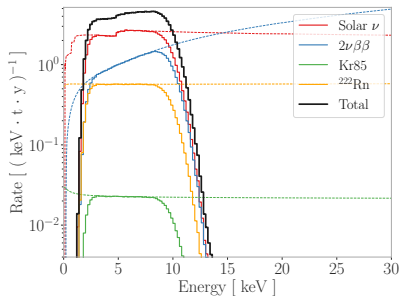
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DARWIN: Simulation pipeline



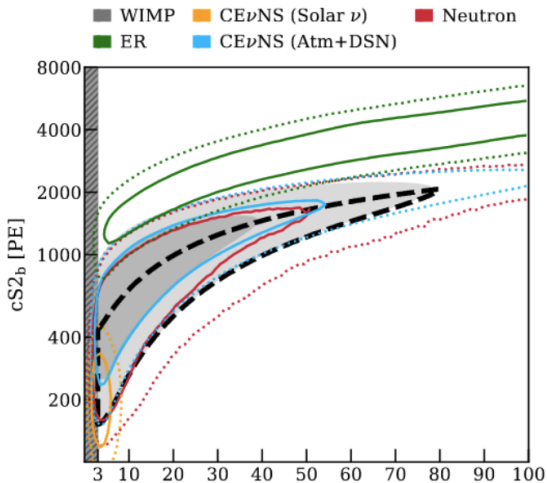
Backgrounds



- Intrinsic and extrinsic.
- Coherent neutrino scattering provides dominant background for WIMP searches.

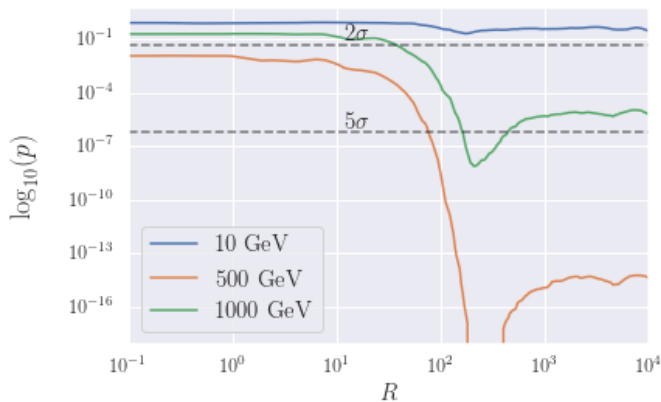
Binned likelihood based approach

$$\mathcal{L}(\mathbf{x}) = \prod_{i=1}^{bins} \frac{\lambda_i^{n_i}}{n_i!} e^{-\lambda_i} \quad : \text{ER veto (99.98\%), fidiucilization etc.}$$



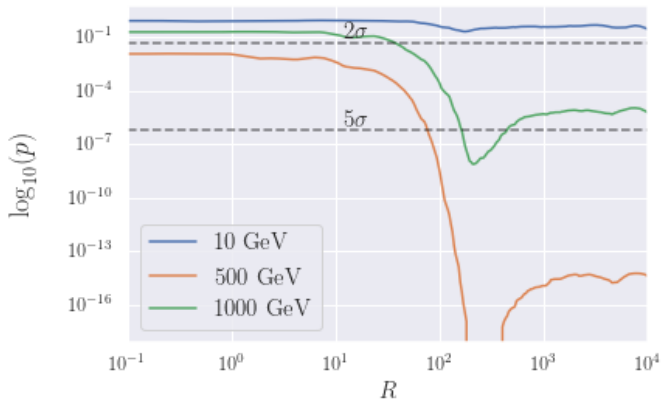
Effect of R

- Explore effect of the R parameter.
- Three mock data sets corresponding to 10, 500 and 1000 GeV at fixed $\sigma = 10^{-45} \text{cm}^2$, 5 t -yr exposure.
- Best result for $R \sim 170$, but generally free to choose!



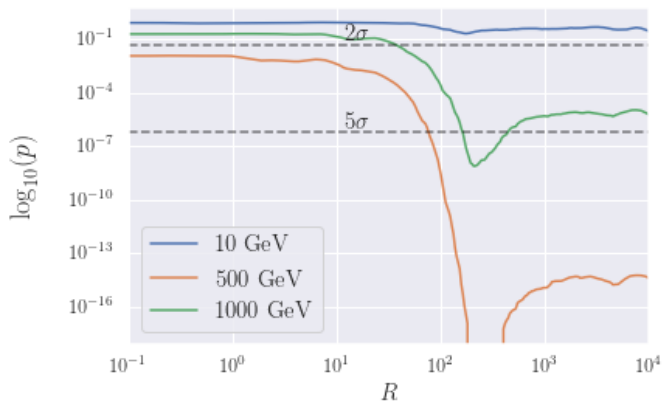
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Forecasting sensitivity

- Probability to accept/reject \mathcal{H}_0 after some exposure.
- Model independent.
- Simulate $\sim 10^4$ realisations of $-2 \ln \mathcal{L}(\mathbf{TS} | \mathcal{H}_0)$ to ascertain the asymptotic form of \mathcal{H}_0 .

$$p = \int_{q_{\text{med}}}^{\infty} dq \mathcal{H}_0(q) .$$

Median sensitivity

- Probability to accept/reject \mathcal{H}_0 after some exposure.
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