Theory status of b-hadron lifetimes

Aleksey Rusov

CPPS, Theoretische Physik 1, Universität Siegen

Collaborative Research Center TRR 257
Particle Physics Phenomenology after the Higgs Discovery

Workshop "Implications of LHCb measurements and future prospects"
CERN, Geneva, 25 - 27 October 2023

based on ArXiv: 2301.07698, ArXiv: 2208.02643

Introduction

b-hadron lifetimes

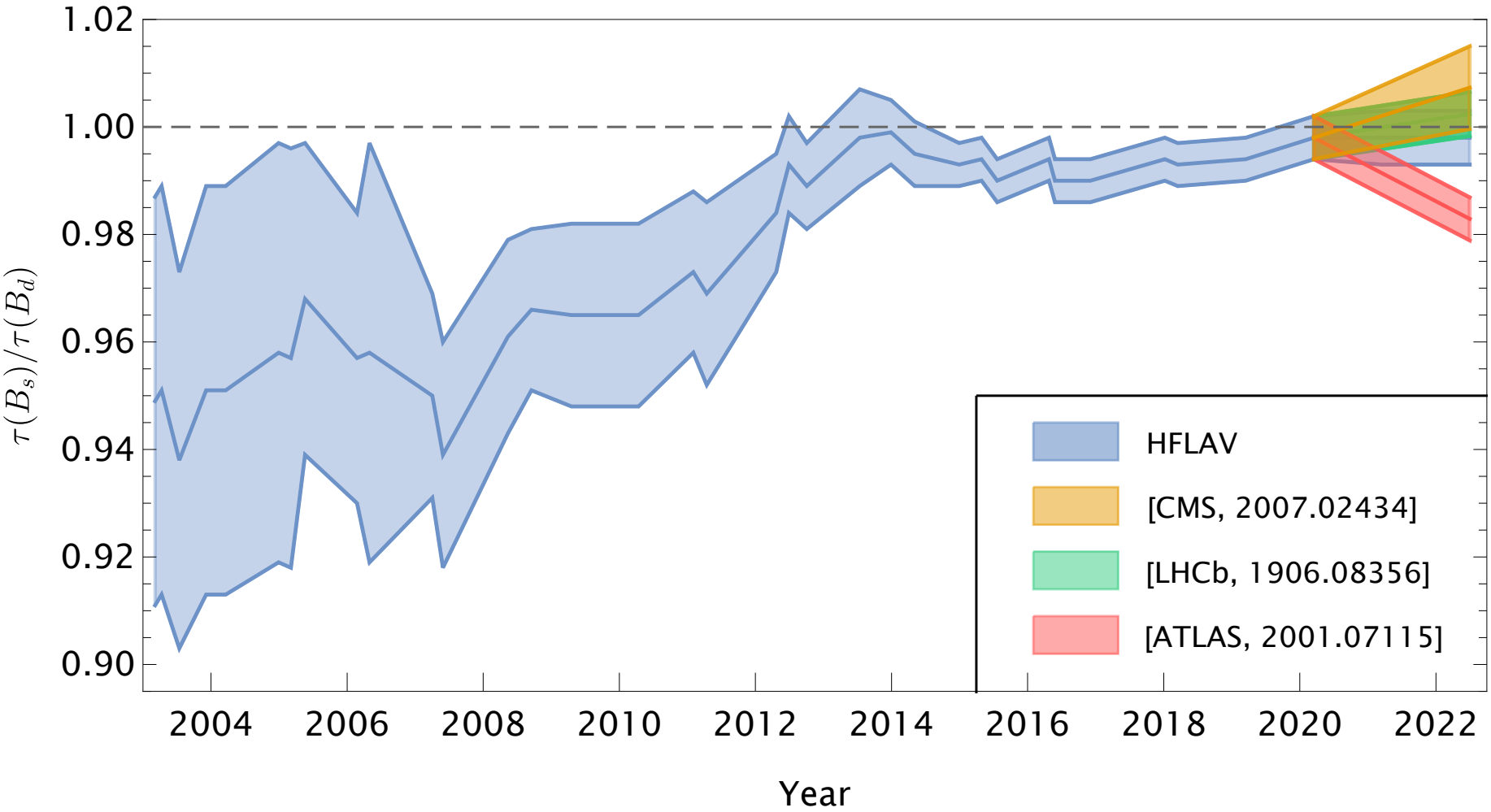
- Lifetimes of b hadrons are measured precisely at the experiment
- HFLAV and PDG 2023

	B^+	B_d^0	B_s^0
τ [ps]	1.638 ± 0.004	1.519 ± 0.004	1.521 ± 0.005
$\tau(X)/\tau(B_d^0)$	1.076 ± 0.004	1	1.002 ± 0.004

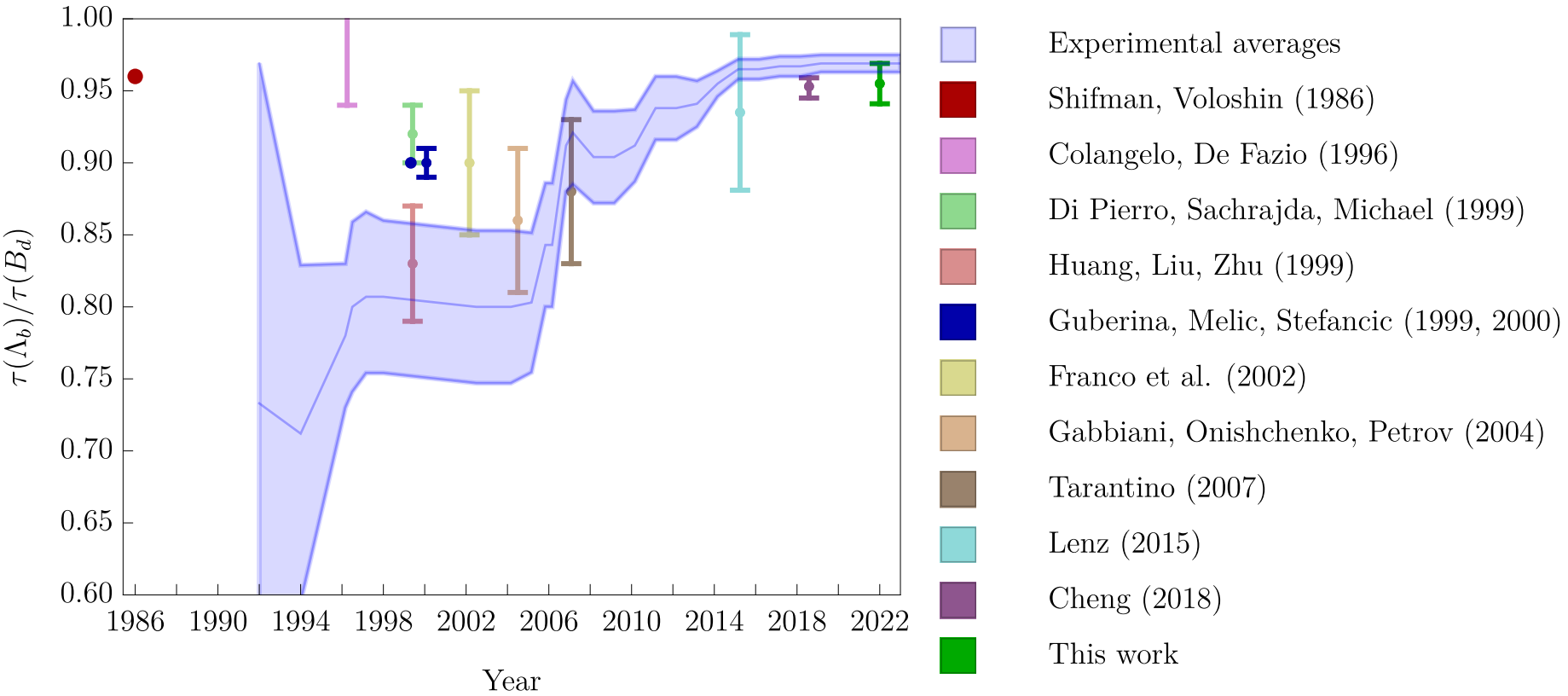
	Λ_b^0	Ξ_b^0	Ξ_b^-	Ω_b^-
τ [ps]	1.471 ± 0.009	1.480 ± 0.030	1.572 ± 0.040	$1.64^{+0.18}_{-0.17}$
$\tau(X)/\tau(B_d^0)$	0.969 ± 0.006	0.974 ± 0.020	1.035 ± 0.027	$1.080^{+0.118}_{-0.112}$

History of $\tau(B_s^0)/\tau(B_d^0)$

[ArXiv: 2208.02643]



History of $\tau(\Lambda_b^0)/\tau(B_d^0)$



[ArXiv: 2301.07698]

Theory of lifetimes

Heavy hadron lifetimes: theory

- Total width of a hadron $\mathcal{B}$ is given by

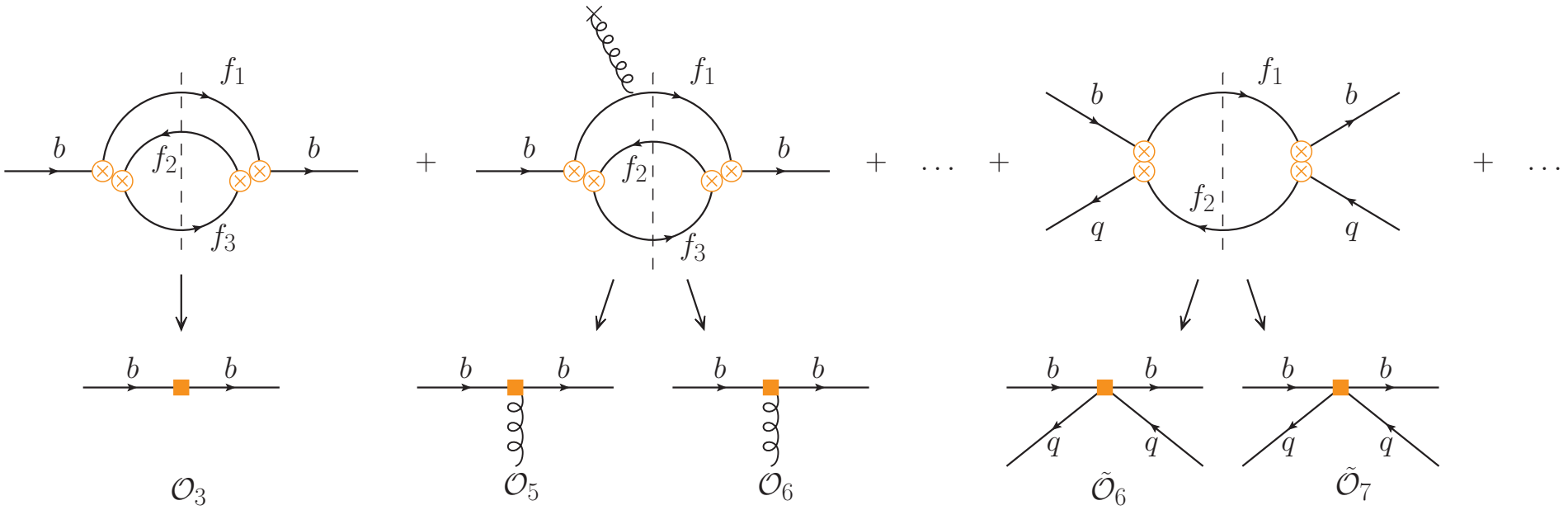
$$\Gamma(\mathcal{B}) = \frac{1}{2m_{\mathcal{B}}} \sum_X \int_{\rm PS} (2\pi)^4 \delta^{(4)}(p_{\mathcal{B}} - p_X) \, |\langle X(p_X)|\mathcal{H}_{\rm eff}|\mathcal{B}(p_{\mathcal{B}})\rangle|^2$$

$$\text{Optical Theorem} \quad = \frac{1}{2m_{\mathcal{B}}} \, {\rm Im} \, \langle \mathcal{B}(p_{\mathcal{B}})| \, i \int d^4x \, T \left\{ \mathcal{H}_{\rm eff}(x), \mathcal{H}_{\rm eff}(0) \right\} |\mathcal{B}(p_{\mathcal{B}})\rangle$$

- Computed using heavy quark expansion (HQE) in powers of $\Lambda/m_b \ll 1$
- Ratio of lifetimes $\dfrac{\tau(\mathcal{B}_1)}{\tau(\mathcal{B}_2)} = \dfrac{\Gamma_b + \delta\Gamma_{\mathcal{B}_2}}{\Gamma_b + \delta\Gamma_{\mathcal{B}_1}} = 1 + (\delta\Gamma_{\mathcal{B}_2} - \delta\Gamma_{\mathcal{B}_1})\,\tau(\mathcal{B}_1)$
- May be sensitive to New Physics contributions

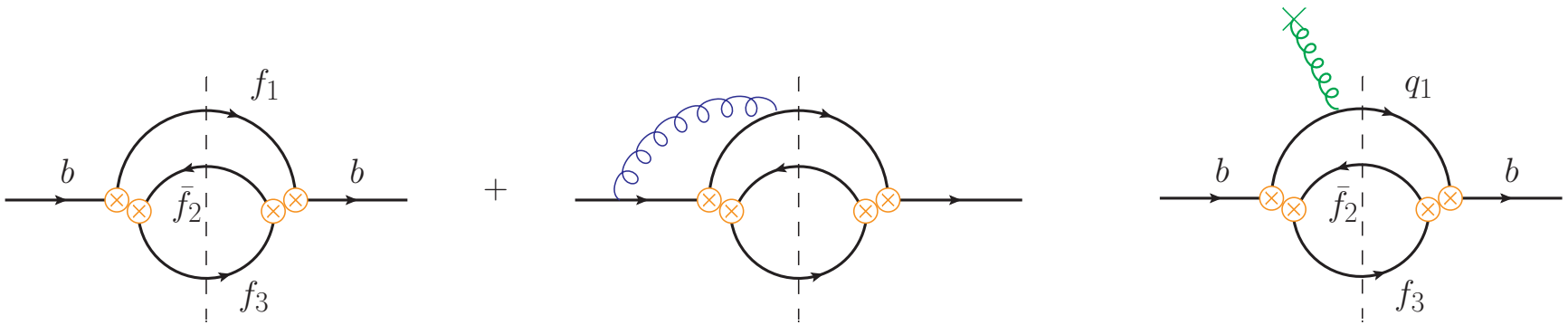
$$\frac{\tau(\mathcal{B}_1)}{\tau(\mathcal{B}_2)} = \frac{\Gamma_b + \delta\Gamma_{\mathcal{B}_2}}{\Gamma_b + \delta\Gamma_{\mathcal{B}_1}} = 1 + \left(\delta\Gamma^{\rm SM}_{\mathcal{B}_2} - \delta\Gamma^{\rm SM}_{\mathcal{B}_1}\right)\tau(\mathcal{B}_1) + \left(\delta\Gamma^{\rm NP}_{\mathcal{B}_2} - \delta\Gamma^{\rm NP}_{\mathcal{B}_1}\right)\tau(\mathcal{B}_1)$$

HQE: diagrams



$$\Gamma(\mathcal{B}) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \ldots \right]$$

Two-quark contribution

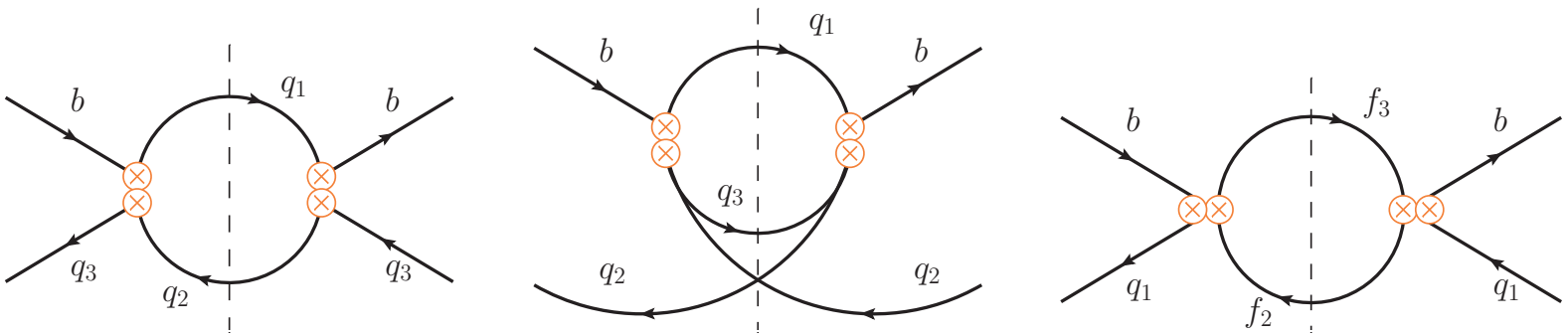


- Perturbatively calculable short-distance coefficients

$$\Gamma_i = \Gamma_i^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_i^{(1)} + \dots$$

  - ⋆ Universal for all heavy hadrons containing a b-quark
- Matrix elements of two-quark operators $\langle \mathcal{O}_5 \rangle$, $\langle \mathcal{O}_6 \rangle$, ...
  - ⋆ Depend on b-hadron spectator quark(s)

Four-quark contribution



- Perturbatively calculable short-distance coefficients

$$\tilde{\Gamma}_i = \tilde{\Gamma}_i^{(0)} + \frac{\alpha_s}{4\pi}\tilde{\Gamma}_i^{(1)} + \dots$$

  - ⋆ Dependent on b-hadron spectator quark(s)
- Matrix elements of four-quark operators $\langle\tilde{\mathcal{O}}_6\rangle$, $\langle\tilde{\mathcal{O}}_7\rangle$, ...
  - ⋆ Depend on b-hadron spectator quark(s)

Theory status of b-hadron lifetimes

Status of short-distance coefficients and recent progress

	Semi-leptonic				Non-leptonic		
	LO	NLO	N^2LO	N^3LO	LO	NLO	N^2LO
Γ_3	✓	✓	✓	✓*	✓	✓	✓
Γ_5	✓	✓			✓	✓$^\bullet$	
Γ_6	✓	✓			✓$^\diamond$	🕒	
Γ_7	✓				🕒		
Γ_8	✓						
$\tilde{\Gamma}_6$	✓	✓	🕒		✓	✓	🕒
$\tilde{\Gamma}_7$	✓	🕒			✓	🕒	

$^\diamond$ [Lenz, Piscopo, AR, 2004.09527], [Mannel, Moreno, Pivovarov, 2004.09485]

* [Fael, Schönwald, Steinhauser, 2011.13654] $^\bullet$ [Mannel, Moreno, Pivovarov, 2304.08964 (for $m_c = 0$)]

✓ - known ✓ - partly known 🕒 - in progress or planned [Karlsruhe, Siegen]

Status of matrix elements and recent progress

	B-mesons	b-baryons
$\langle \mathcal{O}_5 \rangle$	Spectroscopy relations V_{cb} fit to semileptonic data	Spectroscopy relations
$\langle \mathcal{O}_6 \rangle$	EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$ V_{cb} fit to semileptonic data	EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$
$\langle \tilde{\mathcal{O}}_6 \rangle$	HQET sum rules* Lattice QCD$^\diamond$	NRCQM QCD sum rules$^\bullet$
$\langle \tilde{\mathcal{O}}_7 \rangle$	VIA; HQET sum rules	Scaling estimate

* [King, Lenz, Rauh, 2112.03691]

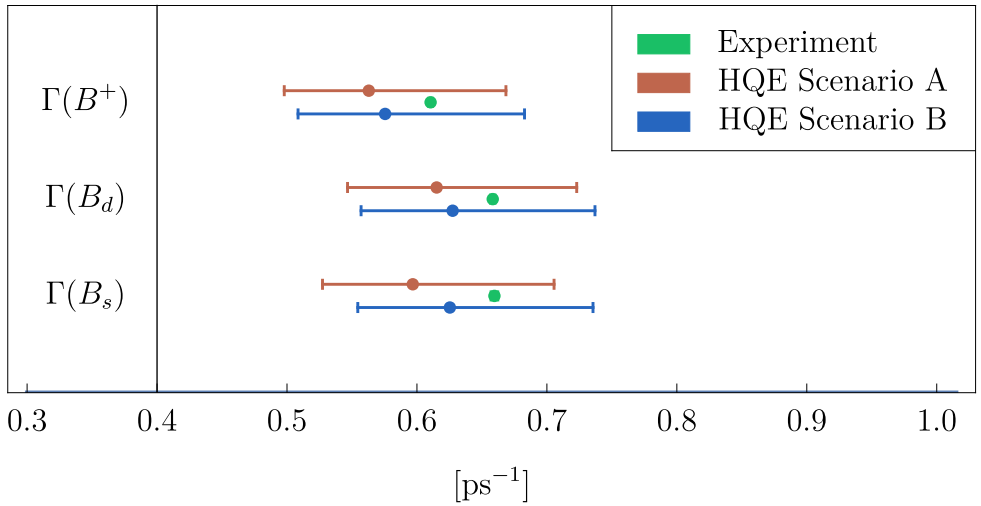
$^\diamond$ [Black, Witzel (in progress)]

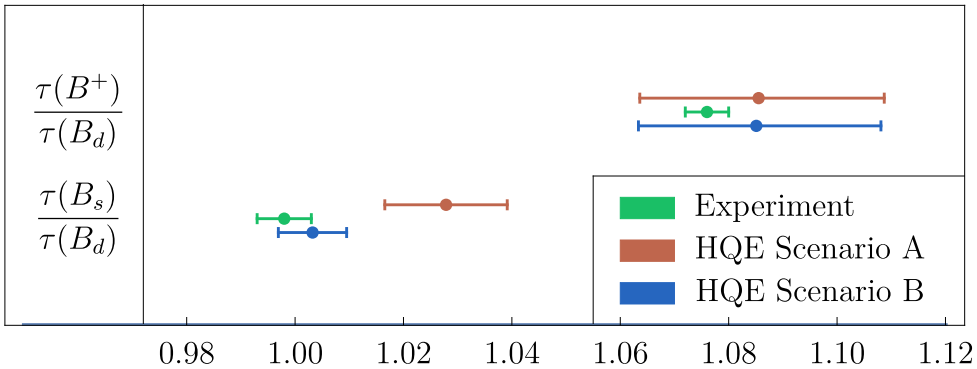
$^\bullet$ only for Λ_b^0

Results

B-meson lifetimes and ratios

[Lenz, Piscopo, AR, 2208.02643]





Scenario A

larger ρ_D^3, larger ~~$SU(3)_F$~~

(ρ_D^3 from V_{cb} fit by [Bordone, Capdevila, Gambino, 2107.00604])

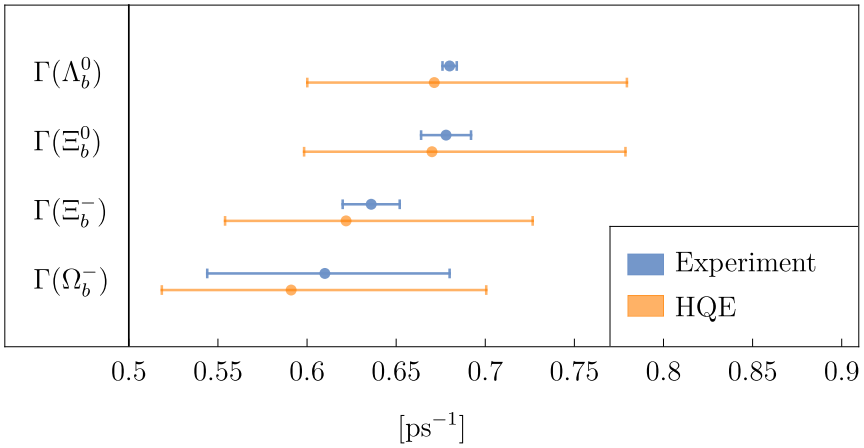
Scenario B

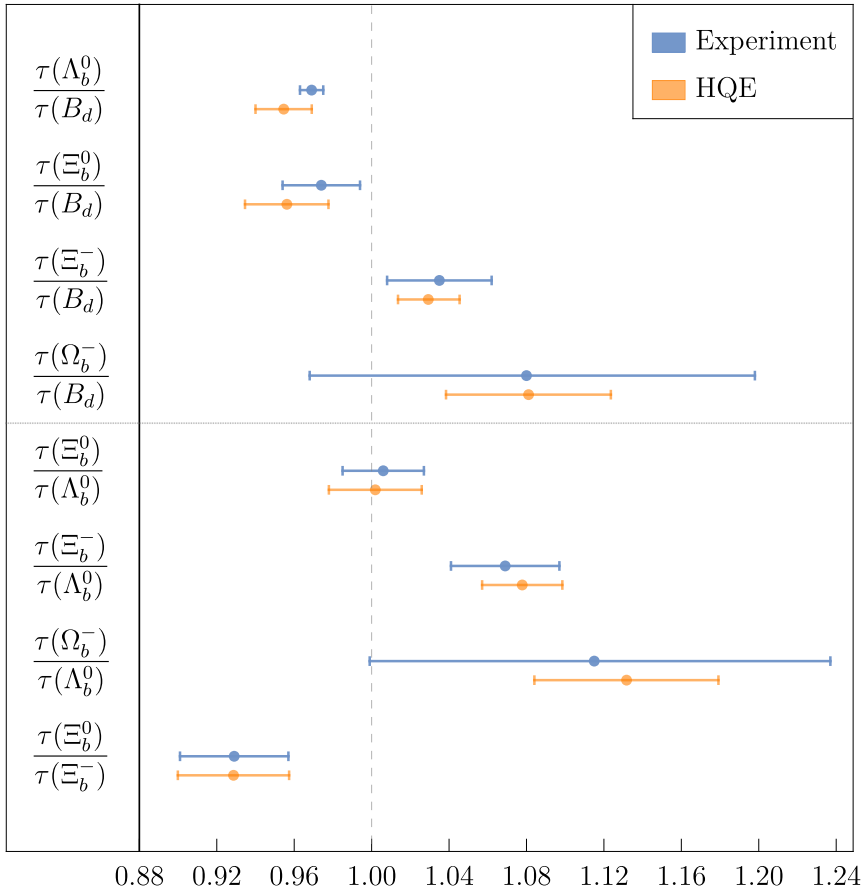
smaller ρ_D^3, smaller ~~$SU(3)_F$~~

(ρ_D^3 from V_{cb} fit by [Bernlochner et al., 2205.10274])

B-meson lifetimes and ratios

[Gratrex, Lenz, Melic, Nisandzic, Piscopo, AR, ArXiv: 2301.07698]





- Excellent agreement with data
- No indication of sizeable quark-hadron duality violation

Conclusion and outlook

Conclusion and outlook

- HQE predictions for b-hadron lifetimes in good agreement with data
- HQE also works for c-hadron lifetimes (though with very large uncertainties)

[King, Lenz, Piscopo, Rauh, Rusov, Vlahos, 2109.13219]
[Gratrex, Melic, Nisandsic, 2204.11935], [Cheng, Liu, 2305.00665]

- Further improvements

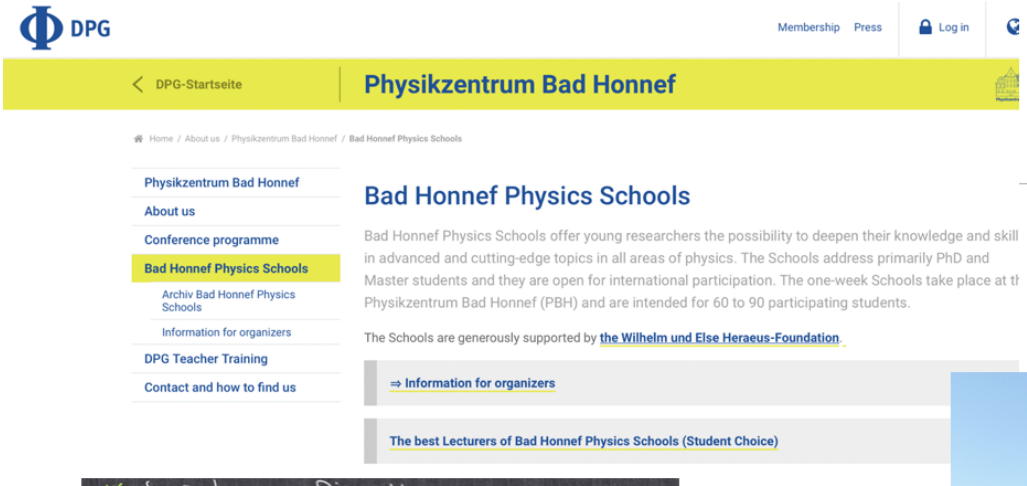
	Non-leptonic		
	LO	NLO	N^{2}LO
Γ_3	✓	✓	🕒
Γ_5	✓	🕒	
Γ_6	✓	🕒	
Γ_7	🕒		
$\tilde{\Gamma}_6$	✓	✓	🕒
$\tilde{\Gamma}_7$	✓	🕒	

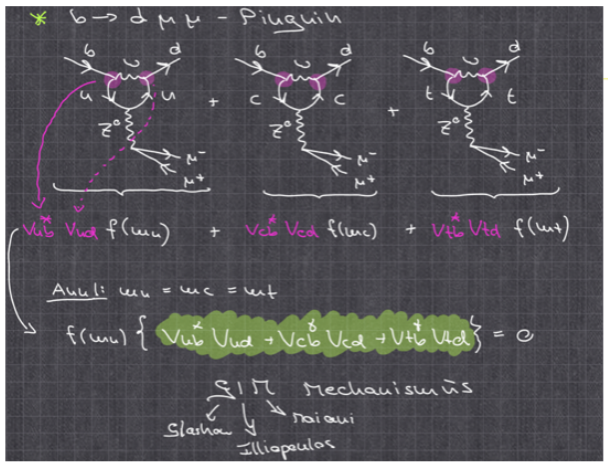
	B-mesons	b-baryons
$\langle \mathcal{O}_5 \rangle$		
$\langle \mathcal{O}_6 \rangle$	ρ_D^3 from fit ?	
$\langle \tilde{\mathcal{O}}_6 \rangle$	LQCD 🕒	LQCD
$\langle \tilde{\mathcal{O}}_7 \rangle$	HQET SR 🕒	

🕒 - in progress or planned

🕒 - complete determination in progress

- Bad Honnef Physics School "Color meets Flavor", March 2024





www.dpg-physik.de/veranstaltungen/2024/color_meets_flavor

Backup

Definition of non-perturbative parameters

- Matrix elements of dimension-5 two-quark operators

$$
\begin{aligned}
2m_B\,\mu_\pi^2(\mathcal{B}) &= -\langle \mathcal{B}(p_\mathcal{B})|\bar{b}_v(iD_\mu)(iD^\mu)b_v|\mathcal{B}(p_\mathcal{B})\rangle \\
2m_\mathcal{B}\,\mu_G^2(\mathcal{B}) &= \langle \mathcal{B}(p_\mathcal{B})|\bar{b}_v(iD_\mu)(iD_\nu)(-i\sigma^{\mu\nu})b_v|\mathcal{B}(p_\mathcal{B})\rangle
\end{aligned}
$$

- Matrix elements of dimension-6 two-quark operators

$$
\begin{aligned}
2m_\mathcal{B}\,\rho_D^3(\mathcal{B}) &= \langle \mathcal{B}(p_\mathcal{B})|\bar{b}_v(iD_\mu)(iv\cdot D)(iD^\mu)b_v|\mathcal{B}(p_\mathcal{B})\rangle \\
2m_\mathcal{B}\,\rho_{LS}^3(\mathcal{B}) &= \langle \mathcal{B}(p_\mathcal{B})|\bar{b}_v(iD_\mu)(iv\cdot D)(iD_\nu)(-i\sigma^{\mu\nu})\,b_v|\mathcal{B}(p_\mathcal{B})\rangle
\end{aligned}
$$

- Dimension-6 four-quark operators

$$\mathcal{O}_1^q = (\bar{h}_v^i\gamma_\mu(1-\gamma_5)q^i)(\bar{q}^j\gamma^\mu(1-\gamma_5)h_v^j) \qquad \mathcal{O}_2^q = (\bar{h}_v^i(1-\gamma_5)q^i)(\bar{q}^j(1+\gamma_5)h_v^j)$$

$$\mathcal{O}_3^q = (\bar{h}_v^i\gamma_\mu(1-\gamma_5)q^j)(\bar{q}^j\gamma^\mu(1-\gamma_5)h_v^i) \qquad \mathcal{O}_4^q = (\bar{h}_v^i(1-\gamma_5)q^j)(\bar{q}^j(1+\gamma_5)h_v^i)$$

- Matrix elements of dimension-6 four-quark operators for B-mesons

$$\langle B_q|\mathcal{O}_n^q|B_q\rangle = f_{B_q}^2 M_{B_q}^2\, B_n(B_q)$$

Definition of non-perturbative parameters

- Matrix elements of dimension-6 four-quark operators for b-baryons in non-relativistic constituent quark model (NRCQM)

$$\frac{\langle \Lambda_b|\mathcal{O}_1^q|\Lambda_b\rangle}{2M_{\Lambda_b}} = -y_{\tilde{q}}\,\frac{4}{3}\,\frac{M_{\Sigma_b^*}-M_{\Sigma_b}}{M_{B^*}-M_B}\,|\Psi^B(0)|^2$$

$$\frac{\langle \Xi_b^0|\mathcal{O}_1^u|\Xi_b^0\rangle}{2M_{\Xi_b}} = \frac{\langle \Xi_b^-|\mathcal{O}_1^d|\Xi_b^-\rangle}{2M_{\Xi_b}} = -y_{\tilde{q}}\,\frac{4}{3}\,\frac{M_{\Xi_b^*}-M_{\Xi_b'}}{M_{B^*}-M_B}\,|\Psi^B(0)|^2$$

$$\frac{\langle \Xi_b^-|\mathcal{O}_1^s|\Xi_b^-\rangle}{2M_{\Xi_b}} = \frac{\langle \Xi_b^0|\mathcal{O}_1^s|\Xi_b^0\rangle}{2M_{\Xi_b}} = -y_s\,\frac{4}{3}\,\frac{M_{\Xi_b^*}-M_{\Xi_b'}}{M_{B_s^*}-M_{B_s}}\,|\Psi^{B_s}(0)|^2$$

$$\frac{\langle \Omega_b^-|\mathcal{O}_1^s|\Omega_b^-\rangle}{2M_{\Omega_b}} = -y_s\,6\,\frac{4}{3}\,\frac{M_{\Omega_b^*}-M_{\Omega_b}}{M_{B_s^*}-M_{B_s}}\,|\Psi^{B_s}(0)|^2$$

$$|\Psi^{B_q}(0)|^2 = \frac{F_{B_q}^2(\mu_0)}{12} \qquad y_q = \frac{m_b^b\, m_q^b}{m_b^m\, m_q^m} \qquad \langle\mathcal{B}|\mathcal{O}_{2,3,4}^q|\mathcal{B}\rangle \sim \langle\mathcal{B}|\mathcal{O}_1^q|\mathcal{B}\rangle$$