Final-state interactions in the CP asymmetries of charm-meson two-body decays

Eleftheria Solomonidi

In collaboration with Antonio Pich & Luiz Vale Silva

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Implications of LHCb measurements and future prospect

IFIC
INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

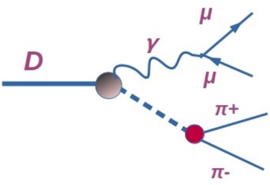
CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

Introduction

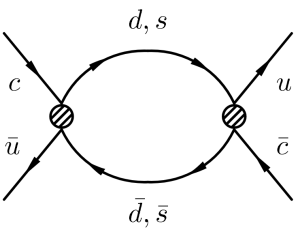
Charm Physics in the limelight

- Complementary to down-type sector for CKM tests
- Experimental programme is growing (LHCb, Belle II, BESIII)
- Theoretical efforts have to keep up

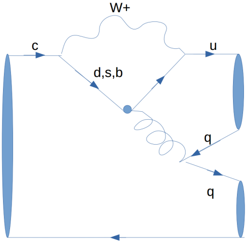
Rare decays



Mixing



CP violation in decays



Compared to K, B Physics:

- Different masses $\Rightarrow$ different loop effects
  & different mixing sizes $\Rightarrow$ different phenomenology!

$$m_d,\ m_s,\ m_b << m_W \Rightarrow \lambda_d F(x_d) + \lambda_s F(x_s) + \lambda_b F(x_b) \approx 0$$

A new anomaly or an incomplete theory prediction?

$$\Delta A_{CP}^{exp} \equiv A_{CP}(D^0 \to K^+K^-) - A_{CP}(D^0 \to \pi^+\pi^-) = [-1.54 \pm 0.29] \cdot 10^{-3}$$

$$\Delta \alpha_{CP}^{dir,exp} = [-1.57 \pm 0.29] \cdot 10^{-3}$$ [LHCb 2019]

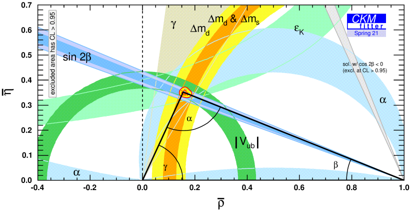
$$A_{CP}(D^0 \to K^+K^-) = [6.8 \pm 5.4(\text{stat}) \pm 1.6(\text{syst})] \cdot 10^{-4}$$ [LHCb 2022]

$$\alpha_{CP}^{dir}(D^0 \to \pi^+\pi^-) = [23.2 \pm 6.1] \cdot 10^{-4}$$

Is the SM theoretical prediction in agreement?

Is it NP? [see e.g. Hiller et al. 2023, Chala, Lenz, Rusov, Scholtz 2019]

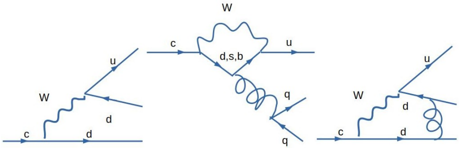
- CP violation from the SM induced by only 1 phase; CKM controlled by 4 independent parameters
- Weak sector (CKM parameters) probed by nuclear, π, K & B physics



- Strong sector introduces uncertainties

Weak and strong, short and long distance

$$\mathscr{A}(D^0 \to f) = A(f) + i r_{CKM} B(f)$$
$$\mathscr{A}(\overline{D^0} \to f) = A(f) - i r_{CKM} B(f)$$
$$\alpha_{CP}^{dir} \approx 2 \underbrace{r_{CKM}}_{\text{weak phases}} \frac{|B(f)|}{|A(f)|} \cdot \sin \arg \underbrace{\frac{A(f)}{B(f)}}_{\text{strong phases}}$$



At $\mu \sim \mu_c$:

$$\mathscr{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\Sigma_{i=1}^{2} C_i(\mu) \left(\lambda_d Q_i^d(\mu) + \lambda_s Q_i^s(\mu) \right) - \lambda_b (\Sigma_{i=3}^{6} C_i(\mu) Q_i(\mu) + C_{8g}(\mu) Q_{8g}(\mu)) \right]$$

$\lambda_q = V_{cq}^* V_{uq}\,, \quad q = d, s, b.$
$|\lambda_d| \approx |\lambda_s| = \mathscr{O}(\lambda)$

$$\boxed{\lambda_d + \lambda_s + \lambda_b = 0}$$

$$\boxed{r_{CKM} = Im \frac{V_{cb}^* V_{ub}}{V_{cd}^* V_{ud}} \approx 6.2 \cdot 10^{-4}}$$

Penguin operators

$$Q_3 = (\bar{u}c)_{V-A} \Sigma_q (\bar{q}q)_{V-A}$$
$$\boxed{Q_4 = (\bar{u}_j c_i)_{V-A} \Sigma_q (\bar{q}_i q_j)_{V-A}}$$
$$Q_5 = (\bar{u}c)_{V-A} \Sigma_q (\bar{q}q)_{V+A}$$
$$\boxed{Q_6 = (\bar{u}_j c_i)_{V-A} \Sigma_q (\bar{q}_i q_j)_{V+A}}$$

Current-current operators

$$Q_1^q = (\bar{q}c)_{V-A} (\bar{u}q)_{V-A}$$
$$Q_2^d = (\bar{q}_j c_i)_{V-A} (\bar{u}_i q_j)_{V-A}$$
$$(q = d, s)$$

$C_{4,6} < 0.1\, C_2,\ 0.03\, C_1$ *(GIM mechanism at play)*

$$\mathscr{A}(D^0 \to f) = A(f) + i r_{CKM} B(f)$$

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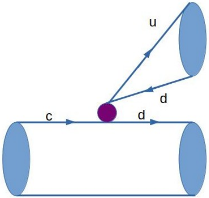

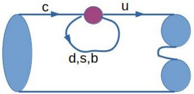

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Problem: hadronic matrix elements

$$\langle hh | Q_i | D^0 \rangle$$

Charm scale is special!

$\Lambda_{\chi PT} \approx m_\rho < m_D = 1865$ MeV

$\frac{\Lambda_{QCD}}{m_c} \approx \mathscr{O}(1)$

See also: Khodjamirian, Petrov Phys. Lett. B, 774:235–242, 2017, Brod, Kagan, Zupan Phys. Rev. D, 86:014023, 2012, Schacht, Soni Phys. Lett. B, 825:136855, 2022, Franco, Mishima, Silvestrini JHEP, 05:140, 2012, Buccella, Paul, Santorelli Phys. Rev. D, 99(11):113001, 2019,Hiller, Jung, Schacht Phys. Rev. D, 87(1):014024, 2013

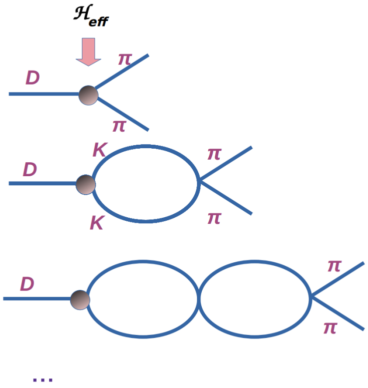
Data-driven approach - principles

A way to look at the problem: rescattering

Idea: implement long-distance QCD effects through resummation of rescattering "bubbles"

- Strong process, blind to the weak dynamics
- Isospin (u↔d) is a good symmetry of strong interactions
- In I=0, two channels:

$$S_{strong} = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to KK \\ KK \to \pi\pi & KK \to KK \end{pmatrix}$$



Rescattering & unitarity

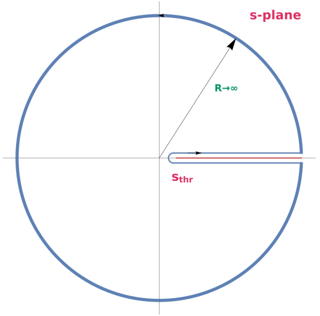
- S matrix is **unitary**, as well as strong sub-matrix
- Optical theorem-like relation: $\mathrm{Im}\,\langle hh|Q_i|D\rangle = \sum_n \langle hh|T|n\rangle \langle n|Q_i|D\rangle^*$
- For I=0: $\begin{pmatrix} A(D\to\pi\pi) \\ A(D\to KK) \end{pmatrix} = \underbrace{\begin{pmatrix} \eta e^{i2\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{i2\delta_2} \end{pmatrix}}_{S_{strong}} \cdot \begin{pmatrix} A^*(D\to\pi\pi) \\ A^*(D\to KK) \end{pmatrix}$
- The phases of the decay amplitudes are related to the rescattering phases **for which data and data-driven parameterizations exist**
- Watson's theorem (elastic rescattering limit):
  $\arg A(D\to\pi\pi) = \arg A(\pi\pi\to\pi\pi) \mod \pi$
- With inelasticities: more complicated, phase-shifts dependent on magnitudes of the amplitudes too

Analyticity and unitarity consequences: Dispersion relations

- **Analyticity**: fundamental, model-independent property related to **causality**
- Amplitudes analytical except for a right-hand cut
- Cauchy's theorem:
  $A(s) = \frac{1}{2\pi i} \oint_C ds' \frac{A(s')}{s'-s}$ leads to

$$\mathrm{Re}A(s) = \frac{1}{\pi} \int_{s_{thr}}^{\infty} ds' \frac{\mathrm{Im}A(s')}{s'-s}$$

(Dispersion relation)



Single channel case:

Implement unitarity of S-matrix :

$$\underbrace{\mathrm{Re}A_I(s)}_{\text{Re at a point}} = \frac{1}{\pi} \underbrace{PV \int_{s_{thr}}^{\infty} ds' \frac{\tan \delta_I(s')}{s'-s} \mathrm{Re}A_I(s')}_{\text{integral of Re along the physical region}}$$

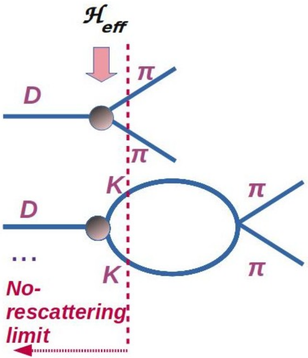
\+ one subtraction ⇒

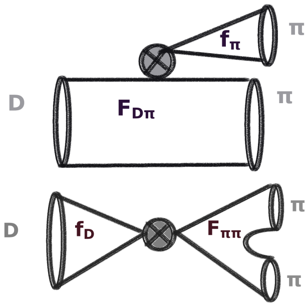
Solution:

$$|A_I(s)| = \underbrace{A_I(s_0)}_{\text{ampl. without rescattering}} \times \underbrace{exp\{\frac{s-s_0}{\pi} PV \int_{4M_\pi^2}^{\infty} dz \frac{\delta_I(z)}{(z-s_0)(z-s)}\}}_{\text{Omnès factor } \Omega}$$

Matrix elements *without* rescattering

The no-rescattering limit coincides with the limit of $N_c \to \infty$
$\Rightarrow$ we are left with the matrix elements from *factorization* - no strong phase induced





(Same for $D \to KK$)

- Decay constant and form factors come from lattice and data ($\pi\pi$ FF through χPT)
- Non-perturbative QCD information within each current naturally included

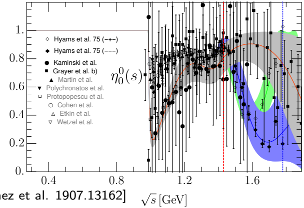
Data-driven approach: details

Summary of our method

- Factor out the eff. weak Hamiltonian (weak process & short-distance QCD)
- Implement final state interaction (FSI) dynamics in the isospin limit (no $SU(3)$ assumption needed)
- Isospin blocks:
  - I=0, unitarity with 2 channels: $\pi\pi$ and KK
  - I=1 with KK elastic rescattering
  - I=2 with $\pi\pi$ elastic rescattering
- Isospin-zero amplitudes treated with dispersion relations (DRs) calculated numerically (based on Moussallam et al. [hep-ph/9909292])
- Use inelasticity and phase-shift parameterizations [Pelaez et al., 1907.13162],[Pelaez et al., 2010.11222] up to energies $\sim m_D$ - extrapolate for higher & consider uncertainties
- For I=1 and 2, extract |Omnès factors| from Br's of $A(D^+ \to \pi^+\pi^0) \sim A_{I=2}, A(D^+ \to K^+\overline{K^0}) \sim A_{I=1}$; phases left unconstrained
- Decay-specific physical input (in the subtraction constant of DRs): large N_C limit of the eff. weak Hamiltonian

Choice of Omnès factors

For the isospin=0 channels we **calculate** numerically the Omnès matrix at $s = m_D^2$

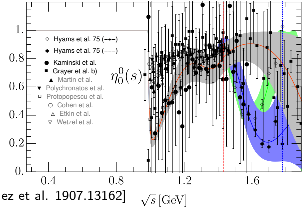


[Pelaez et al. 1907.13162]

	solution I	solution II	solution III
$\eta_0^0,\, m_\eta^* = 1$	$\Omega^{(0)} = \begin{pmatrix} 0.80\, e^{+1.60\, i} & 1.01\, e^{-1.69\, i} \\ 0.56\, e^{-1.50\, i} & 0.59\, e^{+2.07\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.39\, e^{+1.64\, i} & 0.59\, e^{-2.13\, i} \\ 0.51\, e^{-1.31\, i} & 0.56\, e^{+2.43\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.71\, e^{+0.53\, i} & 1.35\, e^{-2.67\, i} \\ 0.38\, e^{-0.98\, i} & 0.42\, e^{+2.65\, i} \end{pmatrix}$
$\eta_0^0 - \delta\eta_0^0,\, m_\eta^* = 1$	$\Omega^{(0)} = \begin{pmatrix} 0.56\, e^{+1.84\, i} & 0.61\, e^{-1.73\, i} \\ 0.57\, e^{-1.41\, i} & 0.58\, e^{+2.25\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.42\, e^{+1.75\, i} & 0.54\, e^{-2.05\, i} \\ 0.51\, e^{-1.33\, i} & 0.55\, e^{+2.43\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.35\, e^{+1.13\, i} & 0.74\, e^{-2.47\, i} \\ 0.50\, e^{-1.18\, i} & 0.55\, e^{+2.48\, i} \end{pmatrix}$
$\eta_0^0 - \delta\eta_0^0,\, m_\eta^* = 2$	$\Omega^{(0)} = \begin{pmatrix} 0.58\, e^{+1.80\, i} & 0.64\, e^{-1.74\, i} \\ 0.58\, e^{-1.37\, i} & 0.61\, e^{+2.26\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.43\, e^{+1.64\, i} & 0.58\, e^{-2.10\, i} \\ 0.52\, e^{-1.25\, i} & 0.57\, e^{+2.48\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.40\, e^{+1.01\, i} & 0.80\, e^{-2.50\, i} \\ 0.50\, e^{-1.11\, i} & 0.56\, e^{+2.53\, i} \end{pmatrix}$
$\eta_0^0 - \delta\eta_0^0,\, m_\eta^* = 3$	$\Omega^{(0)} = \begin{pmatrix} 0.60\, e^{+1.76\, i} & 0.66\, e^{-1.74\, i} \\ 0.60\, e^{-1.33\, i} & 0.63\, e^{+2.26\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.44\, e^{+1.53\, i} & 0.61\, e^{-2.16\, i} \\ 0.52\, e^{-1.17\, i} & 0.59\, e^{+2.53\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.45\, e^{+0.91\, i} & 0.86\, e^{-2.53\, i} \\ 0.50\, e^{-1.04\, i} & 0.57\, e^{+2.58\, i} \end{pmatrix}$
sol. B': $\lvert g_0^0 \rvert$	$\Omega^{(0)} = \begin{pmatrix} 2.01\, e^{+1.39\, i} & 2.47\, e^{-1.76\, i} \\ 0.37\, e^{-0.33\, i} & 0.54\, e^{+3.05\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 1.91\, e^{+0.60\, i} & 2.78\, e^{-2.55\, i} \\ 0.31\, e^{-0.23\, i} & 0.45\, e^{+3.30\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 2.20\, e^{+0.43\, i} & 3.55\, e^{-2.72\, i} \\ 0.35\, e^{+0.03\, i} & 0.57\, e^{+3.40\, i} \end{pmatrix}$
sol. C': $\lvert g_0^0 \rvert$	$\Omega^{(0)} = \begin{pmatrix} 1.83\, e^{+1.38\, i} & 2.65\, e^{-1.76\, i} \\ 0.34\, e^{-0.40\, i} & 0.57\, e^{+3.00\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 1.80\, e^{+0.59\, i} & 3.11\, e^{-2.56\, i} \\ 0.29\, e^{-0.24\, i} & 0.49\, e^{+3.24\, i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 2.09\, e^{+0.43\, i} & 3.94\, e^{-2.72\, i} \\ 0.32\, e^{-0.03\, i} & 0.61\, e^{+3.34\, i} \end{pmatrix}$

Choice of Omnès factors

For the isospin=0 channels we **calculate** numerically the Omnès matrix at $s = m_D^2$



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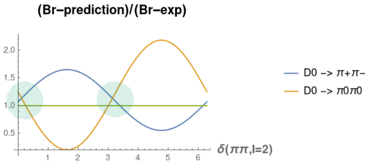
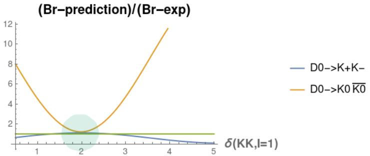
- We examine the branching fraction predictions for the decays $\pi^+\pi^-$, $\pi^0\pi^0$, K^+K^-, $K^0\overline{K^0}$ based on each Omnès matrix ($\Leftrightarrow$ rescattering input) separately
- Only a few of them give simultaneously correct Br values for all channels:





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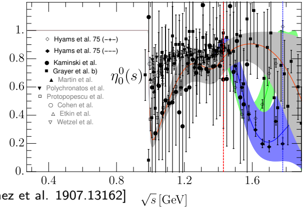

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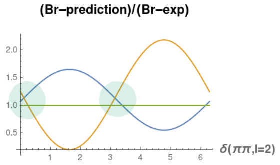
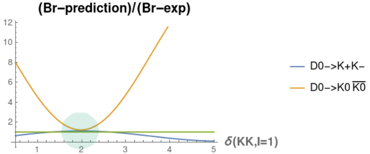
$\eta_0^0 - \delta\eta_0^0,\, m_\eta^* = 1$	$\Omega^{(0)} = \begin{pmatrix} 0.56\,e^{+1.84\,i} & 0.61\,e^{-1.73\,i} \\ 0.57\,e^{-1.41\,i} & 0.58\,e^{+2.25\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.42\,e^{+1.75\,i} & 0.54\,e^{-2.05\,i} \\ 0.51\,e^{-1.33\,i} & 0.55\,e^{+2.43\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.35\,e^{+1.13\,i} & 0.74\,e^{-2.47\,i} \\ 0.50\,e^{-1.18\,i} & 0.55\,e^{+2.48\,i} \end{pmatrix}$
$\eta_0^0 - \delta\eta_0^0,\, m_\eta^* = 2$	$\Omega^{(0)} = \begin{pmatrix} 0.58\,e^{+1.80\,i} & 0.64\,e^{-1.74\,i} \\ 0.58\,e^{-1.37\,i} & 0.61\,e^{+2.26\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.43\,e^{+1.64\,i} & 0.58\,e^{-2.10\,i} \\ 0.52\,e^{-1.25\,i} & 0.57\,e^{+2.48\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.40\,e^{+1.01\,i} & 0.80\,e^{-2.50\,i} \\ 0.50\,e^{-1.11\,i} & 0.56\,e^{+2.53\,i} \end{pmatrix}$
$\eta_0^0 - \delta\eta_0^0,\, m_\eta^* = 3$	$\Omega^{(0)} = \begin{pmatrix} 0.60\,e^{+1.76\,i} & 0.66\,e^{-1.74\,i} \\ 0.60\,e^{-1.33\,i} & 0.63\,e^{+2.26\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.44\,e^{+1.53\,i} & 0.61\,e^{-2.16\,i} \\ 0.52\,e^{-1.17\,i} & 0.59\,e^{+2.53\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 0.45\,e^{+0.91\,i} & 0.86\,e^{-2.53\,i} \\ 0.50\,e^{-1.04\,i} & 0.57\,e^{+2.58\,i} \end{pmatrix}$
sol. B': $\lvert g_0^0 \rvert$	$\Omega^{(0)} = \begin{pmatrix} 2.01\,e^{+1.39\,i} & 2.47\,e^{-1.76\,i} \\ 0.37\,e^{-0.33\,i} & 0.54\,e^{+3.05\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 1.91\,e^{+0.60\,i} & 2.78\,e^{-2.55\,i} \\ 0.31\,e^{-0.23\,i} & 0.45\,e^{+3.30\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 2.20\,e^{+0.43\,i} & 3.55\,e^{-2.72\,i} \\ 0.35\,e^{+0.03\,i} & 0.57\,e^{+3.40\,i} \end{pmatrix}$
sol. C': $\lvert g_0^0 \rvert$	$\Omega^{(0)} = \begin{pmatrix} 1.83\,e^{+1.38\,i} & 2.65\,e^{-1.76\,i} \\ 0.34\,e^{-0.40\,i} & 0.57\,e^{+3.00\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 1.80\,e^{+0.59\,i} & 3.11\,e^{-2.56\,i} \\ 0.29\,e^{-0.24\,i} & 0.49\,e^{+3.24\,i} \end{pmatrix}$	$\Omega^{(0)} = \begin{pmatrix} 2.09\,e^{+0.43\,i} & 3.94\,e^{-2.72\,i} \\ 0.32\,e^{-0.03\,i} & 0.61\,e^{+3.34\,i} \end{pmatrix}$

- We examine the branching fraction predictions for the decays $\pi^+\pi^-$, $\pi^0\pi^0$, K^+K^-, $K^0\overline{K^0}$ based on each Omnès matrix ($\Leftrightarrow$ rescattering input) separately
- Only a few of them give simultaneously correct *Br* values for all channels:





Results

Rescattering quantified

With the branching fractions correctly reproduced
the Omnès matrix looks like:

$$\Omega_{I=0} = \begin{pmatrix} 0.58e^{1.8i} & 0.64e^{-1.7i} \\ 0.58e^{-1.4i} & 0.61e^{2.3i} \end{pmatrix}$$

The **physical solution** is

$$\begin{pmatrix} \mathbf{A}(D \to \pi\pi_{I=0}) \\ \mathbf{A}(D \to KK_{I=0}) \end{pmatrix} = \underbrace{\Omega_{I=0}}_{\text{rescattering}} \cdot \underbrace{\begin{pmatrix} \mathbf{A}_{(\text{large } N_C)}(D \to \pi\pi_{I=0}) \\ \mathbf{A}_{(\text{large } N_C)}(D \to KK_{I=0}) \end{pmatrix}}_{\text{non-rescattering}}$$

It turns out:

Significant rescattering between the two final states!

penguin insertions $\approx$ tree insertions
(of current-current operators, for I=0 reduced matrix elements)

Equivalently:

$$|\langle \pi\pi | (\overline{d}c)(\overline{u}d) | D^0 \rangle| \approx |\langle KK | (\overline{d}c)(\overline{u}d) | D^0 \rangle|,$$
$$|\langle KK | (\overline{s}c)(\overline{u}s) | D^0 \rangle| \approx |\langle \pi\pi | (\overline{s}c)(\overline{u}s) | D^0 \rangle|$$

Sources of CP violation

Remember: Difference of weak phases AND strong phases needed

For $D \to \pi\pi$ we then have: $((\bar{d}c)(\bar{u}d), (\bar{s}c)(\bar{u}s) \sim$ current-current operators$)$

- I=2: $\lambda_d \times \langle \pi\pi_{I=2} | (\bar{d}c)(\bar{u}d) | D \rangle$
- I=0:
  $\lambda_d \times \langle \pi\pi_{I=0} | (\bar{d}c)(\bar{u}d) | D \rangle + \lambda_s \times \langle \pi\pi_{I=0} | (\bar{s}c)(\bar{u}s) | D \rangle - \lambda_b \times \langle \pi\pi_{I=0} |_{\text{penguin operators}} | D \rangle$

If rescattering was elastic it would be

- $\langle \pi\pi_{I=0} | (\bar{s}c)(\bar{u}s) | D \rangle = 0$ AND
- $\arg \langle \pi\pi_{I=0} |_{\text{penguin operators}} | D \rangle = \arg \langle \pi\pi_{I=0} | (\bar{d}c)(\bar{u}d) | D \rangle$

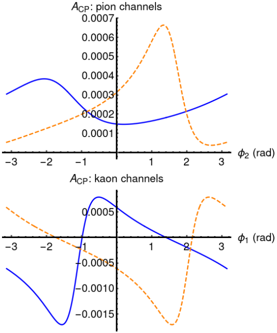
$\Rightarrow$ single source of CPV *would be* the interference between $I = 0$ and $I = 2$

Instead, multiple sources of CPV present in this process

$\alpha_{CP}(\pi\pi)(0-0)$, $\alpha_{CP}(\pi\pi)(2-0)$ for $D^0 \to \pi^+\pi^-$, $\pi^0\pi^0$
$\alpha_{CP}(KK)(0-0)$, $\alpha_{CP}(KK)(1-0)$ for $D^0 \to K^+K^-$, $K^0\overline{K^0}$

Some small cancellations present - do not affect the final result

CP asymmetries



charged meson channels
neutral meson channels

$$\Delta\alpha_{CP}^{dir,exp} = (-1.57 \pm 0.29) \cdot 10^{-3}$$

With $\delta(I=2,\pi\pi)$, $\delta(I=1,KK)$ around the chosen values, we predict:

$$\Delta\alpha_{CP}^{dir,theo} \sim 5 \cdot 10^{-4}!!$$

and $\alpha_{CP}^{dir}(D^0 \to \pi^+\pi^-) \approx 3 \cdot 10^{-4}$, $\alpha_{CP}^{dir}(D^0 \to K^+K^-) \approx -2 \cdot 10^{-4}$

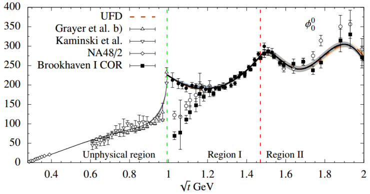
$$\alpha_{CP}^{dir} \approx 2 \underbrace{r_{CKM}}_{\sim 6\cdot 10^{-4}} \underbrace{\frac{|B(f)|}{|A(f)|}}_{\sim 1/3} \cdot \underbrace{\sin\arg\frac{A(f)}{B(f)}}_{\sim 1}$$

NB: Short-distance penguins also not negligible for the CP asymmetries: C_6 small but annihilation insertion very large so that $C_6 \langle Q_6 \rangle_{\text{large } N_C} \sim C_1 \langle Q_1 \rangle_{\text{large } N_C}$

NB: $SU(3)$ breaking manifested through differences in the $\pi\pi$ and KK rescattering parameters; similar level to breaking observed in decay constants, form factors

With fewer uncertain strong parameters (preliminary)

- $\pi\pi$, KK inelasticity has large uncertainties
- Use only one low-energy strong phase for isospin 0: $\pi\pi + KK$ phase



[Pelaez et al. 2010.11222]

- Assumption: 2-channel unitarity → CPT/unitarity theorem also applying

Sum rule for $\alpha_{CP}(\pi\pi)(0-0)$, $\alpha_{CP}(KK)(0-0)$

$\Rightarrow \alpha_{CP}(\pi\pi)(0-0) \cdot \alpha_{CP}(KK)(0-0) < 0$

- We manage to constrain:

$$\begin{aligned} 0 &< \alpha_{CP}(\pi\pi)(0-0) \lesssim 5 \times 10^{-4} \\ -3 \times 10^{-4} &\lesssim \alpha_{CP}(KK)(0-0) < 0 \end{aligned}$$

- The CP asymmetry from $I = 2/0$ interference is not constrained, but would require very large values of isospin-0 Omnès matrix elements ⇒ some dynamics not manifested in the data

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- **We still estimate the CP asymmetry for the $\pi^+\pi^-$ too small compared to the experimental value!**
- CPV in $D^0 \rightarrow \pi^0\pi^0$ predicted to be of similar magnitude *(could some experiments look there?)*
- Future directions: diferent isospin-2 scenarios, more channels in isospin-0?
- But these are naively not expected to change the picture...
- Cross-checks in other channels are crucial

Thank you very much!

BACKUP

Isospin-2 and -1 fixing

$$\mathscr{A}(D^+ \to \pi^+\pi^0) = \frac{3}{2\sqrt{2}} A^{\pi}_{I2}$$

$$\mathscr{A}(D^+ \to K^+\overline{K^0}) = A^{K}_{I1}$$

We fix $|A^{\pi}_{I2}|$, $|A^{K}_{I1}|$ from the Br's and use them in e.g.

$$\mathscr{A}(D^0 \to \pi^+\pi^-) = -\frac{1}{2\sqrt{3}} A^{\pi}_{I2} + \frac{1}{\sqrt{6}} A^{\pi}_{I0}$$

If I=2 elastic then $A^{\pi}_{I2} = \Omega_{I=2} A_{fac,I=2}$
If inelastic $A^{\pi}_{I2} = \Omega_{I=2} A_{fac,I=2} +$ (mixing) but we use directly
$A^{\pi}_{I2} = |A^{\pi}_{I2}| exp\{i\delta^{\pi\pi}_{I=2}\}$, phase left free

Naive estimate of final state interaction effects

$$\begin{pmatrix} A^{I=0}_{\pi\pi} \\ A^{I=0}_{KK} \end{pmatrix} = S_S^{1/2} \cdot \begin{pmatrix} A^{I=0}_{\pi\pi,\text{bare}} \\ A^{I=0}_{KK,\text{bare}} \end{pmatrix}$$

bare amplitudes: from factorisation (no strong phases)
Reproduces correctly Watson's theorem
What unitarity gives:

$$\begin{pmatrix} A^{I=0}_{\pi\pi} \\ A^{I=0}_{KK} \end{pmatrix} = S_S \cdot \begin{pmatrix} (A^{I=0}_{\pi\pi})^* \\ (A^{I=0}_{KK})^* \end{pmatrix}$$

No direct solution for the amplitudes, just relates them to the rescattering phases:

$$argA^{I=0}_{\pi\pi} = \delta_1 + arccos\sqrt{\frac{(1+\eta)^2 - \left(\frac{|A^{I=0}_{KK}|}{|A^{I=0}_{\pi\pi}|}\right)^2 (1-\eta^2)}{4\eta}}$$

$$argA^{I=0}_{KK} = \delta_2 + arccos\sqrt{\frac{(1+\eta)^2 - \left(\frac{|A^{I=0}_{\pi\pi}|}{|A^{I=0}_{KK}|}\right)^2 (1-\eta^2)}{4\eta}}$$

Numerical solution of 2-channel case

$$\begin{pmatrix} ReA^{\pi}(s) \\ ReA^{K}(s) \end{pmatrix} = \frac{s - s_0}{\pi} PV \int_{s_{thr}}^{\infty} ds' \frac{(ReT)^{-1}(ImT)(s')}{(s'-s)(s'-s_0)} \begin{pmatrix} ReA^{\pi}(s') \\ ReA^{K}(s') \end{pmatrix} + \begin{pmatrix} ReA_0^{\pi}(s_0) \\ ReA_0^{K}(s_0) \end{pmatrix}$$

- We discretise following the method from [Moussallam et al. hep-ph/9909292] into

$$\begin{pmatrix} ReA^{\pi}(s_i) \\ ReA^{K}(s_i) \end{pmatrix} = \frac{s_i - s_0}{\pi} \sum_j \hat{w}_j \frac{(ReT)^{-1}(ImT)(s_j)}{(s_j - s_i)(s_j - s_0)} \begin{pmatrix} ReA^{\pi}(s_j) \\ ReA^{K}(s_j) \end{pmatrix} + \begin{pmatrix} ReA_0^{\pi}(s_0) \\ ReA_0^{K}(s_0) \end{pmatrix}$$

- This creates an **invertible** matrix which gives *a* (discrete) solution
- Subtleties taken care of as in [Moussallam et al. hep-ph/9909292]
- To pick the *fundamental* solutions, we fix the vector at an unphysical point $s < 0$ and
  - check they behave as $\frac{1}{s}$ for large s
  - make sure the numerical determinant behaves as the (known) analytical determinant

Isospin decomposition

- $\pi\pi$ states can have isospin=0,2. KK can have isospin=0,1.

$$\begin{pmatrix} A(\pi^+\pi^-) \\ A(\pi^0\pi^0) \\ A(K^+K^-) \\ A(K^0\overline{K}^0) \end{pmatrix} = \begin{pmatrix} -\frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{6}} & 0 & 0 \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} A_\pi^2 \\ A_\pi^0 \\ A_K^1 \\ A_K^0 \end{pmatrix}$$

CPV in I=0

$$\begin{pmatrix} A^\pi \\ A^K \end{pmatrix} = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \begin{pmatrix} Re\lambda_d T^\pi + ... \\ Re\lambda_s T^K + ... \end{pmatrix}$$

$$\begin{pmatrix} B^\pi \\ B^K \end{pmatrix} = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \begin{pmatrix} Im\lambda_d T^\pi + \sum_i Im\lambda_{d_i} P_i^\pi \\ Im\lambda_s T^K + \sum_i Im\lambda_{d_i} P_i^K \end{pmatrix}$$

Can consider either $Im\lambda_d = 0$ or $Im\lambda_s = 0$, not both simultaneously
$\Rightarrow$ In α_{CP}^{dir} there always exists a term $\sim T^\pi T^K$, both for $\pi\pi$ and for KK

Some numerical inputs

- $C_1 = 1.18$, $C_2 = -0.32$, $C_3 = 0.011$, $C_4 = -0.031$, $C_5 = 0.0068$, $C_6 = -0.032$ ($\mu = 2$ GeV)
- $\lambda_d = V^*_{cd} V_{ud} \approx 0.22$
- $\overline{m_c}(2GeV) = 1.097 GeV$
- Compare $m_D = 1865$ MeV to $\Lambda_{\chi PT} \approx m_\rho = 775$ MeV