Born-Oppenheimer EFT (BOEFT) for Hybrids & Doubly heavy tetraquark

Implications of LHCb measurements and future prospects
CERN

Oct 26, 2023

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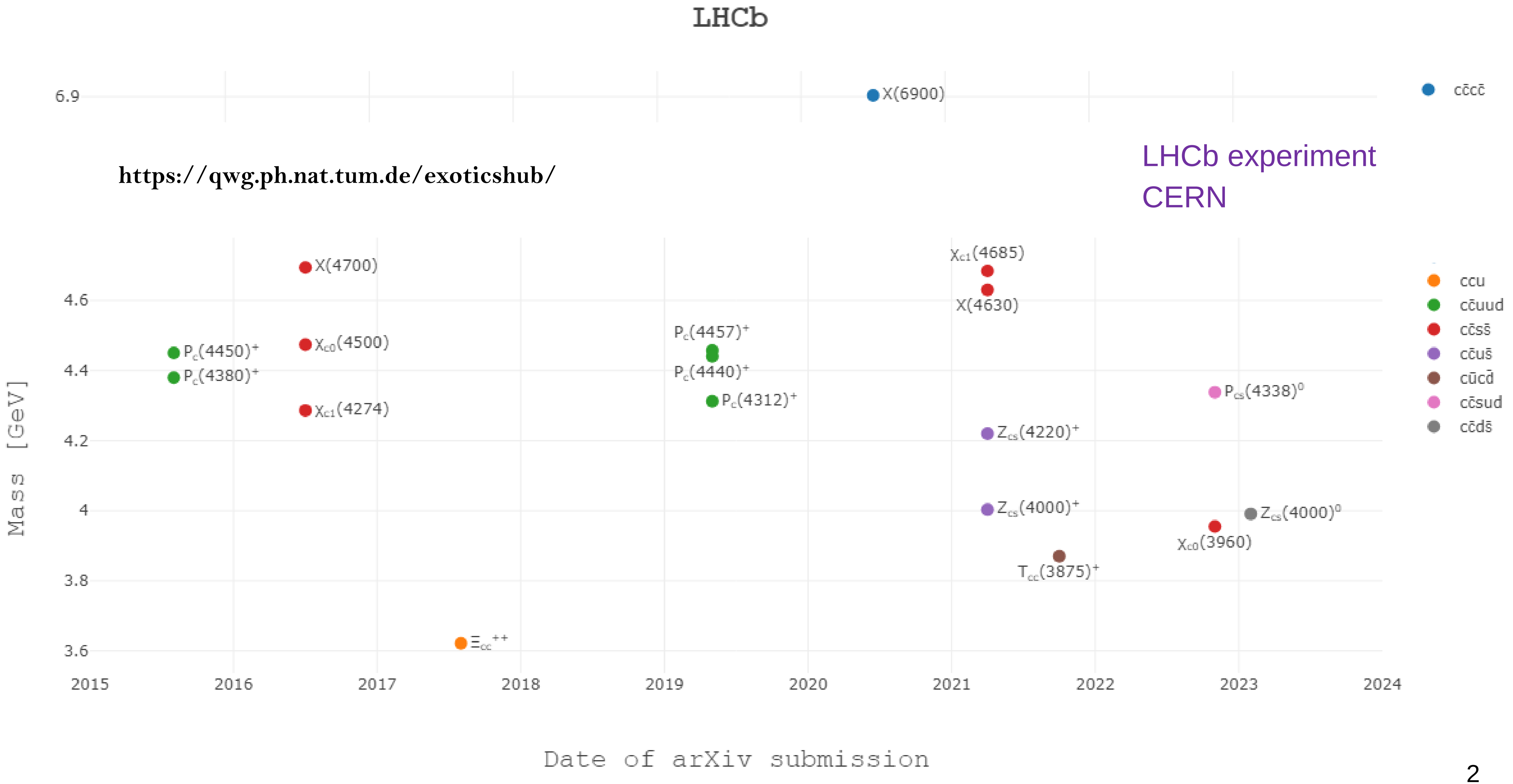
Based on 2212.09187, 2312.xxxx with N. Brambilla & A.Vairo and 2006.08650 with E. Braaten.

Technical University of Munich

DFG Deutsche Forschungsgemeinschaft

Exotic Hadrons at LHCb



https://qwg.ph.nat.tum.de/exoticshub/

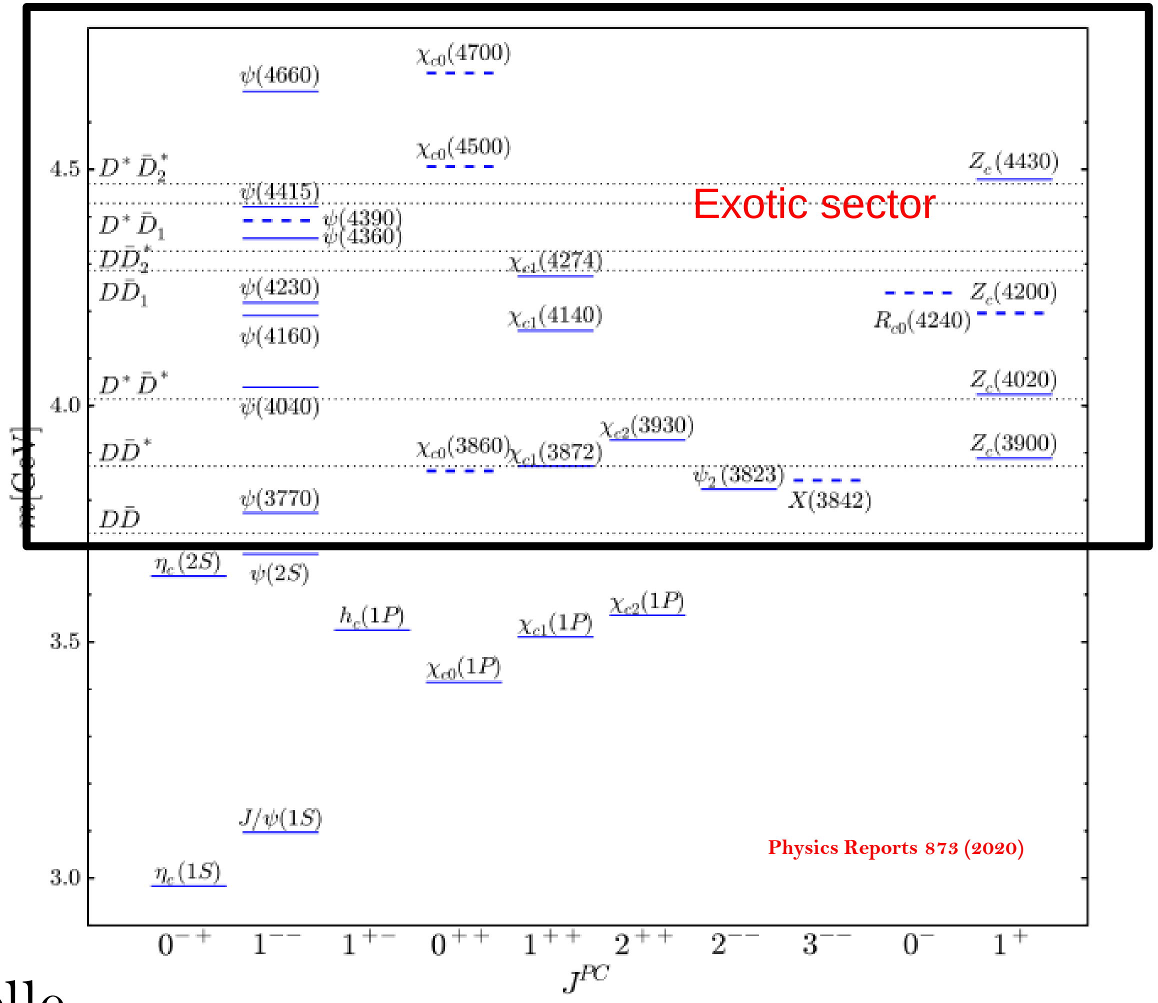
LHCb experiment
CERN

Exotic Hadron

- States beyond traditional classification of mesons and baryons
- Exotic states: XYZ mesons (heavy-quark sector)
  - ✓ States that don't fit traditional $Q\bar{Q}$ spectrum.
  - ✓ Exotic quantum numbers:
    - $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ etc. are exotic
    - Charged Z_c and Z_b states: minimal 4-quark state:

      $Z_c(4430)^{\pm}$ $Z_b(10650)^{\pm}$ **Tetraquarks**

For review see Brambilla et al. Phys. Reports. 873 (2020)



- X(3872): First exotic state discovered in 2003 by Belle.

  Phys. Rev. Lett. 91, 262001 (2003)

- Dozens of XYZ mesons discovered since 2003.

Exotic Hadron

Castellà , Soto Phys. Rev. D. 102, (2020)

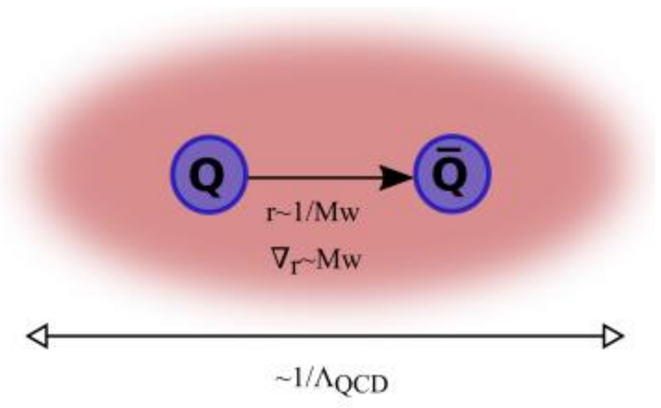
Brambilla, AM, Vairo arXiv 2312.xxxx

- Exotic hadron ($\boldsymbol{Q\bar{Q}X}, \boldsymbol{QQX}, \ldots\ldots$), $\boldsymbol{X}$: any combination of light quark and gluons to obtain color singlet hadron

:

- Hierarchy of scales:

$$m \gg mv \gtrsim \Lambda_{\rm QCD} \gg mv^2$$



❖ Mass of heavy quark: $\boldsymbol{m}$ ❖ Energy scale for light d.o.f: $\boldsymbol{\Lambda_{\rm QCD}}$

❖ Relative separation between heavy quarks: $r \sim 1/mv$

❖ Heavy Quark K.E scale: mv^2

BOEFT: Effective theory based on Born-Oppenheimer Approximation

Heavy quark: slow-degrees of freedom $\quad$ X: fast-degrees of freedom

BOEFT: EFT focused at energy scale mv^2

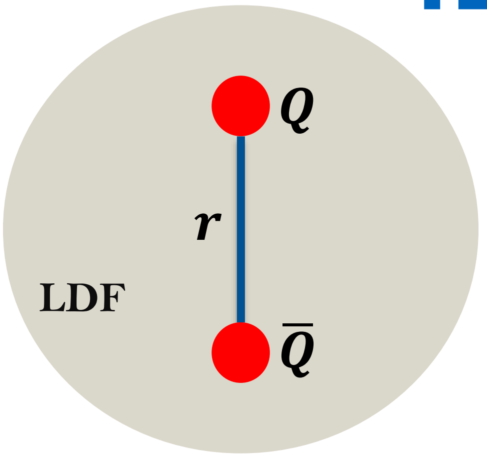
- Time-scale for dynamics of $Q\bar{Q}$: $\sim \dfrac{1}{mv^2} \gg \dfrac{1}{\Lambda_{\rm QCD}}$

Born-Oppenheimer (BO) Approximation

Braaten, Langmack, Smith Phys. Rev. D. 90, 014044 (2014)

Quantum #'s

- **BOEFT potentials ($V_\Gamma(r)$):** All effect of LDF (light quarks, gluons) **captured as** potential between 2 heavy quarks



- **Static limit (m⟶ ∞):** BOEFT potential are the static energies

  Cylindrical symmetry due to preferred quark-antiquark axis

- $V_\Gamma(r)$: **Γ** labelled by cylindrical symmetry ($D_{\infty h}$) representation (diatomic molecules):

  ✓ Absolute value of component of angular momentum of light d.o.f

  $$|\boldsymbol{r}\cdot\boldsymbol{K}| \equiv \Lambda = \mathbf{0}, \mathbf{1}, \mathbf{2}, \ldots\ldots\ldots (\textbf{or } \boldsymbol{\Sigma}, \boldsymbol{\Pi}, \boldsymbol{\Delta}, \boldsymbol{\Phi}, \ldots\ldots)$$

  ✓ Product of charge conjugation and parity ($\boldsymbol{CP}$):

  $$\boldsymbol{\eta} = \mathbf{+1}\ (\mathbf{g}),\ \mathbf{-1}\ (\boldsymbol{u})$$

  ✓ $\boldsymbol{\sigma}$: Eigenvalue of reflection about a plane containing static sources.

  $$\sigma = P\,(-1)^{K_{\text{light}}} = \pm 1$$

$$\Gamma \equiv \Lambda_\eta^\sigma$$

Braaten, Langmack, Smith Phys. Rev. D. 90, 014044 (2014)

- r⟶ 0: **Spherical symmetry restored**: Labelled by light d.o.f quantum #'s $\kappa = K^{PC}$.

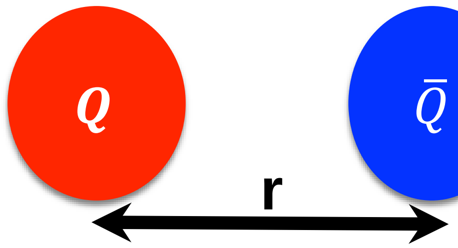
Brambilla, Pineda , Soto, and Vairo Nucl. Phys. B566 (2000)   Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015), 114019

Exotic Hadron

Brambilla, AM, Vairo arXiv 2312.xxxx

- Exotic hadron $(Q\bar{Q}X, QQX, \ldots\ldots)$, X is light d.o.f.

$Q\bar{Q}X$



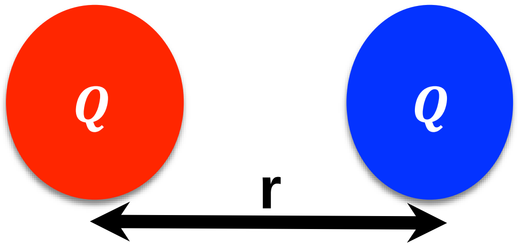
color: $3 \otimes \bar{3} = 1 \oplus 8$

X= gluon → **Hybrid**

X= $q\bar{q}$ → **Tetraquark / Molecule**

X= qqq → **Pentaquark / Molecule** and so on

QQX



color: $3 \otimes \bar{3} = \bar{3} \oplus 6$

X= q → **Double heavy baryon**

X= $\bar{q}\bar{q}$ → **Tetraquark**

X= $q\bar{q}q$ → **Pentaquark** and so on

$$\mathcal{O}_K(t,\, \mathbf{r},\, \mathbf{R}) \longrightarrow Z_{\Psi_K}(r,\, \Lambda_{\rm QCD})\Psi_K(t,\, \mathbf{r},\, \mathbf{R})$$

NRQCD BOEFT

BOEFT can address all these states with inputs from Lattice QCD on the BOEFT potentials

BOEFT: Hybrids

Brambilla, Lai, AM, Vairo arXiv:2212.09187

Brambilla, Krein, Castellà , Vairo Phys. Rev. D. 97, (2018)

- BOEFT Lagrangian:

$$L_{\mathrm{BOEFT}} = L_\Psi + L_{\Psi_{\kappa\lambda}}$$

Quarkonium:

$$L_\Psi = \int d^3\mathbf{R} \int d^3\mathbf{r}\, \mathrm{Tr}\left[\Psi^\dagger(\mathbf{r},\mathbf{R},t)\left(i\partial_t + \frac{\nabla_r^2}{m_Q} - V_\Psi(r)\right)\Psi(\mathbf{r},\mathbf{R},t)\right]$$

Trace over spin indices.

Hybrid:

$$L_{\Psi_{\kappa\lambda}} = \int d^3\mathbf{R} \int d^3\mathbf{r} \sum_{\kappa\lambda\lambda'} \mathrm{Tr}\left\{\Psi^\dagger_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)\left[i\partial_t - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda}\frac{\nabla_r^2}{m_Q}P^i_{\kappa\lambda'}\right]\Psi_{\kappa\lambda'}(\mathbf{r},\mathbf{R},t)\right\}$$

r: relative coordinate

R: COM coordinate

Hybrid potential:

$$V_{\kappa\lambda\lambda'}(r) \equiv P^{i\dagger}_{\kappa\lambda} V^{ij}_\kappa(r) P^j_{\kappa\lambda'} = V^{(0)}_{\kappa\lambda}(r)\delta_{\lambda\lambda'} + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m_Q} + \dots$$

Static potential

Incudes spin-dependent potentials

- Hybrid spin-dependent potentials: at order $1/m_Q$ (contrary to quarkonium $O(1/m_Q^2)$)

Brambilla, Lai, Segovia, Castellà, Vairo Phys. Rev. D. 101, (2020)

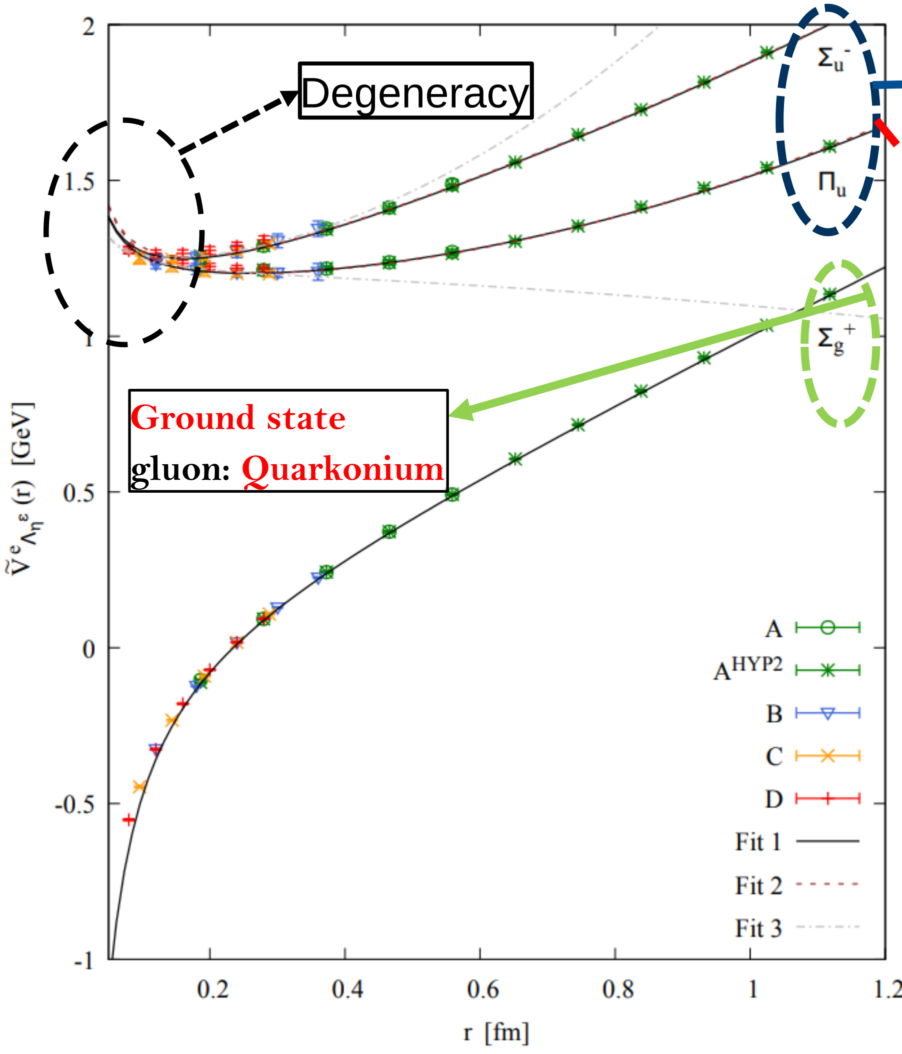
Brambilla, Lai, Segovia, Castellà, Vairo Phys. Rev. D. 99, (2019)

Soto, Valls, arXiv 2302.01765

$Q\bar{Q}$ pair: Static Energies

- **Static limit (m→ ∞):** heavy quarks are fixed in position. Interquark potential given by LDF energy.

Schlosser and Wagner Phys. Rev. D. 105, (2022)



First excited state gluon configuration: Hybrid

Gluonic operators characterizing hybrids

Λ_η^σ	K^{PC}	O_n
Σ_u^-	1^{+-}	$\hat{r}\cdot B,\ \hat{r}\cdot(D\times E)$
Π_u	1^{+-}	$\hat{r}\times B,\ \hat{r}\times(D\times E)$
$\Sigma_g^{+\prime}$	1^{--}	$\hat{r}\cdot E,\ \hat{r}\cdot(D\times B)$
Π_g	1^{--}	$\hat{r}\times E,\ \hat{r}\times(D\times B)$
Σ_g^-	2^{--}	$(\hat{r}\cdot D)(\hat{r}\cdot B)$
Π_g'	2^{--}	$\hat{r}\times((\hat{r}\cdot D)B + D(\hat{r}\cdot B))$
Δ_g	2^{--}	$(\hat{r}\times D)^i(\hat{r}\times B)^j + (\hat{r}\times D)^j(\hat{r}\times B)^i$
Σ_u^+	2^{+-}	$(\hat{r}\cdot D)(\hat{r}\cdot E)$
Π_u'	2^{+-}	$\hat{r}\times((\hat{r}\cdot D)E + D(\hat{r}\cdot E))$
Δ_u	2^{+-}	$(\hat{r}\times D)^i(\hat{r}\times E)^j + (\hat{r}\times D)^j(\hat{r}\times E)^i$

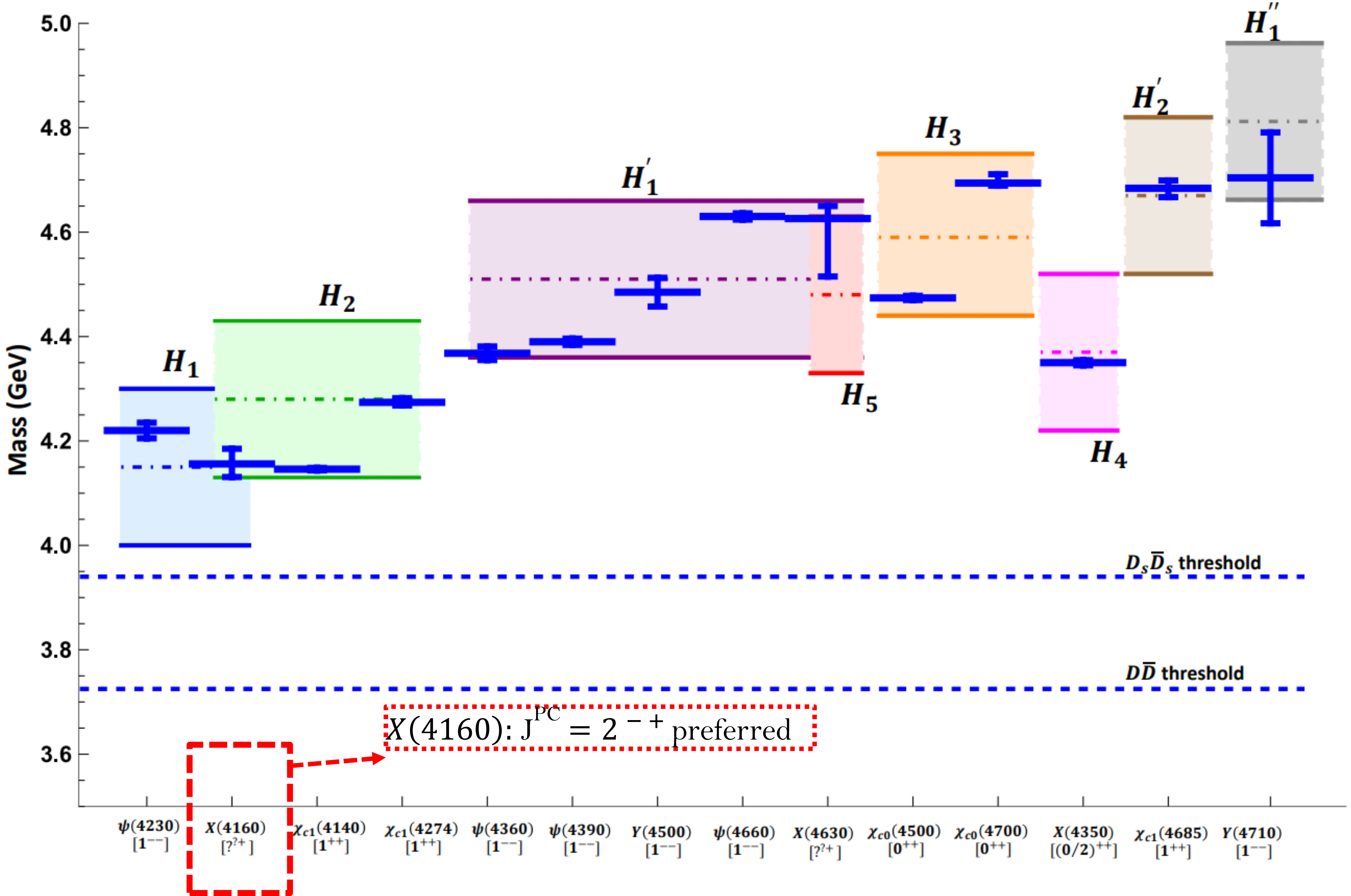
Foster and Micheal (UKQCD collaboration), Phys. Rev. D 59, 094509 (1999)

Brambilla, Pineda, Soto and Vairo, Rev. Mod. Phys 77, (2005)

Focus on these two for low lying hybrids

BOEFT: Hybrids

- **Charmonium hybrids**: comparison with experimental results:



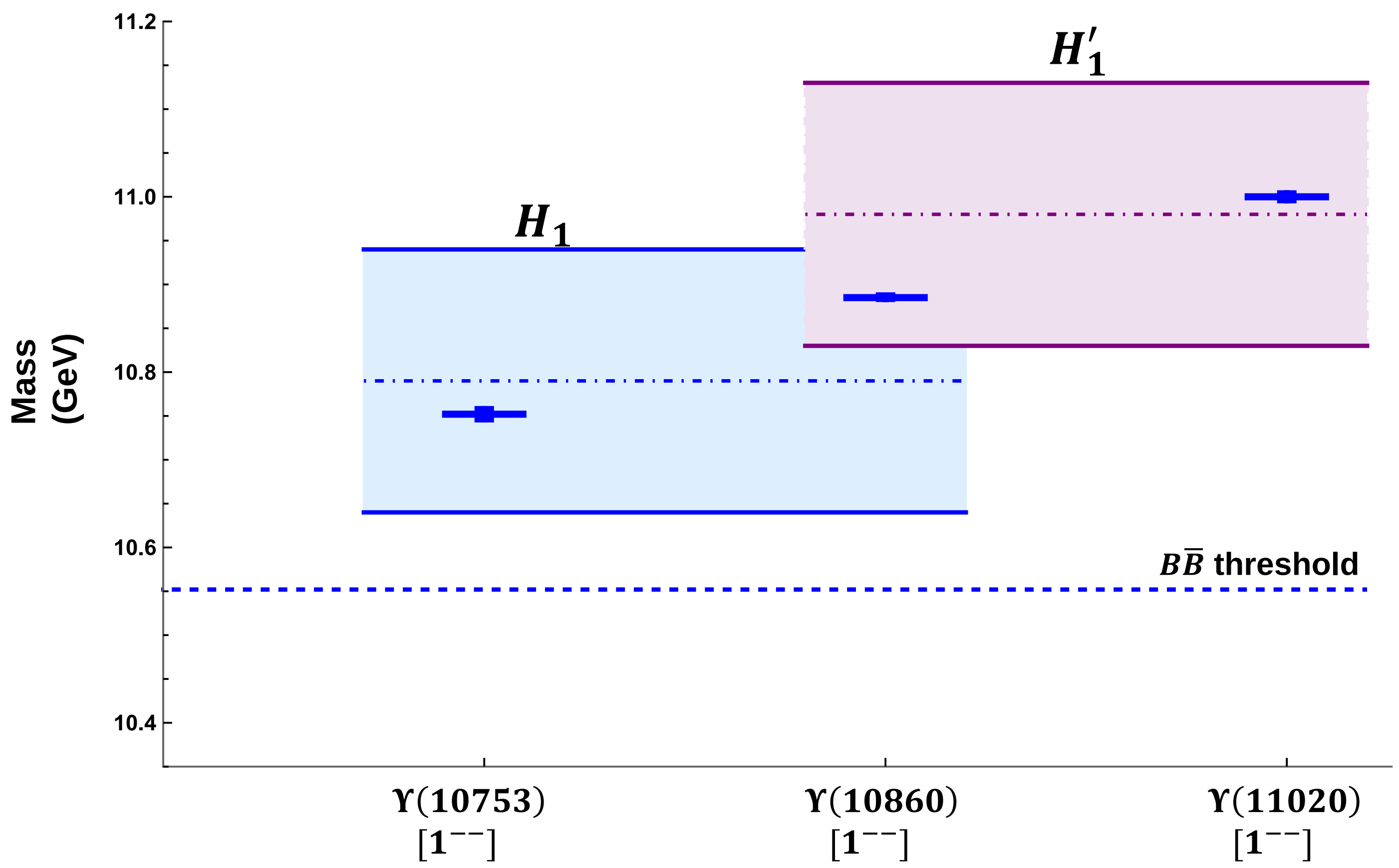
	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

PDG 2022

Brambilla, Lai, AM, Vairo arXiv:2212.09187

BOEFT: Hybrids

- **Bottomonium hybrids**: comparison with experimental results:



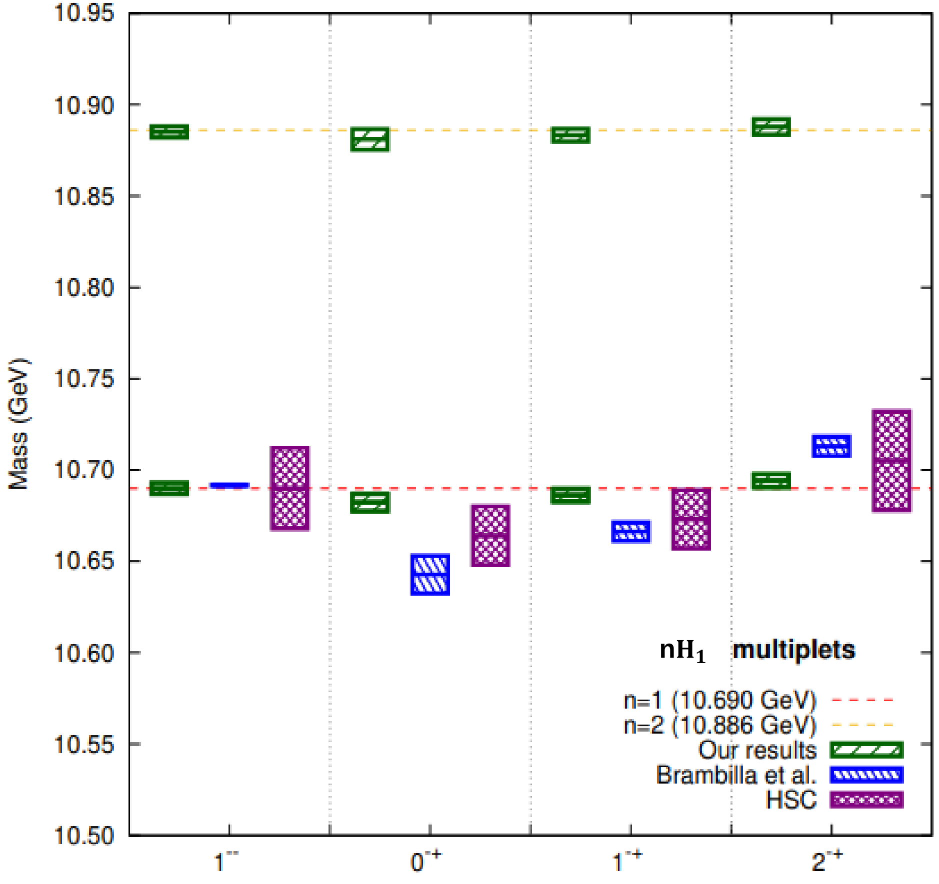
PDG 2022

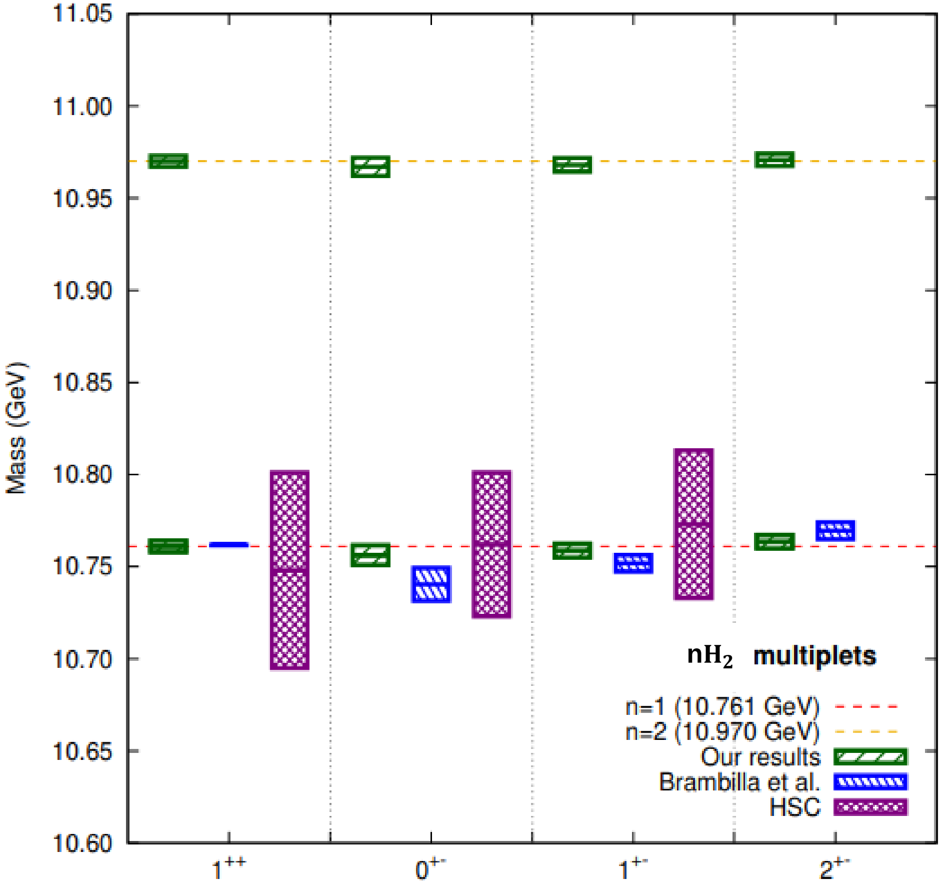
	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Brambilla, Lai, AM, Vairo arXiv:2212.09187

BOEFT: Hybrids

- **Including spin-dependent hybrid potentials:**





	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Our results refer to Soto & Valls arXiv 2302.01765

Brambilla, Lai, Segovia, Castellà, Vairo
Phys. Rev. D. 101, (2020)

Brambilla, Lai, Segovia, Castellà, Vairo
Phys. Rev. D. 99, (2019)

Soto, Valls, arXiv 2302.01765

Lattice data from
Hadron Spectrum collaboration JHEP 02 (2021) 214

BOEFT: Decays

- Semi-inclusive process: $H_m \to Q_n + X$; H_m: low-lying hybrid, Q_n: low-lying quarkonium (states below threshold) and X: light hadrons.

✓ ΔE: Large energy difference ⇒ $\Delta E \equiv E_{H_m} - E_{Q_n} \gtrsim 1$ **GeV**.

✓ Assume hierarchy of scales: $\Lambda_r \gg \Delta E \gg \Lambda_{QCD} \gg mv^2$

Energy scale related to decay

$$\Lambda_r^{-1} \equiv |\langle Q_n|\mathbf{r}|H_m\rangle|$$

- In BOEFT, all energy scales above **mv²** are integrated out. So, scale ΔE must be integrated out. This gives imaginary contribution to hybrid potential:

Optical theorem: $$\sum_n \Gamma(H_m \to Q_n) = -2\,\mathrm{Im}\,\langle H_m|V|H_m\rangle$$

DISCLAIMER!!!
Decay to open-flavor threshold states not accounted here.

- Imaginary piece of hybrid potential: determined from matching **pNRQCD** and **BOEFT** effective theories.

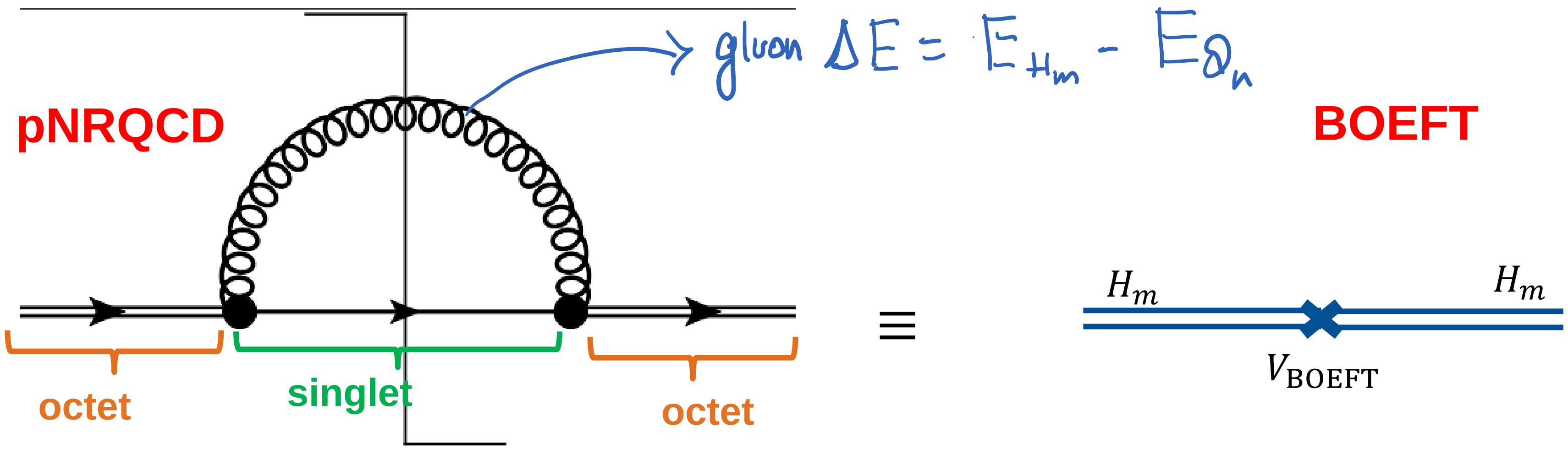
Brambilla, Lai, AM, Vairo arXiv:2212.09187

Hybrid Decays

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- $\boldsymbol{H_m \to Q_n + X}$ **:** ΔE (energy gap) >> Λ_{QCD}: gluon **resolves color configuration of** $Q\bar{Q}$ pair in hybrid and quarkonium:

Color configuration of $Q\bar{Q}$ pair:
Quarkonium -------> Singlet
Hybrid ------> Octet



- Quarkonium and Hybrid fields in short-distance limit $\mathbf{r} \to \mathbf{0}$ (matching condition)

$$S\left(\mathbf{r},\mathbf{R},t\right) \to Z_{\Psi}^{1/2}(\mathbf{r})\,\Psi(\mathbf{r},\mathbf{R},t),$$

$$P^{i\dagger}_{\kappa\lambda}O^a\left(\mathbf{r},\mathbf{R},t\right)G^{ia}_{\kappa}(\mathbf{R},t) \to Z_{\kappa}^{1/2}(\mathbf{r})\,\Psi_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)$$

singlet (S) and octet (O)

$\boldsymbol{G_\kappa^{ia}}$**: Gluon fields**

Hybrid Decays

- Spin-conserving decay due to $\boldsymbol{r}\cdot\boldsymbol{E}$ term :

$$\Gamma(H_m \to Q_n) = \frac{4\alpha_s(\Delta E)\,T_F}{3N_c}\, T^{ij}\,(T^{ij})^\dagger\,\Delta E^3$$

DISCLAIMER!!!
Decay to open-flavor threshold states not accounted here.

$$|S_H = 1 > ---\to |S_Q = 1 >$$
$$|S_H = 0 > ---\to |S_Q = 0 >$$

$$T^{ij} \equiv \langle H_m | r^j | Q_n \rangle = \int d^3\mathbf{r}\, \Psi^{i\dagger}_{(m)}(\mathbf{r})\, r^j\, \Phi^{Q\bar{Q}}_{(n)}(\mathbf{r})$$

$$\langle H_m | \boldsymbol{r} | Q_n \rangle = \sqrt{T^{ij}(T^{ij})^\dagger}$$

$\Psi^i_{(m)}$: Hybrid wf

Φ^Q_n : Quarkonium wf

R. Oncala, J. Soto, Phys. Rev. D96, 014004 (2017).

J. Castellà, E. Passemar, Phys. Rev. D104, 034019 (2021)

$|\chi_H\rangle$: Hybrid spin wf

$|\chi_Q\rangle$: Quarkonium spin wf

- Spin-flipping decay due to $\boldsymbol{S}.\boldsymbol{B}$ term:

$$|S_H = 1 > ---\to |S_Q = 0 >$$
$$|S_H = 0 > ---\to |S_Q = 1 >$$

$$T^{ij} \equiv \langle H_m | \left(S_1^j - S_2^j\right) | Q_n \rangle = \left[\int d^3\mathbf{r}\, \Psi^{i\dagger}_{(m)}(\mathbf{r})\, \Phi^Q_{(n)}(\mathbf{r})\right] \langle \chi_H | \left(S_1^j - S_2^j\right) | \chi_Q \rangle$$

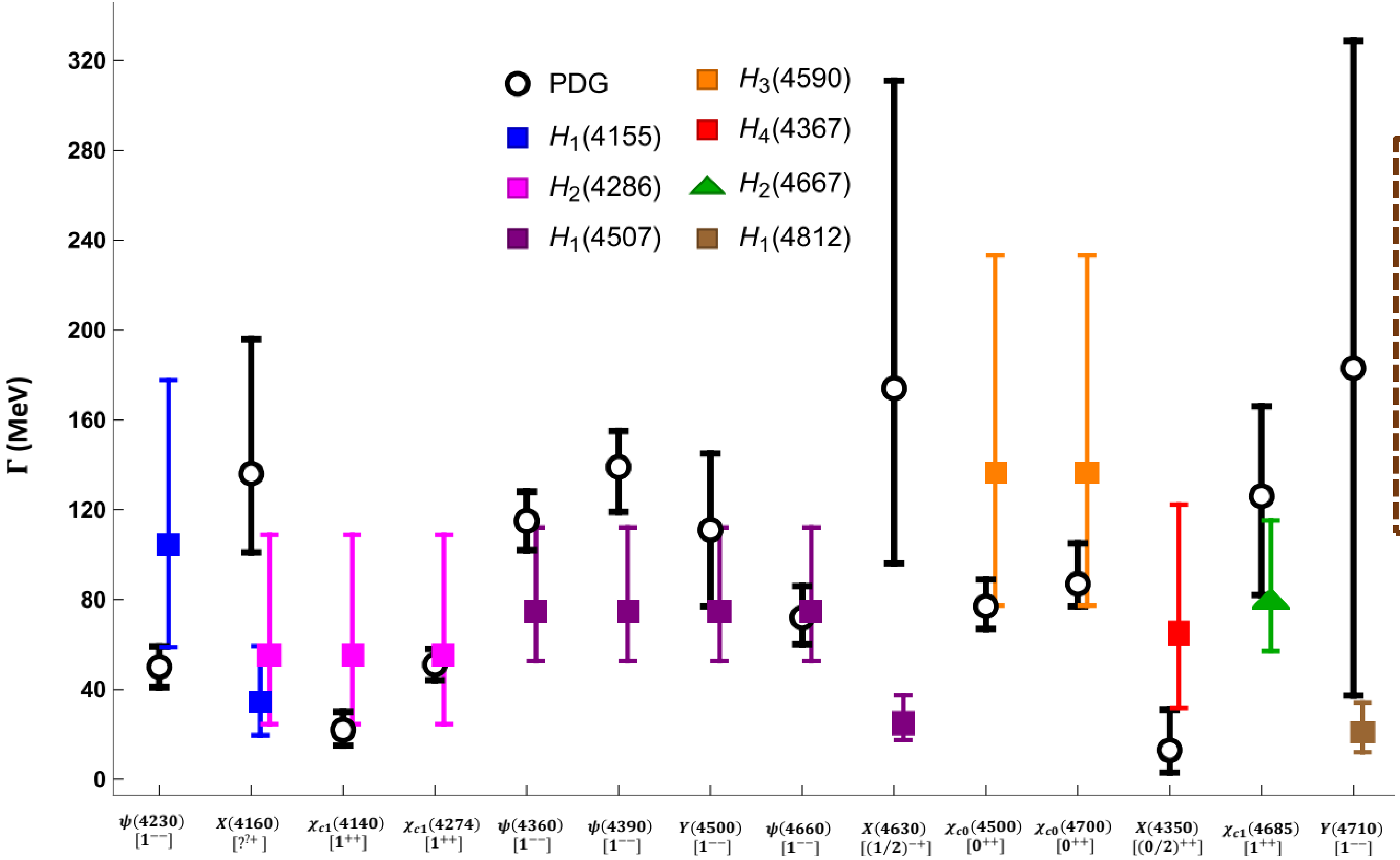
- Depends on overlap of quarkonium and hybrid wavefunctions.
- Based on hierarchy: $\Lambda_r \gg \Delta E \gg \Lambda_{QCD}$

Brambilla, Lai, AM, Vairo arXiv:2212.09187

Results

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Comparison: charm exotic states with corresponding charmonium hybrid state:



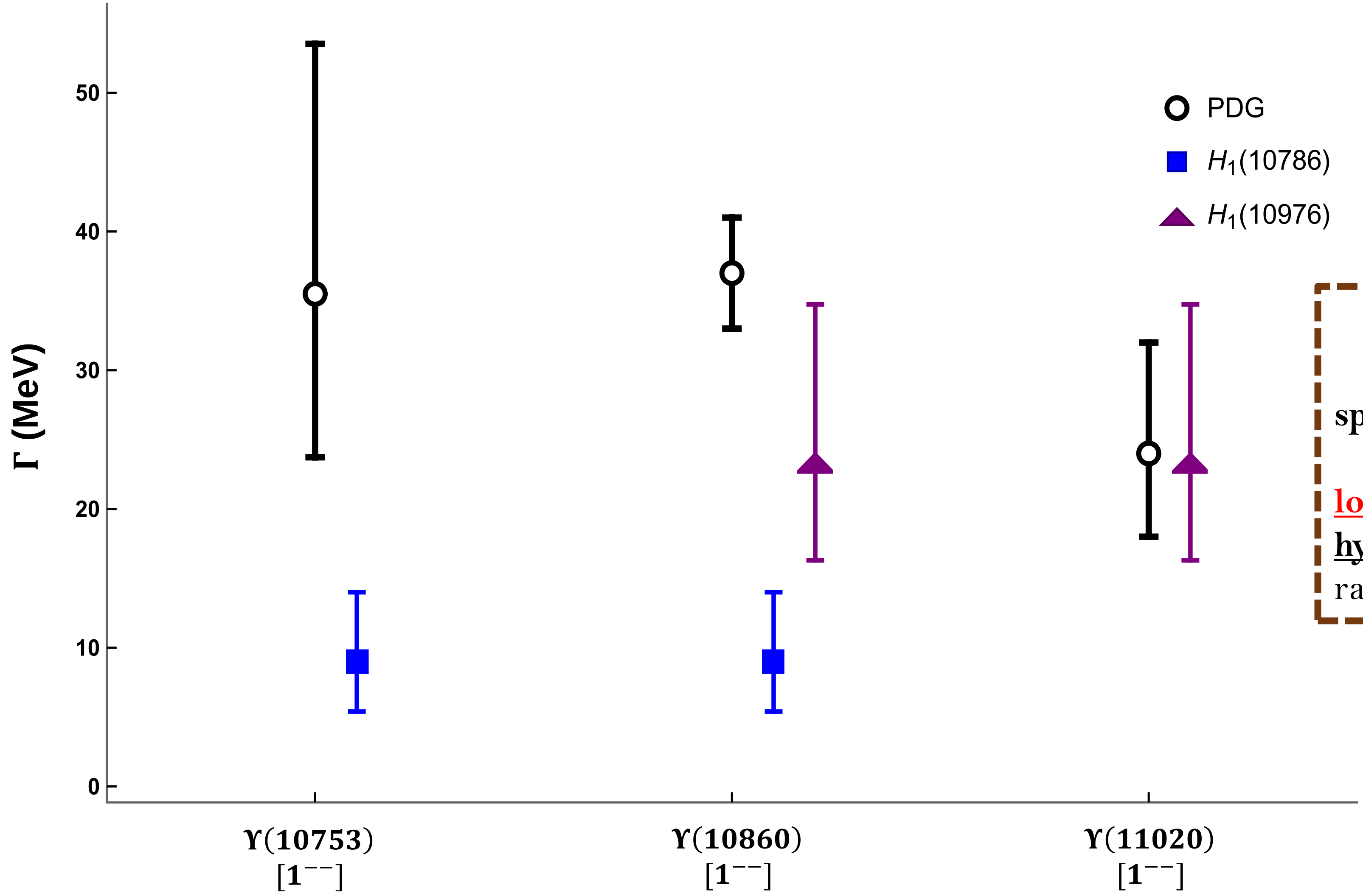
$$H_m \to Q_n + X$$

spin-conserving + spin-flipping decays
=
lower bound on the total decay widths of hybrids which is compared with inclusive rate of physical states in PDG.

Results

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Comparison: bottom exotic states with corresponding bottomonium hybrid state:



$$H_m \to Q_n + X$$

spin-conserving + **spin-flipping decays**

=

lower bound on the total decay widths of hybrids which is compared with inclusive rate of physical states in PDG.

Hybrid Decays

- **Hybrid decays to meson-pair threshold states:** $\Delta E \lesssim \Lambda_{QCD}$

Conventional Wisdom: Hybrid decays to two S-wave mesons forbidden! $H_m \nrightarrow D^{(*)}\bar{D}^{(*)}$

Kou & Pene, Phys Lett B 631 (2005) Page, Phys Lett B 407 (1997) Farina, Tecocoatzi, Giachino, Santopinto & Swanson, Phys Rev D 102 (2020)

Born Oppenheimer quantum numbers for hybrids and ground state meson pair **does allow for decay to two s-wave mesons**. Bruschini 2306.17120

	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Most quarkonium hybrids can decay into pair of s-wave mesons !!!

forbidden for decay into pair of s-wave mesons

Recent lattice computation for $c\bar{c}$ hybrid 1^{-+} **decay** to

$D_1\bar{D}$: 258(133) MeV $D^*\bar{D}$: 88(18) MeV Shi et al 2306.12884

$D^*\bar{D}^*$: 150(118) MeV

Computing these decays of hybrid to threshold states in BOEFT framework ?

Hybrid: Summary

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Hybrids ($Q\bar{Q}g$): Color singlet state of color octet $Q\bar{Q}$ + gluon. (Q = c, b)

✓ **Isoscalar neutral mesons (Isospin=0)**

✓ Candidates for hybrids based on **mass, quantum numbers,** and **decays** to **quarkonium**:

Charm sector:

- $\boldsymbol{X(4160)}$: could be **charm hybrid** $\boldsymbol{H_1[2^{-+}](4155)}$.
- $\mathbf{X(4630)}$: could be **charm hybrid** $\boldsymbol{H_1[(1/2^{-+})](4507)}$.
- $\boldsymbol{\psi(4390)}$: could be **charm hybrid** $\boldsymbol{H_1[1^{--}](4507)}$.
- $\boldsymbol{\psi(4710)}$: could be **charm hybrid** $\boldsymbol{H_1[(1^{--})](4812)}$.
- $\boldsymbol{\chi_{c1}(4685)}$: could be **charm hybrid** $\boldsymbol{H_2[(1^{++})](4667)}$.

Bottom sector:

- $\boldsymbol{\Upsilon(10753)}$: could be **bottom hybrid** $\boldsymbol{H_1[(1^{--})](10786)}$.

DISCLAIMER!!!

All the above interpretation can differ accounting for decays to meson-pair threshold states and hybrid-quarkonium mixing.

$QQ\bar{q}\bar{q}$

First doubly charmed tetraquark seen by LHCb

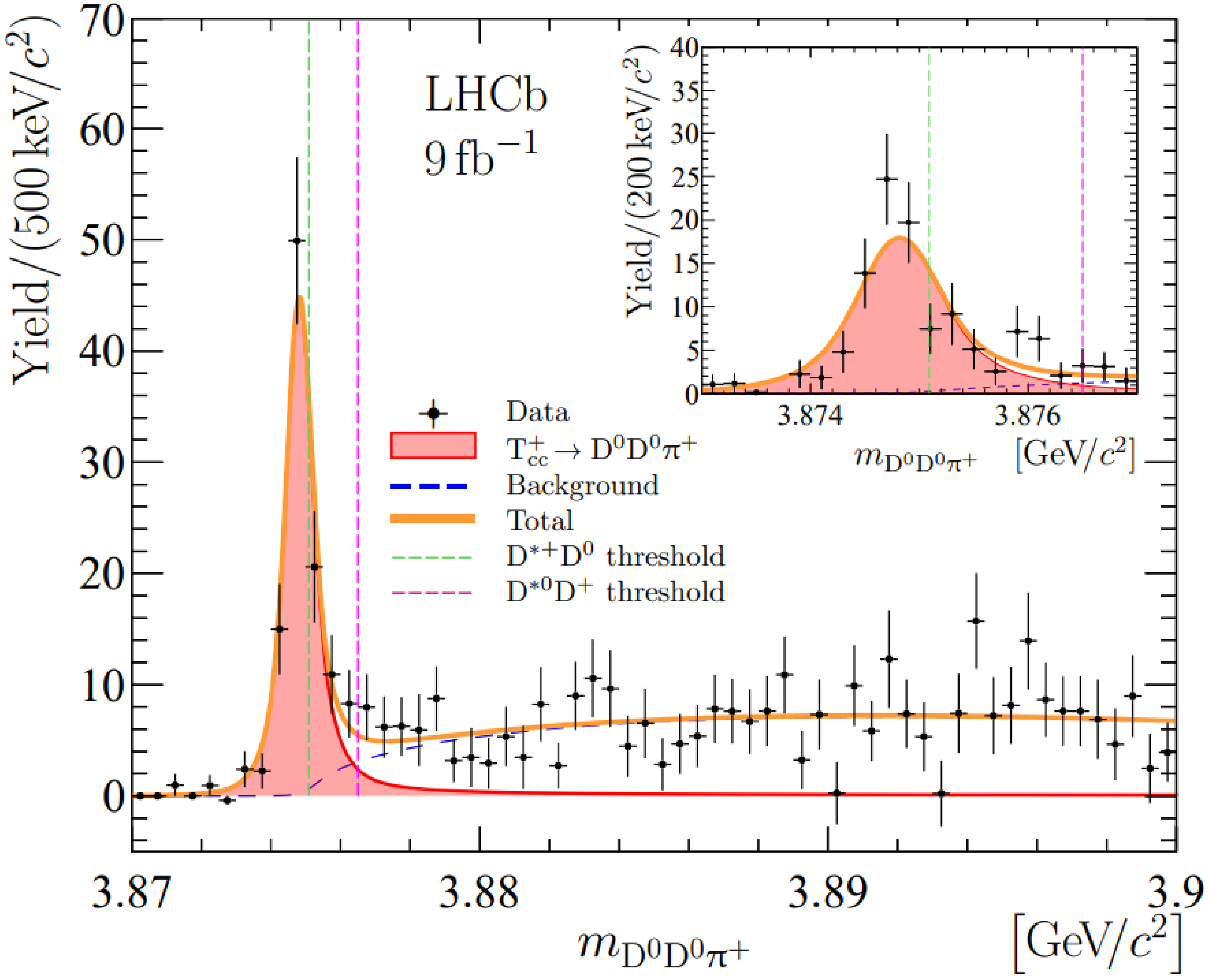
$$T_{cc}^{+}(3875) \to D^0 D^0 \pi^+$$

- Exotic quark content $cc\bar{u}\bar{d}$
- Consistent with **isoscalar** with **J^P=1$^+$**

Mass below $D^{*+}D^0$ threshold and very narrow

$$m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0}) = -0.27 \pm 0.06 \text{ MeV}.$$

$T_{cc}^{+}(3875)$



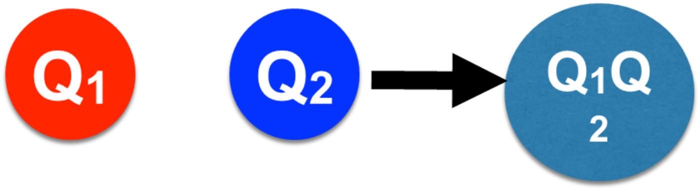
LHCb (Nature Phys. 18 (2022) 7, 751; Nature Comm. 13 (2022) 3351)

$QQ\bar{q}\bar{q}$

Brambilla, AM, Vairo arXiv 2312.xxxx

doubly heavy core

spin: $1/2 \otimes 1/2 = \boxed{0 \oplus 1}$



color: $3 \otimes 3 = 6 \oplus \boxed{3^*}$

flavor: $\{Q_1Q_2\}$ $[Q_1Q_2]$

J^P: 1^+ 0^+

light antiquarks

$\{qq'\}$, 1^+ $[qq']$, 0^+

Defines the Born-Oppenheimer static potentials $\Sigma_g^+, \{\Sigma_g^-, \Pi_g\}$

doubly heavy tetraquarks

QQ color state	Light spin K^{PC}	Static energies	Isospin I	l	J^P $S_Q=0$	J^P $S_Q=1$
anti-triplet $\bar{3}$	0^+	$\{\Sigma_g^+\}$	0	0	—	1^+
				1	1^-	—
	1^+	$\{\Sigma_g^-, \Pi_g\}$	1	0	0^-	—
				1	1^-	$(0,1,2)^+$

J^P for T_{cc}^+

Limited lattice inputs available on Born-Oppenheimer static potentials $\Sigma_g^+, \{\Sigma_g^-, \Pi_g\}$

$QQ\bar{q}\bar{q}$

Assuming attractive Coulomb potential forms a compact diquark in 3* color state, use

1. Heavy quark-diquark symmetry (HQDQ)
2. pNRQCD effective theory

} predict Masses of doubly heavy tetraquark states

Braaten, He, AM, arXiv:2006.08650

- **HQDQ symmetry:** Light quark only sees the diquark as point source $\overline{\mathbf{3}}$ color charge. Cannot resolve individual heavy quarks in the diquarks.

- Relevant terms for Triplet field $\mathbf{T}$ in pNRQCD Lagrangian:

  - **kinetic term:** $T^\dagger \left[\frac{\boldsymbol{D}_R^2}{2m_R} + \frac{\boldsymbol{\nabla}_r^2}{2m_r} \right] T$

    $m_r = \frac{m_1 m_2}{m_1 + m_2} \qquad m_R = m_1 + m_2$

  - **spin dependent term:** $T^\dagger(\boldsymbol{S}_1/m_{Q_1} + \boldsymbol{S}_2/m_{Q_2}) \cdot \boldsymbol{B}\, T$

  - **orbital angular momentum term (irrelevant for L=0 diquark):**

    $(m_{Q_1}^2 + m_{Q_2}^2)/[m_{Q_1} m_{Q_2}(m_{Q_1} + m_{Q_2})]\; T^\dagger \boldsymbol{L} \cdot \boldsymbol{B}\, T$

[Brambilla, Vairo, and Rosch, Phys. Rev. D72, 034021 (2005)]

$QQ\bar{q}\bar{q}$

Braaten, He, AM, arXiv:2006.08650

- Hamiltonian for doubly heavy hadrons (QQq, $QQ\bar{q}\bar{q}$) :

$$H_\ell^{Q_1Q_2} = (m_{Q_1} + m_{Q_2}) + \mathcal{E}_{\ell,Q_1Q_2} + \frac{\mathcal{S}_{\ell,Q_1Q_2}}{8\mu_{Q_1Q_2}} \boldsymbol{S}\cdot\boldsymbol{j}_\ell.$$

$$\mathcal{E}_{\ell,Q_1Q_2} = \mathcal{E}_{Q_1Q_2} + \mathcal{E}_\ell + \frac{\mathcal{K}_\ell}{2\,(m_{Q_1} + m_{Q_2})}$$

HQDQ symmetry: $QQ\bar{q}\bar{q} \leftrightarrow \bar{Q}\bar{q}\bar{q}$ $\quad$ $QQq \leftrightarrow \bar{Q}q$

- HQDQ symmetry can **relate the parameters for tetraquarks** with **parameters** for doubly heavy baryons Q_1Q_2q, heavy baryons $\bar{Q}\bar{q}\bar{q}$ and mesons $\bar{Q}q$

cc

$$\mathcal{E}_{\ell,cc} = \mathcal{E}_{\ell',cc} + \frac{m_b - 2m_c}{2(m_b - m_c)}(\mathcal{E}_{\ell,\bar{c}} - \mathcal{E}_{\ell',\bar{c}}) + \frac{m_b}{2(m_b - m_c)}(\mathcal{E}_{\ell,\bar{b}} - \mathcal{E}_{\ell',\bar{b}})$$

$$m_{s,\ell,cc} = \frac{m_b - 2m_c}{2(m_b - m_c)} m_{s,\ell,\bar{c}} + \frac{m_b}{2(m_b - m_c)} m_{s,\ell,\bar{b}}$$

$$\mathcal{S}_{\ell,QQ} = \mathcal{S}_{\ell,\bar{Q}} = \mathcal{S}_{\bar{\ell},Q}.$$

bb

$$\mathcal{E}_{\ell,bb} = \mathcal{E}_{\ell',bb} - \frac{m_c}{2(m_b - m_c)}(\mathcal{E}_{\ell,\bar{c}} - \mathcal{E}_{\ell',\bar{c}}) + \frac{2m_b - m_c}{2(m_b - m_c)}(\mathcal{E}_{\ell,\bar{b}} - \mathcal{E}_{\ell',\bar{b}})$$

$$m_{s,\ell,bb} = -\frac{m_c}{2(m_b - m_c)} m_{s,\ell,\bar{c}} + \frac{2m_b - m_c}{2(m_b - m_c)} m_{s,\ell,\bar{b}},$$

$$\mathcal{S}_{\ell,QQ} = \mathcal{S}_{\ell,\bar{Q}} = \mathcal{S}_{\bar{\ell},Q}.$$

$Q_1Q_2q, \bar{Q}\bar{q}\bar{q}$ and $\bar{Q}q$ parameters determined by minimizing χ^2 fit to lattice data and PDG

$QQ\bar{q}\bar{q}$

Results for masses of doubly heavy tetraquarks:

Braaten, He, AM, arXiv:2006.08650

cc & bb tertaquarks

flavor	J^P	Eichten-Quigg	this work	threshold
$cc[\bar{u}\bar{d}]$	1^+	3978	3963 ± 13	3875
$cc[\bar{s}\bar{u}]$	1^+	4156	4158 ± 15	3975
$cc\{\bar{u}\bar{d}\}$	$0^+, 1^+, 2^+$	$4146 + (0, 21, 64)$	$4136 + (0, 22, 66) \pm 13$	$3734 + (0, 141, 0)$
$cc\{\bar{s}\bar{u}\}$	$0^+, 1^+, 2^+$		$4268 + (0, 22, 66) \pm 13$	$3833 + (0, 142, 0)$
$cc\,\bar{s}\bar{s}$	$0^+, 1^+, 2^+$		$4400 + (0, 22, 66) \pm 15$	$3937 + (0, 144, 0)$
$bb[\bar{u}\bar{d}]$	1^+	**10482**	$\mathbf{10476 \pm 25}$	10604
$bb[\bar{s}\bar{u}]$	1^+	**10643**	$\mathbf{10655 \pm 25}$	10692
$bb\{\bar{u}\bar{d}\}$	$0^+, 1^+, 2^+$	$10674 + (0, 7, 21)$	$10672 + (0, 7, 21) \pm 25$	$10559 + (0, 45, 0)$
$bb\{\bar{s}\bar{u}\}$	$0^+, 1^+, 2^+$		$10793 + (0, 7, 21) \pm 25$	$10646 + (0, 45, 0)$
$bb\,\{\bar{s}\bar{s}\}$	$0^+, 1^+, 2^+$		$10915 + (0, 7, 21) \pm 25$	$10734 + (0, 49, 0)$

No stable cc tetraquarks!! Prediction based on HQDQ symmetry incompatible with $T_{cc}^+(3875)$

Stable Tetraquarks: Masses below Strong decay threshold

HQDQ symmetry captures short-distance limit of Born-Oppenheimer potentials

Going beyond **HQDQ symmetry** will require solving Schrodinger equation (multi-channel ?) with full information on Born-Oppenheimer potentials $\Sigma_g^+, \{\Sigma_g^-, \Pi_g\}$

Brambilla, AM, Vairo arXiv 2312.xxxx

Summary/Outlook

- Born-Oppenheimer EFT: Tool based on QCD and Born-Oppenheimer approximation to study Exotic states.

- BOEFT: model-independent & systematic framework with inputs from lattice QCD.

- BOEFT prediction for hybrids:

Charm sector:

- $\boldsymbol{X(4160)}$: could be **charm hybrid $H_1[2^{-+}](4155)$.**
- $\mathbf{X(4630)}$: could be **charm hybrid $H_1[(1/2^{-+})](4507)$.**
- $\boldsymbol{\psi(4390)}$: could be **charm hybrid $H_1[1^{--}](4507)$.**
- $\boldsymbol{\psi(4710)}$: could be **charm hybrid $H_1[(1^{--})](4812)$.**
- $\boldsymbol{\chi_{c1}(4685)}$: could be **charm hybrid $H_2[(1^{++})](4667)$.**

Bottom sector:

- $\boldsymbol{\Upsilon(10753)}$: could be **bottom hybrid $H_1[(1^{--})](10786)$.**

- Doubly heavy teraquark $QQ\bar{q}\bar{q}$ states can be addressed in BOEFT framework with inputs from lattice QCD required for Born-Oppenheimer potentials $\boldsymbol{\Sigma_g^+}, \{\boldsymbol{\Sigma_g^-}, \boldsymbol{\Pi_g}\}$.

- Stay tuned for results.!!

Work in progress based on BOEFT

- Addressing hybrid-quarkonium mixing

Hybrid states in the same energy range as quarkonium can mix (same quantum #'s). O (1/m) term in BOEFT.

Impact on decay: $H_m \leftrightarrow Q'_m \to Q_n + X$; n & m denotes quantum #'s

Ex. $H_1 \left[1^{--}\right](4155) \leftrightarrow c\bar{c}\left[1^{--}\right](3S)$

Effect on decay: $H_m \leftrightarrow Q^{'}_m \to (\eta_c, J/\psi, \cdots) + (\gamma, \cdots)$

- Understanding the tetraquark **spectrum** both **doubly heavy** and **quarkonium tetraquarks**.

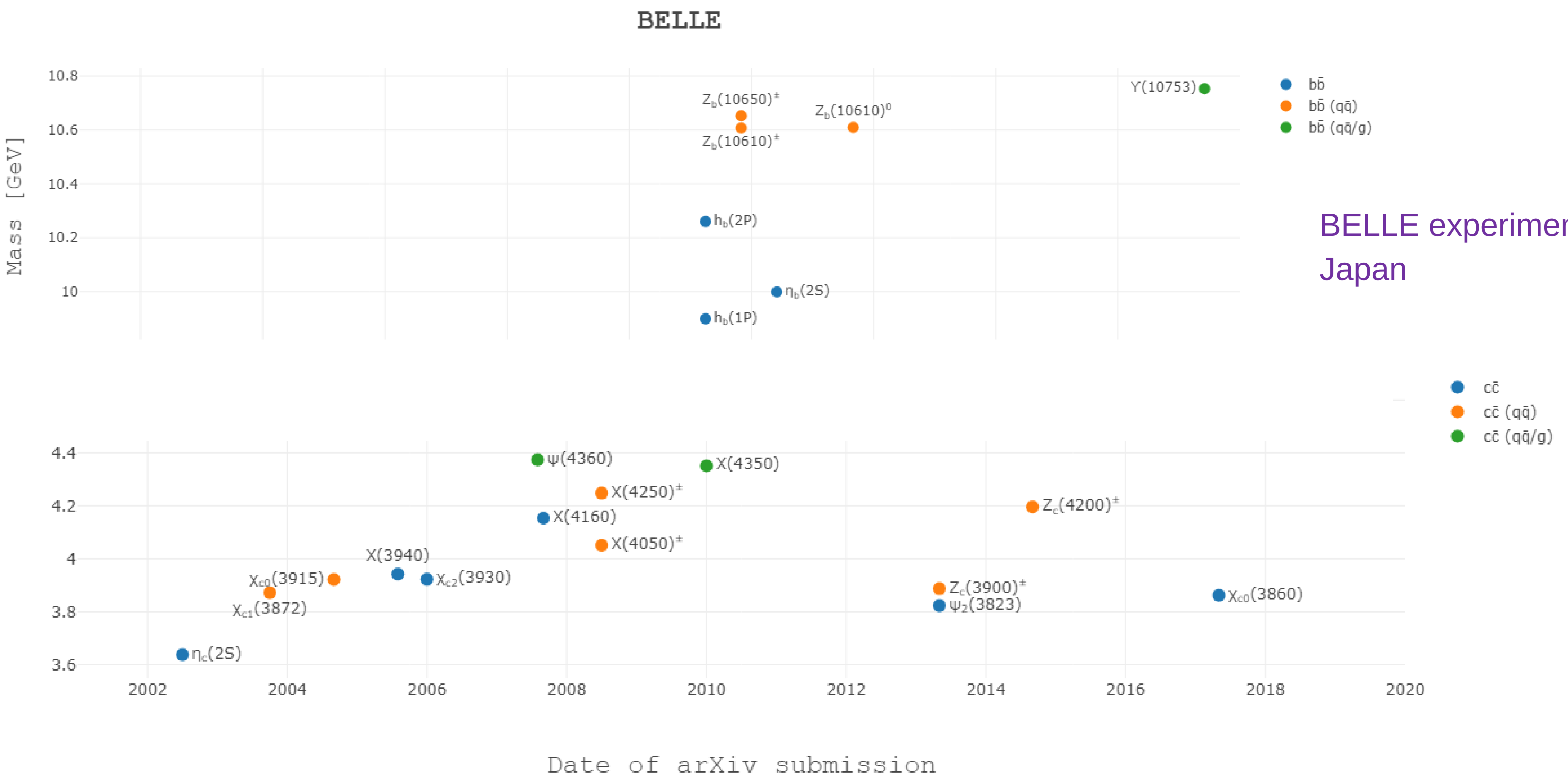
- Computing quarkonium hybrid **decays** to **pair of heavy mesons** by solving **multi-channel Schrodinger equation**.

BOEFT : Aim is to have unified framework for XYZ exotics !!!.

Backup Slides

New Hadrons: XYZ mesons

https://qwg.ph.nat.tum.de/exoticshub/

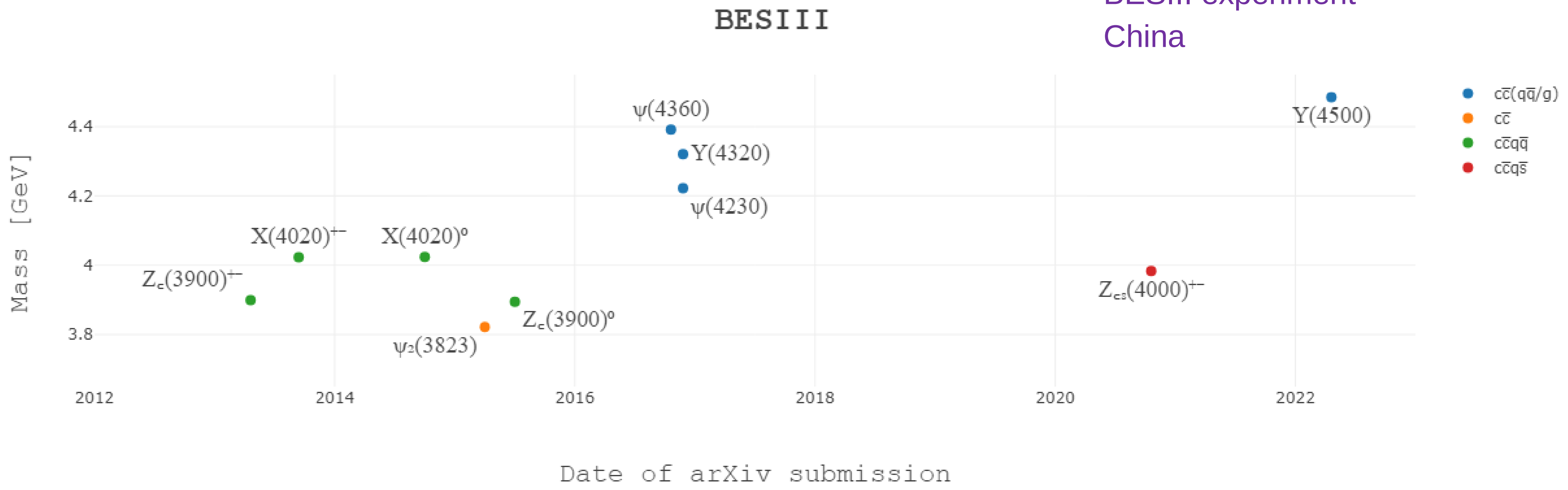


BELLE experiment
Japan

New Hadrons: XYZ mesons

https://qwg.ph.nat.tum.de/exoticshub/

BESIII experiment
China



Observation of many newer heavy hadrons are expected in the near future !!

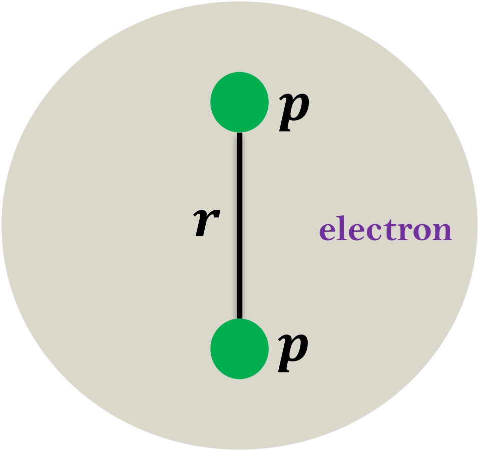
BIG QUESTION:

How to understand XYZ states ? Can we theoretically predict the spectrum ??

Born-Oppenheimer Philosophy

- Sharp difference between time or energy scales of heavy & light degrees of freedom.

Ex. H_2^+ molecule: 2 protons & 1 electron. $m_p \sim 1\,\mathrm{GeV} \gg m_e \sim 0.5\,\mathrm{MeV}$



Protons (nuclei) move very slowly compared to electrons and can be considered **static** (fixed) when considering the motion of the electrons

Electrons instantaneously adjust as **r** changes

1. Solve electron Schrödinger eq. for fixed **r**

$$H_{\mathrm{el}}(\mathbf{r})\,|\psi_{\mathrm{el}}^i;\mathbf{r}\rangle = E_{\mathrm{el}}^i(r)\,|\psi_{\mathrm{el}}^i;\mathbf{r}\rangle$$

2. Solve nuclei (proton) Schrödinger eq. with $\boldsymbol{E_{\mathrm{el}}^i(r)}$ as **potential**.

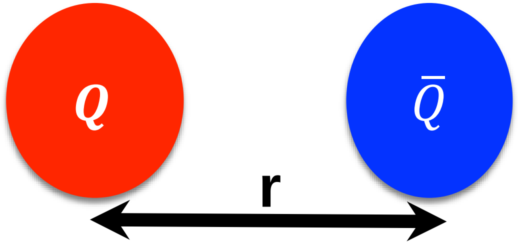
QCD states with 2-heavy quarks (XYZ mesons): analogous of molecules in atomic systems !!!

Heavy quarks ↔ nuclei **Gluons & light quarks ↔ electrons**

Lattice input required for gluon and light-quark energies

Exotic Hadron

- Consider an exotic hadron $\boldsymbol{Q\bar{Q}X}$, $\boldsymbol{X}$: any combination of light quark and gluons to obtain color singlet hadron



spin: $1/2 \otimes 1/2 = 0 \oplus 1$

color: $3 \otimes \bar{3} = 1 \oplus 8$

Let's say $\boldsymbol{Q\bar{Q}}$ pair in **octet color**, then the operator characterizing $\boldsymbol{Q\bar{Q}X}$

$$\mathcal{O}_K(t,\,\mathbf{r},\,\mathbf{R}) \longrightarrow \mathbf{Z_{H_K}}(\mathbf{r})\mathbf{O^a}(\mathbf{t},\,\mathbf{r},\,\mathbf{R})\mathbf{H^a_K}(\mathbf{t},\,\mathbf{R}) \longrightarrow \mathbf{Z_{\Psi_K}}(\mathbf{r},\,\boldsymbol{\Lambda}_{\mathrm{QCD}})\boldsymbol{\Psi}_\mathbf{K}(\mathbf{t},\,\mathbf{r},\,\mathbf{R})$$

NRQCD

Weakly coupled pNRQCD

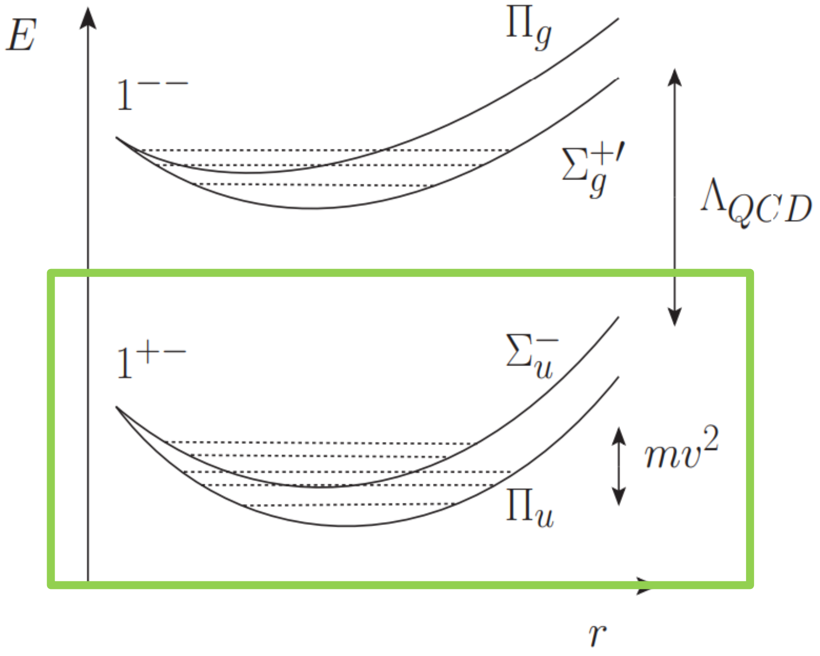
strongly coupled pNRQCD / BOEFT

Note: Operators $\mathbf{H^a_K}$: labelled by light d.o.f quantum number $\mathbf{K}$. Characterizes BOEFT potentials.

- In case of quarkonium, $\boldsymbol{Q\bar{Q}}$ pair in color singlet and $\mathrm{K}^{\mathrm{PC}} = 0^{++}$ (scalar), $H_{0^{++}} = \mathbb{I}$

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015) Castellà , Soto Phys. Rev. D. 102, (2020) Brambilla, AM, Vairo arXiv 230x.xxxx

BOEFT: Hybrids



- Degeneracy at short distances $\boldsymbol{r \to 0}$, mixes hybrid states corresponding to Σ_u^- and Π_u potential
- Coupled Schrödinger Eq: Dynamics of $Q\bar{Q}$ at scale $mv^2 \ll \Lambda_{QCD}$

Schrödinger equation

$$\left[-P^{i\dagger}_{\kappa\lambda}\frac{\nabla_r^2}{m}P^i_{\kappa\lambda'} + V_{\kappa\lambda\lambda'}(r)\right]\Psi^n_{\kappa\lambda'}(\boldsymbol{r}) = E_n\Psi^n_{\kappa\lambda}(\boldsymbol{r})$$

$\kappa = 1^{+-}$

$\lambda = 0, \pm 1$

Hybrid Spectrum:

Multiplet	J^{PC}	$M_{c\bar{c}g}$	$M_{b\bar{b}g}$
H_1	$\{1^{--}, (0,1,2)^{-+}\}$	4155	10786
H_1'		4507	10976
H_1''		4812	11172
H_2	$\{1^{++}, (0,1,2)^{+-}\}$	4286	10846
H_2'		4667	11060
H_2''		5035	11270
H_3	$\{0^{++}, 1^{+-}\}$	4590	11065
H_3'		5054	11352
H_3''		5473	11616
H_4	$\{2^{++}, (1,2,3)^{+-}\}$	4367	10897
H_5	$\{2^{--}, (1,2,3)^{-+}\}$	4476	10948

Λ- doubling: opposite parity states non-degenerate.

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)   Brambilla, Lai, AM, Vairo arXiv:2212.09187

Exotic: Hybrid candidates

State (PDG)	State (Former)	M (MeV)	Γ (MeV)	J^{PC}	Decay modes
$\chi_{c1}(4140)$	$X(4140)$	4146.5 ± 3.0	19^{+7}_{-5}	1^{++}	$\phi J/\psi$
$X(4160)$		4153^{+23}_{-21}	136^{+60}_{-35}	$?^{??}$	$\phi J/\psi$, $D^* \bar{D}^*$
$\psi(4230)$	$Y(4230)$ $Y(4260)$	4222.7 ± 2.6	49 ± 8	1^{--}	$\pi^+\pi^- J/\psi$, $\omega \chi_{c0}(1P)$, $\pi^+\pi^- h_c(1P)$
$\chi_{c1}(4274)$	$Y(4274)$	4286^{+8}_{-9}	51 ± 7	1^{++}	$\phi J/\psi$
$X(4350)$		$4350.6^{+4.7}_{-5.1}$	13^{+18}_{-10}	$(0/2)^{++}$	$\phi J/\psi$
$\psi(4360)$	$Y(4360)$ $Y(4320)$	4372 ± 9	115 ± 13	1^{--}	$\pi^+\pi^- J/\psi$, $\pi^+\pi^- \psi(2S)$
$\psi(4390)$[a]	$Y(4390)$	4390 ± 6	139^{+16}_{-20}	1^{--}	$\eta J/\psi$, $\pi^+\pi^- h_c(1P)$
$\chi_{c0}(4500)$	$X(4500)$	4474 ± 4	77^{+12}_{-10}	0^{++}	$\phi J/\psi$
$Y(4500)$[b]		4484.7 ± 27.5	111 ± 34	1^{--}	
$X(4630)$[c]		4626^{+24}_{-111}	174^{+137}_{-78}	$?^{?+}$	$\phi J/\psi$
$\psi(4660)$	$Y(4660)$ $X(4660)$	4630 ± 6	72^{+14}_{-12}	1^{--}	$\pi^+\pi^- \psi(2S)$, $\Lambda_c^+ \bar{\Lambda}_c^-$, $D_s^+ D_{s1}(2536)$
$\chi_{c1}(4685)$[d]		4684^{+15}_{-17}	126^{+40}_{-44}	1^{++}	$\phi J/\psi$
$\chi_{c0}(4700)$	$X(4700)$	4694^{+17}_{-5}	87^{+18}_{-10}	0^{++}	$\phi J/\psi$
$Y(4710)$[e]		4704 ± 87	183 ± 146	1^{--}	
$\Upsilon(10753)$		$10752.7^{+5.9}_{-6.0}$	36^{+18}_{-12}	1^{--}	$\pi\pi\Upsilon(1S,2S,3S)$
$\Upsilon(10860)$	$\Upsilon(5S)$	$10885.2^{+2.6}_{-1.6}$	37 ± 4	1^{--}	$\pi\pi\Upsilon(1S,2S,3S)$, $\pi^+\pi^- h_b(1P,2P)$, $\eta\Upsilon(1S,2S)$, $\pi^+\pi^-\Upsilon(1D)$ (see PDG listings)
$\Upsilon(11020)$	$\Upsilon(6S)$	11000 ± 4	24^{+8}_{-6}	1^{--}	$\pi\pi\Upsilon(1S,2S,3S)$, $\pi^+\pi^- h_b(1P,2P)$, (see PDG listings)

- ✓ Isoscalar neutral meson states above the open-flavor thresholds which are potential candidates for hybrids
- ✓ Table adapted from PDG 2022
- ✓ Y(4500): New state recently seen by BESIII experiment.

M. Ablikim et al, Chin.Phys.C,46,111002(2022).

- ✓ X(4630): New state recently seen by LHCb experiment.
- ✓ $\chi_{c1}(4685)$: New state recently seen by LHCb experiment.

R. Aaji et al, Phys. Rev. Lett. 127, 082001 (2021)

- ✓ Y(4710): New state recently seen by BESIII experiment.

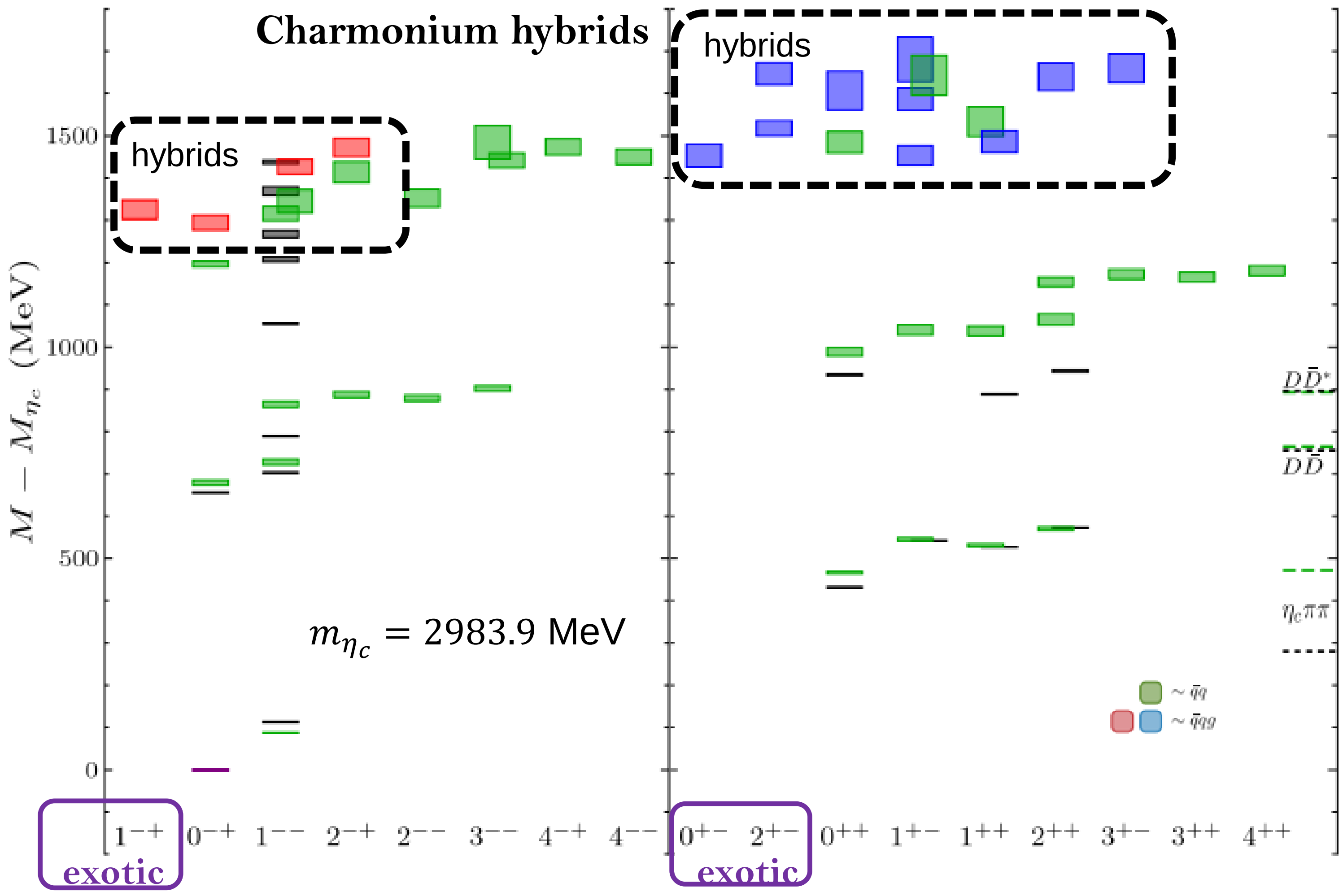
M. Ablikim et al, arXiv: 2211.08561.

Brambilla, Lai, AM, Vairo arXiv:2212.09187

BOEFT: Hybrids

- Lattice results for charm hybrids ($m_\pi \approx 240$ MeV) :

Results agree within error bars



Lattice data from
Hadron Spectrum collaboration JHEP 12 (2016) 89

Box represents uncertainties in lattice computations

- $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ etc. are exotic quantum #'s

	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

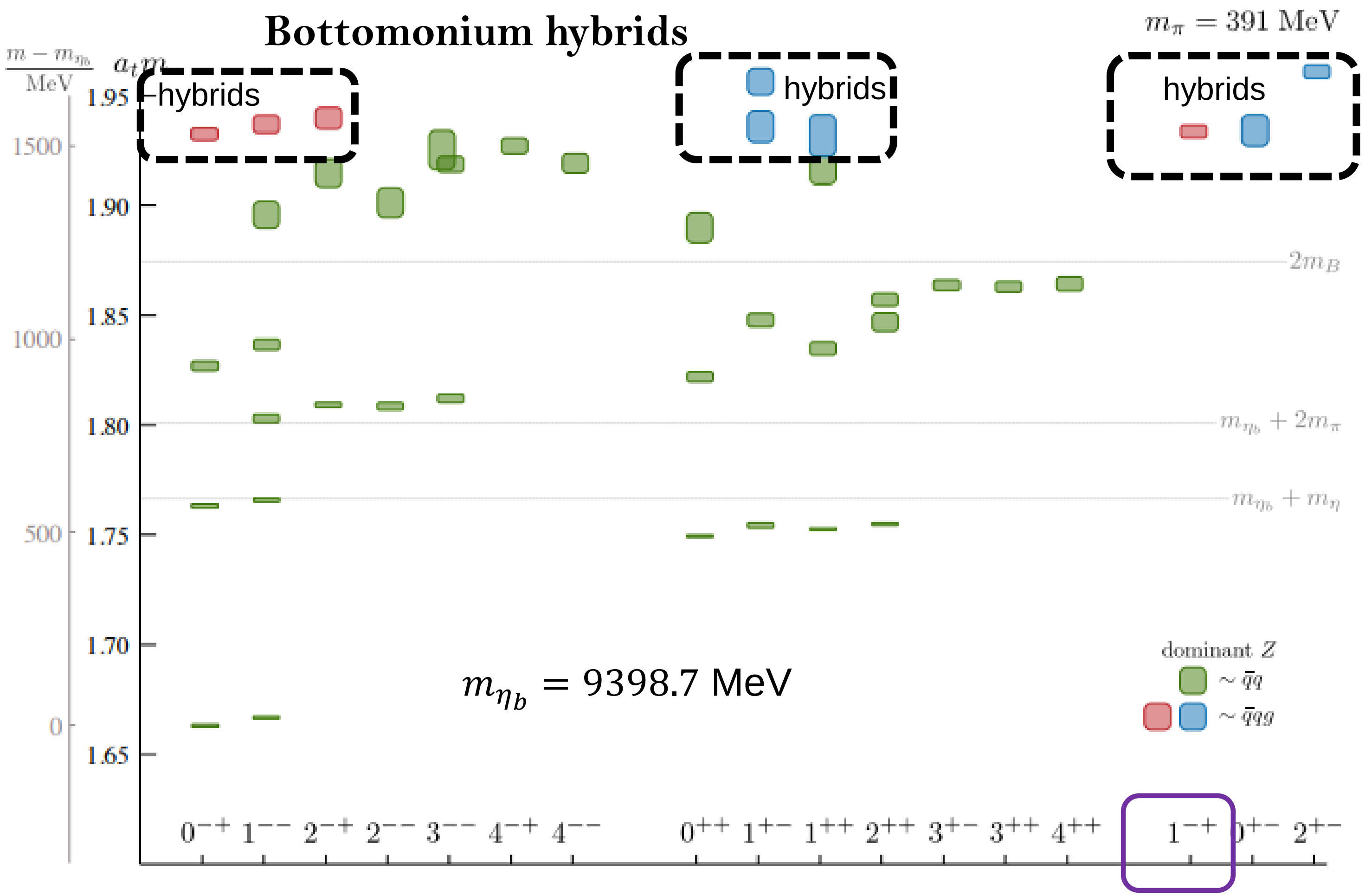
Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)

Brambilla, Lai, AM, Vairo arXiv:2212.09187

BOEFT: Hybrids

- Lattice results for bottom hybrids ($m_\pi \approx 391$ MeV):

Results agree within error bars



Lattice data from
Hadron Spectrum collaboration JHEP 02 (2021) 214

Box represents uncertainties in lattice computations

- $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ etc. are exotic quantum #'s

	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

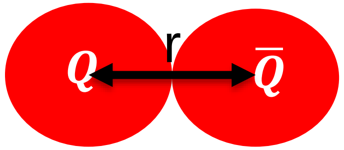
Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92, (2015)

Brambilla, Lai, AM, Vairo arXiv:2212.09187

Hybrid Decays

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Color configuration of $Q\bar{Q}$ pair ($\mathbf{r} \to \mathbf{0}$): quarkonium and hybrid in short-distance limit



Quarkonium -----> Singlet

Hybrid -----> Octet

- Quarkonium and Hybrid fields in short-distance limit $\mathbf{r} \to \mathbf{0}$ (matching condition)

Fields:

$$S\,(\mathbf{r}, \mathbf{R}, t) \to Z_\Psi^{1/2}(\mathbf{r})\,\Psi(\mathbf{r}, \mathbf{R}, t),$$

$$P^{i\dagger}_{\kappa\lambda} O^a\,(\mathbf{r}, \mathbf{R}, t)\, G^{ia}_\kappa(\mathbf{R}, t) \to Z_\kappa^{1/2}(\mathbf{r})\,\Psi_{\kappa\lambda}(\mathbf{r}, \mathbf{R}, t)$$

singlet (S) and octet (O)

G^{ia}_{κ}: **Gluon fields**

Potentials:

$$E_{\Sigma_g^+}(r) = V_s(r) + b_{\Sigma_g^+}\, r^2 + \ldots,$$

$$E_{\Sigma_u^-, \Pi_u}(r) = V_o(r) + \Lambda + b_{\Sigma,\Pi} r^2 + \ldots$$

V_s & V_o: singlet and octet potential

Λ: gluelump mass

For decay rate computation, **start with effective theory of singlet and octet fields** and match to **BOEFT of quarkonium and hybrid fields**

Hybrid Decays

- pNRQCD Lagrangian:

Brambilla, Lai, AM, Vairo arXiv:2212.09187

Weakly-coupled pNRQCD Lagrangian

$$L_{\text{pNRQCD}} = \int d^3R \Big\{ \int d^3r \Big(\text{Tr}\left[\text{S}^\dagger \left(i\partial_0 - h_s \right) \text{S} + \text{O}^\dagger \left(iD_0 - h_o \right) \text{O} \right]$$

$$+ g\text{Tr}\left[\text{S}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\text{O} + \text{O}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\text{S} + \frac{1}{2}\text{O}^\dagger \boldsymbol{r}\cdot\{\boldsymbol{E},\text{O}\} \right] + \frac{g}{4m}\text{Tr}\left[\text{O}^\dagger \boldsymbol{L}_{Q\bar{Q}}\cdot[\boldsymbol{B},\text{O}] \right]$$

$$+ \frac{gc_F}{m}\text{Tr}\left[\text{S}^\dagger (\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,\text{O} + \text{O}^\dagger (\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot\boldsymbol{B}\,\text{S} + \text{O}^\dagger \boldsymbol{S}_1\cdot\boldsymbol{B}\,\text{O} - \text{O}^\dagger \boldsymbol{S}_2 \text{O}\cdot\boldsymbol{B} \right] - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\, a} \Big\}$$

- **Spin preserving decays $[O(r^2)]$**
- **Spin flipping decays $[O(1/m^2)]$**

Perturbative computation can be performed to compute decay rate of hybrid decays to low-lying quarkonium !!!

Perturbative computation at order $\boldsymbol{O(r^2)}$ or $\boldsymbol{O(1/m^2)}$

Singly Heavy Hadrons

- Hamiltonian (predict masses with accuracy upto ~ 5 MeV):

$$H_\ell^Q = m_Q + \mathcal{E}_{\ell,Q} + \frac{\mathcal{S}_{\ell,Q}}{2m_Q} \boldsymbol{S} \cdot \boldsymbol{j}_\ell.$$

- Parameters determined by minimizing χ^2 fit to data (PDG 2020)

Q	ℓ	$\mathcal{E}_{u/d,Q}$ [MeV]	$m_{s,Q}$ [MeV]	$\mathcal{S}_Q$ [GeV^2]	dof	χ^2/dof
c	$\bar{q},\ \frac{1}{2}^-$	313.4 ± 2.0	102.3 ± 3.5	0.472 ± 0.012	3	0.28
b	$\bar{q},\ \frac{1}{2}^-$	306.4 ± 0.2	87.6 ± 0.5	0.455 ± 0.004	2	1.20
c	$[q\,q'],\ 0^+$	626.5 ± 1.1	182.9 ± 1.4		1	1.09
b	$[q\,q'],\ 0^+$	612.5 ± 3.2	174.7 ± 4.0		1	0.40
c	$\{q\,q'\},\ 1^+$	837.7 ± 0.7	124.0 ± 0.8	0.147 ± 0.003	9	0.85
b	$\{q\,q'\},\ 1^+$	820.8 ± 2.2	117.7 ± 2.3	0.136 ± 0.028	5	0.30

Braaten, He, AM, arXiv:2006.08650

Doubly Heavy Baryons

- Hamiltonian for doubly heavy hadrons (QQq, $QQ\bar{q}\bar{q}$) :

$$H_\ell^{Q_1Q_2} = (m_{Q_1} + m_{Q_2}) + \mathcal{E}_{\ell,Q_1Q_2} + \frac{\mathcal{S}_{\ell,Q_1Q_2}}{8\mu_{Q_1Q_2}} \boldsymbol{S} \cdot \boldsymbol{j}_\ell.$$

$$\mathcal{E}_{\ell,Q_1Q_2} = \mathcal{E}_{Q_1Q_2} + \mathcal{E}_\ell + \frac{\mathcal{K}_\ell}{2\,(m_{Q_1} + m_{Q_2})}$$

- Parameters determined by minimizing χ^2 fit to lattice data for DHB.

[Briceno, Lin, Bolton, Phys. Rev. D86, 094504 (2012), Alexandrou, Drach, Jansen, Kallidonis, Kostou, , Phys. Rev. D90, 074501 (2014), Brown, Detmold, Meinel, Orignos, , Phys. Rev. D90, 094507 (2014), Mathur, Padmanath, , Phys. Rev. D99, 031501 (2019)]

Q_1Q_2	ℓ	$\mathcal{E}_{u/d,Q_1Q_2}$ [MeV]	m_{s,Q_1Q_2} [MeV]	$\mathcal{S}_{Q_1Q_2}$ [GeV^2]	dof	χ^2/dof
cc	$q, \frac{1}{2}^+$	319.5 ± 11.0	124.9 ± 13.4	0.363 ± 0.024	12	0.29
$[bc]$	$q, \frac{1}{2}^+$	275.8 ± 37.2	55.0 ± 47.0		0	
$\{bc\}$	$q, \frac{1}{2}^+$	309.3 ± 27.3	73.5 ± 34.3	0.181 ± 0.046	2	8×10^{-5}
bb	$q, \frac{1}{2}^+$	152.0 ± 25.1	130.0 ± 33.6	0.472 ± 0.075	2	2×10^{-5}

Braaten, He, AM, arXiv:2006.08650

Hybrid-quarkonium mixing (in progress)

Brambilla, AM, Vairo,.... (in progress)

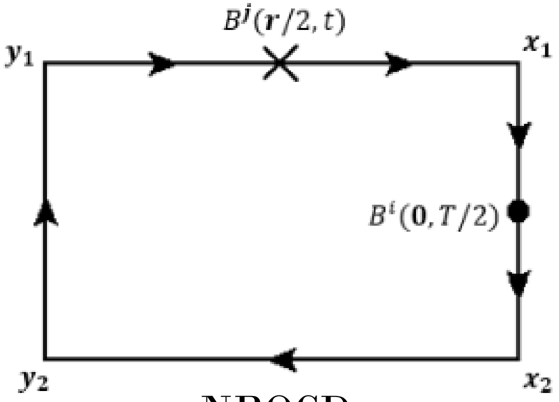
- Hybrid states in the same energy range and same quantum #'s as quarkonium can mix.
- Mixing impact spectrum and decay properties of hybrid. Implications on hybrid interpretation for exotics.

Oncala & Soto, Phys. Rev. D. 96, (2017)

Ex. $H_1\left[1^{--}\right](4155) \leftrightarrow c\bar{c}\left[1^{--}\right](3S)$

Effect on decay: $H_m \leftrightarrow Q_m^{'} \rightarrow (\eta_c, J/\psi, \cdots) + (\gamma, \cdots)$

- Hybrids with gluon quantum # $\boldsymbol{\kappa = 1^{+-}}$, mix with quarkonium through heavy-quark spin dependent operator. Mixing potential at $O(1/m)$ in BOEFT.
- Mixing potential $V_{\kappa\lambda}^{\rm mix}$: determined from matching NRQCD and BOEFT at $O(1/m)$



NRQCD

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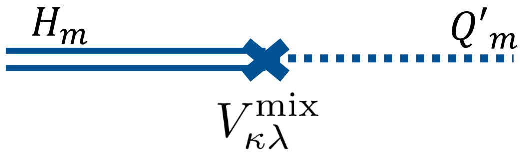

BOEFT

Expression after matching:

$$V_{|\lambda|}^{\rm mix} = -\frac{gc_F}{2m_Q}\, {}^{(0)}_{\lambda}\langle 1|B^j\left(\mathbf{r}/2, 0\right)|0\rangle^{(0)} P_\lambda^j,$$

Above expression can be computed on lattice if we identify:

$$|0\rangle^{(0)} = |\Sigma_g^+\rangle$$

$$|1\rangle_{\lambda=0}^{(0)} = |\Sigma_u^-\rangle, |1\rangle_{|\lambda|=1}^{(0)} = |\Pi_u\rangle$$

What is an XYZ Meson?

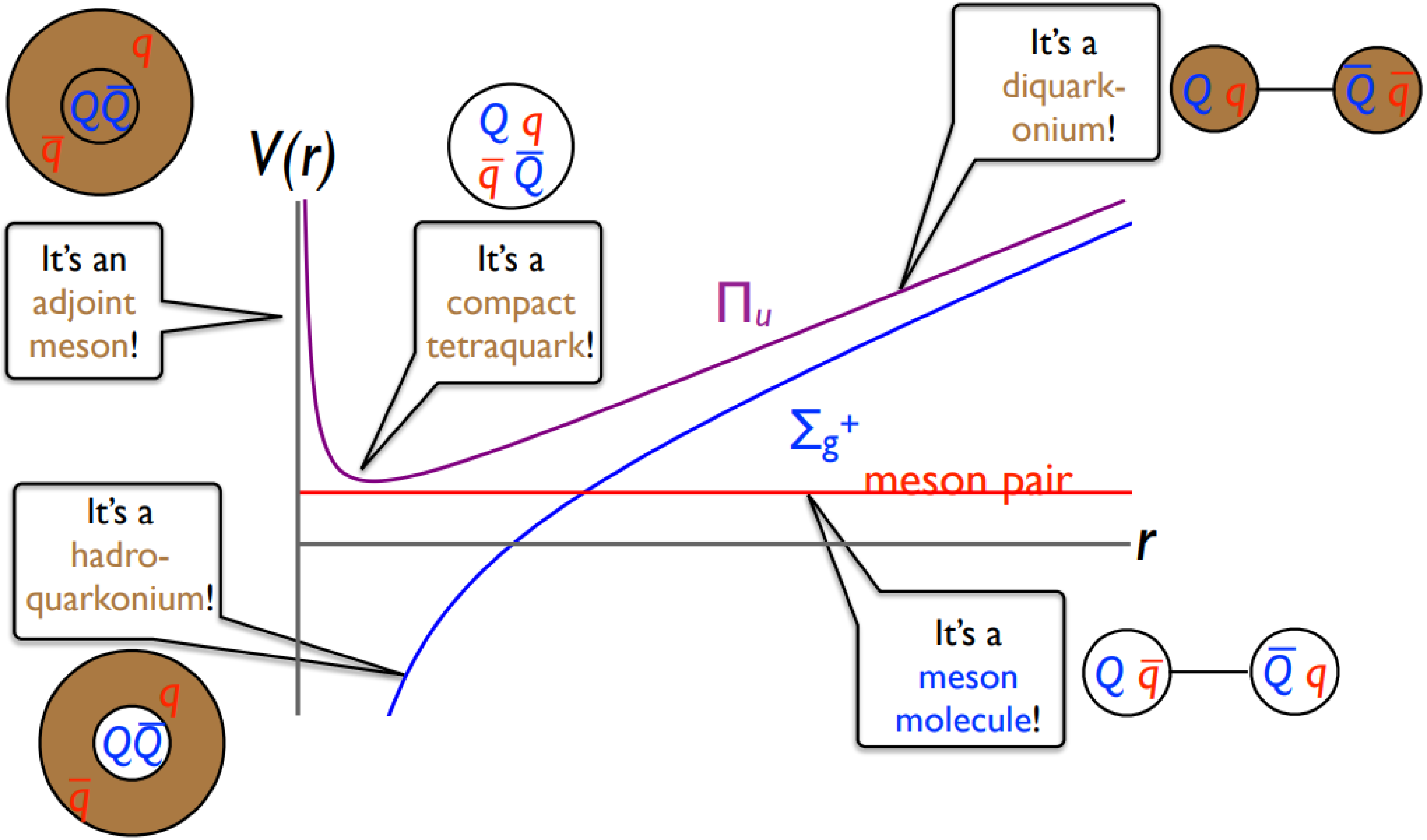

Figure from Eric Braaten talk:
Charm 2020 conference

Each model describes some region of the Born-Oppenheimer wavefunction